If you are typing your answers into the computer the use of mathematical typing software is not required. Answers such as $4x^3$ can be written as 4x^3 but not 4x3 and $\frac{3x+2}{x+1}$ written as (3x+2)/(x+1) but not 3x+2/x+1 and $\frac{1}{2}x$ must not be written as 1/2x.

DO NOT use x for multiplication. You **do not** need to type out all stages of your answer just the key parts. Intermediate working can be completed on paper which will **not be marked**. You should **not use a calculator** or any other calculation device.

Paper D Total = 100 marks

1.

In this question A is an integer and answers can be left in **terms of A** when needed.

$$f(x) = x^2 - 2Ax + A^2$$

$$g(x) = x^2 - A^2$$

$$x \in \mathbb{R}$$

a)

i) Write $f(x)$ as the product of 2 linear factors.

ii) Write $g(x)$ as the product of 2 linear factors.

2 marks

b) $\frac{1}{f(x)} + \frac{1}{g(x)}$ can be written in the form $\frac{Bx}{h(x)}$ where B is an integer;

find B and $h(x)$.

3 marks

c) Solve the equation $[f(x)]^2 = [g(x)]^2$

3 marks

d) Write down

i) the range of $f(x)$.

ii) the range of $g(x)$.

2 marks

e) Describe the single transformation that would map $g(x)$ onto $f(x)$.

2 marks

12 marks

2. For the function $s(x) = \frac{kx+1}{x-3}$ $x \neq 3, k \in \mathbb{R}$

a) Find the values of k for which each of the following equations has no solution

i) $s(x) = 1$ ii) $s(x) = -x$ iii) $s(x) = k$

7 marks

b) Find k such that $s(\sqrt{3})$ is a rational number.

2 marks

c) i) Find $t(x)$ such that the composite function $s(t(x)) = x$

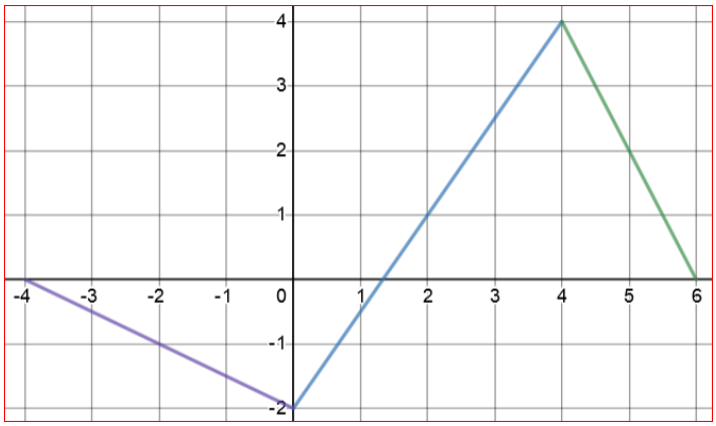
ii) Write down the domain of $t(x)$.

5 marks

14 marks

3.

The continuous function $h(x)$ is defined on the domain $-4 \leq x \leq 6$ as shown in the diagram below.



a)

i) Write down $h(4)$

ii) Find $h(1)$

2 marks

b) Solve the equation $h(x) = 0$

2 marks

c) Find $h'(-2)$

1 mark

d) Find $\int_{-4}^{6} h(x)dx$

2 marks

e) A new function $p(x)$ is defined as $p(x) = \frac{h(x)}{h(x)+x}$

Find $p'(2)$

3 marks

f) Evaluate the following integrals giving your answers **exactly**

i) $\int_{-1}^{1} \frac{h'(x)}{h(x)}dx$ ii) $\int_{-2}^{4}[(h(x)]^3 h'(x)dx$ iii) $\int_{2}^{6} xh'(x)dx$

12 marks

22 marks

4. A curve is defined by the equation

$$3x^2 + 4\,y^2 + 6xy = 9$$

a) Find $\frac{dy}{dx}$ in terms of x and y

3 marks

b) Find the equations of any tangents to the curve where $x = 0$

4 marks

c) Find the equations of any tangents to the curve that are vertical giving your answers **exactly**.

4 marks

11 marks

5.

a) For the following geometric series what are the conditions on x for there to be a convergent sum to infinity?

$$1 + cos^2x + cos^4x + cos^6x + \cdots$$

2 marks

b) Assuming these conditions have been met write down the sum to infinity in terms of sinx.

4 marks

c) Evaluate the sum to infinity when $x = \frac{\pi}{6}$

1 mark

d) Write down a similar geometric series with sum to infinity of $\frac{1}{cos^2x}$

2 marks

e) i) Hence find $3 + cos^4x + sin^4x + cos^6x + sin^6x + cos^8x + sin^8x + \cdots$ in terms of $sin2x$

ii) Comment on the values of x for which your expression would be valid.

5 marks

f) i) Write down the first 3 terms of $(1 - sin^2x)^n$ in ascending powers of $sinx$.

ii) Hence write down the full expansion of $(1 - sin^2x)^3$

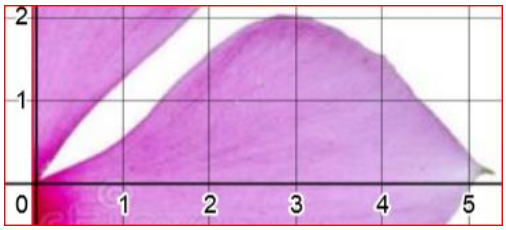
iii) Assuming your exapansion in i) is valid when $n < 0$ explain how this confirms your answer to d)

7 marks

21 marks

6.

A Biologist needs to find the surface area of a flower with 5 petals, She decides to model the shape of the top half of a petal as a cubic curve using a graphing programme.



a) Give two reasons why she might have chosen a cubic curve rather than a parabola.

2 marks

b) She considers a general cubic $y = ax^3 + bx^2 + cx + d$ passing through the points $(0,0), (3,2)\ and\ (5,0)$

i) Explain why making sure the outline went through the origin was particularly useful.

ii) Using the other two points write down two equations in a,b and c that she needs to solve.

iii) Eliminate c from these two equations to obtain an equation in terms of a and b only.

iv) Explain why she cannot derive a solution from just these equations.

5 marks

c) Looking at the shape more carefully the Biologist decides it might have a maximum at (3,2) and derives another equation.

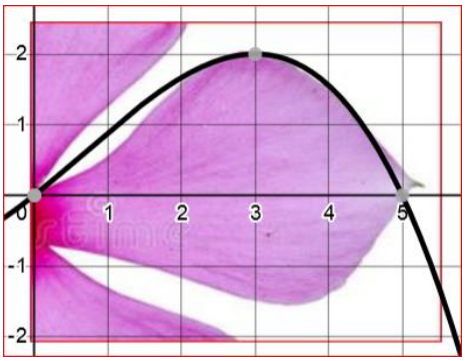
i) Find this new equation

ii) Obtain another equation in terms of a and b only.

iii) Solve your equations to find a,b and c.

8 marks

d) The Biologist's model gives the curve shown.



i) She does the following integration

$10\int_0^5(ax^3+bx^2+cx+d)dx$ using her values of a, b, c, d. Explain whether you think this might be and under or an over estimate of the area that she needs giving reasons for your answer.

ii) Give a reason why a quartic model might have been more appropriate.

iii) The Biologist discovers that she has not input the picture correctly into the graphing package and that the length of the leaf is 3 cm and not 5 cm. How should she adjust her answer?

5 marks

20 marks

Total 100