You can work on paper and scan your solutions or type out your answers onto the computer. Use of mathematical typing software is not required. You should **not use a calculator** or any other calculation device.

Paper D Total = 100 marks

1.

In this question $\boldsymbol{a}$ **and** $\boldsymbol{b}$ are non-zero positive integers and answers should be left in **terms of** $\boldsymbol{a}$ **and/or** $\boldsymbol{b}$ when needed.

$$f(x) = a^2x^2 - 25b^2$$
$$g(x) = 2ax^2 + 16b^2$$

a)

i) Write $f(x)$ as the product of 2 linear factors.

ii) Write $f(x) + g(x) + x^2$ as the product of 2 linear factors.

5 marks

b) Given that $g(x) - f(x)$ is constant (independent of x)

i) Determine the value of a

ii) For this value of a write $\frac{g(x)+10abx+9b^2}{f(x)}$ in the form $\frac{cx+d}{ex+f}$ where c, d, e and f are integers and may be written in terms of b

6 marks

Total 11 marks

2.

Each of the following equations is written in terms of a where a is a real number.

i) Solve each equation exactly giving your answers in terms of a.

ii) State the set of values of a that ensure that x is a real number.

a) $3e^{2ax} + 2e^{ax} - 1 = 0$

5 marks

b) $\log_{10}(a^2x) - \log_{10}(x+1) = 2$

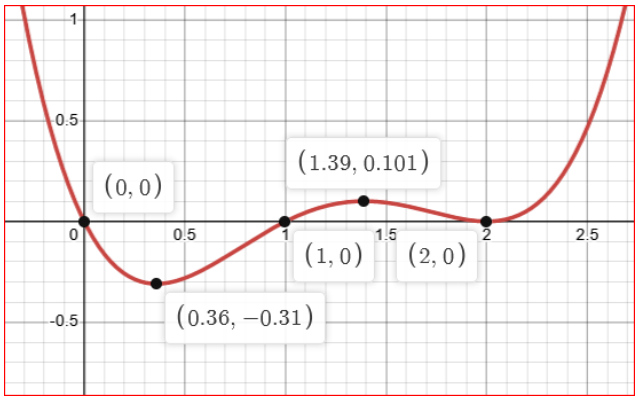
4 marks

c) $ax^2 - 4ax + 1 = 0$

5 marks

Total 14 marks

3.

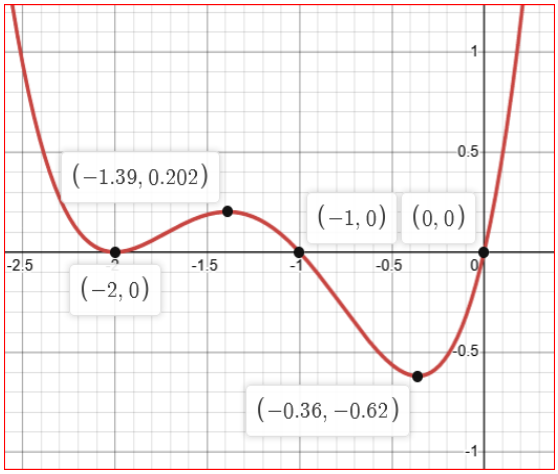


The diagram above shows the graph of $y = f(x)$.

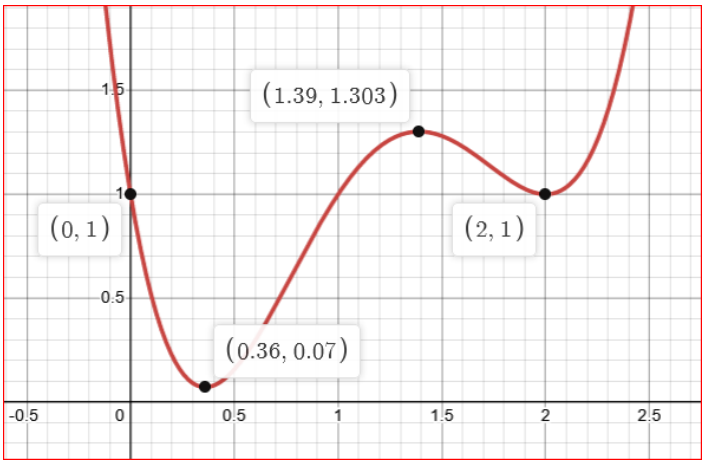
The graph meets the x-axis at three points with coordinates $(0,0)$, $(1,0)$ and $(2,0)$. The point with coordinates $(2,0)$ is a turning point. The graph has two other turning points with approximate coordinates $(1.39\ , 0.101)$ and $(0.36\ , -0.31)$.

a) The following graphs each represent transformations of the graph of $y = f(x)$. For each graph write down the transformations of $f(x)$ making sure that any combined transformations are written in the correct order and write the equation for each graph in the form $y = Af(Bx + C) + D$ where A, B, C and $D \in \mathbb{R}$ ie A, B, C and D are real numbers.

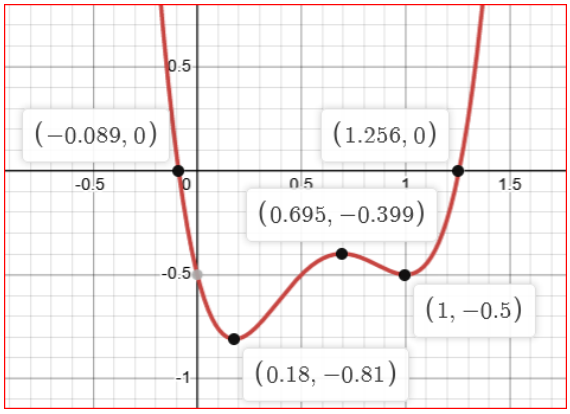
i)



ii)



iii)



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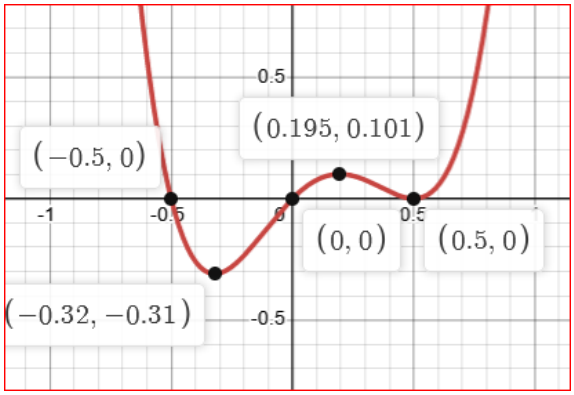
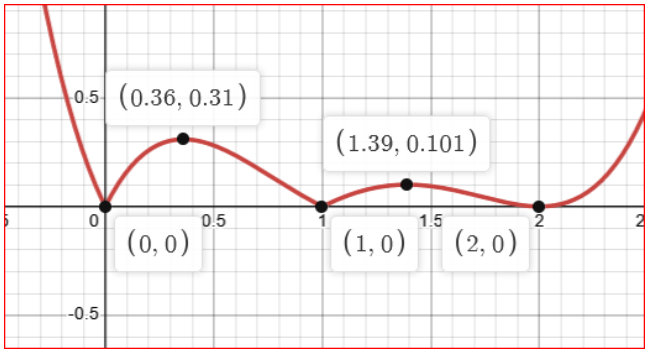


12 marks

b) Write the equation of the following graph in terms of $f(x)$



1 mark

It is now given that $f(x) = kx(x-a)(x-b)^2$ where a , $b \in \mathbb{R}$

c) Write down the value of i) a and ii) b

2 marks

d) Use the values of a and b from part c) to find $f'(x)$ giving your answer in the form $k(x-b)g(x)$ where $g(x)$ is a quadratic expression.

4 marks

e) **Hence** write down an approximate value for $\frac{7+\sqrt{17}}{8}$

2 marks

Total 21 marks

4.

The table shows values of $f(x)$ and $g(x)$where $x = 1, x = 2$ and $x = 4$

	$x = 1$	$x = 2$	$x = 4$
$f(x)$	1	4	$\frac{3}{2}$
$g(x)$	5	1	15

a) Find the values of

i) $f(2x)$ when $x = 1$

ii) $(f(x))^2$when $x = 2$

iii) $f\big(f(x)\big)$ when $x = 2$

iv) $g\big(f(x)\big)$ when $x = 2$

4 marks

The table below shows the values of the derivatives of $f(x)$ and $g(x)$where $x = 1, x = 2$ and $x = 4$

	$x = 1$	$x = 2$	$x = 4$
$f'(x)$	3	$\frac{3}{2}$	- 1
$g'(x)$	$\frac{3}{5}$	4	5

b) Find $h'(2)$ when

i) $h(x) = \frac{g(x)}{f(x)}$

ii) $h(x) = f(g(x))$

6 marks

c) Given that $f(x) > 0, g(x) > 0$ for all values of x and that $f(x)$and $g(x)$ are continuous and differentiable find the values of the following integrals (giving your answers exactly).

i) $\int_1^4 g'(x)dx$

ii) $\int_1^2 f'(x)g(x) + g'(x)f(x)\,dx$

iii) $\int_2^4 x\, g''(x)dx$

8 marks

Total 18 marks

5.

A curve C is defined using a common parameter θ such that

$x = ksin\theta$, $y = 1 + 2kcos2\theta$ for $0 \leq \theta \leq \pi$ where k is a real number such that $0 < k < \frac{1}{2}$. Answers to this question may be left in terms of k where needed.

a) i) Find $\frac{dx}{d\theta}$ in terms of θ. ii) Find $\frac{dy}{d\theta}$ in terms of θ.

iii) Hence find $\frac{dy}{dx}$ in terms of θ.

3 marks

b) i) Find the smallest value of θ such that the tangent to the curve C is parallel to $y + 4x = 0$.

ii) Hence find the equation of a tangent to the curve C that is parallel to $y + 4x = 0$.

7 marks

c) i) Find the equation of the curve C in the form $y = A + Bx^2$ where A and B are real numbers.

ii) Write down the domain of the function $y = f(x)$.

iii) Write down the range of the function $y = f(x)$.

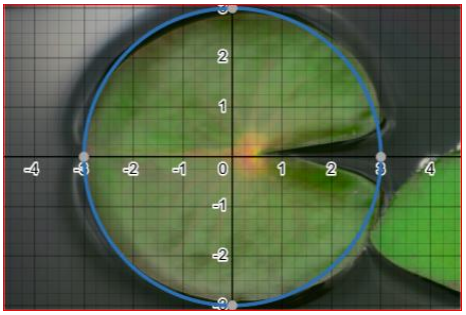
5 marks

d) Find the area enclosed by the curve C and the x – axis.

3 marks

Total 18 marks

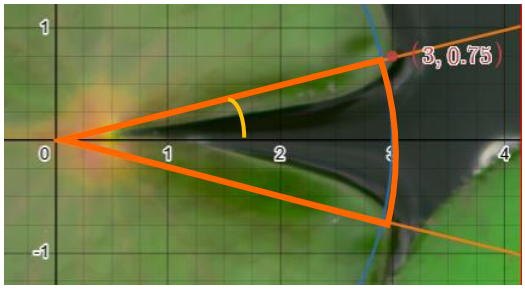
6.



A biologist is modelling a lily pad in order to calculate its area.

They start by modelling it as a circle centre the origin and with radius 3.

a) Write down the cartesian equation of the circle. *1 mark*



b) Initially the biologist models the area of the circle to remove as a sector of the circle bounded by two lines both through the origin one passing through the point $(3, \frac{3}{4})$ and the other through $(3, -\frac{3}{4})$

i) Write down the value of $tanA$ where A is the angle between the line through $(3, \frac{3}{4})$ and the x – axis. A is measured in radians.

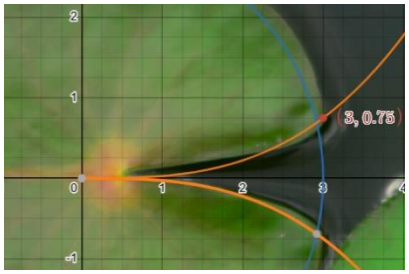
ii) Hence find the exact value of $tan2A$

iii) The biologist uses angle A and calculates the area $9\pi - 9A$

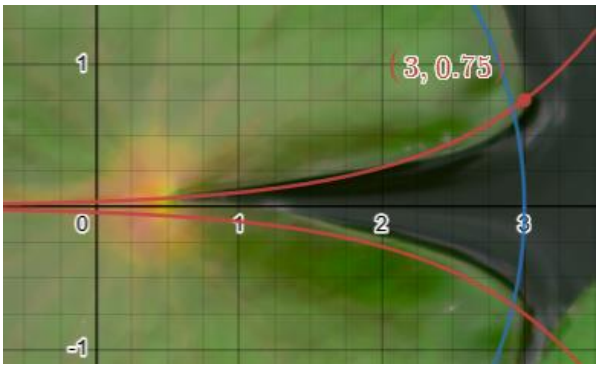
Will using $9\pi - 9A$ provide an overestimate or an underestimate of the area of the lily pad?

4 marks

c) The biologist tries to fit a curve in order to get a better approximation for the area and uses $y = Bx^3$ where $B > 0$. Find the value of B so that the equation $y = Bx^3$ passes through the point $(3, \frac{3}{4})$.



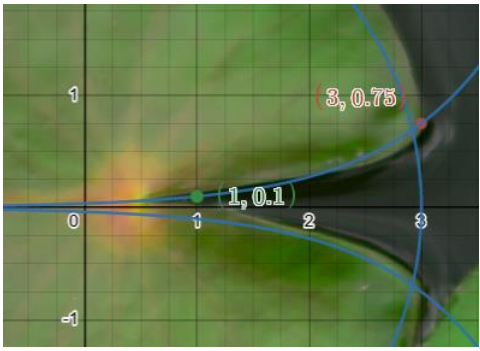
2 marks



d) Still not happy the Biologist tries $y = Ce^x$ where $C > 0$. Find the value of C so that the equation $y = Ce^x$ passes through the point $(3, \frac{3}{4})$. Give your answer as an exact value.

2 marks

e) The last model that the Biologist tries is $y = De^{Ex}$ where $D, E > 0$. Find the value of D and E so that the equation $y = De^{Ex}$ passes through the point $(3, \frac{3}{4})$ and the point $(1, \frac{1}{10})$. Give your answers as exact values.



6 marks

f) The diameter of one of the lily pads is 1.8m. After calculating the area using the model the biologist multiplies it by a scaling factor. Determine the value of the scaling factor.

3 marks

Total 19 marks