

Paper D 2017 Markscheme

1.

factorise $2x^2 - 11x + 15$ or $4x^2 - 25$

$(2x - 5)(x - 3)$ $(2x - 5)(2x + 5)$

combine fractions with same numerator

$$\frac{3(2x + 5) - 2(x - 3)}{(2x - 5)(x - 3)(2x + 5)}$$

$$\frac{4x + 21}{(2x - 5)(x - 3)(2x + 5)}$$

M1

A1 A1

M1

A1 for simplified answer

5 marks

2.

a)

$$2 - 4x = x^2 - 1$$

$$x^2 + 4x - 3 = 0$$

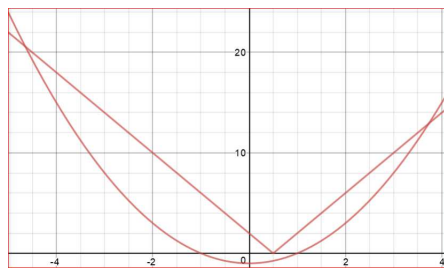
$$(x + 2)^2 - 4 - 3 = 0$$

$$x = -2 \pm \sqrt{7}, y = 10 - / + 4\sqrt{7}$$

b)

$$-2 - \sqrt{7} < x < -2 + \sqrt{7}$$

c)



$$-2 + 4x = x^2 - 1$$

$$x^2 - 4x - 1 = 0$$

$$(x - 2)^2 - 4 - 1 = 0$$

$$x = 2 \pm \sqrt{5}$$

$$-2 - \sqrt{7} < x < 2 + \sqrt{7}$$

a)

M1 can be earned in b)

M1

A1 exact answers

b) A1 A1 FT a) allow decimals

SC1 for outside region or \leq

c)

M1

A1

A1 allow decimals

8 marks

3.

a)

finding a factor eg $x = 1$ is a solution so $(x - 1)$ a factor

factorising 1 linear and 1 quadratic eg $(x - 1)(x^2 - x - 2)$

$$(x - 1)(x - 2)(x + 1)$$

$$x = 1, -1, 2$$

b)

using a log law correctly $\ln x + 2 = 2 \ln x$

$$\ln x = 2 \quad x = e^2$$

c)

$$2e^{2x} + 5e^x - 3 = 0$$

$$(2e^x - 1)(e^x + 3) = 0$$

$$2e^x - 1 = 0$$

$$x = \ln(1/2) \text{ oe}$$

d)

$$\sqrt{2} \sin x = \sin x / \cos x$$

$$\sin x (\sqrt{2} \cos x - 1) = 0$$

$$\sin x = 0 \quad \cos x = 1/\sqrt{2}$$

$$x = 0, \pi, 2\pi \quad x = \pi/4, 7\pi/4$$

a)

M1

M1 A1 or full solution with factor theorem

A1

b)

M1

M1 A1

c)

recognising quadratic M1

factorising oe M1

A1

A1 (A0 if 2 solutions)

d)

$\tan x = \sin x / \cos x$ M1

M1

A1 degrees lose A1

A1 A1

16 marks

4.

a)

$f(2) = 0$ so $(x - 2)$ is a factor

$$f(x) = (x - 2)(x^2 + x + 1)$$

b)

$$\text{discriminant} = 1 - 4 = -3 < 0$$

so quadratic has no further roots or

c)

$$A(x - 2)(x^2 + x + 1) + (Bx + C)(x - 2) + D(x^2 + x + 1) = x^3 + 3x^2 - 2x - 2$$

$$x^3 \text{ term } A = 1$$

$$x^2 \text{ term } -A + B + D = 3 \quad B + D = 4$$

$$x \text{ term } -A + C - 2B + D = -2 \quad C - 2B + D = -1$$

$$c \text{ term } -2A - 2C + D = -2 \quad -2C + D = 0$$

$$D = 2C \quad B + 2C = 4 \quad 2B + 4C = 8$$

$$3C - 2B = -1 \quad 7C = 7$$

$$C = 1 \quad D = 2 \quad B = 2$$

$$1 + \frac{2x+1}{h(x)} + \frac{2}{g(x)}$$

$$d) \quad Ax + \ln(x^2 + x + 1) + D \ln(x - 2) + c$$

a)

M1

A1 A1

b)

M1

R1

c)

M1 A1 for each
valid attempt at A
B C D

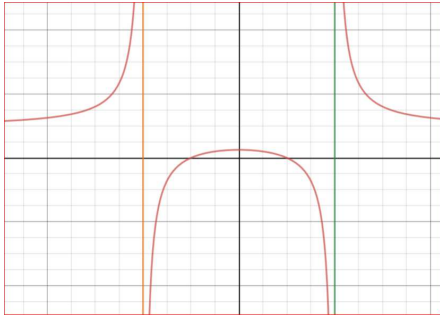
A1

d)

A1 ft A1 cao A1 FT

17 marks

5.

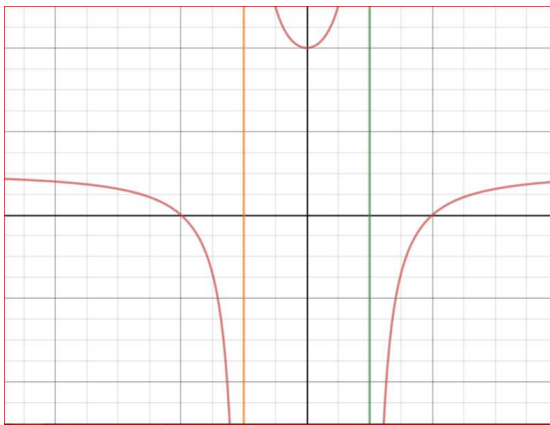


a) asymptotes $x = \pm b$ $y = 1$

intersections $(0, a^2/b^2)$ $(\pm a, 0)$

b) $a^2/b^2 < y \leq 1$

c)



$1 \leq y < a^2/b^2$

a) shape A1

A1 A1

A1 A1

b) A2 (A1 if part correct)

c) A1 A1

9 marks

6.

a)
 $1.6 \times 0.8 = 1.28$

b)
 $2 + 3.2 + 2.56 = 7.76$

c) i) 2×0.8^n or $1.6 \times 0.8^{n-1}$
ii) $2 + 3.2 + 3.2 \times 0.8 + 3.2 \times 0.8^2 + \dots + 3.2 \times 0.8^{n-1}$
 $= 2 + 3.2 (1 - 0.8^n) / 0.2$ oe

d) $2 \times 0.8^n \leq 0.02$
 $0.8^n < 0.01$
 $n \ln 0.8 < \ln 0.01$
 $n > \ln 0.01 / \ln 0.8 = 20.63$
so 21 bounces

e) In limit as n tends to ∞ the distance travelled tends to
 $2 + 16 = 18$ from below

a)
A1

b)
M1 for 5.2A1

c) i) A1
ii) M1 GP with
 $r = 0.8$ A1 (any
correct form)

d) M1
M1 correct use
of logs or T
and I
A1FT if power
of n involved
A1

e) R1

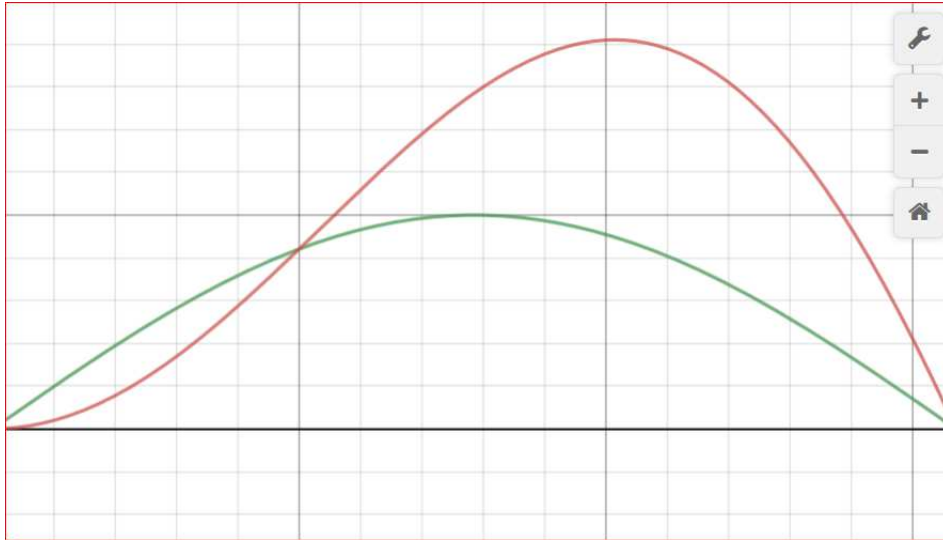
11 marks

7.

a) i) $x \cos x + \sin x$ ii) $(\sin x - x \cos x) / \sin^2 x$

b) i) $-x \cos x + \int \cos x dx = -x \cos x + \sin x + (c)$

ii) $e^{\sin x} + (c)$



c) $x \sin x = \sin x \quad x = 1 \text{ or } \sin x = 0 \quad (0,0) \quad (1, \sin 1) \quad (\pi, 0) \text{ or on sketch}$

d) $m = 1 \quad \text{so } (y - \pi/2) = (x - \pi/2) \text{ ie } y = x$

e) $\int_0^{\pi/2} x dx - \int_0^1 \sin x dx - \int_1^{\pi/2} x \sin x dx$
 $= \frac{\pi^2}{8} + [\cos x]_0^1 - [-x \sin x + \cos x]_1^{\pi/2} = \frac{\pi^2}{8} + \sin 1 - 2$

f) $x \cos x + \sin x = 0$

$x = -\tan x$

any valid reason eg $x > \pi/2 \quad \tan x < 0$

a) i) M1 A1 ii) M1 A1

b) i) M1A1A1

iii) M1 A1

c) sketch
 A1A1 showing
 relationship

intercepts A1 for
 one A1 for 3

d) M1 A1

e) M1 at least 2 parts
 A1

A1 integration A1
 cao

f) M1

R1

21 marks

8.

$$\begin{aligned} a) \quad V &= \pi \int_{-5}^5 x^2 dy = \frac{\pi}{625} \int_{-5}^5 y^4 + 150y + 5625 dy \\ &= \frac{\pi}{625} \left[\frac{y^5}{5} + 75y^2 + 5625y \right]_{-5}^5 \\ &= 112\pi \end{aligned}$$

$$\begin{aligned} b) \quad \text{Volume of cylinder height 10cm radius 4} &= 160\pi \\ (160\pi - 112\pi) / 160\pi \times 100 &= 30\% \end{aligned}$$

a) M1 A1

M1 d A1

M1 dA1

b)

A1 dimensions

M1 A1FT

9 marks

9.

$$dh/dt = 2 \text{ or } h = 2t$$

$$\tan \theta = h/7 \text{ or } 2t/7$$

$$\sec^2 \theta \, d\theta/dt = dh/dt \div 7 \text{ or chain rule}$$

$$d\theta/dt = 2 \cos^2 \theta / 7 = 1/7 \text{ rad/s}$$

a) A1

A1

M1

A1

4 marks