High frequency trading, news and market fragmenation

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- HFTs try to minimise so-called “latencies:" the time it takes for them to receive “news” (e.g., a quote update) from trading platforms, process this information, and react to it by sending new orders (market orders, limit orders, cancellations) based on this information.

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- As market conditions change, HFT firms can adjust their holdings very frequently. Kirilenko et al. (2010) find that HFTs reduce half of their holdings in about two minutes on average.

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- Authorities have been concerned with new algorithms referred to as “news aggregation" that search the internet, news sites and social media for selected keywords, and fire off orders in milliseconds before the information is widely disseminated. (*FBI joins SEC in computer trading probe*, Financial Times, March 5, 2013)

HFTs and news

- Brogaard, Hendershott, and Riordan (2014) find that HFTs' aggressive orders are correlated with public news.
- They also present evidence that HFTs' orders predict price changes over very short horizons and account for 25-42% of trading volume.
- On the other hand, directional HFTs realize most of their profits on aggressive orders over relatively long horizons (e.g. over the day, Carrion (2013) and Brogard et al. (2014)), suggesting they are concerned with the long run value of the asset.

“News trading and speed” by Foucault et al. (J. of F. (2016))

- Trading takes place continuously over [0,1], and the liquidation value of the asset follows

$$v_t = v_0 + \int_0^t \sigma_v dB_t.$$

- HFT speculator is risk neutral and observes v, whereas the competitive risk neutral dealer observes z, which is v with some independent Gaussian noise.
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- In this version of the Kyle model they consider "two" filtrations for the dealer:
  1. *Fast speculator:* $\sigma(z_s, y_s; s < t) \vee \sigma(dy_t)$.
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- They find two different equilibria corresponding to these 'filtrations.'

Our model

- The interest rate is set to 0 and a single risky asset is traded. The fundamental value of this asset at time equals $V = \sigma_V \eta + \mu$, where η is a standard Normal.
- All market participants observe a public Gaussian signal X^M that converges to η at time 1.
- The HFT receives a private signal X^I, which is also Gaussian and converging to η.
- The noise traders' cumulative demand follows a Brownian motion B indepedent from everything above.
- In addition to the public signal X^M, the market makers also observe the net order flow

$$Y = B + \theta,$$

where θ is the HFT's strategy.

Equilibrium

- The market makers compete in a Bertrand fashion and set the price $S_t = E[V|\mathcal{F}_t^M]$ at all times, where $\mathcal{F}^M$ is their filtration.
- We shall focus on Markovian equilibrium in which price is given by $H(t, X_t, X_t^M)$, where X follows

$$dX_t = w(t, X_t)dY_t + r(t, X_t, X_t^M)dt \tag{1}$$

  for deterministic functions w and r chosen by the market makers in equilibrium. Moreover, the pricing rule H is strictly increasing in the 'processed signal' X of market makers.
- Given the pricing rule the trader choses θ to maximize her expect profit W_1^θ at time 1:

$$W_1^\theta = (V - S_{1-})\theta_{1-} + \int_0^{1-} \theta_{s-} dS_s. \tag{2}$$

Comparison with Kyle (1985)

- In the classical Kyle model, there is no public signal and the insider observes η at the beginning.
- In equilibrium $X = Y$

$$dY_t = dB_t + \frac{\eta - Y_t}{1 - t} dt.$$

- Y is a Brownian motion in its own filtration but converges to η.
- $S_t = \sigma_V Y_t + \mu$.
- Thus, the insider trades not to be “detected locally” but to drive prices so that $S_1 = V$ at the end of the trading horizon.

Signals X^M and X^I

- We model the public signal as a bridge process:

$$X_t^M = \int_0^t \sigma_M(s) dB_s^M + \int_0^t \sigma_M^2(s) \frac{\eta - X_s^M}{1 - \Sigma_M(s)} ds \qquad (3)$$

  such that $\Sigma_M(t) = \int_0^t \sigma_M^2(s) ds$ and $\Sigma_M(1) = 1$.
- Similarly,

$$X_t^I = X_0^I + \int_0^t \sigma_I(s) dB_s^I + \int_0^t \sigma_I^2(s) \frac{\eta - X_s^I}{1 - \Sigma_I(s)} ds \qquad (4)$$

  where $X_0^I \sim N(0, c^2)$, $\Sigma_I(t) = c^2 + \int_0^t \sigma_I^2(s) ds$ with $\Sigma_I(1) = 1$.
- X^M and X^I are martingales in their own filtrations.

Signal quality

- In fact
$$X_t^M = \beta^1_{\Sigma_M(t)} \quad X_t^I = \beta^2_{\Sigma_I(t)}$$
for some Brownian motions β^i. Thus, Σ_M (resp. Σ_I) is the speed of convergence to η of the public (resp. private) signal.
- Moreover, given $\mathcal{F}_t^{X^M}$ (resp. $\mathcal{F}_t^{X^I}$), $\eta \sim N(X_t^M, 1 - \Sigma_M(t))$ (resp. $\eta \sim N(X_t^I, 1 - \Sigma_I(t))$.
- Note, however, that (3) and (4) are not the Doob-Meyer decomposition of the signals in respective filtrations of the market maker or the speculator.

Speculator's valuation

- The $\mathcal{F}^I$-decomposition of X^I and X^M are given by

$$X^I_T = X^I_0 + \int_0^t \sigma_I(s)d\beta^I_s + \int_0^t \sigma_I^2(s)\frac{Z_s - X^I_s}{1-\Sigma_I(s)}ds \qquad (5)$$

$$X^M_T = \int_0^t \sigma_M(s)d\beta^M_s + \int_0^t \sigma_M^2(s)\frac{Z_s - X^M_s}{1-\Sigma_M(s)}ds, \qquad (6)$$

  where β^I and β^M are independent $\mathcal{F}^I$-Brownian motions and $Z_t := \mathbb{E}[\eta|\mathcal{F}^I_t]$.
- Moreover, $Z_t = \lambda_0(t)X^I_t + \lambda_1(t)X^M_t$, where

$$\lambda_0(t) = \frac{1-\Sigma_Z(t)}{1-\Sigma_I(t)},\ \lambda_1(t) = \frac{1-\Sigma_Z(t)}{1-\Sigma_M(t)},\ \text{with}$$
$$\Sigma_Z = \frac{\Sigma_I(t)+\Sigma_M(t)-2\Sigma_I(t)\Sigma_M(t)}{1-\Sigma_I(t)\Sigma_M(t)}, \quad t\in[0,1). \qquad (7)$$

- $\eta \sim N(Z_t, 1-\Sigma_Z(t))$. Moreover, $\Sigma_Z \geq \max\{\Sigma_I, \Sigma_M\}$.

Market makers' response

- Since we expect the system to be Gaussian we conjecture w in (1) depends only on t and

$$r(t, x, u) = r_1(t)x + r_2(t)u.$$

- Price should also be an affine function of X and X^M. Thus,

$$H(t, x, u) = \mu + \sigma_V(\beta_1(t)x + \beta_2(t)u).$$

, and we set $\beta_1 \equiv 1$ to avoid over parametrisation.

Speculator's optimal strategy

- Suppose that the market makers use $H(t, x, u) = \mu + \sigma_V(x + \beta_2(t)u)$. As in the earlier Kyle models the speculator's strategy is optimal if the insider's strategy is continuous and of finite variation, and prices converge to the true value:

$$H(t, X_t, X_t^M) \to V \text{ as } t \to 1.$$

- HJB equation for speculator's optimality imposes the following restriction on coefficients:

$$\begin{aligned} r_1(t) = \frac{w'(t)}{2w(t)} &= \frac{w'(t)}{w(t)} + \beta_2(t)\frac{\sigma_M^2(t)}{1 - \Sigma_M(t)} \\ r_2(t) &= \beta_2(t)\left((1 - \beta_2(t))\frac{\sigma_M^2(t)}{1 - \Sigma_M(t)} - \frac{\beta_2'(t)}{\beta_2(t)}\right). \end{aligned} \tag{8}$$

Market maker's valuation in equilibrium

- We conjecture that the speculator's strategy is linear in X, X^M and Z so that the conditional distribution of η for the market makers is Gaussian.
- The experience suggests

$$d\theta_t = \alpha(t)(\mu + \sigma_V Z_t - H(t, X_t, X_t^M))dt$$

for some deterministic α.

- Thus, in market maker's filtration

$$\begin{aligned} dX_t &= (r_1(t)X_t + r_2(t)X_t^M + w(t)\alpha(t)(\mu + \sigma_V \widehat{Z}_t - H(t, X_t, X_t^M))dt \\ &\quad + w(t)dB_t^M \\ dX_t^M &= \sigma_M(t)dW_t^M + \sigma_M^2(t)\frac{\widehat{Z}_t - X_t^M}{1 - \Sigma_M(t)}dt, \end{aligned}$$

where $\widehat{Z}_t := \mathbb{E}[Z_t|\mathcal{F}_t^M] = \mathbb{E}[\eta|\mathcal{F}_t^M]$, and (B^M, W^M) is a 2-dimensional $\mathcal{F}^M$-Brownian motion.

Equilibrium

- Note that competition among market makers implies $\widehat{Z}_t = X_t + \beta_2(t)X_t^M$.
- If we combine this with the equations from linear filtering for the condtional distribution of η, we obtain the following system, where $v(t)$ is the $\mathcal{F}_t^M$-conditional variance of Z_t:

$$\begin{aligned} \sigma_Z^2(t) - v'(t) &= w^2(t) + \frac{\sigma_M^2(t)(v(t) + 1 - \Sigma_Z(t))^2}{(1 - \Sigma_M(t))^2} \\ w'(t) &= -2w(t)\frac{\sigma_M^2(t)(v(t) + 1 - \Sigma_Z(t))}{(1 - \Sigma_M(t))^2} \end{aligned} \quad (9)$$

- The above implies w and $v + 1 - \Sigma_Z$ are decreasing as soon as the solution exists.
- Note that once we find w we immediately obtain β_2 from (8).

Solution of (9)

- Note that the initial condition of v in (9) is known and equals the variance of $Z_0 = X_0^I$. However, $w(0)$ is not known yet and has to be determined in equilibrium.
- However, one can show that

$$v(t) + 1 - \Sigma_Z(t) = w^{\frac{1}{2}}(t)\left(w^{-\frac{1}{2}}(0) - \int_0^t w^{\frac{3}{2}}(s)ds\right). \quad (10)$$

- Since $v(1) = 0$, this imposes a condition on w and its initial condition.
- Since w depends on v which depends on w, and in particular its initial condition, this leads to a fixed point problem.
- We can show the existence of a solution to the system (9) by stipulating that

$$w^{-\frac{1}{2}}(0) = \int_0^1 w^{\frac{3}{2}}(s)ds. \quad (11)$$

Equilibrium

- Given that we have a solution (w^*, v^*) to the system (9) by imposing (11), an equilibrium exists, in which the speculator's strategy is given by

$$d\theta_t^* = \frac{w^*(t)}{v^*(t)}(Z_t - X_t - \beta_2(t)X_t^M))dt$$

- Moreover, the equilibrium dynamics of X follow

$$dX_t = (r_1(t)X_t + r_2(t)X_t^M)dt + w(t)dB_t^M$$

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  from the point of view of market makers.
- Moreover, $X_1 + \beta_2(1)X_1^M = \eta$.
- The HFT's expected profit is given by

$$\sigma_V \int_0^1 w^*(t)dt.$$

Some concluding remarks

- We have constructed a model that can accommodate an HFT taking advantage of low latency.
- One can show that $w^*(t) \to 0$ as $t \to 1$. That is, the market makers giving a less weight to the total demand as the liquidation time get closer to limit the speculation opportunities for the HFT.
- w^* only depends on the function Σ_M (and not Σ_I.) Thus, the price impact, i.e. $\sigma_V w^*$, depends only on the variance of the asset payoff and the quality of public information.
- Trading intensity of the HFT is given by $\frac{w^*}{v^*}$, whereas in the classical Kyle model it equals $\frac{1}{v_0}$, with v_0 being the corresponding conditional variance in the Kyle model. Thus, the presence of the public news makes the speculator's relative trading intensity go down.