Some unwisdom about philosophy/foundations of quantum theory from the perspective of operator algebras

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A comforting thought – by Thomas Mann

"[Lotte] fancied her unwisdom, taking it as a sign that she was still young and unchanged despite the years, and with a covert smile rejoicing in the same. In a farewell letter Someone [Goethe] had written: "And I, dear Lotte, rejoice to read in your eyes that you believe I shall never change." There it is, the faith of our youth; we never, at bottom relinquish it; never, however old we grow, do we tire of reconfirming its truth, of reassuring ourselves that we are still the same, that growing old is but a physical, outward phenomenon and naught can avail to alter that innermost, foolish self of ours which we have carried about so long. And herein lies the blight and shamefaced secret of our dignified old age." (T. Mann: Lotte in Weimar)

In original German

Charlotte empfand "... Unweisheit als jugendlich, als Beweis und Merkmal innerster Unverwüstlichkeit, Unveränderlichkeit durch die Jahre ... und sich mit heimlichem Lächeln darin gefiel. Was jemand [Goethe] ihr einst geschrieben, auf einem Abschiedszettel: 'Und ich, liebe Lotte, bin glücklich in Ihren Augen zu lesen, Sie glauben ich werde nie verändern.' -, ist der Glaube unserer Jugend, von dem wir im Grunde niemals lassen, und dass er Stich gehalten habe, dass wie immer diesselben geblieben, dass Altwerden ein Körperlich-Äusserliches sei und nichts vermöge über die Beständigkeit unseres innersten, dieses närrischen, durch die Jahrzenhte hindurchgeführte Ich, ist eine Beobachtung, die anzustellen unseren höheren Tagen nicht missfällt -, sie ist das heiterveschämte Geheimnis unserer Alterswürde."

In Hungarian

"[Lotte] az ésszerűtlenséget fiatalosnak érezte, mint annak jelét és bizonyítékát, hogy belső természete időskorában is változatlan, elpusztíthatatlan maradt, és titkon mosolyogva tetszelgett ebben a tudatban. Amit egy búcsúlevéllben írt neki egykor valaki [Goethe]: 'és én, kedves Lotte, boldog vagyok, mert azt olvasom ki a szeméből, hisz benne, hogy sohasem fogok megváltozni' – az a fiatalságunk hite, amitől voltaképpen sohasem tágítunk, hogy pedig ez a hit megállta a próbát, hogy mindvégig ugyanazok maradtunk, hogy az öregedés külső jelenség, és legbenső valónk, bolondos, évtizedeken át hordozott énünk állandóságán semmi ki nem foghat, ez jóleső megfigyelés életünk alkonyán – öregkori méltóságunk derűsen szemérmes titka."

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Hidden variables

Proposition (von Neumann 1933)

- *No dispersion free states on $\mathcal{B}(\mathcal{H})$*
  - *Consequently: States with non-zero dispersion cannot be thought of as averages of dispersion-free states of the quantum system*

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Redei FoP 1986

- Can states on a C^*-algebra $\mathcal{A}$ with non-zero dispersion be thought of
  - as averages of states with less dispersion
  - where the states with less dispersion might be states of an underlying (hidden) quantum system represented by another C^*-algebra $\mathcal{B}$ related to $\mathcal{A}$ in some specific manner?

Formally

$$L\colon \mathcal{B} \rightarrow \mathcal{A}$$

positive, linear unit preserving

hiding map

Dual L^* of L

takes states ϕ on $\mathcal{A}$

into hidden states

$L^*(\phi) \doteq \phi \circ L$ on $\mathcal{B}$

Can $L^*(\phi)$ be decomposed into states on $\mathcal{B}$
with less dispersion than the dispersion of ϕ?

No-go theorems and their interpretation

A No-go theorem stating that a decomposition is not possible if L preserves structural property Φ of algebra $\mathcal{B}$ means:

- C^*-algebras sharing structural property Φ belong to the same uncertainty class
- Reduction of uncertainty measured by dispersion is not possible without destroying structure Φ preserved by L

The problem of hidden variables

$$\Updownarrow$$

Determining the equal uncertainty classes of C^*-algebras

Proposition (Redei FoP 1986; JMP 1987, 1989)

Reduction is not possible in the following cases:

- *L is a Jordan algebra homomorphism*
- *$\mathcal{A}$ is uniformly hyperfinite, $\mathcal{A} \subseteq \mathcal{B}$ subalgebra, $L\colon \mathcal{B} \to \mathcal{A}$ conditional expectation*
- *If $\mathcal{A}$ and $\mathcal{B}$ are quasilocal von Neumann algebras of AQFT and*
    - *L maps local algebras to local algebras*
    - *L commutes with representations of Poincare groups on $\mathcal{A}, \mathcal{B}$*
    - *+ some technical conditions*

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  - *$+$ some technical conditions*

Problems

- How about L preserving CCR/other structural properties ?
- How about maps L preserving structure of observables if observables are allowed to be non-selfadjoint? ($\to$ Bryan)
- How about the problem in the category of von Neumann algebras with L normal?
  - Does the type of von Neumann algebra matter?

Entropic hidden variables

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$\Downarrow$

Replacing dispersion with entropy
in the operator algebraic formulation of the hidden variable problem
(Redei Synthese 1987)

$\Downarrow$

The problem of entropic hidden variables

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Determining the equal entropic uncertainty classes of C^*-algebras

Entropic hidden variables

Proposition (Redei Synthese 1987, PL-A, 1989)

Reduction of entropic uncertaintly is not possible in the following cases:

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Operations

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A completely positive, linear, unit preserving map

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Example (Operations)

- States
- Conditional expectations
  - Projection postulate
- Kraus operations $T(A) = \sum_i K_i^* A K_i$
- Accardi-Cecchini ϕ-preserving conditional expectation

Operations and no-signaling

A major technical difficulty about operations is that there is no general representation theorem for them (not all operations are Kraus operations)

This entails

Corollary (Redei & Valente SHPMP 2010)

Einstein locality (local commutativity) in AQFT does not entail no-signaling for all operations

$\Downarrow$

The concept of no-signaling with respect to general operations had to re-defined (operational C^*-separability)

Proposition (Redei & Valente SHPMP 2010)

Operational C^*-separability holds in AQFT for strictly spacelike separated double cone algebras

A small research programme

States are a special class of operations

$\Downarrow$

Many definitions involving states can be meaningfully (re)formulated for operations

$\Downarrow$

Many questions/problems involving states re-appear
as questions/problems about operations

Entanglement

Definition (entangled state)

If $\mathcal{A}, \mathcal{B}$ are C^*-subalgebras of C^*-algebra $\mathcal{C}$, state ϕ on $\mathcal{C}$ is called $(\mathcal{A}, \mathcal{B})$-entangled if it is not in the w^*-closure of the convex hull of product states across $\mathcal{A}, \mathcal{B}$

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If $\mathcal{A}, \mathcal{B}$ are C^*-subalgebras of C^*-algebra $\mathcal{C}$, then operation T on $\mathcal{C}$ is an $(\mathcal{A}, \mathcal{B})$-entangled operation if it is outside of the BW-closure of convex combinations of product operations across $\mathcal{A}, \mathcal{B}$

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$$T_i \stackrel{BW}{\rightarrow} T$$

iff for all $X \in \mathcal{A}$ and all states ϕ

$$\phi(T_i(X)) \rightarrow \phi(T(X))$$

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- What is the status of operational entanglement in AQFT?
  - Conjecture: Analogous to the status of entangled states – proof?

Categorial subobject independence as morphism co-possibility

Definition (Redei SHPMP 2014; Z. Gyenis & Redei CMP 2017)

Let $\mathfrak{C}$ be a class of objects with two classes of morphism

$$Mor_{\mathfrak{C}} \text{ and } Hom_{\mathfrak{C}}$$

such that $(\mathfrak{C}, Hom_{\mathfrak{C}})$ is a subcatgeory of $(\mathfrak{C}, Mor_{\mathfrak{C}})$

Two $Hom_{\mathfrak{C}}$-subobjects O_1, O_2 of object O are called $Mor_{\mathfrak{C}}$-independent if any two $Mor_{\mathfrak{C}}$-morphisms m_1 and m_2 on the $Hom_{\mathfrak{C}}$-subobjects O_1, O_2 are implementable by a single $Mor_{\mathfrak{C}}$-morphism m on object O

Example

Example (Redei & Summers IJTP 2010)

Operational C^*-independence:

- Subobjects in the category of C^*-algebras $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ with respect to the injective C^*-algebra homomorphisms as morphisms $hom_{\mathfrak{Alg}}$
- The class of operations $Op_{\mathfrak{Alg}}$ as the class of morphisms to define $hom_{\mathfrak{Alg}}$-subobject independence

$$\Downarrow$$

the notion of

$Op_{\mathfrak{Alg}}$-independence of $hom_{\mathfrak{Alg}}$-subobjects

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the notion of

$Op_{\mathfrak{Alg}}$-independence of $hom_{\mathfrak{Alg}}$-subobjects

is meaningful

Physical interpretation of $Op_{\mathfrak{Alg}}$-independence:
Any two (inter)action (with)on the subsystems are jointly possible as an (inter)action (with)on the large system

Categorial subsystem independence

One can recover the major subsystem independence concepts that occur in local quantum physics by choosing special subclasses of the class of all non-selective operations $Op_{\mathfrak{Alg}}$ (Redei FoP 2010):

- States as a subclass of operations $\rightarrow$ C^*-independence
- Normal states as the subclass of operations $\rightarrow$ W^*-independence
- Normal operations as subclass of operations $\rightarrow$ operational W^*-independence
- Product versions of these specific independence concepts obtained by considering $Op_{\mathfrak{Alg}}$-independence in the product sense with respect to the respective subclasses of operations
  - C^*-and W^*-independence in the product sense
  - operational C^*-and W^*-independence in the product sense
- Logical independence of von Neumann lattices (Redei FoP 1995; IJTP 1995; Z. Gyenis & Redei CMP 2017)

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$Op_{\mathfrak{Alg}}$-independence serves as a general, categorial frame in which subsystem independence can be formulated and analyzed

Operator-valued correlations

Definition

If T is an operation on $\mathcal{C}$ then

- $A, B \in \mathcal{C}$ are called correlated in T if

$$T(AB) \neq T(A)T(B) \tag{1}$$

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- $A, B \in \mathcal{C}$ are called correlated in T if

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- T on $\mathcal{C}$ is called correlated if there exists $A, B \in \mathcal{C}$ that are correlated in T

Screening-off operator-valued correlations

Definition

Operation S on $\mathcal{C}$ screens off the correlation

$$T(AB) \neq T(A)T(B) \tag{2}$$

if

$$(T \circ S)(AB) = (T \circ S)(A)(T \circ S)(B) \tag{3}$$

Note: This definition does not require that $\mathcal{S}$ is a product operation (= screens off all correlations)

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An operation that screens off an operator-valued correlation is the general notion of common cause of a correlation in quantum context

Natural questions

Problem

If $\mathcal{A}, \mathcal{B}$ are C^*-subalgebras of $\mathcal{C}$ under what conditions is the triplet $(\mathcal{A}, \mathcal{B}; \mathcal{C})$

operationally common cause complete

in this sense:

- For any operation T on $\mathcal{C}$ which is correlated on $A \in \mathcal{A}$ and $B \in \mathcal{B}$ there exists an operation on $\mathcal{C}$ that screens of the correlation in T between A and B

Some very specific results

Results on operational common cause closedness are known only for cases where

- the correlated operatios are correlated states
- the screener-off operations are special selective operations given by Bayesian conditionalization

Call this: simple common cause closedness

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Proposition (Two cases of simple common cause completeness)

- *$\mathcal{A} = \mathcal{B} = \mathcal{C}$ are commutative algebras determined by purely non-atomic classical probability measure spaces (B. Gyenis & Redei FoP 2004; Z. Gyenis & Redei PoS 2011)*
- *$\mathcal{A} = \mathcal{B} = \mathcal{C}$ are von Neumann algebras with projection lattices with faithful states determining a measure theoretically purely non-atomic quantum probability space (Z. Gyenis & Redei Erkenntnis 2014; Kitajima & Redei SHPMP 2015)*

Screening off superluminal operator valued correlation in QFT?

One can amend the concept of screening off operation in the context of categorial quantum field theory (Brunetti & Fredenhagen & Verch CMP 2003) by requiring the screener-off operation to belong to a local algebra located in the intersection of the causal pasts of the (spacelike separated) local algebras containing the correlated observables (Redei SHPMP 2014)

And one can ask

Problem

Can correlations between spacelike separated operators given by genuine operations be screened of by an operation localized the causal pasts of the (spacelike separated) local algebras containing the correlated operators?

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The answer is not known – not even in case of correlations given by states

We only know:

Proposition (Redei & Summers FoP 2002; Hofer & Redei & Szabo 2013)

Correlations given by states betwen observables in spacelike separated spacetime regions in the context of the Haag-Kastler AQFT can be screened of by selective operations (given by Bayesian conditionalization) localized in the union of the causal pasts of the spacelike separated regions containing the correlated observables.

Informal conclusion

Operator algebra theory offers a flexible and powerful framework in which foundational-philosophical problems of quantum theory can be nicely and clearly articulated and related to results in mainstream mathematical physics

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There are a number of mathematically well-formulated problems still open

$\Downarrow$

Hope to continue to work with you on such problems
for some more years to come!