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Non-renewable natural capital and the social cost of carbon in wealth accounting *

Rintaro Yamaguchi† Matthew Agarwala‡ Giles Atkinson§

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Abstract

Fossil fuels represent a significant portion of the wealth of resource-rich nations. However, their valuation as non-renewable natural capital in inclusive or comprehensive wealth accounting to indicate sustainability does not embody the external costs of climate change damages. This study consistently incorporates the social cost of carbon (SCC) into the value of depletion of non-renewable natural capital for wealth accounting of resource-rich nations. We show generalised shadow prices of depletion under different resource allocation mechanisms (RAMs) in the presence of the SCC under declining extraction and the unburnable natural capital stock constraint. In our application to oil, depletion is valued differently across RAMs of user-cost shadow pricing and weighted average shadow pricing, depending on how rent, SCC, and decarbonisation develop in the future. The sustainability implication of the choice of RAM is even more significant in the presence of SCC.

Key words: genuine savings; natural capital; fossil fuel; inclusive wealth accounting; social cost of carbon; sustainable development

JEL codes: C43, D63, O47, Q01, Q54

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1 Introduction

Wealth accounting studies, exemplified by World Bank (2011, 2018, 2021, 2024) and UNU-IHDP and UNEP (2012, 2014), continue to make it clear that fossil fuels represent a substantial portion of the estimated value of natural capital and, especially for countries with abundant reserves, of national wealth. For example, in 2018, this component of natural capital represents about 35% of total wealth in the Middle East and North Africa, according to World Bank (2021). For these and other national economies, how wealth is sustained as these fossil fuel resources are depleted remains an important policy question.

Yet, given ever-growing concerns about the social costs of fossil fuel use, the approach taken in most wealth accounting studies to valuing (changes in) fossil fuel assets looks increasingly curious.[1] That is, the value of fossil fuel assets is understood without explicit reference to their broader social costs. This contrasts with the approach taken in a social cost-benefit analysis of a fossil fuel development project, which would typically include explicit consideration of the global climate damage resulting from the depletion of those reserves.[2]

Integrating such perspectives more fully into wealth accounting could start with two (related) questions. First, looking forward, what is the social value of *stocks*—i.e. fossil fuel assets—that should be included in an extended balance sheet? Second, looking back, when resources are depleted, how does this emphasis on social value change the understanding of asset depreciation and hence the *flow* of reinvestment needed to sustain wealth? Our aim in this paper is to address both questions using a consistent framework for wealth accounting. This, we argue, is crucial to ensure a more rounded understanding of the contribution of fossil fuel assets to national wealth.

Our contribution proceeds by incorporating the social cost of carbon (SCC) into the capital resource economy model under a resource allocation mechanism (RAM) that sets the rules for how initial resource stocks are mapped onto a future extraction trajectory (Arrow et al., 2003; Hamilton and Ruta, 2009). This is important because it determines how depletion is valued along this trajectory.

Currently, the World Bank's adjusted net savings (ANS) subtract the flow of CO_2 damages from net savings of emitting countries in the *responsibility*-based approach, while Arrow et al. (2012) and UNEP's accounts adjust the comprehen-

[1]See Appendix A for the related literature.

[2]The ongoing debate about stranded fossil fuel assets is also relevant here. According to one prominent estimate, global compliance with the Paris Agreement could effectively render one-third of oil reserves, one-half of gas reserves and four-fifths of coal reserves valueless (McGlade and Ekins, 2015; IEA, 2023).

sive wealth index of countries suffering CO_2 damages in the *vulnerability*-based approach.

However, to the extent that sustainability indicators are intended to be forward-looking, it may be theoretically inconsistent to include carbon damage only retrospectively. Carbon emissions do not appear out of nowhere, but have been buried underground in other forms. Terrestrial carbon stocks should also be reflected in the net present value (NPV) of fossil fuels if they are expected to be released into the atmosphere. Accounting for carbon in balance sheets has tended to focus on renewable (biotic) natural capital, focusing on the capitalisation of ecosystem services including carbon storage. By analogy, fossil fuel stocks which are currently unused under-the-ground might be construed in a similar way, given these are currently providing avoided damage services.[3] The mirror image of this is that, along the depletion path assumed in valuing fossil fuel assets, the implied release of this carbon is a liability. It strikes us that this is important to take account of.

Here, we immediately face another pair of important questions: *i) which shadow price should we use, or equivalently, which RAM should we have in mind?; and ii) to whom should we attribute these prices?* Regarding i) which price question, while Arrow et al. (2012) and earlier papers such as Hamilton and Clemens (1999) propose using the current resource rent to value depletion, Hamilton and Ruta (2009, 2017) and Hamilton (2016) propose using the average rent over the lifetime of the resource (i.e. N/S, where N is the present value of rents over its lifetime and S is the physical resource stock). Wei (2015) suggests valuing depletion by the present value of the last unit extracted: the so-called El Serafy or user-cost approach (El Serafy, 1981, 1989). We significanly generalise the expression of shadow prices to value depletion by using discount factors with intuitive interpretations.

As even this relatively small handful of approaches tends to produce markedly different estimates of asset depreciation (Atkinson and Hamilton, 2007), the question naturally arises as to which method is the most appropriate. One answer, in theory, is whichever approach best corresponds to the change in intergenerational well-being indicated by either the value of the change in wealth (i.e. capital stocks) using constant capital shadow prices, or the value of the change in real wealth using a Divisia capital shadow price index (Asheim and Weitzman, 2001). In this respect, Hamilton and Ruta (2017) support Hamil-

[3]In addition, negative emission technologies such as carbon capture and storage (CCS) are becoming more realistic in the pursuit of net-zero economies. Abandoned oil fields are prominent subsoil sites with the potential to host stored carbon.

ton (2016) by showing that the average unit price (N/S) is consistent with the shadow price in a RAM with fixed resource life years. In this set-up, the change in the total value of the resource stock would not be equal to the real value of the change in the resource stock.

As we will show, factoring in the SCC to the underlying RAM adds another dimension as it implies very different assessments of resource depletion value under decarbonisation (i.e., declining depletion and unburnable resource stock). The marginal shadow price may or may not be higher than the average price, depending on how rents, SCC, and decarbonisation evolve in the future. The value of resource depletion using resource rents would be an overestimate if carbon damages were not deducted, as the opportunity cost of depleting the resource may be lower in the future when the economy is more decarbonised along the path promised under the Paris Agreement. Moreover, while Hamilton and Ruta (2017) show that the shadow price of oil means the value of either the last drop of oil or the average drop, we show in our Proposition that, in the user-cost pricing, the change in real value and total value would deviate under decarbonisation; and that in the weighted average pricing the nearer future rent or SCC matters more to the shadow price than the farther future rent or SCC if a large decline in extraction is expected.

Since our shadow price now includes the SCC, answering i) which shadow price would involve the choice of the appropriate SCC in light of the objective of the analysis (Kotchen, 2018) and of the RAM. As opposed to the global SCC (GSCC), a domestic or national SCC (NSCC) is the present value of the country-level future damages caused by additional CO_2 emissions anywhere.[4] NSCC is therefore consistent with actual sustainability experience in national accounting. In contrast to this vulnerability-based approach is the responsibility-based approach, which allocates the SCC to emitting countries (Hamilton and Atkinson, 1996).[5]

We argue that an upstream variant of this responsibility-based approach—the extraction-based approach—is also relevant, as well as production- and consumption-based approaches in the literature.[6] On one level, this can be

[4]Several studies disaggregate GSCC into NSCCs using integrated assessment models, such as Ricke et al. (2018). Asheim and Yamaguchi (2026) do so using the GIVE model that has country-level resolutions of climate change damage (Rennert et al., 2022).

[5]Despite the reality of no carbon taxes, the rationale for doing so is to attach property rights to pollutees and implement carbon pricing according to the polluter pays principle (Hamilton, 2012). However, Asheim and Yamaguchi (2025) show that emitting countries paying for their emissions does not necessarily indicate that sustainability is properly reflected by subtracting GSCC from adjusted net savings.

[6]As Atkinson et al. (2012) show, contrasting these two approaches often highlights the

viewed as reflecting—in wealth accounts—the risks that global climate policy poses to fossil fuel-rich countries and it seems fitting that this risk is reflected in wealth accounts, as well as more broadly in discussions of stranded assets. Moreover, a growing body of literature on supply-side environmental policy (Asheim et al., 2019) asserts that controlling fossil fuel suppliers would be more efficient and promising than demand-side policies, which seem politically even more challenging nowadays.

Assessing whether wealth is being sustained cannot only focus on whether saving covers off depletion of nonrenewable natural capital, especially when resource use entails substantial social cost. Whether this social cost is the national responsibility of resource producers is more arguable, although clearly this process starts with the depletion of fossil fuels. A stronger interpretation, however, is that we incorporate carbon damage that fossil fuel-producing countries might be required to pay for in some way to help contribute to sustaining global wealth. We believe that the responsibility-based approach, including our extraction-based accounting, should use the GSCC, as the following schematic diagram shows, unless the underlying objective of the accounting is to study only the domestic consequences.

$$\text{stage:}\quad \underbrace{\text{extraction}\quad\text{production}\quad\text{consumption}}_{\substack{\text{responsibility}\\ \text{-based}\\ \Downarrow\\ \text{GSCC}}}\quad \underbrace{\text{damage}}_{\substack{\text{vulnerability}\\ \text{-based}\\ \Downarrow\\ \text{NSCC}}}$$

Our empirical application suggests that the value of depletion widely differs both across RAMs and regions. However, the break-even initial SCC that would result in a zero depletion value appears to be somewhere between USD 50/tCO_2 and USD 100/tCO_2 for Europe, Africa and the Asia-Pacific region, regardless of the underlying RAM. Notably, for the CIS and the Middle East, the break-even SCC is lower than USD 50/tCO_2 under the weighted average shadow pricing, while it is higher than USD 50/tCO_2 under the user-cost shadow pricing that looks at the opportunity cost of crowding out the additional resource at the year of exhaustion. Given that recent estimates of the SCC tend to be much higher than these values, capturing the SCC would immensely change the valuation of fossil fuels.

The rest of this paper is organised as follows. In Section 2, we study a familiar optimal RAM to guide us to what (not) to be accounted for sustain-

responsibility of higher-income countries that import commodities.

ability accounting. In suboptimal RAMs frequently studied in the accounting of non-renewable resources, Section 3 generalises shadow prices to value depletion. Section 4 sums up our results and interprets the previous practical wealth accounting. Section 5 applies these RAMs to the valuation of oil as natural capital across nations. Section 7 concludes.

2 Motivating the inclusion of SCC: a benchmark optimal RAM

2.1 Model

It is helpful to start from an optimal economy, in which the social cost of carbon is fully internalised. In what follows, we use a global economy model with no national borders and no spillovers. In such a world, oil extraction is determined to balance its contribution to production with its scarcity and externality costs. Following Hamilton and Clemens (1999) and Aronsson and Löfgren (2010), let the social well-being at t take the form

$$V := \int_t^\infty U(C,X)e^{-\delta(\tau-t)}d\tau \tag{1}$$

where C denotes consumption and $\delta > 0$ expresses the pure rate of time preference. The arguments in the utility function are the consumption and externality of carbon stock, X. In integrated assessment models, it is commonly assumed that carbon damage affects well-being via the decline in output. However, climate change also affects utility as the loss of amenity, which justifies our formulation in (1). We assume that the marginal utility of consumption is positive but declining, and the marginal disutility of carbon is negative and increasing, that is, $U_C > 0$, $U_{CC} < 0$, $U_X < 0$, and $U_{XX} < 0$. The cross derivatives are assumed to be negative ($U_{CX} < 0$), implying that the marginal utility of consumption does not increase in the presence of pollution externality.

We have three capital assets in the model. Conventional capital increases as a result of the output equation with resource use, R:

$$\dot{K} = F(K,R) - C, \tag{2}$$

where $F_K > 0, F_R > 0, F_{KK} < 0$, and $F_{RR} < 0$. This implicitly assumes that there is no extraction cost incurred. The second class of capital is non-renewable natural capital, which is depleted by resource use:[7]

$$\dot{S} = -R \tag{3}$$

The third capital has a negative value; global carbon stock in the atmosphere increases as oil extraction increases, and it dissipates at the rate of γ:

$$\dot{X} = \psi R - \gamma X, \tag{4}$$

where ψ is a conversion parameter from ton of oil to tCO_2. Then, the current-value Hamiltonian reads

$$\mathcal{H} = U(C, X) + \lambda_K (F(K, R) - C) - \lambda_S R + \lambda_X (\psi R - \gamma X), \tag{5}$$

where λ_i stands for the shadow prices for capital i. The first-order conditions for optimality include

$$\begin{aligned} U_C &= \lambda_K, && (6)\\ \lambda_K F_R - \lambda_S + \psi\lambda_X &= 0, && (7)\\ -\lambda_K F_K &= \dot{\lambda}_K - \delta\lambda_K, && (8)\\ 0 &= \dot{\lambda}_K - \delta\lambda_S, && (9)\\ -U_X + \gamma\lambda_X &= \dot{\lambda}_X - \delta\lambda_X, && (10) \end{aligned}$$

along with the transversality conditions. In particular, equation (7) indicates that the natural capital shadow price consists not only of its marginal production but also of its externality to utility. In a fully optimal economy, it holds that

$$\delta V = \mathcal{H} = U(C, X) + U_C \left(\dot{K} - \left(F_R + \frac{\psi\lambda_X}{\lambda_K} \right) R + \frac{\lambda_X}{\lambda_K} (\psi R - \gamma X) \right). \tag{11}$$

In other words, genuine savings under optimality works out to be

$$\begin{aligned} G &= \dot{K} - \left(F_R - \frac{-\psi\lambda_X}{\lambda_K} \right) R + \frac{\lambda_X}{\lambda_K} (\psi R - \gamma X) \\ &= \dot{K} - F_R R + \gamma \frac{-\lambda_X}{\lambda_K} \end{aligned} \tag{12}$$

The two expressions of genuine savings in equation (12) suggest that accounting separately for oil stock depletion and carbon stock accumulation, on the one hand (12-1), and accounting for oil at the gross resource price, on the other hand (12-2), are equivalent from a global perspective. Equation (12-2) simply

[7]We do not assume resource discovery or any other addition to the non-renewable resource stock (Arrow et al., 2003; Hamilton and Atkinson, 2013; Pezzey, 2024).

records oil depletion at its current gross resource price (F_R), net of gains from carbon dissipation. It is useful to imagine an oil producer which is asked to buy a permit for extraction that completely offsets its associated carbon emission. In other words, if oil depletion is determined considering the externality, then accounting for oil depletion using the gross rental price and accounting for the carbon damage caused by oil use would involve double counting.

Expressed as a global aggregate, equation (12-1) shows that it makes perfect sense to include carbon damages simultaneously in both resource depletion and associated carbon accumulation in order to account for sustainability. In practice, sustainability accounting has so far only accounted for carbon accumulation because it has focused on *realised*, ex-post sustainability (Hamilton and Clemens, 1999; Arrow et al., 2012). As equation (12-1) suggests, a more complete, forward-looking treatment would be to include the carbon externality in both resource depletion and carbon accumulation, provided that the carbon damage is partially internalised, and in both the resource stock and the carbon stock, as long as oil is expected to be extracted in the future and to the extent that the carbon damage is expected to affect the total value of natural capital as the NPV of its future flow of services.[8]

It is important to understand the intuition behind the inclusion of SCC in the valuation of resource depletion, as a shift from resource rent accounting to net resource rent accounting including SCC would 'improve' the genuine savings of an oil-producing country. As we will argue, the shadow price of non-renewable natural capital indicates either the marginal value of the last unit or the average value over the operating period, depending on the underlying RAM. Since resource depletion also implies the lost opportunity cost to future generations of using the additional unit of resource, it is intuitive that resource depletion would be overstated if SCC were excluded. Thus, this is not a paradox once we note that the depleted resource becomes less valuable if SCC is internalised in the future.

2.2 Social cost of carbon

In the climate change literature, the shadow price of carbon capital relative to the produced capital $\sigma := -\lambda_X/\lambda_K$ is often called the social cost of carbon (SCC). The SCC is defined by the present value of all future damage costs by increasing one unit of emission in the present, $\frac{-\partial V}{\partial(\psi R)}/\frac{\partial V}{\partial C}$. In this expression,

[8]This argument further suggests that an even more complete accounting would be to include the future carbon stock, which could be related to the forward-looking cost of carbon emissions under the baseline scenario. This is left for future research.

the carbon damage cost (incurred by emission, not resource use) is divided by the marginal utility of consumption to account for damage in the consumption numeraire.

Using equations (6), (8), and (10), the Hotelling rule for SCC says that it increases at a rate less than the interest rate (neglecting dissipation):

$$\frac{\dot{\sigma}}{\sigma} = F_K + \gamma - \frac{U_X}{\lambda_X} \tag{13}$$

which can be converted to the expression of the SCC:

$$\sigma = \int_t^{\infty} \left(\frac{-U_X}{U_C} \; e^{-\int_t^z (F_K+\gamma)dy} \right) dz. \tag{14}$$

2.3 Resource rent

From optimality conditions (6)-(10), it can be shown that the Hotelling rule is extended to

$$\frac{\dot{F_R}}{F_R} = F_K + \frac{\psi}{F_R} \left(\gamma\sigma - \frac{-U_X}{U_C} \right). \tag{15}$$

The negative externality of the carbon stock acts to slow down the increase in the resource rent. Ignoring stock dissipation γ, the resource rent increases when the opportunity cost of holding the resource is greater than the instantaneous social cost of oil use, in which case it is optimal to deplete the resource. As the instantaneous social cost of oil use increases and approaches the real rate of return on capital (F_K), the resource rent stops changing and the extraction of the resource comes to a halt. At this point, the social cost of oil use is too high to justify further extraction. We also observe from (13) and (15) that in this optimum economy, the change rate of the resource rent is smaller than the interest rate, assuming that the carbon dissipation rate is negligible, justifying our setting in the next section.

Following Hamilton and Ruta (2009, 2017), we define the total value of the natural resource stock as the net present value of the total rent, N. From equation (12), we can write

$$N = \int_t^{\infty} (F_R - \sigma\psi) R \; e^{-\int_t^z F_K dy} dz \tag{16}$$

To the extent that the SCC is expected to be woven into the private, as well as social, profits from non-renewable resources in the future, their total value as the net present value of profits also embodies the SCC. The time derivative of the value of the total resource stock is

$$\dot{N} = F_K N - (F_R - \sigma\psi) R, \tag{17}$$

which constitutes what is sometimes called the fundamental equation of asset equilibrium.

3 Sub-optimal RAMs under decarbonisation

3.1 Preliminaries

The RAM we saw in section 2 is familiar, as it involves the optimum of internalising the SCC in an infinite horizon. In what follows, we move closer to a practical, real-world wealth accounting. Following Arrow et al. (2003), a RAM can be formalised by mapping the initial state variables to future control and state variables:

$$\alpha : \{K(t), S(t), X(t)\} \to \{K(z), S(z), X(z), C(z), R(z)\}_{z=t}^{\infty}$$

where generalised capital stock K, non-renewable natural capital S, atmospheric carbon stock X, consumption C, and extraction R are determined for all time periods. Imagine also that in this α, the interest rate r is constant.

In the green national accounting literature, two RAMs have been proposed for the constant extraction regime: in $\alpha =$SW, the constant extraction quantity is fixed and the remaining years are determined (El Serafy, 1989; Wei, 2015), while in $\alpha =$HR, the end date is fixed first and the constant extraction quantity is determined (Hamilton and Ruta, 2009, Sec. 5). Hamilton and Ruta (2017) show that these two RAMs yield different shadow prices both of which are consistent with the definitions of Dasgupta and Mäler (2000), and, when applied to the Hartwick (1977) investment rule, different consequences in terms of consumption change.

In the following section, we present a generalised model for accounting for the depletion of non-renewable natural capital. This model incorporates important changes and relaxes the restrictive settings of previous studies. Firstly, we allow for variation in annual extraction by assuming an exponential decline rate of $\phi \geq 0$. Although constant extraction is consistent with current accounting practices and convenient assumptions, it is not entirely realistic. In particular, given the international community's commitment to net-zero economies, it is unlikely that oil will continue to be extracted until it is depleted. Although accounting conventions and convenience may take precedence over reality, it would be sensible to gradually relax the restrictive assumption of constant extraction while remaining within the bounds of what is permissible under accounting conventions. Importantly, this is in line with the SEEA's (United Nations et al.,

2012) recommendations, which state that non-constant extraction paths could be used to estimate resource stock values. This assumption is also partially adopted by Hamilton (2016) when he illustrates how average pricing can be used to value exhaustible resources in a RAM where a constant share of the total resource stock value is depleted. However, he does not develop a full accounting model that derives shadow prices under declining extraction, which we do here.

Secondly, as implied by our analysis in Section 2, we introduce time-varying SCC when computing effective resource rents. Thirdly, as with future SCC, we also assume that resource rents could rise exponentially. This will help us to consider the impact of the effective discount rate.

Fourthly, we take account of the existence of a global carbon budget, which implies that not all subsoil resources can be used up. This global carbon budget translates into a burnable oil budget in terms of volume, which can be conceptually allocated to countries (McGlade and Ekins, 2015; IEA, 2023). Suppose that such a burnable oil stock is $S_t - S_T \geq 0$, which might be determined either exogenously or endogenously, as discussed later. [9] More concretely, we have:

Assumptions.

- The initial oil stock is given by S_t;
- The burnable oil stock is determined from the carbon budget, so that S_T ($\leq S_t$) is the oil stock that should be left unburnt at T.
- Let the initial extraction be $R(t) = R_t$;
- The fixed declining rate of extraction is $\phi \geq 0$, so that annual extraction is $\dot{S}(z) = -R(z) = -R_t e^{-\phi(z-t)}$ for $z > t$;
- The unit resource rent is growing at the rate $0 < g_n \leq r$, so that $n(t) = n_t e^{g_n t}$ given the initial resource rent, n_t;
- The unit SCC, $\tau(z) = \tau_t e^{g(z-t))}$ for $z > t$, is exogenously given,[10] and constant or increasing at a rate of $0 \leq g$ ($< r$).[11]

[9] The burnable oil stock can be generalised to the used or employed capital, as opposed to the available or potentially usable capital. What is relevant to actual welfare improvement may be the change in utilised capital, while potential welfare improvement may be related to the change in available capital (Yamaguchi, 2020). Fossil fuels are physically there, but we might not have capabilities to tap into all of them due to concerns about climate change.

[10] By this, we implicitly assume that the country in question is small in the oil market.

[11] The recent analytic IAM literature shows that the SCC is linear in the initial output and grows at the rate of output growth, supporting our assumption here. If the global damage is

For later purposes, we define two *discount factors* that are applied to rents and SCCs. The discount factors become relevant because extraction of a non-renewable natural capital in the current reporting period crowds out extraction opportunities in the future, which are valued less because of discounting and the changes in rents and SCCs.[12] The discount factors depend on the depletion time T, since it —together with the RAM (α) assumed— determines when and how extraction is crowded out. In α = SW, it is crowded out at the end of the extraction period; in α = HR, the extraction reduction is spread over the remaining depletion life until T, implying equal burden sharing among all the future generations. Seen from the other side, current addition of oil increases the value of the natural capital stock on the margin, which is measured by the rent at the terminal date in α =SW, and by the the period-average rent in α =HR. In contrast, under the optimal RAM in Section 3, we saw that the current rent with zero discounting can be used to value current depletion.

To the extent that natural capital is non-renewable, we could only obtain rents from it for a finite period of time. After the exhaustion of the resource, we need to depend on other assets that yield dividends indefinitely to sustain consumption and well-being. The proportion of reinvestment in other assets is the user cost. We may consume only the residual as a true income, in the tradition of Hicks (1946) or stationary equivalent of future consumption (Weitzman, 1976). Thus, the NPV of a finite series of constant rents (normalised to unity so that $R(t) = 1$ in this subsection 3.1) should be equated with the NPV of an infinite series of perpetual annuity, Y:

$$N(t) = \int_t^T e^{-r(z-t)}dz = \int_t^\infty Y e^{-r(z-t)}dz \tag{18}$$

which bring us to

$$N(t) = \frac{1 - e^{-r(T-t)}}{r} = \frac{Y}{r}. \tag{19}$$

If natural capital were renewable, so that there is no exhaustion date, then the exponential term would disappear, leading trivially to $Y = 1$ (i.e., we can consume total rent). In other words, the exponential term is the cost of using up the non-renewable resource, or *user cost*:

$$1 - Y = e^{-r(T-t)} =: \Omega^{\mathrm{SW}}(r(T-t)), \tag{20}$$

which is the NPV of the rent at the terminal date. The residual true income is

$$Y = 1 - e^{-r(T-t)}. \tag{21}$$

the fixed share, ξ, of global GDP, and the global GDP grows at g, then the global SCC can be written as $\sigma = \xi\, GDP_{\mathrm{initial}}/(r-g)$ and hence, $\dot{\sigma}/\sigma = g$.

[12] We thank Sjak Smulders for the suggestion of and insights regarding the discount factors.

From (19) we confirm that Y is the return or interest on wealth, $Y = rN(t)$. Moreover, dividing both sides of (19) by the extraction years, we have

$$\frac{N(t)}{T-t} = \frac{1-e^{-r(T-t)}}{r(T-t)} = \frac{Y}{r(T-t)} =: \Omega^{\mathrm{HR}}(r(T-t)), \tag{22}$$

which is the *period-average capitalised true income* of the resource stock. With (20) and (22) in mind, we have

Definition. *El-Serafy-Wei and Hamilton-Ruta discount factors are defined by:*

$$\Omega^{\alpha}(x) = \begin{cases} e^{-x} & \text{if } \ \alpha = \mathrm{SW} \\ \frac{1-e^{-x}}{x} & \text{if } \ \alpha = \mathrm{HR} \end{cases} \tag{23}$$

where $x > 0$ and the underlying RAM, α = SW and HR, refer to El Serafy (1989) and Wei (2015), and Hamilton and Ruta (2009), respectively. Moreover,

$$\Omega^{\alpha}(0) = 1 \quad \text{for all } \alpha. \tag{24}$$

We can immediately characterise the SW and HR discount factors as follows:

$$\Omega^{\mathrm{SW}\prime}(x) = -e^{-x} < 0, \quad \Omega^{\mathrm{HR}\prime}(x) = \frac{e^{-x}(1+x)-1}{x^2} < 0, \tag{25}$$

$$0 < \Omega^{\mathrm{SW}}(x) < \Omega^{\mathrm{HR}}(x) < 1 \quad \text{for } x > 0. \tag{26}$$

3.2 Basic model

We can write the physical quantity of the usable resource stock at the initial date as

$$S_t - S_T = \int_t^T R_t e^{-\phi(z-t)} dz = \begin{cases} R_t \ \frac{1-e^{-\phi(T-t)}}{\phi} & \text{if } \phi > 0 \ , \\ R_t(T-t) & \text{if } \phi = 0 \ , \end{cases} \tag{27}$$

alternatively, the *resource extraction constraint* can be written as

$$\begin{aligned} R_t &= \begin{cases} (S_t - S_T)\frac{\phi}{1-e^{-\phi(T-t)}} & \text{if } \phi > 0 \ , \\ (S_t - S_T)\frac{1}{T-t} & \text{if } \phi = 0 \end{cases} \\ &= \frac{S_t - S_T}{\Omega^{\mathrm{HR}}(\phi(T-t))(T-t)}. \end{aligned} \tag{28}$$

Even further, this can be solved for the extraction years:

$$T - t = \begin{cases} -\frac{1}{\phi} \ln\left(1 - \frac{\phi(S_t - S_T)}{R_t}\right) & \text{if } \phi > 0 \\ \frac{S_t - S_T}{R_t} & \text{if } \phi = 0. \end{cases} \tag{29}$$

The total value of the resource stock is

$$
\begin{aligned}
N(t) &= \int_t^T (n_t e^{g_n(z-t)} - \tau_t e^{g(z-t)}) R_t e^{-(r+\phi)(z-t)} dz \\
&= \begin{cases} R_t \left(n_t \frac{1-e^{-(r+\phi-g_n)(T-t)}}{r+\phi-g_n} - \tau_t \frac{1-e^{-(r+\phi-g)(T-t)}}{r+\phi-g} \right) & \text{otherwise} \\ R_t \left(n_t (T-t) - \tau_t \frac{1-e^{-(r+\phi-g)(T-t)}}{r+\phi-g} \right) & \text{if } r+\phi = g_n \end{cases} \\
&= R_t (T-t) \left(n_t \Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t \Omega^{\mathrm{HR}}(r_\tau(T-t)) \right),
\end{aligned} \tag{30}
$$

where $r_n := r + \phi - g_n$ and $r_\tau := r + \phi - g$. Using the resource extraction constraint (28), this can be rewritten as a proportion of the burnable stock:

$$
N(t) = (S_t - S_T) \frac{n_t \Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t \Omega^{\mathrm{HR}}(r_\tau(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))}. \tag{31}
$$

Note that the value of the resource stock including SCC (31) may be negative, depending on the initial rent, interest rate, SCC, and their growth rates.

In addition, the *effective average price* would be defined as[13]

$$
\frac{N(t)}{S_t - S_T} = \frac{n_t \Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t \Omega^{\mathrm{HR}}(r_\tau(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))}. \tag{32}
$$

The time derivative of the total value of the natural capital stock (30) can be given by

$$
\begin{aligned}
\dot{N}(t) &= r_n \int_t^T n_t R_t e^{-r_n(z-t)} dz - r_\tau \int_t^T \tau_t R_t e^{-r_\tau(z-t)} dz - (n_t - \tau_t) R_t \\
&= -R_t \left(n_t \Omega^{\mathrm{SW}}(r_n(T-t)) - \tau_t \Omega^{\mathrm{SW}}(r_\tau(T-t)) \right),
\end{aligned} \tag{33}
$$

the former of which is the return on wealth net of current use, which is referred to as the fundamental equation of asset equilibrium by Hartwick and Hageman (1993, p.215). The latter relationship clarifies that the user cost—or the social net present value of marginal extraction at the terminal date—is associated with measuring the change in the value of total stock.

In what follows, we look into detailed decision-making processes of each RAM. In RAM $\alpha = \mathrm{SW}$, the initial and ensuing extraction quantities are chosen. Given the burnable stock, the extraction years are determined as a result. In RAM $\alpha = \mathrm{HR}$, in contrast, the time horizon is fixed, and the burnable stock is also given. All the extraction quantities are determined as a result.

[13]The qualifier effective is put here as $S_t - S_T$, not S_t, is the effectively usable stock.

3.3 RAM SW: user cost shadow pricing

In this RAM, since R_t and $S_t - S_T$ are given, T is endogenous, meaning that additional burnable oil contributes to extended operation years of oil. The shadow price of the burnable oil stock is

$$p^{\mathrm{SW}}(t) := \left.\frac{\partial N(t)}{\partial S(t)}\right|_{\alpha=\mathrm{SW}} = \frac{\partial N(t)}{\partial T}\frac{dT}{dS(t)} = R_t\left(n_t e^{-r_n(T-t)} - \tau_t e^{-r_\tau(T-t)}\right)\frac{dT}{dS(t)}, \tag{34}$$

where, using (29),

$$\frac{dT}{dS(t)} = \frac{1}{R_t - \phi(S_t - S_T)}, \tag{35}$$

which captures the effect of extending the terminal date by having additional capital. Thus, we obtain

$$\begin{aligned} p^{\mathrm{SW}}(t) &= \frac{R_t}{R_t - \phi(S_t - S_T)}\left(n_t e^{-r_n(T-t)} - \tau_t e^{-r_\tau(T-t)}\right) \\ &= \frac{R_t}{R_t - \phi(S_t - S_T)}\left(n_t \Omega^{\mathrm{SW}}(r_n(T-t)) - \tau_t \Omega^{\mathrm{SW}}(r_\tau(T-t))\right). \end{aligned} \tag{36}$$

The essential interpretation of the shadow price as the user cost carries over: the net benefit of the last drop of burnable oil at the terminal date. However, the shadow price is now also weighted by the degree of decarbonisation, $R_t/(R_t - \phi(S_t - S_T))$. The steeper the decline in extraction and/or the more burnable oil, the weight becomes larger. A corollary is that the marginal shadow price is positive (negative) if and only if the present value rent at the terminal date is larger (smaller) than the present value SCC at the terminal date.

It is important to note that in this RAM, the value of the change in real wealth is not equal to the value of the change in total wealth any more. Comparing (33) and (36), we have

$$p^{\mathrm{SW}}(t)\dot{S}(t) = -p^{\mathrm{SW}}(t)R_t = \underbrace{-\ R_t\left(n_t \Omega^{\mathrm{SW}}(r_n(T-t)) - \tau_t \Omega^{\mathrm{SW}}(r_\tau(T-t))\right)}_{\dot{N}(t)}\ \frac{R_t}{R_t - \phi(S_t - S_T)} \tag{37}$$

which is equal to $\dot{N}(t)$ only when $\phi(S_t - S_T) = 0$, in contrast to Hamilton and Ruta (2017, eq(8)). Using $\dot{N}$ instead of $p^{\mathrm{SW}}\dot{S}$ to value resource depletion would over/underestimate sustainability, particularly so when significant decarbonisation is expected.

3.4 RAM HR: weighted average shadow pricing

Alternately, we can consider another RAM where $S_t - S_T$ and T are given, so that R_t is endogenously determined. From (30), the shadow price of the burnable oil stock is

$$
\begin{aligned}
p^{\mathrm{HR}}(t) := \left.\frac{\partial N(t)}{\partial S(t)}\right|_{\alpha=\mathrm{HR}} &= \frac{\partial N(t)}{\partial R(t)}\frac{dR(t)}{dS(t)} \\
&= \begin{cases} \left(n_t \frac{1-e^{-(r+\phi-g_n)(T-t)}}{r+\phi-g_n} - \tau_t \frac{1-e^{-(r+\phi-g)(T-t)}}{r+\phi-g}\right)\frac{dR(t)}{dS(t)} & \text{otherwise} \\ \left(n_t(T-t) - \tau_t \frac{1-e^{-(r+\phi-g)(T-t)}}{r+\phi-g}\right)\frac{dR(t)}{dS(t)} & \text{if } \phi = 0 \text{ and } r = g_n \end{cases} \\
&= (T-t)\left(n_t\Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t\Omega^{\mathrm{HR}}(r_\tau(T-t))\right)\frac{dR(t)}{dS(t)}
\end{aligned}
\tag{38}
$$

where, using (28),

$$
\begin{aligned}
\frac{dR(t)}{dS(t)} &= \begin{cases} \frac{\phi}{1-e^{-\phi(T-t)}} & \text{if } \phi > 0\ , \\ \frac{1}{T-t} & \text{if } \phi = 0 \end{cases} \\
&= \frac{1}{\Omega^{\mathrm{HR}}(\phi(T-t))(T-t)},
\end{aligned}
\tag{39}
$$

so that the marginal contribution of the resource stock is equal to the decarbonisation-weighted average unit. Thus, the marginal shadow price is given by:

$$
p^{\mathrm{HR}}(t) = \frac{n_t\Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t\Omega^{\mathrm{HR}}(r_\tau(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))} = \frac{N(t)}{S_t - S_T}, \tag{40}
$$

which is exactly equal to the effective average shadow price, (32). This is because, with the terminal date being fixed, an additional resource stock implies an incremental resource flow uniformly spread in every period, so that N is linear in $S_t - S_T$.

The value of depletion is

$$
\begin{aligned}
p^{\mathrm{HR}}(t)\dot{S}(t) &= -\frac{N(t)}{S_t - S_T}R_t = -\frac{n_t\Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t\Omega^{\mathrm{HR}}(r_\tau(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))}\frac{S_t - S_T}{\Omega^{\mathrm{HR}}(\phi(T-t))(T-t)} \\
&= -\frac{N(t)}{\Omega^{\mathrm{HR}}(\phi(T-t))(T-t)}.
\end{aligned}
$$

Finally, differentiating both sides of (40) confirms that the change in the value of total stock reads

$$
\dot{p}^{\mathrm{HR}}(t)(S_t - S_T) + p^{\mathrm{HR}}(t)(S_t \dot{-} S_T) = \dot{N}(t).
$$

The change in the value of the total wealth might deviate from the value of the change in the real wealth, as the former includes capital gain, extending Hamilton and Ruta (2017, eq(11)).

3.5 Summing up

We now summarise our central results. As we discussed around equation (12-1) in the optimal RAM, we need complete, forward-looking shadow prices that includes not only the scarcity cost but the social cost of potential emission. The accounting rule for depletion using such shadow prices is:

Proposition. *Under RAM $\alpha =$ SW and HR, depletion is valued by*

$$p^{\mathrm{SW}}(t)\dot{S}(t) = -R_t \left(n_t\Omega^{\mathrm{SW}}(r_n(T-t)) - \tau_t\Omega^{\mathrm{SW}}(r_\tau(T-t))\right) \frac{R_t}{R_t - \phi(S_t - S_T)},$$
$$p^{\mathrm{HR}}(t)\dot{S}(t) = -R_t \left(\frac{n_t\Omega^{\mathrm{HR}}(r_n(T-t)) - \tau_t\Omega^{\mathrm{HR}}(r_\tau(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))}\right), \tag{41}$$

respectively, where $p^{\mathrm{SW}}(t)$ and $p^{\mathrm{HR}}(t)$ are respective appropriate shadow prices, $r_n := r + \phi - g_n$, $r_\tau := r + \phi - g$, and the initial resource extraction at t, R_t, needs to satisfy the resource extraction constraint (28). Specifically, when $\phi = 0$, the value of depletion can be expressed in a reduced form:

$$p^{\alpha}(t)\dot{S}(t) = -R_t \left(n_t\Omega^{\alpha}(r_n(T-t)) - \tau_t\Omega^{\alpha}(r_\tau(T-t))\right) \quad \text{for all } \alpha. \tag{42}$$

Corollary. *The two shadow prices are never equal, due to $\Omega^{\mathrm{SW}} < \Omega^{\mathrm{HR}}$ in (26), even when $\phi = 0$ or for zero discounting. In particular, we can establish that $p^{\mathrm{SW}}(t) < p^{\mathrm{HR}}(t)$ when $\phi = 0$ and $\tau_t = 0$.*

In addition, the presence of the unburnable stock S_T affects only the SW shadow price, p^{SW}. S_T does not affect the HR weighted average shadow price, p^{HR}, as it only affects R.

4 Comparing different approaches in wealth accounting: ANS, CWON, IWR, and our RAMs

Our formulation enables previous studies to be expressed as variants, facilitating the identification of structural differences in their sustainability assessments at a theoretical level.

The previous studies that appear in Table 1 do not assume rent growth, SCC, and unburnable stock. Only Hamilton (2016) suggests declining extraction in his model.

RAM previous studies	Parameter settings (Common to all: $g_n = g = \tau = S_T = 0$)	Shadow price to value depletion
SW (user cost pricing)		
El Serafy (1989), Arrow et al. (2003), Wei (2015), CWON (2006–)	$\phi = 0$	$p^{\mathrm{SW}} = n_t \Omega^{\mathrm{SW}}(r(T-t)) = n_t e^{-r(T-t)}$ where $T - t = S_t/R_t$
HR (weighted average pricing)		
Hamilton and Ruta (2009), ANS (1999–)	$\phi = 0$	$p^{\mathrm{HR}} = n_t \Omega^{\mathrm{HR}}(r(T-t)) = n_t \frac{1-e^{-r(T-t)}}{r(T-t)}$
Hamilton (2016), SEEA	$0 < \phi < 1$	$p^{\mathrm{HR}} = n_t \frac{\Omega^{\mathrm{HR}}((r+\phi)(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))} = n_t \frac{1-e^{-(r+\phi)(T-t)}}{r+\phi} \frac{\phi}{1-e^{-\phi(T-t)}}$
Optimal (efficient pricing)		
Arrow et al. (2012), IWR	$\phi = 0$	$n_t \Omega^{\alpha}(0) = n_t$

Table 1: Valuing depletion in previous studies in the green national/wealth accounting literature in our framework

Note: p^{SW} and p^{HR} refer to El Serafy-Wei user cost shadow price (36) and Hamilton-Ruta (weighted) average shadow price (40), respectively. In α =SW, the remaining years of extraction is given by the reserve-production ratio, so that $T - t = S_t/R_t$. In α =HR, $T - t$ is given.

Using the shadow price defined as the marginal contribution to well-being of an additional stock (Dasgupta and Mäler, 2000), Arrow et al. (2003, Section 5.3) and Wei (2015) derive the shadow price—and recover the user cost (El Serafy, 1989)—of a non-renewable resource, assuming a RAM where an additional stock would extend the exhaustion date, so that $T - t$ is determined by S_t/R_t.

Before valuing depletion, the *Changing Wealth of Nations* (CWON) first measures the NPV of the discounted rents (30) with $\phi = g_n = \tau_t = 0$: $N(t) = \int_t^T n_t R_t e^{-r(z-t)} dz$ where T is given by $t + S_t/R_t$, and then takes their time difference to arrive at the value of depletion. This essentially gives us the value of depletion expressed by $\dot{N}(t) = -n_t R_t e^{-r(S_t/R_t)}$, which is a special case of $\alpha =$ SW depletion, (33). This is divergent from the approach adopted by Adjusted Net Savings (ANS) in the *World Development Indicators* by the World Bank, which now aligns with the recommendation of SEEA concerning average shadow pricing (i.e., $N(t)/S(t)$).

Hamilton (2016) goes further to applying the unit value of depletion defined by the SEEA Central Framework (United Nations et al., 2012)—average shadow pricing in our terminology—to sustainability analysis of declining extraction. He would have obtained shadow prices that look like the one in Table 1.[14]

In their applied work, Arrow et al. (2012) and their application in *Inclusive*

[14] It was not until SEEA Central Framework (United Nations et al., 2012) that a unique recommendation for valuing depletion has been proposed, partly because it was not clear how depletion should be valued in accounting, produced or non-produced, stocks or inventories (Edens, 2013).

Wealth Report (IWR) use the current rent of the resource under question. This can be interpreted as pricing by F_R under instantaneous efficiency regarding the arbitrage between using and holding the resource asset expressed in equation (7) with no externality (i.e., $\psi = 0$). Another potential interpretation would be that they are using SW or HR shadow prices with zero discount rate, $n_t\Omega^\alpha(0) = n_t$, although this would not be consistent with their assumptions of discount rates elsewhere in their study.

From (26), we have

$$\Omega^{\mathrm{SW}}(r(T-t)) < \Omega^{\mathrm{HR}}(r(T-t)) < \frac{\Omega^{\mathrm{HR}}((r+\phi)(T-t))}{\Omega^{\mathrm{HR}}(\phi(T-t))} < 1,$$

which implies that, under the common parameter setting of $g_n = g = \tau = S_T = 0$, the absolute values of depletion have the theoretical relationship

$$\text{CWON} < \text{ANS} < \text{SEEA} < \text{IWR} \tag{43}$$

for the same quantity of resource use.[15]

Hamilton and Ruta (2009) and Hamilton (2016) compare the change in real wealth, $p^{\mathrm{HR}}\dot{S}$, and the change in total wealth including capital gain, $\dot{N}$. As (37) indicates, this essentially translates into the comparison of the change in real wealth under p^{HR} and p^{SW}, respectively, under $\phi = \tau = 0$. In Figure 1, Hamilton and Ruta (2009, Table 1) and Hamilton (2016, Fig.1) are recovered as the relationship between the discount factors for $\alpha = \mathrm{HR}$ and SW for $r = 0, 2, 4\%$, as functions of $T - t$ on the horizontal axis. Figure 1 can be re-read as the ratio of the value of depletion between IWR ($\Omega^{\mathrm{SW}}(0) = 1$), ANS ($\Omega^{\mathrm{HR}}(r(T-t))$ and $\Omega^{\mathrm{SW}}(r(T-t)$.Their difference widens as the discount rate and/or the remaining lifetime expand.

How does the assumption of decarbonisation and SCC come into play here? First, $\phi(S_t - S_T) > 0$ in the SW shadow price (41) means that the clear relationship like (43) may not hold. Second, the HR shadow price is raised upward by decarbonisation weighting in the denominator in (41), other things being equal. The shadow price is a weighted average price in the middle of the path to net-zero emissions, as a nearer future rent and SCC are more important than a farther future rent and SCC. Third, obviously, the rent net of SCC means that shadow prices, or the value of depletion, would be smaller.

[15] Note that we have simplified the notation here for this stylised, theoretical relationship between CWON-like, ANS-like, SEEA-like, and IWR-like approaches. It does not ensure any empirical relationship either in these reports, as the underlying assumptions of these approaches are not equal.

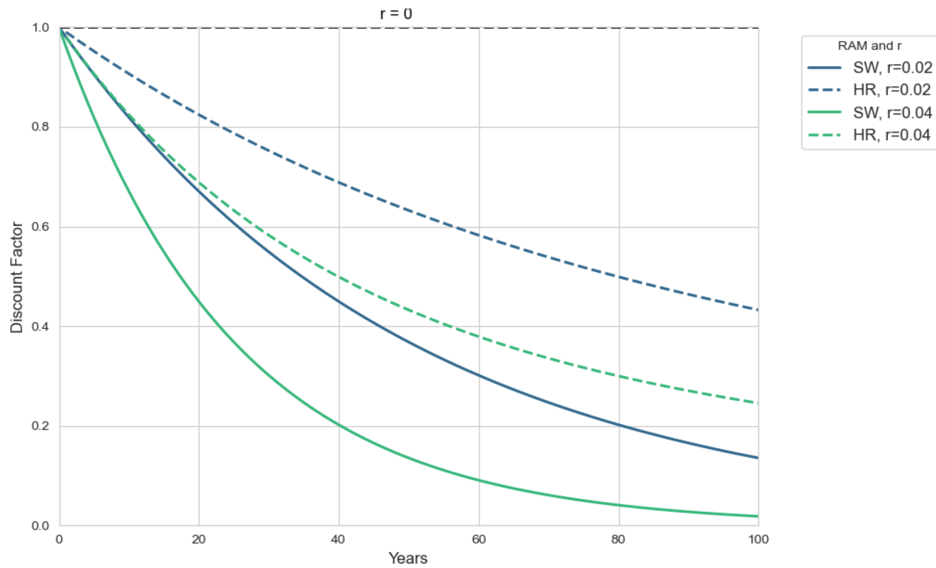

Figure 1: Discount factors $\Omega^{\mathrm{SW}}(r(T-t))$ and $\Omega^{\mathrm{HR}}(r(T-t))$ for $r = 0, 2, 4\%$

5 Empirical application to oil as non-renewable natural capital

Having characterised several realistic sub-optimal RAMs of constant and declining extraction, we are ready to apply them to the valuation of fossil fuels as non-renewable natural capital across nations. The central purpose of this exercise is to demonstrate how different RAMs result in different estimates of resource depletion.

5.1 Data and assumptions

We use oil as an illustration. It is straightforward to extend the exercise to other non-renewable natural capital, such as natural gas and coal. All the data are drawn from the Energy Institute *Statistical Review of World Energy 2023*. Current production is the recent five-year average of 2018–2022. Current price is also taken as the recent five-year average of 2018–2022, following the *Changing Wealth of Nations* (World Bank, 2018, 2021). For all the RAMs, resource rent is assumed to be constant until the terminal date (i.e., $g_n = 0$), following the existing literature, so the Hotelling rule does not hold. Regional rental rates are calculated using the average from 2013 to 2017 in the World Bank's *WDI* data, ranging from 0.125 in North America to 0.673 in the Middle East and North Africa. Appendix 2 justifies the use of these data in relation to the value of the resource and the user cost of produced capital.

We report only on seven regional aggregates (North America, South and Central America, Europe, CIS, Middle East, Africa, and Asia and Pacific) in the following subsection, but all the country data are available in the Appendix.

The consumption discount rate, r, is assumed to be constant at either 2% or 4%. The former is suggested as a base case in many recent studies (Drupp et al., 2018; Rennert et al., 2022), while the latter is standard in practical accounting (World Bank, 2018, 2021, 2024). The qualitative results do not change across these discount rates, so we focus on the 4% case in the following. The increasing rate of SCC is set at $g = 1.5\%$ per year. In the simplest analytical model of SCC, the growth rate of real SCC is equal to the growth rate of consumption without climate change damage (see footnote 11). Since the consumption discount rate can be decomposed to $r = \delta + \eta g > g$ by the Ramsey formula, these assumptions are internally consistent as long as the elasticity of marginal utility $\eta \geq 1$.

We also assume that countries will decrease their resource extraction at the rate of $\phi = 0.1\%$. Given the current production R_t and the effective reserve $S_t - S_T$, which is assumed to be 90% of S_t (i.e., 10% of oil reserve should be left unburnt), this will give us the remaining life years of $T - t$ according to (29). These assumptions, as well as the regional current data, are summarised in Table 2.

Region \ unit	R_t million t	S_t billion t	S_T billion t	$(T-t)^*$ years	$T-t$ years	rental rate	rent USD/t
North America	1,082	36.1	3.6	33	31	0.125	70
S.& Cent. America	321	50.8	5.1	159	154	0.417	234
Europe	160	1.8	0.2	11	10	0.378	212
CIS	691	19.9	2.0	29	26	0.378	212
Middle East	1,389	113.2	11.3	81	76	0.673	377
Africa	360	16.6	1.7	46	42	0.673	377
Asia Pacific	354	6.1	0.6	17	16	0.464	260

constant discount rate $r = 0.04$
decarbonisation rate $\phi = 0.001$
SCC growth rate $g = 0.015$
unburnt rate $S_T = 0.1 S_t$
$\phi = 0$ and $S_T = S_t$ assumed for the no decarbonisation case $(T-t)^*$

Note: Production (R_t), reserve (S_t), and price data are taken from the Energy Institute *Statistical Review of World Energy 2023*.

Table 2: Summary of current data and assumptions

Figure 2: Value of total resource stock (unit: USD trillion) under the constant discount rate of 4%

5.2 Value of total resource stock

Figure 2 shows the estimated total values of oil as a non-renewable natural capital stock in seven regions, according to (30) or (31), for the cases of no decarbonisation ($\phi = 0$ and $S_T = 0$) and decarbonisation with initial GSCCs being USD 0/tCO_2, USD 50/tCO_2, and USD 100/tCO_2, respectively. The no decarbonisation case corresponds to the NPV of rents, as customarily computed in wealth accounting. Figure 2 illustrates that the initial SCC of USD 50/tCO_2 would translate into negative total value for North, South and Central Americas, while for other regions it looks like the break-even initial SCC falls somewhere between USD 50/tCO_2 and USD 100/tCO_2.

5.3 Value of depletion

Figure 3 illustrates the value of depletion, which differs across different RAMs and initial SCCs. Each region has eight different cases, four for SW pricing and another four for HR pricing.

Moving from the first bar of the SW pricing with no decarbonisation ($\phi = S_T = \tau = 0$) to the second bar of decarbonisation but no SCC ($\phi = 0.5\%$,

$S_T = 0.1S_t$, and $\tau = 0$), there is only a moderate decline in the absolute value of depletion. Setting the initial SCC of USD 50/tCO_2 in the third bar yields varied implications: for Europe, Africa, and Asia Pacific, this means even more moderate value of depletion as the opportunity cost would be lower, while for the other four regions (North America, South and Central America, Europe, CIS, and Middle East) this changes the sign of the value of depletion. The fourth bar, which marks the initial SCC of USD 100/tCO_2, shows positive values of depletion for all regions. Under the SW pricing, the exhaustion price is used, and by the time of exhaustion, the SCC is much higher than the initial SCC.

The fifth and sixth pairs of bars represent the values of depletion under the HR pricing with no decarbonisation ($\phi = S_T = \tau = 0$) and decarbonisation with no SCC ($\phi = 0.5\%$, $S_T = 0.1S_t$, and $\tau = 0$), respectively. Again, there is a slight decrease in the absolute value of depletion, from the fifth to the sixth. For no decarbonisation and SCC=0, the absolute values of depletion under the HR pricing are larger than those under the SW pricing. This is expected from our Corollary: $p^{\mathrm{SW}}(t) < p^{\mathrm{HR}}(t)$ when $\phi = 0$ and $\tau_t = 0$ due to (26), meaning that the rent at the year of exhaustion is smaller than the period-average rent in the absence of SCC.

Finally, the seventh and eighth bars show the values of depletion in the HR pricing for the initial SCCs of USD 50/tCO_2 and USD 100/tCO_2, respectively. The HR value of depletion is negative for all regions except the Americas, CIS and the Middle East, which experience positive depletion values under SW pricing. This is because, under HR pricing, the weighted average price is used, so that the high SCC at the time of exhaustion is moderated compared to the SW pricing.

Once again, the break-even initial SCC that would result in a depletion value of zero appears to be somewhere between USD 50/tCO_2 and USD 100/tCO_2 for Europe, Africa and the Asia-Pacific region, regardless of the underlying RAM. Notably, for the CIS and the Middle East, the break-even SCC is higher than USD 50/tCO_2 under SW shadow pricing but lower than USD 50/tCO_2 under HR shadow pricing. For the Americas, the break-even SCC is lower than USD 50/tCO_2 under any RAM, partly due to the relatively low estimated rental rate of 12.5%. However, given that recent estimates of SCCs are much higher than this level (Rennert et al., 2022), the results cast doubt on the social acceptability of using fossil fuels in the Americas.

Figure 4 shows the corresponding depletion value for the capped resource lifetime. Under the 30-year cap, the discount factors are larger for both SW and HR shadow prices, enlarging the absolute values of depletion. In the Middle

Figure 3: Value of depletion of oil (unit: USD billion) under the constant discount rate of 4%

East, the value of depletion is now negative under the HR shadow pricing with the initial SCC of USD $50/tCO_2$, reflecting the nearer future SCC.

Figure 4: Value of depletion of oil under the constant discount rate of 4% (upper panel) and with the cap on the life years of 30 years (bottom panel) (unit: USD billion)

6 Conclusion

While fossil fuel assets continue to be an empirically significant component of national wealth for a number of countries, connecting this asset value—in the theory and practice of wealth accounting—to concerns about the climate change liability that is intertwined with resource depletion and use is an important challenge.

We have shown generalised shadow prices of depletion using discount factors for user-cost and average-price resource allocation mechanisms (RAMs). Moreover, the crucial element of this social value is the social cost of carbon (SCC), as it changes the way RAMs determine the value of subsoil assets and their depletion. Comparing with optimal economies, we have shown that relevant RAMs regarding rent, SCC, and decarbonisation pathways yield different shadow prices and values of subsoil assets and their depletion.

Our analysis suggests several important changes to wealth accounting in practice. First, the wealth of fossil fuel-rich countries should be revised on the conservative side if we expect (partial) internalization of SCC in the real world. Second, the change in wealth of these nations should also be revised in a manner consistent with their stock estimates. Paradoxical as it may seem, this may imply an improvement in genuine savings flows if carbon damages are adequately included in stock estimates. This reflects that the opportunity cost of current depletion is lower if the rising SCC is internalised. This revision would also treat terrestrial and atmospheric carbon in a consistent way. Third, from a methodological standpoint, accounting for non-renewable natural capital should be taken seriously, probably with more RAMs or scenarios, having UN SEEA's (United Nations et al., 2014) recommendations in mind. Declining extraction, which we addressed here, is a needed and actually recommended extension by SEEA.

Looking forward, our treatment of this issue might be updated and extended in ways that endogenise key parameters. First, the SCC could be calibrated consistently with the discount rate, rather than with two exogenously given parameters. Second, the SCC could also be endogenised to respond to the extraction pathways of the RAM in question. That said, this may well require complex modelling of the strategic behaviour of fossil fuel-producing countries. Third, the carbon content of specific oil wells is not considered in the current analysis.

Of course, in bottom-line sustainability assessments, these values of depletion are combined with the value of the change in other capital, as well as with the value of carbon emissions. In the final analysis, the SCC adjustment of our depletion accounting might be offset to a certain extent. The degree of the cancellation is varied, depending on how much of national depletion is assigned to domestic consumption as opposed to exports. They are empirically important and should be our next research agenda.

A Related literature

A number of existing studies have focused on estimating genuine saving (or adjusted net saving) and integrating the costs of carbon emissions into these metrics of how the (real) value of wealth is changing over time. More recently, other studies show the exact forward-looking terms that need to be incorporated to capture the mitigation and adaptation costs of carbon emissions of self and other countries in the future, in addition to the social cost of carbon from current carbon emissions (Asheim and Yamaguchi, 2026, 2025). Practical measurement has tended to focus on one of two approaches, both using the SCC, but differing in terms of whether this is allocated (i.e. debited) to the emitting country responsible for the social cost or to the affected country where climate damage takes place.

An exemplar of the former is World Bank (various) which subtracts the social cost of carbon damage from the genuine savings (i.e., adjusted net savings; the change in real wealth) of emitting countries. This follows the approach of Hamilton and Atkinson (1996) and Hamilton (2012) where this debit can be interpreted as the notional liability attributable to the polluting country. Other studies examine the sustainability implications of such payments by emitting countries, showing the importance of timing of receipt and payment of compensation to affected countries, given capital gains on these payments. By contrast, Arrow et al. (2012) and UNU-IHDP and UNEP take a different approach in deducting (future) climate change damage in a country arising from that year's global emissions of carbon dioxide.

In principle, both approaches are plausible adjustments to genuine savings. But each relies on different institutional assumptions on the extent to which the SCC is internalised in real markets (Arrow et al., 2003; Fenichel and Abbott, 2014; Fenichel et al., 2018). Given the reality that actual economies only partially internalise the SCC, perhaps a presumption of realism (rather than normative wishful thinking) suggests the damage approach is the most appropriate. That said, subsequent to the Paris Agreement, there is increasing pressure to internalise carbon in the price of fossil fuel resources, albeit to an uncertain extent. In practice, therefore, both approaches provide useful information when viewed in context.

The essence of these contributions is to estimate how carbon emissions lead to changing (global) wealth, and to attribute this to specific national economies. As mentioned, such work has been important, therefore, in assessments of how and whether these economies currently are saving enough for the future. Fewer studies have also sought to reflect such insights additionally in national balance sheets.

However, in an interesting application of user-cost shadow pricing, Barbier and Burgess (2017) value the global carbon budget, in essence, by treating this as non-renewable natural capital and accounting for its 'depletion'. In doing so, they use an El-Serafy-like user cost; i.e., valuing the last unit of the carbon budget. The authors stress that this user-cost approach differs from the social cost of carbon. Our paper can be seen as a reconciliation between the scarcity cost and the damage cost of carbon in wealth accounting, since we include both resource rent and the social cost of carbon in accounting for the total value of fossil fuels.

Most prominently perhaps, a number of studies examine wealth accounting for the carbon storage services provided by biotic or renewable natural capital, such as soil, trees and forests, and oceans (World Bank, 2021; Atkinson and Gundimeda, 2006). Plausible answers, in this respect, rest on competing notions of baseline and counterfactuals, compared to which we can account for prospective gains in the future. There are also concerns about potential double-counting, since the benefit of avoided emissions might be captured in the value of produced and human capital, and so already be measured in estimates of total wealth. This problem would be more pronounced if carbon emissions into the atmosphere were recorded twice as a depletion of terrestrial carbon sink and degradation of the atmospheric environment.

B Value of the mine and the value of the resource

Given that the profit from the resource is a joint product of non-renewable natural capital and produced capital, Cairns (2019) demonstrates that the value of the resource is determined by deducting the value of the relevant produced capital from the NPV of the profit stream. Once the investment decision has been made, the extraction volume is capped, making SW shadow pricing plausible, whereas HR shadow pricing would be more appropriate for long-term decision-making prior to the initial investment cost being incurred. Therefore, the true value of the resource with uncommitted investment should deduct the value of the relevant produced capital instead of the mine.

Denoting the unit profit by π and the value of produced capital by $\Phi(K)$, the value of the resource is written as

$$v(N(t)) = \int_t^T \pi(z)R(z)e^{-r(z-t)}dz - \Phi(K(t)). \tag{44}$$

Cairns (2019) also argues that $v(N(t))$ is the discounted value of user costs attributed to the resource, while $\Phi(I(t))$ is the discounted value of user costs attributed to the produced capital. This means that the value of the resource now reads

$$v(N(t)) = \int_t^T \pi(z)R(z)e^{-r(z-t)}dz - \int_t^T \xi(z)K(z)e^{-r(z-t)}dz, \tag{45}$$

where $\xi(t) = \delta + \Xi - \dot{\lambda}_K/\lambda_K$ is the user cost or shadow rental on produced capital (Dasgupta, 2009) and Ξ is the depreciation rate of produced capital. $\delta - \dot{\lambda}_K/\lambda_K$ can be taken as the normal returns to produced capital, as it is equal to F_K in a first-order condition (8) in our optimal model.

This implies in principle that estimating the shadow rental on produced capital associated with the extraction R in each period would render $v(N(t))$ equal to $N(t)$. Empirical application therefore needs to carefully use the resource rent $n(z)R(z)$ that can approximate $\pi(z)R(z) - \xi(z)K(z)$ for $t \leq z \leq T$.

The World Bank has made progress in including user costs attached to produced capital in their resource rent calculation. The latest edition of CWON (World Bank, 2024) estimates capital stocks in the fossil fuel sector, which are multiplied by their rates of return plus depreciation rates to obtain the user costs. World Bank (2024, p.100) finds that "[g]lobally, the inclusion of user costs increased rents for oil, gas, and metals and minerals" but this effect "varies across regions and may reflect differing levels of capital expenditure and industrial investment." Our use of the World Bank estimates of rents in our application does not fully reflect their latest update, but constitutes a step in the right direction as the Bank's effort continues.

C Sensitivity analysis

Below in Figures 5 and 6 we show some sensitivity analysis of the values of stock and depletion regarding the discount rate of 2% as opposed to 4%.

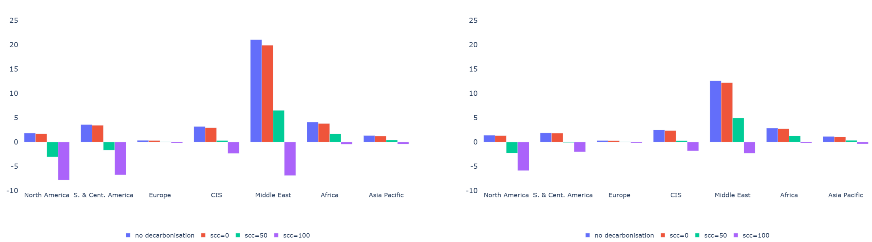

Figure 5: Value of total resource stock (unit: USD trillion) under the constant discount rates of 2% (left panel) and 4% (right panel)

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Figure 6: Value of depletion of oil (unit: USD billion) under the constant discount rates of 2% (upper panel) and 4% (bottom panel)