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Mean-Spirited Growth

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*We contribute to the Solow-Swan strain of economic growth literature by integrating national distributional aspects into a dynamic model of economic growth. We show that country specific optimal growth policies balance distributional "preferences" against savings propensities of different percentiles of the emerging income distribution. We then present international comparisons of growth decomposition into distributional statistics and propose new measures of economic performance. Historical economic performance reveals heterogeneous patterns of inequality increasing (e.g. US, UK, Germany, China) and decreasing (e.g. Ireland, France, Netherlands, Vietnam) economic growth around the world. These comparisons reveal the implicit efficiency-equity trade-offs in each country's set of national economic growth and social policies.**

Keywords: Inequality, economic growth, distribution, the elasticity of marginal utility, preferences.

JEL codes: D63, O43, P17, P27

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1 Introduction

Over the last 80 years, per-capita gross domestic product (GDP) growth has become the go-to measure of economic "success" at the national level. As a result, National Accounts dutifully report this measure of economic performance, we compare countries on this basis, and even allocate development assistance by this measure.[1] Yet, per-capita GDP growth, $\dot{G}c$, is increasingly viewed as a poor measure of growth in wellbeing. Its failure to reflect future well-beings is well demonstrated and understood, it being a flow measure that ignores important aspects of the stock of wealth (Weitzman, 1976; Hamilton and Hepburn, 2017). But even in its own right as a measure of economic flow, GDP per capita ignores how that flow is distributed. This omission has also received considerable attention in light of concern about changing levels of economic inequality witnessed in recent history (Stiglitz, Sen, and Fitoussi, 2010; Piketty, 2015; Piketty, Yang, and Zucman, 2017).

In this paper we demonstrate how a simple adaption of the standard model of neoclassical growth to include societal distributional objectives affects the optimal growth path. A country's optimal growth path then depends on societal inequality aversion. In turn, an economy's performance must be gauged against a measure that embodies the dynamic effects of growth on the distribution on income and aversion to inequality. This measure, we argue, could be used alongside the usual metrics to compare country's growth paths and their individual performances over time.

Unsurprisingly, we demonstrate considerable cross-country variation in our inequality adjusted growth measure, reflecting how different countries have taken different growth development paths alongside which inequality has evolved. Rawl's (1971) conception – institutions, and concept – the outcome of justice, suggest different places have different distributive institutions and hence outcomes. Critically, for non-symmetric distributions – as income and consumption tend to be – mean, median and modal measures of centre differ. $\dot{G}c$ is, roughly, a measure of change in the mean of the income distribution and does not report on changes in the median and mode of a non-symmetric distribution. It provides no information about the distribution nor does any representative agent sort of conception based on it or any other single datapoint. It is even possible that measures of $\dot{G}c$ can be positive without a change

[1] Whether evaluating countries within the purview of the European Bank for Reconstruction and Development (EBRD), Heavily Indebted Poor Countries (HIPC), or New Partnership for Africa's Development (NEPAD) initiatives, one finds per-capita GDP growth an important metric of success. It serves as such explicitly but also signals to non-governmental organisations where to direct limited resources. Sufficient growth to nudge a country from developing, to in-transition, to developed-economy status may result in the loss of substantial aid.

in consumption for much of the population – statistical prosperity without a real correspondence. The possibility of a disconnect between growth statistics and shared prosperity has, naturally, resulted in increasing calls to move beyond $\dot{G}c$ (e.g., Stiglitz, Sen, and Fitoussi, 2010).

Growth theory has, however, focussed on the mean because it is indispensable – it makes complex questions about society and welfare tractable. In terms of growth and wellbeing, it is then generally assumed that improvements for the representative, generally mean agent imply improvements for the population it represents. In national statistics, this has always been proxied for by per-capita mean income. Within the realm of theoretical exploration where this assumption is explicit, or the limitations imposed by it so well known that it is practically explicit, the limited representativeness of $\dot{G}c$ is clear. However, the practice of placing great importance on $\dot{G}c$ when its representative agent origin is not explicit, for instance in national accounts and development policy, has led to discontent with and debate about the usefulness of $\dot{G}c$.

Because the representative agent assumption is explicit, rising inequality does not contradict the Solow-Swan factor decomposition strain of growth literature (Solow, 1956) as this literature does not say anything at all about the matter. In effect though, the representative agent is too simple – it represents a uniform population.[2] One is tempted to appeal to the Kaldor-Hicks compensation criteria to suggest the representative agent method is sufficient. For instance, Mankiw, Romer, and Weil (1992) suggest their form of factor decomposition is descriptive of "international variation in the standard of living." Yet the non-normal, non-symmetric nature of income implies it is only descriptive of variation in the mean agent's standard of living and not that of the other $N-1$ members of a potentially diverse, N size population. Another way to view this matter is that the factor decomposition strain of economic growth makes no claim about the ownership of factors of production, yet policy based on it presumably assumes a desirable dispersal of returns.

The distributional dimension means that a level of $\dot{G}c$ can be achieved in different ways. The lognormal distribution is particularly useful in visualising this. It has tractable analytical properties too and generally fits income and consumption data well. It is well known that the mean, median, and mode of it take forms $\bar{c} = exp(\mu + \frac{1}{2}\sigma^2)$, $\tilde{c} = exp(\mu)$, and $c^{Mo} = exp(\mu - \sigma^2)$. Extending from Emmerling, Groom, and Wettingfeld (2017), the respective growth rates

[2] Alternately, it can be represented by a degenerate distribution.

are $\bar{g}_t = \frac{1}{t}\Delta\mu_{0,t} + \frac{1}{t}\frac{1}{2}\Delta\sigma^2_{0,t}$, $\tilde{g}_t = \frac{1}{t}\Delta\mu_{0,t}$ and $g^{Mo}_t = \frac{1}{t}\Delta\mu_{0,t} - \frac{1}{t}\Delta\sigma^2_{0,t}$ where $\Delta\mu_{0,t} = \mu_t - \mu_0$ and $\Delta\sigma^2_{0,t} = \sigma^2_t - \sigma^2_0$. Here, we observe it is possible that increases in σ^2 can drive growth in mean income, increasing inequality. But inequality-driven growth also implies that for many workers – those earning and consuming around the mode – the economic outlook worsens. The most common, modal, experience would then conflict with national statistics that indicate "economic growth."[3] Next, an empirical exercise suggests the importance of the distributional complication.

2 An Empirical Exploration

The factor decomposition literature suggests that levels of $\dot{Gc}$ can be achieved in different ways – combinations of capital, labour, productivity, etc. Using the lognormal assumption, levels of $\dot{Gc}$ can also be achieved through combinations of growth in μ and σ^2 which indicate how distribution is structured in an economy. As with factors of production, policies and institutions can facilitate growth in either one or both distributional dimensions. Growth in μ and σ^2 highlight the trade-off between growth in the median and the right tail of the distribution, respectively, for overall per-capita growth. We perform growth distributional decomposition on some internationally comparable datasets containing mean and median income data which suggest how different distributional policies impact growth.

We utilise Luxembourg Income Survey (LIS), Organisation for Economic Co-operation and Development (OECD), World Bank, and World Inequality Database (WID) datasets to derive growth paths that countries have taken. The LIS and OECD provide comparable estimates based on extensive, representative household surveys. The World Bank dataset –also survey-based – includes data on income and consumption for a more diverse set of countries. The WID data instead includes percentile incomes across the distribution for a few countries using a shared methodology. For each country, we assume a lognormal distribution relationship, and then decompose the median to estimate μ followed by the mean to estimate residual σ^2. Figure 1 presents the LIS and OECD data decomposition into the share of economic growth contributable to median (μ-driven) and inequality (σ^2-driven) growth.[4] We can observe that despite using similar household-scale measures, the OECD data generally suggests less

[3] The mean-modal spread also exceeds the mean-median gap sometimes proposed as a measure of inequality suggesting the latter is insufficient in capturing the issue.

[4] The underlying data is presented in Table 27 of appendix *A5.1 Data Tables Supporting Figure 36 and Figure 37.*

unequal growth. More importantly, differences within each dataset are apparent. Nations with similar mean growth rates achieve it by different distributional processes. Notable inequality-increasing outliers are the United States, Germany, and Canada over the last 30 to 40 years. In contrast, Ireland, France, the Netherlands, and Hungary experienced growth in combination with reductions in inequality.[5] Of course, growth and distributional statistics cannot identify all sources of unrest – discontent over immigration and essential commodity and fuel prices (in Hungary, France) can lead to turmoil even when national statistics suggest improvements in general wellbeing.

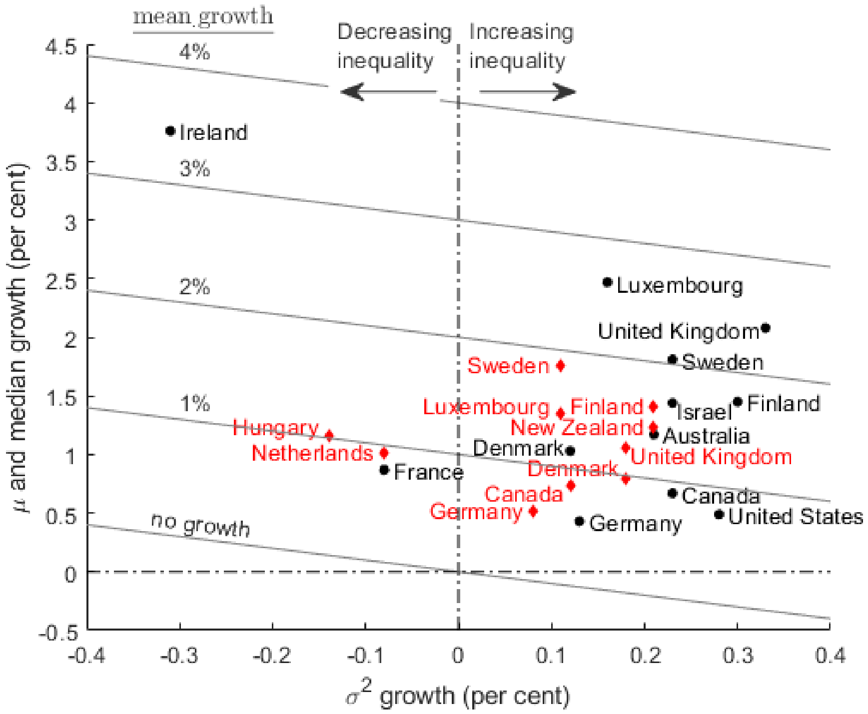


FIGURE 1. LIS AND OECD DATA ANNUAL GROWTH RATE DECOMPOSITION

Notes: Luxembourg Income Study (LIS) estimates (black dot markers) based on PPP-adjusted equivalised disposable household income from surveys conducted between 1979 and 2013. OECD estimates (red diamond markers) based on CPI-adjusted household income in national currencies from surveys conducted between 1975 and 2017. OLS estimates of annual growth rates in μ, σ^2, and mean growth for countries listed in Table 4 in appendix *A1 Data Tables Supporting Figure 1 and Figure 2.*

Source: Author calculations based on data from OECD (2019), and Thewissen, Nolan, and Roser (2016) which is derived from the LIS Database.

Within the preceding LIS and OECD datasets, all countries experienced $\dot{G}c$ on average along various distributional trajectories. Decomposition of the World Bank dataset in Figure 2 provides a more varied picture as it describes nations with more varied institutions, endowments, and states of development.[6] We might qualitatively make some observations.

[5] Also noted, for instance, in Garbinti, Goupille-Lebret, and Piketty (2018) in the case of France.

[6] The underlying data for Figure 37 is presented in Table 28 in appendix *A5.1 Data Tables Supporting Figure 36 and Figure 37.*

Along the rightmost edge of the data mass signifying greater inequality-driven growth, we find the United States, Germany, Bangladesh, urban India, and South Africa. These countries are known for increasing concern over inequality, but generally stable political regimes over the period. On the other extreme edge signifying decreasing inequality-driven growth, we find Mexico and several Central and South American countries. These are known for stronger communal traditions but also political stability over the period. We might suggest that while distribution is but one factor in economic "success", too unequal growth in either direction leads to social disquiet. We observe that, at least mechanically, there is an equity-efficiency trade-off that occurs. We can also observe that negative growth is not experienced equally as in the case of Croatia – that growth by itself is not a driver of inequality.

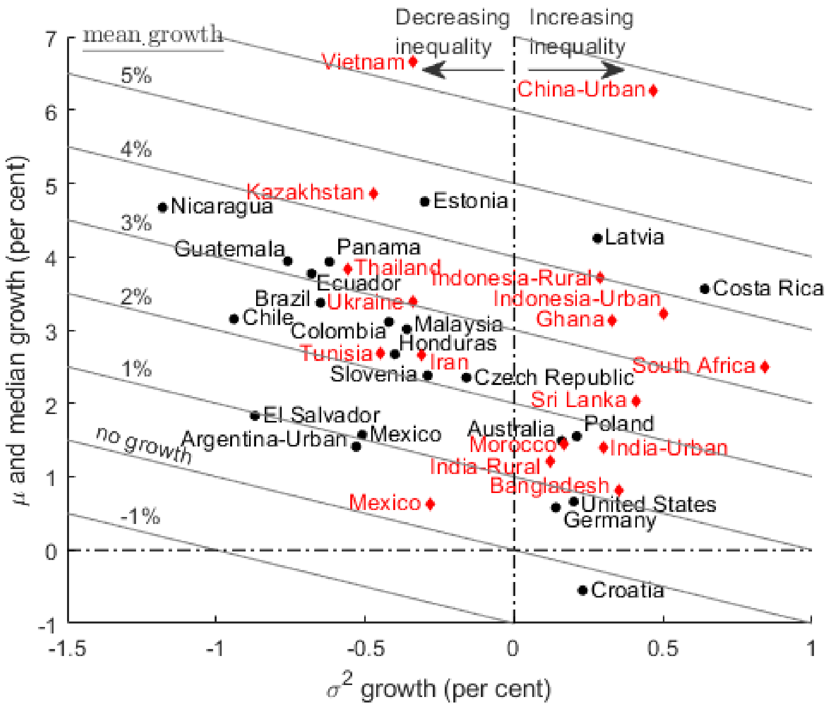


FIGURE 2. WORLD BANK DATA ANNUAL GROWTH RATE DECOMPOSITION

Notes: World Bank income-based survey data (black dot markers) and consumption-based data (red diamond markers). OLS estimates of annual growth rates in μ, σ^2, and mean growth for countries listed in Table 5 of appendix *A1 Data Tables Supporting Figure 1 and Figure 2*. Estimates based on PPP-adjusted household per-capita income and consumption expenditures.

Sources: Author calculations based on data from World Bank (2019).

National policies are not necessarily designed to share the returns to production in any socially preferred way. It is difficult to make the case that socially destructive levels of inequality are actually in the public's interest. However, when national policies do embody "preferences", different trajectories in μ and σ^2 also result. Observed growth paths embody the political costs of growth, distribution and historical ownership of policy. Rich histories have, perhaps, led to traditions of sharing the returns to economic growth in each country that we would generally lump together as "preferences." Growth in μ versus σ^2 is one salient result. However, how does an economy move from preferences over sharing to embarking on a growth path? A review of the literature on growth, distribution, and measuring inequality precedes a dynamic growth model that incorporates distribution.

3 Literature Review

An entire academic industry discusses inequality and growth and produces supplemental statistics to $\dot{G}c$. These generally do not acknowledge the representative agent source of the issue but do tend to explore the non-symmetric distributional departure of measures of centre. Concern for inequality in general is then followed by proposals for alternative measures of performance and the construction of alternative measures of performance. Yet, these measures do not typically recognise the central role of societal preferences for equity and fairness in either determining the existing distribution, or identifying what trajectory of growth and distribution that is desirable in a social welfare sense.

For instance, one typical statistic for inequality is the spread between per-capita (mean) income and the median, with $median < mean$ indicating inequality (Barro 2000). Measuring the spread of means and medians is important where inequality is concerned because an economic policy that focusses on one or the other – the mean versus median debate, still neglects the distributional issue.[7] Yet the spread, or growth in the spread, is merely descriptive and ignores notions of fairness, institutional constraints, or other "social preferences" over distribution.

Another popular distributional measure is the Gini coefficient (Gini, 1912; Dalton, 1920) in the form summed over the Lorenz curve (Lorenz, 1905). Despite its popularity, Atkinson (1970) discusses why it is insufficient for the task. Among the criticisms, it too neglects social preferences, and we append that it only provides a static snapshot. In the context of economic growth, a dynamic measure is more relevant.

Another approach has been to consider the distributional implications of economic growth, sometimes quite formally, without arriving at a statistic. The implication is that either more or less inequality is beneficial, not that a certain amount of inequality is preferred. Early endeavours of this variety include Kuznets (1955, 1973), Stiglitz (1969), Solow (1974), and

[7] Note, however, the median has an important distributional quality that it is robust to increases in the tail.

Leontief (1983). Despite highlighting the important role of different savings rates along the income distribution, these works fail to provide a functional statistic that reflects social preferences and the dynamics of growth.

Advancements in the study of growth and inequality have also progressed alongside general methodological improvements. Van der Ploeg (1983) provides a fascinating implementation of a predator-prey model to employment and growth. Benhabib and Rustichini (1996) offer a game-theoretic, political economy model of appropriation among interest groups. However, as with their predecessors, these were academic not practical endeavours.

Others have instead focused on poverty reduction resulting in more actionable poverty indices. The most introductory of these is a measure of the share of the population living below some poverty line – the headcount ratio. The poverty gap index, Watts index (Watts, 1964; Zheng, 1993), Sen-Shorrocks-Thon index (Sen, 1976; Shorrocks, 1995), and Foster-Greer-Thorbecke indices (Foster, Greer, and Thorbecke, 1984) all improve on the headcount notion by incorporating the intensity of poverty. Other measures such as the Multidimensional Poverty Index (MPI) take a more general perspective on wellbeing as advocated for by Sen (1999). The MPI – for better or worse – summarises progress in the Human Development Index through a subjective weighting scheme to arrive at a measure of welfare (UNDP, 2010; Alkire and Jahan, 2018). One cannot seriously argue that mass poverty is ever socially optimal, and these indices share the merit of focussing the discussion on the most severe of distributional shortcomings. This study does not seek to supplant such measures. Rather, we focus on the where the issue is not extreme, pervasive poverty, but rather general discontent with the distribution of growth?

Interest in differences in inequality and growth have resulted in several empirical explorations in recent decades. International comparisons have been particularly fashionable such as by Barro (1991, 2000). These generally link economic growth to measures of inequality such as the Gini coefficient to arrive at some conclusion on whether inequality supports or hinders growth. See overviews in, for example, Shin (2012), Cingano (2014), and Grigoli and Robles (2017). This body of research has been less informative in total, however, as some find inequality positively, and others negatively, related to economic growth.

Persson and Tabellini (1994), like others, find that income inequality is harmful to growth. However, they attempt a plausible, non-savings-based explanation linking income inequality to policies that do not protect property rights and thus do not protect the private returns to investment. Halter, Oechslin, and Zweimüller (2014) add a time dimension – inequality increases economic growth in the short run but reduces it in the long term, perhaps linked to

Persson and Tabellini's institutional argument. Banerjee and Duflo (2003) instead suggest a more complicated relationship – that changes in inequality in either direction reduce growth. Then in a manner not unlike the argument that higher earners save more, Jones (2019) suggests a model where those with the highest incomes drive innovation. Finally, Jorgenson (2018) reiterates the many arguments to date that we should measure welfare rather than income because it captures notions of inequality, poverty, and distribution that income cannot.

Yellen (2016), Chetty et al. (2017), and Fried, Novan, and Peterman (2018) have also weighed in on the enduring challenge of inequality and economic policy. These have generally lamented the state of inequality rather than chart a path forward. Perhaps this is because attempts to simplify the distributional discussion have not always been well-received (Piketty 2015). The level of criticisms levelled at incorporating, or even discussing inequality and economic growth are striking given the discussed explicit limitations of the factor decomposition growth literature.

After reviewing the wealth of work, lack of consensus, and intensity of criticism, one may want to just jump to the punchline as in Stiglitz (2016) and prescribe a broad, generally common sense, but expensive set of remedies. Perhaps a more practical approach begins with an observation that factors at both the top and bottom of the income distribution impact economic growth. As such, no single statistic captures whether economic growth benefits a society (Voitchovsky 2005, 2009). Instead, practitioners should seek to complement $\dot{G}c$ with appropriate context on distribution and societal preferences in a policy-relevant manner.

Our goal is twofold. The first is to illustrate the importance of societal preferences in arriving at an optimal growth path for the entire distribution, not just mean income. The second is to arrive at a useful complementary statistic to $\dot{G}c$ that takes into account the distributional decomposition of $\dot{G}c$ illustrated in Figures 1 and 2, and embodies society's preferences over these components, such as the growth of mean of income and its spread. We begin by building on the Ramsey-Cass-Koopmans (RCK) model, which fuses the Ramsey rule for how much a society should save, with neoclassical growth theory (Ramsey 1928; Cass 1965; Koopmans 1963). Within our framework, aggregate savings are modelled to depend on income levels of individuals in society. Our central model calibrates this relationship to recent empirical work on the relationship between inequality, income and savings, which supports the premise that the wealthy save a larger share of income. U.S. tax data covering a century puts firm numbers in support of the belief that there are differences in savings rates (Saez and Zucman 2016). As a general matter of allocating scarce personal resources, we expect the trade-off between current consumption and savings to hold beyond the U.S. Closely related, research using a global poll of over 1.7 million people suggests there is some relatively consistent income satiation point where savings rates go from zero for much of the population, to positive for high earners (Jebb, Tay, Diener, and Oishi 2018). The stepped savings function is one example that makes the essential efficiency-equity trade-off clear: inequality increases savings increases growth. Alternative savings relationships are also investigated and discussed for robustness.

4 Incorporating Consumption Distribution into Growth Theory

We have made claims on how neoclassical growth theory and related indicator statistics fall short of policy need. We now put foundations from the literature to work by, in effect, making a single change to the Solow-Swan paradigm. While a more sophisticated approach to studying economic growth and inequality than using a representative agent is needed, it should transparently integrate economic distribution yet result in clear, concise policy recommendations. We build on recent advancements in Emmerling, Groom, and Wettingfeld (2017) and then incorporate a representative distribution, rather than a representative agent, into an efficient capital market assumption. The resulting model suggests how growth contributes to mean and all other consumption points through competing influences.

We assume the constant relative risk aversion (CRRA) utility form (Atkinson, 1970) which has particularly desirable analytical properties, and the lognormal form (McAlister, 1879) of representative consumption (income minus savings) distribution (Arrow et al. 2014; Pinkovskiy and Sala-i-Martin 2009; Battistin, Blundell, and Lewbel 2009).[8] We then implement the consumption side of the RCK model in a simplified form as objective:[9, 10]

(1) $$\max_{\{\mu(t),\sigma^2(t),t>0\}} W_0 = \int_{t=0}^{\infty} \left(\int_0^{C_{max}} u\big(c_i(t)\big)\, dc(t) \right) e^{-\rho t} dt,$$

where we abstract from uncertainty, note that $c_i(t) \sim LN(\mu(t), \sigma^2(t))$, and that for any agent, i:

[8] Limitations to the representative consumer approach beyond what have already been discussed can be found in Caselli and Ventura (2000).

[9] It may be helpful to note $\nabla W_0(\mu_0, \dots \mu_T, \sigma_0^2, \dots, \sigma_T^2) = \left(\frac{\partial W_0}{\partial \mu(0)}, \dots, \frac{\partial W_0}{\partial \mu(T)}, \frac{\partial W_0}{\partial \sigma^2(0)}, \dots, \frac{\partial W_0}{\partial \sigma^2(T)}\right)^{\mathsf{T}}$ where $\frac{\partial W_0}{\partial \mu(t)} = e^{-\rho t} \int_0^{C_{max}} u\big(c_i(t)\big) \left(\frac{ln\big(c_i(t)\big) - \mu(t)}{\sigma^2(t)}\right) dc(t)$ and $\frac{\partial W_0}{\partial \sigma^2(t)} = e^{-\rho t} \int_0^{C_{max}} u\big(c_i(t)\big) \left(\frac{\big(ln\big(c_i(t)\big) - \mu(t)\big)^2}{2\sigma^4(t)} - \frac{1}{2\sigma^2(t)}\right) dc(t)$ for $t = (0, \dots, T)$.

[10] We have abstracted from population growth which is traditionally included in the RCK model in this application.

(2) $$u(c_i(t)) = \begin{cases} c_i(t)^{1-\eta}(1-\eta)^{-1}, & if\ \eta > 0, \eta \neq 1 \\ \ln(c_i(t)), & if\ \eta = 1 \end{cases}$$

which incorporates the elasticity of marginal utility of consumption, η. That is, rather than concerning ourselves with the welfare function of a representative agent, we use the objective as in Emmerling, Groom, and Wettingfeld (2017) as the welfare over the distribution as a whole while incorporating preferences over distribution represented in η. Note that Emmerling et al. (2017, p79) show that this social welfare function over individuals in society can be *represented* by a single agent consuming Atkinsons's (1970) equally distributed equivalent income, c_{ede}. This can be thought of as the representative agent if so desired.[11]

However the welfare of society is viewed, the objective in this economy is to choose the distribution of income or "institutional structure" to maximise our objective subject, in essence, to η and the initial distribution. However, it is impossible to write a satisfactory function for something as ambiguous and far-reaching as the structures of society that distributes the returns from production. Instead, we abstract heroically from these factors and proxy for them by jumping to the structure's convenient distributional representation - values of μ and σ^2. Taking this approach focusses in on the question at hand, ensuring the essential connections between societal preferences over inequality, the income distribution and optimal growth, remain front and central.

The objective is subject to a system of differential equations describing developments in capital and consumption possibilities:

(3) $$\dot{k}(t) = \left[\int_0^{C_{max}} s\big(c_i(t)\big)dc(t)\right] - \delta k(t),$$

(4) $$\dot{\bar{c}}(t) = -\frac{u\prime(\bar{c}(t))}{\bar{c}(t)u\prime\prime(\bar{c}(t))}\bar{c}(t)\big(f'\big(k(t)\big) - \delta - \rho\big),$$

where $k(0) > 0$, the production function $f(k(t))$ is of simple Cobb-Douglas form, and the savings rate can be dependent on position in the distribution.[12]

[11] This follows from the definition of the equally distributed equivalent income: $u(c_{ede}) = \int_\theta u(c_i)\, dH(\theta)$, where $H(\theta)$ is the density function for consumption in society at a particular time.

[12] The per-capita Cobb-Douglas production function adopted is of form $f(k(t)) = Ak(t)^\alpha$ with total factor productivity multiplier A, and output elasticity of capital α. In practice it is fitted to initial period economic parameters.

Equation (4) is a modified Keynes-Ramsey Rule (Ramsey, 1928) which we can restate as $\dot{\bar{c}}(t) = \frac{\bar{c}(t)}{\eta}\left(f'\left(k(t)\right) - \delta - \rho\right)$ where the elasticity of marginal utility with respect to consumption at the mean is $\eta = -\frac{\bar{c}(t)u''(\bar{c}(t))}{u'(\bar{c}(t))}$. The implication is that we weight changes in consumption by an assumed common social preference containing the inverse marginal utility of consumption. This links the literature on inequality preferences to growth and can be contrasted against Negishi (1960). A convenient distributional assumption also links savings and growth as we specify the savings function to include the recent literature on unequal savings rates and satiation points as:

$$s(c_i(t), b) = \begin{Bmatrix} 0, & if\ c_i(t) < b \\ \psi(c_i(t)), & if\ c_i(t) \geq b \end{Bmatrix} \tag{5}$$

where b is some point of satiation – an ancillary condition on utility signifying whether necessary consumption is satisfied in each period before the agent saves for later consumption (e.g. continued consumption in retirement). The motivation for b stems from observations in Saez and Zucman (2016) and Jebb, Tay, Diener, and Oishi (2018), and it can be modelled either relative to the distribution or absolute The embedded function $\psi(\cdot)$ conceptually satisfies $\psi'(\cdot) > 0$ and at least initially $\psi''(\cdot) > 0$, but in practice requires less sophistication to get the expected results. In appendix *A3 Implications of Alternative Savings Functional Forms*, we explore different interpretations of this functional form – the implications of choosing a uniform savings rate as well as of setting b as absolute rather than relative. In principle, and motivated by the country level institutions in the background, a dystopian, or revolt constraint can also be specified – for instance that median consumption in any period cannot be lower than the initial poverty level. The rational is that if much of the population is pushed into abject poverty, one may expect a collapse of the sort of institutions that make economic activity predictable and modellable.

The operation and design of the model is purposefully simple. An "economy" arrives at an efficient outcome by balancing the influence of the concavity of the distribution-weighted utility function from consumption for all agents, against savings-driven growth supported by the upper tail.[13] The state equations $\dot{\bar{c}}(t)$, $\dot{k}(t)$, and the impact of η are in turn dependent on the distribution through savings and initial distributional conditions $\mu(0)$ and $\sigma^2(0) > 0$.[14]

5 A Simulation

In the model, "social preference" over income distribution has a pervasive influence on distribution and growth. Some have made ethical arguments why there might be a social preference at all, while others just assume that such preferences exist (Stern, 2007; Dasgupta, 2008; Tol, 2010). An alternative is to suppose that institutions supporting a set of economic growth paths are constrained by popular mandate. A distribution process then emerges that appears as if society has preferences over the degree of inequality. We can, at a minimum, interpret η as a summary parameter of complex institutional arrangements that underpin distribution in society – either way, an η is implied.

The appropriate value of the social preference indicator η is a matter of ongoing debate and methodological development. See, for example, Evans (2005), and Groom and Maddison (2019).[15] Emmerling, Groom, and Wettingfeld (2017) note that estimates of η vary from 0.4 to 4 depending on the context, and rarely fall outside the range $\eta = [1, 2.5]$. Notably, some argue for $\eta = 1, 2$ as holding special significance (Buchholz and Schumacher 2010; Dasgupta 2008; Groom and Maddison 2019; Stern 2007; and Tol 2010). $\eta = 1$ suggests a social preference for consumption maximisation – an emphasis on developing mean consumption potential, while larger values suggest greater inequality aversion. Some instead specifically advocate for $\eta = 2$ on the premise of emphasising balanced economic growth – an emphasis on median growth. We remain detached from the normative debate and rather explore the implications of inequality preferences on optimal economic distribution.

To form a concrete example of the impact of η, we start from initial conditions $\mu(0), \sigma^2(0)$ derived from U.S. national statistics on median and mean household income (U.S. Census Bureau, 2019) and calibrate parameters in the preceding model from popular sources.[16] We then perform repeated simulations to obtain preferences for 30-year growth paths in a μ and σ^2 space for select parameter values in $\eta = [0.5, 4]$. By the lognormal distributional assumption,

[13] It can be shown that for concave, symmetric utility forms that minimizing variance maximizes utility.

[14] These constraints can be operationalized as the 2T vector $(\mu(0), \dots, \mu(T), \sigma^2(0), \dots, \sigma^2(T))^{\intercal}$ where $\mu(t) = \ln(\bar{c}(t)) - \frac{1}{2}\sigma^2(t)$ and $\sigma^2(t) = 2(\ln(\bar{c}(t)) - \mu(t))$ for $t = (0, \dots, T)$ derived from the mean of the lognormal distribution.

[15] η, the elasticity of marginal utility with respect to consumption, is interpretable as social inequality aversion among other uses.

[16] Penn World Table 9.0 (Feenstra, Inklaar, and Timmer 2015) is used to calibrate the production function and other parameters as well as Saez and Zucman (2016) and Jebb, Tay, Diener, and Oishi (2018).

we can represent results as average annual growth rates of mean, median, and modal incomes in the same space as in Figure 3. Optimised economic progress after 30-years falls along the dashed line resulting from the mean under the lognormal assumption. The direction and extent of pathways in (σ^2, μ) space differ by preference over η. After an initial correction from the real-world trend, growth pathways generally fall along what we may approximate as a vector. We also include a projected growth path based on the 1967-2017 U.S. trend. Naturally, the real trend includes all sorts of shocks not included in the η-based simulation.

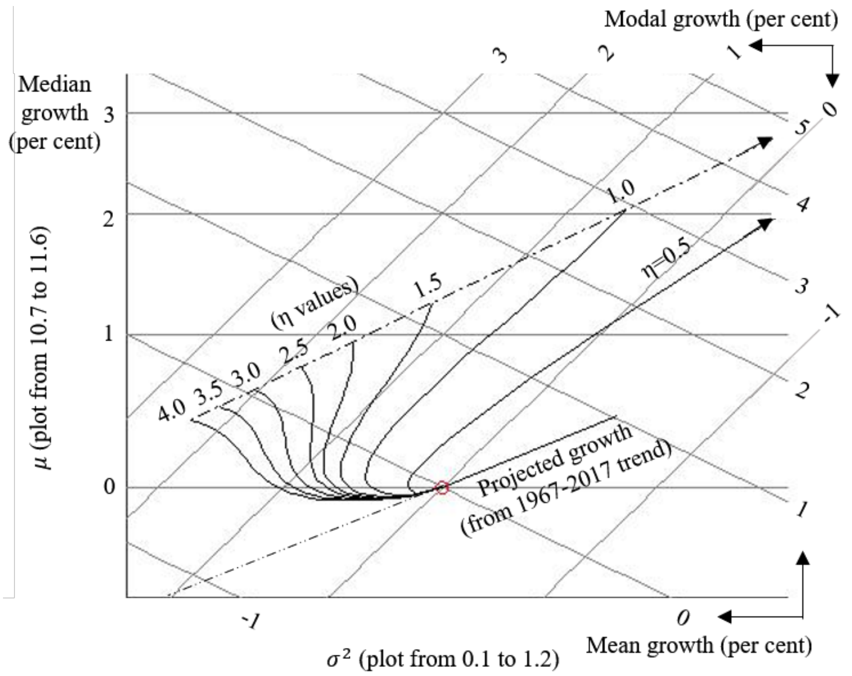


FIGURE 3. OPTIMAL μ AND σ^2 PARAMETERS OVER 30 YEARS CONDITIONAL ON η

Notes: Optimal growth paths in (μ, σ^2) space for the U.S. depending on the social preference for inequality represented by η (higher value interpretable as a preference for less inequality) and projected U.S. growth based on the 1967-2017 trend. These are overlaid on a contour map of the implied annualised mean, median, and modal growth rates.

Sources: Author calculations based on repeated simulations and data from the U.S. Census Bureau (2019).

We must remain aware of the lognormal distributional assumption when interpreting Figure 3 and the results in general. One issue is that the lognormal approximates the bulk of any income or consumption distribution well, but not ultrawealthy outliers. We could use the gamma distribution or more sophisticated alternatives, but the capacity to represent growth in a space where mean, median, and modal implications can be represented simultaneously would be lost. Instead, the point is to have an intuition for when critical economic and social parameters, like inequality preference, drive the results, and not the distributional assumption.

It is also important to discuss the implications of our welfare function. Over the 30-year planning horizon, the decisionmaker weighs each agent *i*'s benefit in each period *t* against every other agent's benefit in every period (see discussion in, for instance, Gollier, 2011). The planning horizon is critical because the η parameter summarises inequality preferences between every *i* in every *t*. Preference for less inequality (higher η) implies not just a preference for more equal distribution in each period, but also greater intertemporal redistribution to earlier periods to balance later gains from economic growth, regardless of the discount rate. Thus, a forward-thinking, multiyear planning society with inequality aversion may behave as if discounting even if $\rho = 0$.

Some observations: After the initial correction of distributional policy to match preferences, planning horizon, and lack of real world instability, we see the $\eta = 2$ path settles into distribution-preserving growth ($\bar{g}_t = \tilde{g}_t = g_t^{Mo}$). The $\eta = 1$ path also settles into what would be mean-maximising growth in a short run, noniterative model. However, a preference for greater inequality, ($\eta < 1$), results in higher average mean growth through higher capital accumulation from savings. This higher annualised growth rate, however, is driven by gains enjoyed in later years at the expense of lower consumption in earlier ones.

Similarly, a preference for economic policy focused on median rather than mean income, advocated for instance by Aghion et al. (2013), European Commission (2014), and Stiglitz (2012) based on inequality aversion, might maximise short-run but not long-run median growth. Policy emphasising greater inequality can also result in higher median growth (eventually) through savings. The issue is whether we can first tolerate lower median growth and then greater long-run inequality. Under the lognormal assumption, another trade-off is a reduction in, or possibly negative, modal growth.

Instead of choosing to monitor median income as our metric, suppose instead a society acts to maximise median income growth. That is, suppose that society chooses policies suggesting an η (recall, however, this is a parameter) that maximizes growth in $\tilde{c} = exp(\mu)$. From a sufficiently long-run perspective, we might then choose to maximise growth in the mean, $\bar{c} = exp(\mu + \frac{1}{2}\sigma^2)$ too, since growth in σ^2 has a positive effect on growth in μ through savings. But this would require ignoring intergenerational differences and instead suppose that growth in the annualised mean is sufficiently rewarding. But this conflicts with what η is indicating which is a preference over inequality between all *i* and *t*. The point is also not to choose η, a parameter indicating social preferences. The optimal distributional policy instead follows from and indicates an η parameter representing preferences.

More generally, emphasising either mean or median growth does not necessarily result in increased total wellbeing based on the CRRA utility form. For many social preference values where $\eta \neq 1, 2$, planning for growth in either mean or median statistics is socially suboptimal. Instead, for any social preference over distribution and inequality an entirely different optimal growth path is preferred. A national statistic emphasising economic growth in any form alone is not informative on whether national preferences over distribution are being addressed.

A static statistic is also insufficient to describe progress. Along the optimal growth paths in (μ, σ^2) space, a mean-median relationship, or in other such comparisons, one value tends to grow comparatively faster in most preference scenarios. Current statistics on distribution do not adequately reflect that even stable social preferences result in changing levels of inequality.

We also note that projected U.S. growth continuing from the 1967-2017 trend – generally predictable in the long-run as a linear estimate fits the 1967-2017 data well – is not generally short-run optimal under plausible inequality preferences indicated by an η closer to 1.5 or 2. Instead, the current policy suggests an as-if social preference more unequal. The result is a sacrifice of growth potential by any measure of centre – mean, median, or mode.[17] The result is that any policy "revolution" to a more equitable preference-based growth policy begins with economic upheaval. We find support for this in Banerjee and Duflo (2003) where any preference change incurs a temporary loss in $\dot{G}c$. A policy conflict then occurs – a representative government, knowing η, cannot justify a growth trend suggesting an inequitable social preference, yet policymakers would be hesitant to embark on a radical policy realignment if $\dot{G}c$ is the metric of economic success.

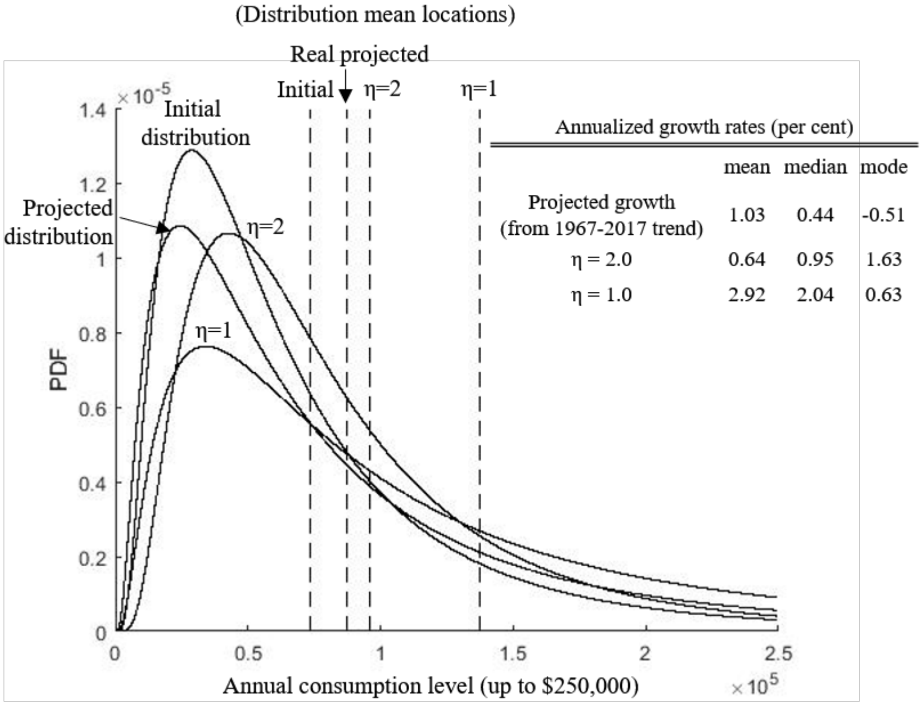
Finally, we can put comparisons on perhaps more familiar ground as a set of comparative distributions in Figure 4. These emphasise how different values of η result in different consumption distributions over time.[18] One might appeal to the Keynes-Ramsey Rule rearranged as $\frac{\dot{\bar{c}}(t)}{\bar{c}(t)} = \bar{g}_t = \frac{1}{\eta}\left(f'\left(k(t)\right) - \delta - \rho\right)$ to understand their relationship.[19] A preference for less equality, or greater inequality, (smaller η) results in a preference for more income growth in the right tail. Under the lognormal assumption, this necessarily results in a reduction in modal income growth and a fattening (higher frequency) in at least some upper-intermediary (middle) incomes. There is a greater dispersal across incomes too – less common experience across households which might impact national cohesion. Unfortunately, this

[17] Graphically, a trend maximizing short run growth in one measure of center, for instance the mean, would run perpendicular to the mean growth contour lines – traversing them by least distance.

[18] Assuming national policy impacting distribution is based on preferences represented by η.

[19] One may also want to appeal to this in attempting to estimating values of η empirically. That is, suppose η implied in any period of economic activity can be estimated as a ratio of net marginal product and mean consumption growth. This is not explored here, however.

outcome again highlights the inadequacy of both mean and median measures. Both measures may register growth, while a subset of agents around the mode are pushed into poverty.



	mean	median	mode
Projected growth (from 1967-2017 trend)	1.03	0.44	-0.51
η = 2.0	0.64	0.95	1.63
η = 1.0	2.92	2.04	0.63

FIGURE 4. CONSUMPTION DISTRIBUTION AT 30 YEARS, BY η VALUE

Notes: Consumption distributions and measures of centre for select values of η. Initial distribution based on recent U.S. national statistics and projected distribution based on 1967-2017 U.S. growth trend. A smaller η value, representing a greater preference for inequality, results in higher growth in inequality and the right tail. Und the lognormal assumption, the modal statistic may decrease even if the median and mean increase as projected based on current U.S. policy – a particularly adverse effect for much of society.

Sources: Author calculations based on simulations and data from the U.S. Census Bureau (2019).

Whatever national preferences over distribution and growth are, a complementary national statistic to $\dot{G}c$ appears warranted. It can inform whether growth policy is placing the economy on a preferred or at least familiar path. Policymakers and the public can then judge whether the path is desirable in part based on whether they take a normative or positive perspective. We conclude by presenting candidates for the purpose.

6 Economic Performance in Terms of Distribution

The emphasis of this paper is the importance of incorporating social preferences over distribution into economic growth policy. At a minimum, policies over distribution result in growth paths as if there is a preference, even if it is one that does not benefit most of society. A statistic indicating when we have departed from a distributional growth path, preference, etc. complements $\dot{G}c$ because it says something about whether growth policy will be regarded as equitable or at least results in outcomes that match expectations. Policy fitness, in turn, informs on whether policy supports social stability. It can also say something about $\dot{G}c$ – whether it will exceed or fall short of what the factor decomposition growth literature predicts because of the savings-distribution relationship.

Whether a growth path matches social preferences is the important characteristic for monitoring distribution, not which path we are on. One way to develop a statistic that informs on this is to incorporate η as a benchmark, and contrast it against implied-η measures or other comparisons. Incorporation results in an internationally comparable statistic while retaining national context because we ask how closely growth policies match each society's preference in the aggregate. We then compare how on-target each society's policies are, rather than comparing growth when preferences over how growth is achieved may be radically different.

To incorporate the benchmark, consider a growth statistic over the distribution of form

$$\nu_{(\cdot)} = D(t) - D(\eta) \tag{6}$$

where $D(t)$ reports a measure of economic performance in *t*, $D(\eta)$ identifies what the society would prefer in the same units based on social preferences, and a positive (negative) value of $\nu_{(\cdot)}$ suggests greater (lesser) inequality growth than preferred. One approach is to state $D(t)$ as the value of η implied by distributional growth in each period, $\eta(t)$, e.g. implied-η. We can then compare this to $D(\eta) = \eta$ – society's generally constant social preference. The statistic on growth and distribution is then

$$\nu_{(\eta)} = \eta(t) - \eta \tag{7}$$

An advantage of benchmark $D(\eta) = \eta$ is that each η value identifies a growth path with an approximate rate of divergence, and so has a dynamic perspective "baked in". One issue is that we must measure the abstract quality $\eta(t)$ consistently and frequently. Another issue is the interpretation of the units of measure as ν_η is in units of divergence from preferences. However, this would hardly be our first economic measure in abstract units.

Alternatives in somewhat more familiar units readily emerge. An equivalent measure to an η growth path is the explicit rate of divergence of two points on the income distribution, and mean and median statistics are often available. Suppose $D(\eta)$ is the growth rate of divergence in mean and median implied by social preference value η, and $D(t)$ the observed rate of divergence in the same period. The new statistic follows as

(8) $$\nu_{mean/median} = g(t)_{mean/median} - g(\eta)_{mean/median}$$

If the lognormal assumption fits the distribution, we might estimate $\nu_{mean/median}$ from its mean and median analytical values as $\nu(\mu, \sigma^2, \eta)$ where $g(t)_{mean/median} = \frac{1}{t}\frac{1}{2}\Delta\sigma^2_{0,t}$ which is growth attributable to a spread in the distribution. If $g(\eta)_{mean/median}$ is based on $\eta = 2$, signifying a preference for equal growth such that $\bar{g}_t = \tilde{g}_t = g_t^{Mo}$, this implies $g(\eta)_{mean/median} = 0$ and $\nu_{mean/median} = \frac{1}{t}\frac{1}{2}\Delta\sigma^2_{0,t}$ but this is an exception.

We may also want to revisit the normal coefficient of variation (CV) based on its familiarity in the field and general usefulness in discussing distribution. The appropriate statistic, however, is the divergence from preferences over the growth rate of the coefficient of variation, g_{CV}:[20]

(9) $$\nu_{(CV)} = g(t)_{CV} - g(\eta)_{CV}$$

Finally, the Gini coefficient is a popular and intuitive measure of inequality, and we would be remiss to leave it out here. As in the preceding forms, a comparison of growth in the observed Gini coefficient, $g(t)_{Gini}$, to the implied socially optimal trend in a society, g_{Gini}, is most informative:

(10) $$\nu_{(Gini)} = g(t)_{Gini} - g(\eta)_{Gini}.$$

As an empirical starting point, Table 1-Table 3 report average annual values for the four proposed measures of $D(t)$ using the set of countries in five datasets. These values are also equivalent to $\nu_{(\cdot)}(\eta = 2)$ – the values of $\nu_{(\cdot)}$ if we assume the social preference is an equal growth rate across the distribution. Except for the directly η-based statistic, these values are normalised by mean growth to report the rate of divergence, or inequality growth, per-per cent growth. The η measure does not require normalisation because the statistic informs on the η path of the economy without regard to the magnitude of growth. When sufficiently long time-series are available as in the WID dataset, we also report values for significant periods of development. Because the measures are composed from OLS estimates on different qualities of the data – annual Gini coefficients, mean, median, and residual values – not all $D(t)_{(\cdot)}$ estimates are statistically significant and reported for all countries. Consequently, the global

[20] From the normal coefficient of variation $CV = \frac{\sigma}{\mu}$, growth is $g_{CV} = \frac{\dot{CV}}{CV} = \frac{\mu\dot{\sigma} - \dot{\mu}\sigma}{\mu\sigma}$.

picture is not representative – only nations with sufficiently stable growth and data collection initiatives are included – those where at least one distributional measure can be calculated at generally acceptable confidence levels.

TABLE 1—GROWTH AND DISTRIBUTION TREND ESTIMATES, LIS AND OECD DATA

	Period [a]	$\bar{g}$ [b]	$\frac{g_{mean/median}}{\bar{g}}$	$\frac{g_{CV}}{\bar{g}}$	$\frac{g_{Gini}}{\bar{g}}$	η
LIS data						
Australia	1981-2010(8)	1.38**	0.16	0.52	0.40**	1.77**
Canada	1981-2010(10)	0.90	0.25	1.03	0.59	1.59
Denmark	1987-2010(7)	1.15	0.10**	0.81**	-	1.86
Finland	1987-2010(7)	1.75	0.17	1.14	0.75	1.74
France	1978-2010(7)	0.79**	-0.10*	-0.46**	-0.63*	2.12**
Germany	1984-2010 (7)	0.56**	0.23**	0.88**	0.79**	1.64**
Ireland	1987-2010(8)	3.45	-0.09**	-0.40	-0.17**	2.11
Israel	1986-2010(7)	1.67	0.14	0.29	0.62	1.80
Italy	1986-2010 (11)	0.63**	-	-	0.60*	-
Luxembourg	1985-2010(8)	2.63	0.06**	-	0.26	1.92**
Norway	1979-2010 (8)	2.28	-	-	0.21**	-
Spain	1980-2010 (8)	2.15	-	-0.31**	-	-
Sweden	1981-2005(6)	2.04	0.11**	0.97	0.44**	1.84
United Kingdom	1979-2010(9)	2.41	0.14	0.37	0.29**	1.80
United States	1979-2013(10)	0.77	0.37	1.12	0.70	1.33
OECD data						
Canada	1976-2017 (42)	0.85	0.14	0.54	0.31	1.80
Denmark	1985-2016 (16)	0.99	0.19	1.36	0.57	1.71
Finland	1986-2017 (32)	1.63	0.13	0.82	0.55	1.81
Germany	1985-2016 (14)	0.60	0.13	0.43	0.97	1.82
Hungary	1991-2016 (15)	1.02**	-0.14**	-0.60**	-0.30**	2.15**
Israel	1990-2017 (14)	1.88	-	-	0.17	-
Italy	1984-2016 (17)	0.33*	-	-	1.18*	-
Luxembourg	1986-2016 (15)	1.46	0.07	0.23	0.46	1.90
Netherlands	1977-2016 (17)	0.92	-0.09	-0.44	0.17*	2.11
New Zealand	1985-2014 (10)	1.45	0.15**	-	0.32*	1.78
Norway	1986-2017 (14)	2.19	-	-	0.17**	-
Sweden	1975-2017 (15)	1.86	0.06	0.29	0.49	1.92
United Kingdom	1975-2017 (23)	1.23	0.14	0.30	0.42	1.79
United States	1995-2017 (13)	0.54	-	1.26	0.76	-

Notes: $g_{mean/median}$ is the growth rate of divergence of mean and median income as an annual percentage change that is approximated by estimating the change in $\ln(mean/median)$. g_{CV} is the growth rate in the coefficient of variation approximated by estimating $ln(CV)$. g_{Gini} is the growth rate of the Gini coefficient. We approximate η by the growth vector approach (Turk, 2020). A limitation of the CRRA utility function is that only $\eta > 0$ are considered valid and so lower values imply policies diverging from CRRA utility-based preferences. All estimates reported are statistically significant at the 1-per cent level or better using Huber–White standard errors unless otherwise noted (** 5 per cent level; * 10 per cent level). Statistical significance of all inequality estimates is the least of the two estimated values used in their composition.

[a] Number of observations, in parentheses. [b] Average annual growth rate, in per cent.

Sources: Author calculations based on data from Thewissen, Nolan, and Roser (2016) which is derived from the Luxembourg Income Study (LIS) Database; and OECD (2019).

TABLE 2—GROWTH AND DISTRIBUTION TREND ESTIMATES, WORLD BANK DATA

	Period [a]	$\bar{g}$ [b]	$\frac{g_{mean}/_{median}}{\bar{g}}$	$\frac{g_{CV}}{\bar{g}}$	$\frac{g_{Gini}}{\bar{g}}$	η
<u>Income basis</u>						
Argentina--Urban	1987-2017 (27)	0.88*	-0.60	-1.07	-0.59*	2.46
Australia	1981-2014 (10)	1.65	0.10	0.14	0.20	1.87
Bolivia	1990-2017 (19)	2.83	-	-0.58*	-	-
Brazil	1981-2017 (33)	2.72	-0.24	-0.36	-0.14	2.24
Canada	1981-2013 (11)	1.09	-	-	0.25**	-
Chile	1987-2017 (14)	2.20	-0.43	-0.58	-0.35	2.37
Colombia	1992-2017 (19)	2.69	-0.16*	-0.29**	-0.15*	2.17
Costa Rica	1981-2017 (31)	4.20	0.15	-	0.09**	1.77
Croatia	1988-2015 (8)	-0.32**	-0.73	-3.90	-4.16**	-0.51
Czech Republic	1993-2015 (14)	2.19	-0.08	-0.40	-	2.09
Ecuador	1987-2017 (20)	3.09	-0.22*	-0.40**	-0.24**	2.23
El Salvador	1991-2017 (23)	0.96	-0.90	-1.47	-1.42	2.56
Estonia	1993-2015 (14)	4.45	-0.07*	-0.27	-0.16**	2.08
Germany	1991-2015 (18)	0.72	0.19	0.54**	0.54	1.71
Guatemala	1986-2014 (5)	-	-	-	-	2.24
Honduras	1989-2017 (28)	2.27	-0.18	-0.34	-0.11**	2.19
Israel	1986-2016 (10)	1.94	-	-	0.23**	-
Latvia	1993-2015 (15)	4.53	0.06**	-	0.19**	1.92
Lithuania	1993-2015 (13)	6.54	-	-0.13**	-	-
Malaysia	1984-2015 (12)	2.65	-0.14	-0.32	-0.17	2.15
Mexico	1989-2016 (15)	1.05*	-0.49	-0.68	-0.53*	2.40
Nicaragua	1993-2014 (6)	3.49	-0.34	-0.55	-0.36	2.32
Panama	1989-2017 (24)	3.31	-0.19	-0.33	-0.21	2.20
Poland	1985-2015 (15)	1.76	0.12	0.35	0.54	1.83
Slovenia	1993-2015 (13)	2.09	-0.14**	-0.66**	-	2.15
United States	1986-2016 (10)	0.86	0.24	0.43	0.31	1.62
Uruguay	1981-2017 (14)	0.95	-	-	-0.24*	-
<u>Consumption basis</u>						
Bangladesh	1983-2016 (9)	1.17	0.30**	0.75**	0.61	1.48**
China--Rural	1990-2015 (13)	6.09	-	-	0.09	-
China--Urban	1990-2015 (13)	6.74	0.07	0.08**	0.19	1.90
Cote d'Ivoire	1985-2015 (10)	-2.36		-0.19**	-	-
Georgia	1996-2017 (22)	1.67**	-	-	-0.13*	-
Ghana	1987-2016 (7)	3.46	0.10**	-	0.20	1.87
India--Rural	1983-2011 (6)	1.33	0.09**	-	-	1.88
India--Urban	1983-2011 (6)	1.69	0.18	0.21**	0.33	1.73
Indonesia--Rural	1984-2017 (25)	4.00	0.07	-	0.20	1.90
Indonesia--Urban	1984-2017 (25)	3.73	0.13	0.11	0.23	1.81
Iran, Islamic Republic of	1986-2016 (11)	2.35	-0.13	-0.34	-0.23	2.15
Kazakhstan	1996-2017 (18)	4.39	-0.11	-0.43	-0.40	2.12
Mauritania	1987-2014 (7)	2.24	-	-0.74**	-0.54**	-
Mexico	1984-2016 (15)	-	-	-	-	2.53
Morocco	1984-2013 (6)	1.61**	0.11*	-	-	1.85*
Pakistan	1987-2015 (12)	2.90	-	-0.12*	-	-
South Africa	1993-2014 (7)	3.34	0.25**	-	0.11**	1.59**
Sri Lanka	1985-2016 (8)	2.45	0.17	0.20*	0.29	1.75
Thailand	1981-2017 (23)	3.28	-0.17	-0.40	-0.22	2.19
Tunisia	1985-2015 (7)	2.23	-0.20	-0.55	-0.38	2.21
Turkey	1987-2016 (17)	2.95	-	-0.19**	-	-
Ukraine	1992-2016 (19)	3.03	-0.11**	-0.50	-0.47	2.13
Vietnam	1992-2016 (10)	6.30	-0.05**	-0.26	-	2.07

Notes: $g_{mean/median}$ is the growth rate of divergence of mean and median income as an annual percentage change, approximated by estimating the change in $\ln(mean/median)$. g_{CV} is the growth rate in the coefficient of variation approximated by estimating $ln(CV)$. g_{Gini} is the growth rate of the Gini coefficient. We approximate η by the growth vector approach (Turk 2020). A limitation of the CRRA utility function is that only $\eta > 0$ are considered valid and so lower values imply policies diverging from CRRA utility-based preferences. All estimates reported are statistically significant at the 1-per cent level or better using Huber–White standard errors unless otherwise noted (** 5 per cent level; * 10 per cent level). Statistical significance of all inequality estimates is the least of the two estimated values used in their composition.

[a] Number of observations, in parentheses. [b] Average annual growth rate, in per cent.

Sources: Author calculations based on data from World Bank (2019).

TABLE 3—GROWTH AND DISTRIBUTION TREND ESTIMATES, WID DATA

	Period [a]	$\bar{g}$ [b]	$\frac{g_{mean/median}}{\bar{g}}$	$\frac{g_{CV}}{\bar{g}}$	$\frac{g_{Gini}}{\bar{g}}$	η
Brazil [c]	2001-2015 (15)	1.25 [Γ]	-0.14 [Γ]	-0.20**	-0.23 [Γ]	2.16 [Γ]
China [d]	1978-2015 (38)	5.63	0.23	0.05	0.24	1.63
Transition	1978-1999 (22)	4.65	0.24	0.08	0.34	1.62
Present	2000-2015 (16)	9.29	0.15	-0.02	0.06	1.78
Cote d'Ivoire [e]	1988-2014 (6)	-1.48**	-0.41	-0.31	-	-0.36
Egypt [f]	1999-2015 (6)	1.45	-0.15	-0.25	-0.33	2.17
France [g]	1900-2014 (83/102)	2.21	-0.27	-0.24	-0.14	2.27
LIS comparison	1978-2010 (26/33)	2.43	0.75	0.33	0.05	0.41
Pre-WWII	1900-1939 (20/27)	0.68 [Γ]	-0.67 [Γ]	-	-0.40 [Γ]	2.83
Post-war	1946-1969 (22/24)	4.52	-	-0.10	0.08	-
Transition	1970-1999 (25/30)	1.40 [Γ]	-0.06 [Γ]	-	-	2.87
Present	2000-2014 (11/15)	2.47	1.11	0.37	-0.06	-0.12
India [h]	1951-2013 (54/63)	1.84	-	-0.08	0.15	1.89* [Γ]
Postcolonial	1951-1964 (12/14)	2.06	-0.38	-0.48	-0.31	2.34
Transition	1965-1999 (32/35)	1.86	0.06	-0.03	0.12	1.92
Present	2000-2013 (10/14)	3.74	0.20	0.10	0.53	1.68
Palestine [f]	1996-2011 (10)	-1.33*	-0.29	-0.28	-0.28*	-0.28**
Russia [i]	1961-2015 (25/36)	-	-	-	-	-
USSR	1961-1989 (3/10)	2.38 [Γ]	-0.05 [Γ]	-	-0.27 [Γ]	2.06 [Γ]
Transition	1990-1999 (6/10)	-5.38 [Γ]	-	-	-1.44 [Γ]	-
Present	2000-2015 (16)	2.89	-0.79	-0.36	-0.22	2.53
Turkey [f]	1994-2016 (16)	2.06 [Γ]	-	-0.32*	-0.38 [Γ]	2.28** [Γ]
United States [j]	1962-2014 (51)	1.47	0.29	0.08	0.37	1.50
LIS comparison	1979-2013 (35)	1.44	0.21**	0.03	0.42	1.68**
Transition	1962-1980 (17)	1.62	-	-0.06	-	-
Reaganomics	1981-1992 (12)	2.53	0.66	0.27	0.33	0.60
Present	1993-2014 (22)	1.28 [Γ]	0.07 [Γ]	-0.29**	0.35 [Γ]	1.90 [Γ]

Notes: Annual estimates used as the basis of these measures are derived from percentile distributional data. $g_{mean/median}$ is the growth rate of divergence of mean and median income as an annual percentage change, approximated by estimating the change in $\ln(mean/median)$. g_{CV} is the growth rate in the coefficient of variation approximated by estimating $ln(CV)$. g_{Gini} is the growth rate of the Gini coefficient. We approximate η by the growth vector approach (Turk 2020). A limitation is the CRRA utility function is that only $\eta > 0$ are considered valid and so lower values imply policies diverging from CRRA utility-based preferences. All estimates reported are statistically significant at the 1-per cent level or better using Huber–White standard errors unless otherwise noted (** 5 per cent level; * 10 per cent level). Statistical significance of all inequality estimates is the least of the two estimated values used in their composition.

[a] Number of observations, in parentheses, where a lognormal/gamma distribution fits the data sufficiently well based on the two-sample Kolmogorov-Smirnov test (Massey, 1951). [b] Average annual growth rate, in per cent. [Γ] Denotes estimates based on the gamma distribution when lognormal-based estimates do not fit the data sufficiently well – these estimates are explored in appendix *A2 The Gamma Distribution Alternative*.

Sources: Author calculations based on World Inequality Database (WID) data discussed in: [c] Morgan (2017); [d] Piketty, Yang, and Zucman (2017); [e] Czajka (2017); [f] Alvaredo, Assouad, and Piketty (2018); [g] Garbinti, Goupille-Lebret, and Piketty (2018); [h] Chancel and Piketty (2017); [i] Novokmet, Piketty, and Zucman (2018); and [j] Piketty, Saez, and Zucman (2016).

Whatever the form of $D(t)$, a couple approaches can be taken to setting the benchmark $D(\eta)$. One may be called a proactive or normative approach. That is, set a measure of performance based on careful societal introspection (whom do we want to be as a society?) and define the measure of performance as a comparison to it. In a sense, we decide what is "fair" and then evaluate growth in those terms. It is suggested we may refer to this as the European approach.

One may also take a positivist approach. That is, look at where countries are and impute the social preference associated with their status. From a policy perspective, what a practitioner does is make apparent the extent of the trade-off between μ and σ^2, or mean and median growth, evident in a country's existing trajectory. We may then ask whether something different – a policy intervention, ought to be done when current and long-run distributional paths differ. It is suggested that we may refer to this as the U.S. approach, though these are generalisations.

In the U.S. approach, historical measures of $D(t)$ replace $D(\eta)$ in future periods without requiring the exogenous discovery of underlying social preferences. In the European approach, one may take these values as a starting point, but need not take the current state of the economy indicated, revealed preference estimates of η, or long-run or short-run implied-η estimates, or other historical indicators of distribution as the benchmark of performance. $D(\eta)$ may in fact differ substantially from that implied by existing policies. In any event, even among experts one method cannot be agreed on (Drupp, Freeman, Groom, and Nesje, 2018), and both have some merit.

7 Concluding Remarks

If a population has no preference over distribution, GDP growth (not even requiring it in per-capita form) is a sufficient statistic as implicit in it is that more consumption potential without regard to distribution is preferred. A standard argument stemming from the Kaldor-Hicks compensation criteria is to make the pie as large as possible and then figure out how to distribute it later. If the public is confident in the state's omniscience and the elite's generosity, then perhaps GDP growth remains sufficient, whatever distributional preferences are. However, Sen (2000) argues that Kaldor-Hicks is not a defensible criterion if there is no mechanism and no intent for gainers to compensate losers. Suppose, given the existence of extreme wealth, poverty, and discontent among the poor and marginalised, that we side with Sen and cannot trust in unmonitored distribution.

A statistic that includes an agnostic representation of preferences is useful in monitoring economic progress. Not taking a stance on what the social preference value ought to be is an advantage. A preference for more inequality may, for instance, be representative of how rapidly a country prefers to modernise through capital accumulation, i.e. China's capital hungry modernisation initiatives. It may also be the product of longstanding religious or cultural beliefs that one would be on questionable ethical grounds to reject outright, i.e. Saudi Arabia's theocratic-supported monarchy. In comparison, measures like the static Gini coefficient imply that a decrease in inequality is preferable. The proposition of new statistic $\nu_{(\cdot)}$ makes no such claim on optimal distribution and instead represents the deviation from any distribution-growth path.

$\nu_{(\cdot)}$ also offers the advantage of representing the dynamic nature of growth better as it uses the rate of divergence in measures of centre. Static alternatives such as the mean-median spread, mean/median ratio, coefficient of variation, Gini, and measures of poverty are only appropriate if the mean and median grow in proportion – which we have shown is unlikely. Static measures cannot represent the changing nature of distribution with economic growth. At best we would end up comparing static values in two periods without guidance on whether the change is beneficial.

$\nu_{(\cdot)}$ also challenges that the method of conducting international comparisons changes. Formulating $\nu_{(\cdot)}$ as relative to national preferences has the advantage of informing on how well national policies fit. It is a measure of deviation rather than placing international values side-by-side without context. This suggestion is not to say that comparing components $D(t)$ or $D(\eta)$ between countries does not make for good discussion. But keeping the statistic as the composite $\nu_{(\cdot)}$ is more informative when comparing, say, an OECD member against an LDC with different institutional arrangements, production potential, and social traditions.

In the model, we have treated η as a constant parameter, but like target $\dot{Gc}$, the unemployment rate, or inflation, it is almost certainly subject to change over time. For instance, societies may be willing to sacrifice equality in favour of rapid modernisation over some period, and then transition to preferring a more equal dispersion once some quality of life potential is attainable through redistributions or economic reorganisation. In operation, consider if economic growth is distributed more unequally than preferred because society's preferences have shifted to a more equitable distribution (η has increased). Then $\nu_{(\cdot)} > 0$ and an interpretation is that future growth is bought at a higher price – greater sacrifice – than desired in the present given the remaining needs of society today. If instead society changes to favour individual success – a chance at attaining personal wealth and status become paramount (η decreases), then $\nu_{(\cdot)} < 0$ implies a more equal distribution is occurring than fitting with the society that now prefers to gamble. Estimating η and $D(\eta)$ well is essential work, and exploring how the η summary of preferences changes over time is an interesting avenue for further research. These are some of the many questions this research leads to, rather than answers.

To conclude, we cannot discard measures of the magnitude of growth such as $\dot{Gc}$ because they say something vitally important. They tell us how the per-capita consumption potential of society is changing – whether the capacity for better lives is increasing. But by summarising the welfare distribution into a preference-free measure of centre, they do not inform on how consumption is operationalised. Moving from a representative agent to a representative distribution basis when modelling economic growth reveals what we give up in using the representative agent assumption. Clearly, at least one more statistic is needed. This statistic(s) should portray the dynamic nature of distribution and growth and embody social preferences over trade-offs inherent in national distributional policies. As a practical matter, a companion to $\dot{G}c$ should also be simple to formulate and interpret. The proposals on $\nu_{(\cdot)}$ generally meet these conditions and are based on our distributional-growth model. The model itself and the statistics based on it perhaps make sense of some inconsistencies in economic growth and social outcomes. For instance, improvements in measures of economic growth but not measures of national satisfaction, or vice versa, may result from how poorly or well economic distributions match preferences.

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Appendices

A1 Data Tables Supporting Figure 1 and Figure 2

TABLE 4— DECOMPOSITION OF GROWTH INTO DISTRIBUTION PARAMETERS, LIS AND OECD

	Period [a]	$\bar{g}$ [b]	μ-driven growth [b]	σ^2-driven growth [b]
LIS data				
Australia	1981-2010 (8)	1.38**	1.17**	0.21
Canada	1981-2010 (10)	0.90	0.67	0.23
Denmark	1987-2010 (7)	1.15	1.03	0.12**
Finland	1987-2010 (7)	1.75	1.45	0.30
France	1978-2010 (7)	0.79**	0.87	-0.08*
Germany	1984-2010 (7)	0.56**	0.43*	0.13**
Ireland	1987-2010 (8)	3.45	3.76	-0.31**
Israel	1986-2010 (7)	1.67	1.44	0.23
Luxembourg	1985-2010 (8)	2.63	2.47**	0.16**
Sweden	1981-2005 (6)	2.04	1.81	0.23**
United Kingdom	1979-2010 (9)	2.41	2.08	0.33
United States	1979-2013 (10)	0.77	0.49	0.28
OECD data				
Canada	1976-2017 (42)	0.85	0.73	0.12
Denmark	1985-2016 (16)	0.99	0.80	0.18
Finland	1986-2017 (32)	1.63	1.41	0.21
Germany	1985-2016 (14)	0.60	0.52	0.08
Hungary	1991-2016 (15)	1.02**	1.16**	-0.14**
Luxembourg	1986-2016 (15)	1.46	1.35	0.11
Netherlands	1977-2016 (17)	0.92	1.01	-0.08
New Zealand	1985-2014 (10)	1.45	1.23	0.21**
Sweden	1975-2017 (15)	1.86	1.76	0.11
United Kingdom	1975-2017 (23)	1.23	1.05	0.18

Notes: Luxembourg Income Study (LIS) estimates in PPP-adjusted equivalised disposable household income, and OECD estimates in CPI-adjusted household income, of the annual rate of long-run growth. Statistically significant at the 1-per cent level or better using Huber–White standard errors unless otherwise noted (** 5 per cent level; * 10 per cent level).

[a] Number of observations in the period in parentheses. [b] Average annual growth rate, in per cent.

Sources: Author calculations based on data from OECD (2019), and Thewissen, Nolan, and Roser (2016), which is derived from the Luxembourg Income Study (LIS) Database.

TABLE 5— DECOMPOSITION OF GROWTH INTO DISTRIBUTION PARAMETERS, WORLD BANK

	Period [a]	$\bar{g}$ [b]	μ-driven growth [b]	σ^2-driven growth [b]
Income				
Argentina--Urban	1987-2017 (27)	0.88*	1.41**	-0.53
Australia	1981-2014 (10)	1.65	1.49	0.16
Brazil	1981-2017 (33)	2.72	3.37	-0.65
Chile	1987-2017 (14)	2.20	3.15	-0.94
Colombia	1992-2017 (19)	2.69	3.11	-0.42*
Costa Rica	1981-2017 (31)	4.20	3.56	0.64
Croatia	1988-2015 (8)	-0.32**	-0.55	0.23
Czech Republic	1993-2015 (14)	2.19	2.35	-0.16
Ecuador	1987-2017 (20)	3.09	3.77	-0.68*
El Salvador	1991-2017 (23)	0.96	1.83	-0.87
Estonia	1993-2015 (14)	4.45	4.75	-0.30*
Germany	1991-2015 (18)	0.72	0.58	0.14
Guatemala	1986-2014 (5)	-	3.94*	-0.76
Honduras	1989-2017 (28)	2.27	2.67	-0.40
Latvia	1993-2015 (15)	4.53	4.25	0.28**
Malaysia	1984-2015 (12)	2.65	3.01	-0.36
Mexico	1989-2016 (15)	1.05*	1.57**	-0.51
Nicaragua	1993-2014 (6)	3.49	4.67	-1.18
Panama	1989-2017 (24)	3.31	3.93	-0.62
Poland	1985-2015 (15)	1.76	1.55	0.21
Slovenia	1993-2015 (13)	2.09	2.38	-0.29**
United States	1986-2016 (10)	0.86	0.66	0.20
Consumption				
Bangladesh	1983-2016 (9)	1.17	0.82**	0.35**
China--Urban	1990-2015 (13)	6.74	6.27	0.47
Ghana	1987-2016 (7)	3.46	3.13	0.33**
India--Rural	1983-2011 (6)	1.33	1.22	0.12**
India--Urban	1983-2011 (6)	1.69	1.39	0.30
Indonesia--Rural	1984-2017 (25)	4.00	3.71	0.29
Indonesia--Urban	1984-2017 (25)	3.73	3.23	0.50
Iran, Islamic	1986-2016 (11)	2.35	2.66	-0.31
Kazakhstan	1996-2017 (18)	4.39	4.86	-0.47
Mexico	1984-2016 (15)	-	0.63**	-0.28
Morocco	1984-2013 (6)	1.61**	1.44*	0.17*
South Africa	1993-2014 (7)	3.34	2.50**	0.84**
Sri Lanka	1985-2016 (8)	2.45	2.04	0.41
Thailand	1981-2017 (23)	3.28	3.84	-0.56
Tunisia	1985-2015 (7)	2.23	2.68	-0.45
Ukraine	1992-2016 (19)	3.03	3.38	-0.34**
Vietnam	1992-2016 (10)	6.30	6.65	-0.34**

Notes: Estimates of the annual rate of long-run growth in 2011 PPP($) adjusted household per-capita income and consumption expenditures. All OLS estimates reported are statistically significant at the 1-per cent level or better using Huber–White standard errors unless otherwise noted (** 5 per cent level; * 10 per cent level).

[a] Number of observations in the period in parentheses. [b] Average annual growth rate, in per cent.

Source: Author calculations based on data from the World Bank (2019).

A2 The Gamma Distribution Alternative

Because we derive μ and σ^2 values from fitting a lognormal distribution directly on WID percentile values, we use this opportunity to present decomposition under a competing distributional assumption. The gamma distribution, $c_i(t) \sim \Gamma(k(t), \theta(t))$, provides one such alternative. It follows from a similar decomposition process to the lognormal: from the gamma mean, $k\theta$, growth decomposes as $\bar{g} = g_k + g_\theta$. For context, a decrease in shape parameter (k) increases the skewness of the gamma distribution, $2/\sqrt{k}$. Table 6 provides lognormal and gamma decomposition for the WID percentile distribution data.

TABLE 6— DECOMPOSITION OF GROWTH INTO LN AND Γ PARAMETERS, WID DATA

	Period [a]	Lognormal distribution			Gamma distribution		
		$\bar{g}$ [b]	μ-driven growth [b]	σ^2-driven growth [b]	$\bar{g}$ [b]	θ-driven growth [b]	k-driven growth [b]
Brazil [c]	2001-2015 (15)	1.24**	1.54	-	1.25	0.86**	0.39
China [d]	1978-2015 (38)	5.63	4.34	1.29	5.36	8.02	-2.66
Transition	1978-1999 (22)	4.65	3.54	1.10	4.45	7.45	-3.00
Present	2000-2015 (16)	9.29	7.89	1.40	8.69	10.20	-1.51
Cote d'Ivoire [e]	1988-2014 (6)	-1.48**	-2.09	0.61	-1.84	-	-
Egypt [f]	1999-2015 (6)	1.45	1.67	-0.22	1.30	0.38	0.92
France [g]	1900-2014	2.21	2.81	-0.60	2.43	1.87	0.56
Pre-WWII	1900-1939 (20/27)	-	0.91	-0.70	0.68	-	0.92**
Post-war	1946-1969 (22/24)	4.52	4.39	-	4.58	4.56	-
Transition	1970-1999 (25/30)	-	1.67	-1.36	1.40	-	-
Present	2000-2014 (11/15)	2.47	-0.26	2.73	0.18	-	-
India [h]	1951-2013 (54/63)	1.84	1.82	-	2.05	2.59	-0.55**
Postcolonial	1951-1964 (12/14)	2.06	2.84	-0.78	2.31	-	-
Transition	1965-1999 (32/35)	1.86	1.75	0.11	1.86	1.71	-
Present	2000-2013 (10/14)	3.74	2.98	0.76	4.42	12.69	-8.27
Palestine [f]	1996-2011 (10)	-1.33*	-1.71	0.38	-1.27*	-	-0.77
Russia [i]	1961-2015 (25/36)	-	-	-	0.60	2.42	-1.85
USSR	1961-1989 (3/10)	-	-	-	2.38	1.62	0.76
Transition	1990-1999 (6/10)	-	-	-	-5.38	-	-9.59
Present	2000-2015 (16)	2.89	5.17	-2.29	4.09	2.37	1.72
Turkey [f]	1994-2016 (16)	2.19	3.03	-	2.06	-	1.51**
United States [j]	1962-2014 (51)	1.47	1.03	0.43	1.42	2.28	-0.86
Transition	1962-1980 (17)	1.62	1.53	-	1.50	1.43	0.07
Reaganomics	1981-1992 (12)	2.53	0.86	1.68	1.77	3.84	-2.07
Present	1993-2014 (22)	0.81	1.13	-	1.28	1.55	-0.28

Notes: Growth and distribution of pre-tax income among individuals age 20 and over. Lognormal and gamma distributions fit to annual percentile income thresholds by maximum likelihood estimation. Under the lognormal distributional assumption, analytical values of $\bar{c} = exp(\mu + \frac{1}{2}\sigma^2)$ and $\tilde{c} = exp(\mu)$ enable decomposition of growth. Decomposition under a gamma distributional assumption follows from the mean, $k\theta$, when $c_i(t) \sim \Gamma(k(t), \theta(t))$ and thus $\bar{g} = g_k + g_\theta$. A decrease in shape parameter (k) suggests an increase in the skewness of the gamma distribution as $2/\sqrt{k}$. Statistically significant at the 1-per cent level or better using Huber–White standard errors unless otherwise noted (** 5 per cent level; * 10 per cent level).

[a] Number of years within the period where the lognormal/gamma distribution fits well (two-sample Kolmogorov-Smirnov test) in parentheses. [b] Average annual growth rate, in per cent.

Sources: Author calculations based on data discussed in [c] Morgan (2017); [d] Piketty, Yang, and Zucman (2017); [e] Czajka (2017); [f] Alvaredo, Assouad, and Piketty (2018); [g] Garbinti, Goupille-Lebret, and Piketty (2018); [h] Chancel and Piketty (2017); [i] Novokmet, Piketty, and Zucman (2018); [j] Piketty, Saez, and Zucman (2016), and available from World Inequality Database (WID), WID.world.

A3 Implications of Alternative Savings Functional Forms

The savings functional form we employ – a judgement call with surprisingly limited implications – has received enough attention in seminars to warrant an appendix. Our colleague Roger Fouquet observes that our differential savings rate model is akin to supposing a uniform savings rate but that higher-income members of society earn a higher return on savings in the aggregate production process. We might suppose these higher returns result from a more scrutinous selection of investments (the use of investment services) and might decline as we move down the income scale. Satiation point b then would denote the point where abnormally high returns begin to occur on average.

Other colleagues suggest using a constant savings rate akin to setting $b = 0$ and specifying total savings as $s(c_i(t)) = \bar{s} \sum_{i=1}^{N} c_i(t)$ for $i = 1{:}\,N$ (all) of the population, and a distributional and time constant savings rate $\bar{s}$ which we can also interpret as the average national savings rate. We compare this to the current, differential savings function which is approximately stepped with both constant lower (below b), and upper (above b) savings rates of 0 and $\hat{s}$. Suppose total consumption below b is C_L, above is C_U, and $0 \leq b < 1$ is the population share below b and is relative (constant). Then the differential case savings total is $(1-b)\hat{s}C_U = I_U$ and uniform case savings are $\bar{s}(bC_L + (1-b)C_U) = I_{L+U}$. Setting $I_U = I_{L+U}$ and writing as $\bar{s}$ identifies the rate equivalent to $\hat{s}$:

(11) $$\bar{s} = \hat{s}\left(\frac{b}{(1-b)}\frac{C_L}{C_U} + 1\right)^{-1}$$

Suppose we set $\bar{s}$ according to its parameters in the initial period and then compare the implications for total, savings-driven growth. From the previous paper, we observe for $\eta = 2$ growth paths, $\frac{C_L}{C_U}$, is constant, for $\eta > 2$ is increasing, and for $\eta < 2$ is decreasing. Further, the change in this ratio occurs slowest for η values closest to two and increases slowly as we follow η further away. For pathways around $\eta = 2$ we can specify a value of $\bar{s}$ leading to similar results, with higher η the $\bar{s}$-based model would suggest higher growth and for lower η less growth. Since most long-run national estimates are around $\eta = [1.5, 2]$, our savings specification is more superficial novelty than departure.

Another matter is the implication of setting the satiation point as a relative versus absolute bound. That is, we might choose between modelling households as saving once they have surpassed some percentile in the distribution, e.g., 80th, versus some level, e.g., $100,000 in real take-home income or consumption. One may be predisposed to assume an absolute satiation point is more appropriate as it is not uncommon to say “if I can just make six-figures, I would save for retirement.” However, recent evidence on savings (Saez and Zucman, 2016) suggest the relative based distribution may be more appropriate. Also note that if the absolute condition were true, and always so, then in say the year 1900 it would have also been true. Undoubtedly, most households have become financially better off in the interim 120 or so years, yet most of the population continues to save as though they reside below the satiation point. The results of this paper have assumed a relative satiation point, but we present a comparison in Figure 5 in the form of a 100-year simulation repeated for preferences denoted by $\eta = 1$, and only differing by whether b is relative or absolute and set at the same initially equivalent values. In the short term – within 30-years or so – little difference emerges. However, over the extended period, differences in consumption and distribution become substantial. Under a relative assumption, greater inequality must be emphasised to accommodate growth, while under an absolute assumption, the capital saturation point maximising mean income is reached earlier. The form of satiation point may be a minor point, however, as one cannot generally expect a 100-year projection to carry much weight.

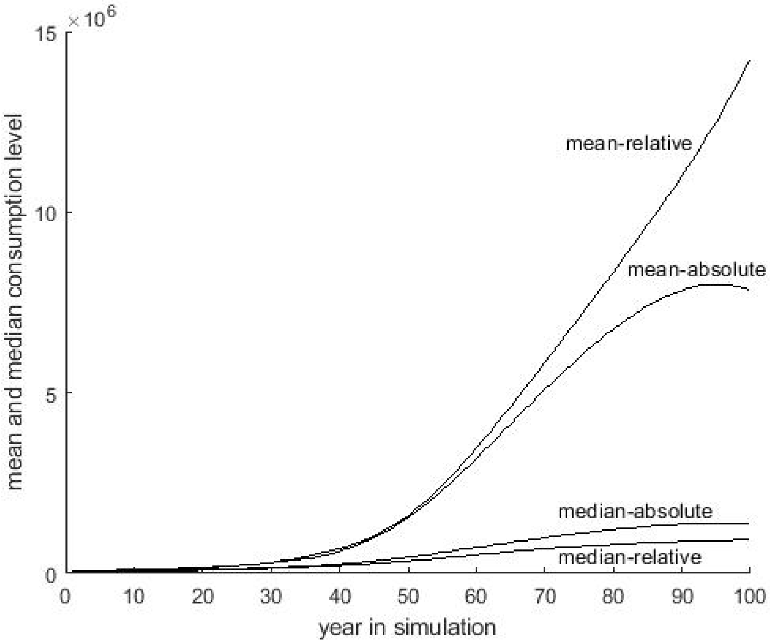

FIGURE 5. MEAN AND MEDIAN TRAJECTORIES OVER 100 YEARS

Notes: Differences in growth of mean and median consumption levels (η=1 assumed) by whether the consumption satiation point (point of positive savings) is relative or absolute. The impact of absolute versus relative is small in foreseeable years (the next three or so decades) but has a substantial impact further out. Under a relative satiation assumption, higher inequality is necessary to drive growth resulting in a more extensive spread. Under an absolute assumption, an economy reaches steady-state capital and consumption levels over a shorter time. As a practical matter, it is likely inaccurate to have confidence in economic projections one-hundred years out given the unpredictability of advancements and limitations.