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Policy Interaction and the Transition To Clean Technology

Ghassane Benmir* Josselin Roman†

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Abstract

Using a stochastic general equilibrium model with financial frictions and a two-sector production economy (i.e. green and dirty sectors), we assess the differences between a first-best and second-best environmental fiscal policy in successfully meeting the Paris Agreement accords, as well as investigate the role and efficiency of macroprudential and monetary policies under the presence of a carbon market (such as the European Trading Scheme (ETS) market). We first find that a second-best instrument is needed in the Euro Area to be aligned with the Paris Agreement, but that it leads to two distortions: i) a welfare wedge resulting from the medium/long-term impacts of the sub-optimal policy, and ii) a risk premium distortion arising from the short-term volatility of the ETS market. To dampen these two effects and prevent potential shocks to CO_2 price levels from distorting the functioning of monetary policy through a rise in risk premia, we show how macroprudential and monetary tools contribute to closing these two wedges on welfare and risk premium, respectively. We find that sectoral macroprudential weights on loans favorable to the green sector boost green capital and output, reducing the effect of the sub-optimal carbon policy on welfare. With respect to the risk premium, we find that quantitative easing (QE) is able to close the wedge, as a QE rule would allow authorities to drastically reduce the effect of price volatility on risk premia. In addition, we find that macroprudential policy is needed to provide an incentive to central banks to engage in large scale green asset purchase programs. This work aims to provide central banks and similar institutions with the tools to contribute to climate change mitigation and demonstrates the importance of including these institutions in the push to reduce global emission levels.

Keywords: Climate Change, Two-Sector Economy, Macroprudential Policy, Quantitative Easing, Welfare, Risk Premium.

JEL: Q58, E32, E52.

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1 Introduction

Climate change has shifted from a fringe issue to a worldwide emergency. Our understanding of the phenomena and our willingness to act have developed significantly, in part paralleling the ways in which climate change is being experienced around the globe. It has become a hot topic where academics, industry, and lay people alike are finding common ground. As such, growing academic awareness is leading to important literature in the domain. The implementation of a strategy for the substantial reduction of greenhouse gases (GHG) at the global level has become a major priority. Since the Rio Conference in 1992, a debate has raged in academic and political circles over the growth-environmental trade-off. Discussions focus on the means by which economic activities could align with environmental concerns instead of being hindered by assumed mutual exclusivity. In practice, especially in the short and medium terms, however, financial and economic activity on one side, and environmental policy on the other, are in tension. A need for both medium/long- and short-term policies aimed at bridging environmental quality and economic efficiency, as well as addressing financial stability, are in dire need, in order to foster economic sustainability. Of special concern are climate actions that may strongly impact macroeconomic activity, given the potentially high added cost of GHG offsetting. With the substantial effects of climate actions on the overall economy, a growing body of research from the field of macroeconomics and macro-finance, among others, are now tackling these issues.

An increasing interest in a "Green Financial System"—as outlined in the Paris "One Planet Summit" held in December 2017, where "[E]ight central banks and supervisors established a Network of Central Banks and Supervisors for Greening the Financial System (NGFS)"—is putting climate change challenges at the heart of the macro-financial system. NGFS [2019] recently published a call for action in which it outlined the role central banks can play in monitoring and mitigating climate change, considering the adverse impact it could have on financial stability. Integrating climate change challenges within the macro-monetary and macroprudential frameworks is increasingly gaining momentum within institutions such as the European Central Bank (ECB), thus making research that combines macroeconomics and environmental concerns extremely relevant to policy makers. Bolton et al. [2020] recently advocated in a joint publication from the BIS and *Banque de France* for "better coordination of fiscal, monetary and prudential and carbon regulations", which is perfectly in line with the focus of our article.

Tackling climate change challenges requires innovating classic research paths, which tend to favor the use of models that capture only one of the following: environmental externality, macroeconomic fluctuations, or monetary and financial policy. However, as underlined by Rudebusch and Swanson [2012], this limited approach is reductive, and indicates that macroeconomic modeling suffers from theoretical incompleteness. Policy recommendations (based on such models) that aim to mitigate GHG effects should be able to capture macroeconomic variations, monetary and financial policy, as well as environmental constraints, as these are tightly linked.

Apart from a few exceptions (see for instance Fischer and Springborn [2011] and Heutel [2012]) who paved the way by investigating linkages between CO_2 emissions and business cycle fluctuations, most papers in this literature focused on the long-term implications of climate risk on macroeconomics (e.g. Nordhaus [2008], Golosov et al. [2014], Acemoglu et al. [2016], and Van der Ploeg et al. [2020], among others) and do not assess the interaction of fiscal and financial/monetary policies.

Given this gap in the environmental-macroeconomic-financial literature and therefore a lack of approach that encompasses these literatures, our paper seeks to assess the interaction between environmental policies, namely: i) fiscal, ii) monetary, and iii) macroprudential, each of which is aimed at reducing CO_2 emissions by using a heterogeneous macroeconomic production economy. To the best of our knowledge, this is the first article to look at the interaction between environmental, monetary, and macroprudential policies in a dynamic stochastic general equilibrium (DSGE). In the spirit of our work, recent papers by Carattini et al. [2021] and Diluiso et al. [2020] explore similar questions and comforts our findings. Our paper falls within at least three strands of literature. We first build on the canonical versions of New Keynesian (NK) models such as Woodford [2003], Smets and Wouters [2003] or Christiano et al. [2005] to derive the core of our economy[1]. Second, we add environmental components as in Heutel [2012] among others to introduce the environmental constraints, which allows for the analysis of the dynamics of the economy under the presence of the CO_2 externality. However, as opposed to Heutel [2012], we differentiate between green and dirty firms instead of using one sole representation for firms, thus borrowing from the multi-sector literature of Woodford [2003] and Carvalho and Nechio [2016] among others. Finally, we include balance sheet constrained financial intermediaries as in Gertler and Karadi [2011]. Because we introduce a macroprudential authority that can alter this constraint, we also draw on Pietrunti [2017].

Our first finding is that a second-best instrument such as the one used in the European Trading Scheme (ETS) is needed in order to meet the Paris Agreement targets. However, we show that relying on a second-best instrument induces both i) a welfare loss as the cost of carbon under a second-best instrument is higher than the optimal policy, and ii) a risk premium distortion as the sub-optimal policy is subject to high price volatility and sudden changes (Figure I). In order to allow for more flexibility and to ease the welfare burden as well as close the wedge on the risk premium, other policies are greatly needed. On one hand, we show that a sectoral weight macroprudential policy is able to reduce the wedge gap without imposing infeasible regulatory weights on assets held by financial intermediaries. On the other hand, we show that a non-conventional policy such as quantitative easing (QE) is able to close the wedge on the risk premium, which otherwise could alter the monetary policy transmission (Doh et al. [2015]). Thus, macroprudential and monetary policies could play an important role in offsetting the welfare impact stemming from climate change fiscal policy, and reducing the risk premium level and fluctuations, respectively. In particular, we find that sectoral macroprudential weights on loans favorable to the green sector boost green capital and output—meaning that there is a gain in welfare compared to the sub-optimal policy economy without macroprudential policy—while keeping emissions to output ratio at

[1] Note that for simplicity we abstract from wages rigidities.

the desired level. With respect to QE, we simulate an increase in CO_2 prices via a shock on the price level and show how the rise in risk premia level and volatility is then closed by QE policy rules. Our actual findings could be further reinforced if we were to see a transition to a greener economy favoring the green sector over the dirty sector, as illustrated in our simulated transition in Figure II and Figure III, and as argued in the work of Acemoglu et al. [2016], where the focus is on the long-term transition strategies. More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero emissions.

Merging these different sets of policy tools will not only help contribute to this burgeoning field of research and address the gaps identified above, but will also set the path for new analysis in macroeconomics, environmental policy, and monetary policy. The proposed approach can help shape policy and empower central banks among other institutions to address one of the most pressing issues of our time.

This paper is organized as follows: section 2 presents the model, section 3 explains the calibration, section 4 displays the results and section 5 concludes.

2 The Model

Using the NK-DSGE framework as a foundation, the present paper investigates the potential role of fiscal policy, central bank unconventional monetary policy, and macroprudential policy, in mitigating climate change impacts on macro and financial aggregates. We first model our two-sector economy following Woodford [2003] for the labor specific component within the household, and the two-sector production economy following Carvalho and Nechio [2016]. Then, we model the environmental component following Nordhaus [2008] and Heutel [2012], among others. Finally, drawing on Gertler and Karadi [2011], we model the financial intermediaries and the banking sector.

In a nutshell, the economy modeled is described using a discrete set up with time $t \in$

$(0, 1, 2, \ldots \infty)$. The production sectors produce two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms via financial intermediaries. Public authorities decide on the fiscal and environmental policy, while the central bank decides on the monetary and macroprudential policy.

2.1 The Household

At each period, the representative households supply two types of labor to the sectors of which our economy is comprised (i.e 'green' and 'dirty' sectors denoted by $k \in \{g, d\}$[2]), while they also consume and save. Households have two choices to save: lending their money either to the government or to financial intermediaries that will finance firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households cannot lend their money to a financial intermediary owned by one of their members. Household members who are workers supply labor and return their salaries to the household to which they belong.

Agents can switch between the two occupations over time. There is a fraction f of agents who are bankers and a probability θ_B that a banker remains a banker in the next period. Thus, $(1-f)\theta_B$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to the household, which will use them as start-up funds for the new banker.

Households solve the following maximization problem:

$$\max_{\{C_t, L_{t,k}, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L_{t+i,k}^{1+\varphi} \right] \tag{1}$$

s.t.

$$C_t + B_{t+1} = \sum_k \left(\frac{W_{t,k}}{P_t} L_{t,k} + \Pi_{t,k} \right) + \frac{T_t}{P_t} + R_t B_t, \tag{2}$$

[2]where g refers to the green sector and d to the dirty sector.

where $\beta \in (0,1)$ is the discount factor, parameters σ, $\varphi > 0$ shape the utility function of the representative household associated with risk consumption C_t, and labor in each sector k is $L_{t,k}$. The consumption index C_t is subject to external habits with degree $h \in [0;1)$ while $\chi_k > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked for each sector k. Labor supply $L_{t,k}$ in each sector is remunerated at nominal wage $W_{t,k}$. $\Pi_{t,k}$ is profits from the ownership of firms (both financial and non-financial) that will serve as start-up funds for the new banker and T_t is lump sum taxes. As we assume that intermediary deposits and government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired.

Solving the first order conditions and denoting ϱ_t as the marginal utility of consumption, the labor/supply and consumption/savings equations are:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{3}$$

$$\varrho_t = \chi_k \frac{L_{t,k}^{\varphi}}{\frac{W_{t,k}}{P_t}}, \tag{4}$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{5}$$

where the stochastic discount factor is the expected variation in marginal utility of consumption: $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$.

2.2 The Firms

2.2.1 The Final Firms

Using the multi-sector framework from Carvalho and Nechio [2016], and under non-perfect competition, we assume that production is comprised of two sectors. Our representative final firms produce a final good $Y_{t,k}$ in these two competitive sectors. Using no more than capital and labor to produce the intermediate good Y_{jt} (where $j \in (0,1)$ is the continuum of intermediate goods firms), intermediate firms supply the final sectors. In other words, the "bundling" of intermediate goods within the two sectors leads to a final good.

The final economy good is a constant elasticity of substitution aggregate of the two sectors:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}} Y_{t,d}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{6}$$

with $\theta \in (1, \infty)$ the elasticity of substitution between the two sectors, and $\varkappa$ the weight of each sector. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j in each of the two sectors k at prices $P_{jt,k}$:

$$\max_{Y_{jt}} \Pi_t^{\text{Final}} = P_t Y_t - \varkappa \int_0^1 P_{jt,g} Y_{jt,g} dj - (1-\varkappa) \int_0^1 P_{jt,d} Y_{jt,d} dj, \tag{7}$$

where the aggregation of green and dirty firms reads as:

$$Y_{t,k} = \int_0^1 \left(Y_{jt,k}^{1-\frac{1}{\theta_k}} \right)^{\frac{1}{1-\frac{1}{\theta_k}}}. \tag{8}$$

However, while we assume a constant elasticity of substitution between the final sectors, we consider a different elasticity of substitution θ_k between differentiated intermediate goods of the two sectors. As the goods of the two sectors entail different costs, a different elasticity of substitution is considered. This assumption, which shapes the marginal cost structure, is based both on theoretical work of Tucker [2010] as well as on the empirical findings of Chan et al. [2013] and Chegut et al. [2019], where it is found that green projects entail higher marginal cost (7-13 percent higher costs for green projects in the construction industry compared to non green projects depending on the 'greeness' of the project, and 5-7 percent higher costs in the cement and iron & steel sectors, respectively).

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}} \right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} Y_t. \tag{9}$$

Under perfect competition and free entry, the price of the final good is denoted P_t, while the price $P_{t,k}$ is the price index of sector-k intermediate goods. Finally, the price $P_{jt,k}$ is the price charged by firm j from sector k.

The prices of final aggregate goods and for each sector are given by:

$$P_t = \left(\varkappa P_{t,g}^{1-\theta} + (1-\varkappa) P_{t,d}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{10}$$

$$P_{t,k} = \left(\int_0^1 P_{jt,k}^{1-\theta_k} dj\right)^{\frac{1}{1-\theta_k}}. \tag{11}$$

2.2.2 The Intermediate Firms

Our economy is comprised of two categories of firms: i) green corresponding to environmentally-friendly firms with a stock of capital k_g, and ii) dirty with higher emissions rate of a stock of capital k_d relying on CO_2 intensive components.

The representative firms j in each sector k of the modeled economy seek profit maximization by making a trade-off between the desired level of capital and labor. Furthermore, the firms will incur externality costs and choose the level of abatement to maximize their profit. As presented in Heutel [2012] real business cycle model, the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externality deteriorates the environment and the stock of pollutant alters production possibilities of firms. However, we differ from Heutel [2012] insofar incorporating the damages from the stock of emissions through the level of temperature as follows:

$$Y_{jt,k} = d(T_t^{Temp}) \varepsilon_t^{A_k} K_{jt-1,k}^{\alpha} L_{jt,k}^{1-\alpha},\ \alpha \in (0,1), \tag{12}$$

where $d(T_t^{Temp})$ is a convex polynomial function of order 2 displaying the temperature level ($d(T_t^{Temp}) = ae^{-(bT_t^{Temp^2})}$), with $(a,b) \in \mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017].

And where, global temperature $d(T_t^{Temp})$ is linearly proportional to the level of cumulative emissions as argued by Dietz and Venmans [2019]:

$$T_t^{Temp} = \upsilon_1^{Temp}(\upsilon_2^{Temp} X_{t-1} - T_{t-1}^{Temp}) + T_{t-1}^{Temp}, \tag{13}$$

with υ_1^{Temp} and υ_2^{Temp} chosen following Dietz and Venmans [2019].

In addition, α is the classical elasticity of output with respect to capital, and $\varepsilon_t^{A_k}$ is a sector-specific technology shock that follows an $AR(1)$ process: $\varepsilon_t^{A_k} = \rho_{A_k}\varepsilon_{t-1}^{A_k} + \sigma_{A_k}\eta_t^{A_k}$, with $\eta_t^{A_k} \sim \mathcal{N}(0,1)$. Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1-\gamma_d)X_{t-1} + E_{jt} + E^*, \tag{14}$$

where E_{jt} is the flow of emissions from both the green and dirty firms ($E_{jt} = \varkappa E_{jt,g} + (1-\varkappa)E_{jt,d}$) at time t and γ_d is the decay rate. E^* represents the rest of the world emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere.

The emissions level is modeled by a nonlinear technology (i.e. abatement technology μ) that allows for reducing the inflow of emissions:

$$E_{jt,k} = (1-\mu_{jt,k})\varphi_{t,k} Y_{jt,k}. \tag{15}$$

The emissions $E_{jt,k}$ at firm level are proportional to the production $Y_{jt,k}$ with $\varphi_{t,k}$ the fraction of emissions to output.[3] Also, emissions could be reduced at the firm level through an abatement effort $\mu_{jt,k}$. The firms are allowed to invest in an abatement technology, which is assumed to be different between the green and dirty sectors, thus incurring the firms' direct costs.

We model the direct abatement effort costs following Heutel [2012]:

$$Z_{jt,k} = f(\mu_{jt,k})Y_{jt,k}, \tag{16}$$

[3] Contrary to Lontzek et al. [2015], we consider $\varphi_{t,k} = \varphi_k$ constant overtime and calibrate it using Euro Area emission to GDP levels.

where

$$f(\mu_{jt,k}) = \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}},\ \ \theta_1 > 0,\ \theta_2 > 1, \tag{17}$$

with $\theta_{1,k}$ and $\theta_{2,k}$ representing the cost efficiency of abatement parameters for each sector.

Thus the profits of our representative intermediate firms in each sector $\Pi_{jt,k}$ will be impacted by the presence of the environmental externality. The revenues are the real value of intermediate goods $Y_{jt,k}$, while the costs arise from wages $W_{t,k}$ (paid to the labor force $l_{jt,k}$), investment in capital $K_{jt,k}$ (with returns $R^K_{t,k}$), abatement $\mu_{jt,k}$ (the firms are enduring), and any environmental damages captured by emissions $E_{jt,k}$ (environmental taxes).

$$\begin{aligned}\Pi_{jt,k} &= \frac{P_{jt,k}}{P_t}Y_{jt,k} - \frac{W_{t,k}}{P_t}L_{jt} - \frac{R^K_{t,k}}{P_t}K_{jt,k} - \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}}Y_{jt,k} - \frac{\tau_{et,k}}{P_t}E_{jt,k} \\ &= \left(\frac{P_{jt,k}}{P_t} - MC_{t,k}\right)Y_{jt,k},\end{aligned} \tag{18}$$

As firms are not free to update prices each period, they first choose inputs so as to minimize cost, given a price, subject to the demand constraint.

The cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital and labor, as well as the abatement and output, respectively:

$$\frac{R^K_{t,k}}{P_t} = \alpha\Psi_{jt,k}\frac{Y_{jt,k}}{K_{jt,k}}, \tag{19}$$

$$\frac{W^K_{t,k}}{P_t} = (1-\alpha)\Psi_{jt,k}\frac{Y_{jt,k}}{L_{jt,k}}, \tag{20}$$

$$\frac{\tau_{et,k}}{P_t} = \frac{\theta_{1,k}\theta_{2,k}}{\varphi_t}\mu_{jt,k}^{\theta_{2,k}-1}, \tag{21}$$

$$MC_{jt,k} = MC_{t,k} = \Psi_{t,k} + \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \frac{\tau_{et,k}}{P_t}(1-\mu_{t,k})\varphi_t, \tag{22}$$

where $\Psi_{jt,k} = \Psi_{t,k}$[4] is the marginal cost component related to the same capital-labor ratio all firms of each sector choose (footnote (4)). This marginal cost component is common to

[4] $\Psi_{jt,k} = \Psi_{t,k} = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{1}{\varepsilon_t^{A,k}d(T_t^{Temp})}\left(\frac{W_{t,k}}{P_t}\right)^{1-\alpha}\left(\frac{R^K_{t,k}}{P_t}\right)^{\alpha}.$

all intermediate firms, however, it is different across sectors.

Equation (21) is the optimal condition on abatement: abating CO_2 emissions is optimal when its marginal gain equals its marginal cost. This highlights the key role of emissions in shaping price dynamics where the production of one additional unit of goods reduces the profits of firms, which in turn is partially compensated for by the marginal gain from emitting GHG in the atmosphere.

In addition, abatement effort $\mu_{t,k}$ is common to all firms of the same sector, as the environmental cost, which firms of the same sector are subject to, is constant across sectors.

Furthermore, as the impact of the environmental externality is not internalize by the firms (i.e. they take X_t and T_t^{Temp} as given), the shadow value of the environmental externality is zero.

The total marginal cost captures both abatement and emissions costs as shown above in equation (22). Also, we note that in the case of the laissez-faire scenario, $MC_{t,k} = \Psi_{t,k}$ as the firms are not subject to emissions and abatement constraints.

In addition, the monopolistic firms engage in infrequent price setting à la Calvo. We assume that intermediate goods producers for each sector re-optimize their prices $P_{jt,k}$ only at the time when a price change signal is received. The probability (density) of receiving such a signal h periods from today is assumed to be independent from the last time the firm received the signal. A number of firms ξ will receive the price-change signal per unit of time. All other firms keep their old prices. Thus, the profit maximization of our intermediate firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \Pi_{jt+i,k} \tag{23}$$

$$\text{s.t. } Y_{jt+i,k} = \left(\frac{P_{jt,k}}{P_{t+i,k}}\right)^{-\theta_k} \left(\frac{P_{t+i,k}}{P_{t+i}}\right)^{-\theta} Y_t,$$
$$\text{and, } Y_{jt+i,k} = d(T_t^{Temp})\varepsilon_t^{A_k} K_{jt-1+i,k}^{\alpha} L_{jt+i,k}^{1-\alpha}.$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor, or as commonly called in the macro-finance literature, the pricing kernel (for i=1 we note $M_{t,t+1} = \beta\Lambda_{t,t+1}$ as in Jermann [1998]).

The NK Philips Curve pricing equations are as follows:

$$p^*_{t,k} = \frac{P^*_{t,k}}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \mathrm{MC}_{t+i,k} \Im_{t+i,k}}{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \Im_{t+i,k}}, \tag{24}$$

where

$$\begin{aligned} \Im_{t+i,k} &= \left(\frac{1}{P_{t+i,k}}\right)^{-\theta_k} \left(\frac{P_{t+i,k}}{P_{t+i}}\right)^{-\theta} P_t^{\theta} Y_{t+i} \\ &= {P_{t+i,k}}^{\theta_k - \theta} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} Y_{t+i}, \end{aligned} \tag{25}$$

or equivalently:

$$\begin{aligned} p^*_{t,k} &= \frac{P^*_{t,k}}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}}, \qquad (26) \\ \text{with:} S_{t,k} &= P_{t,k}^{\theta_k - \theta} \Psi_{t,k} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^{\theta} S_{t+1,k}, \\ \text{and:} \Theta_{t,k} &= P_{t,k}^{\theta_k - \theta} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^{\theta - 1} \Theta_{t+1,k}, \\ \text{and:} \Upsilon_{t,k} &= P_{t,k}^{\theta_k - \theta} \left[\theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \frac{\tau_{et,k}}{P_t}(1 - \mu_{t,k}) \varphi_t\right] Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^{\theta} \Upsilon_{t+1,k}, \end{aligned}$$

with inflation $\pi_t = P_t / P_{t-1}$.

The pricing equation above is obtained simply by equating the dynamic marginal revenues to the dynamic marginal costs, thus, yielding an optimal pricing condition p^*. As in each period a fraction ξ of the intermediate firms of each sector choose their optimal price P^*_k, we can rewrite the final firms goods price P_k as a weighted average of the last period's price level and the price set by firms adjusting in the current period: $P_{t,k} = (\xi P_{t-1,k}^{1-\theta_k} + (1-\xi) P_{t,k}^{*1-\theta_k})^{\frac{1}{1-\theta_k}}$. In addition, please note that the j-index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t, will make the same decisions.

2.2.3 Capital Producing Firms

We assume that households own capital producing firms and receive profits. Green and dirty firms buy specific types of capital from intermediate goods firms at the end of period t and then repair depreciated capital and create new capital. They then sell both the new and re-furbished capital. The relative price of a unit of new capital is either $Q_{t,g}$ or $Q_{t,d}$. We suppose that there are flow adjustment costs associated with producing new capital. Accordingly, capital producing firms face the following maximization problem:

$$\max_{\{I^n_{t,k}\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s,k} - 1) I^n_{t+s,k} - f_k(.)(I^n_{t+s,k} + \bar{I}_k) \right\} \tag{27}$$

$$\text{with} \quad I^n_{t,k} = I_{t,k} - \delta K_{t,k}, \tag{28}$$

$$K_{t,k} = K_{t-1,k} + I^n_{t,k}, \tag{29}$$

$$\text{and} \quad f_k(.) = \frac{\eta_i}{2} \left(\frac{I^n_{t+s,k} + \bar{I}_k}{I^n_{t+s-1,k} + \bar{I}_k} - 1 \right)^2, \tag{30}$$

where $I^n_{t,k}$ and $I_{t,k}$ are net and gross capital created, respectively, $\bar{I}_k$ is the steady state investment for each kind of firm, $\delta K_{t,k}$ is the quantity of re-furbished capital, and η_i the inverse elasticity of net investment to the price of capital. Thus, we get the following value for $Q_{t,k}$:

$$Q_{t,k} = 1 + f_k(.) + f'_k(.) \left(\frac{I^n_{t,k} + \bar{I}_k}{I^n_{t-1,k} + \bar{I}_k} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f'_k(.) \left(\frac{I^n_{t+1,k} + \bar{I}_k}{I^n_{t,k} + \bar{I}_k} \right)^2 \right\}. \tag{31}$$

2.3 Financial Intermediaries

We modify the setup of Gertler and Karadi [2011] to allow financial intermediaries to invest in both green and carbon-intensive firms. In our baseline framework, we model the incentive constraint as in Pietrunti [2017] allowing for a realistic implementation of macroprudential policy through regulatory weights on loans.

A representative bank's balance sheet can be depicted as:

$$Q_{t,g}S_{t,g} + Q_{t,d}S_{t,d} = N_t + B_t, \tag{32}$$

where $S_{t,g}$ and $S_{t,d}$ are financial claims on green and dirty firms and $Q_{t,g}$ and $Q_{t,d}$ their respective relative price. Note that $S_{t,k} = K_{t,k}$, as firms from both sectors do not face frictions when requesting financing. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, the banks' equity capital evolves as follows:

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,d}Q_{t-1,d}S_{t-1,d} - R_t B_{t-1}, \tag{33}$$

$$N_t = (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d} + R_t N_{t-1}, \tag{34}$$

where $R^k_{t,k} = \frac{R^K_{t,k}/P_t - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denote the gross rate of return on a unit of the bank's assets from $t-1$ to t for sector k.[5]

The goal of a financial intermediary is to maximize its equity over time. Thus, we can write the following objective function:

$$V_t = E_t\left\{\sum_{i=1}^{\infty} \Delta\beta^i \Lambda_{t,t+i}(1-\theta_B)\theta_B^{i-1}N_{t+1}\right\}, \tag{35}$$

where Δ is a parameter allowing to adjust the bankers' discount factor. We introduce a regulator in charge of the supervision of financial intermediaries. Drawing on Pietrunti [2017], we assume that this regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t \geq \lambda_t(\lambda_g Q_{t,g}S_{t,g} + \lambda_d Q_{t,d}S_{t,d}), \tag{36}$$

with λ_t the risk weight on loans and λ_g and λ_d specific weights that can be applied to loans

[5] Note that the depreciated capital has a value of one as adjustment costs only apply to net investment.

for green and/or dirty firms. The regulator can modify these weights, altering the constraint weighing on banks and thus the financial frictions in our economy. In our baseline version of the model, however, we consider the case where λ_g and λ_d are both equal to one, and we calibrate $\bar{\lambda}$[6] to match the steady state capital ratio of European banks. We guess that the value function is linear of the form $V_t = \Gamma_t N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_{t,g}, S_{t,d}} E_t \left\{ \Delta \beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \right\}, \tag{37}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t$. Maximization subject to constraint (36) yields the following first order and slackness conditions:

$$\Delta \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,k} - R_{t+1}) \right\} = \nu_t \lambda_k \lambda_t, \tag{38}$$

$$\nu_t \left[\Gamma_t N_t - \lambda_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}) \right] = 0, \tag{39}$$

where ν_t is the multiplier for constraint (36). One interesting result is that we get:

$$N_t \geq \Xi_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}), \tag{40}$$

where $\Xi_t = \lambda_t / \Gamma_t$ is the capital ratio for banks and λ_g and λ_d represent potential rewards or penalties on the weights required by the regulator on green and dirty loans, respectively.[7] Finally, we rewrite the value function to find Γ_t:

$$\begin{aligned} V_t &= \lambda_t \nu_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}) + \Delta \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \right\} \\ \Gamma_t N_t &= \nu_t \Gamma_t N_t + \Delta \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \right\} \\ \Gamma_t &= \frac{1}{1 - \nu_t} \Delta \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \right\}. \end{aligned} \tag{41}$$

[6] Where the 'bar' variable represent the steady state level.

[7] For instance, if $\lambda_g < 1$ banks will need to hold less capital for loans they grant to green firms compared to dirty firms.

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B[(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d}] + (\theta_B R_t + \omega)N_{t-1}, \tag{42}$$

with $\omega \in [0;1)$ the proportion of funds transferred to entering bankers.

2.4 Public Authorities

2.4.1 Central Bank

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t - \bar{\imath} = \rho_c\left(i_{t-1} - \bar{\imath}\right) + (1 - \rho_c)\left[\phi_\pi\left(\pi_t - \bar{\pi}\right) + \phi_y\left(Y_t - Y_{t-1}\right)\right], \tag{43}$$

where $\bar{\imath}$ is the steady state of the nominal rate i_t, $\rho_c \in [0,1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is the output gap stance penalizing deviations of output from its previous period level Y_{t-1}. Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t\left\{\pi_{t+1}\right\}. \tag{44}$$

Because we want to replicate the current economic conditions as closely as possible, we will calibrate our model such that the nominal rate would be extremely low by historical standards (1 percent at the steady state). This drastically limits the scope of conventional monetary policy, as the central bank can not set its nominal interest rate below zero.

2.4.2 Government

The government sets a budget constraint according to the following rule:

$$T_t + \frac{\tau_{et}}{P_t}E_t + s_{t,g}\psi_{t,g}K_{t,g} + s_{t,d}\psi_{t,d}K_{t,d} = G_t, \tag{45}$$

with the public expenditure G_t finding its source from taxes T_t, revenue from emissions tax $\tau_{et} E_t$ and from public financial intermediation on both green and dirty firms $s_{t,g}\psi_{t,g}K_{t,g}$ and $s_{t,d}\psi_{t,d}K_{t,d}$ (with $s_{t,k}$ the spread between each sector's risky rate and the riskless rate). The government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \tag{46}$$

2.5 Normalization and Aggregation

It is also common in most NK models that in equilibrium, factors and goods markets clear as shown below.

First, the market-clearing conditions for aggregate capital, investment, labor, and wages, in the two sector economy read as[8]: $K_t = \sum_k g(\varkappa) \int_0^1 K_{jt,k} dj$, $I_t = \sum_k g(\varkappa) \int_0^1 I_{jt,k} dj$, $L_t = \sum_k g(\varkappa) \int_0^1 L_{jt,k} dj$, and $W_t = \sum_k g(\varkappa) \int_0^1 W_{jt,k} dj$.

Similarly, global aggregate emissions and aggregate emissions cost are two weighted sums of sectoral emissions $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k} dj$, and sectoral emissions cost $Z_t = \sum_k g(\varkappa) \int_0^1 Z_{jt,k} dj$, respectively.

As presented in Gali and Monacelli [2008], the Calvo $D_{pt,k}$ price dispersion is essentially a measure of distortion introduced by dispersion in relative prices. This shows that there is an additional distortion associated with relative price fluctuations owing to price stickiness. The Calvo $D_{pt,k}$ price dispersion is bounded below at 1, where 1 would be the value in the case of flexible prices, where all firms choose the same price. The price dispersion in our two-sector economy reads as:

$$\int_0^1 Y_{jt,k} \mathrm{dj} = \int_0^1 \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_{t,k} \mathrm{dj} = D_{pt,k} Y_{t,k}, \tag{47}$$

with $D_{pt,k}$ the aggregate loss of efficiency induced by price dispersion of the intermediate goods. In other words, it also reads as $D_{pt,k} = (1-\xi)\left(\frac{P_{t,k}}{P_t}\right)^{(\theta_k-\theta)} \left(p^*_{t,k}\right)^{-\theta_k} +$

[8]Where $g(\varkappa) = \varkappa$ for sector the green sector g and $(1-\varkappa)$ for the dirty sector d.

$\xi\left(\frac{P_{t,k}}{P_t}\right)^{-\theta}\pi_{t,k}^{\theta_k}D_{pt-1,k}$.

Furthermore, as outlined in Annicchiarico and Di Dio [2015], in addition to the departures from the canonical NK model[9], our two-sector environmental components are impacted by the price dispersion as following:

$$E_{t,k} = (1-\mu_{t,k})\varphi_k D_{pt,k} Y_{t,k}, \tag{48}$$

$$Z_{t,k} = \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} D_{pt,k} Y_{t,k}. \tag{49}$$

Finally, the resource constraint of the economy reads as follows:

$$Y_t = C_t + G_t + I_t + \sum_k f_k(.)(I_{t+s,k}^n + \bar{I}_k) + Z_t. \tag{50}$$

2.6 Climate Externality and Financial-Economics Inefficiencies

2.6.1 Competitive Equilibrium

To pin down the optimal policy[10], we solve for the Competitive Equilibrium ("CE"). The CE in this economy is defined as follows:

Definition 1 *A competitive equilibrium consists of an allocation* $\{C_t, L_{t,k}, K_{t,k}, E_{t,k}, X_t, T_t^{Temp}\}$, *a set of prices* $\{P_t, R_t, R_{t,k}^k, W_{t,k}\}$ *and a set of policies* $\{\tau_{et,k}, T_t, B_{t+1}\}$ *such that*

- *the allocations solve the consumers', firms', and banks'* [11] *problems given prices and policies,*
- *the government budget constraint is satisfied in every period,*
- *temperature change satisfies the carbon cycle constraint in every period, and*

[9] Where: $Y_{t,k} = d(T_t^{Temp})\varepsilon_t^{A_k} K_{t-1,k}^{\alpha} L_t^{1-\alpha} D_{pt,k}^{-1}$ and $\Pi_{t,k} = (1 - MC_{t,k}D_{pt,k})Y_{t,k}$.

[10] As we consider a closed economy, we assume that cooperation takes place in such a way to avoid free-riding and potential carbon leakages. This is achieved by setting E^* to a constant.

[11] The banks' problem remains the same as the one presented in the financial intermediaries section.

- *markets clear.*

Definition 2 *The optimal solution sets the carbon tax $\tau_{et,k}$ as an optimal policy $\tau^*_{et,k}$, which maximizes the total welfare in equation Equation 1*[12,13]*:*

$$\frac{\tau_{et,k}{}^*}{P_t} = g(\varkappa) SCC_t. \tag{51}$$

with SCC_t the social cost of carbon:

$$SCC_t = \eta\beta\frac{\lambda_{t+1}}{\lambda_t} SCC_{t+1} + (\upsilon_1^{Temp}\upsilon_2^{Temp})\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T, \tag{52}$$

and with,

$$\S_t^T = (1-\upsilon_1^{Temp})\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^{Temp})}{\partial T_t^{Temp}} K_{t-1,k}^{\alpha} L_{t,k}^{1-\alpha} \tag{53}$$

2.6.2 Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 3 *The public authorities, however, do not optimally set the carbon price as highlighted in definition 2. In the EU area, public authorities target an emissions level/price that is consistent with their objective of a 55% emissions reduction by 2030. In practice, this means gradually increasing the cost of carbon through the reduction of emission quotas distributed to firms within specific sectors. We model this situation by assuming that the fiscal authority targets a specific carbon price that can be hit by exogenous shocks:*

$$\frac{\tau_{et,k}}{P_t} = \epsilon_{t,k}^{Tax} Carbon\ Tax. \tag{54}$$

This flexible representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives. That said, the targeted CO_2 level/price is assumed to be constant at the business cycle frequency.

[12]Where $g(\varkappa) = \varkappa$ for sector the green sector g and $(1-\varkappa)$ for the dirty sector d.

[13]The full derivation of the CE can be found in the technical appendix

2.6.3 Welfare Distortion

Definition 4 *The welfare distortion arises when there is a difference between the optimal environmental policy and the targeted policy consistent with the EU objectives:*

$$\frac{\tau_{et,k}{}^*}{P_t} \neq \frac{\tau_{et,k}}{P_t} \tag{55}$$

When $\frac{\tau_{et,k}{}^*}{P_t} \neq \frac{\tau_{et,k}}{P_t}$*, the welfare cost is higher, thus inducing a loss in household lifetime consumption*[14]*:*

$$\Delta_{\{\tau-\tau^*\}} Welfare < 0 \tag{56}$$

where the welfare could be decomposed into consumption and labor components as follows:

$$\begin{aligned} Wedge_C \propto & (1-g)(\varepsilon_t^{A,k}\bar{L}^{1-\alpha})(d(T_t^{Temp})K_{t-1,k}^{\alpha} - d(T_t^{Temp})^* K_{t-1,k}^{\alpha}{}^*) - (f(K_{t,k}) - f(K_{t,k})^*) \\ & - ((\varepsilon_t^{A,k}\bar{L}^{1-\alpha})(d(T_t^{Temp})K_{t-1,k}^{\alpha} f(\mu_{t,k}) - d(T_t^{Temp})^*)K_{t-1,k}^{\alpha}{}^* f(\mu_{t,k})^*) \\ Wedge_L \propto & \sum_k (1-\alpha)(\varepsilon_t^{A,k}\Psi_{t,k} d(T_t^{Temp})K_{t-1,k}^{\alpha} - \varepsilon_t^{A,k}\Psi_{t,k}{}^* d(T_t^{Temp*})K_{t-1,k}^{\alpha}{}^*)\bar{L}^{-\alpha} \end{aligned}$$

Proposition 1 *To reduce the welfare wedge on consumption and labor supply, we propose a sectoral macroprudential policy targeting a capital ratio requirement as in Basel III.*

Implementing a higher policy rate compared to an optimal policy clearly decreases the damages from temperature to production $d(T_t^{Temp}) < d(T_t^{Temp})^*$. Similarly, abatements are costlier under the higher policy rate. This results in a loss of welfare, but prevents potential climate risks in the future that are not internalized by firms nor by households. The sectoral-maroprudential policy, which will lower the capital requirement for green assets, will in turn trigger a rise in green firms' capital. As green firms are less subject to the carbon price, the increase in the relative size of the green sector, compared to the dirty one, will lead to a welfare gain.

[14] A full decomposition of the welfare effect is presented in Appendix C.

2.6.4 Risk Premium Wedge

Using a second-best instrument, such as a market of carbon permits, introduces short-term price volatility and sudden price changes that have a direct impact on risk premia.

Definition 5 *When the carbon pricing is modeled following a second-best instrument, the variances of corporate risk premia are higher, as compared to an economy where the instrument used isn't subject to high levels of volatility.*[15] *In the case of a positive carbon price shock, the marginal cost of firms increases as they are now subject to higher CO_2 prices. This in turn will raise the risk premium:*[16]

$$RP_{t,k} = R^k_{t,k} - R_t \tag{57}$$

$$= f(\Psi_{t,k}, Y_{t,k}, K_{t-1,k}, Q_{t,k}) - R_t \tag{58}$$

Proposition 2 *Volatility in risk premia stemming from carbon price fluctuations could potentially distort the functioning of monetary policy operations. To prevent this situation, we propose a short-term monetary policy: a QE rule, which reacts to changes in risk premia in order to ensure financial stability.*

The risky rate reacts to changes coming both from the firms' side and the financial side. In this case, the goal is to cut the link between the rise of the marginal cost (triggered by an increase in the carbon price) and the impact on the risk premium. One way to do so is to act on the financial side to compress the risk premium. Similar to models where a rise in risk premia comes from an exogenous shock on the quality of capital (e.g. crisis simulation in Gertler and Karadi [2011]), the central bank is able to circumvent this effect by stepping into the loan market.

[15] We will use the shock on the price level $\epsilon^{Tax}_{t,k}$ in Equation 55 in the quantitative analysis section.

[16] The impact is symmetric in the case of a negative carbon price shock. Furthermore, whether the shock is positive or negative, it implies higher volatility for the marginal cost and the risk premium.

2.7 Set of Policies

Environmental Policy

When acting optimally, the decentralized planner would set the environmental policy as shown in Definition 2 ($\tau^*_{et,k}$ is set equal to the social cost of carbon $g(\varkappa)SCC_{t,k}$). However, as highlighted in the previous section, the EU authorities depart from the optimal policy and set the environmental policy following the ETS system in such a way so as to be able to ratify the Paris Agreement and follow the EU emissions reduction objectives ($\tau_{et,k} \neq \tau^*_{et,k}$).

Sectoral Macroprudential Weights

There is a macroprudential regulator with the ability to modify the regulatory constraint weighing on banks detailed in subsection 2.3. We include in our baseline model a general macroprudential rule akin to a Countercyclical Capital Buffer, a defined in Basel III:

$$\lambda_t = \bar{\lambda} + \rho_\lambda \lambda_{t-1} + (1-\rho_\lambda)\phi_\lambda \Big(\frac{K_{t-1}}{\frac{1}{T}\sum_i Y_{t-i}} - \frac{K_{t-2}}{Y_{t-2}} \Big), \tag{59}$$

where λ_t reacts to change in the average credit to GDP ratio in the last four quarters, net of the last period. This rule forces banks to hold more capital when the credit to GDP gap is growing. For the purpose of our research we also allow the macroprudential authority to adjust the sectoral weights on loans λ_g and λ_d to favor either the green or the dirty sector. This will tighten or loosen the regulatory constraint on banks, forcing them to reduce their stock of loans or letting them lend more freely to a specific sector.

Quantitative Easing

The current low rates environment implies that central banks must prove innovative to keep fulfilling their mandates in a liquidity trap environment. A common alternative to nominal interest rate setting is the use of assets purchase programs, also referred to as QE. In the model section, we show how the value of loans to both dirty and green firms are determined. We now introduce a central bank that can substitute for financial intermediaries in financing these firms. Much like the Corporate Sector Purchase Program in the Euro Area, the central bank has the ability to fund non-financial firms in order to reduce corporate spread, steer private investment, and ultimately keep inflation within range of its target. Then for each type of firm k we now have:

$$Q_{t,k}S_{t,k} = Q_{pt,k}S_{pt,k} + Q_{gt,k}S_{gt,k}, \tag{60}$$

with $Q_{gt,k}S_{gt,k}$ the total real value of loans to firms of type k held by the central bank. $Q_{pt,k}S_{pt,k}$ is the total real value of loans to firms of type k held by financial intermediaries as defined in 2.3. As in Gertler and Karadi [2011], we model this intervention by assuming that the central bank holds a portion $\psi_{t,k}$ of total loans to non-financial firms belonging to each sector:

$$Q_{gt,k}S_{gt,k} = \psi_{t,k}Q_{t,k}S_{t,k}. \tag{61}$$

For simplicity, we abstract from monitoring costs. We assume that the central bank follows a counter-cyclical credit policy rule that reacts to the variations in the anticipated spread ($\text{RP}_{t,k} = R_{t+1,k} - R_t$) in order to decide the share of assets $\psi_{t,k}$ it holds. This rule is defined as follows:

$$\psi_{t,k} = \rho_{u_k}\psi_{t-1,k} + (1 - \rho_{u_k})(\phi_k^s(\text{RP}_{t,k} - \bar{\text{RP}}_k)), \tag{62}$$

where $\rho_{u_k} \in [0, 1)$ is the rule smoothing coefficient. Note that in our baseline model $\psi_{t,k} = 0$ so that the central bank allows financial intermediaries to be the sole source of funding for firms.

3 Model Solving

In order to solve for the medium/long-run pathways scenario, we use the extended path algorithm, which allows for both integrating deterministic trends and stochastic shocks. This approach aims to keep the ability of deterministic methods to provide accurate account of non-linearities, where usual local approximation techniques don't perform as well under the presence of such non-linearities (Adjemian and Juillard [2013]). As for addressing the short-term business cycle simulations, we use perturbation methods as they are usually performed in the macro literature.

4 Calibration

Calibrated parameters are reported in Table I, Table II, and Table III. For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio at 40 percent[17], the share of hours worked per day at one third in each sector, and the capital intensity in the production function α at 0.33. The inverse elasticity of net investment to the price of capital η_i is set at 1.728 as in Gertler and Karadi [2011] and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2, as argued by Stern [2008] and Weitzman [2007]. We set the discount factor at 0.9975 to get a steady state real interest rate of 1 percent. This choice is motivated by the low interest rate environment witnessed in recent years.

Regarding the environmental part, we calibrate the damage function according to Dietz and Stern [2015][18]. The global temperature parameters υ_1^{Temp} and υ_2^{Temp} are set following Dietz and Venmans [2019] to pin down the 'initial pulse-adjustment timescale' of the climate system. The level of the remainder of the world's emissions E^* is set at 1.55 in order to replicate the global level of carbon in the atmosphere of 840 gigatons. To calibrate the share of the green sector, what we consider green in our model is a sector with a carbon performance allowing for an emission target aligned with the Paris Agreement of 2 degrees Celsius or below. We use sectoral data made available by the Transition Pathway Initiative to set the share of green firms $\varkappa$ to 30 percent. Furthermore, as argued by De Haas and

[17] We match the level of the Euro Area.

[18] We perform a sensitivity analysis using values from Nordhaus and Moffat [2017] and Weitzman [2012] in the next section.

Popov [2019], CO_2 emissions intensity differs largely between sectors and industries. We use the carbon intensity parameters φ_d and φ_g to match the observed ratio of emissions to output for the Euro Area (EA) at 21 percent[19]. Assuming that the carbon intensity in the green sector is one third of what it is in the dirty sector, we find that $\varphi_d = 0.33$ and $\varphi_g = 0.11$. The abatement parameters $\theta_{d,1}$, $\theta_{d,2}$ and $\theta_{g,2}$, which pin down the abatement costs for each sector are set as in Heutel [2012]. We then proceed to set $\theta_{g,1}$ to match the drop in emissions induced by the introduction of the carbon tax in the EA. More precisely, we retrieve the value of $\theta_{g,1}$ in such a way so as to be consistent with a reduction of emissions of 14.3 percent between 2009 and 2019[20], which is associated with an increase in the tax from 0 to 24.9 euro (the average price of ETS in 2019). In our model, this leads to a value of $\theta_{g,1}$ of 0.02, which means that the abatement technology is cheaper in the green sector. The decay rate of emissions δ_x is set at 0.21 percent. Finally, θ_d, the dirty firms' marginal cost parameter, is calibrated as in Smets and Wouters [2007] to replicate the mean markup and marginal cost levels observed in the economy, while θ_g, as highlighted in subsection 2.2, is calibrated such that the green marginal cost is 6 percent higher than the dirty marginal cost as argued by Chan et al. [2013] and Chegut et al. [2019].

As for the financial parameters, we set the probability of remaining a banker θ_B at 0.98, meaning that 2 percent of bankers default every quarter, which is slightly less than in Gertler and Karadi [2011]. $\bar{\lambda}$ is calibrated at 0.0177 to generate a spread of 80 basis points between risky and risk-less assets. This value is taken from Fender et al. [2019]. The authors also find that the spread between green and dirty bonds recently disappeared. Thus, we target the same steady state for R_g and R_d. Δ is a parameter allowing the introduction of a different discount factor in the bankers' objective function relative to households and is set to 0.99. The proportional transfer to the entering banker ω is set to 0.004 in order to match a capital ratio of approximately 14.4 percent in the EA. Finally, the monetary rule parameters are set as in Smets and Wouters [2003].

[19] We compute this value as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from FRED.

[20] We remove the first and the last year of data.

Regarding shocks processes, we fit an AR(1) to the ETS data to find the standard deviation and auto-regressive parameter in the tax shock (0.64 and 0.95, respectively). For the auto-regressive parameter of the QE shock, we set it at 0.66 to account for the reinvestment of proceeds and the slow exit of assets from the central bank's balance sheet.

5 Quantitative Analysis

5.1 Fiscal Environmental Policy Scenario

The goal of this section is to find and analyze a theoretical fiscal pathway consistent with the objective of the EU for 2030. To this end, we use a reduced version of the model that does not include financial intermediaries and investment adjustment costs. This is without a loss of generality, since we are interested in the firms' trade-off between tax and abatement in the medium run, and not in the impact of financial frictions on the business cycle. This reduced form does however ease the computational burden. We thus find the trajectory of the carbon price that leads to a desired reduction in emissions (either 40 percent or 55 percent relative to the level of 1990). We then highlight the negative impact of a sub-optimal carbon price on welfare.

5.1.1 Growth, carbon price, and the EU objectives

Figure IV shows what carbon price trajectories would be needed to be on track for achieving the Paris Agreement in the EU, according to two different growth scenarios. The blue dashed line is the central scenario with a growth trend of 0.8 percent, corresponding to the average real growth rate of the EU area from 2000 to 2020. The green solid line is a scenario with a growth trend of 1.6 percent. We also add stochastic components drawn from random disturbances to TFP and the carbon price[21]. This allows us to simulate a

[21] We set the standard deviation of the TFP shock to .008 as estimated by Benmir et al. [2020], while we set the standard deviation of the carbon price shock to 0.086 as to match the ETS carbon price quarterly volatility between 2008 and 2019

realistic transition scenario, where trends in growth and carbon price are anticipated, but shocks can distort these deterministic processes in the short run. Depending on the growth scenario, reducing emissions by 40 percent compared to 1990 level would require a carbon price between 60€ and 80€ per ton of CO_2 (ignoring the impact of temporary shocks). We also find that the price of carbon needs to follow the growth of output to be able to shrink the flow of emissions to the desired level.

Figure V uses the central growth scenario to compare the Paris Agreement trajectory to the net-zero trajectory[22]. We find that being on the path toward net-zero emissions would require a carbon price as high as 100€ per ton of CO_2 by 2030. This ambitious goal would have a negative impact on output and consumption. Note, however, that our model takes the abatement technology as given. With improving technology, the EU could reach the same target with a lower carbon price, but the mechanisms to trigger this improvement in the abatement technology are left for further research.

5.1.2 Welfare implications

The loss in output and consumption associated with the rising carbon price also have consequences on welfare. Figure VI plots the trajectory of the second-best environmental policy chosen by the EU compared to the optimal environmental policy. This optimal policy trajectory is not able to meet the Paris Agreement, and certainly not the net-zero target. However, the carbon price needed to achieve the Paris Agreement is found to alter the welfare, as the household utility of consumption tends to deteriorate when a tax policy is introduced since the utility of consumption does not capture the effects of climate change directly[23]. Figure VI shows that the welfare loss increases over time as the carbon price keeps rising, thus introducing a distortion with respect to the first-best allocation. We will see in the next section that this effect can be partially offset by sectoral macroprudential weights.

[22]The EU recently announced its willingness to reach net-zero by 2050, which means reducing emissions by 55 percent in 2030 compared to 1990.

[23]Benmir et al. [2020] show how the welfare improves if the households internalize the externality ($u_{xc} \neq 0$).

As reported in our sensitivity analysis (Table IV), the optimal price of carbon in 2020 depends on the specification of the damages. Hence, different calibrations from the literature give an initial range of carbon price from 12.9€ to 40.6€ . Interestingly, this result is obtained while keeping β and σ constant, which are parameters known to drive the level of the optimal price of carbon in the Integrated Assessment Models (IAM) literature[24]. For the remainder of the paper, we set the damages parameters *à la* Dietz.

5.2 Introducing Macroprudential Policy

In order to reduce the welfare gap induced by the sub-optimal policy, we investigate the role macroprudential policy could play. We report in Table V the effect of a simple drop in the weight on green loans in the regulatory constraint. The idea is that the regulator wants to give an incentive to banks to invest in green loans rather than dirty loans. For financial intermediaries, it means they have to hold less equity to maintain the same level of loans as in the green sector. In other words, we expect this shift in λ_g to increase green capital K_g at the steady state and hence to lead to a greener economy. We will also see in the next section that it modifies the behavior of banks when they face shifts in risk premia.

To investigate the role of targeted weighted macroprudential policy, we look at the steady state implied by a carbon price of 100€ , which is a likely value for 2030 projections outlined in the previous section. We then compare the results of the model under this carbon price to the optimal one, and to a model with a high tax but where macroprudential policy is active. We then set λ_g to 0.7, maintaining λ_d unchanged at 1.

We find that a 100€ carbon price leads to a consumption loss of roughly 1.3 percent, but it allows for reaching the targeted reduction in emissions. In this context, the implementation of a lower macroprudential weight for the green sector boosts the green capital stock by more than 3.1 percent, resulting in a rise in green output of 1.02 percent. The result is that the consumption loss induced by the high carbon price is reduced by approximately 30 percent, while the output to emissions ratio remains constant. However, this goes hand in hand with

[24] See Nordhaus [2008] and Stern [2008] for example.

a decrease in the rate on green loans, inducing a spread between dirty and green rates. In our setup, this will have consequences on the behavior of banks that have to maximize their objective function.

5.3 Quantitative Easing and the Policy Mix

5.3.1 Risk Premia Stabilization

We now introduce quantitative easing. As defined above, the central bank has the ability to substitute to financial intermediaries in financing either green or dirty firms. We first show how this policy affect the response of risk premia to tax shocks. As the EU decided to implement its environmental fiscal policy through carbon permits, there is an inherent variance in the price of carbon[25]. Estimating the parameters of the shock on the ETS series and simulating the model allow us to analyze how these unexpected variations in the carbon price could affect firms and banks.

Figure VII presents the responses of risk premia to a positive shock to the tax level. We compare two scenarios: i) a model with only environmental policy (ETS like system), ii) a model with an environmental policy (ETS like system) and QE. The exercise confirms that QE rules are efficient instruments in mitigating the impact of tax shocks on risk premia. The increase in the spreads is almost completely offset by an increase in $\psi_{t,k}$ of less than 0.5 percent and the volatility observed in the other scenario is drastically reduced. The mechanism at play here is the same as in the case of exogenous financial shocks on risk premia, except that the initial rise in risk premia is triggered by the shock on the carbon price and its subsequent effect on firms' marginal costs. By stepping in to directly lend to firms, the central bank is able to restore the equilibrium on the loans market and avoid potential negative effects coming from the rise of spreads. Table VI confirms that the variance of risk premia is significantly reduced in the presence of QE rules.

Figure VIII shows how the central bank would react if the sectoral macroprudential

[25] Table VI displays the moments of risk premia and marginal costs for both sectors following a positive shock on carbon prices.

weight favorable to the green sector were to be implemented. In this case, the shock on carbon price has a bigger impact on the dirty risk premium, and a smaller impact on the green risk premium. This is because banks have an incentive to hold green assets rather than dirty assets, creating a stabilization effect on the green spread, but amplifying the reaction of the dirty spread to the shock. Thus, the central bank adjusts its reaction by buying fewer green assets, while still being able to completely offset the impact of the shock on risk premia. Dirty asset purchases, however, are higher when the green sector is favored by a lower macroprudential weight.

5.3.2 Asset Purchase Program Scenario

Although QE rules can be used as a short-term instrument to partially offset financial shocks (that could be stemming from shocks to the carbon price as previously shown), asset purchases are often part of large scale planned programs. The idea of integrating environmental criteria in the portfolio choices of central banks is currently gaining momentum. In particular, the ECB President Christine Lagarde recently advocated for a green strategic shift in the conduct of unconventional monetary policy[26]. In practice, the ECB is already buying green corporate bonds ("20 percent of all available green bonds" according to President Lagarde), but has yet to differentiate between green and dirty bonds in its policy framework. Considering green bonds' issuance is rapidly growing and european governments are also gradually emitting more green bonds to finance the transition to a less carbon-intensive economy, it is worth investigating the impact central bank asset purchases directed toward green projects could have.

The scenario studied here is a series of four positive 2 percent shocks on ψ_t^k. This is akin to a purchase program decided by a monetary authority and results in the central bank holding a bit more than 12 percent of either green or dirty assets at the peak of the program. As we want to replicate a planned purchased program, we deactivate the reaction to the spread by setting ϕ_k to 0. We calibrate the auto-regressive parameter to 0.66 so that the

[26]https://www.ecb.europa.eu/press/inter/date/2020/html/ecb.in200723 0606f514ed.en.html

assets bought slowly exit the central bank's balance sheet.

Figure IX and Figure X display the reaction of selected variables to a series of positive dirty and green QE shocks, respectively. We plot the responses when both the QE and the tax are active (blue line), and when the QE, the tax, and the macroprudential policy are all active (red dotted line).

A first interesting finding—and a crucial one—for a central bank is that dirty and green QE both induce a rise in the inflation rate. These programs both lead to an increase in the inflation rate of roughly 1.6 percent at an annual rate, absent any other shock. The effect on inflation is slightly weakened when sectoral macroprudential weights are active. It is a prerequisite that green QE has a positive impact on inflation in order to become a potential monetary policy tool, and these results indicate that a green QE could also be justified on the ground of low inflation expectations.

A second result is that there is no apparent reason for a central bank to implement green QE rather than dirty QE. This can be explained by the fact that the two assets are perfectly substitutable for financial intermediaries, meaning that their responses to either green or dirty QE will be exactly the same. When introducing sectoral weights on loans, however, public authorities can alter this mechanism.[27] In this case, a trade-off appears between higher GDP growth and lower emissions. With both types of QE, the introduction of macroprudential policy favorable to the green sector allows the reduction of emissions relative to output. However, opting for green QE leads to a greater drop in emissions, at the cost of a smaller boost to GDP and inflation. This trade-off would disappear in the event that the green sector expands enough to be as big or bigger than the dirty one. Policy makers could then achieve both higher output and lower emissions with the above-mentioned policy coordination.

Figure II and Figure III represent the transition paths where the weight of the greener sector is gradually increasing, thus making the greener sector predominant. Moving toward a greener economy not only decreases substantially emissions, which in turn decreases the

[27] Similarly, Ferrari and Nispi Landi [2021] break the perfect substitutability by introducing a quadratic cost related to the holding of green bonds.

environmental policy (i.e. the tax), it also helps achieve the so sought-after decoupling of emissions and output. The emissions to output ratio $E_Y = E/Y$ falls almost linearly with an increase in the weight of the green sector and drive the level of the tax to a lower level than the one needed to pledge the Paris agreement.

6 Conclusion

We developed a macro-environmental-financial DSGE model with both endogenously-constrained financial intermediaries and heterogeneous firms. We then used the model to assess the effects of various policies and their interactions on carbon emissions.

We find that a second-best instrument of about 100€ per ton of carbon is needed to be aligned with the net-zero target. However, the actual implementation of this second-best instrument induces two inefficiencies. The first inefficiency is linked to the need of an increasingly higher price of carbon (compared to the optimal) to meet the EU targets. This decoupling generates a growing welfare loss. To address this wedge, we show that a sectoral macroprudential policy is efficient in partially offsetting the welfare loss while reaching the emissions target. The second inefficiency is related to the market design of the environmental fiscal policy in the EU area. The present volatility in the ETS is shown to affect firms' marginal costs and thus to alter risk premia. We find that QE rules that react to changes in risk premia are able to completely offset movements in spread levels and volatility, allowing for a smooth transmission of monetary policy.

Turning to QE programs, we find that macroprudential policy is needed to provide an incentive to central banks to engage in green QE. Choosing between dirty and green QE then implies a trade-off between higher output and lower emissions. This trade-off would disappear in the event that the green sector grows enough to be as big as or bigger than the dirty sector. More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero.

We hope that this article will pave the way for more research on the interaction between environmental, monetary, and macroprudential policies. Many extensions could be conducted using our framework. In particular, we think that further research could be devoted to the impact of non-linearities within the financial sector on the dynamics of the model and to the role that endogenous TFP could play in fostering the emergence of greener output growth. We also believe it could be fruitful to examine how to capture the environmental quality on the welfare of households in more direct ways than in existing models.

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A Appendix: Tables

TABLE I
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
<u>Standard Parameters</u>		
β	Discount factor	0.9975
α	Capital intensity	0.33
δ	Depreciation rate of capital	0.025
h	Habits formation parameter	0.8
σ	Risk aversion	2
φ	Disutility of labor	1
η_I	Capital adjustment cost	1.728
$\varkappa$	% of Green firms in the economy	30
θ	Price elasticity	5
θ_g	Price elasticity in sector G	11
θ_d	Price elasticity in sector D	7
ξ	Price stickiness (Calvo parameter)	3/4
$\bar{L}$	Labor supply	1/3
$\bar{g}/\bar{y}$	Public spending share in output	0.4

TABLE II

Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Environmental Parameters		
$\bar{e_d}/\bar{y_d} = \varphi_d$	Emissions-to-output ratio in sector D	0.33
$\bar{e_g}/\bar{y_g} = \varphi_g$	Emissions-to-output ratio in sector G	0.11
γ_d	CO_2 natural abatement	0.0021
$\theta_{1,g}$	Abatement cost parameter for sector G	0.02
$\theta_{2,g}$	Abatement cost parameter for sector G	2.7
$\theta_{1,d}$	Abatement cost parameter for sector D	0.05
$\theta_{2,d}$	Abatement cost parameter for sector D	2.7
υ_1^{Temp}	Temperature parameter	0.5
υ_2^{Temp}	Temperature parameter	0.00125
a	Damage function parameter	1.004
b	Damage function parameter	0.02

TABLE III

Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Banking Parameters		
ω	Proportional transfer to the entering bankers	0.004
Δ	Parameter impacting the discount factor of bankers	0.99
$\bar{\lambda}$	Steady state risk weight on loans	0.0177
ρ_λ	Smoothing macropru rule coefficient	0.9
ϕ_λ	Credit gap policy parameter	0.2
θ_B	Probability of staying a banker	0.98
ρ_c	Smoothing monetary rule coefficient	0.8
ϕ_y	Output policy parameter	0.2
ϕ_Π	Inflation policy parameter	1.5

TABLE IV

Sensitivity of the optimal carbon price to the damage function calibration

	$b = 0.01$	$b = 0.02$	$b = 0.04$
Optimal Carbon Price (€)	12.9	23.5	40.6

Notes: The figures reported in the table show the sensitivity of the optimal price of carbon to different level of calibration for the damage function. $b = 0.01$ corresponds to Nordhaus and Moffat [2017], $b = 0.02$ corresponds to Dietz and Stern [2015] and $b = 0.04$ corresponds to Weitzman [2012].

TABLE V

Steady state values –Tax versus Tax and Macroprudential Policy

	Optimal Policy	EU Tax Policy	EU Tax and MacroPru
Consumption	0.9309	0.9191	0.9223
Aggregate Output	1.9521	1.9351	1.9443
Green Output	1.0099	1.0079	1.0182
Dirty Output	0.9789	0.9647	0.9647
Aggregate Emissions	0.2104	0.0917	0.0919
Green Sector Emissions	0.0916	0.0458	0.0462
Dirty Sector Emissions	0.2613	0.1114	0.1114
Green Capital Stock	10.2486	10.0984	10.4131
Dirty Capital Stock	9.3233	8.8432	8.8431
Green Real Rate	1.0045	1.0045	1.0039
Dirty Real Rate	1.0045	1.0045	1.0045
Agg. Tax (in euros)	22	100	100
Tax as % of GDP in Green Sector	0.1804	0.8952	0.8952
Tax as % of GDP in Dirty Sector	0.5306	2.276	2.276

TABLE VI

Risk premia volatility under the carbon price shock

	Mean	Standard Deviation	Variance
Tax Model			
EP_g	0.2542	0.4279	0.1831
EP_d	0.2548	0.4279	0.1831
MC_g	0.9079	0.0039	0.0000
MC_d	0.8587	0.0097	0.0001
Tax and QE Rules Model			
EP_g	0.2121	0.0128	0.0002
EP_d	0.2121	0.0128	0.0002
MC_g	0.9087	0.0036	0.0000
MC_d	0.8569	0.0118	0.0001

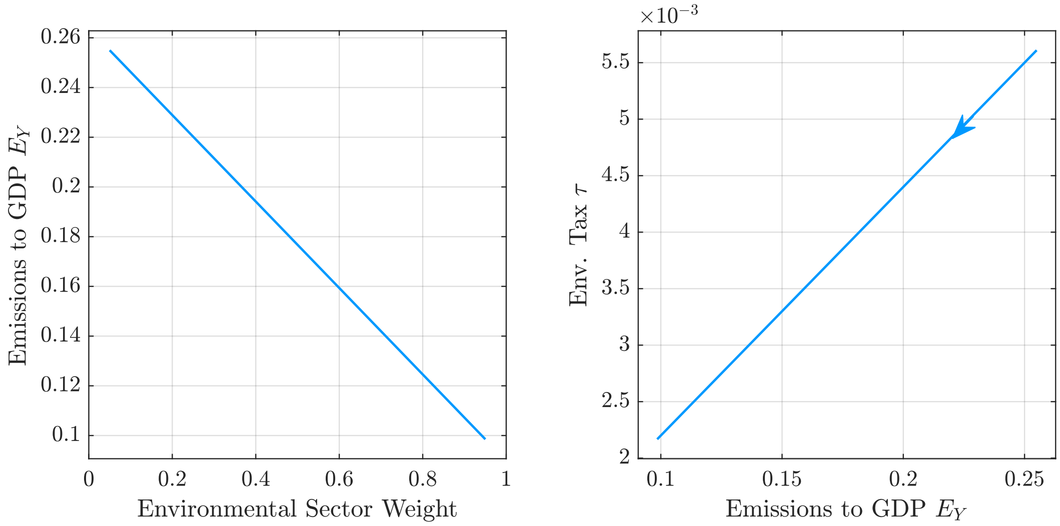
Notes: The figures reported in the table show the moments of selected variables following a positive carbon price shock.

B Appendix: Figures

FIGURE I. ETS Price in Euros

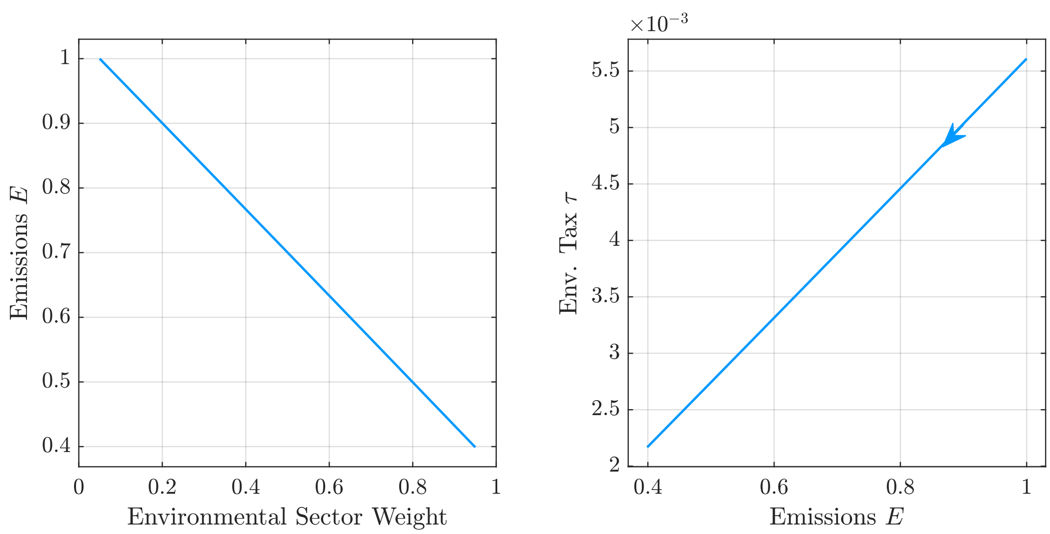
Source: Ember Climate.

FIGURE II. Sectoral weights, carbon intensity, and the environmental policy



Notes: The graph on the left reports the interaction between emissions to output and sectoral weights. The right graph reports how sectoral weight through emissions to output drives the carbon tax.

FIGURE III. Sectoral weights, emission levels (normalized to one), and the environmental policy



Notes: The graph on the left reports the interaction between emissions and sectoral weight. The right graph reports how sectoral weights shape the carbon tax.

FIGURE IV. Transition scenarios with three different growth assumptions

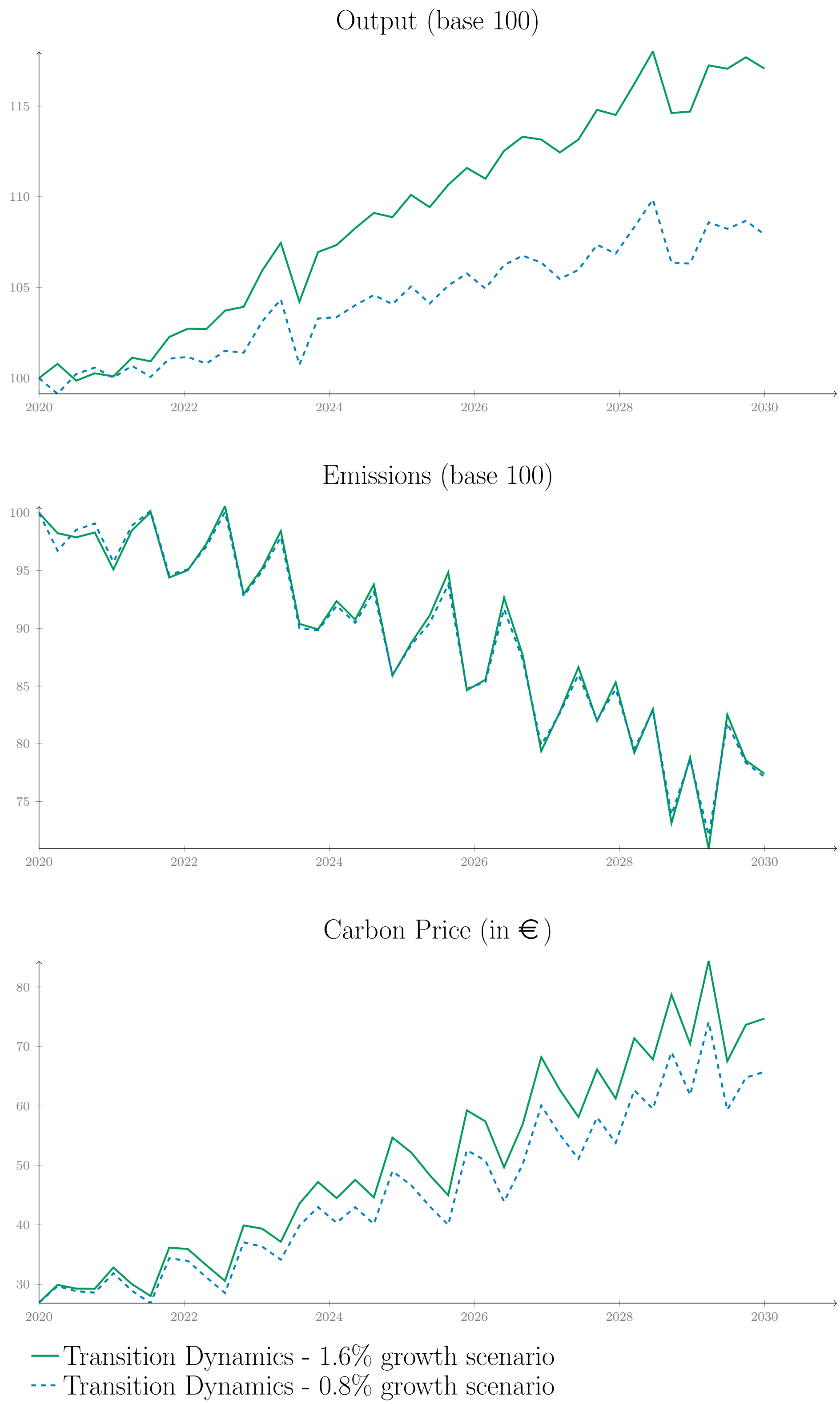


FIGURE V. 40% and 55% emissions reduction compared to 1990, according to the central transition scenario (0.8% annual growth trend)

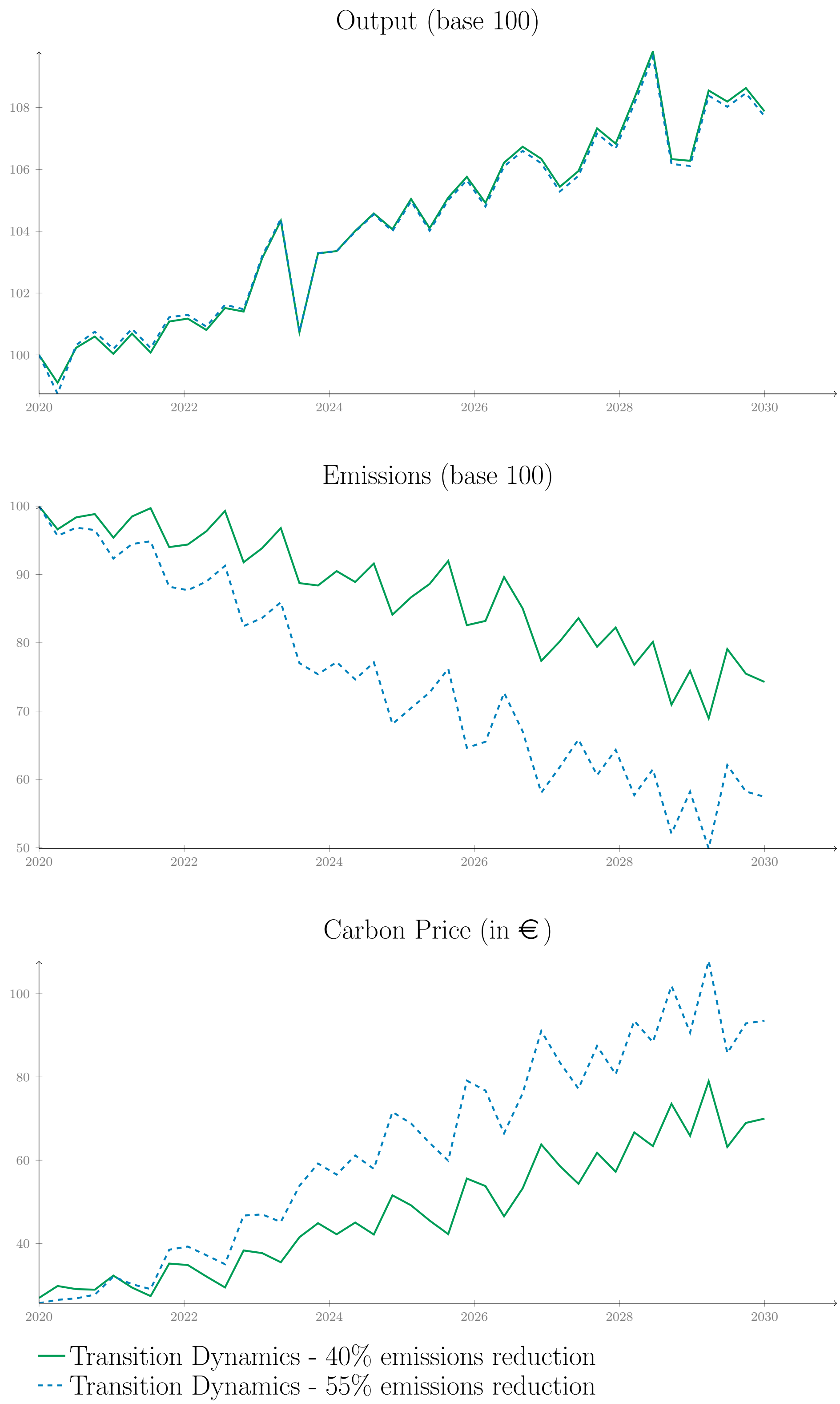


FIGURE VI. Central transition scenario (0.8% annual growth trend) compared to the welfare optimal path

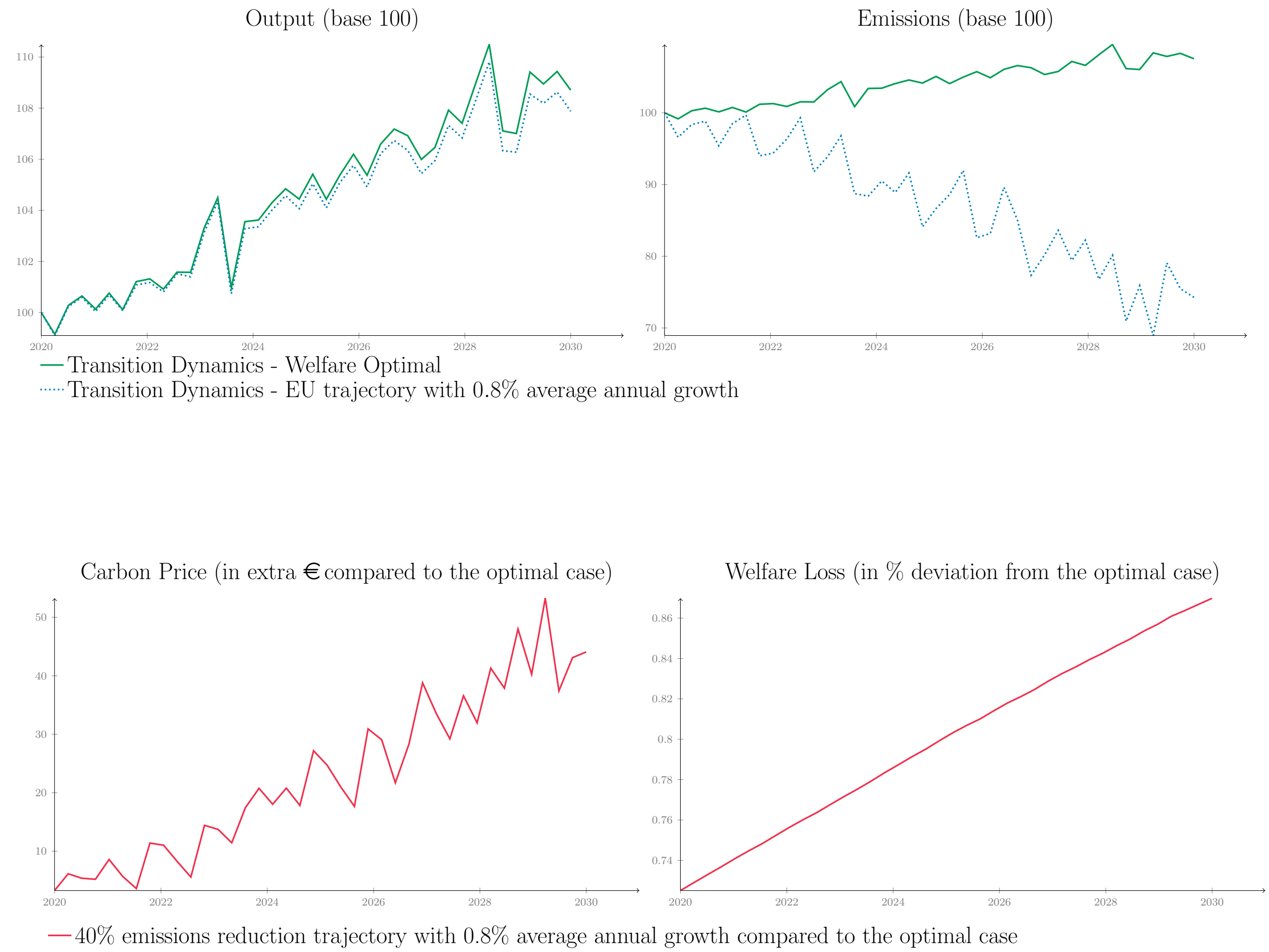


FIGURE VII. Effect of a positive tax shock (ε_t^{τ}) on selected variables with and without QE policy rules, no sectoral macroprudential policy - deviations from steady state.

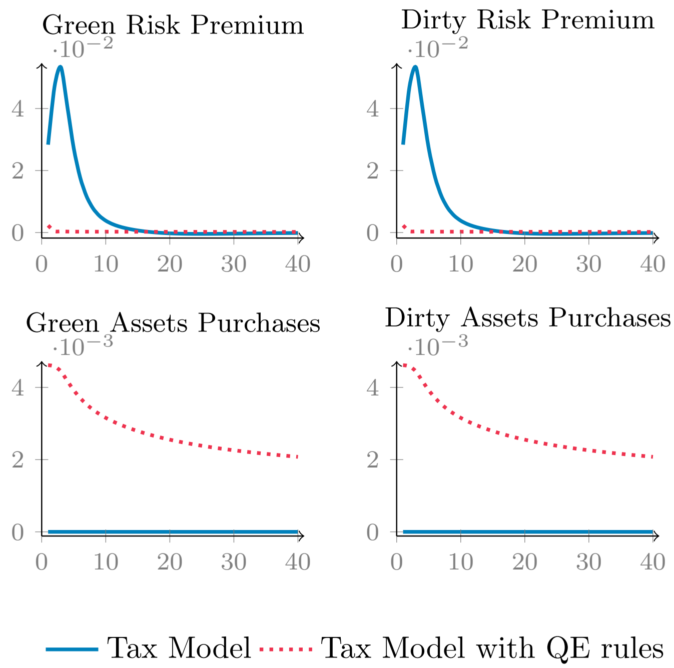


FIGURE VIII. Effect of a positive tax shock (ε_t^{τ}) on selected variables with and without QE policy rules, active sectoral macroprudential policy - deviations from steady state.

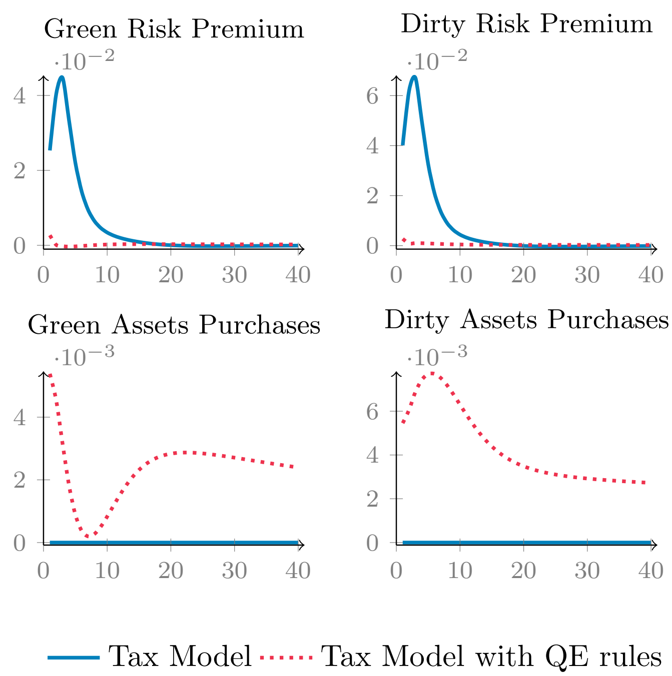


FIGURE IX. Effect of a series of positive dirty QE shock ($\varepsilon_t^{\psi_d}$) on selected variables - percentage deviations from steady state.

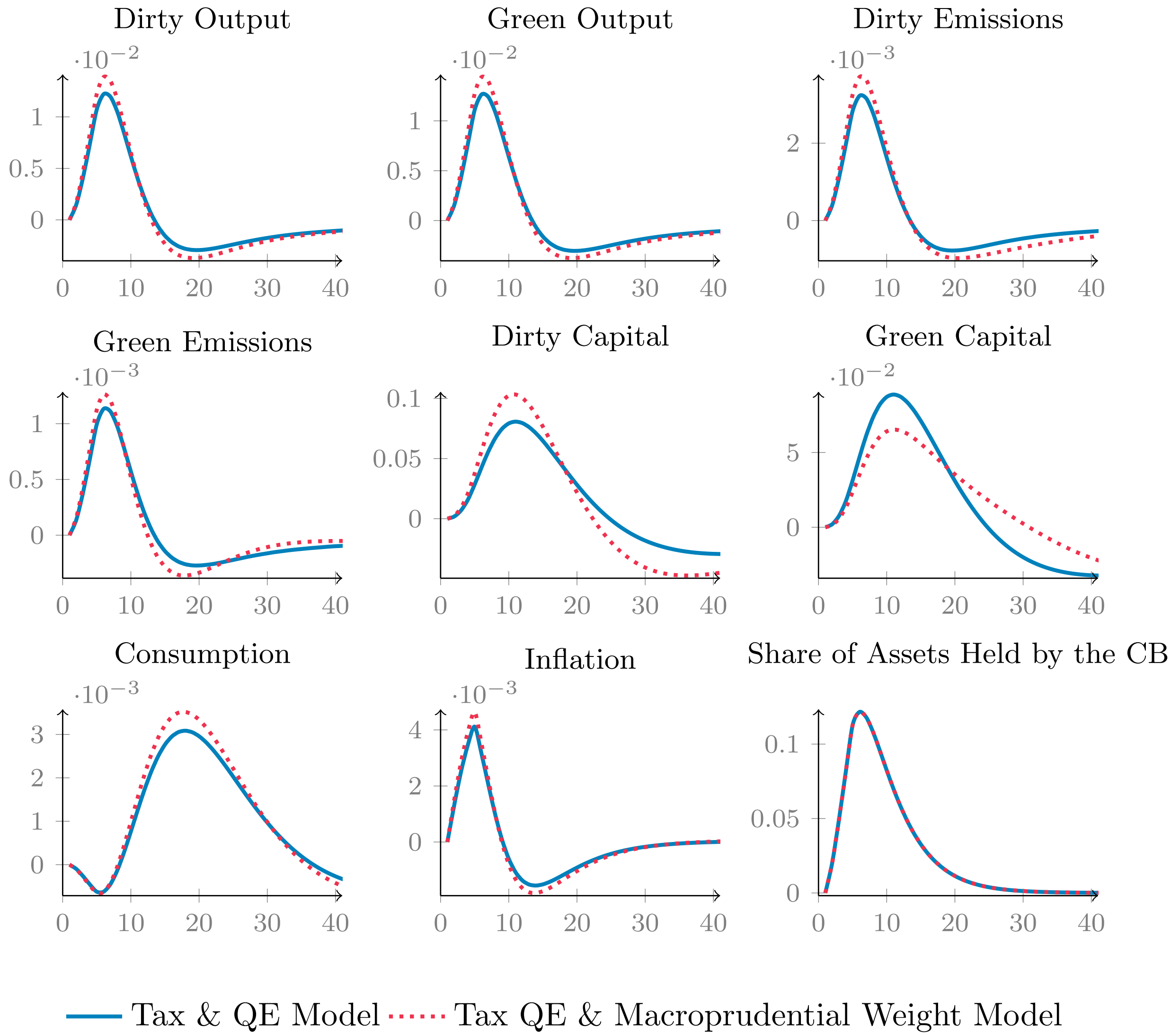
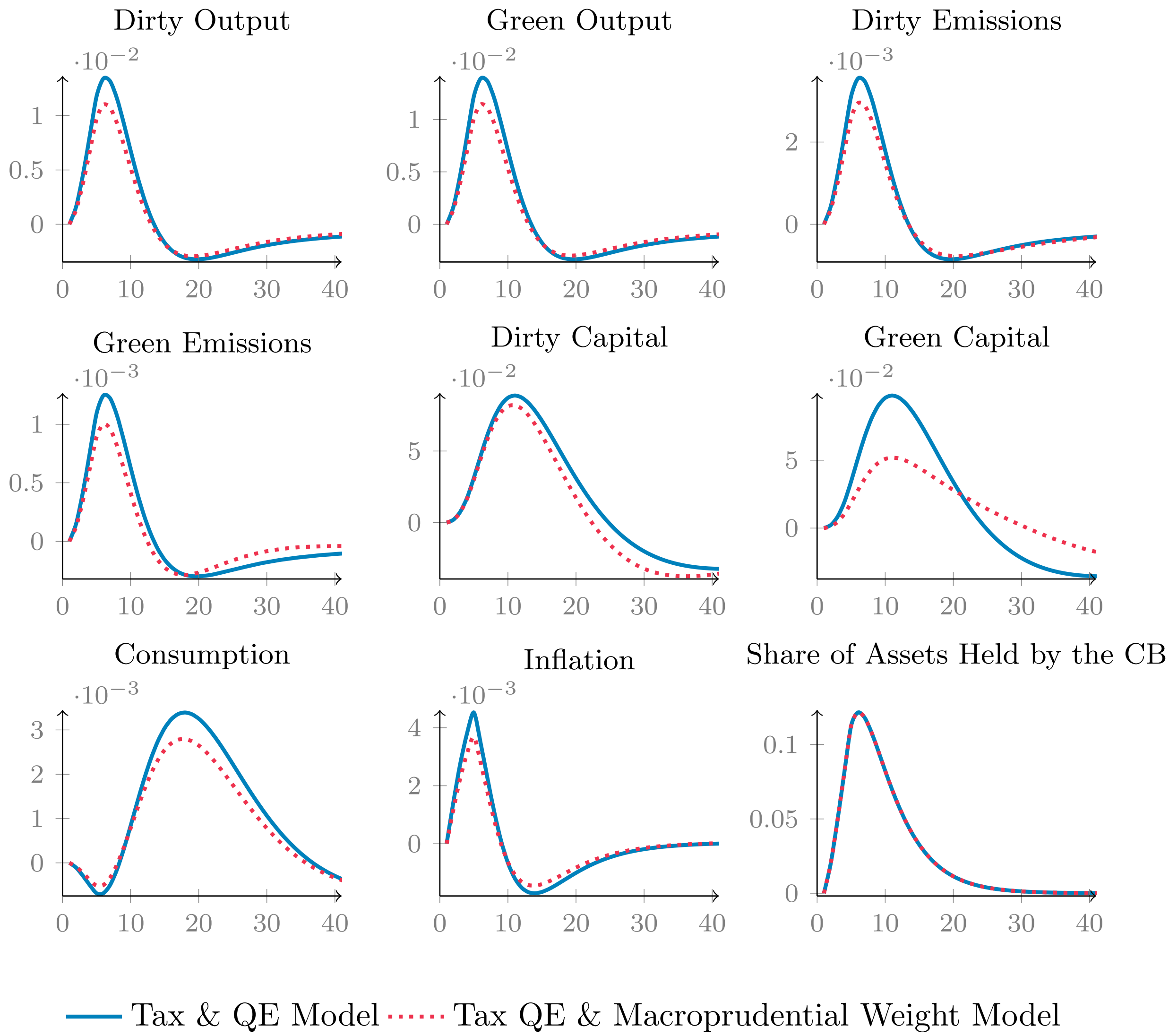


FIGURE X. Effect of a series of positive green QE shock ($\varepsilon_t^{\psi_g}$) on selected variables - percentage deviations from steady state.



(For the Online Appendix)

C Appendix: Climate Externality and Inefficiencies

The planners social problem for the households reads as following:

$$
\begin{aligned}
&\max E_t \sum_{i=0}^{\infty} \beta^i \Bigg(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L_{t+i,k}^{1+\varphi} \\
&+ \lambda_t \bigg(\sum_k \left(\frac{W_{t,k}}{P_t} L_{t,k} + \Pi_{t,k} \right) + \frac{T_t}{P_t} + R_t B_t - C_t - B_{t+1} \bigg) \\
&+ \lambda_t \sum_k q_{t,k} \left(\frac{P_{t,k}}{P_t} Y_{t,k} - \frac{W_{t,k}}{P_t} L_{t,k} - \frac{R_t^K}{P_t} K_{t,k} - f(\mu_{t,k}) Y_{t,k} - \Pi_{t,k} \right) \\
&+ \lambda_t \sum_k \Psi_{t,k} (\varepsilon_t^{A,k} d(T_t^{Temp}) K_{t-1,k}^{\alpha} L_{t,k}^{1-\alpha} - Y_{t,k}) \\
&+ \lambda_t \sum_k \varrho_{t,k} (g(\varkappa) E_{t,k} - E_t) \\
&+ \lambda_t \S_t^X (X_t - \eta X_{t-1} - E_t) \\
&+ \lambda_t \S_t^T (T_t^{Temp} - \upsilon_1^{Temp} (\upsilon_2^{Temp} X_{t-1} - T_{t-1}^{Temp}) - T_{t-1}^{Temp}) \\
&+ \lambda_t \sum_k \S_{t,k}^E (E_{t,k} - (1-\mu_{t,k}) \varphi_{t,k} Y_{t,k}) \Bigg),
\end{aligned}
$$

where the Social Cost of Carbon SCC_t is $\S_t^X$, and $\Psi_{t,k}$ the marginal cost component related to the firms problem.

The first order conditions determining the SCC_t are the ones with respect to $T_t^{Temp}, X_t, E_{t,k}, \mu_{t,k}$ and $\Pi_{t,k}$:

$$\lambda_t \S_t^T = \beta(1-\upsilon_1^{Temp})\lambda_{t+1}\S_{t+1}^T - \lambda_t \sum_k \Psi_{t,k}\varepsilon_t^{A,k} \frac{\partial d(T_t^{Temp})}{\partial T_t^{Temp}} K_{t-1,k}^{\alpha} L_{t,k}^{1-\alpha}$$

$$\lambda_t \S_t^X = \beta(\upsilon_1^{Temp}\upsilon_2^{Temp})\lambda_{t+1}\S_{t+1}^T + \beta\eta\lambda_{t+1}\S_{t+1}^X$$

$$\lambda_t \S_{t,k}^E = g(\varkappa)\lambda_t \S_t^X$$

$$\lambda_t q_{t,k} f'(\mu_{t,k}) = \varphi_{t,k}\lambda_t \S_{t,k}^E$$

$$\lambda_t = \lambda_t q_{t,k}.$$

Rearranging these FOCs we obtain the following SCC_t:

$$\S_t^T = (1-\upsilon_1^{Temp})\Lambda_{t,t+1}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k} \frac{\partial d(T_t^{Temp})}{\partial T_t^{Temp}} K_{t-1,k}^{\alpha} L_{t,k}^{1-\alpha}$$

$$\S_t^X = (\upsilon_1^{Temp}\upsilon_2^{Temp})\Lambda_{t,t+1}\S_{t+1}^T + \eta\Lambda_{t,t+1}\S_{t+1}^X$$

$$\S_{t,k}^E = g(\varkappa)\S_t^X$$

$$f'(\mu_{t,k}) = \varphi_{t,k}\S_{t,k}^E$$

The competitive equilibrium problem for the firms reads as following:

$$\begin{aligned}
&\max E_t \sum_{i=0}^{\infty} \Bigg(\left(\frac{P_{t,k}}{P_t} Y_{t,k} - \frac{W_{t,k}}{P_t} L_{t,k} - \frac{R_t^K}{P_t} K_{t,k} - f(\mu_{t,k}) Y_{t,k} - \frac{\tau_{et,k}}{P_t} E_{t,k} - \Pi_{t,k} \right) \\
&+ \lambda_t \sum_k \Psi_{t,k}(\varepsilon_t^{A,k} d(T_t^{Temp}) K_{t-1,k}^{\alpha} L_{t,k}^{1-\alpha} - Y_{t,k}) \\
&+ \lambda_t \sum_k \S_{t,k}^F (E_{t,k} - (1-\mu_{t,k})\varphi_{t,k} Y_{t,k}) \Bigg)
\end{aligned}$$

The first order conditions determining the tax rate $\frac{\tau_{et,k}}{P_t}$ are the ones with respect to $E_{t,k}$ and

$\mu_{t,k}, L_{t,k}$:

$$\S_t^F = \frac{\tau_{et,k}}{P_t}$$
$$f'(\mu_{t,k}) = \S_t^F \varphi_{t,k}$$

Thus, from both the household and firm FOCs, we get:

$$\S_t^F = \frac{\tau_{et,k}}{P_t}$$
$$\S_t^F = \S_t^E$$
$$f'(\mu_{t,k}) = \S_t^E \varphi_{t,k}$$
$$\S_t^T = (1 - \upsilon_1^{Temp})\Lambda_{t,t+1}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k} \frac{\partial d(T_t^{Temp})}{\partial T_t^{Temp}} K_{t-1,k}^{\alpha} L_{t,k}^{1-\alpha}$$
$$\S_t^X = (\upsilon_1^{Temp}\upsilon_2^{Temp})\Lambda_{t,t+1}\S_{t+1}^T + \eta\Lambda_{t,t+1}\S_{t+1}^X$$
$$\S_{t,k}^E = g(\varkappa)\S_t^X$$

The competitive equilibrium problem for the banks remains the same.

C.1 Welfare Distortion

When $\frac{\tau_{et,k}}{P_t}^* < \frac{\tau_{et,k}}{P_t}$, the welfare cost is higher, thus requiring a loss in households lifetime consumption:

$$\Delta_{\{\tau-\tau^*\}}\text{Welfare} = E_t \sum_{i=0}^{\infty} \beta^i \left(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{(C_{t+i}^* - hC_{t+i-1}^*)^{1-\sigma}}{1-\sigma} \right.$$
$$\left. - \sum_k \left(\frac{\chi_k}{1+\varphi} L_{t+i,k}^{1+\varphi} - \frac{\chi_k}{1+\varphi} L_{t+i,k}^{1+\varphi\,*} \right) \right),$$

Thus, to understand the how the welfare is impacted by either policy, we dis-tangle the effect on consumption and the effect on labor.

First let's focus on the consumption impact[28], we can reduce the the problem to the following[29]:

$$\text{Wedge}_{\text{C}} = \left(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{(C^*_{t+i} - hC^*_{t+i-1})^{1-\sigma}}{1-\sigma} \right) \propto \Delta C_t$$
$$\propto \Delta Y_t - \Delta I_t - \Delta G_t - \Delta Z_t$$
$$\propto \Delta(1-g)Y_t - \Delta I_t - \Delta Z_t$$

Thus, the total effect on consumption reads as following:

$$\text{Wedge}_{\text{C}} \propto (1-g)(Y_t - Y_t^*) - (I_t - I_t^*) - (Z_t - Z_t^*)$$

Without a loss of generality, we can focus on one sector (as the aggregate variables are CES functions) an draw the same conclusion for the model with both sectors:

$$\begin{aligned} \text{Wedge}_{\text{C}} \propto & (1-g)(\varepsilon_t^{A,k}\bar{L}^{1-\alpha})(d(T_t^{Temp})K^{\alpha}_{t-1,k} - d(T_t^{Temp})^* {K^{\alpha}_{t-1,k}}^*) \\ & - (f(K_{t,k}) - f(K_{t,k})^*) \\ & - ((\varepsilon_t^{A,k}\bar{L}^{1-\alpha})(d(T_t^{Temp})K^{\alpha}_{t-1,k}f(\mu_{t,k}) - d(T_t^{Temp})^*){K^{\alpha}_{t-1,k}}^* f(\mu_{t,k})^*) \end{aligned}$$

Comparing now the impacts of higher tax rate to an optimal tax, we can first clearly see that the damages from higher temperature will be lower under the higher tax rate than under the optimal tax $d(T_t^{Temp}) < d(T_t^{Temp})^*$ as temperature is increasing since emissions are reduced at a higher rate. Similarly, abatement is higher under the higher tax rate. As such, we propose a sectoral-maroprudential policy, which will target a capital ratio per sector. Such policy would increase capital holdings in each sector or in a specific sector, thus, closing the welfare wedge part linked to household consumption and compensating for the loss in GDP linked to higher costs of abatement.

[28] At the steady state we set $\bar{L}$=1/3, thus $L_{t,k} = \bar{L}$.

[29] First by utilizing the fact that the utility function is strictly increasing. Then by using the economy budget constraint: $Y_t = C_t + I_t + G_t + Z_t$, $G_t = gY_t$, and $Z_t = f(.)Y_t$

Turning now to the labor impact:

$$\begin{aligned}\text{Wedge}_{\text{L}} = \sum_k \frac{\chi_k}{1+\varphi}(L_{t+i,k}^{1+\varphi} - L_{t+i,k}^{1+\varphi\ *}) &\propto \sum_k (W_{t,k} - W_{t,k}^*) \\ &\propto \sum_k (1-\alpha)\Delta\Psi_{t,k} Y_{t.k}/\bar{L} \\ &\propto \sum_k (1-\alpha)(\varepsilon_t^{A,k}\Psi_{t,k} d(T_t^{Temp}) K_{t-1,k}^{\alpha} \\ &- \varepsilon_t^{A,k}\Psi_{t,k}{}^* d(T_t^{Temp*}) K_{t-1,k}^{\alpha}{}^*)\bar{L}^{-\alpha}\end{aligned}$$

As for the consumption wedge, a sectoral-maroprudential policy targeting a capital ratio per sector would reduce the wedge by increasing the capital holding.

C.2 Premium Distortion

The risk premium is defined as:

$$\begin{aligned}EP_{t,k} &= R_{t,k}^K - R_t \\ &= \alpha L_{t,k}^{1-\alpha} \epsilon_t^{t,k} \Psi_{t,k} d(T_t^{\text{Temp}}) K_{t-1}^{\alpha-1} - R_t\end{aligned}$$

At the steady state, as we chose labour to march hours worked in the economy, the previous expression simplifies to:

$$\begin{aligned}EP_{t,k} &= \alpha \bar{L}^{1-\alpha} \epsilon_t^{t,k} \Psi_{t,k} d(T_t^{\text{Temp}}) K_{t-1}^{\alpha-1} - R_t \\ &= \alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t-1,k}} - R_t\end{aligned}$$

Thus, relying on a second best instrument such as the ETS carbon pricing system where carbon prices are subject to sudden changes and important volatility exposure, the risk premium would be subject to direct impact of such changes as the marginal cost component $\Psi_{t,k}$ and is directly linked to the carbon prices:

- $\Psi_{t,k}$, which represents the maginal cost component related to the labour and capital used by firms, would increase as a result of the increasing abatement level and tax level ($\Psi_{t,k}$=$MC_{t,k}$-$\theta_{1,k}\mu_{t,k}^{\theta_{2,k}}$-$\tau_{t,k}(1-\tau_{t,k})\varphi_k$).

Thus, $EP_{t,k}$ would increase following an increase in a price of CO_2.

In order to counter balance such an increase, we could set a QE rule which will react to the risk premium increase and would restore the initial level by impacting the capital holding.