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Policy Interactions and the Transition to Clean Technology

Ghassane Benmir* Josselin Roman†

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Abstract

Using a DSGE model with financial frictions and a two-sector production economy (i.e. green and dirty sectors), we assess different types of fiscal, monetary, and macroprudential policies aimed at reducing CO2 emissions. We show that CO2 emissions and CO2 mitigation policies induce two inefficiencies: risk premium and welfare distortions, respectively. We first find that a substantial carbon tax is needed in the Euro Area to be aligned with the Paris Agreement, but that it leads to a significant welfare loss. To dampen this effect and prevent potential shocks to emissions from distorting the functioning of monetary policy through a rise in risk premia, we explore monetary and macroprudential tools. We find that sectoral time-varying macroprudential weights on loans favorable to the green sector boost green capital and output with a minimal welfare cost. With respect to QE, we find that a carbon tax improves the benefits of both green and dirty asset purchases. Regarding the impact of the environmental externality, we show that a QE rule would allow authorities to drastically reduce the effect of emissions on risk premia. This work aims to provide central banks and similar institutions with the tools to contribute to climate change mitigation, and demonstrates the importance of including these institutions in the push to reduce global emission levels.

Keywords: Climate Change, Two-Sector Economy, Optimal Fiscal Policy, Zero-Lower-Bound, Macroprudential Policy, Quantitative Easing.

JEL: Q58, E32, E52.

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1 Introduction

Climate change has shifted from a fringe issue to a worldwide emergency. Our understanding of the phenomena and our willingness to act have developed significantly, in part paralleling the ways in which climate change is being experienced around the globe. It has become a hot topic where academics, industry, and lay people alike are finding common ground. As such, growing academic awareness is leading to important literature in the domain. The implementation of a strategy for the substantial reduction of greenhouse gases (GHG) at the global level has become a major priority. Since the Rio Conference in 1992, a debate has raged in academic and political circles over the growth-environmental trade-off. Discussions focus on the means by which economic activities could align with environmental concerns instead of being hindered by assumed mutual exclusivity. In practice, especially in the short and medium terms, however, financial and economic activity on one side, and environmental policy on the other, are in tension. A need for short-term policies aimed at bridging environmental quality and economic efficiency, as well as addressing financial stability, are in dire need, in order to foster economic sustainability. Of special concern are climate actions that may strongly impact macroeconomic activity, given the potentially high added cost of GHG offsetting. With the substantial effects of climate actions on the overall economy, a growing body of research from the field of macroeconomics and macro-finance, among others, are now tackling these issues.

A growing interest in a "Green Financial System"—as outlined in the Paris "One Planet Summit" held in December 2017, where "[E]ight central banks and supervisors established a Network of Central Banks and Supervisors for Greening the Financial System (NGFS)"—is putting climate change challenges at the heart of the macro financial system. NGFS [2019] recently published a call for action in which it outlined the role central banks can play in monitoring and mitigating climate change, considering the adverse impact it could have on financial stability. Integrating climate change challenges within the macro-monetary and macro-prudential frameworks is increasingly gaining momentum within institutions such as the European Central Bank (ECB), thus making research that combines macroeconomics and climate change environmental concerns extremely relevant to policy makers. Earlier this year, Bolton, Despres, Pereira Da Silva, Samama, and Svartzman [2020] advocated in a joint publication from the BIS and *Banque de France* for "better coordination of fiscal, monetary and prudential and carbon regulations", which is perfectly in line with the findings in our article.

Tackling climate change challenges requires innovating classic research paths, which tend to favor the use of models that capture only one of the following: environmental variables, macroeconomic fluctuations, or monetary and financial policy. However, as underlined by Rudebusch and Swanson [2012], this limited modeling approach is reductive, and indicates that macroeconomic modeling suffers from theoretical incompleteness. Policy recommendations (based on such models) that aim to mitigate GHG effects should be able to capture macroeconomic variations, monetary and financial policy, as well as environmental constraints, as these are tightly linked.

Given this gap in the environmental-macroeconomic-monetary-macroprudential approach, our paper seeks to assess the interactions among environmental policies, namely: i) fiscal, ii) monetary, and iii) macroprudential, each of which is aimed at reducing CO2 emissions by using a heterogeneous macroeconomic production economy. To the best of our knowledge, this is the first article to look at the interaction between environmental, monetary, and macroprudential policies in a DSGE model under both a non-zero-lower-bound (non-ZLB) environment and a ZLB environment[1]. Going forward, it will be an important component to successfully fight climate change. Our paper falls within at least three strands of literature. We first build on the canonical versions of New Keynesian (NK) models such as Woodford [2003], Christiano, Eichenbaum, and Evans [2005] or Smets and Wouters [2003] to derive the core of our economy[2]. Second, we add environmental components as in Heutel [2012] among others to introduce the environmental constraints, which allows for the anal-

[1]The ZLB environment corresponds to an environment where nominal interest rates are close to zero and can't be further lowered by central banks.

[2]Note that for simplicity we abstract from wages rigidities.

ysis of the dynamics of the economy under the presence of the CO2 externality. However, as opposed to Annicchiarico and Di Dio [2015], we differentiate between green and dirty firms instead of using one sole representation for firms, thus borrowing from the multi-sector literature of Woodford [2003] and Carvalho and Nechio [2016] among others. Finally, we include balance sheet constrained financial intermediaries as in Gertler and Karadi [2011]. Because we introduce a macroprudential authority that can alter this constraint, we also draw on Pietrunti [2017] and Gertler, Kiyotaki, and Queralto [2012].

One of our main findings is that an environmental tax efficiency on emission reduction heavily depends on the abatement efficiency (i.e. low transition cost). Moreover, the optimal Ramsey tax policy from a welfare perspective is found to be of a small magnitude, suggesting that a tax policy isn't enough for the climate change mitigation strategy to be succesful and socially acceptable. Thus in order to allow for more flexibility, and to ease the welfare burden, other policies are greatly needed. Furthermore, as shown in Figure 1 and Figure 2 an increase in emission to output ratio raises the risk premium, which in turn could alter the monetary policy transmission (Doh, Cao, Molling, et al. [2015]). Monetary and macroprudential policies could therefore play a major role in offsetting climate change and closing the inefficiency gap induced by this environmental externality. In particular, we find that sectoral time-varying macroprudential weights on loans favorable to the green sector boost green capital and output, meaning that there is a lower emissions to output ratio. Combining this policy with a carbon tax is also shown to be welfare enhancing compared to a tax only scenario. With respect to quantitative easing (QE), we find that a carbon tax improves the benefits of both green and dirty asset purchases. However, we find that macroprudential policy is needed to provide an incentive to central banks to engage in green QE. This means that the choice between dirty and green QE implies a trade-off between higher output and lower emissions. Our actual findings could be further reinforced if we were to see a transition to a greener economy favoring the green sector over the dirty sector, as illustrated in our simulated transition in Figure 3 and Figure 4, and as argued in the work of Acemoglu, Akcigit, Hanley, and Kerr [2016], where the focus is on the long-term transition strategies. Regarding the impact of the environmental externality, we show that QE rules are more efficient than macroprudential policy in closing the premium inefficiency gap. Thus, asset purchases could be used as a short term countercyclical tool while sectoral macroprudential policy could play a more structural role.

Merging these different sets of policy tools will not only help contribute to this burgeoning field of research and address the gaps identified above, but will also set the path for new analysis in macroeconomics, environmental policy, and monetary policy. The proposed approach can help shape policy making and empower central banks among other institutions to address one of the most pressing issues of our time.

This paper is organized as follows: section 2 presents the model, section 3 explains the calibration, section 4 displays the results and section 5 concludes.

2 The Model

Using the NK-DSGE framework as a foundation, the present paper investigates the potential role of fiscal policy, central bank monetary policy, and macroprudential policy, in mitigating climate change challenges. We first model our two-sector economy following Woodford [2003] for the labor specific component within the household, and the two-sector production economy following Carvalho and Nechio [2016]. Then, we model the environmental component following Nordhaus [2008] and Heutel [2012], among others. Finally, drawing on Gertler and Karadi [2011], we model the financial intermediaries and the banking sector.

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \dots \infty)$. The production sectors produce two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms via financial intermediaries. Public authorities decide on the fiscal and environmental policy, while the central bank decides on the monetary and macroprudential policy.

2.1 The Household

At each period, the representative households supply two types of labor to the sectors of which our economy is comprised (i.e 'green' and 'dirty' sectors denoted by $k \in \{g, d\}$[3]), while they also consume and save. Households have two choices to save: lending their money either to the government or to financial intermediaries that will finance firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households cannot lend their money to a financial intermediary owned by one of their members. Household members who are workers supply labor and return their salaries to the household to which they belong.

Agents can switch between the two occupations over time. There is a fraction f of agents who are bankers and a probability θ_B that a banker remains a banker in the next period. Thus, $(1-f)\theta_B$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to the household, which will use them as start-up funds for the new banker.

Households solve the following maximization problem:

$$\max_{\{C_t, L_{t,k}, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \frac{\varepsilon^B_{t+i}}{\varepsilon^B_t} \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L^{1+\varphi}_{k,t+i} \right] \tag{1}$$

s.t.

$$C_t + B_{t+1} = \sum_k \left(\frac{W_{t,k}}{P_t} L_{t,k} + \Pi_{t,k} \right) + \frac{T_t}{P_t} + R_t B_t, \tag{2}$$

where $\beta \in (0, 1)$ is the discount factor, parameters σ, $\varphi > 0$ shape the utility function of the representative household associated with risk consumption C_t, and labor in each sector k is $L_{t,k}$. The consumption index C_t is subject to external habits with degree $h \in [0; 1)$ while $\chi_k > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked for each sector k. Labor supply $L_{t,k}$ in each sector is remunerated at nominal wage

[3]where g refers to the green sector and d to the dirty sector.

$W_{t,k}$. $\Pi_{t,k}$ is profits from the ownership of firms (both financial and non-financial) that will serve as start-up funds for the new banker and T_t is lump sum taxes. As we assume that intermediary deposits and government bonds are one period bonds, R_tB_t is interest received on bonds held. B_{t+1} is bonds acquired. Finally, ε_t^B is a preference shock that follows an $AR(1)$ process: $\varepsilon_t^B = \rho_B\varepsilon_{t-1}^B + \sigma_B\eta_t^B$, with $\eta_t^B \sim \mathcal{N}(0,1)$.

Solving the first order conditions and denoting ϱ_t as the marginal utility of consumption, the labor/supply and consumption/savings equations are:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B}(C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{3}$$

$$\varrho_t = \chi_k \frac{L_{t,k}^{\varphi}}{W_{t,k}/P_t}, \tag{4}$$

$$1 = \beta E_t \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \Lambda_{t,t+1} R_{t+1}, \tag{5}$$

where the stochastic discount factor is the expected variation in marginal utility of consumption: $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$.

2.2 The Firms

2.2.1 The Final Firms

Using the multi-sector framework from Carvalho and Nechio [2016], and under non-perfect competition, we assume that production is comprised of two sectors as outlined above: green and dirty, indexed by $k \in \{g, d\}$, where representative firms produce a final good $Y_{t,k}$ in these two competitive sectors, using no more than capital and labor to produce the intermediate good Y_{jt} where $j \in (0,1)$ is the continuum of intermediate goods firms. The "bundling" of intermediate goods within the two sectors leads to a final good. Goods are symmetric and act under perfect competition. The final economy good is a constant elasticity of substitution aggregate of the two sectors:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}} Y_{t,d}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{6}$$

with $\theta \in (1, \infty)$ the elasticity of substitution between the two sectors, and $\varkappa$ the weight of each sector. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j in each of the two sectors k at prices $P_{jt,k}$:

$$\max_{Y_{jt}} \Pi_t^{\text{Final}} = P_t Y_t - \varkappa \int_0^1 P_{jt,g} Y_{jt,g} dj - (1-\varkappa) \int_0^1 P_{jt,d} Y_{jt,d} dj, \tag{7}$$

where the aggregation of green and dirty firms reads as:

$$Y_{t,k} = \int_0^1 \left(Y_{jt,k}^{1-\frac{1}{\theta_k}} \right)^{\frac{1}{1-\frac{1}{\theta_k}}}. \tag{8}$$

However, while we assume a constant elasticity of substitution between the final sectors, we consider a different elasticity of substitution θ_k between differentiated intermediate goods of the two sectors. As the goods of the two sectors entail different costs, a different elasticity of substitution is considered. This assumption, which shapes the marginal cost structure, is based both on theoretical work of Tucker [2010] as well as on the empirical findings of Chegut, Eichholtz, and Kok [2019] and Chan, Li, and Zhang [2013], where it is found that green projects entail higher marginal cost (7-13 percent higher costs for green projects in the construction industry compared to non green projects depending on the 'greeness' of the project, and 5-7 percent higher costs in the cement and iron & steel sectors, respectively).

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}} \right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} Y_t. \tag{9}$$

Under perfect competition and free entry, the price of the final good is denoted P_t, while the price $P_{t,k}$ is the price index of sector-k intermediate goods. Finally, the price $P_{jt,k}$ is the price charged by firm j from sector k.

The prices of final aggregate goods and for each sector are given by:

$$P_t = \left(\varkappa P_{t,g}^{1-\theta} + (1-\varkappa)P_{t,d}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{10}$$

$$P_{t,k} = \left(\int_0^1 P_{jt,k}^{1-\theta_k} dj\right)^{\frac{1}{1-\theta_k}}. \tag{11}$$

2.2.2 The Intermediate Firms

As our economy is comprised of two categories of firms green corresponding to environmentally-friendly firms with a stock of capital k_g and dirty with higher emissions rate of a stock of capital k_d relying in CO2 intensive components.

The representative firms j in each sector k of the modeled economy seek profit maximization by making a trade-off between the desired level of capital and labor. Furthermore, the firms will incur externality costs and choose the level of abatement to maximize their profit. As presented in Heutel [2012] RBC model, the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externalities deteriorate the environment and the stock of pollutant alters production possibilities of firms as follows:

$$Y_{jt,k} = (1 - d(X_t))\varepsilon_t^{A_k} K_{jt-1,k}^{\alpha} L_{jt,k}^{1-\alpha}, \, \alpha \in (0,1), \tag{12}$$

where $d(X_t)$ is a convex polynomial function of order 2 displaying the stock of pollution ($d(X) = a + bX + cX^2$, with $(a,b,c) \in \mathbb{R}^3$, which is borrowed from Nordhaus [2008]).

In addition, α is the classical elasticity of output with respect to capital, and $\varepsilon_t^{A_k}$ is a sector-specific technology shock that follows an $AR(1)$ process: $\varepsilon_t^{A_k} = \rho_{A_k}\varepsilon_{t-1}^{A_k} + \sigma_{A_k}\eta_t^{A_k}$, with $\eta_t^{A_k} \sim \mathcal{N}(0,1)$. Furthermore, the carbon emissions stock X follows a law of motion:

$$X_t = (1 - \gamma_d)X_{t-1} + E_{jt} + E^*, \tag{13}$$

where E_{jt} is the flow of emissions from both the green and dirty firms ($E_{jt} = \varkappa E_{jt,g} + (1 - \varkappa)E_{jt,d}$) at time t and γ_d is the decay rate. E^* represents the rest of the world emissions.

The amount of emissions is modeled by a nonlinear technology such as abatement costs that would reduce the inflow of emissions:

$$E_{jt,k} = (1 - \mu_{jt,k})\varphi_{t,k} Y_{jt,k}. \tag{14}$$

The emissions E at firm level are proportional to the production Y with $\varphi_{t,k}$ the proportion of emissions to output.[4].Also, emissions could be reduced through an abatement effort μ. The firms are allowed to invest in an abatement effort, which is assumed to be different between the green and dirty sectors, thus incurring the firms' direct costs.

We model the direct abatement effort costs following Heutel [2012]:

$$Z_{jt,k} = f(\mu_{jt,k})Y_{jt,k},\ \theta_1 > 0,\ \theta_2 > 1, \tag{15}$$

where

$$f(\mu_{jt,k}) = \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}}, \tag{16}$$

with $\theta_{1,k}$ and $\theta_{2,k}$ representing the cost efficiency of abatement parameters for each sector.

Thus the profits of our representative intermediate firms in each sector Π_{jt} will correspond to the difference between the revenues of the intermediate firms and their costs. The revenues are the real value of intermediate goods $Y_{jt,k}$, while the costs generate from wages $W_{t,k}$ (paid to the labor force $l_{jt,k}$), investment in capital $K_{jt,k}$ (with returns $R^K_{t,k}$), abatement $\mu_{jt,k}$ (the firms are enduring), and any environmental damages captured by emissions $E_{jt,k}$.

$$\begin{aligned}\Pi_{jt,k} &= \frac{P_{jt,k}}{P_t}Y_{jt,k} - \frac{W_{t,k}}{P_t}L_{jt} - \frac{R^K_{t,k}}{P_t}K_{jt,k} - \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}}Y_{jt,k} - \frac{\tau_{et,k}}{P_t}E_{jt,k} \\ &= \left(\frac{P_{jt,k}}{P_t} - MC_{t,k}\right)Y_{jt,k}.\end{aligned} \tag{17}$$

As firms are not free to update prices each period, they first choose inputs so as to

[4]Contrary to Lontzek, Cai, Judd, and Lenton [2015], we consider $\varphi_{t,k} = \varphi_k$ constant overtime and calibrate it using Euro Area emission to GDP levels.

minimize cost, given a price, subject to the demand constraint.

The cost-minimization problem yields the real marginal cost, which can be expressed following first-order conditions with respect to the firm's optimal choice of output and abatement, as follows:

$$\Psi_{jt,k} = \Psi_{t,k} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{1}{\varepsilon_t^{A,k}(1-d(X_t))} \left(\frac{W_{t,k}}{P_t}\right)^{1-\alpha} \left(\frac{R_{t,k}^K}{P_t}\right)^{\alpha}, \tag{18}$$

$$\varphi_t \frac{\tau_{et,k}}{P_t} Y_{jt,k} - \theta_{1,k}\theta_{2,k}\mu_{jt,k}^{\theta_{2,k}-1} Y_{jt,k} = 0, \tag{19}$$

$$MC_{jt,k} = MC_{t,k} = \Psi_{t,k} + \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \frac{\tau_{et,k}}{P_t}(1-\mu_{t,k})\varphi_t, \tag{20}$$

where $\Psi_{jt,k} = \Psi_{t,k}$ is the marginal cost component related to the same capital-labor ratio all firms of each sector choose (18). This marginal cost component is common to all intermediate firms, however, it is different across sectors.

Equation (19) is the optimal condition on abatement: abating CO2 emissions is optimal when its marginal gain equals its marginal cost. This highlights the key role of emissions in shaping price dynamics where the production of one additional unit of goods reduces the profits of firms, which in turn is partially compensated for by the marginal gain from emitting GHGs in the atmosphere.

In addition, as abatement effort μ is common to all firms of the same sector, as is the cost of abatement to which firms of the same sector are subject, the total marginal cost captures both abatement costs and emission costs (20).

Also, we note that in the case of the laissez-faire scenario, $MC_{t,k} = \Psi_{t,k}$ as the firms are not subject to emissions and abatement constraints.

In addition, the monopolistic firms engage in infrequent price setting à la Calvo. We assume that intermediate goods producers for each sector re-optimize their prices $P_{jt,k}$ only at the time when a price change signal is received. The probability (density) of receiving such a signal h periods from today is assumed to be independent from the last time the firm received the signal. A number of firms ξ will receive the price-change signal per unit of time.

All other firms keep their old prices. Thus, the profit maximization of our intermediate firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \sum_k \Pi_{jt+i,k} \tag{21}$$

$$s.t.\ \ Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t, \tag{22}$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor, or as commonly called in the macro-finance literature, the pricing kernel (for i=1 we note $M_{t,t+1} = \beta\Lambda_{t,t+1}$ as in Jermann [1998]).

The NK Philips Curve pricing equations[5] are as follows:

$$p^*_{t,k} = \frac{P^*_{t,k}}{P_t} == \frac{\theta_k}{\theta_k - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \mathrm{MC}_{t+i,k} \Im_{t,k+i}}{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \Im_{t+i,k}}, \tag{23}$$

where

$$\begin{aligned} \Im_{t+i,k} &= \left(\frac{1}{P_{t+i,k}}\right)^{-\theta_k} \left(\frac{P_{t+i,k}}{P_{t+i}}\right)^{-\theta} P_t^{\theta} Y_{t+i} \\ &= {P_{t+i,k}}^{\theta_k - \theta} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} Y_{t+i}, \end{aligned} \tag{24}$$

or equivalently:

[5] For the full mathematical derivations and algebra, please refer to the online appendix.

$$p^*_{t,k} = \frac{P^*_{t,k}}{P_t} = \frac{\theta_k}{\theta_k - 1}\frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}}, \tag{25}$$

$$\text{with:} S_{t,k} = P^{\theta_k-\theta}_{t,k}\Psi_{t,k}Y_t + \frac{\varrho_{t+1}}{\varrho_t}\xi\beta\mathbb{E}_t\pi^{\theta}_{t+1}S_{t+1,k}, \tag{26}$$

$$\text{and:}\Theta_{t,k} = P^{\theta_k-\theta}_{t,k}Y_t + \frac{\varrho_{t+1}}{\varrho_t}\xi\beta\mathbb{E}_t\pi^{\theta-1}_{t+1}\Theta_{t+1,k}, \tag{27}$$

$$\text{and:}\Upsilon_{t,k} = P^{\theta_k-\theta}_{t,k}\left[\theta_{1,k}\mu^{\theta_{2,k}}_{t,k} + \frac{\tau_{et,k}}{P_t}(1-\mu_{t,k})\varphi_t\right]Y_t + \frac{\varrho_{t+1}}{\varrho_t}\xi\beta\mathbb{E}_t\pi^{\theta}_{t+1}\Upsilon_{t+1,k}, \tag{28}$$

with inflation $\pi_t = P_t/P_{t-1}$.

The pricing equation below is obtained simply by equating the dynamic marginal revenues to the dynamic marginal costs, thus, yielding an optimal pricing condition p^*. As in each period a fraction ξ of the intermediate firms of each sector choose their optimal price P^*_k, we can rewrite the final firms goods price P_k as a weighted average of the last period's price level and the price set by firms adjusting in the current period: $P_{t,k} = (\xi P^{1-\theta_k}_{t-1,k} + (1-\xi)P^{*1-\theta_k}_{t,k})^{\frac{1}{1-\theta_k}}$. In addition, please note that the j-index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t, will make the same decisions.

2.2.3 Capital Producing Firms

We assume that households own capital producing firms and receive profits. Green and dirty firms buy specific types of capital from intermediate goods firms at the end of period t and then repair depreciated capital and create new capital. They then sell both the new and re-furbished capital. The relative price of a unit of new capital is either $Q_{t,g}$ or $Q_{t,d}$. We suppose that there are flow adjustment costs associated with producing new capital.

Accordingly, capital producing firms face the following maximization problem:

$$\max_{\{I^n_{t,k}\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s,k} - 1) I^n_{t+s,k} - f_k(.)(I^n_{t+s,k} + \bar{I}_k) \right\} \tag{29}$$

$$\text{with} \quad I^n_{t,k} = I_{t,k} - \delta K_{t,k}, \tag{30}$$

$$K_{t,k} = K_{t-1,k} + I^n_{t,k}, \tag{31}$$

$$\text{and} \quad f_k(.) = \frac{\eta_i}{2} \left(\frac{I^n_{t+s,k} + \bar{I}_k}{I^n_{t+s-1,k} + \bar{I}_k} - 1 \right)^2, \tag{32}$$

where $I^n_{t,k}$ and $I_{t,k}$ are net and gross capital created, respectively, $\bar{I}_k$ is the steady state investment for each kind of firm, $\delta K_{t,k}$ is the quantity of re-furbished capital, and η_i the inverse elasticity of net investment to the price of capital. Thus, we get the following value for $Q_{t,k}$:

$$Q_{t,k} = 1 + f_k(.) + f'_k(.) \left(\frac{I^n_{t,k} + \bar{I}_k}{I^n_{t-1,k} + \bar{I}_k} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f'_k(.) \left(\frac{I^n_{t+1,k} + \bar{I}_k}{I^n_{t,k} + \bar{I}_k} \right)^2 \right\}. \tag{33}$$

2.3 Financial Intermediaries

We modify the setup of Gertler and Karadi [2011] to allow financial intermediaries to invest in both green and carbon-intensive ('dirty') firms. We model the incentive constraint as in Pietrunti [2017] instead of Gertler et al. [2012] as this specification allows for a more realistic implementation of macroprudential policy through regulatory weights on loans.[6]

A representative bank's balance sheet can be depicted as:

$$Q_{t,g} S_{t,g} + Q_{t,d} S_{t,d} = N_t + B_t, \tag{34}$$

where $S_{t,g}$ and $S_{t,d}$ are financial claims on green and dirty firms and $Q_{t,g}$ and $Q_{t,d}$ their respective relative price. On the liability side, N_t is the banks' net worth and B_t is debt to

[6] As a robustness exercise (see C), we use Gertler et al. [2012] modeling specification and show that our results regarding the model dynamics remain unchanged. In subsection 4.3, we also compare the two policies.

households. Over time, the banks' equity capital evolves as follows:

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,d}Q_{t-1,d}S_{t-1,d} - R_tB_{t-1}, \tag{35}$$

$$N_t = (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d} + R_tN_{t-1}, \tag{36}$$

where $R_{t,k} = \frac{R^K_{k,t}/P_t-(Q_{t,k}-\delta)}{Q_{t-1,k}}$ denote the gross rate of return on a unit of the bank's assets from $t-1$ to t for sector k.[7]

The goal of a financial intermediary is to maximize its equity over time. Thus, we can write the following objective function:

$$V_t = E_t\left\{\sum_{i=1}^{\infty}\Delta\beta^i\Lambda_{t,t+i}(1-\theta_B)\theta_B^{i-1}N_{t+1}\right\}, \tag{37}$$

where Δ is a parameter allowing to adjust the bankers' discount factor. We introduce a regulator in charge of the supervision of financial intermediaries. Drawing on Pietrunti [2017], we assume that this regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t \geq \lambda_t(\lambda_gQ_{t,g}S_{t,g} + \lambda_dQ_{t,d}S_{t,d}), \tag{38}$$

with λ_t the risk weight on loans and λ_g and λ_d specific weights that can be applied to loans for green and/or dirty firms. As will be made clear below, the regulator can modify these weights, altering the constraint weighing on banks and thus the financial frictions in our economy. These weights follow prudential policy rules containing auto-regressive structures[8], as the policy changes operated by the regulator following a shock are quite persistent overtime. In our baseline version of the model, however, we consider the case where λ_g and λ_d are both equal to one, and we calibrate $\bar{\lambda}$[9] to match the steady state capital ratio of European banks.

[7] Note that the depreciated capital has a value of one as adjustment costs only apply to net investment.

[8] The rules are detailed below in the macroprudential policy section.

[9] Where the 'bar' variable represent the steady state level.

We guess that the value function is linear of the form $V_t = \Gamma_t N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_{t,g}, S_{t,d}} E_t \left\{ \Delta\beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \right\}, \tag{39}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t$. Maximization subject to constraint (38) yields the following first order and slackness conditions:

$$\Delta\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,g} - R_{t+1}) \right\} = \nu_t \lambda_g \lambda_t, \tag{40}$$

$$\Delta\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,d} - R_{t+1}) \right\} = \nu_t \lambda_d \lambda_t, \tag{41}$$

$$\nu_t \left[\Gamma_t N_t - \lambda_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}) \right] = 0, \tag{42}$$

where ν_t is the multiplier for constraint (38). One interesting result is that we get:

$$N_t \geq \Xi_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}), \tag{43}$$

where $\Xi_t = \lambda_t / \Gamma_t$ is the capital ratio for banks and λ_g and λ_d represent potential rewards or penalties on the weights required by the regulator on green and dirty loans, respectively.[10] Finally, we rewrite the value function to find Γ_t:

$$\begin{aligned} V_t &= \lambda_t \nu_t (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}) + \Delta\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} N_t \right\} \\ \Gamma_t N_t &= \nu_t \Gamma_t N_t + \Delta\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_t N_t \right\} \\ \Gamma_t &= \frac{1}{1 - \nu_t} \Delta\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \right\}. \end{aligned} \tag{44}$$

[10]For instance, if $\lambda_g < 1$ banks will need to hold less capital for loans they grant to green firms compared to dirty firms.

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B[(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d}] + (\theta_B R_t + \omega)N_{t-1}, \tag{45}$$

with $\omega \in [0; 1)$ the proportion of funds transferred to entering bankers.

2.4 Public Authorities

2.4.1 Central Bank

Policy Rate Setting

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t - \bar{\imath} = \rho_c\left(i_{t-1} - \bar{\imath}\right) + (1 - \rho_c)\left[\phi_\pi\left(\pi_t - \bar{\pi}\right) + \phi_y\left(Y_t - Y_{t-1}\right)\right], \tag{46}$$

where $\bar{\imath}$ is the steady state of the nominal rate i_t, $\rho_c \in [0, 1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is the output gap stance penalizing deviations of output from its previous period level Y_{t-1}. Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t\left\{\pi_{t+1}\right\}. \tag{47}$$

Because we want to replicate the current economic conditions as closely as possible, we will calibrate our model such that the nominal rate would be extremely low by historical standards (1 percent at the steady state). This drastically limits the scope of conventional monetary policy, as the central bank can not set its nominal interest rate below zero. The Zero Lower Bound (ZLB) implies non linear responses to shocks that affect the path of the nominal rate and we must take it into account. To do so, we will use the non-linear technique of simulation developed by Guerrieri and Iacoviello [2015].

Quantitative Easing

The ZLB also implies that central banks must prove innovative to keep fulfilling their mandates in a liquidity trap environment. A common alternative to nominal interest rate setting is the use of assets purchase programs, also referred to as Quantitative Easing (QE). In the previous section, we showed how the value of loans to both dirty and green firms are determined. We now introduce a central bank that can substitute for financial intermediaries in financing these firms. Much like the Corporate Sector Purchase Program (CSPP) in the Euro Area, the central bank has the ability to fund non-financial firms in order to reduce corporate spread, steer private investment, and ultimately keep inflation in range with its target. Then for each type of firm k we now have:

$$Q_{t,k}S_{t,k} = Q_{pt,k}S_{pt,k} + Q_{gt,k}S_{gt,k}, \tag{48}$$

with $Q_{gt,k}S_{gt,k}$ the total real value of loans to firms of type k held by the central bank. Note that $S_{t,k} = K_{t,k}$, as firms from either sector, do not face frictions when requesting financing. $Q_{pt,k}S_{pt,k}$ is the total real value of loans to firms of type k held by financial intermediaries as defined in 2.3. As in Gertler and Karadi [2011], we model this intervention by assuming that the central bank holds a portion $\psi_{t,k}$ of total loans to non-financial firms belonging to each sector:

$$Q_{gt,k}S_{gt,k} = \psi_{t,k}Q_{t,k}S_{t,k}. \tag{49}$$

For simplicity, we abstract from monitoring costs. We assume that the central bank follows a counter-cyclical credit policy rule that reacts to the variations in the anticipated spread ($\text{EP}_{t+1,k} = R_{t+1,k} - R_{t+1}$) in order to decide the share of assets $\psi_{t,k}$ it holds. This rule is defined as follows:

$$\psi_{t,k} = \rho_{u_k}\psi_{t-1,k} + (1-\rho_{u_k})(\phi^s_k(\text{EP}_{t+1,k} - \bar{\text{EP}}_k)) + \varepsilon^{\psi_k}_t, \tag{50}$$

where $\rho_{u_k} \in [0,1)$ is the rule smoothing coefficient and $\varepsilon^{\psi_k}_t$ represents a shock to the credit policy following an $AR(1)$ shock process: $\varepsilon_t^{\psi_k} = \rho_{\psi_k}\varepsilon_{t-1}^{\psi_k} + \sigma_{\psi_k}\eta_t^{\psi_k}$,with $\eta_t^{\psi_k} \sim \mathcal{N}(0,1)$. The latter is motivated by credit policy shocks, which are not directly motivated by spread gaps. Note that in our baseline model $\psi_{t,k} = 0$ so that the central bank allows financial intermediaries be the sole source of funding for firms.

2.4.2 Macroprudential Authority

As briefly explained above, there is a macroprudential regulator with the ability to modify weights on loans in the regulatory constraint. Following Pietrunti [2017], we include in our baseline model a general macroprudential rule akin to a *Countercyclical Capital Buffer* (CCyB), as defined in Basel III:

$$\lambda_t = \bar{\lambda} + \rho_\lambda \lambda_{t-1} + (1-\rho_\lambda)\phi_\lambda\Big(\frac{K_{t-1}}{\frac{1}{T}\sum_i Y_{t-i}} - \frac{K_{t-2}}{Y_{t-2}}\Big), \tag{51}$$

where λ_t reacts to change in the average credit to GDP ratio in the last four quarters, net of the last period. This rule forces banks to hold more capital when the credit to GDP gap is growing. For the purpose of our research we also introduce a specific rule for each sector. This allows the macroprudential authority to respond to changes in risk premia, which in turn respond to changes in emission levels. The rules read as follows:

$$\lambda_{t,g} = \bar{\lambda}_g + \rho_{\lambda_g}\lambda_{t-1,g} + \phi_{\lambda_g}(\mathrm{EP}_{t,g} - \bar{\mathrm{EP}}_g), \tag{52}$$

$$\lambda_{t,d} = \bar{\lambda}_d + \rho_{\lambda_d}\lambda_{t-1,d} + \phi_{\lambda_d}(\mathrm{EP}_{t,d} - \bar{\mathrm{EP}}_d). \tag{53}$$

$\lambda_{t,g}$ and $\lambda_{t,d}$ react to the variations in the spread in each sector ($\mathrm{EP}_{t,k}$), respectively, and the rules are smoothed with an auto-regressive process.

2.4.3 Government

The government sets a budget constraint according to the following rule[11]:

$$T_t + \tau_{et} E_t + s_{t,g}\psi_{t,g} K_{t,g} + s_{t,g}\psi_{t,d} K_{t,d} = G_t, \tag{54}$$

with the public expenditure G_t finding its source from taxes T_t, revenue from emissions tax $\tau_{et} E_t$ and from public financial intermediation on both green and dirty firms $s_{t,g}\psi_{t,g} K_{t,g}$ and $s_{t,d}\psi_{t,d} K_{t,d}$. The government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \tag{55}$$

Environmental Policy

The government decides to either ratify or not ratify (or renege on) the Paris Agreement. When the government is not operating an environmental policy (i.e the laissez-faire equilibrium) the tax τ_{et} is set equal to 0. Otherwise, when the government tries to hold to the COP 21 Agreement (i.e. a GHG emission reduction) $\tau_{et} > 0$. This is explained further in the results section.

2.5 Normalization and Aggregation

It is also common in most NK classical models that in equilibrium, factors and goods markets clear as shown below.

First, the market-clearing conditions for aggregate capital, investment, labor, and wages, in the two sector economy reads as[12]: $K_t = \sum_k g(\varkappa) \int_0^1 K_{jt,k} dj$, $I_t = \sum_k g(\varkappa) \int_0^1 I_{jt,k} dj$, $L_t = \sum_k g(\varkappa) \int_0^1 L_{jt,k} dj$, and $W_t = \sum_k g(\varkappa) \int_0^1 W_{jt,k} dj$.

Similarly global aggregate emissions and aggregate emissions cost are two weighted sums of sectoral emissions $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k} dj$, and sectoral emissions cost $Z_t =$

[11] In the baseline version of the model (without tax and QE), the budget constrain collapses to $T_t = G_t$.

[12] Where $g(\varkappa) = \varkappa$ for sector the green sector g and $(1 - \varkappa)$ for the dirty sector d.

$\sum_k g(\varkappa) \int_0^1 Z_{jt,k} dj$, respectively.

As presented in Gali and Monacelli [2008], the Calvo $D_{pt,k}$ price dispersion is essentially a measure of distortion introduced by dispersion in relative prices. This shows that there is an additional distortion associated with relative price fluctuations owing to price stickiness. The Calvo $D_{pt,k}$ price dispersion is bounded below at 1, where 1 would be the value in the case of flexible prices, where all firms choose the same price. The price dispersion in our two-sector economy reads as:

$$\int_0^1 Y_{jt,k} \mathrm{dj} = \int_0^1 \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_{t,k} \mathrm{dj} = D_{pt,k} Y_{t,k}, \tag{56}$$

with $D_{pt,k}$ the aggregate loss of efficiency induced by price dispersion of the intermediate goods[13]. In other words, it also reads as $D_{pt,k} = (1-\xi)\left(\frac{P_{t,k}}{P_t}\right)^{(\theta_k - \theta)} \left(p^*_{t,k}\right)^{-\theta_k} + \xi \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} \pi_{t,k}^{\theta_k} D_{pt-1,k}$.

Furthermore, as outlined in Annicchiarico and Di Dio [2015], in the addition to the classical changes operating in an NK model[14], our two-sector environmental economy we introduce is also affected at the aggregate level by the price dispersion. The emissions as well as the abatement cost reads as:

$$E_{t,k} = (1 - \mu_{t,k}) \varphi_k D_{pt,k} Y_{t,k}, \tag{57}$$

$$Z_{t,k} = \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} D_{pt,k} Y_{t,k}. \tag{58}$$

In addition, the resource constraint of the economy reads as follows:

$$Y_t = C_t + G_t + I_t + \sum_k f_k(.)(I^n_{t+s,k} + \bar{I}_k) + Z_t. \tag{59}$$

[13] Please refer to the online appendix for the full computation.

[14] Where: $Y_{t,k} = (1 - d(X_t))\varepsilon_t^{A_k} K^{\alpha}_{t-1,k} L_t^{1-\alpha} D^{-1}_{pt,k}$ and $\Pi_{t,k} = (1 - MC_{t,k} D_{pt,k}) Y_{t,k}$.

3 Calibration

Calibrated parameters are reported in Table 1. For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio at 40 percent[15], the share of hours worked per day at one third in each sector, and the capital intensity in the production function α at 0.33. The inverse elasticity of net investment to the price of capital η_i is set at 1.728 as in Gertler and Karadi [2011] and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2, as argued by Stern [2008] and Weitzman [2007]. We set the discount factor at 0.99751 to get a steady state real interest rate of 1 percent. This choice is motivated by the low interest rate environment we have witnessed in recent years.

The environmental component parameters, and specifically the damage function parameters d_0, d_1, and d_2 of the model are set as in Nordhaus [2008] and Heutel [2012]. The global level of the remainder of the world's emissions E^* is set at 2.8 in order to replicate the steady state level of the stock of emissions $X_t = 1520$ GTons (from the pre-industrial period to 2018)[16]. To calibrate the share of the green firms/sector, what we consider green in our model is a sector with a carbon performance allowing for an emission target aligned with the Paris Agreement of 2 degrees Celsius or below. We use sectoral data made available by Transition Pathway Initiative[17] to set the share of green firms $\varkappa$ to 30 percent. Furthermore, for the intensity of emissions to GDP for each sector, as argued by De Haas and Popov [2019], CO2 intensity differs largely between sectors and industries. Using the European Environmental Agency CO2 emissions intensity data[18] as well as the OECD GDP data, we observe a carbon intensity level of 35-40 percent for the last few of years. Thus the carbon intensity for each sector should satisfy the following equation $\varkappa\varphi_g + (1-\varkappa)\varphi_d = 0.4$. We set φ_d to ensure the observed CO2 to GDP ratio of about 50 percent (in the energy and

[15] We match the level of the Euro Area.

[16] https://ourworldindata.org/grapher/cumulative-co2-emissions-region?stackMode=absolute

[17] https://www.transitionpathwayinitiative.org/tpi/sectors

[18] https://www.eea.europa.eu/data-and-maps/figures/ghg-emission-intensity-of-european

industrial services). Setting a value for the dirty sector carbon intensity automatically then yields a value for φ_g of 15 percent for the green firms as their level of emissions is much lower. The abatement parameters $\theta_{k,1}$ and $\theta_{k,2}$, which pin down the abatement costs for each sector are set as in Heutel [2012] for the dirty sector[19], and are assumed to be higher for the green sector. As highlighted in the McKinsey cost curve for GHG abatement[20], the cost for abating an additional unit increases steadily (or even arguably exponentially) as cheaper technologies are used first. As our green firms are considered to have already benefited from these technologies, they incur higher abatement costs than the dirty firms. The decay rate of emissions δ_x is set at 0.21 percent. Finally, θ_d the dirty firms' marginal cost parameter is calibrated as in Smets and Wouters [2007] to replicate the mean markup and marginal cost levels observed in the economy, while θ_g as highlighted in the final firm section of the model is calibrated such that the difference in the marginal cost between the two sectors is 6 percent higher as argued by Chan et al. [2013] and Chegut et al. [2019].

As for the financial parameters, we set the probability of staying a banker θ_B at 0.98, meaning that 2 percent of bankers default every quarter, which is slightly less than in Gertler and Karadi [2011]. $\bar{\lambda}$ is calibrated at 0.0177 to generate a spread of 80 basis points between risky and risk-less assets. This value is taken from Fender, McMorrow, Sahakyan, and Zulaica [2019]. The authors also find that the spread between green and dirty bonds recently disappeared. Thus, we target the same steady state for R_g and R_d. Δ is a parameter allowing the introduction of a different discount factor in the bankers' objective function relative to households and is set to 0.99. The proportional transfer to the entering banker ω is set to 0.004 in order to match a capital ratio of approximately 14.4 percent in the Euro Area. Finally, the monetary rule parameters are set as in Smets and Wouters [2003] and the macroprudential rule parameters as in Pietrunti [2017].

The AR(1) parameters for all the shocks, namely the two technology, and preference shocks are calibrated as in Smets and Wouters [2003].

[19]A sensitivity analysis is also conducted for different values of $\theta_{d,1}$.

[20]https://www.mckinsey.com/business-functions/sustainability/our-insights/a-cost-curve-for-greenhouse-gas-reduction

4 Quantitative Analysis

4.1 Dynamics of the Model at the ZLB

Before moving to the analyses of each of the policies and their interactions, this section highlights the model's dynamics under the non-ZLB environment (i.e linear) versus the ZLB environment (i.e. non-linear). Shocks are calibrated so that the ZLB would bind for a few periods and start at period one.

Figure 5 and Figure 6 present the responses to a 13 percent positive green technology shock and dirty technology shock, respectively. The autoregressive parameter is set to 0.82 in line with Smets and Wouters [2003]. As our economy is comprised of two sectors, we allow for the possibility of different technology shocks, which affect each sector differently.

As expected following any technology shock, inflation decreases significantly more under the ZLB environment than the under non-ZLB environment, while the interest rate falls less in the ZLB than in the non-ZLB environment, as the central bank is unable to significantly lower its policy rate to counter balance the decreasing inflation rates resulting from the positive technology shocks.

Under the linear model, both the green and dirty TFP shocks raise more the aggregate output as well as the consumption as compared to the non-linear model. The small magnitude of the distortionary effect introduced by the ZLB environment (red line as compared to the blue line) is not persistent as the ZLB does not bind for a long period in the chosen example. Likewise, under the non-ZLB environment, the aggregate emissions both fall and rise more than in the non-linear model following the green and dirty technology shocks, respectively. These emissions dynamics are mainly driven by the sectoral shock. The green TFP shock increases emissions in the green sector while decreasing it in the dirty sector and vise versa.

Figure 7 presents the responses to a -6.5 percent negative preference shock with an autoregressive parameter of 0.95 as in Christiano, Motto, and Rostagno [2014]. Similar to the technology shocks, the aggregate output and the aggregate emissions increase more under the linear environment than under the non-linear, while consumption and interest rate decrease more under the non-linear than under the linear environment, as the ZLB acts as a constraint to the central bank. The same dynamics are observed in each sector regarding output and emissions. The ZLB, by halting investment, further decreases the output and slow the recovery in each sector leading to a higher decrease in emissions.

For all the following sections we use the ZLB environment as the baseline model, and we contrast it to the fiscal, macroprudential, and monetary policies. This is motivated by the fact that current nominal rates are at or near the ZLB in most developed countries, and likely to stay at this level for a prolonged period of time. We will also use the exact same calibrations for shocks in the next section to allow for a precise comparison between different specifications of the model.

4.2 Fiscal Environmental Policy Scenario

4.2.1 A Fiscal Policy To Meet the Paris Agreement

To compare the economic variations, we contrast a laissez-faire scenario where no environmental policy is implemented with a scenario where the government is inline with the COP 21 Agreement (i.e. a GHG emission reduction target of 20 percent), and thus implements an environmental policy.

Technically, this means that the business-as-usual policy would set $\tau_{e,t} = \mu_t = 0$ indicating that firms are not investing in any abatement technology to reduce emissions nor is there an enforced policy controlling for emissions production; while the environmental policy regime sets a tax on emissions at a fixed level aiming at reducing by 20 percent the emissions level. As we have two types of sectors, we allow the green firms to emit less, however they incur a higher abatement cost for each extra unit than the dirty firms as former are already using green technology, thus making it more difficult for them to abate an extra unit for at cheaper cost.

The steady state level of abatement is therefore determined in such way that the total amount of emissions abated from both sectors—while accounting for their heterogeneities in abatement possibilities, costs, and emission intensity to GDP—totals the Paris Agreement target. Moreover, setting cost parameters at levels such as those found by Heutel [2012], Annicchiarico and Di Dio [2015], and Benmir, Jaccard, and Vermandel [2020] yields an aggregate environmental tax between 10 and 30 percent of total output depending on the cost efficiency (Table 3). While we use the scenario where it is efficient, from a cost perspective, to abate (i.e. $\theta_{d,1} = 0.8$) as a baseline for our analysis in the following section, we keep in mind that different economic shocks (such as Covid-19) could slow down the abatement effort and increase the costs of abatement in the short/medium term.

Figure 8 and Figure 9 compare the responses of both aggregate and sectoral productions Y, Y_g, and Y_d, emissions E (also both aggregate and sectoral), consumption C, inflation π, and real interest R in the case of a positive technology green shock and a positive technology dirty shock similar to the simulation described in the previous section.

On one hand, in our baseline model, a green TFP shock raises the green output as the green sector finds itself more productive compared to the dirty sector, which sees its output fall. However, as the impact of the shock is more significant in the green sector (even though it represents only 30 percent of our economy) relative to the dirty sector, the aggregate output increases.

On the other hand, the aggregate emissions fall driven mainly by the dirty sector production drop. Although a rise of the green sector production increases the emissions level as green firms are more productive, the fact that they are CO2 friendly (i.e. a low emission to GDP intensity), makes this emissions increase less pronounced proportionally to the dirty sector emissions decrease as the firms in the former sector are experiencing a slow in productivity.

In turn, this rise in production both in the case of green and dirty TFP shocks contributes to an increase in household consumption as they see their wealth increase.

Turning to the effect of an introduction of an environmental policy (i.e. an environmental tax) as shown in green and blue in Figure 8 for the case of the green TFP shock, the decline in emissions in the green sector provoked by the environmental policy is characterized by a small decrease in the green output (slightly more pronounced in the case of higher abatement costs as shown with the blue line) as compared to the laissez-faire equilibrium. By reducing the emissions in the green sector, the tax mitigates the climate damages firms face, which in turn contributes to a decrease in firms' output. From the dirty sector perspective, the fall in output, which is a result of a more efficient green sector, decreases emissions (although by less than the baseline scenario), as dirty sector firms are less competitive than green sector firms. Finally, at the aggregate level, as the intensity of emissions to GDP of the dirty sector is far more significant than the green sector, the aggregate emissions decreases by less under the tax policy as compared to the laissez-faire.

Conversely, a dirty TFP shock, as represented in Figure 9, has the opposite effects on global emissions than that of a green technology shock. As argued by Heutel [2012] among others, the emissions decrease when an environmental policy is introduced, thus retrieving the pro-cyclicality aspect of an environmental tax. Thus, it is optimal to increase the tax during booms, and to lower it during recessions, as a consumption sacrifice could become very costly. In this case, the introduction of the tax has a negative effect on consumption as it decreases by 0.8 to 10 percent depending on the cost efficiency (Table 2 and (Table 3).

Relative to the cases of the green and dirty technology shocks, the preference shock in Figure 10 shows that the policy has a similar impact. In the laissez-faire equilibrium, the negative shock generates a decrease in consumption almost similar in magnitude to the environmental tax scenario, as households are less impatient and therefore prefer to postpone consumption to the next periods. The shock also leads to an initial decrease in the level of emissions as firms' production fall driven by the consumption drop. However, as the household inter-temporal trade-off increases savings, and thus investments, firms' output rises consequently in both sectors rapidly thereafter. The policy helps reduce the quantity of emissions by a considerable percentage relative to the laissez-faire scenario without distorting the economy driven mainly by the efficiency of abatement costs. As the abatement costs rise it is expected that the tax would distort the household consumption.

Finally, we also note that as in the case of the technology shock, the preference shock generates a fall in the policy rate as well as inflation as firms are more productive.

4.2.2 The Optimal Environmental Fiscal Policy

In this section we investigate the role abatement costs play in the effectiveness of an optimal policy à la Ramsey, where a benevolent government (Ramsey planner) maximizes the expected discounted utility of households, given the constraints of the decentralized economy. We consider the case of a Ramsey planner controlling optimally the tax rate on emissions. As a common practice, we assume that the government is able to commit to the contingent policy rule it announces at time 0 (i.e. ex-ante commitment to a feedback policy, so as to have the ability to dynamically adapt the policy to the changed economic conditions). In what follows we first consider the case of a Ramsey planner choosing environmental regulation for different levels of environmental abatement costs Z_t. We start from the optimality conditions for households and firms and then reduce the number of constraints to the Ramsey planner's optimal problem by substitution. The dynamic responses of the Ramsey plans are computed by taking second order approximations of the set of first order conditions around the steady state.

Ramsey Optimal Response Under Environmental Externality Policy

The Ramsey optimal tax rate on emissions, which maximizes welfare, namely τ_{et}, only appears in the abatement function u. Thus, the Lagragian problem reads as follows:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L_{t+i,k}^{1+\varphi} \right.$$
$$\left. + \sum_i \lambda_{i,t,k} [\mathrm{FOC}_{i,t,k} + BC_{i,t,k}] \right),$$

with $\{\lambda_{i,t,k}\}$ representing the sequences of the Lagrange multipliers both on the first order conditions of the competitive economy $\{\text{FOC}_{i,t,k}\}$ and the constraints $\{BC_{i,t,k}\}$.

The planner will choose the sequences

$\{Q_{k,t}, C_t, Y_{t,k}, L_{t,k}, I^n_{t,k}, K_{t+1,k}, E_{t,k}, X_{t,k}, \Theta_{t,k}, N_t, \nu_t, \Gamma_t, \Upsilon_{t,k}, D_{pt,k}, \Pi_{t,k}, p^*_{t,k}\}_{t=0}^{\infty}$ and $\{\tau_{et,k}\}_{t=0}^{\infty}$ given the exogenous processes. The first-order conditions as usually outlined are optimal from a "timeless perspective" in such a way as to prevent the Ramsey planner from reneging on prior announcements.

Findings

The model is solved through Dynare using the Ramsey setup and applying perturbation methods. As seen in Annicchiarico and Di Dio [2015] the optimal level of environmental tax policy is found to be of small order.

As the fraction of dirty firms is much higher in the economy than that of green firms, and as the former have higher intensity of emissions to output, the Ramsey social planner will optimize over the dirty sector and then set the same level of the tax to both sectors.

The level of the optimal tax, which maximizes the welfare is found to be of a small magnitude (as mentioned), as the household welfare tends to deteriorate when a tax policy is introduced since the utility of consumption does not capture the effects of climate change directly (Benmir et al. [2020] show how the welfare improves if the marginal utility of consumption to emissions $u_{ec} \neq 0$.[21]). The negative effects of the environmental externality are captured through the production of the firms and then impact the household via the potentially shrinking profits. However as the magnitude of the latter is of a small proportion, the optimal solution is found to be of the order of .3 percent.

The optimal tax is found not to be very sensitive to the abatement efficiency, as it is of a small magnitude, as shown in Table 4.

Since the optimal tax is shown to be of a small magnitude, increasing the 10 to 30 percent baseline tax[22] could further distort the welfare, indicating a need to seek other policy

[21] This utility specification could be explored in future research.

[22] The tax levels found under abatement technologies scenarios of $\theta_{1,d} = 0.8$ and $\theta_{1,d} = 10.8$, respectively.

instruments in addition to fiscal ones. In order to achieve higher targets of CO2 emissions reduction—which are otherwise necessary to offset climate change—calls for innovative approaches and policies that could ease the burden on tax payers should be sounded.

Furthermore, political instability (e.g. US withdrawal from the Paris Agreement) and price volatility (e.g. oil price decrease following Covid-19) among other factors, could lead to a sudden drop in firms' abatement efforts as highlighted by Dai, Zhang, and Wang [2017] and Hepburn, O'Callaghan, Stern, Stiglitz, and Zenghelis [2020]. This slowdown in climate change mitigation in turn would generate an increase in the emission to output ratio as abatement levels would experience a decrease. By simulating a small increase in emission to output ratio (of a magnitude of .04 percent)[23]—which we obtain as a result of a 1 percent decrease in abatement levels through an identical negative AR(1) process shock on abatement technologies U_g and U_d—we show in Figure 1 that such a rise in emission to output increases significantly the risk premium in both sectors by about 6 basis points (annually)[24]. Furthermore, Figure 2 highlights the same response to a 1 percent decrease in emissions abatement, however this is under a general macroprudential rule enforced by the regulator where $\lambda_t = h(t)$, as opposed to Figure 1, where we abstract from any macroprudential rule (i.e. setting $\lambda_t = \bar{\lambda}$). The presence of a macroprudential rule decreases the risk premium to a 4 annual basis points, thus suggesting the potential role of such regulation in closing the inefficiency gap. This policy mechanism will be further discussed in the following sections. In turn, as argued by Doh et al. [2015], this is seen to alter monetary policy transmission. Thus, the initial inefficiency gap induced by the CO2 externality that the environmental tax seeks to address remains unsolved by the introduction of the tax alone. As such, macroprudential and monetary policies could play an important role in closing the inefficiency gap and helping to achieve mitigation goals[25].

[23] As to not significantly alter the decoupling dynamics empirically measured in the US and the EU, among other countries and economic areas (see Dai et al. [2017]).

[24] We note that the impact on the risk premium significantly depends on the efficiency of abatement (i.e. abatement cost). For example, a small increased of 50 percent in abatement costs in each sector would raise the risk premium to more than 8 basis points.

[25] Keeping inline with the Tinbergen Principle (i.e. one inefficiency one instrument).

4.3 Introducing Macroprudential Policy

4.3.1 Macroprudential Policy à la Pietrunti [2017]

We start by investigating the effect of macroprudential policy as developed in subsection 4.3. We first show the effect of a simple drop in the weight on green loans in the regulatory constraint. This corresponds to changing the steady state values in (52) and (53) and setting the reaction parameters to 0. The idea is that the regulator wants to give an incentive to banks to invest in green loans rather than dirty loans, but does not respond to changes in risk premia. For financial intermediaries, it means they have to hold less net worth to maintain the same level of loans to the green sector. In other words, we expect this shift in $\bar{\lambda}_g$ to increase K_g at the steady state and hence to lead to a greener economy. To perform the following exercises, we now set $\bar{\lambda}_g$ to 0.7, maintaining $\bar{\lambda}_d$ unchanged at 1[26]. We first show the impact on steady state values in Table 5[27]. In particular, we see that a decrease in the green loans weight of 30 percent leads to an increase of the green capital stock of more than 3.1 percent, resulting in a rise in output of 1.03 percent. However, this goes hand in hand with a decrease in the rate on green loans, inducing a spread between dirty and green rates. In our setup, it will have consequences on the behavior of banks that have to maximize their objective function.

We then simulate the responses of our model to a shock to the emission to output ratio as in the previous section under three scenarios. The blue line in Figure 11 is the model with the environmental tax, the dotted red line is the model with fixed but different weights on loans, and the dashed green line is the model with variables weights as presented in the model section. Interestingly, the model with fixed weights induces a trade-off between the two sectors. We are able to slightly stabilize the green risk premium at the cost of exacerbating the effect of the shock on the dirty risk premium. In the model with variable weights, however, the rise in the green premium is cut by almost half, while for the dirty

[26] Note that it does not impact the steady state level of the capital ratio.

[27] This table also displays steady states values for a green macroprudential policy within the tax/subsidy setup of Gertler et al. [2012] as discussed in the next section.

premium it remains unchanged. This is because steady state weights are different in the model with variable weights, as in the model with fixed weights. Introducing time-varying macroprudential weights that favor the green sector thus only helps reduce the effect of the shock to the emission to output ratio on the green risk premium.

4.3.2 Macroprudential Policy à la Gertler et al. [2012]

In this section, we conduct the same exercise with a different way of modeling macroprudential policy. We now use a tax/subsidy scheme as in Gertler et al. [2012]. The idea in this case is that the government levies a tax on banks' assets to subsidize the use of outside equity[28]. For this scenario we calibrate the taxes constant τ_g^s and τ_d^s in such a way to retrieve an increase in the steady state level of the equity ratio $\bar{x}_k$ from 9 to 16%, which is close to what is seen in Gertler et al. [2012][29]. The second scenario is a macroprudential policy favoring the green sector. To do so, the government subsidizes both green assets and outside equity by levying a tax on dirty assets. As the goal here is to create heterogeneity between sectors, but not to drastically affect bankers' balance sheets, we target an aggregate level of equity ratio similar to the starting point of 9-10 percent[30].

Figure 12 displays the results of our model to a shock to the emission to output ratio under three scenarios. The blue line is the model with only the environmental tax, the dotted red line is the model with taxes on both assets, and the dashed green line is the model with the tax on dirty assets and the subsidy on green assets. We retrieve the same impact on risk premia when macroprudential policy is not active, which shows the robustness of our baseline model. Regarding macroprudential policy, it seems that the tax/subsidy scheme of Gertler et al. [2012] is more efficient in reducing the impact on spreads. Interestingly, the green macroprudential policy allows to further reduce the impact on the dirty risk premium but barely affects the green risk premium. This can be explained by the fact that banks are better capitalized when it comes to dirty assets, which makes them less sensible to market

[28]The full specification of the model can be found in C.
[29]This means setting $\tau_g^s = .0033$ and $\tau_d^s = .0033$, close to the calibration of Gertler et al. [2012].
[30]This means setting $\tau_g^s = -.004$ and $\tau_d^s = .005$, yielding $\bar{x}_g = 1.63\%$ and $\bar{x}_d = 19.82\%$.

price variations. On the steady state side, however, Table 5 shows that the tax/subsidy scheme does not allow for a boost to the green sector's capital and output, whereas this was the case in the weights scheme developed in the previous subsection.

In Table 6, we perform a counterfactual exercise where we set the environmental tax to match the reduction in the emission to output ratio induced by the introduction of sectoral macroprudential weights. From a welfare perspective, combining a carbon tax with a green macroprudential policy is more efficient as it is less distrortionary (-0.53 percent) than relying only on a tax policy (-1.18 percent) that achieves the same degree of emission to output reduction. Furthermore, in a robustness exercise presented in the same Table 6, we find that the higher the stock of emissions, the more interesting it is to combine a tax and a green macroprudential policy. This suggests that the interaction of these two policies would not only be beneficial today, but would also lead to a greater welfare enhancement if it were to be implemented in the future. Finally, we show that reducing the green assets' weight to 0.5 instead of 0.7 implies a 4 basis points lower emission to output ratio and improves the tradeoff between emissions and welfare.

4.4 Quantitative Easing and the Policy Mix

4.4.1 Risk Premia and the Policy Mix

We now introduce quantitative easing. As defined above, the central bank has the ability to substitute to financial intermediaries in financing either green or dirty firms. We first show how this policy would compare to macroprudential policy when it comes to dampening the impact of emissions shocks on risk premia.

Figure 15 plots the responses of risk premia to a shock to the emission to output ratio. We compare three scenarios: i) a model with only environmental tax as in Figure 1, ii) a model with tax and time-varying weights as in Figure 11, iii) a model with tax, time-varying weights, and QE. We find that QE is better suited to offset the impact of emission shocks on risk premia. The reaction of spreads is divided by three and the volatility observed in the other two scenarios is drastically reduced. Considering this result and the findings in subsection 4.3, it clearly appears that time-varying sectoral macroprudiental policy could be implemented to foster medium-term growth in the green sector (thus lowering the emission to output ratio with minimum impact on welfare), while quantitative easing could be used to offset short-term variations in spreads stemming from shocks to emissions, thus altering monetary policy transmission.

4.4.2 Asset Purchase Program Scenario

The scenario studied here is a series of four positive 2 percent shocks on ψ_t^k. This is akin to a purchase program decided by a monetary authority and results in the central bank holding a bit more than 12 percent of either green or dirty assets at the peak of the program. As we want to replicate a planned purchased program, we deactivate the reaction to the spread by setting ϕ_k to 0. We calibrate the auto-regressive parameter to 0.66 so that the assets bought slowly exit the central bank's balance sheet.

Figure 13 and Figure 14 display the reaction of selected variables to a series of positive dirty and green QE shocks, respectively. We plot the responses when only the QE is active (blue line), when both the QE and the tax are active (red dotted line), and when the QE, the tax, and the macroprudential policy are all active (green dashed line). For this exercise, we only consider our baseline macroprudential policy with time-varying sectoral weights.

A first interesting finding and a crucial one for a central bank is that dirty and green QE both induce a rise in the inflation rate. These programs both lead to an increase in the inflation rate of roughly 1.6 percent to 2.5 percent at an annual rate, absent any other shock. The effect on inflation is slightly weakened when sectoral macroprudential rules are active. It is a prerequisite that green QE has a positive impact on inflation in order to become a potential monetary policy tool, and these results indicate that a green QE could also be justified on the ground of low inflation expectations.

A second result is that the introduction of a carbon tax has a positive environmental effect on the impact of QE. It keeps exactly the same effect on output and inflation, but reduces total emissions. However, without introducing macroprudential policy, there is no apparent reason for a central bank to implement green QE rather than dirty QE. This can be explained by the fact that both assets have the exact same yields at the steady state and their level of risk is seen to be the same by financial intermediaries, meaning that the two assets are completely interchangeable for them.

When introducing time-varying weights on loans, however, public authorities can alter this mechanism. In this case, a trade-off appears between higher GDP growth and lower emissions. With both types of QE, the introduction of a tax and macroprudential policy allows the reduction of emissions relative to output. However, opting for green QE leads to a greater drop in emissions, at the cost of a smaller boost to GDP and inflation. Once again, this trade-off would disappear in the event that the green sector expand enough to be as big or bigger than the dirty one. Policy makers could then achieve both higher output and lower emissions with the above mentioned policy coordination.

Figure 3 and Figure 4 represent the transition paths where the weight of the greener sector is gradually increasing, thus making the greener sector predominant. Moving toward a greener economy not only decreases substantially emissions, which in turn decreases the environmental policy (i.e. the tax), it also helps achieve the so sought after decoupling of emissions and output. The emissions to output ratio $E_Y = E/Y$ falls almost linearly with an increase in the weight of the green sector and drive the level of the tax to a low level equivalent to the optimal tax level found under the Ramsey equilibrium.

Turning to the welfare analysis of QE, highlighted in Table 6, we find that QE has no effect on welfare. As we have just seen, asset purchases, be they green or dirty, have the same impact on both sectors when there is no active macroprudential policy. The policy is thus unable to strengthen the green sector compared to the dirty sector and subsequently trigger a decrease in the emission to output ratio by itself. To achieve a given level of emission to output, policy makers would then need to set the environmental tax at the same level as in the case where there would be no QE implemented. This confirms that asset purchases should be used as a short term counter-cyclical tool, while sectoral macroprudential policy could play a more structural role.

5 Conclusion

We developed a macro-environmental-financial DSGE model with both endogenously constrained financial intermediaries and heterogenous firms. We then used the model to assess the effects of various policies and their interactions on carbon emissions.

Within our framework there are trade-offs between maximizing output and consumption and reducing the impact of the economy on climate change. In particular, we find that a 10 percent environmental tax (as total percentage of output)—demanding a level of abatement of 10 percent and 20 percent from the green and dirty sectors, respectively—is needed in order to be aligned with the Paris Agreement target. However, these tax and abatement levels heavily depend on the abatement efficiency (i.e. low transition cost) and are found to be of smaller magnitude under the optimal Ramsey setup. As mitigation efforts needed to offset the negative effects of CO2 emissions exceed those of 20 percent reduction used as a baseline policy in our model and pledged in the Paris Agreement, and as the short/medium term tax effects on the welfare are shown to be distortionary, a fiscal policy alone is not sufficient. Thus, there is a strong need for additional tools. As the externality is shown to impact the risk premium, and possibly alter monetary policy transmission channels, short-term policy tools are of high importance and should be used in future mitigation strategies. Looking at the role monetary and macroprudential policies can play, we find that sectoral time-varying macroprudential weights on loans favorable to the green sector boost green capital and output, implying a lower emissions to output ratio with a minimal welfare cost. Turning to QE, we find that a carbon tax improves the benefits of both green and dirty asset purchases. However, macroprudential policy is needed to provide an incentive to central banks to engage in green QE. Choosing between dirty and green QE then implies a trade-off between higher output and lower emissions. This trade-off would disappear in the event that the green sector grows enough to be as big as or bigger than the dirty sector. Regarding the impact of the environmental externality, we show that QE is more efficient than macroprudential policy in closing the premium inefficiency gap. On the other hand, green macroprudential policy is more suitable to support the transition to clean technology as it does not distort the welfare significantly.

We hope that this article will pave the way for more research on the interaction between environmental, monetary, and macroprudential policies. Many exercises could be conducted using our framework. In particular, we think that further research could be devoted to the impact of non-linearities within the financial sector on the dynamics of the model and to the role that endogenous TFP could play in fostering the emergence of greener output growth. We also believe it could be fruitful to examine how to capture the environmental quality on the welfare of households in more direct ways than in existing models.

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A Appendix: Tables

	Calibrated parameters	Values
Standard Parameters		
β	Discount factor	0.9975
α	Capital intensity	0.33
δ	Depreciation rate of capital	0.025
h	Habits formation parameter	0.8
σ	Risk aversion	2
φ	Disutility of labor	1
η_I	Capital adjustment cost	1.728
$\varkappa$	% of Green firms in the economy	30%
θ	Price elasticity	5
θ_g	Price elasticity in sector G	11
θ_d	Price elasticity in sector D	7
ξ	Price stickiness (Calvo parameter)	3/4
$\bar{L}$	Labor supply	1/3
$\bar{g}/\bar{y}$	Public spending share in output	0.4
Environmental Parameters		
$\bar{e_d}/\bar{y_d} = \varphi_d$	Emissions-to-output ratio in sector D	0.15
$\bar{e_g}/\bar{y_g} = \varphi_g$	Emissions-to-output ratio in sector G	0.5
γ_d	CO_2 natural abatement	1-0.9979
$\theta_{1,g}$	Abatement cost parameter for sector G	2.41
$\theta_{2,g}$	Abatement cost parameter for sector G	2.7
$\theta_{1,d}$	Abatement cost parameter for sector D	.8
$\theta_{2,d}$	Abatement cost parameter for sector D	2.7
a	Damage function parameter	1.3950e-3
b	Damage function parameter	-6.6722e-6
c	Damage function parameter	1.4647e-8
Banking Parameters		
ω	Proportional transfer to the entering bankers	0.004
Δ	Parameter impacting the discount factor of bankers	0.99
$\bar{\lambda}$	Steady state risk weight on loans	0.0177
ρ_λ	Smoothing macropru rule coefficient	0.9
ϕ_λ	Credit gap policy parameter	0.2
θ_B	Probability of staying a banker	0.98
ρ_c	Smoothing monetary rule coefficient	0.8
ϕ_y	Output policy parameter	0.2
ϕ_Π	Inflation policy parameter	1.5

Table 1

Calibrated parameter values (quarterly basis)

	Steady state values		
	Baseline	Tax	% Change
Aggregate Output	1.9893	1.9934	0.2%
Green Output	1.0282	1.0303	0.2%
Dirty Output	0.9984	1.0005	0.2%
Aggregate Emissions	0.3957	0.3218	-19%
Green Sector Emissions	0.1542	0.1390	-10%
Dirty Sector Emissions	0.4992	0.4001	-20%
Emissions to Output Ratio	0.1989	0.1614	19%
Consumption	0.9451	0.9383	-0.8%
Green Sector Abatement	-	0.1	N/A
Dirty Sector	-	0.2	N/A
Aggregate Tax as % of GDP	-	11.13	N/A
Tax as % of GDP in Green	-	12.03	N/A
Tax as % of GDP in Dirty	-	11.20	N/A

Table 2

Steady state values –Baseline versus Tax Policy

	Abatement Efficiency		
	$\theta_{1,d} = 0.8$	$\theta_{1,d} = 10.8$	$\theta_{1,d} = 30.8$
Consumption	0.9384	0.8475	0.6659
Environmental Tax	11%	28%	62%

Table 3

Abatement Cost Sensitivity Analysis

Notes: The figures reported represent the steady states level sensitivity results of the model with financial intermediaries à la Pietrunti [2017] for different abatement costs (for the dirty sector) under a tax scenario aiming at reducing the emission levels by 20%. We use different values of abatement cost in the dirty sector only as it is the dominant sector in our economy. The results are similar for a same sensitivity analysis using a similar strategy.

	Baseline (No abatement)	Efficient Abatement	Costly Abatement
	-	$\theta_{1,d} = .8$	$\theta_{1,d} = 10.8$
Tax as % of GDP in Green	-	0.50	0.52
Tax as % of GDP in Dirty	-	0.29	0.30
Green Abatement Level	-	0.01	0.01
Dirty Abatement Level	-	0.02	0.006
Aggregate Emissions	0.395	0.398	0.391
Green Sector Emissions	0.154	0.151	0.151
Dirty Sector Emissions	0.499	0.482	0.493

Table 4

Ramsey optimal tax sensitivity to the abatement costs in the dirty sector

	Steady state values					
	Tax 1	Tax 2	MacroPru 1	MacroPru 2	% Change 1	% Change2
Aggregate Output	1.9934	2.0120	2.0029	2.00947	0.4765	-0.1257
Green Output	1.0303	1.0399	1.0409	1.0391	1.0288	-0.0770
Dirty Output	1.0005	1.0098	1.0005	1.0081	0	-0.1683
Aggregate Emissions	0.3218	0.3248	0.3222	0.3243	0.1243	-0.0167
Green Sector Emissions	0.1390	0.1403	0.1405	0.1402	1.0791	-0.0712
Dirty Sector Emissions	0.4002	0.4039	0.4001	0.4032	-0.0250	-0.1733
Emission to Output Ratio	0.1614	0.1614	0.1609	0.1614	-0.3098	0
Consumption	0.9383	0.9424	0.9415	0.9419	0.3410	-0.0530
Green Capital Stock	10.5831	10.8847	10.9145	10.85	3.1314	-0.3188
Dirty Capital Stock	9.6890	9.9652	9.6889	9.9141	-0.001	-0.51
Green Real Rate	1.0045	1.0040	1.0039	1.004	-0.0597	0
Dirty Real Rate	1.0045	1.0040	1.0045	1.004	0	0
Aggregate Tax as % of GDP	11.03	11.13	11.10	11.13	-0.2695	0
Tax as % of GDP in Green	12.20	12.03	12.15	12.04	-0.9975	0.0831
Tax as % of GDP in Dirty	11.13	11.20	11.20	11.20	0.6289	0

Table 5

Steady state values –Tax versus Tax and Macroprudential Policy

Notes: The figures reported under Tax 1 and MacroPru 1 represent the simulation results of the model with financial intermediaries à la Pietrunti [2017] under a tax policy scenario and a macroprudential policy scenario, respectively, while Tax 2 and MacroPru 2 represent the simulation results for the same policy scenarios, however, with financial intermediaries à la Gertler et al. [2012].

		Welfare		
		Mean	Std. Deviation	% Change to Baseline
	Actual Stock of Emission			
	Baseline Model	-8.6636	0.0000	-
	Model with Tax Policy	-8.7643	0.0037	-1.16%
	Model with Macropudential Policy	-8.7352	0.0034	-0.82%
	Model with QE Policy	-8.7643	0.0037	-1.16%
	A 50% Increase in the Stock of Emission			
	Baseline Model	-9.1859	0.0000	-
$E/Y = .1609$	Model with Tax Policy	-9.2820	0.0039	-1.04%
	Model with Macropudential Policy	-9.2514	0.0036	-0.71%
	Model with QE Policy	-9.2820	0.0039	-1.04%
	A 100% Increase in the Stock of Emission			
	Baseline Model	-10.0457	0.0000	-
	Model with Tax Policy	-10.1401	0.0042	-0.93%
	Model with Macropudential Policy	-10.1070	0.0040	-0.61%
	Model with QE Policy	-10.1401	0.0042	-0.93%
	Actual Stock of Emission			
	Baseline Model	-8.6636	0.0000	-
	Model with Tax Policy	-8.7663	0.0037	-1.18%
	Model with Macropudential Policy	-8.7198	0.0034	-0.64%
	Model with QE Policy	-8.7663	0.0040	-1.18%
	A 50% Increase in the Stock of Emission			
	Baseline Model	-9.1859	0.0000	-
$E/Y = .1605$	Model with Tax Policy	-9.2840	0.0039	-1.06%
	Model with Macropudential Policy	-9.2351	0.0036	-0.53%
	Model with QE Policy	-9.2840	0.0039	-1.06%
	A 100% Increase in the Stock of Emission			
	Baseline Model	-10.0457	0.0000	-
	Model with Tax Policy	-10.1423	0.0043	-0.96%
	Model with Macropudential Policy	-10.0892	0.0039	-0.43%
	Model with QE Policy	-10.1423	0.0043	-0.96%

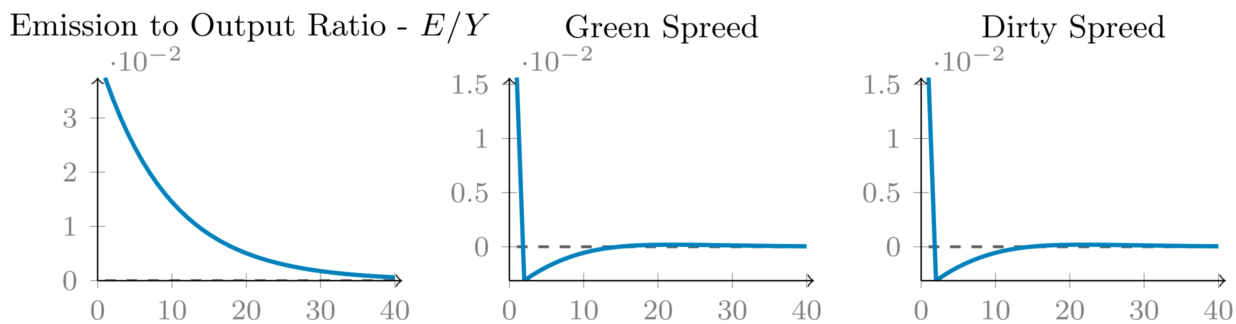
Table 6

Welfare Analysis Under Different Stock of Emission Scenarios

Notes: The figures reported represent the simulation results of the model with financial intermediaries à la Pietrunti [2017] to a negative abatement shock under a tax policy scenario, a macroprudential policy scenario, and a QE scenario. To allow for a comparison between all the scenarios we target a similar emission to output.

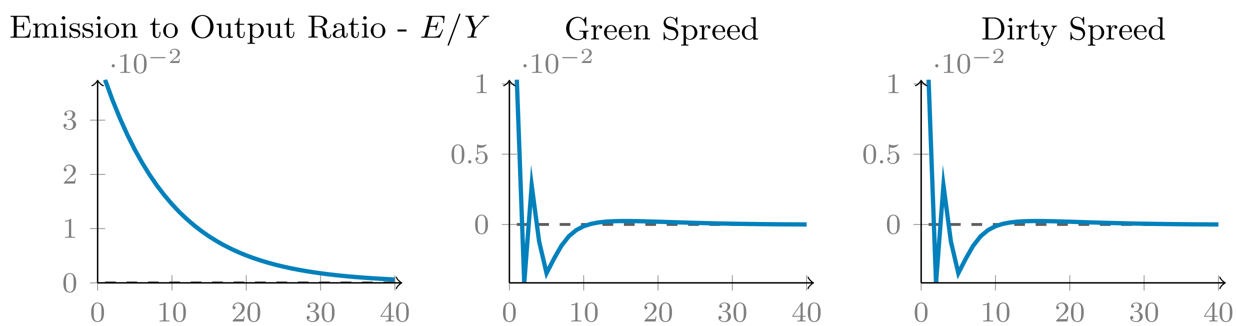
B Appendix: Figures

Figure 1: Effect of a negative abatement shock on the spread in an economy with no Macro-prudential rule - percentage deviations from steady state.



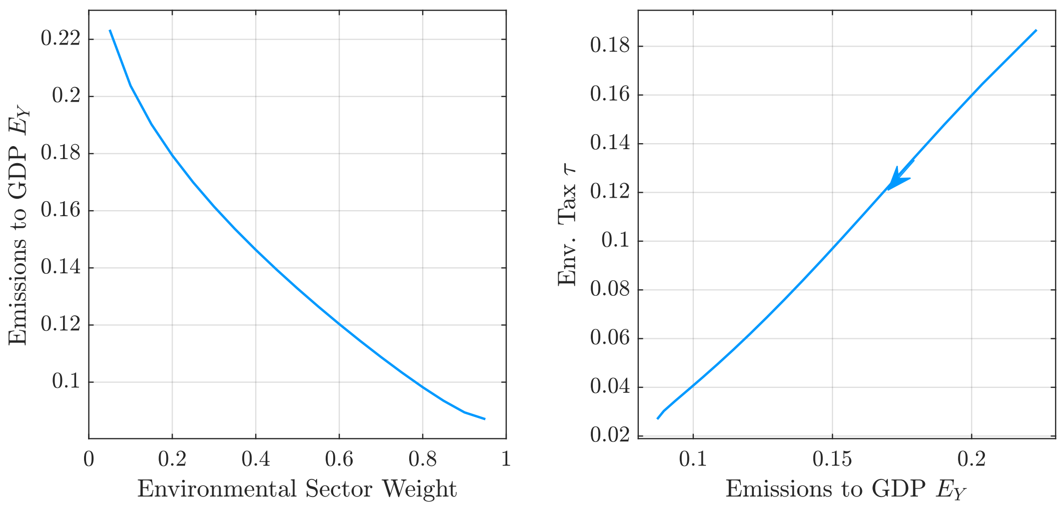
Notes: The simulation is performed under a scenario where abatement decreases by 1 percent (i.e. a 1 percent shock on both U_g and U_d). The risk premium is presented in quarterly deviations from its steady state.

Figure 2: Effect of a negative abatement shock on the spread - percentage deviations from steady state.



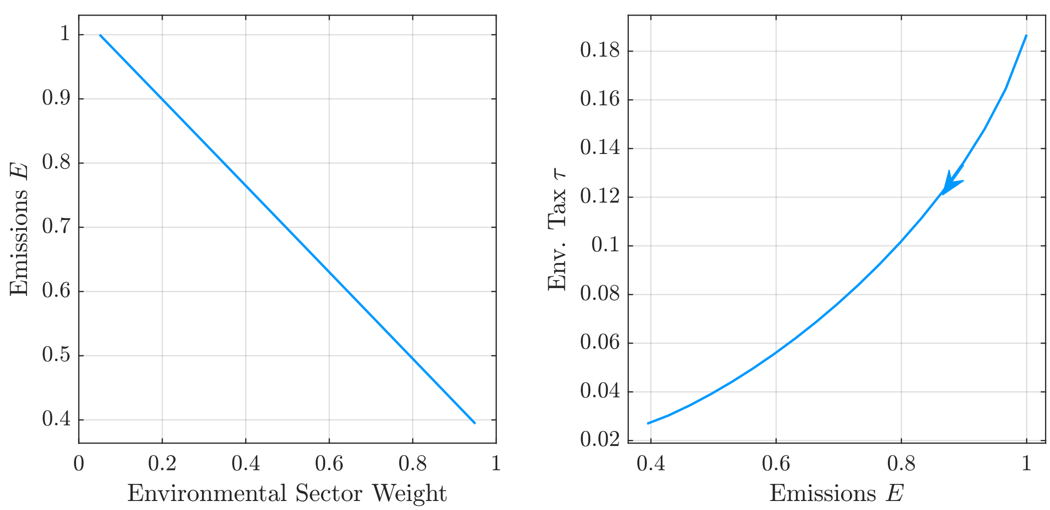
Notes: The simulation is performed under a scenario where abatement decreases by 1 percent (i.e. a 1 percent shock on both U_g and U_d). The risk premium is presented in quarterly deviations from its steady state.

Figure 3: Sectoral weights, carbon intensity, and the environmental policy



Notes: The graph on the left reports the interaction between emissions to output and sectoral weights. The right graph reports how sectoral weight through emissions to output drives the carbon tax.

Figure 4: Sectoral weights, emission levels (normalized to one), and the environmental policy



Notes: The graph on the left reports the interaction between emissions and sectoral weight. The right graph reports how sectoral weights shape the carbon tax.

Figure 5: Effect of a positive green technology shock ($\varepsilon_t^{A_g}$) on selected variables between the linear and non-linear models - percentage deviations from steady state.

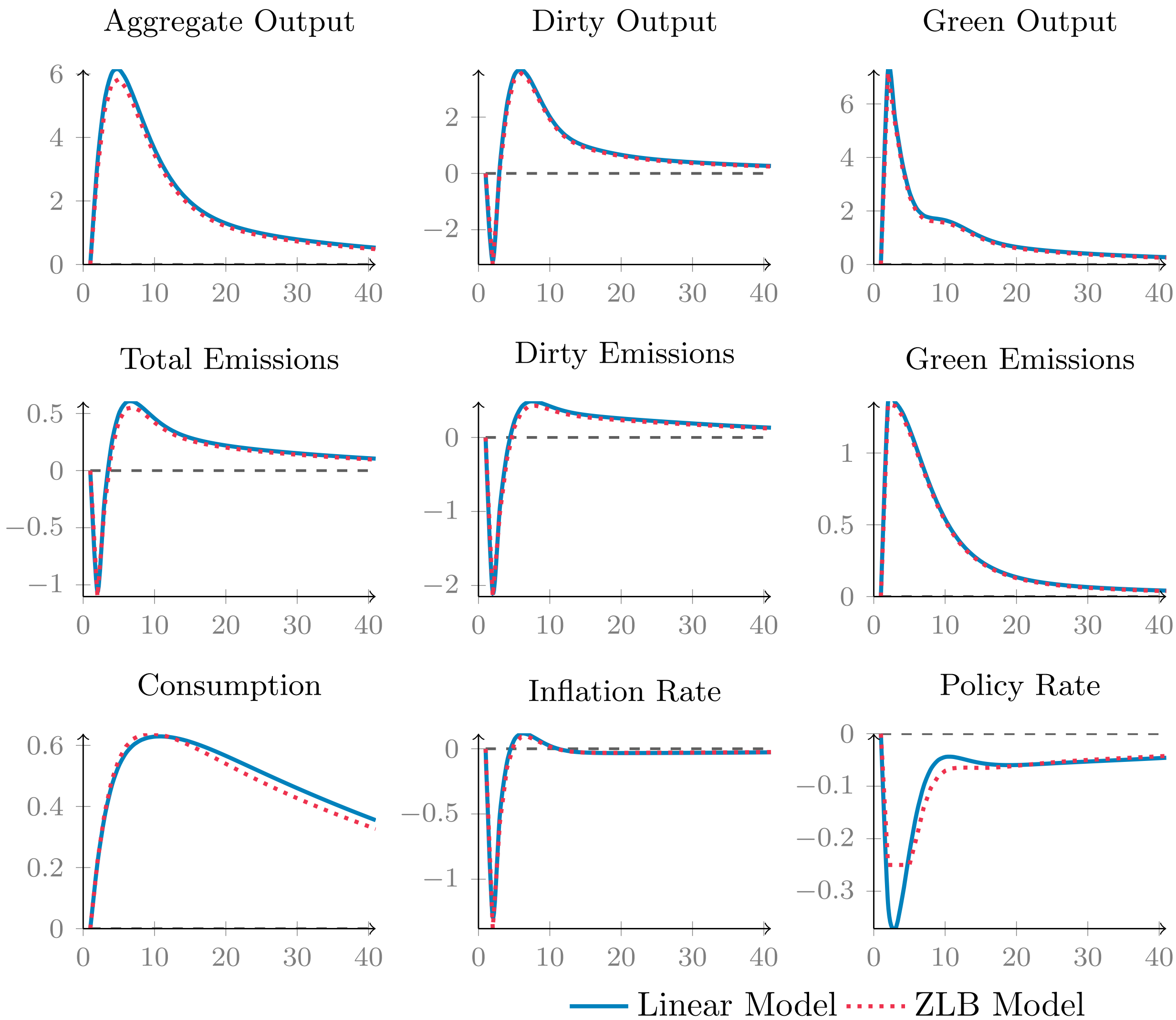


Figure 6: Effect of a positive dirty technology shock ($\varepsilon_t^{A_d}$) on selected variables between the linear and non-linear models - percentage deviations from steady state.

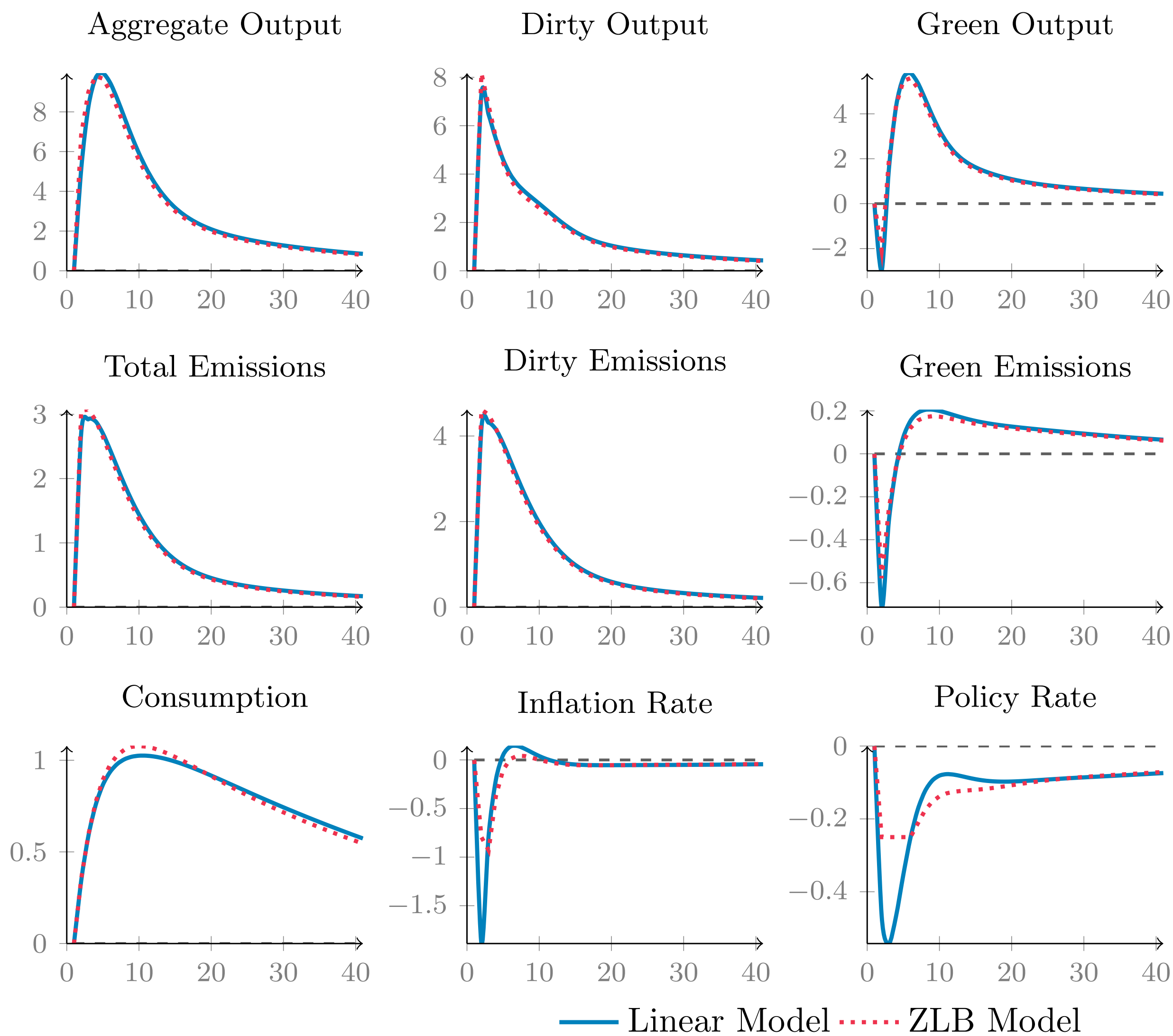


Figure 7: Effect of a negative preference shock (ε_t^B) on selected variables between the linear and non-linear models - percentage deviations from steady state.

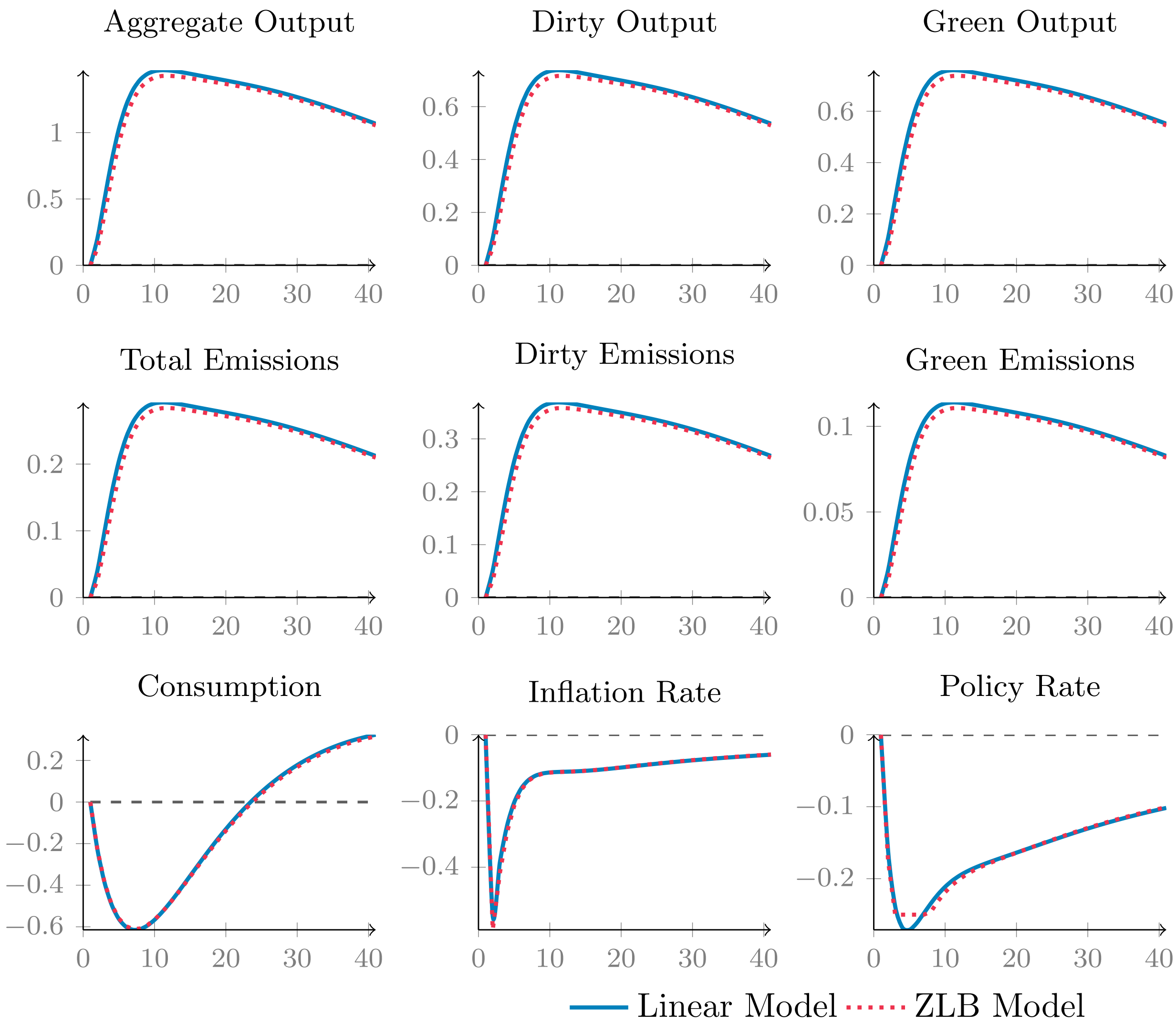


Figure 8: Effect of a positive green technology shock ($\varepsilon_t^{A_g}$) on selected variables between the baseline and tax policy scenarios - percentage deviations from steady state.

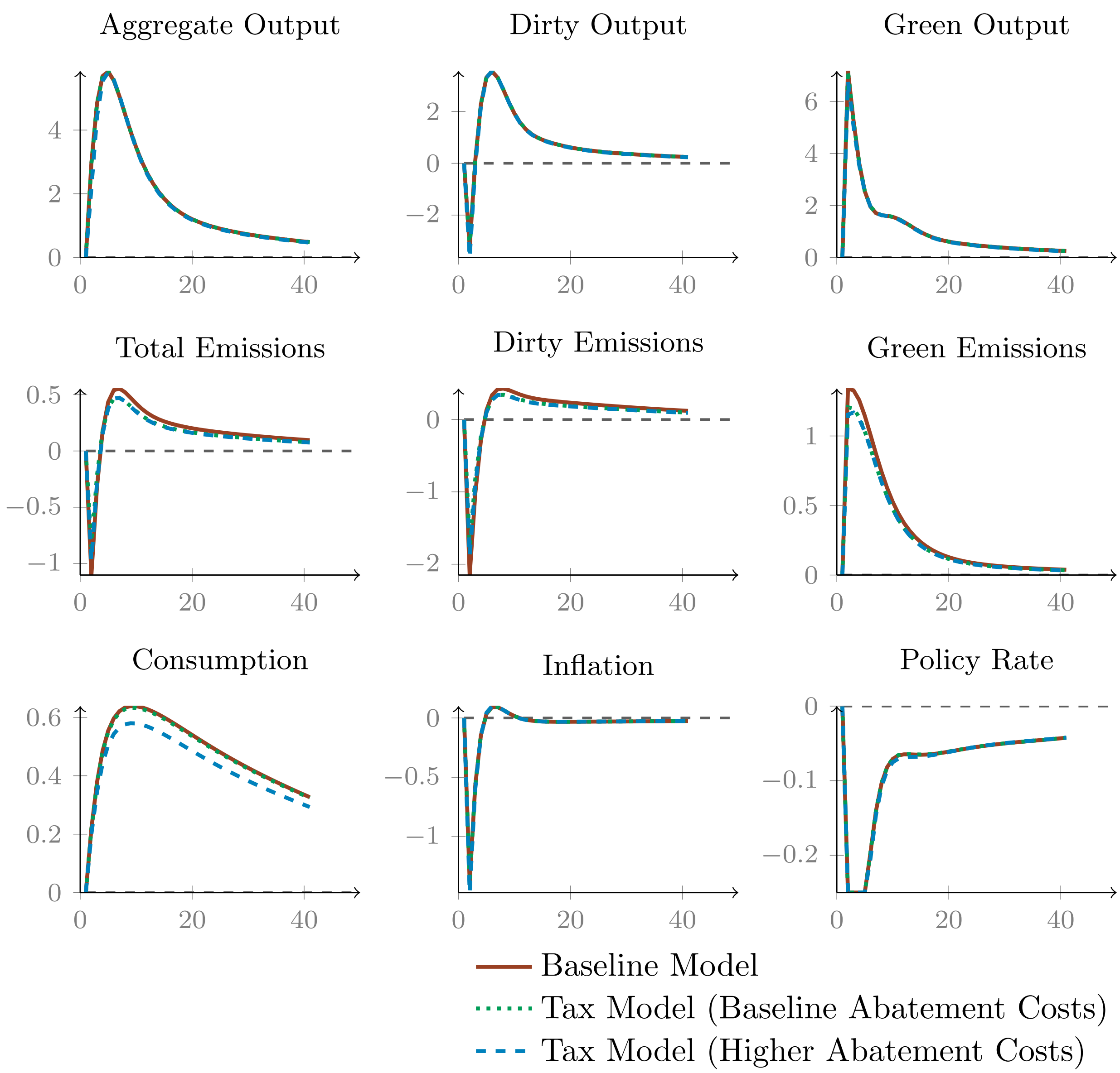


Figure 9: Effect of a positive dirty technology shock ($\varepsilon_t^{A_d}$) on selected variables between the baseline and tax policy scenarios - percentage deviations from steady state.

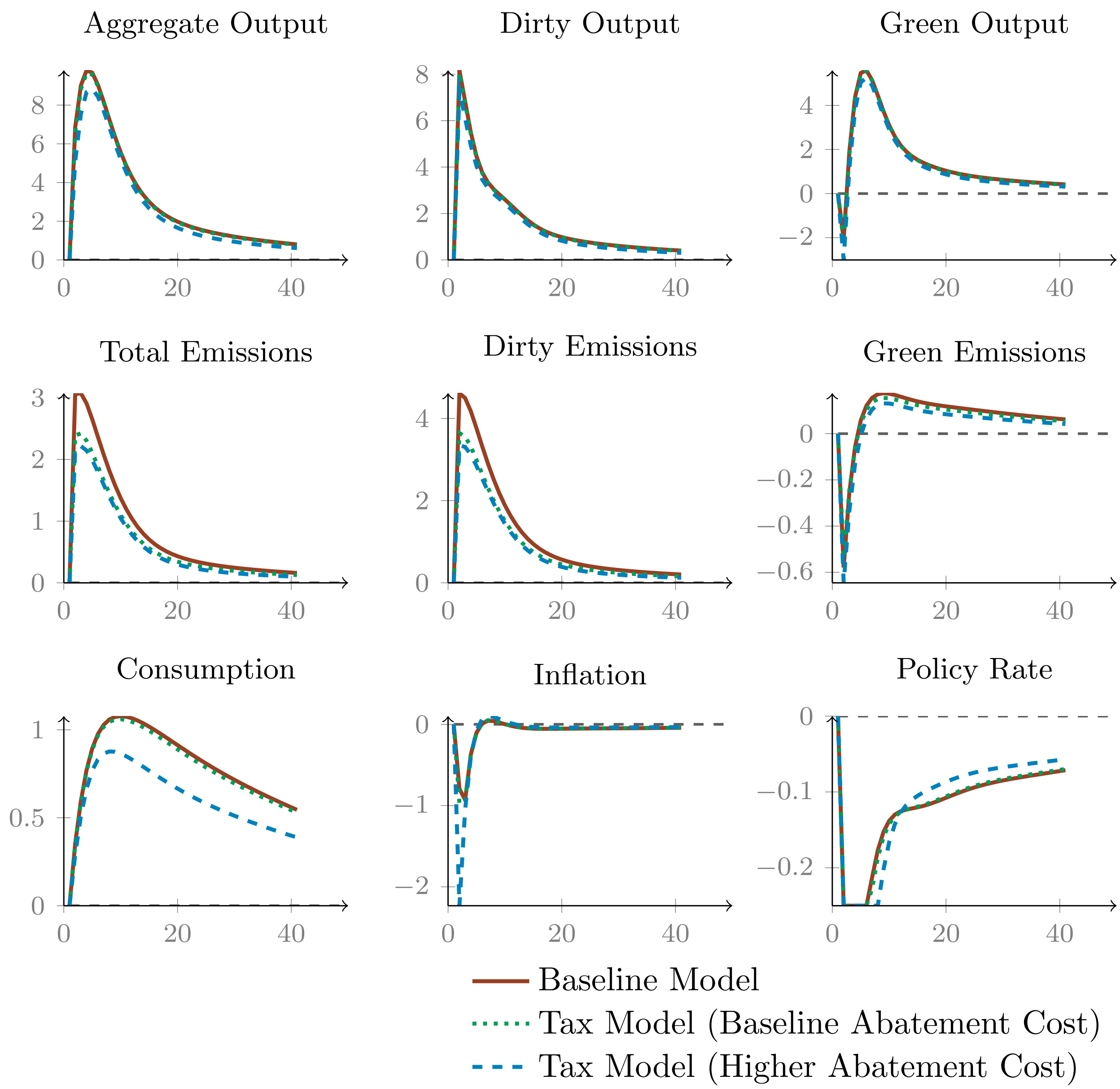


Figure 10: Effect of a negative preference shock (ε_t^B) on selected variables between the baseline and tax policy scenarios - percentage deviations from steady state.

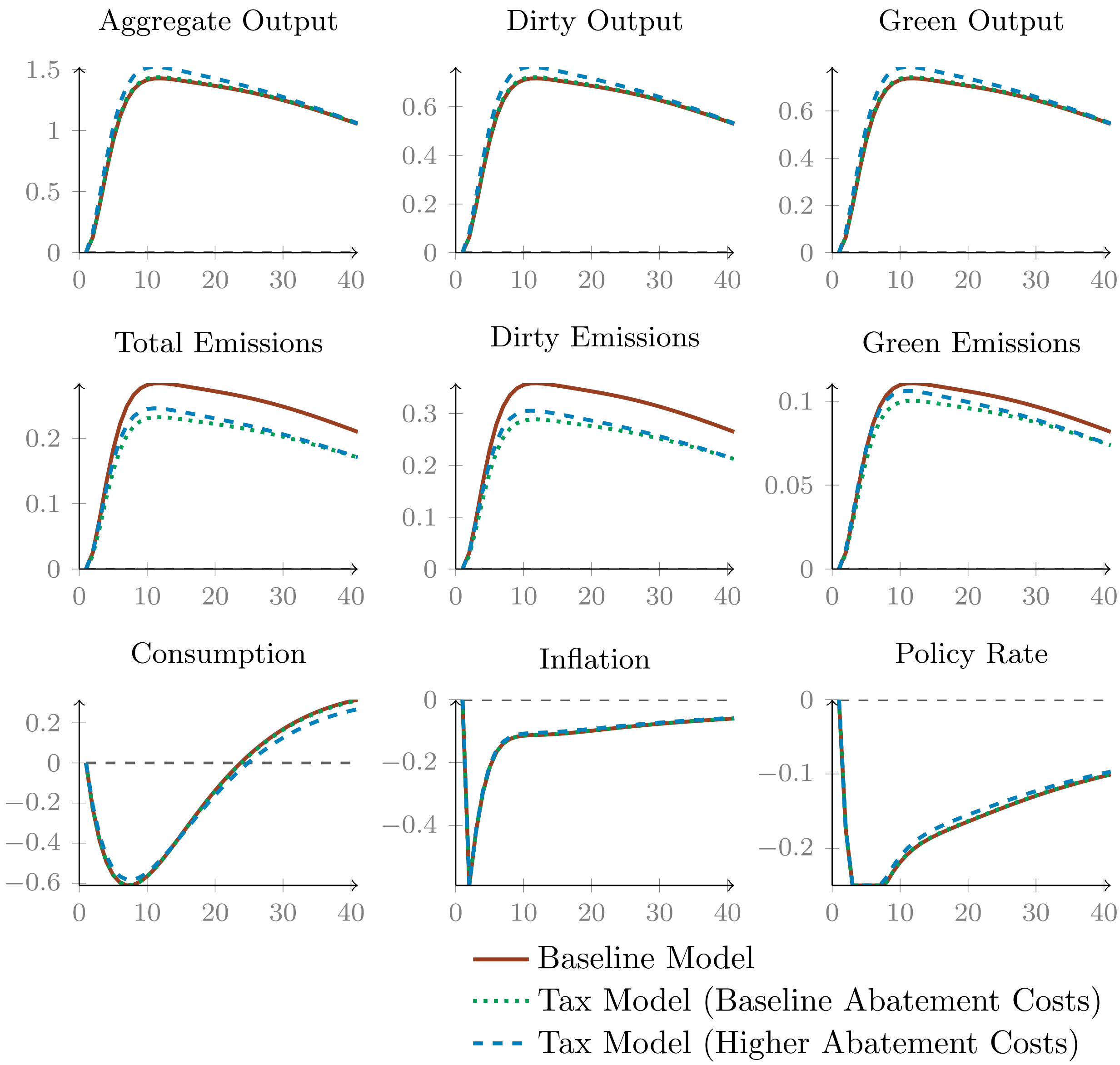


Figure 11: Effect of a negative emission abatement shock ($\varepsilon_t^{U_k}$) on selected variables between the tax policy and macroprudential policy scenarios - percentage deviations from steady state. "Pietrunti (2017) Macroprudential Specification".

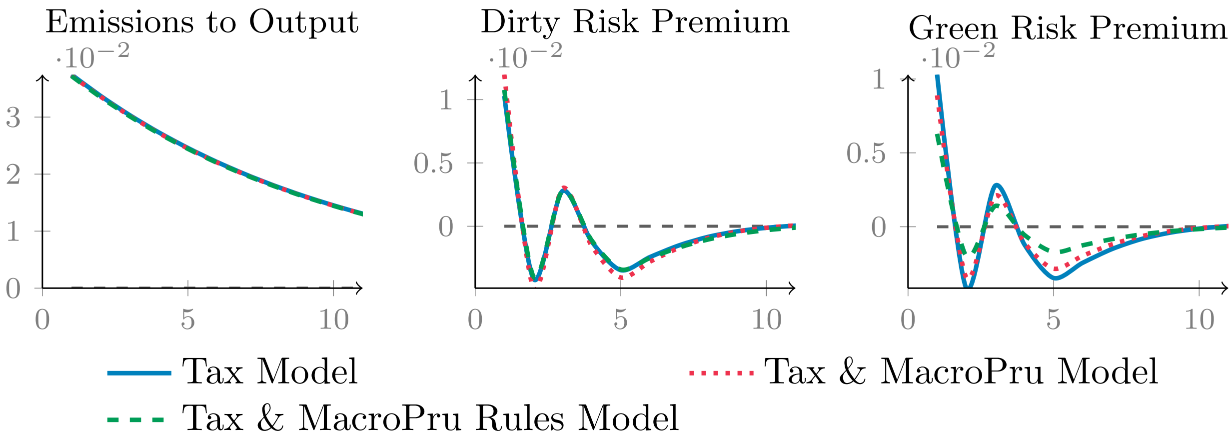


Figure 12: Effect of a negative emission abatement shock ($\varepsilon_t^{U_k}$) on selected variables between the tax policy and macroprudential policy scenarios - percentage deviations from steady state. "Gertler et al. (2012) Macroprudential Specification"

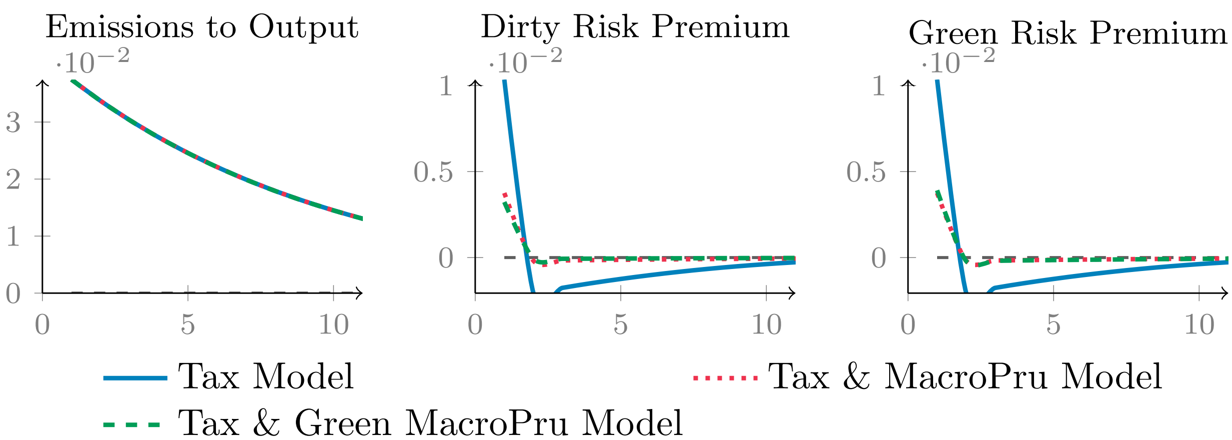


Figure 13: Effect of a series of positive dirty QE shock ($\varepsilon_t^{\psi_d}$) on selected variables - percentage deviations from steady state.

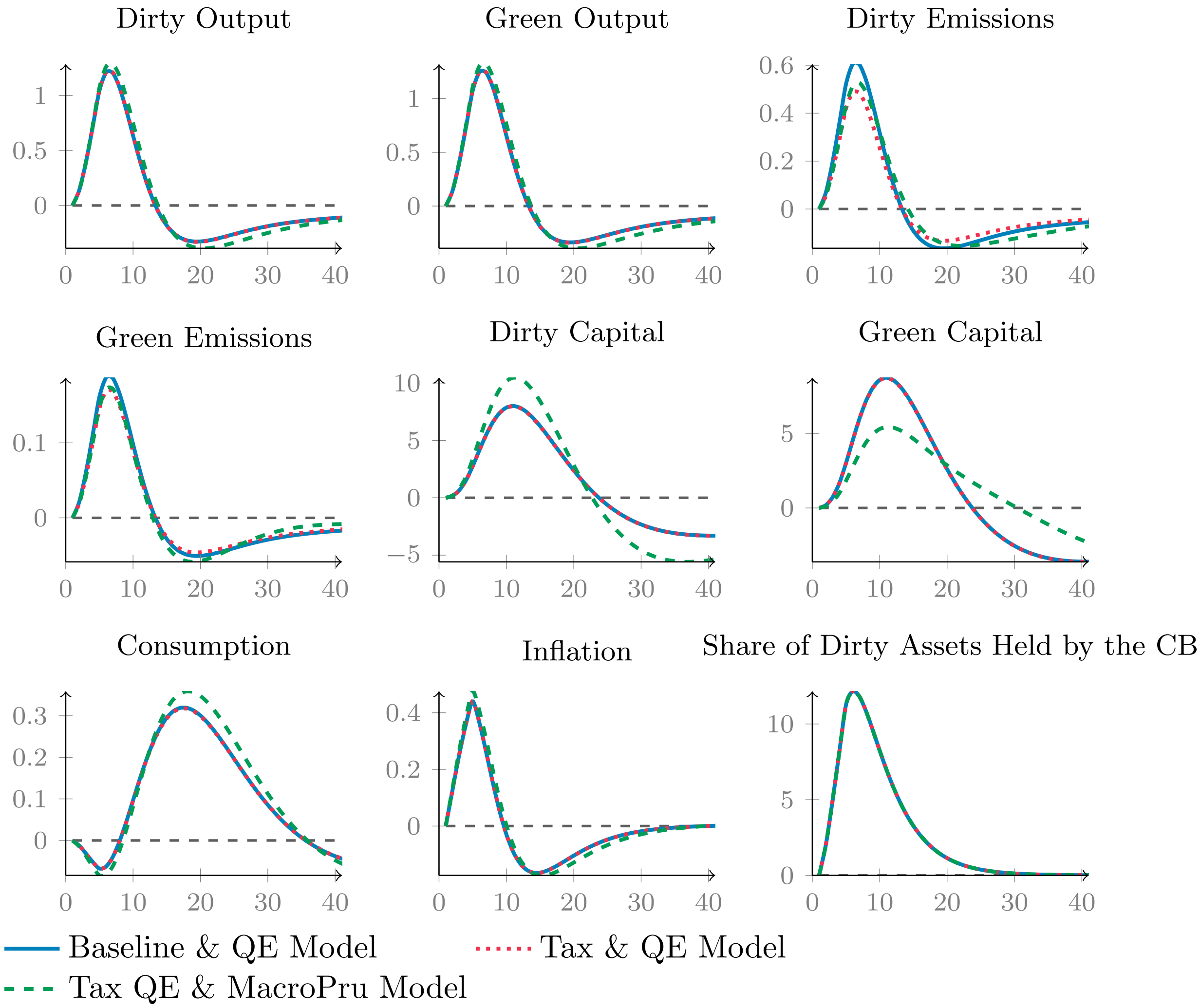


Figure 14: Effect of a series of positive green QE shock ($\varepsilon_t^{\psi_g}$) on selected variables - percentage deviations from steady state.

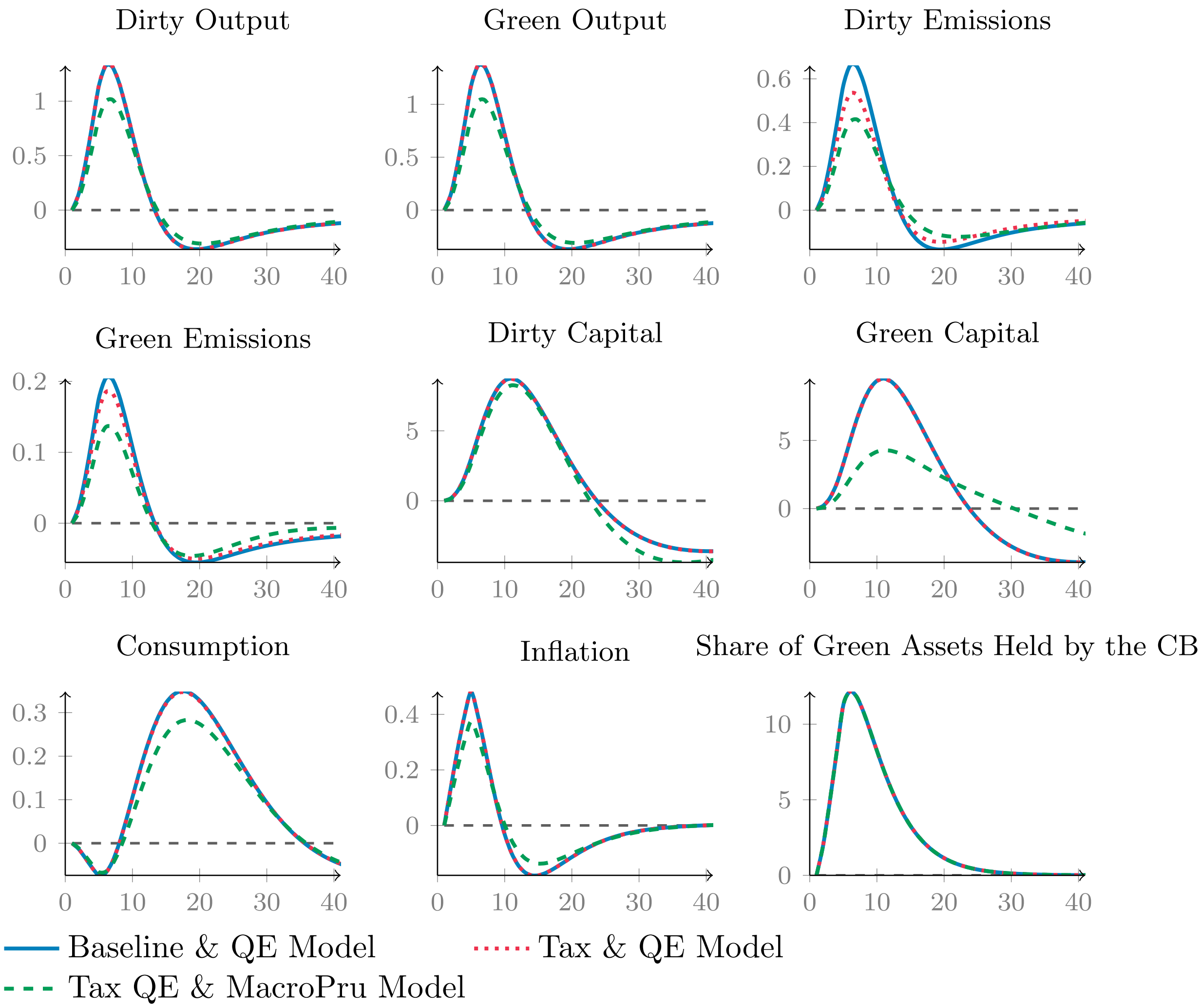
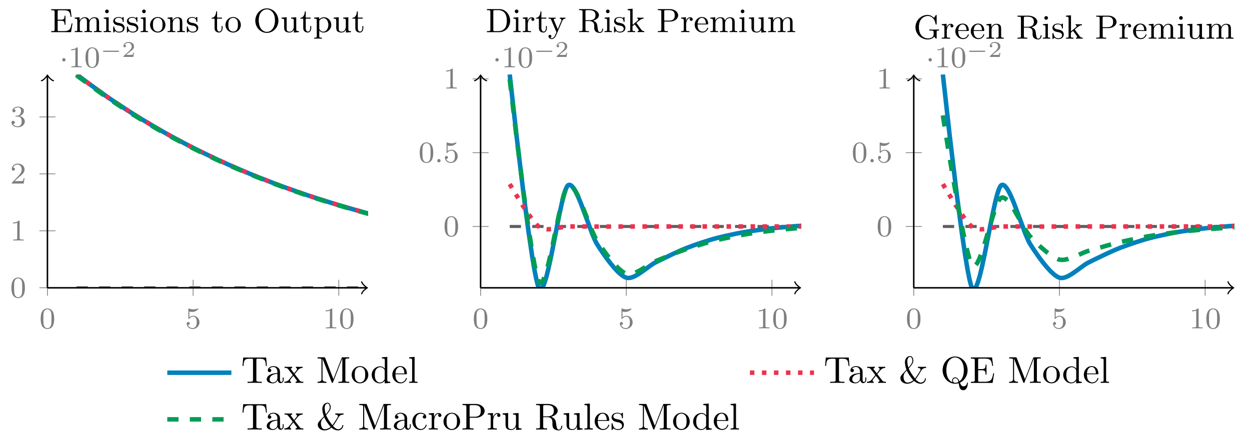


Figure 15: Effect of a negative abatement shock ($\varepsilon_t^{U_k}$) on the risk premium - percentage deviations from steady state.



C Appendix: Robustness Check

In this section we compare our banking modeling approach, which is built à la Pietrunti [2017], with the proposed model specification of Gertler et al. [2012]. The sections treating Firms (2.2) and Public Authorities (2.4) (except Macroprudential Authority (2.4.2)) remain unchanged, while Household (2.1), Financial intermediaries (2.3) and Macroprudential Authority (2.4.2) are adjusted as shown in the following sections. We confirm that our findings regarding the broad dynamics of the model at the ZLB hold with this specification. For the additional parameters calibrations, we take the values of Gertler et al. [2012].

C.1 The Household

We modify our initial setup to allow for a supply of funds to banks through deposits and equity. Deposits, which are non-contingent, risk-less loans to banks, are remunerated at the same rate as government bonds. Given that they are both one-period riskless bonds, they are perfect substitutes. Following Gertler et al. [2012], equity funded by households are modeled as perfectly state-contingent debt and will be called *outside* equity hereafter.[31] We differ from their setup in that we allow households to provide two types of outside equity to banks that will be used to finance either green or dirty firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. The rest of the setup remains unchanged for the representative household.

Th new household maximization problem reads:

$$\max_{\{C_t, L_{t,k}, B_{t+1}, \bar{e}_{t+1,k}\}} E_t \sum_{i=0}^{\infty} \beta^i \frac{\varepsilon^B_{t+i}}{\varepsilon^B_t} \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L^{1+\varphi}_{k,t+i} \right] \tag{60}$$

[31] As opposed to *inside* equity, which are banks' retained earnings.

s.t.

$$C_t + B_{t+1} + \sum_k Q_{et,k}\bar{e}_{t+1,k} =$$
$$\sum_k \left(\frac{W_{k,t}}{P_t} L_{k,t} + \Pi_{k,t} \right) + \frac{T_t}{P_t} + R_t B_t + \sum_k \left[\frac{R^K_{t,k}}{P_t} + (1-\delta) Q_{et,k} \right] \psi_{t,k} e_{t,k}, \tag{61}$$

The sole difference between the above budget constraint and the one used in the main model section is the introduction of equities. As in Gertler et al. [2012], we normalize the units of outside equity $e_{t,k}$ to allow for the equity in each sector to be a claim to the future returns of one unit of the asset that the bank holds. $R^K_{t,k}$ represents the nominal flow of returns generated by one unit of the specific sectoral bank's assets. $Q_{et,k}$ is the price of each type of outside equity. $\psi_{t,k}$ represents the shock on capital quality as in Gertler and Karadi [2011].

The new first order conditions read

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ \frac{\varepsilon^B_{t+1}}{\varepsilon^B_t} (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{62}$$

$$\varrho_t = \chi_k \frac{L^{\varphi}_{t,k}}{W_{t,k}/P_t}, \tag{63}$$

$$1 = \beta E_t \frac{\varepsilon^B_{t+1}}{\varepsilon^B_t} \Lambda_{t,t+1} R_{t+1}, \tag{64}$$

$$1 = \beta E_t \frac{\varepsilon^B_{t+1}}{\varepsilon^B_t} \Lambda_{t,t+1} R_{et+1,k}, \tag{65}$$

where the stochastic discount factor (i.e. the expected variation in marginal utility of consumption) and the returns on sectoral equity $R_{et+1,k}$ read, respectively:

$$\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}, \tag{66}$$

$$R_{et+1,k} = \frac{[\frac{R^K_{t+1,k}}{P_{t+1}} + (1-\delta) Q_{et+1,k}] \psi_{t+1,k}}{Q_{et,k}}. \tag{67}$$

C.2 Financial Intermediaries

We modify the setup of Gertler et al. [2012] to allow financial intermediaries to invest in both green and carbon-intensive ('dirty') firms. They also issue two types of outside equity, depending on the type of firms they want to lend to. A representative bank's balance sheet can be depicted as[32]:

$$Q_{t,g}S_{t,g} + Q_{t,d}S_{t,d} = N_t + B_t + Q_{et,g}e_{t,g} + Q_{et,d}e_{t,d}, \tag{68}$$

where $S_{t,g}$ and $S_{t,d}$ are financial claims on green and dirty firms and $Q_{t,g}$ and $Q_{t,d}$ their respective relative price. On the liability side, N_t is the banks' net worth (also referred to as *inside* equity), B_t is debt to households, and $Q_{et,k}e_{t,k}$ is outside equity for each sector k[33]. Over time, the banks' equity capital evolves as follows :

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,d}Q_{t-1,d}S_{t-1,d} - R_{et,g}Q_{et-1,g}e_{t-1,g} - R_{et,d}Q_{et-1,d}e_{t-1,d} - R_tB_{t-1}. \tag{69}$$

Using equations (68) and (69) we can rewrite the banks' equity capital law of motion as follows:

$$\begin{aligned} N_t =&(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d} \\ &- (R_{et,g} - R_t)Q_{et-1,g}e_{t-1,g} - (R_{et,d} - R_t)Q_{et-1,d}e_{t-1,d} + R_tN_{t-1}, \end{aligned} \tag{70}$$

where $R_{t,k} = \frac{R^K_{k,t}/P_t - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denote the gross rate of return on a unit of the bank's assets from $t-1$ to t for sector k.[34]

The goal of a financial intermediary is to maximize the expected present value of the

[32] As shown in Gertler et al. [2012] the results still hold for all the aggregate banking sector as well.

[33] The outside equity $e_{t,k}$ is the same as the equity held by household as we are interested in equilibrium demand by the household matches the supply from banks.

[34] Note that the depreciate capital has a value of one as adjustment costs only apply to net investment.

future terminal dividend. Thus, we can write the following objective function:

$$V_t = E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1-\theta_B) \theta_B^{i-1} N_{t+1} \right\}. \tag{71}$$

Following Gertler et al. [2012], we assume that managers may divert a fraction of assets to their household once a bank obtains funds. As such possibility arise, households set limitations to the funds they lend to banks. Furthermore, the fraction of funds that could be diverted depends on the composition of the banks' balance sheets. In particular, as highlighted by Gertler et al. [2012], it is assumed "that at the margin it is more difficult to divert assets funded by short term deposits than by outside equity". While, short term deposits constrain the bank to meet a non-contingent payment, dividend payments on the other hand, are tied to the performance of the bank's assets, which is difficult for outsiders to monitor.

Let $x_{t,k}$ denote the fraction of bank assets funded by outside equity for each sector:

$$x_{t,k} = \frac{Q_{et,k} e_{t,k}}{Q_{t,k} S_{t,k}}. \tag{72}$$

Then we assume that after the bank has obtained funds it may divert the fraction $\lambda(x_t)$ of assets:

$$\lambda(x_{t,k}) = \lambda \left(1 + \lambda_1 x_{t,k} + \frac{\lambda_2}{2} x_{t,k}^2 \right). \tag{73}$$

We assume that households require that the discounted value of the bankers' net worth should be greater than or equal to the value they would be able to divert:

$$V_{t,k} \geq \lambda(x_{t,k}) Q_{t,k} S_{t,k}. \tag{74}$$

Using equation (70) and (72) we rewrite again the evolution of the net worth:

$$\begin{aligned} N_t =& (R_{t,g} - x_{t-1,g} R_{et,g} - (1 - x_{t-1,g}) R_t)) Q_{t-1,g} S_{t-1,g} \\ &+ (R_{t,d} - x_{t-1,d} R_{et,d} - (1 - x_{t-1,d}) R_t)) Q_{t-1,d} S_{t-1,d} + R_t N_{t-1}. \end{aligned} \tag{75}$$

Thus for easing the resolution of the maximization problem, we introduce $N_{t,k}$ the net worth for each sector k such that:

$$N_t = \sum_k N_{t,k}, \tag{76}$$

and

$$N_{t,k} = (R_{t,k} - x_{t-1,k}R_{et,k} - (1 - x_{t-1,k})R_t))Q_{t-1,k}S_{t-1,k} + R_t N_{t-1,k}. \tag{77}$$

Thus, the franchise value of the bank at the end of period $t-1$ should satisfy the following Bellman equation for each sector k:

$$V_{t-1,k}(S_{t-1,k}, x_{t-1,k}, N_{t-1,k}) = E_{t-1}\beta\Lambda_{t-1,t}\left\{(1-\theta_B)N_{t,k} + \theta_B \max_{S_{t,k},x_{t,k}}\left[V_{t,k}(S_{t,k}, x_{t,k}, N_{t,k})\right]\right\}, \tag{78}$$

where θ_B is the banks probability to keep existing. We guess as in Gertler et al. [2012] that the value function is linear of the form:

$$V_{t,k}(S_{t,k}, x_{t,k}, N_{t,k}) = (\mu_{st,k} + x_{t,k}\mu_{et,k})Q_{t,k}S_{t,k} + \nu_{t,k}N_{t,k}. \tag{79}$$

In order to solve the above maximization problem with the conjectured value function linear form, we set the leverage ratio for each sector $\Phi_{t,k}$ (i.e. the maximum ratio of bank assets to net worth) such as:

$$\Phi_{t,k} = \frac{Q_{t,k}S_{t,k}}{N_{t,k}}, \tag{80}$$

which indicates that when the borrowing constraint binds, the total quantity of private assets that a bank can intermediate is limited by its net worth $N_{t,k}$.

Maximization of the Bellman function subject to constraint (74) yields the following first order and slackness conditions:

$$\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1,k}(R_{t+1})\right\} = \nu_{t,k}, \tag{81}$$

$$\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1,k}(R_{t+1,k} - R_{t+1})\right\} = \mu_{st,k}, \tag{82}$$

$$\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1,k} (R_{t+1,k} - R_{et+1,k}) \right\} = \mu_{et,k}, \tag{83}$$

where ν_t is the multiplier for constraint (74), $\beta\Lambda_{t,t+1}$ is the banks' stochastic discount factor and $\Omega_{t+1,k} = 1 - \theta_B + \theta_B[\nu_{t+1,k} + \Phi_{t+1,k}(\mu_{et+1,k} + x_{t+1,k}\mu_{et+1,k})]$ the shadow value of a unit of net worth to the bank at $t+1$. Furthermore, we can rewrite the leverage ratio following the above first order conditions:

$$\Phi_{t,k} = \frac{\nu_{t,k}}{\lambda(x_{t,k}) - (\mu_{st,k} + x_{t,k}\mu_{et,k})}. \tag{84}$$

Solving the first order conditions on $x_{t,k}$ and $S_{t,k}$ we rewrite the fraction of assets financed by outside equity in each sector as the ratio of the excess value from substituting outside equity for deposit finance μ_{et} to the excess value on assets over the deposit μ_{st} as follows:

$$x_{t,k} = -\frac{\mu_{st,k}}{\mu_{et,k}} + \left[\frac{\mu_{st,k}}{\mu_{et,k}}^2 + \frac{2}{\lambda_2}\left(1 - \lambda_1 \frac{\mu_{st,k}}{\mu_{et,k}}\right)\right]^{\frac{1}{2}}. \tag{85}$$

Since the leverage ratio does not depend on bank-specific factors, we can aggregate equation (80) to obtain a relation between the aggregate demand for securities by banks $S_{pt,k}$ in each sector and aggregate net worth in the banking sector for each firms sector $N_{t,k}$.

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \sum_k \left\{ (\theta_B + \omega')[R^K_{t,k} + (1-\delta)Q_{t,k}]\psi_{t,k}S_{pt-1,k} - \theta_B[R^K_{t,k} + (1-\delta)Q_{et,k}]\psi_{t,k}e_{t-1,k} \right\} - \theta_B R_t B_{t-1}. \tag{86}$$

C.3 Macroprudential Authority

We introduce a green and dirty tax and subsidy, which help offsetting the banks' incentive to adjust their liability structure. As in Gertler et al. [2012], we consider a tax $\tau_{t,k}$ on the total assets for each sector, which serves as a financing tool for $\tau^s_{t,k}$ the governmental subsidies offered to the banks for each unit of sectoral outside equity issued. The banks new constraint presented in (68) reads as follows:

$$(1+\tau_{t,g})Q_{t,g}S_{t,g} + (1+\tau_{t,d})Q_{t,d}S_{t,d} = N_t + B_t + (1+\tau^s_{t,g})Q_{et,g}e_{t,g} + (1+\tau^s_{t,d})Q_{et,d}e_{t,d}, \quad (87)$$

where we set $\tau^s_{t,k} = \frac{\tau^s_k}{\nu_{t,k}}$ such that the subsidy in each sector k is set to make the net gain to outside equity in each sector from reducing deposits constant in terms of consumption goods.

Furthermore, in the presence of a macroprudential policy, the value function in (79) is modified as follows[35]:

$$V_{t,k}(S_{t,k}, x_{t,k}, N_{t,k}) = ((\mu_{st,k} - \tau_k\nu_{t,k}) + (\mu_{et,k} + \tau^s_k\nu_{t,k})x_{t,k})Q_{t,k}S_{t,k} + \nu_{t,k}N_{t,k}. \quad (88)$$

The new first order condition are simply adjusted as in Gertler et al. [2012] by the tax/subsidy introduced.

[35]Where in equilibrium: $\tau_{t,k} = \tau^s_{t,k}x_{t,k}$.

C.4 Robustness Check: Main Model Dynamic Results

Figure 16: Effect of a positive green technology shock ($\varepsilon_t^{A_g}$) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.

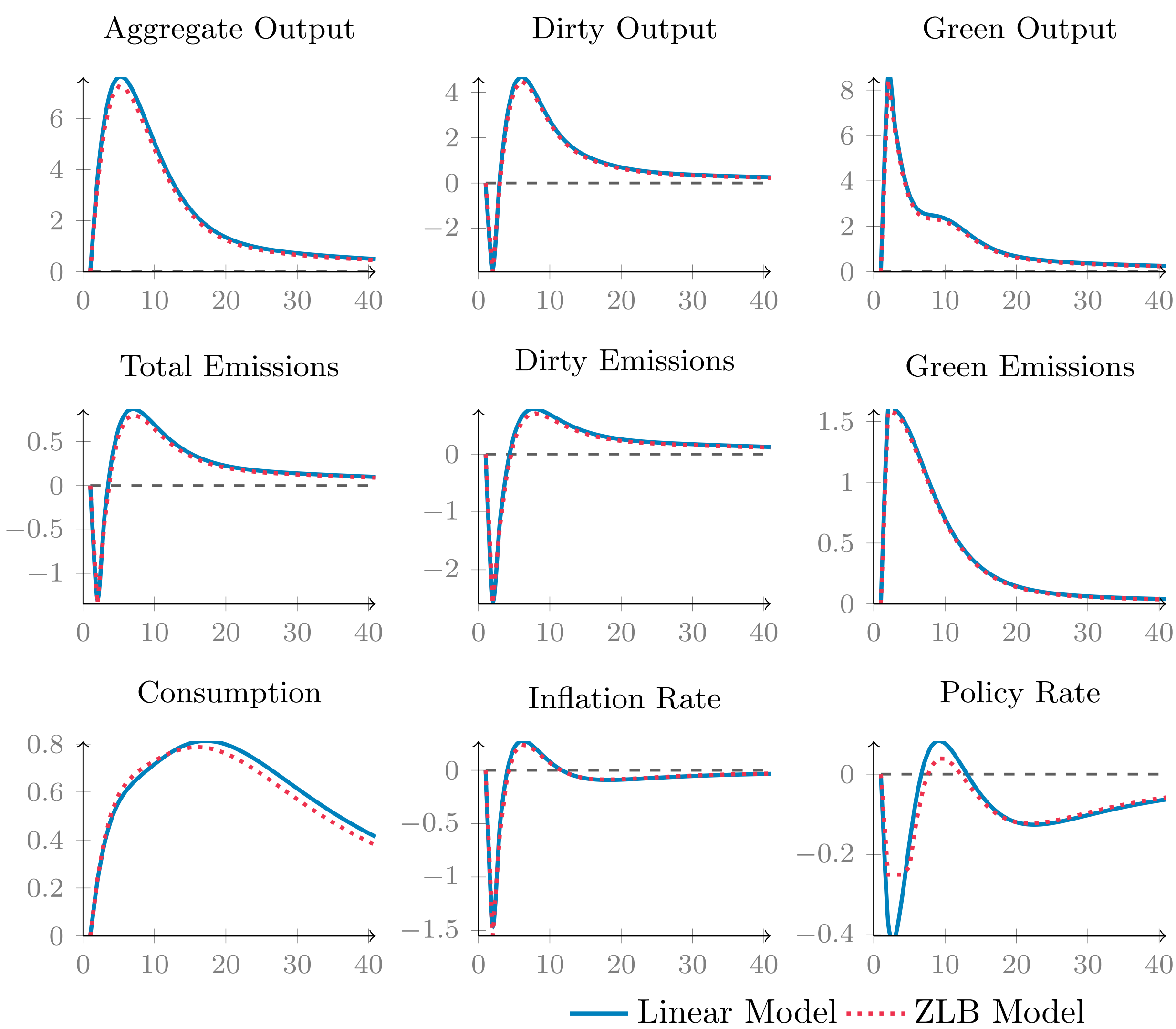


Figure 17: Effect of a positive dirty technology shock ($\varepsilon_t^{A_d}$) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.

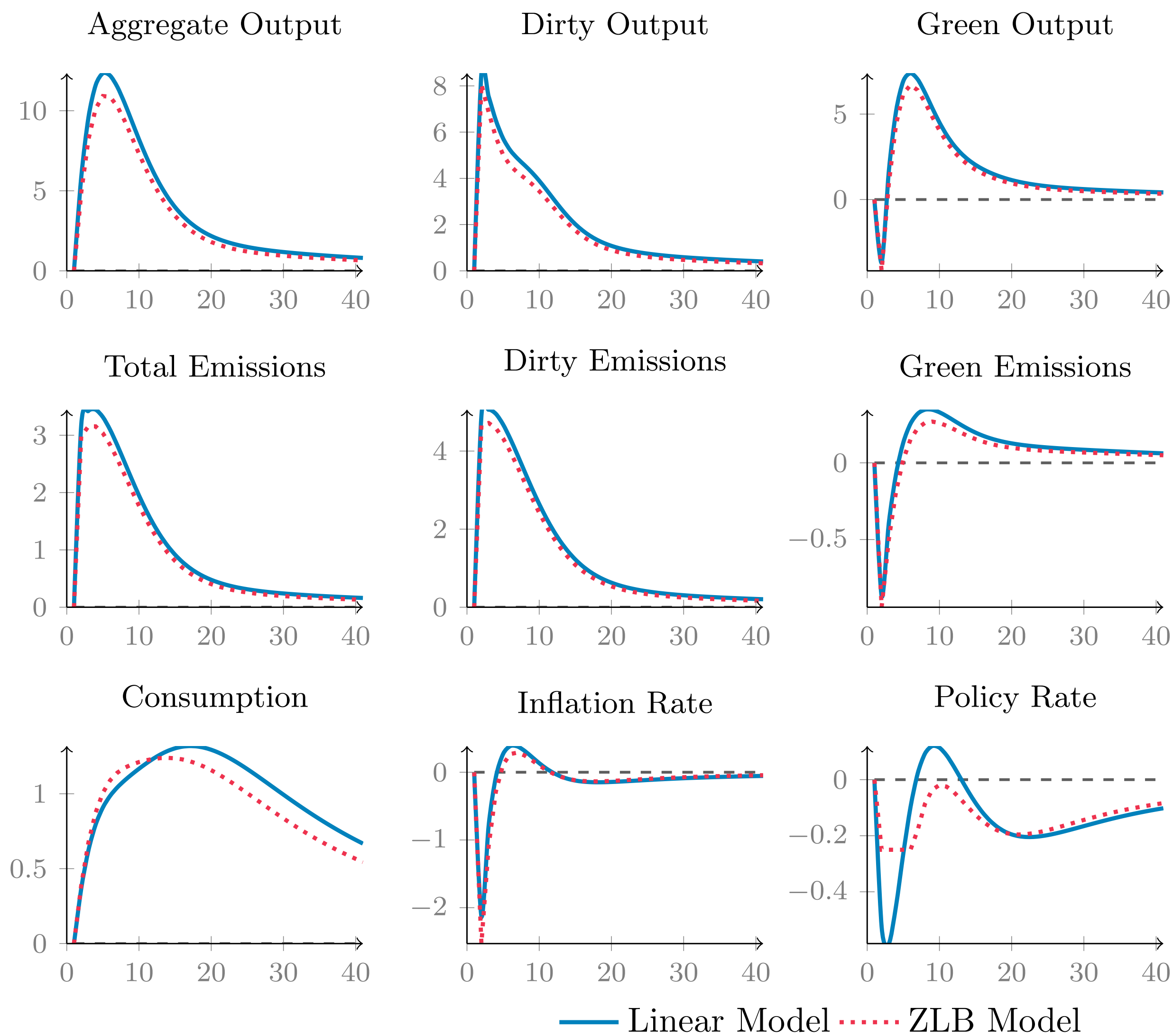
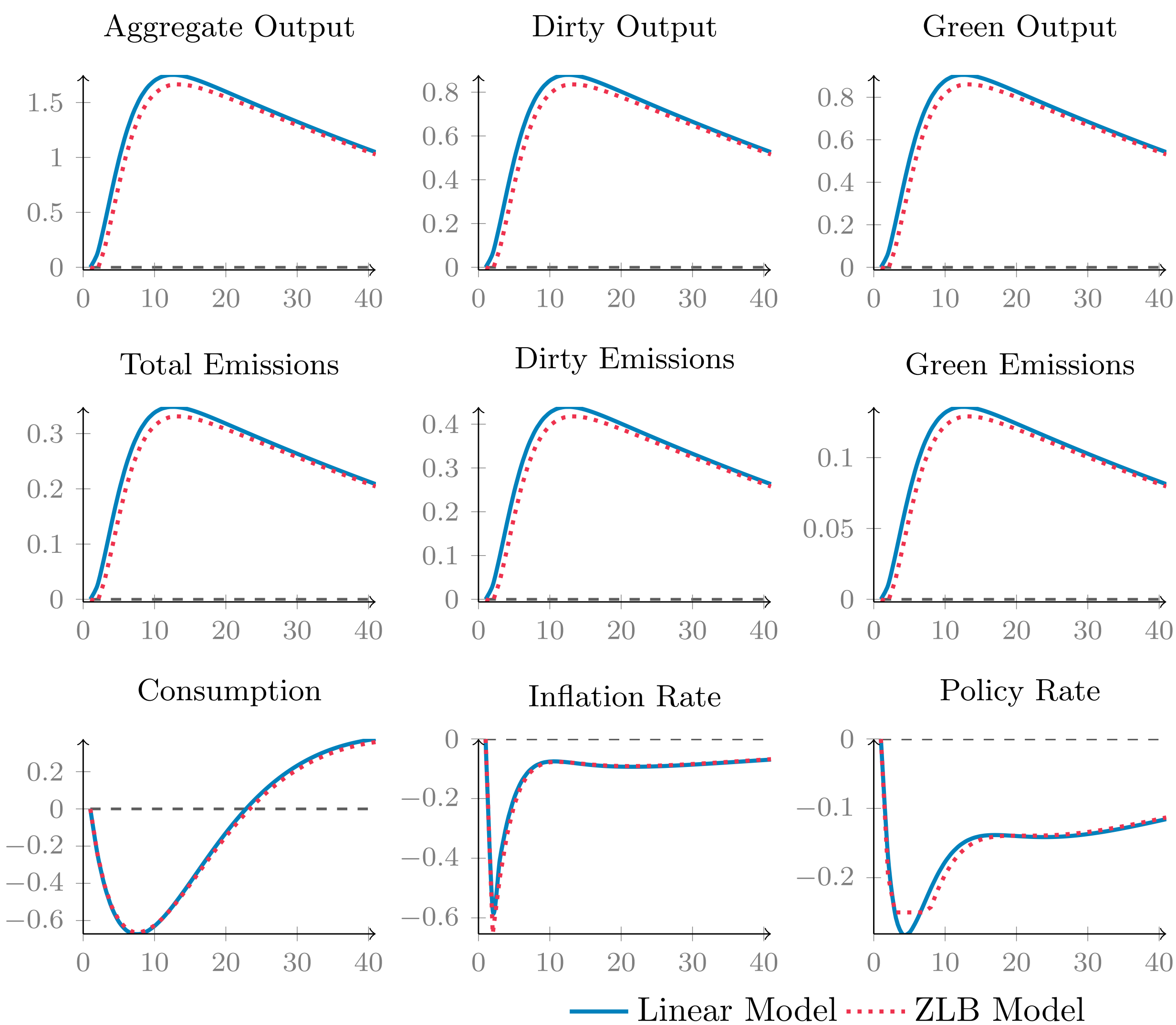


Figure 18: Effect of a negative preference shock (ε_t^B) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.



C.5 Robustness Check: Welfare Analysis Results

	Welfare		
	Mean	Std. Deviation	% Change to Baseline
Actual Stock of Emission			
Baseline Model	-8.6433	0.0000	-
Model with Tax Policy	-8.7384	0.0034	-1.11%
Model with Macropudential Policy	-8.7399	0.0036	-1.12%

Table 7

Welfare Analysis Under Different Stock of Emission Scenarios

Notes: The figures reported represent the simulation results of the model to a negative abatement shock under a tax policy scenario, and a macroprudential policy scenario. To allow for a comparison between all the scenarios we target a similar emission to output.