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# Steering the climate system: an extended comment[*]

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**Abstract**

Lemoine and Rudik (2017) argue that it is efficient to delay reducing carbon emissions, because there is substantial inertia in the climate system. However, this conclusion rests upon misunderstanding the relevant climate physics: there is no substantial lag between $CO_2$ emissions and warming, which policy could rely upon. Applying a mainstream climate physics model to the economics of Lemoine and Rudik (2017) invalidates the article's implications for climate policy: the cost-effective carbon price that limits warming to a range of targets including $2\,°C$ starts high and increases at the interest rate.

*JEL code:* H23, Q54, Q58

# 1 Introduction

The 2015 UN Paris Agreement (United Nations, Framework Convention on Climate Change, 2015) aims to limit global warming to well below $2\,^{\circ}C$ above the pre-industrial level. Analysing how to meet warming targets efficiently is of critical policy importance and economists have not perhaps afforded it the attention it deserves. Lemoine and Rudik (2017), henceforth LR17, explore the implications of inertia in the climate system for cost-effective paths to hold warming to such a target level. Yet economists have tended to focus on optimisation without a temperature constraint, so the new framework presented by LR17 is welcome and likely to trigger a wealth of new research.

LR17 show that if there is a substantial lag between $CO_2$ *emissions* and warming, then warming can be limited to $2\,^{\circ}C$ at much lower cost than standardly concluded by delaying emissions reductions for decades and keeping carbon prices near zero until 2075. Rather than rising at the interest rate according to Hotelling's rule, the least-cost carbon price in LR17 follows an inverse U-shaped path and grows much more slowly than the interest rate throughout the 21st century.

These conclusions are important, all the more so because they diverge markedly from findings in mainstream economic analysis, such as Golosov et al. (2014). They also diverge from the conclusions of recent, high-level policy syntheses (Stiglitz and Stern, 2017; Clarke et al., 2014), according to which global carbon prices start high and rise quickly. These are in the range US\$50-100 per metric ton of $CO_2$ in 2030 along a path that limits warming to $2\,^{\circ}C$ at least cost. By contrast, the least-cost carbon price in 2030 in LR17 is still close to zero (their Figure 1, Panel D). LR17 conclude from their striking results that "it should be a high priority to reassess [standard] models' conclusions using frameworks that take advantage of the braking services provided by the climate system's inertia." (p. 2957)

However, their conclusions are based on a possible misunderstanding of the climate physics literature. LR17 are correct to state that the "climate system displays substantial inertia, warming only slowly in response to additional $CO_2$" (p. 2948), only insofar as this statement relates to the atmospheric *concentration* (i.e. the stock) of $CO_2$. On the other hand, there is no significant inertia between *emissions* (i.e. the flow) of $CO_2$ and resulting warming, and we show that it is this concept of inertia that matters

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for economic policy.[1] That there is no lag between $CO_2$ emissions and resulting warming is well-known and has been established in climate models for ten years (Collins et al., 2013; Ehlert and Zickfeld, 2017; Hare and Meinshausen, 2006; Herrington and Zickfeld, 2014; Joos et al., 2013; Lowe et al., 2009; Matthews and Caldeira, 2008; Matthews et al., 2009; Matthews and Zickfeld, 2012; Matthews and Solomon, 2013; Ricke and Caldeira, 2014; Zickfeld et al., 2012; Zickfeld and Herrington, 2015 and Section 3.1). The calibration used by LR17 involves lags between emissions and warming that are ten times as long as those in standard climate models (see Figure 1).

In this article, we revisit LR17 and introduce climate dynamics that conform to standard climate models to their model of cost-effective $CO_2$ abatement. This requires us to (a) greatly reduce the time lag between $CO_2$ emissions and warming, and (b) correct the excessive decay of atmospheric $CO_2$ in the LR17 model in the long run. We show that doing so invalidates and indeed partially reverses their conclusions. Carbon prices that keep warming below a target level at least cost start around an order of magnitude higher than in LR17. Thereafter they grow approximately at the interest rate, consistent with Hotelling (1931), which is much faster than carbon prices rise in the LR17 model this century.

We also point out that LR17 are incorrect to claim most Integrated Assessment Models (IAMs) compute least-cost carbon prices subject to the constraint that the atmospheric $CO_2$ concentration must not be exceeded (p. 2949, 2956). Rather, most IAMs compute least-cost carbon prices subject to a constraint on cumulative $CO_2$ emissions. Solving for the least-cost carbon price subject to an upper limit on atmospheric $CO_2$ would indeed give an inadequate approximation of the real problem of solving for the least-cost carbon price subject to a temperature constraint. Imposing a constraint on cumulative $CO_2$ emissions does give an adequate approximation of a temperature constraint, as we explain.

The next section compares the physical model of LR17 with a set of carbon-cycle and temperature models employed by Working Group 1 of the 5th Assessment Report of the Intergovernmental Panel on Climate Change or IPCC (IPCC, 2013). It shows large discrepancies between the common

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[1]LR17 only reference Solomon et al. (2009) in support of the claim of substantial inertia, but Solomon et al. (2009) is focused on the question of irreversibility rather than inertia (see also Matthews and Solomon, 2013); results in Solomon et al. (2009) show a rapid response of warming to $CO_2$ emissions, too.

behaviour of the IPCC models and the LR17 model. Section 3 explores the implications of these discrepancies for economic policy, by substituting the IPCC models into the LR17 economic model. It shows that the differences in climate physics lead to large qualitative and quantitative differences in carbon prices and emissions abatement. It also shows that a simpler approach based on a cumulative emissions budget provides a very close approximation of the IPCC models. It then addresses a number of claims made in LR17 about how IAMs are employed. Section 4 concludes.

## 2  Reassessing the LR17 climate model

We attempt to replicate the warming response to $CO_2$ emissions in LR17 using a set of 16 leading models of temperature inertia and 18 leading models of atmospheric $CO_2$ decay from IPCC (2013). In short, we find that not one of the 288 combinations of temperature inertia and $CO_2$ decay in the IPCC models resembles the warming response to $CO_2$ in LR17.

Why not? Let us first look at the decay of atmospheric $CO_2$, then temperature inertia. LR17 model the decay of atmospheric $CO_2$ as

$$\dot{M}_t = E - \delta M_t,$$

where $M_t$ is the increase in the atmospheric $CO_2$ concentration from the pre-industrial level and $E$ is the baseline flow of $CO_2$ emissions into the atmosphere. The difficulty facing this simple representation of the decay of atmospheric $CO_2$ is that the global carbon cycle has multiple timescales and a significant fraction of $CO_2$ emissions will remain in the atmosphere essentially forever. This can be represented by

$$\dot{M}_t = \sum_{i=0}^{3} \dot{M}_t^i = \sum_{i=0}^{3} a_i(E - \delta_i M_t^i) \tag{1}$$

with $\sum_{i=0}^{3} a_i = 1$ and $\delta_0 = 0$ and $M_t = \sum_{i=0}^{3} M_t^i$. Following the use of this specification in IPCC (2013), we use the best fit of Equation (1) to 16 independent, more sophisticated models of the carbon cycle (Joos et al., 2013). This allows us to compare LR17's climate dynamics with a set of more physically realistic carbon-cycle models.

Second, consider the treatment of temperature inertia in response to

the atmospheric concentration of $CO_2$ in LR17. This is modelled as an exponential process towards a steady-state temperature,

$$\dot{T}_t = \phi(sF(M_t) - T),$$

with $T$ being global mean surface warming above the pre-industrial level, $F$ the radiative forcing $(W/m^2)$ resulting from elevated atmospheric $CO_2$, and $s$ a transformation of the parameter known as climate sensitivity, i.e. the long-run equilibrium warming that would result from a doubling of the $CO_2$ concentration.[2] $\phi$ is the crucial thermal inertia parameter.

A single response timescale is insufficient to characterize the response of the surface climate system to radiative forcing. A more representative model comprises two heat reservoirs, one for the warming of the atmosphere and the upper ocean $T$, and one for the warming of the deep ocean $T^o$ :[3]

$$\dot{T}_t = \frac{1}{c}(F(M_t) - bT_t) - \frac{\gamma}{c}(T_t - T_t^o) \tag{2}$$

$$\dot{T}_t^o = \frac{\gamma}{c_o}(T_t - T_t^o). \tag{3}$$

IPCC (2013, ch. 8) employs this simple model, calibrated on the outputs of 18 independent, more sophisticated climate models by Geoffroy et al. (2013), and we do likewise.[4]

We compare the warming response to $CO_2$ emissions in LR17 with the combination of Equations (1) to (3), which we refer to as the *IPCC AR5 impulse-response model* (AR5 refers to the Fifth Assessment Report of IPCC from 2013/14). There are 288 (16 x 18) variants of the IPCC AR5 impulse-response model and three variants of the LR17 model, corresponding with

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[2]Here, $sF$ is the equilibrium climate sensitivity for the radiative forcing corresponding with a doubled $CO_2$ concentration.

[3]Here $c$ and $c_0$ are effective heat capacities per unit area, $\lambda$ is a radiative feedback parameter per unit area for an additional degree of warming and $\gamma$ is a heat exchange coefficient representing the transfer of heat for a difference of 1 degree between upper and lower ocean, see Geoffroy et al. (2013).

[4]The calibrations were based on behaviour of the more sophisticated models under an instantaneous quadrupling of atmospheric $CO_2$ concentrations, which are then held fixed. Further, we assume the same formula for radiative forcing as LR17: $F(M) = \alpha \ln((M + M_{pre})/M_{pre})$. Defining climate sensitivity $cs$ as steady state warming for a doubling of atmospheric carbon emissions, allows to easily compare our formulation of temperature respose $\dot{T} = b/c(cs/\ln 2 * \ln((M + M_{pre})/M_{pre}) - T) - \gamma/c(T - T)$ with LR17's expression $\dot{T} = \phi(cs/\ln 2 * \ln((M + M_{pre})/M_{pre}) - T)$, with $M_{pre}$ the pre-industrial concentration level.

their low, medium and high temperature inertia scenarios. In the experiment, a pulse of 100 GtC is injected into the atmosphere at time zero (taking the atmospheric stock from 850 to 950 GtC), and we compare the models' warming responses.[5]
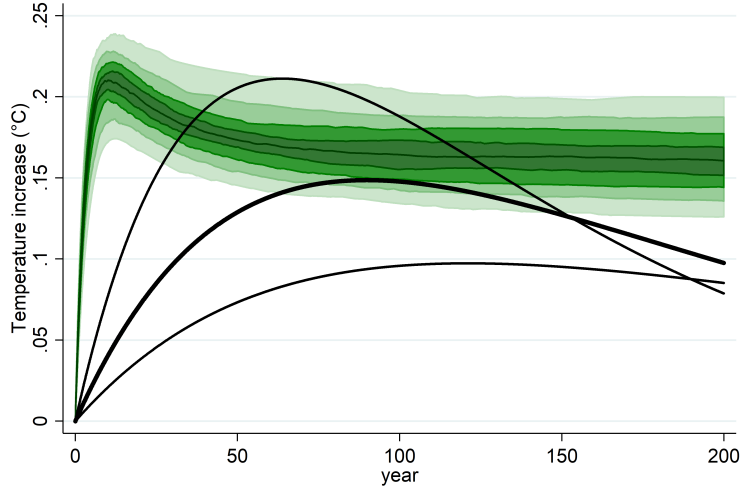
Figure 1 compares the models. The first key result to notice is that all 288 of the IPCC AR5 impulse-response models warm rapidly in response to $CO_2$ emissions. By contrast, the three LR17 models warm up far too slowly. The second key result is that the temperature in the IPCC AR5 impulse-response models remains roughly constant after the rapid initial adjustment. By contrast, in LR17 it decays. So, not a single combination of the 288 reduced-form models of the carbon cycle and thermal response is consistent with the slow temperature response and subsequent decline in LR17. The climate model of LR17 has too much inertia from emissions to temperature and too much carbon decay in the long run. It appears the reason why the delay between emissions and warming is far too long in LR17 is that it is also too long in the DICE model (Nordhaus and Sztorc, 2013; Dietz and Venmans, 2017), on which LR17 calibrate their inertia parameter.[6]

The IPCC AR5 impulse-response models warm quickly in response to $CO_2$ emissions, before temperatures remain constant, because two different natural processes roughly cancel each other out. First, when emissions stop, the atmospheric concentration of $CO_2$ gradually decays, as carbon is absorbed by natural ocean and land sinks. Second, the climate system very slowly approaches a thermal equilibrium with higher levels of atmospheric $CO_2$. The first process ($CO_2$ decay) reduces future temperatures; the second process (thermal inertia) increases future temperatures. To a first order, the timescales and magnitudes of these two processes compensate each other, leading warming to plateau about 10 years after emissions stop (Ricke and Caldeira, 2014; Matthews et al., 2009). Thus ignoring the effect of $CO_2$ decay can lead to the false inference that the very long time it takes for the climate system to reach thermal equilibrium with a higher atmospheric $CO_2$ *concentration* implies a similarly long lag between $CO_2$ *emissions* and warming. This is not the case.

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[5]We employ a climate sensitivity of $3.05°C$ for a doubling of the atmospheric $CO_2$ concentration, consistent with the parametrization of Geoffroy et al. (2013).

[6]However, because DICE has multiple timescales, but LR17's model has only a single time scale, it will behave differently on any periods longer or shorter than the calibration period.

**Figure 1:** The effect of a $CO_2$ emission pulse increasing concentration instantaneoulsy from 389ppm to 436ppm



Black lines represent the climate model in LR17 for their high, medium (bold) and low temperature inertia scenarios. These are not consistent with the space of the pulse for the 288 formally possible combinations of scientifically vetted carbon cycle and thermal inertia models (deciles shown).

We have conducted a number of robustness checks. In Appendix A, we perform a sensitivity analysis with respect to an alternative carbon cycle used by LR17, which is based on Golosov et al. (2014). Again, this variant in LR17 is inconsistent with the temperature responses given by IPCC (2013). We also use the FAIR (Finite Amplitude Impulse Response, Millar et al. (2017)) model, a more recent alternative to the IPCC AR5 impulse-response model, to provide a further check that the assumptions of LR17 are inconsistent with the consensus about climate dynamics (see Appendix D). This also confirms the above findings.

## 3 Implications for economic policy

We now take our physical climate model, i.e. Equations (1)–(3), and embed it within LR17's economic model, in order to evaluate the policy implications. The core finding is that the initial carbon price to minimize the cost of meeting a 2 °C target, rather than being effectively zero, is around 5.6 \$/t$CO_2$. It then follows a qualitatively different path to the least-cost carbon price in LR17, rising at the interest rate, rather than slowly rising over

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the 21$^{\text{st}}$ century, before eventually rising fast, peaking and declining as in LR17. We check this holds for a wide range of calibrations beyond LR17's main scenario.

LR17's objective function is:

$$\min_{A_t} \int_{t_0}^{\infty} C(A(t))e^{-r(t-t_0)}\mathrm{d}t \tag{4}$$

with $C$ cost, $A$ abatement and $r$ the real interest rate.

A solution analogous to the analytical expression for the carbon price in LR17 (their Equation (10)) can be derived by solving Equations (10)–(12) and inserting the solution into Equation (9) in Appendix B:

$$\sum_0^3 a_i \lambda_M^i(t_0) = \sum_0^3 a_i e^{(r+a\delta_i)(t-t_0)}\lambda_M^i(t)+$$
$$1/c \int_{t_0}^t G(z) \sum_0^3 a_i e^{-(r+a_i\delta_i)(z-t_0)}F'(M_z)\mathrm{d}z. \tag{5}$$

As in LR17, the left-hand side of Equation (5) is the present cost of abating an additional unit of $CO_2$ at $t_0$ and the right-hand side is the present benefit of abating that unit. Function $G(t)$, defined in Appendix B, depends on the thermal inertia parameter among other effects. Equation (5) is consistent with LR17 insofar as it is possible that thermal inertia lowers the efficient carbon tax, *ceteris paribus.* However, this says nothing about the size of the effect of thermal inertia: in fact we show that it is negligible.

In order to evaluate the relevance of thermal inertia to least-cost carbon prices and emissions, we follow two different approaches. One is to make an analytical simplification, but a different one to LR17. The other is to analyse the outcome numerically. We take each in turn.

## 3.1 The carbon budget approach

The concept of a carbon budget has become the standard approach to assess global pathways to meet climate targets over the last decade (Allen et al., 2009; Matthews et al., 2009; Meinshausen et al., 2009).[7] Simply put, the car-

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[7]Concentration targets were the standard approach a decade ago, but they have been superseded by carbon budgets (IPCC, 2014b). Carbon budgets are more relevant when

**Figure 2:** Trajectories of the optimal paths



The green and blue paths shown correspond to the $2\,°C$ and concentration targets of LR17 (reproducing their Figure 1). The black path shows the optimal trajectory to reach $2\,°$ C with a standard climate model (the IPCC AR5 impulse-response model), and the red path its approximation by the carbon budget approach. The four panels show emissions net of abatement, temperature, atmospheric $CO_2$ and the carbon price. Economic parameters are as in LR17 (annual consumption discount rate 5.5 %). The LR17 paths result from inaccurate physics (too much thermal inertia and too much decay), so that the IPCC AR5 impulse-response model has higher carbon prices and lower emissions, while yielding higher temperatures.

bon budget is an estimate of the total cumulative $CO_2$ that can be emitted over all time to keep warming below a given threshold. The carbon budget is a physically consistent simplification as it clarifies that the timing of an emissions trajectory is irrelevant. It is based on two central insights from cli-

the goal is to limit warming, such as that expressed in the Paris Agreement.

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mate science (Knutti and Rogelj, 2015; Matthews and Solomon, 2013; Millar et al., 2016): First, as explained above, the emission of a pulse of $CO_2$ produces a one-off step increase in temperature, after a short adjustment period of around 10 years (Matthews and Caldeira, 2008; Joos et al., 2013). Second, this temperature response is largely independent of the existing state of the climate system, such as the atmospheric concentration of $CO_2$, leading to a broadly linear relationship between warming and cumulative $CO_2$ emissions in both modelling studies and in observations of historical climate change (Stocker et al., 2013, p. 103).

For analytical simplification, the carbon budget approach has a convenient formulation and is a credible simplification of a climate model that represents carbon decay and temperature inertia explicitly along the lines of Equations (1)–(3) (see Figure 2). Let $B_t$ denote cumulative emissions, $E$ constant baseline emissions as in LR17 and

$$\dot{B}_t = E - A_t. \tag{6}$$

The carbon budget corresponding to a temperature constraint is given by

$$\zeta B_t \leq \bar{T} \qquad \text{for all} \quad t, \tag{7}$$

where $\zeta$ is the Transient Climate Response to Cumulative Carbon Emissions (TCRE). A possible parametrization is to assume the budget for $2\,°C$ from pre-industrial times to the year 2100 is 1000 GtC and so $\zeta = 0.005 K/GtC$ (Allen et al., 2009).

It is well known that minimising discounted abatement costs subject to Equation (6) gives an optimal price path of

$$C'(A_t) = C'(E)e^{r(t-\bar{t})}, \tag{8}$$

where $\bar{t}$ is the time at which the carbon budget is fully exhausted and emissions are zero. So the carbon price rises at a rate equal to the interest rate (Dietz and Venmans, 2017; van der Ploeg, 2018), is pinned down at the end of the fossil era by the marginal cost of full decarbonization, and the end of the fossil era occurs when the carbon budget is fully exhausted. van der Ploeg (2018) shows how this determines the speed of abatement, the initial carbon price and the time at which emissions are zero, as well as how these

depend on the carbon budget, interest rate and marginal abatement costs. Higher expected growth in the demand for energy shortens the duration of the fossil era as the carbon budget gets exhausted more quickly and implies the carbon price path has to start higher. As LR17 point out, some previous work assumed the optimal carbon price increases at the interest rate plus the decay rate of atmospheric $CO_2$. The carbon budget approach invalidates this.

## 3.2 Comparing LR17, the IPCC AR5 impulse-response model and the carbon budget approach

Figure 2 compares results from (i) our IPCC AR5 impulse-response model, (ii) the carbon budget approach and (iii) the LR17 climate model, using the same economic model in all three cases. We reproduce the emission-concentration and temperature-limit cases of LR17 (their Figure 1). See Appendix C for additional parameters used, the calibration of our initial values and a corresponding carbon budget.

Several major discrepancies emerge. The least-cost path to reach $2\,°C$ in the IPCC AR5 impulse-response model has a very different shape to what is found in LR17: it rises at approximately the interest rate, yields much higher initial and equilibrium carbon prices and does not exceed these temporarily. Our path further implies net zero emissions towards the end of the $21^{st}$ century. As a consequence, significantly higher carbon prices are required throughout. Further, the IPCC AR5 impulse-response model closely approximates the carbon budget approach. Cumulative $CO_2$ emissions until 2100 in the $2\,°C$ scenario of LR17 are ca. 850 GtC. According to IPCC (2013), however, this produces $3°\,C$ warming. By contrast, we impose a budget of 482 GtC between 2005 and 2100.[8]

LR17 model a climate system with greater inertia from emissions to temperature than established climate science suggests. With more realistic (minimal) inertia from emissions to temperature, we find a trivially small difference between the least-cost path that targets the temperature limit and the least-cost path that targets cumulative emissions. The climate model

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[8]The budget imposed brings emissions down to zero around the year 2080, somewhat later than commonly found for $2\,°C$ (IPCC, 2014c) due to a counterfactual decline of emissions from 2005 on, lack of incorporation of non-$CO_2$ forcing and because models assessed by IPCC (2014a), in contrast to our model, represent some of the inertia in the economy and energy system.

of LR17 has too much inertia from concentrations to temperature and too much carbon decay in the long run, which leads to steady-state emissions that are much too high. This is the main driver of the low carbon prices in LR17. This problem is further aggravated, because LR17 abstract from the saturation of carbon sinks, which makes carbon decay even slower when atmospheric $CO_2$ and temperatures rise.[9]

We check robustness of these quantitative differences to lower interest rates and different temperature targets, as LR17 do. Going beyond LR17's sensitivity checks, we also vary the growth rate, mitigation costs and the decarbonization trend, and employ the climate model FAIR (Millar et al., 2017) as a further alternative (details in Appendix D). Most importantly, we find that LR17 significantly underestimate initial carbon prices in all scenarios (Figure 3), so that this difference does not just hold for the specific calibration chosen for Figure 2. By contrast, the carbon budget approach and the IPCC AR5 impulse-response model give very similar initial carbon prices in all cases. For more details see Appendix D.
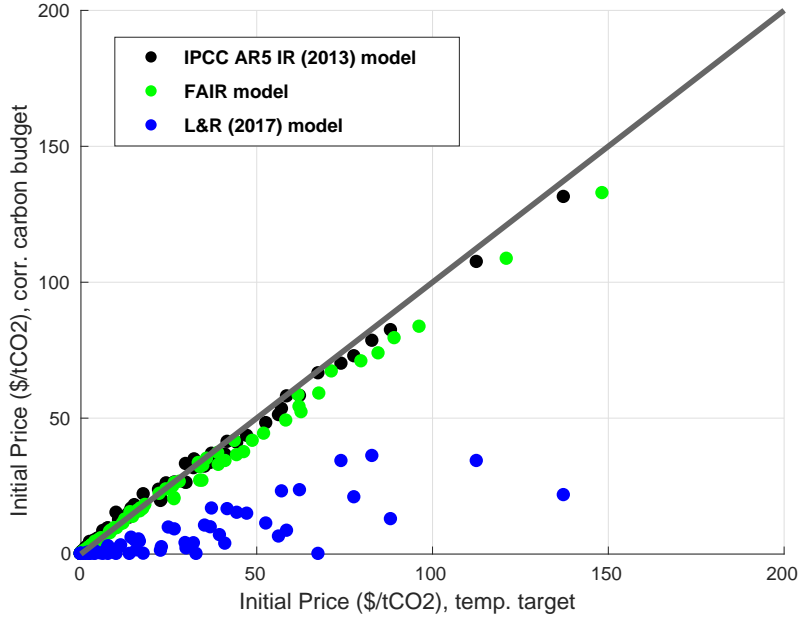
### 3.3   How temperature targets are defined in Integrated Assessment Models

LR17 make two claims about the implementation of temperature targets in "cost-effectiveness Integrated Assessment Models" (CE-IAMs)[10]. First, temperature targets are represented by $CO_2$ concentration limits that must not be exceeded. Second, carbon prices grow exponentially. They show in their modelling that a scenario with both of these properties (named "conventional Hotelling path") is an inefficient implementation of a temperature target. By contrast, the optimal path to this temperature target allows for a temporary overshoot of the steady-state atmospheric $CO_2$ concentration. This comparison is the basis of their conclusion that CE-IAMs drastically overestimate the cost of meeting a 2°C target and also overestimate the optimal near-term carbon price (p. 2949).

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[9]Note that simple minimization of abatement costs leads to an optimal carbon price that is too low at the start and too high in the future, relative to maximization of welfare when both abatement and damage costs are included, because it is indifferent to the timing of the damages. Given that there is minimal delay between emissions and warming, postponing emissions also postpones damages, creating an extra incentive to abate early (Dietz and Venmans, 2017).

[10]They also claim that CE-IAMs do not endogenize savings (p. 2949), which is not true for many of them, see Weyant (2017).

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**Figure 3:** Initial carbon prices



Prices given for a temperature target compared to a carbon budget, for the IPCC AR5 IR model, LR17 and FAIR (Millar et al., 2017), an alternative recent climate model. For the range of assumptions varied, see Appendix D.

However, their first claim is not an accurate portrayal of the current IAM literature. While some older research used concentration targets, the vast majority of published results from CE-IAMs does not rely on them. Contemporary CE-IAMs usually implement temperature targets by limiting the atmospheric concentration of greenhouse gases, radiative forcing, or cumulative emissions *in the year 2100, but not throughout this century.* The level of such limits is set to obtain a certain probability of staying below the given temperature target. Implemented this way, such targets allow for an overshoot of these quantities. For example, 100 out of the relevant 122 scenarios in IPCC (2014a) include a temperature overshoot (see Appendix E for further details). Of the three CE-IAM studies referenced in LR17 (on p. 2949 and p. 2956), Edenhofer et al. (2010) do not implement a not-to-exceed concentration target.[11] Neither do Bauer et al. (2015), who

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[11]Figure 3 and Table 3 of Edenhofer et al. (2010) show that they, in contrast, allow for temporary overshoot of concentrations (and forcing).

"...implement carbon budgets constraining cumulative emissions until 2100 that are consistent with GHG concentrations of 550 ppm $CO_2$-eq and 450 ppm $CO_2$-eq, respectively, at the end of the century." (Bauer et al., 2015, p. 245). Thomson et al. (2011) do implement a not-to-exceed constraint on radiative forcing, which, however, is insufficient for limiting warming to 2°C.

The second claim, that carbon prices are assumed to grow exponentially, is partially correct. Most, but not all, CE-IAMs yield exponential carbon prices (Figure E.11). Some CE-IAMs derive their carbon prices using simple climate models, others assume Hotelling price paths. We have shown above that such a Hotelling price, rising at the interest rate, is the optimal carbon price for a climate target defined as an emissions budget until 2100. We have also demonstrated that this is a very good approximation of a temperature target in 2100.[12]

# 4   Conclusion

The conclusions of LR17 do not hold once a model of the atmosphere consistent with climate scientists' current understanding of the climate system is introduced. LR17 explore the implications of inertia in the climate system for delaying $CO_2$ emissions abatement and claim that "[b]y failing to take advantage of the climate system's inertia, these modeled policies undertake more total abatement than necessary and ramp up policy faster than necessary" (p. 2956). However, this conclusion relies on assuming an excessive lag between emissions and warming, as well as excessive decay of atmospheric $CO_2$ in the long run. Their argument further relies on an inaccurate characterization of how IAMs implement climate targets. Most of these do not implement upper limits to the $CO_2$ concentration, which means the "modelled policies" that LR17 refer to are unrepresentative.

A more accurate representation of climate physics is the carbon budget approach (Allen et al., 2009; Matthews et al., 2009; Meinshausen et al., 2009), which simplifies the derivation of cost-minimizing carbon prices required to keep global mean temperature below 2 °C (van der Ploeg, 2018; Dietz and Venmans, 2017). This approach is also easy to communicate to

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[12]Eliminating the temperature overshoot often found in CE-IAMs could even reverse LR17's conclusion: mitigation pathways without overshoot have higher near-term abatement and carbon prices compared to pathways allowing for overshoot (Clarke et al., 2009).

policy makers. Correcting for these errors, we find that immediate and substantial carbon pricing is required if temperatures are to remain below $2\,°C$. Our results indicate the urgency of implementing ambitious climate policies and are in line with findings that to meet the $2\,°C$ target $CO_2$ emissions must be cut to zero by the second half of this century (IPCC, 2014b).

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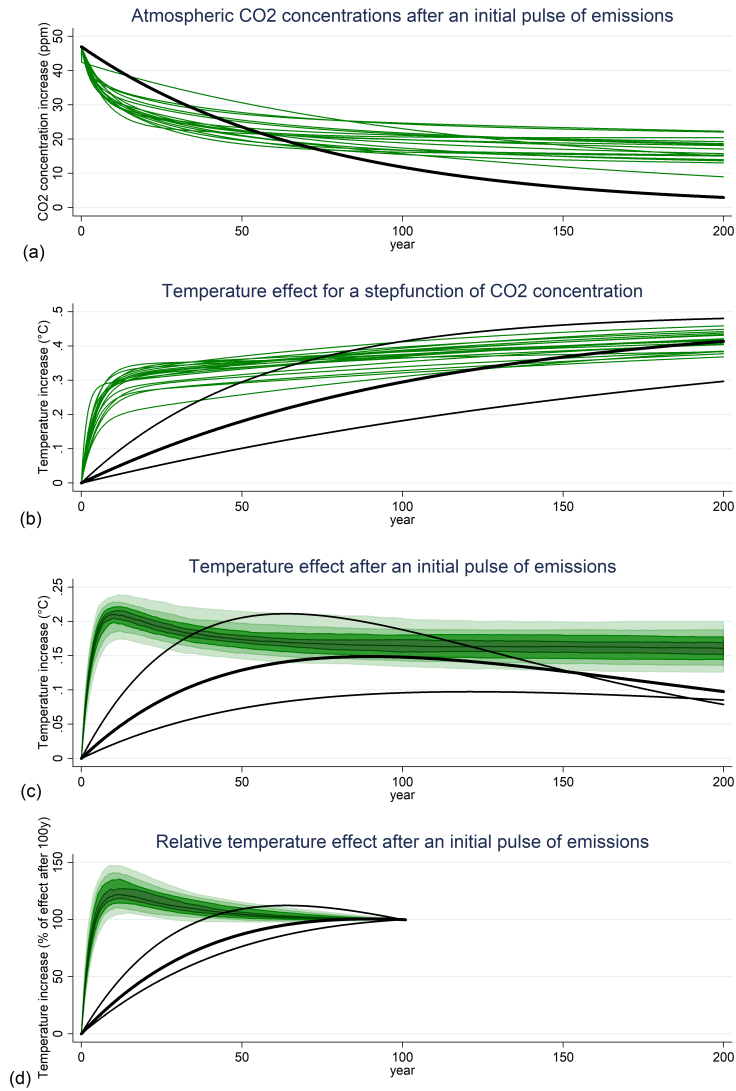
# Online Appendix

## A  Climate model sensitivity analysis

This appendix provides a sensitivity analysis of section 2, which tested the climate representation of LR17 against the IPCC AR5 impulse-response model. First, Figure A.4 contains more results from the experiment. Second, we examine the alternative specification of the carbon cycle due to Golosov et al. (2014), which was also employed by LR17 in their appendix.
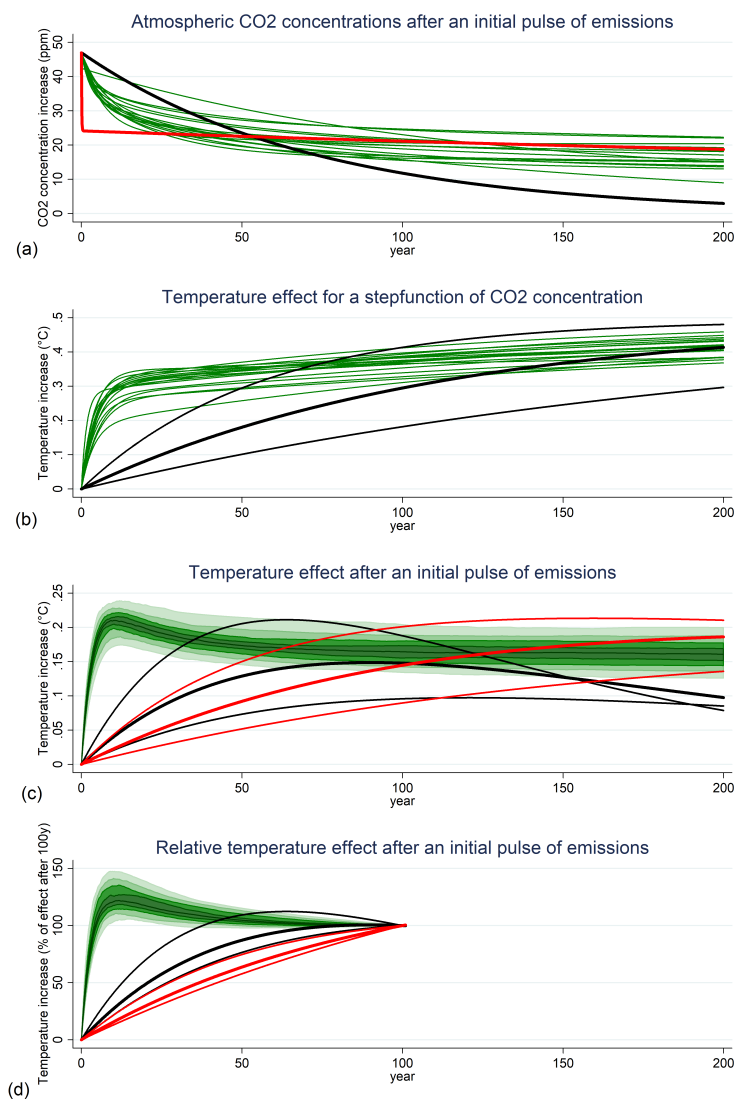
Figure A.4, panel (a), shows the carbon decay for a pulse of emissions of 100 GtC, initially increasing $CO_2$ concentrations from 398 ppm to 436 ppm, fitting Equation (1) to 16 models as in Joos et al. (2013). Figure A.4, panel (b), shows the warming associated with an instantaneous increase in the atmospheric $CO_2$ concentration from 389 ppm to 436 ppm (i.e. atmospheric carbon increases from 850 GtC to 950 GtC without carbon decay) at time zero, fitting Equations (2) and (3) to 18 temperature inertia models as in Geoffroy et al. (2013). This induces radiative forcing of 0.61 $W/m^2$ and results in $0.5°C$ steady-state warming (climate sensitivity of $3°$ for a doubling of $CO_2$). Panels (a) and (b) use the median and multi-model mean parameters of Joos et al. (2013) and Geoffroy et al. (2013) respectively and assume the same climate sensitivity of $3°C$. Using all 288 possible permutations of the IPCC AR5 impulse-response model, we simulate the temperature impact of a pulse of emissions (Figure 1, panel (c), reproducing Figure 1). Panel (d) of Figure 1 expresses the temperature response as a percentage of warming after 100 years. The climate model of LR17 reaches 50% of its year 100 warming after 21 years, while all of the IPCC models reach 50% of their year 100 warming within just two years. Likewise, according to the LR17 model, warming reaches 85% of its year 100 value after 48 years, compared with less than 17.5 years in all the IPCC models. Figure A.5 shows the results from substituting in the carbon decay model of Golosov et al. (2014). When the model of Golosov et al. (2014) is put in, the disparity with the IPCC models is even greater.

**Figure A.4:** The effect of a $CO_2$ emission pulse



Black lines represent the climate model in LR17 for their high, medium and low temperature inertia scenarios. Panel (a) plots the decay of atmospheric $CO_2$ according to the 16 carbon cycle models in Joos et al. (2013) for an emission pulse of 100 GtC. Panel (b) plots the temperature increase for a baseline concentration of 398 ppm according to the 18 temperature inertia models in Geoffroy et al. (2013) for constant forcing. Panel (c) shows the combined effect of the pulse for the 288 combinations of carbon cycle and thermal inertia models as in Figure 1. Panel (d) gives temperature as expressed as a percentage of warming after 100 years instead. Different lines correspond to the deciles of the 288 runs in panel (c) and (d).

**Figure A.5:** The effect of a $CO_2$ emission pulse, including Golosov et al. decay



Atmospheric CO2 concentrations after an initial pulse of emissions

(a)

Temperature effect for a stepfunction of CO2 concentration

(b)

Temperature effect after an initial pulse of emissions

(c)

Relative temperature effect after an initial pulse of emissions

(d)

In addition to Figure 1, red lines represent the climate model in the appendix of LR17, based on (Golosov et al., 2014), for their high, medium and low temperature inertia scenarios

# B    Analytical solution

The first-order conditions with the accurate climate physics remain similar to LR17. Let $\lambda_M^i$ be the shadow variable associated with state $M_i$, and let $\lambda_T$ and $\lambda_T^d$ be the shadow variables associated with the atmospheric temperature and lower ocean temperature respectively. Suppressing time-dependencies,

$$C'(A) = \sum_{i=0}^{3} a_i \lambda_M^i \tag{9}$$

$$\dot{\lambda}_M^i = (r + a\delta_i)\lambda_M^i - \lambda_T \frac{1}{c} F'(M) \tag{10}$$

$$\dot{\lambda}_T = \lambda_T(\frac{b}{c} + \frac{\gamma}{c} + r) - \frac{\gamma}{c_o}\lambda_T^d \tag{11}$$

$$\dot{\lambda}_T^d = -\frac{\gamma}{c}\lambda_T + (r + \frac{\gamma}{c_o})\lambda_T^d. \tag{12}$$

Here we ignore the dependency introduced by limiting temperature to $2\,°\text{C}$, as do LR17 in their analytical formulation of the carbon price (see their p. 2952-3). Equations (10)–(12) describe the evolution of the dynamical system until the temperature constraint binds.

Equations (11) and (12) are a system of linear differential equations that can be solved if the eigenvectors of the matrix of coefficients are linearly independent. This is the case unless (details available upon request):

$$b^2 c_0^2 - 2bcc_0\gamma + 2bc_0^2\gamma + c^2\gamma^2 - 6cc_0\gamma^2 + c_0^2\gamma^2 = 0. \tag{13}$$

We check numerically that this is not the case for the parameter values in Geoffroy et al. (2013). It is nowhere near. This means $\lambda_T(t)$ has an explicit solution of the general form:

$$\lambda_T(t) = \eta_1 \exp(\eta_2 t) + \eta_3 \exp(\eta_4 t) = G(t). \tag{14}$$

Following LR17, Appendix B.3, an explicit solution to Equation (10) can

be obtained, but for each $i$ solving it with the integrated factor method:

$$\lambda_M^i(t_0) = e^{(r+a_i\delta_i)(t-t_0)}\lambda_M^i(t) + 1/c \int_{t_0}^{t} G(z)e^{-(r+a_i\delta_i)(z-t_0)}F'(M_z)\mathrm{d}z. \quad (15)$$

for $i = 1, \ldots, 3$. Insert these expressions into Equation (9) to obtain the analytical solution for the present carbon price, Equation (5).

# C   Numerical solution: Parameters and initial values

The numerical implementation is carried out with GAMS. We use the values employed by LR17 whenever applicable. Parameters for Equation (1) are the mean values from (Joos et al., 2013) ('Best fit to mean trajectory'):

| $a_0$ | $a_1$ | $a_2$ | $a_3$ | $\delta_1$ | $\delta_2$ | $\delta_3$ |
|-------|-------|-------|-------|------------|------------|------------|
| 0.217 | 0.224 | 0.282 | 0.276 | 0.00254 | 0.0274 | 0.232342 |

Further, Equations (2)–(3) are calibrated on mean values from Geoffroy et al. (2013):

|  | C | $C_0$ | b | $\gamma$ | Climate sensitivity (cs) |
|--|---|-------|---|----------|--------------------------|
| Multimodel mean | 7.34 | 105.50 | 1.13 | 0.73 | 3.05 |

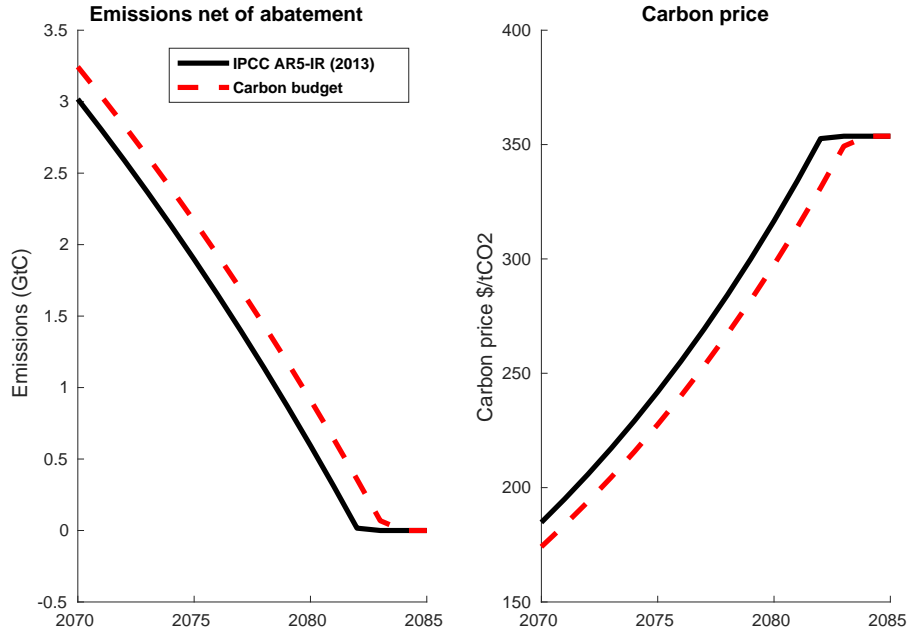Differing slightly from LR17, Geoffroy et al. (2013) define

$$F(M) = cs(b/\ln(2))\ln(M/M_{pre} + 1). \quad (16)$$

The initial values for the carbon pools are derived from an integration of the FAIR simple climate model (Millar et al., 2017) over the historical period until 2005. The model is run in $CO_2$-only mode with all other radiative forcing set to zero. For the initial value of 212.5 GtC of carbon in the atmosphere above the pre-industrial level, $M_0(2005) = 112.413$ GtC, $M_1(2005) = 72.886$ GtC, $M_2(2005) = 23.588$ GtC, $M_4(2005) = 3.4$ GtC.

As an additional initial condition for the temperature model, we specify the deep ocean warming in 2005 to be $0.007° C$ as in DICE.

For the carbon budget approach, we compare the above analysis with the standard assumption that warming of $2\,°C$ accompanies 1000 GtC of cumulative $CO_2$ emissions above pre-industrial (and hence $\zeta = 2/1000$) (Allen

**Figure C.6:** Enlarged section of Figure 2



Around 2075 the trajectories of the optimal path with the IPCC AR5 impulse-response model and its approximation using the carbon budget approach differ in emissions net of abatement and carbon price.

et al., 2009).[13] We compute the remaining deterministic carbon budget from 2005 on to be 482 GtC from the initial condition $M_0(2005)$ that specifies the value of the permanent component of the carbon reservoir and hence the total historical emissions. For carbon budgets corresponding to temperature targets other than $2\,^\circ\text{C}$ we interpolate this linear relationship. Note we do not compute the temperature in the solution to the carbon budget approach as a linear function of the budget, due to the short time lag between emission and temperature increase. In the numerical implementation, we use Equations (1) and (2) instead to compute the temperature path.

Figure C.6 shows how closely the emissions and $CO_2$ price approximations align when the carbon budget approach is implemented in this way.

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[13]We alternatively computed an "internal" carbon budget from the IPCC AR5 impulse-response model and found the fit of the approximation is even closer. The difference is due to how much committed warming is assumed for the base year (details available upon request).
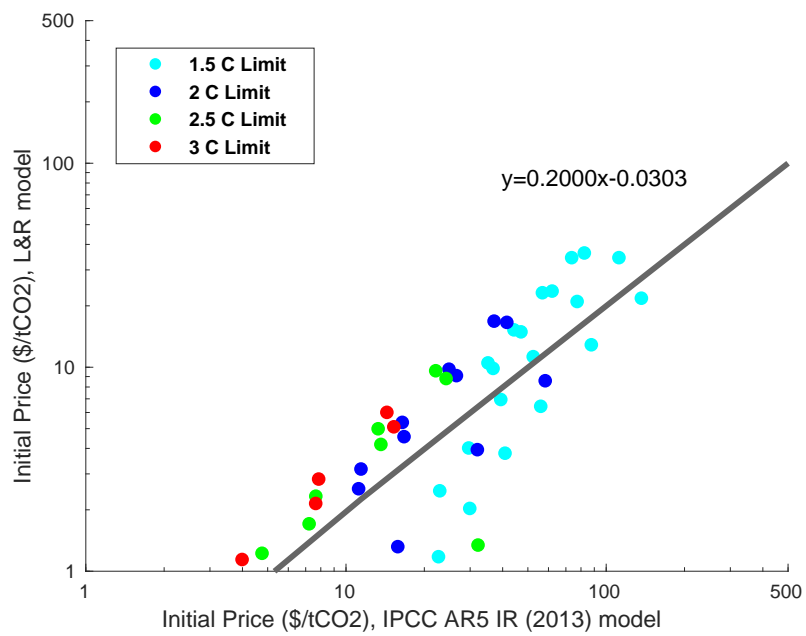
# D  Sensitivity analysis of economic policy implications

We assess the difference between the IPCC AR5 impulse-response model, the carbon budget approach and the temperature case of LR17 under different interest rates and temperature limits. We examine interest rates $r$ of 1.4 %, 3.5 % and 5.5 %. We consider a 2.5 and $3° C$ temperature limit, but also a $1.5° C$ limit. We further test sensitivity to:

- the GDP growth rate, implying increasing emissions, at either 0 or 2 %;

- the decarbonization trend, represented by the parameter $\sigma$ in LR17, Appendix C, either at 0 % or according to DICE (2009 version);

- the mitigation cost as given by $\Psi_t$ in LR17, Appendix C, either at 0 % or according to DICE (2009 version);

- the climate model, using FAIR (Millar et al., 2017) as a more recent alternative to the IPCC AR5 impulse response model (Joos et al., 2013; Geoffroy et al., 2013). FAIR is a simple model that was designed to capture the dependencies on pulse size and background state, for example, the gradual saturation of the capacity of oceans to absorb $CO_2$ that are seen in the Earth System Model response to pulse emissions of $CO_2$.

Table 1 contains the full resulting initial carbon prices for all robustness experiments without the carbon prices when FAIR is used, the latter are given in Table 2. The deviation between our model and the implementation of a target of $2\,°C$ in LR17 is very robust. Figure D.7 illustrates that across sensitivity experiments the LR17 model underestimates initial carbon prices by approximately an order of magnitude, although with some variation. Figure D.8 shows that the correspondence between the IPCC AR5 impulse-response model and the solution for the budget approach in initial carbon prices is particularly close for the $2\,°C$ target.

**Figure D.7:** Comparison of initial carbon prices between the IPCC AR5
IR model and LR17



Across robustness checks, LR17 underestimate initial carbon prices by a factor 5-10.
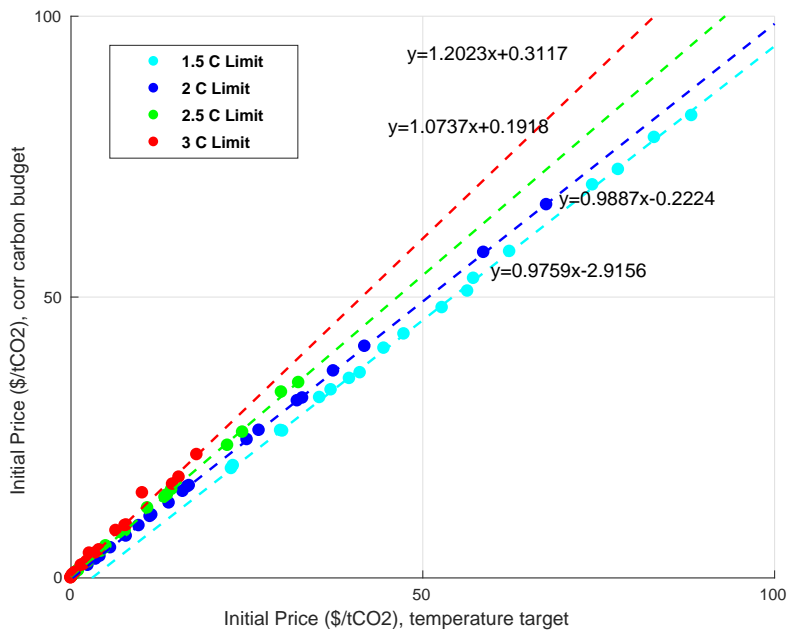
**Figure D.8:** Correspondence between the IPCC AR5 IR model and the budget approach in initial carbon prices for various temperature targets

**Table 1:** Initial carbon prices in $/GtCO$_2$ as given by the IPCC AR5 IR model, the carbon budget approach and LR17 for various temperature targets and carbon budgets

| Scenario | 2°C | 482 GtC | L&R | 1.5°C | 231 GtC | L&R | 2.5°C | 731 GtC | L&R | 3°C | 981 GtC | L&R |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| baseline | 5.6260 | 5.3642 | 0.2221 | 29.8046 | 26.2413 | 3.9865 | 1.0453 | 1.2728 | 0.0046 | 0.1704 | 0.3190 | 0 |
| GDP Growth | 16.5629 | 16.2172 | 5.3226 | 44.4889 | 40.8912 | 15.1262 | 7.7039 | 8.3884 | 2.3136 | 4.0120 | 4.9845 | 1.1323 |
| decarbonization | 4.1268 | 3.8583 | 0 | 30.0866 | 26.1479 | 2.0151 | 0.3315 | 0.4558 | 0 | 0.0091 | 0.0320 | 0 |
| cost reduction | 3.5353 | 3.3470 | 0.0995 | 23.0719 | 19.9878 | 2.4589 | 0.5268 | 0.6578 | 0 | 0.0674 | 0.1367 | 0 |
| GDP Growth, decarbonization | 16.8173 | 16.4269 | 4.5302 | 47.3347 | 43.4275 | 14.7880 | 7.2610 | 7.9789 | 1.6940 | 3.4671 | 4.4375 | 0.7155 |
| GDP Growth, cost reduction | 11.4902 | 11.2034 | 3.1436 | 35.3498 | 32.1064 | 10.4162 | 4.7861 | 5.2753 | 1.2145 | 2.2665 | 2.9058 | 0.5377 |
| decarbonization, cost reduction | 2.4146 | 2.2369 | 0 | 22.8144 | 19.4713 | 1.1686 | 0.1377 | 0.1976 | 0 | 0 | 0.0093 | 0 |
| GDP Growth, decarbonization, cost reduction | 11.2519 | 10.9415 | 2.5168 | 36.9760 | 33.4801 | 9.7892 | 4.2586 | 4.7481 | 0.8142 | 1.8114 | 2.4089 | 0.3035 |
| low DR | 15.9124 | 15.4116 | 1.3088 | 52.7600 | 48.1142 | 11.1758 | 4.9723 | 5.6950 | 0.0715 | 1.4641 | 2.2347 | 0 |
| low DR, GDP Growth | 37.3164 | 36.8458 | 16.6918 | 74.1470 | 69.9687 | 34.1654 | 22.2621 | 23.6016 | 9.5326 | 14.4490 | 16.6800 | 5.9605 |
| low DR, decarbonization | 13.9521 | 13.3433 | 0 | 56.3786 | 51.0577 | 6.3825 | 2.5870 | 3.1885 | 0 | 0.2370 | 0.5800 | 0 |
| low DR, cost reduction | 9.6685 | 9.3017 | 0.5988 | 39.5866 | 35.5061 | 6.8912 | 2.4390 | 2.8630 | 0.0227 | 0.5735 | 0.9455 | 0 |
| low DR, GDP Growth, decarbonization | 41.7643 | 41.2253 | 16.4616 | 82.9306 | 78.4067 | 36.0231 | 24.4034 | 25.9453 | 8.7297 | 15.3642 | 17.9271 | 5.0630 |
| low DR, GDP Growth, cost reduction | 25.0345 | 24.6215 | 9.7003 | 57.2299 | 53.3287 | 22.9985 | 13.3736 | 14.3436 | 4.9524 | 7.9009 | 9.4070 | 2.8070 |
| low DR, decarbonization, cost reduction | 7.8582 | 7.4459 | 0 | 41.1069 | 36.5203 | 3.7589 | 1.0582 | 1.3568 | 0 | 0.0631 | 0.1794 | 0 |
| low DR, GDP Growth, decarbonization, cost reduction | 26.7380 | 26.2708 | 9.0224 | 62.3523 | 58.1125 | 23.4310 | 13.7033 | 14.7731 | 4.1507 | 7.6965 | 9.3244 | 2.1316 |
| ultra-low DR | 58.6658 | 57.9490 | 8.5156 | 112.6419 | 107.4022 | 34.1763 | 32.3707 | 34.7704 | 1.3327 | 17.8977 | 21.9476 | 0.0680 |
| ultra-low DR, decarbonization | 67.6031 | 66.4409 | 0.0009 | 137.3983 | 131.3127 | 21.6213 | 29.9230 | 33.0770 | 0.0015 | 10.1641 | 15.1426 | 0.0008 |
| ultra-low DR, cost reduction | 32.1814 | 31.5302 | 3.9117 | 77.8065 | 72.7094 | 20.8306 | 14.3257 | 15.7454 | 0.4339 | 6.4009 | 8.4309 | 0.0121 |
| ultra-low DR, decarbonization, cost reduction | 32.9201 | 32.0220 | 0 | 88.2250 | 82.3103 | 12.7805 | 10.8955 | 12.4598 | 0 | 2.6338 | 4.4017 | 0 |

**Notes:** This table compares initial carbon prices (in $/tCO$_2$) of the IPCC AR5 IR model ("2 °C"), the carbon budget approach ("482 GtC") and LR17 for a variety of scenarios. The baseline is parametrized by the main scenario of LR17. The further scenarios are modifications of this baseline by changing assumptions as follows. *growth:* 2 % GDP growth, *decarbonization* trend: according to DICE-2009, *cost reduction*: according to DICE-2009, *low discount rate:* 3.5 % and *ulta-low discount rate:* 1.4 %.

**Table 2:** Initial carbon prices in $/GtCO$_2$ as given by the FAIR model and corresponding carbon budgets for various temperature targets and corresponding carbon budgets

| FAIR Scenario | 2 °C | 482 GtC | 1.5°C | 231 GtC | 2.5° C | 731 GtC | 3° | 981 GtC |
|---|---|---|---|---|---|---|---|---|
| baseline | 6.0740 | 5.4851 | 33.9726 | 26.9363 | 1.3253 | 1.3008 | 0.3232 | 0.3262 |
| GDP Growth | 17.4948 | 16.4036 | 48.9071 | 41.6011 | 8.6946 | 8.4628 | 5.1514 | 5.0201 |
| decarbonization | 4.4838 | 3.9741 | 34.6083 | 26.9131 | 0.4606 | 0.4719 | 0.0258 | 0.0338 |
| cost reduction | 3.8433 | 3.4327 | 26.6469 | 20.5877 | 0.6865 | 0.6741 | 0.1393 | 0.1413 |
| GDP Growth, decarbonization | 17.7994 | 16.6290 | 52.1426 | 44.2224 | 8.2718 | 8.0564 | 4.5812 | 4.4735 |
| GDP Growth, cost reduction | 12.1894 | 11.3502 | 39.2510 | 32.7465 | 5.4857 | 5.3278 | 3.0155 | 2.9299 |
| decarbonization, cost reduction | 2.6467 | 2.3132 | 26.6426 | 20.1193 | 0.1990 | 0.2050 | 0.0088 | 0.0114 |
| GDP Growth, decarbonization, cost reduction | 11.9652 | 11.0956 | 41.1700 | 34.1760 | 4.9388 | 4.8009 | 2.4958 | 2.4314 |
| low DR | 16.9324 | 15.6544 | 58.4436 | 49.0572 | 5.8832 | 5.7778 | 2.2617 | 2.2655 |
| low DR, GDP Growth | 39.0845 | 37.1348 | 79.8117 | 70.8457 | 24.3123 | 23.7398 | 17.0985 | 16.7584 |
| low DR, decarbonization | 14.8836 | 13.6172 | 62.7899 | 52.1295 | 3.2216 | 3.2625 | 0.5198 | 0.5960 |
| low DR, cost reduction | 10.3523 | 9.4742 | 44.4388 | 36.3198 | 2.9659 | 2.9125 | 0.9574 | 0.9612 |
| low DR, GDP Growth, decarbonization | 43.8164 | 41.5573 | 89.2613 | 79.3536 | 26.7455 | 26.1038 | 18.3748 | 18.0187 |
| low DR, GDP Growth, cost reduction | 26.3126 | 24.8526 | 62.2178 | 54.1131 | 14.8165 | 14.4418 | 9.6790 | 9.4602 |
| low DR, decarbonization, cost reduction | 8.4476 | 7.6270 | 46.4670 | 37.4310 | 1.3717 | 1.3954 | 0.1578 | 0.1856 |
| low DR, GDP Growth, decarbonization, cost reduction | 28.1554 | 26.5340 | 67.8438 | 58.9862 | 15.2658 | 14.8872 | 9.5861 | 9.3834 |
| ultra-low DR | 61.9388 | 58.4298 | 121.1653 | 108.5663 | 35.9069 | 35.0227 | 22.4347 | 22.0945 |
| ultra-low DR, decarbonization | 71.3956 | 67.1183 | 148.3817 | 132.7255 | 33.5598 | 33.4413 | 14.1594 | 15.3352 |
| ultra-low DR, cost reduction | 34.0334 | 31.8848 | 84.6978 | 73.7747 | 16.2501 | 15.9010 | 8.5841 | 8.5073 |
| ultra-low DR, decarbonization, cost reduction | 34.8613 | 32.4685 | 96.3519 | 83.5615 | 12.6445 | 12.6443 | 4.1424 | 4.4753 |

# E  Implementation of climate targets in Integrated Assessment Modeling scenarios presented in the IPCC AR5 report

Here we provide further detail on the implementation of climate targets in the scenarios used in the Fifth Assessment Report of the IPCC Working Group III (IPCC, 2014a; Clarke et al., 2014; Krey et al., 2014). Since LR17 focus on the $2\,°C$ target, we select scenarios with 2100 radiative forcing of $3.45\ W/m^2$ or lower (see Table 6.2 Clarke et al., 2014). Further, we exclude scenarios assuming delayed action, as well as scenarios from modeling systems that are myopic or use exogenous emission pathways to focus on optimal mitigation paths. Overall, our selection includes 159 scenarios.

Figure E.9 shows a histogram of the difference between peak $CO_2$ concentration and $CO_2$ concentration 2100 as an indicator of overshoot. In 143 out of 153 scenarios this difference is positive, i.e. they exhibit a $CO_2$ concentration peak before the end of the century. Only 10 of these $2\,°C$ scenarios show no peaking of $CO_2$ concentrations during the 21st century. The same is even observed in temperature for the vast majority of scenarios (Figure E.10): 100 out of 122 scenarios have lower end-of-century temperature than peak temperature. Note that for some scenarios $CO_2$ concentration and temperature data are not available.

Figure E.11 shows $CO_2$ price trajectories for this scenario set. Virtually all of them exhibit exponential or close-to-exponential growth of $CO_2$ prices, in line with the Hotelling rule. In many partial equilibrium models the $CO_2$ price grows at 5 % p.a., as this value is chosen for the exogenous discount rate. Intertemporal general equilibrium models have an endogenous interest rate, which typically declines over time due to a slightly lower economic growth.

**Figure E.9:** The difference between peak $CO_2$ concentration and $CO_2$ concentration in 2100 for selected IPCC scenarios
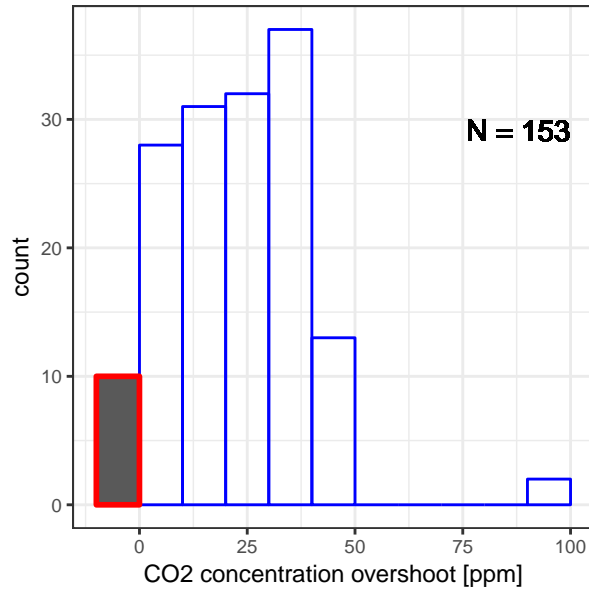


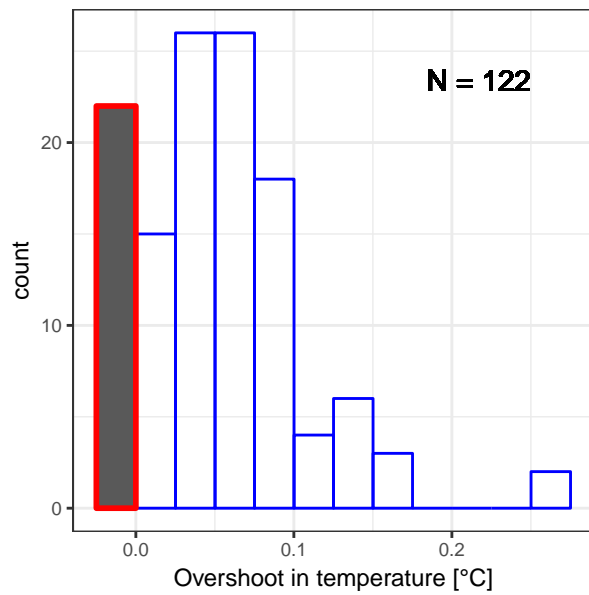**Figure E.10:** The difference between peak warming and warming in 2100 for selected IPCC scenarios

**Figure E.11:** CO$_2$ price trajectories for selected IPCC scenarios