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Revenue Decoupling for Electric Utilities: Impacts on Prices and Welfare

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ABSTRACT

Revenue decoupling (RD) is a regulatory mechanism that allows adjustments of retail electricity rates so that the regulated utility recovers its required revenue despite fluctuations in its sales volume. The U.S. utility data in 2000-2012 reveals that RD is associated with more than 10% higher electricity prices and revenues in two years after RD is implemented relative to similar non-decoupled utilities. Between these comparable utilities, there are no significant differences in the electricity sales, indicating that RD tends to allow larger increases in utility revenues. Theoretically, unexpected sales declines would lead to higher electricity prices while unexpected sales increases would lead to lower prices. RD adjustments have yielded both refunds and surcharges, but the data indicates that electricity prices demonstrate downward rigidity and statistically significant upward adjustments for the utilities subject to RD. Together with the likely negative impacts of RD on low-income (as opposed to high-income) households, this analysis indicates the limitations of decoupling, and fixed-cost recovery practice in general, which involves adjustments in volumetric electricity rates.

Introduction

In an effort to curb pollution externalities associated with energy use, policymakers continue to push for improved energy efficiency and distributed electricity generation. Under the traditional natural-monopoly regulation (i.e., cost-of-service or rate-of-return regulation), however, the volumetric electricity prices are set above the marginal costs and hence the profits tend to increase with the sales volume. Therefore, a utility's interest—to sell more electricity—is misaligned with the regulatory agenda of attaining energy efficiency and conservation[1]. Despite such throughput incentive, the sales of electricity have not been growing over the last decade in the United States, leading to concerns that the utilities are not able to recover the full costs.

Among the potential regulatory options, revenue decoupling (RD) has emerged as an approach to help utilities overcome the disincentive to support the state's energy-efficiency agenda[2]. Revenue decoupling is generally defined as a rate-making mechanism designed to "decouple" the utility's revenues from its sales. By making the utility's revenue independent of sales, RD removes the utility's disincentives to promote customer efforts to reduce energy consumption or to expand distributed generation that often utilizes renewable energy.[3]

Table 1 provides a simple illustration of how RD works.[1] Consider a scenario where the actual sales in the current year are 1 percent lower than the baseline amount of 1 million kWh. Without any revenue adjustment mechanism, this translates to about 1 percent revenue shortfall in the said year. Hence, any shock that lowers demand, be it due to energy efficiency improvement or conservation (or any exogenous income shock), results in lower equity earnings. Under RD, the (volumetric) electricity rate increases so that the required revenue is earned. RD, in effect, provides a mechanisms for customers to receive refunds or pay surcharges based on whether the revenues the utility actually received from customers were greater or smaller than the revenues required to recover the fixed cost.[2]

As of January 2019,15 states and the District of Columbia have implemented RD for electric utilities.[3]Many states implemented RD during and immediately after the U.S. financial crisis in 2000. As a growing number of states have ventured

[1]This illustration is based on a simple full decoupling mechanism. In reality, there are a number of ways to implement RD, but the guiding mechanism is the same (i.e., except for flat distribution which will be discussed later on, all of them have a true-up mechanism that adjusts the electricity rates in order to collect the allowed revenue). For a more complete discussion of RD, see[4].

[2]Note, however, that the difference can occur for many reasons, including weather and economic conditions that are not entirely within the control of the customers nor the utility. In this context, it is apparent that RD insulates the utility from business risks that are now absorbed by the customers[5].

[3]The data is from https://www.nrdc.org/resources/gas-and-electric-decoupling, retrieved on October 8, 2019.

Table 1. An example of how RD works.

	No RD in place	RD in place
Revenue Requirement	$115,384,615	
(Based on expenses, allowed return, taxes)		
Sales Forecast (kWh)	1,000,000,000	
Actual Sales (kWh)	990,000,000	
Unit Price ($/kWh)	0.1154	0 .1166
Decoupling Adjustment ($/kWh)	−−	0 .0012
Actual Revenue	$114,230,769	$115,384,615

Source: The Regulatory Assistance Project (RAP), 2011.

into adopting policies and regulations with energy efficiency objectives, debates on the effectiveness of revenue decoupling emerged. Conservation advocates argue that RD can enhance generation and distribution efficiency by providing utilities the incentives to reduce costs and not through increase in sales[4,6]. They also argue that RD is necessary, if not sufficient, for utilities to promote energy efficiency and/or invest in renewables[7,8]. RD improves a utility's financial situation and lowers risks, thus can potentially reduce the cost of capital[7]. RD is considered to be less contentious, and hence less costly to set rates and conduct cost recovery, than the Loss Revenue Adjustment (LRA). Other policies including LRA requires sophisticated measurement and/or estimation. Moreover, it is easier for state commissions to administer/monitor as opposed to other alternatives[5,7–9]. Recent studies find that the utilities under RD are associated with higher expenditure on demand-side management, indicating larger efforts on energy efficiency improvements[10,11].

Critics of RD, on the other hand, argue that the policy is a blunt instrument to promote energy efficiency, particularly on the part of the utility. Because utilities must rebate the difference between price and costs to consumers, they no longer have an incentive to minimize costs under RD[12]. Knittel[13], for example, showed that RD is not effective in influencing utilities to improve generation efficiency because they do not receive significant economic gains from producing energy more efficiently. Moreover, critics suggest that the policy not only transfers the business risks from the utility to the customers but also may cause customers in one rate class to absorb some of the impact of demand downturns in another class[8]. Residential electric bills, for instance, may increase due to a downturn in industrial demand.

Despite the controversies, little work has been done to provide clear evidence regarding the effects of RD on electricity prices and, in general, economic welfare.[4] One of the potential consequences of RD, given the trend that electricity sales are not growing in many states, is the increase in retail electricity rates. Previous studies on the effects of RD on electricity rates argue that the associated change in electricity rates have been negligible[2,10]. In the U.S. between 2005 and 2012, 23% of the recorded 1,244 RD adjustment cases involve retail rate adjustments between 0 and 1 percent, and more than half of the cases are within the 0-3% range[2]. An issue with this observation is that it captures only the immediate decoupling adjustment similar to the one presented in Table 1. Changes in electricity prices may affect energy users' incentives to invest in energy efficiency improvement (such as efficient appliances or solar panels), which generate feedback effects on the demand for electricity and thus opportunities for further RD adjustments. Thus RD may induce not only immediate electricity rate changes but rate changes over time.

Can we compare electricity prices over time in states with and without RD? Care must be taken because the states and utilities with and without RD may have different economic characteristics, which might explain some of the differences in the prices. In this study, we compare treated investor-owned utilities (those under RD mechanism) versus control-group utilities (those that are not subject to RD) [5] with otherwise similar characteristics to assess the impact of RD on residential electricity rates. Our study design examines utility companies in 17 states that had implemented RD mechanism over the 2000-2012 period and compares their monthly electricity rates with control utilities before and after the RD implementation. We find that decoupling tends to increase the electricity rates rather substantially over months upon implementation, i.e., about 9% on average and about 19% after two years. Using a formal economic model that allows for comparison between RD and non-RD regimes, we provide insights on the potential mechanism behind the observed price effect and policy implications on key issues surrounding residential electricity consumption.

[4]While there exists useful discussions on the performance of RD from various perspectives[12–15], none focused on how decoupling works in the presence of subsidies for distributed generation or the effects of RD on electricity prices and welfare. Comprehensive technical reports and anecdotal evidence are available[2,4]; however, they present divergent views more than clear guiding principles on the potential impact of RD.

[5]We define a utility as an investor-owned electric service provider operating in a particular state, which means that utilities operating in two or more states are treated as unique utilities.

Results

Impacts of RD on residential electricity rates

By simply comparing utilities that were decoupled during the sample period with those that remained non-decoupled, we observed significant divergence in the average residential electricity rates as more utilities get decoupled over time (see panel (a) in Figure 1). Towards the end of 2012, average monthly electricity rates from decoupled utilities increased to $0.19/kWh, which is significantly higher than the average for non-decoupled utilities (about $0.12/kWh). This translates to about a $70 increase in monthly electric bill for an average electric customer, more than 30-fold adjustments compared to the previous estimate of $2.30 per month.[6] The result holds even if we use nominal prices.

Using a simple linear regression that focuses on within-state-utility changes in prices over time and accounts for the potential confounding effects of time-specific shocks that are common to all utilities (i.e. macroeconomic shocks) (Methods), we find an average increase in residential electricity prices associated with RD implementation ($\Delta = 9\%$; $p = 0.0224$; $n = 28,877$). The estimates are similar whether we use nominal prices ($\Delta = \$0.02/kWh$; $p = 0.0024$; $n = 28,953$) or real (inflation-adjusted) prices ($\Delta = \$0.01/kWh$; $p = 0.0586$; $n = 28,953$). The estimated increases in prices are significantly larger than what the previous studies find, which are based on the size of the actual RD adjustment.[2].

A major issue about the estimated effect presented above is the likelihood that utilities that become subject to RD may be systematically different from average utilities in the US. For example, a state in which utilities experience declining sales due to more aggressive environmental policies may be more inclined to implement RD in order for the utilities to recover their fixed costs. Thus simply comparing decoupled and non-decoupled utilities may lead to selection bias. To account for this potential bias in the estimated effect, we compare treated (i.e., decoupled) utilities with those control utilities in the same year-month that had almost identical level and trend in their real prices (in $/kWh) and sales (in MWh) over the 12-month period prior to the implementation of RD. The argument is that in the absence of the policy change, the treated and the control utilities would have behaved similarly, and that any change in the outcome variables for all treated utilities is attributed to the policy change. This procedure generates slightly lower estimates ($\Delta = 7\%$; $p = 0.2671$; $n = 1,175$).

As "Mechanism" below explains, RD may have persistent effects on the electricity prices beyond the the immediate impacts due to rate adjustments. To test the hypothesis that RD impacts may persist over months, we reformulated our method by looking at the differences between the control and the treated groups in each time period, after RD implementation, while maintaining to account for time-invariant utility-specific characteristics (Method). The results, as illustrated in rightmost section of panel (b) in Figure 1, confirm our hypothesis that the effect grows over time, reaching to about 18% two years after the implementation of RD.

We also test the same hypothesis to residential electricity sales and revenues. We find that switching to RD causes no significant effect on residential electricity consumption. In contrast, we see significant increase in revenues after 18 months.

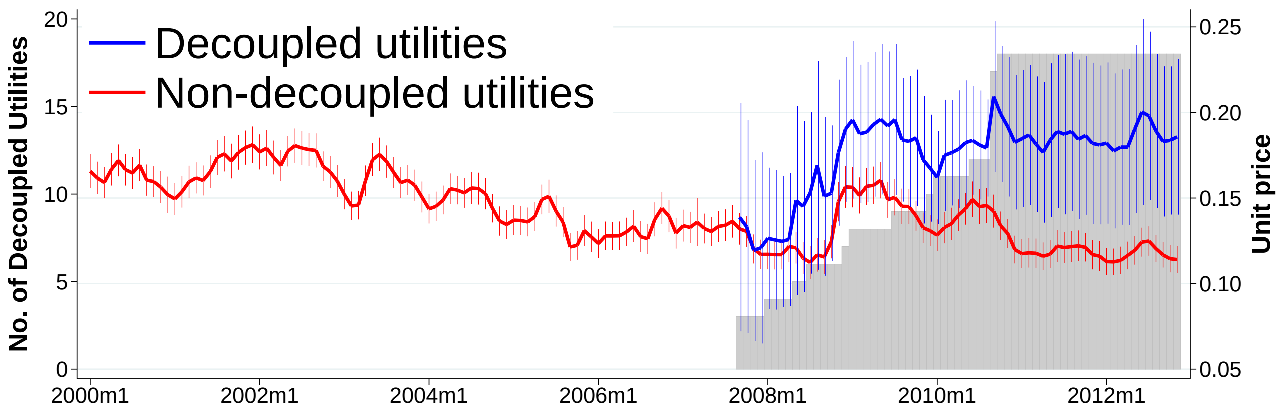
Responses to unexpected changes

Decoupling as a mechanism is supposed to work symmetrically over unexpected increases in sales (that should result in downward price adjustments) and unexpected decreases in sales (that should result in upward price adjustments). A previous study based on RD data in 2005-2012 finds that about 66% of decoupling adjustments were surcharges while the rest 34% resulted in refunds to customers.[2]. Here we test whether decoupling works symmetrically, in terms of magnitude, in events of unexpected changes in sales.

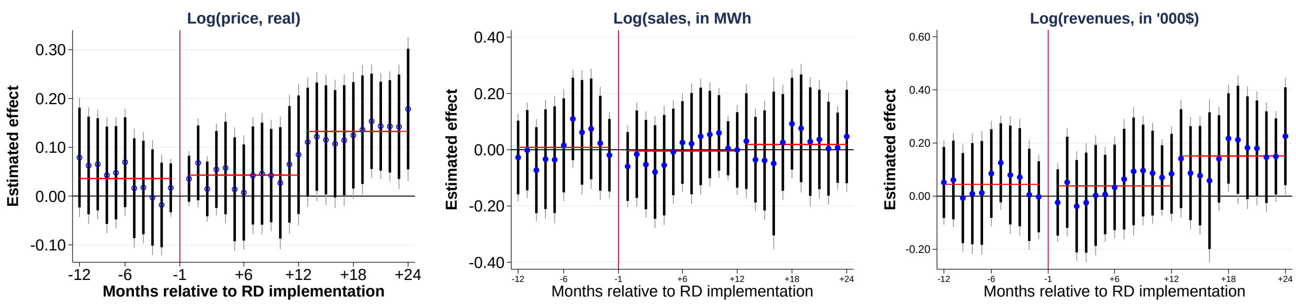
In order to measure unexpected changes in sales, we need to compare actual sales with required revenues. However, we do not observe the revenue requirements of each utility. To come up with an alternative measure for unexpected changes in sales, we compute the average sales growth rate over the previous 6 months or 12 months and compare it with that of the previous month. We then compare the response of treated utilities that had higher-than-usual demand growth (that is, the sales growth rate in the previous month is higher than the growth rate over the previous 6 months or 12 months) versus those that had sales equal to or below the forecasted sales growth. The results are presented in panel (c) in Figure 1. We have two remarkable observations. First, the difference in the estimated effect between those that had higher-than-projected sales growth and those that had lower-than-projected sales growth is very minimal and statistically insignificant. Second, the estimated effect is still positive even for those that had higher-than-projected sales growth. This implies that, at least, utilities experiencing unanticipated sales growth would not have price reductions. Furthermore, there seems to be downward rigidity in electricity prices during periods of unanticipated sales growth such that the customers would still pay higher prices than those who are served by non-decoupled utilities.

[6]This calculation assumes an average monthly consumption of 1,000kWh, following a previous study that assessed the effect of RD implementation on electricity rates[2].

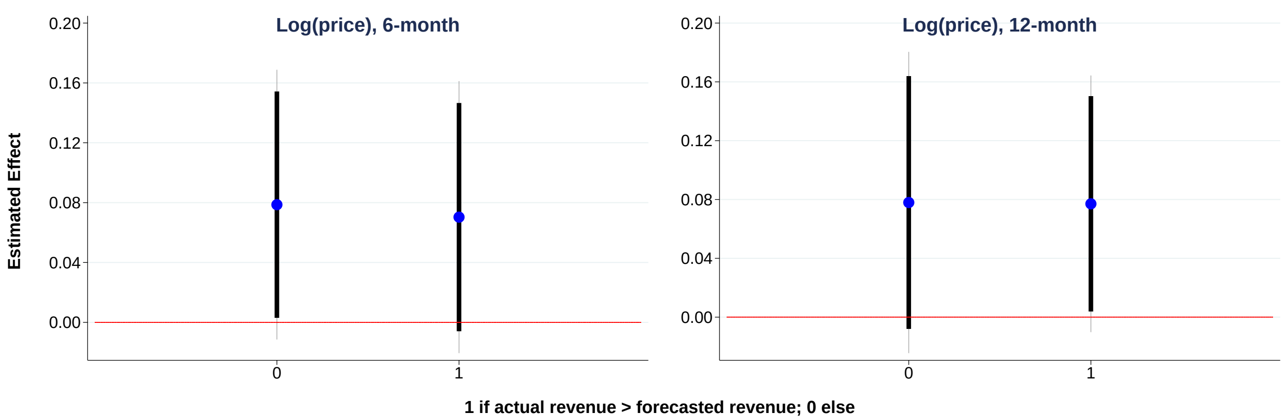
(a) Average monthly electricity prices of decoupled and non-decoupled utilities



(b) Estimated effect of implementing RD, matched sample



(c) Asymmetric price impact of RD, matched sample



Panel (a): The curves represent the estimated average electricity price in $/kWh (right axis), with vertical lines indicating the 95% confidence interval. The shaded vertical bars correspond to the number of decoupled utilities (left axis).
Panel (b): Estimated effects (blue dots - estimated effect relative to 1 month before the RD implementation; thick black vertical lines -90% confidence interval (CI); thin gray vertical lines - 95% CI; horizontal red line - yearly average effect).
Panel (c): Estimated effects (blue dots - estimated effect relative to 1 month before the RD implementation; thick black vertical lines -90% confidence interval (CI); thin gray vertical lines - 95% CI). Forecasted revenue is defined as the difference in log-transformed prices between 6 months and 1 month prior to RD; actual revenue is the difference in log-transformed prices between 2 months and 1 month prior to RD.
All prices are deflated using consumer price index.

Figure 1. Effect of implementing Revenue Decoupling

Mechanism

Supplementary Material (Appendix B.1) explains the formal economic model to describe the economic impacts of revenue decoupling. Here we explain the key insights from the model to explain the impacts in the short run and in the long run.

To explain how RD impacts electricity prices upon unexpected changes in the sales of electricity, it is useful to consider the demand for electricity as well as the supply and the demand of investment in energy efficiency (such as energy-efficient appliances and solar panels). Suppose that there is a supply shock to energy-efficiency investment due to technological innovation (lowering the costs) or policies to encourage such investment (increasing the demand). The induced increase in energy-efficiency investment reduces the demand for electricity. Without RD, the price would stay at the initial level. With RD, the retail electricity price is adjusted upwards (as long as the price elasticity of demand is less than one in absolute value). This is the immediate price impact of RD. However, the increase in electricity price raises the demand for energy efficiency. This secondary impact shifts the demand for electricity further, thereby raising the electricity price further under RD. This explains the positive effect of RD on electricity prices over months after RD implementation (Figure 1).

Figure 1 also indicates that, while there are no differences in the sales (in MWh) growth with or without RD, the revenue grows faster under RD. The above mechanism does not explain these trends. In fact, many states with RD allow for changes in the revenue requirement between rate cases.[7] With such arrangements, utilities are allowed to update the required revenues to recover cost increases due to inflation or capital additions approved by the public utilities commission. Previous studies also find that RD implementation is associated with increases in the utility spending on demand side management.[10,11] Such investments would explain why the revenues grow faster in those states with RD than in those without.

Discussion

Several U.S. states adopted revenue decoupling as one of the many policy measures to provide utilities with incentives to invest in energy efficiency and conservation. Whether decoupling improves efficiency of the electricity sector has been a subject of debate[2,12,14], but few studies have investigated the policy's welfare property theoretically and empirically. By combining the empirical evidence with a formal economic model, we demonstrate below the potential welfare consequences of RD as it links with several pressing welfare issues in the US residential electricity consumption. The detailed theoretical exposition is found in Supplementary Section B.1.

Effect when combined with increased subsidies for distributed generation or energy efficiency. The United States government provides federal tax credits for consumer energy efficiency including investment in solar panels. Many U.S. states also offer state-level tax credits for installing solar panels. For qualified households, these tax credits work as a subsidy for installing solar panels. We examined how the adoption of RD impacts households when the implied subsidies increased. Our model reveals that RD amplifies the negative welfare impact of solar subsidies (see Supplementary Section B.2) through an increase in the unit price of electricity distributed through the grid and the corresponding consumer adjustments for grid-supplied electricity. Under the non-RD regime, an increase in the amount of subsidy, say for solar panels, will create (1) excess burden for a subsidy (called the 'primary welfare effect'[17]) and (2) the 'electricity mark up effect', which is an extra distortion on the use of grid-supplied electricity when price exceeds the marginal costs. Both of these distortionary effects are exacerbated under the RD regime.

Potential Distributional Effect. We also examined how the adoption of revenue decoupling impacts households with and without distributed generation (or solar panels, Supplementary Section B.3). We find that RD will unambiguously benefit those high-income households that can afford to install capital-intensive solar panels and energy efficiency, but adversely affect low-income households that do not. Given inelastic demand for electricity, low-income and presumably credit-constrained households would be adversely affected by the increase in price. This finding is in line with earlier studies that find policies that reduce the cost of solar panels, including production subsidies and tax credits, are generally regressive.[18,19]

Precise welfare expressions would include the share of profits of the utility for each consumer. Because the profit is increasing in a drop in the cost of solar panels or energy efficiency and in the subsidy under RD, this consideration tends to increase the welfare impacts on those with solar panels, and may alleviate the negative welfare impacts on those without solar panels.

Effect of Uncertainty. We consider uncertainty regarding output from solar panels in order to examine how the associated risk is shared between consumers and the utility under the alternative regulation (Supplementary Section B.4). We find that, without RD, any increase in the degree of uncertainty regarding output from solar panels will not change the utility's equilibrium profits nor the consumers' equilibrium expected utility. With RD in place, an increase in the degree of uncertainty will result in an

[7] In 8 out of 12 states with RD studied, revenue changes between rate cases are allowed.[16]

increase in the expected profits of the utility and a decrease in consumers' equilibrium expected utility. Taken together, the results imply that the demand-based risk burden shifts from the utility to the consumers when RD is in place.

Potential welfare effects. Economic efficiency, which incorporates the pollution externalities of electricity generation, implies that the retail prices should be set equal to the social marginal cost (SMC) of electricity services. Increases in electricity prices would lead to lower consumer surplus, but whether it induces negative welfare impacts is not clear once we take into account negative externalities associated with utility-scale electricity generation (damages due to emissions of CO_2 and other air pollution from fossil fuel combustion). On the one hand, under conventional pricing, the electricity price tends to exceed the (private) marginal costs of electricity generation. As discussed earlier, this implies that RD amplifies the distortionary impacts of above-marginal-cost pricing. On the other hand, if the social marginal costs (i.e., including the marginal external costs of electricity generation based on fossil fuel) exceed the retail electricity price, then a price increase due to RD would make the price closer to SMC and generate positive welfare impacts.

A recent paper by Bushnell and Borenstein[20] reveals that, in most of the states that have adopted RD, the marginal price exceeds SMC. To the extent that the price-SMC relationship does not change significantly in the period 2000-2012, this finding indicates that RD tends to generate negative welfare impacts for most states that implemented this policy. This is particularly true for states like California, New York, and Massachusetts where electricity prices exceed SMC. Over time, the grids can become more efficient and cleaner across states. Coupled with RD, these additional investments may necessitate further increases in prices. Therefore, such changes in the grids may magnify the negative welfare effects of RD.

Moving forward: Flat Distribution. The empirical evidence and the policy insights presented above suggest that the current design of RD for electric utilities is not the ideal policy provision to enhance efficiency of the electricity sector nor resort to more renewable energy in the form of distributed power. The question remains: what alternatives would be more efficient while aligning electricity utilities' incentives with societal goals?

There are two main types of designing RD for public utilities. The first one, which is discussed here, applies frequent true-ups on volumetric rates to ensure that the utility's actual revenue is equal to its revenue requirement. The second one, called the straight-fixed variable (SFV) rate design, sets fixed charges (such as the monthly customer charge) to recover the full fixed costs of service delivery while variable costs are recovered through variable charges. At the moment, the second type of RD is more common in natural gas than in electric utilities[21].

Covering revenue shortfalls through the SFV does not come without costs. These costs include the potential increase in consumption with lower volumetric charges and possible distributional concerns when low-earning households would pay fixed monthly charges similar to high-income earners. While our analysis does not promote the use of fixed cost to cover the entire revenue shortfall, we argue, based on the evidence presented above, that fixed charges can be used to cover at least part of the shortfall. Doing so may prevent electricity prices to be so high to increase distortions in the markets for electricity and energy efficiency.

Methods

Data. We use US EIA monthly data for the period covering January 2000 - November 2012 on about 160 unique investor-owned utilities to investigate how RD influenced electricity rates. We drop utilities in California from the sample because decoupling was adopted in the state prior to 2010, the beginning of the sample period. The data contain information about the utilities' sales (in kWh), revenues, and the average electricity prices by end-use sector. We combine the EIA data with information about the timing of revenue decoupling implementation by utilities using data from a previous study[10]. Table A.1 (Supplementary Table) presents the descriptive statistics of the sample.

Analysis. The empirical analysis in identifying the effect of RD on electricity prices consists of the following features. First, we focus on the change from non-RD to RD regime for the same utility operating in a particular state. In particular, we consider those utilities that are observed at least for 12 months prior to the adoption of RD and 24 months thereafter. By focusing on within state-utility changes, we account for the effect of unobserved individual characteristics across utilities that may bias our estimates.

Second, we use difference-in-differences approach (hereafter referred to as DD) to compare electricity prices of decoupled utilities with those that remain in old rate-making schemes. The association between policy changes and subsequent outcomes are easily assessed using pre-post comparisons. This design is valid only if there are no underlying time-dependent trends in outcomes that are correlated with the policy change. In our case, if electricity prices were already increasing for decoupled utilities even before the implementation of RD, then using pre-post study would lead to biased estimates and potentially erroneous association of the change to the implementation of RD. The DD approach solves this issue by taking into account initial difference in prices between decoupled and non-decoupled before the adoption of RD, as well as the difference in prices between the two groups after the policy adoption, thereby implicitly taking into account unobserved factors that may affect prices faced by the treatment or the control group.

Our estimating equation is provided below:

$$p_{it} = \alpha_i + \beta_t + \gamma Post_{it} + \delta RD_{it} + \varepsilon_{it}, \tag{1}$$

where p_{it} is the electricity price charged by utility i in period (month-year) t, $Post$ is equal to 1 when the matched utilities are in the post-RD regime and 0 otherwise, and RD_{it} is a dummy variable that turns to unity when a utility starts to implement decoupling. Coefficients α and β represent utility-state and time fixed effects, respectively, to account for the unobserved utility-state characteristics and month-year specific shocks that are common to all utilities (e.g. macroeconomic shocks). The error term ε is assumed to be i.i.d. Coefficient δ measures the effect of implementing RD on the outcome variable.

One major issue in employing DD is that the estimate of δ could be biased if the control and treatment groups have different pre-treatment characteristics[22]. In our context, this can happen if utilities suffering from a decline in sales, possibly due to increased share in distributed generation or improved energy efficiency among customers, lobby for RD implementation. To address this issue, for each utility in the treatment group, we identify a control utility of similar electricity price trends (measured in log difference between the electricity price a month before and 6 months before) and is operating in the same time period. This procedure allows us to ensure that the matched utilities most likely faced the same macroeconomic conditions and price trends before RD is adopted. This approach, however, reduces our sample significantly. Fortunately, the number of utility-month-year observations are large enough to generate results with confidence.

We assess the performance of our matching procedure by comparing the sample means of the variables used in the matching of treatment and control groups (see Supplementary Table A.3). We find no statistically significant difference in the pre-RD period for the variables that were used in matching, suggesting that our matched sample exhibits parallel pre-treatment trends in prices. Moreover, we also find no statistically significant differences between the means of the two groups for other variables that were not used in the matching (except that residential revenues are different with marginal significance). Thus our procedure is not subject to potential biases associated with selection on unobserved characteristics that affect both assigning of treatment and the outcome of interest.

In order to verify that the estimated effects coincide with the time of the implementation of RD and that the effect is stable over time, we plot the coefficients of the following regression,

$$p_{it} = \alpha_i + \beta_t + \sum \delta_l Month_{lc} + \varepsilon_{it}, \tag{2}$$

where $Month_{lc}$ is a set of indicator variables for lag and lead months relative to the time of implementation of RD by a utility in a particular state and all other variables are as previously defined.

Finally, we test for the symmetry of the electricity price effect of RD in the event of a higher-than-projected sales growth by estimating the following equation:

$$p_{it} = \alpha_i + \beta_t + \gamma_1 Post_{it} + \delta_1 RD_{it} + \delta_2 D_{it} + \gamma_2 (Post_{it} * RD_{it}) + \gamma_3 (Post_{it} * D_{it}) + \delta_3 (Post_{it} * RD_{it} * D_{it}) + \varepsilon_{it}, \tag{3}$$

where $D = 1$ if the previous month's sales growth rate is higher than sales growth for previous 12 months; and $D = 0$ if otherwise. all other variables are as previously defined.

Author contributions statement

A.B. and N.T. both designed the study, A.B. designed the empirical strategy, N.T. built the theoretical model, A.B. and N.T. analysed the results. Both authors reviewed the manuscript.

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Competing Interest

The authors declare no competing interests.

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Supplementary Notes
(for Online Publication)

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Appendix A Supplementary Tables and Figures

Table A.1. Summary Statistics

	Not Decoupled			Decoupled		
	Obs	**Mean**	**SD**	**Obs**	**Mean**	**SD**
Prices ($/kWh)						
Residential	26529	0.10	0.05	2604	0.15	0.08
Commercial	25033	0.09	2.36	2602	0.13	1.20
Industrial	26552	0.09	0.06	2604	0.13	0.07
Total	27076	0.10	1.92	2604	0.13	0.07
Sales (in GWh)						
Residential	26965	339.44	581.02	2604	421.31	423.64
Commercial	26495	242.13	348.25	2604	229.38	309.81
Industrial	26963	319.67	575.54	2604	380.06	448.34
Total	27169	898.52	1,388.56	2604	1,035.95	1,086.13
Revenues (million $)						
Residential	26903	34.43	65.10	2604	53.06	60.19
Commercial	26496	13.60	20.73	2602	16.10	20.28
Industrial	26935	28.03	59.82	2604	42.13	57.17
Total	27126	75.83	137.74	2604	111.86	124.41
No. of unique State-Utilities			192			17
Years			2000-2012			2000-2012

Note: Decoupled utilities are those in a particular state that had adopted RD, which means that the values include pre- and post-RD regime. Non-decoupled utilities are those that had not adopted RD during the sample period.
Source: U.S. Energy Information Administration.

In Table A.1, we observe that the utilities that experienced decoupling have higher average prices than those without decoupling. This observation applies to all sectors (i.e. residential, commercial, and industrial). Decoupled utilities have higher sales, except for commercial customers, and higher revenues for all customers.

Table A.2. Number of States with RD for Electric Utilities.

Since 1990s	2006	2007	2008	2009	2010	2011	2012	2013	2018
1	2	5	5	10	11	12	14	14	17

Source: The Regulatory Assistance Project (RAP), 2011; NRDC, 2019.

Table A.3. Balancing test of matched RD and non-RD utilities.

	Unconditional Mean		
	nonRD	**RD**	**p-value**
Pre-RD Prices (in $/kWh)			
Residential	0.15	0.17	0.597
Commercial	0.14	0.15	0.837
Industrial	0.12	0.12	0.988
Total	0.14	0.15	0.705
Pre-RD Price Trend			
Residential	0.080	-0.010	0.179
Commercial	0.080	0.040	0.196
Industrial	0.060	0.040	0.997
Total	0.200	0.140	0.621
Pre-RD Sales (in GWh)			
Residential	832.28	444.15	0.132
Commercial	435.21	352.33	0.577
Industrial	294.46	177.61	0.446
Total	1567.18	974.32	0.229
Pre-RD Sales Trend			
Residential	0.17	0.13	0.527
Commercial	0.00	0.07	0.259
Industrial	0.05	-0.07	0.525
Total	0.12	0.05	0.246
Pre-RD Revenues (in million $)			
Residential	119.89	59.56	0.074
Commercial	56.81	43.22	0.444
Industrial	24.07	14.30	0.401
Total	201.17	117.11	0.132
Pre-RD Revenue Trend			
Residential	0.20	0.11	0.255
Commercial	0.05	0.08	0.746
Industrial	0.08	-0.04	0.545
Total	0.15	0.06	0.262

Notes: Figures reflect the unconditional means of the matched RD and non-RD utilities during the month before they adopted RD, unless otherwise stated. Trends are measured in log difference. p-values are for testing the statistical significance of the mean difference between the two groups.
Source: U.S. Energy Information Administration.

Appendix B Effects of Revenue Decoupling: Theoretical Results

B.1 Theoretical framework

B.1.1 Consumers

There is a continuum of consumers of measure $N > 0$. Let u_i be consumer i's utility function. Given total electricity consumption e_i and the consumption of numeraire good y_i, the utility is $u_i(e_i, y_i) = v_i(e_i) + y_i$ where $v_i' > 0$ and $v_i'' < 0$. This specification, with zero income elasticity of electricity demand, could be justified in light of some recent empirical findings of zero or very small income elasticity.[8]

Each household chooses how much electricity to purchase from the utility $x_i \geq 0$ and whether to purchase a solar PV ($d_i = 1$) or not ($d_i = 0$). Household i's electricity output from its solar PV is given by $g_i \geq 0$. We abstract from hourly, day-to-day, and seasonal variations in load profiles as well as intermittency of solar electricity outputs. We thus assume grid-supplied electricity (x_i) and electricity from distributed sources (g_i) are perfect substitutes: $e_i = x_i + d_i g_i$. Existence of provisions such as net energy metering might imply that they are indeed almost perfectly substitutable. As long as they are close substitutes, the main arguments of this paper would be valid. We can also interpret d_i as indicating the household's investment in energy efficiency improvement.

We also assume there is no peak-load pricing: consumers face a simple two-part tariff, with a unit volumetric electricity rate $p > 0$ and a fixed payment $f > 0$. Household i maximizes its utility subject to a budget constraint $px_i + f + qd_i + y_i \leq m_i$, where $m_i > 0$ is household i's income and q the (rental) price of a solar panel.[9] The income consists of wage income (where labor endowment is fixed and its supply is assumed to be inelastic) and the household's share of the electric utility's profits. Thus, household i's objective function is given by

$$\max_{x_i \geq 0, d_i \in \{0,1\}} v_i(x_i + d_i g_i) + y_i$$

$$\text{s.t.} \quad px_i + f + qd_i + y_i \leq m_i.$$

The first order condition for utility maximization is given by

$$v_i'(x_i + d_i g_i) = p, \quad d_i = 1 \quad \text{if } g_i \geq q/p, \quad d_i = 0 \quad \text{if } g_i < q/p.$$

Now suppose that households are ordered in terms of PV output: $g_i > g_j$ for all $i, j \in [0, N]$ such that $i < j$. Let $h(n)$ be the total solar output when households 0 to n install solar panels:

$$h(n) \equiv \int_0^n g_i di \quad \text{(and hence } g_n = h'(n)\text{)}.$$

Then all households i with $c_i \geq q/p$ install solar panels and the rest do not. Now we define

$$v(e) = \max_{(e_i)_{0 \leq i \leq N}} \int_0^N v_i(e_i) di \quad \text{s.t.} \quad \int_0^N e_i \leq e.$$

By construction, v is concave with $v' > 0, v'' < 0$. The consumers' utility-maximizing choice satisfies

$$\int_0^N \{v_i(e_i) + y_i\} di = v(e) + M - fN - p(e - h(n)) - qn,$$

where $M \equiv \int_0^N m_i di$, $v'(X) = p$ and $h'(n) = g_n = q/p$. Therefore, maximizing v subject to an aggregate budget constraint $px + qn + y \leq M$ yields the households' utility-maximizing allocation given p, q. The first-order condition is given by

$$v'(e) = v'(x + h(n)) = p; \tag{4}$$

$$h'(n) = \frac{q}{p}. \tag{5}$$

Solving these conditions for x and n yields the demand for grid-supplied electricity, $x(p,q)$, and the demand for solar panels, $n(p,q)$, given the prices p, q.

[8] 23 estimate the income elasticity for California households to be between -0.01 and +0.02.

[9] If x_i represents the annual electricity consumption, then q represents the annual rental price of a solar panel.

B.1.2 Electric Utility

Let $F > 0$ be the fixed cost of providing electricity services (fixed and given at least in the short run). Though not essential for the analysis, assume that the marginal cost $c > 0$ is constant. Thus the utility's service is subject to increasing returns to scale. The utility's profit can then be expressed as

$$\pi = px + Nf - cx - F.$$

B.1.3 Supply of solar panels

We assume that production of solar panels exhibits constant returns to scale and that the solar panels are supplied competitively. We could imagine a small open economy, with a limited option for trading electricity internationally, which faces a constant price of solar panels q.

B.1.4 Regulation with and without decoupling

We consider two regulatory regimes: (1) traditional rate of return regulation with no revenue decoupling (no RD); and (2) the RD regime. With no RD, the electricity price is held fixed between rate cases[10].

Under RD, the electricity price is allowed to change for the utility to earn a fixed, pre-approved level of revenue. We assume that the number of customers N, as well as the fixed fee per customer, f is fixed throughout the analysis. In many cases, the fixed payment is much smaller than the fixed cost of operating the utility. With F redefined appropriately, the rest of the analysis assumes away the presence of the term Nf.[11]

Under the traditional rate-of-return utility regulation, electricity rates are fixed in the short run at the levels approved by the public utilities commissions[24].[12] We can write the regulatory constraint as some fixed price that includes the maximum allowable mark-up over incurred production costs, $\bar{p}$:

$$\bar{p} \leq (1+\alpha)AC = (1+\alpha)\frac{F+cx}{x}.$$

The utility's profit is thus given by

$$\pi = \bar{p}x(\bar{p},q) - cx(\bar{p},q) - F.$$

We assume that $\bar{p} > c$ throughout the analysis. This is based on the observation that the volumetric electricity rates tend to exceed the marginal cost of electricity, and that the monthly fixed fees for residential electricity are not sufficient to cover the fixed cost of electricity services[25]. The same has been observed in residential natural gas markets[26].

While some RD methods include an explicit procedure for changing the level of authorized revenue during years between rate cases, we will only focus on the balancing accounts that guarantee the exact collection of a fixed authorized revenue for a given time period.

Let $\bar{R}$ be the revenue level associated with the initial price level and equilibrium level of x. In this case the electric rate is adjusted so that the revenue is balanced when demand changes: $\bar{R} = px(p,q)$. We can therefore write the utility's profit as

$$\pi = \bar{R} - cx(p,q) - F.$$

In this representation of an equilibrium between rate cases, the decision of the producer is limited: given p,q, it supplies output $x(p,q)$.

B.2 Effects of revenue decoupling

B.2.1 Changes in the cost of solar panels

Effects on electricity price and quantity Here we study the effect of an exogenous change in the price (or the cost) of solar panels q. We first compare the impacts on electricity price and quantity with and without revenue decoupling.

With no revenue decoupling, the equilibrium condition is given by equations (4) and (5). With revenue decoupling in place, the necessary and sufficient condition for an (interior) equilibrium is given by (4) and (5) with $px - \bar{R} = 0$. Total differentiation of the equilibrium conditions in the two cases yield the following proposition about the effect of a decrease in the cost of solar panels on the equilibrium price and quantity of grid-supplied electricity.

[10] Electricity rates are held constant fixed between rate cases, where the utility files before the public utility commission (PUC) for rate adjustments usually due to changes in operating and maintenance costs of electric distribution.

[11] Our focus is on residential electricity markets. We abstract away from electricity markets for industry and commercial sectors, and cross-subsidization across sectors in electricity pricing—issues to be investigated in future studies.

[12] Fuel cost adjustments are allowed between rate cases for many utilities, where the rates are adjusted upon short-term fluctuations in the fuel prices.

Proposition 1 *Without RD, a decrease in the cost of solar panels reduces the equilibrium electricity sales. With RD, a decrease in the cost of solar panels reduces the equilibrium electricity sales, and increases the electricity price, if and only if the demand for electricity is inelastic (i.e., the price elasticity is less than one in absolute value).*

Proof. Total differentiation of (4) and (5) yields

$$v''(x+h(n))dx + v''(x+h(n))h'(n)dn = 0; \tag{6}$$

$$h''(n)dn = \frac{1}{p}dq. \tag{7}$$

From (7), we have $\frac{dn}{dq} = \frac{1}{ph''(n)} < 0$. Substitute this into (6) and we obtain

$$v''(x+h(n))\frac{dx}{dq} + v''(x+h(n))h'(n)\frac{1}{ph''(n)} = 0. \tag{8}$$

It follows that

$$\frac{dx_{noRD}}{dq} = -\frac{h'(n)}{ph''(n)} > 0, \tag{9}$$

which implies that, under the traditional rate-of-return regulation, any decrease in the cost of solar panels reduces the equilibrium output of grid-supplied electricity.

Next we consider the case with RD. Totally differentiate the system (with respect to endogenous variables x, n, p and an exogenous variable q) and obtain

$$\begin{pmatrix} v'' & -1 & v''h' \\ v''h' & 0 & v''(h')^2 + v'h'' \\ p & x & 0 \end{pmatrix} \begin{pmatrix} \frac{dx}{dq} \\ \frac{dp}{dq} \\ \frac{dn}{dq} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{10}$$

Hence, we have

$$\frac{dx_{RD}}{dq} = \frac{v''h'x}{D},$$

where

$$D \equiv \begin{vmatrix} v'' & -1 & v''h' \\ v''h' & 0 & v''(h')^2 + v'h'' \\ p & x & 0 \end{vmatrix} = -v'\{v''(h')^2 + v'h''\} - v'v''h''x.$$

To evaluate these expressions, we derive the price elasticities of demand for electricity and solar panels. Totally differentiate the first order conditions for the consumer's utility maximization (4) and (5) (with respect to x, n and p) to obtain

$$\begin{pmatrix} v'' & v''h' \\ v''h & v''(h')^2 + v'h'' \end{pmatrix} \begin{pmatrix} \frac{\partial x}{\partial p} \\ \frac{\partial n}{\partial p} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{11}$$

Thus we have $\frac{\partial x}{\partial p} = \frac{v''(h')^2 + v'h''}{v'h''v''}$ and hence the price elasticity of demand for utility-generated electricity satisfies

$$\eta_x \equiv \frac{\partial x}{\partial p}\frac{p}{x} = \frac{v''(h')^2 + v'h''}{v''h''x} < 0.$$

Plugging the above elasticity in to dx_{RD}/dq yields

$$\frac{dx_{RD}}{dq} = \frac{\frac{v''h'x}{xv'h''v''}}{-\frac{v''(h')^2 + v'h''}{v''xh''} - 1} = \frac{-\frac{h'}{v'h''}}{1+\eta_x} \begin{cases} > 0 & \text{if } |\eta_x| < 1; \\ \leq 0 & \text{if } |\eta_x| \geq 1. \end{cases} \tag{12}$$

A similar comparative statics on p yields

$$\frac{dp_{RD}}{dq} = \frac{-v'v''h'}{D} = -\frac{p}{x}\frac{dx}{dq} \begin{cases} < 0 & \text{if } |\eta_x| < 1; \\ \geq 0 & \text{if } |\eta_x| \geq 1. \end{cases}$$

■

Therefore, in the empirically relevant case with inelastic electricity demand, the grid-supplied electricity consumption decreases, and the price p increases, as q drops.

Effects on welfare

Now we turn to the welfare effects with and without RD. We assume that the utility's profit is returned to consumers as dividends: household i receives a profit share $s_i\pi$ where $s_i \geq 0$ for all i and $\int_0^N s_i di = 1$. Let W_r denote the representative consumer's welfare under policy regime r ($r \in \{RD, noRD\}$). In the absence of distortions other than the markup in electricity pricing, the welfare is given by

$$W_r = u(x_r + h(n_r)) - p_r x_r - qn_r + [px_r - cx_r - F] = u(x_r + h(n_r)) - cx_r - qn_r - F.$$

Under traditional rate-of-return regulation with no revenue decoupling, we have:

$$\begin{aligned}\frac{dW_{noRD}}{dq} &= v'\frac{dx_{noRD}}{dq} + v'h'\frac{dn_{noRD}}{dq} - n - q\frac{dn_{noRD}}{dq} - c\frac{dx_{noRD}}{dq} \\ &= (\bar{p} - c)\frac{dx_{noRD}}{dq} - n_{noRD}.\end{aligned}$$

If $\bar{p}$ is set close enough to c, the welfare is expected to increase as q declines. However, with a sufficiently large markup, the welfare may decrease as q drops.

Under revenue decoupling, we have:

$$\frac{dW_{RD}}{dq} = (\bar{p} - c)\frac{dx_{RD}}{dp} - n_{RD}.$$

Consider the case where $|\eta_x| < 1$. It follows from (9) and (12) in the proof of Proposition 1 that

$$\frac{dx_{RD}}{dq} = \frac{1}{1 - |\eta_x|}\frac{dx_{noRD}}{dq} > \frac{dx_{noRD}}{dq}.$$

This implies that, with revenue decoupling, the negative effect of a decrease in q on total welfare is exacerbated by the amount of consumer adjustment for x if the electricity demand is inelastic.

Proposition 2 *Without revenue decoupling, the total economic welfare increases as the cost of installing solar panels goes down, provided $\frac{\partial \pi}{\partial q}$ is sufficiently low (or if $\bar{p}$ is set close enough to c). Under revenue decoupling, the negative effect of a decrease in q on total welfare is exacerbated by the amount of consumer adjustment for x, provided that the electricity demand is inelastic.*

B.2.2 Changes in the subsidy for solar installation

With subsidy $s > 0$ per unit of solar panel, the consumer price of solar panels is given by $\bar{q} = q - s$.

Effects on electricity price and quantity Without revenue decoupling, the interior equilibrium satisfies (4) and (5) with $p = \bar{p}$. Under revenue decoupling, the interior equilibrium satisfies (4), (5) and

$$px - \bar{R} = 0.$$

The effect of an increase in the solar subsidy on electricity prices and quantities is the same as that of a decline in the cost of solar panels.

Proposition 3 *Without RD, an increase in the subsidy for solar panels reduces the equilibrium electricity sales. With RD, an increase in the subsidy for solar panels reduces the equilibrium electricity sales, and increases the electricity price, if and only if the demand for electricity is inelastic.*

Proof. For the case with no RD, a simple modification of the analysis in section B.2.1 yields

$$\frac{dx_{noRD}}{ds} = \frac{h'(n)}{ph''(n)} < 0. \tag{13}$$

For the case with RD, totally differentiate the system (with respect to endogenous variables x, n, p and an exogenous variable s) and obtain

$$\begin{pmatrix} v'' & -1 & v''h' \\ v''h' & 0 & v''(h')^2 + v'h'' \\ p & x & 0 \end{pmatrix} \begin{pmatrix} \frac{dx}{ds} \\ \frac{dp}{ds} \\ \frac{dn}{ds} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}. \tag{14}$$

Hence, we have

$$\frac{dx_{RD}}{ds} = \frac{-v''h'x}{D} = \frac{\frac{h'}{v'h''}}{1+\eta_x} \begin{cases} < 0 & \text{if } |\eta_x| < 1; \\ \geq 0 & \text{if } |\eta_x| \geq 1. \end{cases},$$

where D is as defined in section B.2.1. A similar comparative statics on p yields

$$\frac{dp_{RD}}{ds} = \frac{v'v''h'}{D} = -\frac{p}{x}\frac{dx}{dq} \begin{cases} > 0 & \text{if } |\eta_x| < 1; \\ \leq 0 & \text{if } |\eta_x| \geq 1. \end{cases}$$

■

Effects on welfare

Under solar subsidy with policy regime r, the welfare is given by

$$W_r = u(x_r + h(n_r)) - px_r - \bar{q}n_r + [px_r - cx_r - F] - sn_r = u(x_r + h(n_r)) - cx_r - qn_r - F,$$

where $\bar{q} = q - s$. Differentiate the above expression with respect to s:

$$\frac{dW_r}{ds} = v'(x_r + h(n_r)) \left\{ \frac{dx_r}{ds} + h'(n_r)\frac{dn_r}{ds} \right\} - c\frac{dx_r}{ds} - q\frac{dn_r}{ds}$$

$$= (p-c)\frac{dx_r}{ds} + v'(x_r + h(n_r))h'(n_r)\frac{dn_r}{ds} - q\frac{dn_r}{ds} = (p-c)\frac{dx_r}{ds} - s\frac{dn_r}{ds}.$$

With no revenue decoupling, we obtain the following intuitive expression:

$$\frac{dW_{noRD}}{ds} = -(p-c)\eta_{x,q}\frac{x}{\bar{q}} + s\eta_n\frac{n}{\bar{q}}, \tag{15}$$

where $\eta_{x,q}$ is the cross-price elasticity of the demand for electricity with respect to the price of solar panels. The second term is the usual Harberger excess burden formula for a subsidy (called the 'primary welfare effect'[17]). The first term, which would not exist under marginal-cost (or competitive) pricing with $p = c$, captures the effect of a solar subsidy on the demand for solar panels (due to an increase in solar subsidies). We call this the 'electricity markup effect.' To the extent that the electricity price exceeds the marginal cost, the subsidy on solar panels generates an extra distortion on the use of grid-supplied electricity.

Next, we consider the welfare impact under revenue decoupling. It follows from (14) that

$$\frac{dW_{RD}}{ds} = (p-c)\frac{dx_{RD}}{ds} - s\frac{dn_{RD}}{ds}.$$

The appendix shows that we can rewrite the expression to the following:

$$\frac{dW_{RD}}{ds} = -(p-c)\frac{\eta_{x,q}}{1-|\eta_x|}\frac{x}{\bar{q}} + s\frac{-\left\{-\eta_x + \eta_n\frac{qn}{px}\right\}\eta_n\frac{n}{q}}{1-|\eta_x|} + s\frac{-|\eta_n|\frac{n}{q}}{1-|\eta_x|}. \tag{16}$$

The above formula reveals how revenue decoupling amplifies the welfare impact of solar subsidies. The first and the third terms (the electricity markup effect and the primary welfare effect) are negative while the second term is positive. The third term represents the usual Harberger excess burden formula for a subsidy, but it is multiplied by $1/(1-|\eta_x|)$. The first term was also present in the absence of decoupling, but is also now multiplied by $1/(1-|\eta_x|)$. The second term is positive, but the sum of the second and the third term is negative. The second term is likely smaller in magnitude than the first and the third term because it involves a product of elasticities on the numerator. Therefore, depending on the size of the price elasticity of electricity demand, revenue decoupling exacerbates the excess burden due to solar subsidies.

Proposition 4 *With no revenue decoupling, the excess burden due to an increase in the subsidy on solar panels exceeds the primary welfare effect due to a markup in electricity pricing. Under revenue decoupling, both the primary welfare effect and the electricity markup effect are exacerbated when demand is inelastic.*

Proof.

With no RD,

$$\frac{dW_{noRD}}{ds} = (p-c)\frac{h'(n)}{ph''(n)} - s\frac{-1}{ph''(n)} < 0.$$

To interpret this expression, note that $\frac{h'(n)}{ph''(n)} = -\eta_{x,q}\frac{x}{\bar{q}} < 0$ and $\frac{-1}{ph''(n)} = -\eta_n\frac{n}{\bar{q}} > 0$. This yields equation (15).

With RD, the first term on the right-hand side of

$$\frac{dW_{RD}}{ds} = (p-c)\frac{dx_{RD}}{ds} - s\frac{dn_{RD}}{ds}$$

reduces to

$$(p-c)\frac{dx_{RD}}{ds} = (p-c)\frac{-v''h'x}{D} = (p-c)\frac{\frac{h'}{v'h''}}{1+\eta_x} = -(p-c)\frac{\eta_{x,q}}{1-|\eta_x|}\frac{x}{\bar{q}}.$$

The second term satisfies

$$\frac{dn_{RD}}{ds} = \frac{p+v''x}{D} = \frac{(p+v''x)/(xv'v''h'')}{D/(xv'v''h'')} = \frac{\frac{p}{xv'v''h''} + \frac{v''x}{xv'v''h''}}{\frac{-v'\{v''(h')^2+v'h''\}}{xv'v''h''} - \frac{v'v''h''x}{xv'v''h''}}$$

$$= \frac{-\frac{1}{xv''h''}}{1+\eta_x} - \frac{\frac{dn}{dq}\frac{q}{n}\frac{n}{q}}{1+\eta_x} = \frac{-\frac{1}{xv''h''}}{1-|\eta_x|} - \frac{\eta_n\frac{n}{q}}{1-|\eta_x|}.$$

To evaluate the numerator of the first term $-\frac{1}{xv''h''}$, note that

$$\frac{\partial x}{\partial p} = \frac{v''(h')^2 + v'h''}{v'h''v''} = \frac{(h')^2}{v'h''} + \frac{1}{v''},$$

where $h'(n) = q/p$. and $\frac{1}{v'h''} = \frac{\partial n}{\partial q}$. Thus

$$\frac{1}{v''} = \frac{\partial x}{\partial p} - \frac{\partial n}{\partial q}\left(\frac{q}{p}\right)^2.$$

We also have $\frac{1}{h''} = \frac{\partial n}{\partial q}p$. Hence,

$$-\frac{1}{xv''h''} = -\left\{\frac{\partial x}{\partial p}\frac{1}{x} - \frac{\partial n}{\partial q}\left(\frac{q}{p}\right)^2\frac{1}{x}\right\}\frac{\partial n}{\partial q}p = -\left\{\frac{\partial x}{\partial p}\frac{p}{x} - \frac{\partial n}{\partial q}\frac{q}{n}\frac{n}{q}\left(\frac{q}{p}\right)^2\frac{p}{x}\right\}\frac{\partial n}{\partial q}\frac{q}{n}\frac{n}{q}$$

$$= -\left\{-\eta_x + \eta_n\frac{qn}{px}\right\}(-1)\eta_n\frac{n}{q} = \left\{-\eta_x + \eta_n\frac{qn}{px}\right\}\eta_n\frac{n}{q}(<0).$$

From (11), we have $\frac{\partial n}{\partial p} = \frac{-v''h'}{v'h''v''} = -\frac{h'}{ph''}$. Hence

$$\eta_{n,p} \equiv \frac{dn}{dp}\frac{p}{n} = -\frac{h'}{ph''}\frac{p}{n} > 0$$

is the cross-price elasticity of the demand for solar panels with respect to electricity price. Therefore, the welfare impact of a marginal increase in the solar subsidy is given by equation 16. ■

B.2.3 Externalities of electricity generation

We describe how the analysis changes if we assume that the utility's electricity services involve negative externalities due to fossil fuel use for electricity generation. Let $\delta > 0$ represent the marginal external damages associated with the production and delivery of grid-supplied electricity x. We assume that, in the absence of emissions prices, each household does not take into account the external effects of its consumption. The welfare expression under no RD is given by

$$W_{nonRD} = v(x(\bar{p},q) + h(n(\bar{p},q))) - qn(\bar{p},q) - cx(\bar{p},q) - F - \delta(x(\bar{p},q)).$$

Under RD, the welfare is now expressed as:

$$W_{RD} = v(x(p,q) + h(n)) - cx(p,q) - F - \delta x(p,q)$$

Therefore,

$$\begin{aligned}\frac{dW_{nonRD}}{dq} &= v'\frac{dx}{dq} + v'h'\frac{dn}{dq} - n - q\frac{dn}{dq} - c\frac{dx}{dq} - \delta\frac{\partial x}{\partial q} \\ &= (\bar{p} - c - \delta)\frac{\partial x}{\partial q} - n\end{aligned}$$

under no RD while

$$\frac{dW_{RD}}{dq} = \left(\left[v' - c - e\right]\frac{\partial x}{\partial p}\frac{dp}{dq} + \frac{\partial x}{\partial q}\right) - n$$

holds under RD. To the extent that the markup $p - c$ exceeds the marginal external damages δ, the qualitative results are the same as in the previous section.

We now discuss additional results regarding the distributional impacts of decoupling on households with different income levels (and different propensity to purchase solar panels) as well as the effects of decoupling on risk allocations between electricity consumers and producers when there is uncertainty about electricity generation from renewable energy sources.

B.3 Distributional Impacts of Decoupling

We evaluate the distributional impacts of changes in q (or subsidy if that is what underlies the change in $\bar{q}$).

Proposition 5 *Under RD, a decrease in the cost of solar panels (due to technological improvement or government subsidy) is welfare-improving to those consumers who install solar panels, and welfare-reducing to those who did not install solar panels.*

Proof. For those without solar panels, we have

$$\frac{du_i}{dq} = \frac{d}{dq}\{v_i(x_i) - m_i - px_i\} = -\frac{dp}{dq}x_i > 0,$$

when demand is inelastic. (The equality follows from the envelope theorem.)

For those with solar panels, we have

$$\frac{du_i}{dq} = \frac{d}{dq}\{v_i(x_i + g_i) - m_i - px_i - q\} = -\frac{dp}{dq}x_i - 1$$

Note that $\frac{dp}{dq} = \frac{-p\frac{\partial x}{\partial q}}{x(1-|\eta_x)|}$.
Therefore,

$$\begin{aligned}\frac{du_i}{dq} &= \frac{-p\frac{\partial x}{\partial q} - 1 + |\eta_x|}{(1 - |\eta_x)|} \\ &< 0 \text{ if } |\eta_x| < 1.\end{aligned}$$

Precise welfare expressions would include the share of profits of the utility for each consumer. Because the profit is increasing in a (drop in) $\bar{q}$ and in the subsidy under RD, this consideration tends to increase the welfare impacts on those with solar panels, and may alleviate the negative welfare impacts on those without solar panels. ■

B.4 Decoupling under uncertainty

Here we provide an extensions of the model to incorporate uncertainty associated with distributed generation.

Here we consider uncertainty regarding output from solar panels in order to examine how the associated risk is shared between consumers and the utility under the alternative regulation. Given installation n, suppose the output from distributed generation is given by

$$x^d = \theta h(n),$$

where θ is a random variable with a set of nonnegative realizations $\{\theta_s\}$, $s \in S$, such that $E\theta = \bar{\theta}$. The household chooses n before uncertainty is realized and chooses how much electricity to buy from the utility upon realization of uncertainty, i.e., it chooses a state-contingent electricity consumption plan.

The household's problem is

$$\max_{\{x_s\}_{s\in S},n} E[u(e,y)]$$

subject to

$$e_s = x_s + \theta_s h(n), \quad x_s \geq 0, \quad p_s x_s + qn + y_s \leq M \quad \text{for each } s \in S.$$

The objective function in this case is

$$E[v(x+\theta h(n)) - px] + M - qn.$$

The first order conditions for an interior solution are

$$v'(x_s + \theta_s h(n)) = p_s \quad \text{for all } s \in S, \tag{17}$$
$$E[v'(x+\theta h(n))\theta]h'(n) = q. \tag{18}$$

Proposition 6 *Without revenue decoupling, any increase in the variance of θ will not change the utility's equilibrium expected profits.*

Proof. The utility's expected profit under uncertainty without RD can be expressed as:

$$\begin{aligned} E[\pi] &= E[\bar{p}x - \bar{c}x] \\ &= (\bar{p} - \bar{c})E[x]. \end{aligned} \tag{19}$$

Without RD, The electricity price is fixed irrespective of the realization of uncertainty. Note that under this regulatory scheme, consumer demand satisfies $v'(e_s^*) = \bar{p}$ for all s, i.e., $e_s^* = e^*$ for all s. This implies that:

$$e^* = x_s + \theta_s h(n), \quad \forall s \in S. \tag{20}$$

Note further that $E[\theta h(n)] = \bar{\theta} h(n)$ because $E[\theta_s] = \bar{\theta}$. Therefore,

$$\begin{aligned} E[\pi] &= E[(\bar{p} - c)(e^* - \theta h(n))] = (\bar{p} - \bar{c})[e^* - E(\theta)h(n)] \\ &= (\bar{p} - c)[e^* - \bar{\theta}h(n)], \end{aligned} \tag{21}$$

which is independent of the variance of θ. ■

To evaluate the effect of uncertainty under revenue decoupling, we assume that (with slight abuse of notation) $S = \{1,2\}$, $\theta_1 = \theta + \varepsilon$, $\theta_2 = \theta - \varepsilon$, with $p_1 = p_2 = 1/2$, where $\varepsilon \in (0, \theta)$.

Proposition 7 *With revenue decoupling in place, an increase in the variance of θ will result in an increase in the expected profits of the utility.*

Proof. With RD, the utility's expected profit is now expressed as:

$$E[\pi] = E[\bar{R} - \bar{c}x_s] \tag{22}$$

If we take the derivative of (22) with respect to ε, we get

$$\frac{dE[\pi]}{d\varepsilon} = -\bar{c}E[\frac{dx}{d\varepsilon}]. \tag{23}$$

To evaluate $\frac{dx}{d\varepsilon}$, take the derivative of the consumer's expected utility with respect to ε:

$$\frac{dE[U]}{d\varepsilon} = E\left[v'(X_s)\left\{\frac{dx_s^u}{d\varepsilon} + \frac{d\theta_s}{d\varepsilon}h(n)\right\}\right] = E\left[\frac{R}{x_s}\left\{\frac{dx_s^u}{d\varepsilon} + \frac{d\theta_s}{d\varepsilon}h(n)\right\}\right] \tag{24}$$

Total differentiation of the first-order condition for the consumer's utility maximization, $v'(x_s + \theta_s h(n)) = \frac{R}{x_s}$ for $s = 1, 2$, yields

$$\left(v'' + \frac{R}{(x_1)^2}\right)dx_1 + v''(\theta - \varepsilon)h'(n)dn = v''h(n)d\varepsilon \tag{25}$$

$$\left(v'' + \frac{R}{(x_2)^2}\right)dx_2 + v''(\theta + \varepsilon)h'(n)dn = -v''h(n)d\varepsilon \tag{26}$$

Utility maximization also implies $E[v'\theta_s]h'(n) = q$. Thus

$$\sum_s \pi_s[v'(x_s + \theta_s h(n))\theta_s] = \frac{q}{h'(n)} \tag{27}$$

Totally differentiating the above conditions and manipulating terms, we obtain

$$\begin{aligned}\frac{1}{2}\left[v''(\theta - \varepsilon)dx_1 + v''(\theta + \varepsilon)dx_2\right] + \left[v''h(n)[(\theta^2 + \varepsilon^2) + \frac{q}{h'(n)^2}h''(n)\right]dn \\ = \left[-v''h(n)\varepsilon + \frac{1}{2}[v_1' - v_2']\right]d\varepsilon\end{aligned} \tag{28}$$

where $v_1' \equiv v'(e_1)$, $v_2' \equiv v'(e_2)$. Solving for $\frac{dE[U]}{d\varepsilon}$ will entail solving (25), (26), and (28) in a system of equations. Re-writing the problem into a matrix form will yield the following:

$$\begin{pmatrix} v'' + \frac{R}{(x_1)^2} & 0 & v''(\theta - \varepsilon)h'(n) \\ 0 & v'' + \frac{R}{(x_2)^2} & v''(\theta + \varepsilon)h'(n) \\ \frac{1}{2}v''(\theta - \varepsilon) & \frac{1}{2}v''(\theta + \varepsilon) & v''h'(n)(\theta^2 + \varepsilon^2) + \frac{q}{h'(n)^2}h''(n) \end{pmatrix} \begin{pmatrix} \frac{dx_1}{d\varepsilon} \\ \frac{dx_2}{d\varepsilon} \\ \frac{dn}{d\varepsilon} \end{pmatrix} = \begin{pmatrix} v''h(n) \\ -v''h(n) \\ -v''h(n) + \frac{1}{2}[v_1' - v_2']\varepsilon \end{pmatrix}$$

Let D_A be the determinant of the coefficient matrix and D_{xi} be the determinant formed by replacing the ith column of the matrix on the left-hand side with the vector on the left-hand side. Applying Cramer's Rule, we can compute for $\frac{dx_1}{d\varepsilon}$ by:

$$\frac{dx_1}{d\varepsilon} = \frac{D_{x1}}{D_A} \tag{29}$$

Where:

$$D_{x1} = \det\begin{bmatrix} v''h(n) & 0 & v''(\theta - \varepsilon)h'(n) \\ -v''h(n) & v'' + \frac{R}{(x_2)^2} & v''(\theta + \varepsilon)h'(n) \\ -v''h(n)\varepsilon + \frac{1}{2}[v_1' - v_2'] & \frac{1}{2}v''(\theta + \varepsilon) & v''h'(n)(\theta^2 + \varepsilon^2) + \frac{q}{h'(n)^2}h''(n) \end{bmatrix}, \tag{30}$$

$$D_A = \det\begin{bmatrix} v'' + \frac{R}{(x_1)^2} & 0 & v''(\theta - \varepsilon)h'(n) \\ 0 & v'' + \frac{R}{(x_2)^2} & v''(\theta + \varepsilon)h'(n) \\ \frac{1}{2}v''(\theta - \varepsilon) & \frac{1}{2}v''(\theta + \varepsilon) & v''h'(n)(\theta^2 + \varepsilon^2) + \frac{q}{h'(n)^2}h''(n) \end{bmatrix}. \tag{31}$$

To show that $E\left[\frac{dx_s}{d\varepsilon}\right] < 0$ when $D_A < 0$, we note that:

$$\begin{aligned} E\left[\frac{dx_s}{d\varepsilon}\right] = &\frac{1}{2D_A}\left[\frac{R}{(x_2)^2}(v'')^2h'h\theta(\theta + \varepsilon) + \left(v'' + \frac{R}{(x_2)^2}\right)v''hq\frac{h''}{h'^2}\right] \\ &+ \frac{1}{2D_A}\left[-\frac{1}{2}(v_1' - v_2')(v'' + \frac{R}{(x_2)^2})(v''(\theta - \varepsilon)h')\right] \\ &+ \frac{1}{2D_A}\left[-\frac{R}{(x_1)^2}(v'')^2h'h\theta(\theta - \varepsilon) - \left(v'' + \frac{R}{(x_1)^2}\right)v''hq\frac{h''}{h'^2}\right] \\ &+ \frac{1}{2D_A}\left[-\frac{1}{2}(v_1' - v_2')(v'' + \frac{R}{(x_1)^2})(v''(\theta + \varepsilon)h')\right]. \end{aligned} \tag{32}$$

Note that terms in the square brackets can be expressed as:

$$= \left[\frac{p_2}{x_2}(\theta+\varepsilon) - \frac{p_1}{x_1}(\theta-\varepsilon)\right] v'' h' h \theta \tag{33}$$

$$+ \left[\frac{p_2}{x_2} - \frac{p_1}{x_1}\right] v'' h q \frac{h''}{h'^2} \tag{34}$$

$$+ \left[\left(v'' + \frac{R}{(x_2)^2}\right)(\theta-\varepsilon) + \left(v'' + \frac{R}{(x_1)^2}\right)(\theta+\varepsilon)\right]\left[-\frac{1}{2}(v_1' - v_2')v'' h'\right]. \tag{35}$$

Here (33) is positive since $\frac{p_2}{x_2} > \frac{p_1}{x_1}$ (note that $p_1 < p_2$ and $x_1 > x_2^U$) and $(\theta+\varepsilon) > (\theta-\varepsilon)$. For the same reason, (34) is positive. For (35), we assume that $(v'' + \frac{R}{x_s}) < 0$ which makes the sum of the terms in the first bracket to be negative. Since $p_1 < p_2$, the term outside the bracket is negative. This makes the whole expression negative. Overall, $E\left[\frac{dx_s}{d\varepsilon}\right] < 0$ when $D_A < 0$. Therefore, $E[\pi] > 0$ when $D_A < 0$. ■

Proposition 8 *With no RD, an increase in the variance of θ (i.e., having a mean-preserving spread of θ) does not change the household's equilibrium expected utility.*

Proof. Under traditional regulation, we have $p_s = \bar{p}$ for all $s \in S$: between rate cases, the electricity price is fixed irrespective of the realization of uncertainty. In this case, we have

$$e_s = e_{s'} = e^* \text{for all } s, s' \in S,$$

where, e^* solves $v'(e^*) = \bar{p}$, and $\bar{p}\bar{\theta}h'(n) = q$; i.e. $h'(n) = \frac{q}{\bar{p}\bar{\theta}}$, where $\bar{\theta} \equiv E[\theta]$. In this case, the household's utility satisfies

$$E[v(e^*) + M - \bar{p}\{e^* - \theta h(n^*)\}] - qn^* = v(e^*) + M - \bar{p}[e^* - \bar{\theta}h(n*)] - qn^*.$$

Note that e^* and n^* are independent of the variance of θ. Hence, a change in the variance of θ_s has no effect on the household's equilibrium expected utility.[13] ■

Proposition 9 *Under RD, an increase in the variance of θ (or, equivalently, an increase in ε) reduces the expected utility of consumers.*

Proof. Evaluate D_A as defined in the previous proof:

$$\begin{aligned} D_A = &\left(v'' + \frac{R}{(x_1)^2}\right)\left(v'' + \frac{R}{(x_2)^2}\right)\left(v'' h'(n)(\theta^2 + \varepsilon^2) + \frac{q}{h'(n)^2} h''(n)\right) \\ &- \left(\frac{1}{2}v''(\theta-\varepsilon)\right)\left(v'' + \frac{R}{(x_2)^2}\right)\left(v''(\theta-\varepsilon)h'(n)\right) \\ &- \left(\frac{1}{2}v''(\theta+\varepsilon)\right)\left(v''(\theta+\varepsilon)h'(n)\right)\left(v'' + \frac{R}{(x_1^2)^2}\right). \end{aligned} \tag{36}$$

Note that D_A can be simplified:

$$\begin{aligned} D_A \quad = \quad & 0.5\left(v'' + \frac{}{(x_1)^2}\right)\left[\frac{}{(x_2)^2} v'' h'(\theta+\varepsilon)^2 + \left(v'' + \frac{}{(x_2)^2}\right) v'(X_2)(\theta+\varepsilon)h''/h'\right] \\ + \quad & 0.5\left(v'' + \frac{}{(x_2)^2}\right)\left[\frac{}{(x_1)^2} v'' h'(\theta-\varepsilon)^2 + \left(v'' + \frac{}{(x_1)^2}\right) v'(X_1)(\theta-\varepsilon)h''/h'\right]. \end{aligned} \tag{37}$$

Note that $(x_s)(v'' + \frac{R}{(x_s)^2}) = p_s' x_s + p_s = MR_s$. When demand is inelastic, MR is negative because to sell a marginal (infinitesimal) unit the firm would have to lower the selling price so much that it would lose more revenue on the pre-existing units than it would gain on the incremental unit. Thus, under inelastic demand, $v'' + \frac{R}{(x_1)^2} < 0$ (because $x_1 > 0$).

[13] This result is due to the quasilinearity assumption on the utility function, i.e., no income effects. If the household's utility depends nonlinearly on y, then an increase in the variance of θ may impact the household's utility.

For D_{x1}:

$$D_{x1} = \frac{R}{(x_2)^2}(v'')^2 hh'\theta(\theta+\varepsilon) + \left(v'' + \frac{}{(x_2)^2}\right) v''hq\frac{h''}{h'^2} - \left(\frac{1}{2}(v'_1 - v'_2)\right)\left(v'' + \frac{R}{(x_2)^2}\right)(v''(\theta-\varepsilon)h') \tag{38}$$

Applying the same method above, we can compute for $\frac{dx_2}{d\varepsilon}$ by applying $\frac{dx_2}{d\varepsilon} = \frac{D_{x2}}{D_A}$, where

$$D_{x2} = \det\begin{bmatrix} v'' + \frac{R}{(x_1)^2} & v''h(n) & v''(\theta-\varepsilon)h'(n) \\ 0 & -v''h(n) & v''(\theta+\varepsilon)h'(n) \\ \frac{1}{2}v''(\theta-\varepsilon) & -v''h(n)\varepsilon + \frac{1}{2}[V'_1 - V'_2] & v''h'(n)(\theta^2+\varepsilon^2) + \frac{q}{h'(n)^2}h''(n) \end{bmatrix} \tag{39}$$

As for D_{x2}, we have

$$D_{x2} = -\frac{R}{(x_1)^2}(v'')^2 hh'\theta(\theta-\varepsilon) - \left(v'' + \frac{R}{(x_1)^2}\right) v''hq\frac{h''}{h'^2} - \left(\frac{1}{2}(v'_1 - v'_2)\right)\left(v'' + \frac{R}{(x_1)^2}\right)(v''(\theta+\varepsilon)h'). \tag{40}$$

Now evaluate $\frac{dEU}{d\varepsilon}$:

$$\begin{aligned}\frac{dEU}{d\varepsilon} &= E\left[\frac{R}{x_s}\left(\frac{dx_s^u}{d\varepsilon} + \frac{d\theta_s}{d\varepsilon}h(n)\right)\right] \\ &= E\left[\frac{R}{x_s}\frac{dx_s^u}{d\varepsilon}\right] + E\left[v'(X_s)\frac{d\theta_s}{d\varepsilon}h(n)\right],\end{aligned} \tag{41}$$

where $\frac{dx_s^u}{d\varepsilon} = \frac{D_{xs}}{D_A}$.

We first evaluate the $E\left[\frac{R}{x_s}\frac{dx_s^u}{d\varepsilon}\right]$ by substituting (38) and (40) into $\frac{dx_s^u}{d\varepsilon}$:

$$\begin{aligned}E\left[\frac{R}{x_s}\frac{dx_s^u}{d\varepsilon}\right] = &\frac{}{2D_A x_1^u x_2^u}\left[p_2(v'')^2 h'h\theta(\theta+\varepsilon) + x_2\left(v'' + \frac{}{(x_2)^2}\right)\left(v''hq\frac{h''}{h'^2}\right)\right] \\ &+ \frac{}{2D_A x_1^u x_2^u}\left[-\frac{1}{2}(v'_1 - v'_2)x_2\left(v'' + \frac{}{(x_2)^2}\right)(v''(\theta-\varepsilon)h')\right] \\ &+ \frac{}{2D_A x_1^u x_2^u}\left[-p_1(v'')^2 h'h\theta(\theta-\varepsilon) - x_1\left(v'' + \frac{}{(x_1)^2}\right)\left(v''hq\frac{h''}{h'^2}\right)\right] \\ &+ \frac{}{2D_A x_1^u x_2^u}\left[-\frac{1}{2}(v'_1 - v'_2)x_1\left(v'' + \frac{}{(x_1)^2}\right)(v''(\theta+\varepsilon)h')\right],\end{aligned} \tag{42}$$

where the expressions inside the square brackets are all equal to

$$[p_2(\theta+\varepsilon) - p_1(\theta-\varepsilon)](v'')^2 h'h\theta \tag{43}$$

$$+\left[x_2^u\left(v'' + \frac{}{(x_2)^2}\right) - x_1^u\left(v'' + \frac{}{(x_1)^2}\right)\right] v''hq\frac{h''}{h'^2} \tag{44}$$

$$+\left[x_2^u\left(v'' + \frac{}{(x_2)^2}\right)(\theta-\varepsilon) + x_1^u\left(v'' + \frac{}{(x_1)^2}\right)(\theta+\varepsilon)\right]\left(-\frac{1}{2}\right)(v'_1 - v'_2)v''h'. \tag{45}$$

We will show that the terms (43) - (45) are all positive. Given $n > 0$, we have $x_1^u > x_2^u$ and $p_1 < p_2$. The first order condition for x_s satisfies

$$v'(x_s + \theta_s h) = p_s = R/x_s.$$

Totally differentiate both sides with respect to x_s and θ_s:

$$v''dx_s + v''hd\theta_s = -Rx_s^{-2}dx_s, \quad \text{i.e.,} \quad \frac{\partial x_s}{\partial \theta_s} = \frac{-v''h}{v'' + \frac{R}{x_s^2}}.$$

The last expression is negative when $v'' + \frac{R}{x_s^2} < 0$. Because $\theta_1 = \theta - \varepsilon < \theta + \varepsilon = \theta_2$, we have $x_1^u > x_2^u$ and $p_1 < p_2$.

The term (43) is positive because $p_1 < p_2$ and $\theta - \varepsilon < \theta + \varepsilon$ while term (44) implies $[v''(x_2 - x_1) + (p_2 - p_1)]v''hq\frac{h''}{h'^2} > 0$. Term (45) is positive when $v'' + \frac{R}{x_s^2} < 0$. Therefore, $\frac{dx_s}{d\varepsilon} < 0$ if $D_A < 0$.

Next, we can evaluate the last term of equation (41).

$$E[v'\frac{d\theta_s}{d\varepsilon}h] = \frac{1}{2}[v_1'(-h) - v_2'(h)]$$
$$= \frac{1}{2}[p_2 - p_1]h > 0. \tag{46}$$

Therefore, we need to evaluate the sum of the two terms in (41).

$$\frac{dEU}{d\varepsilon} = E[v'\frac{dx_s}{d\varepsilon}] + E[v'\frac{d\theta_s}{d\varepsilon}h] = E[v'\frac{dx_s}{d\varepsilon}] + D_A(x_1 - x_2)h. \tag{47}$$

We also have

$$D_A(x_1 - x_2)h$$
$$= \frac{1}{2}\left[\left(v'' + \frac{R}{(x_1^U)^2}\right)\frac{R}{(x_2^U)^2}v''h'(\theta+\varepsilon)^2 + \left(v'' + \frac{R}{(x_2^U)^2}\right)\frac{R}{(x_1^U)^2}v''h'(\theta-\varepsilon)^2\right](x_1 - x_2)h \tag{48}$$
$$-\frac{1}{2}x_2\left(v'' + \frac{R}{(x_1^U)^2}\right)\left(v'' + \frac{R}{(x_2^U)^2}\right)hq\frac{h''}{h'^2} + \frac{1}{2}x_1\left(v'' + \frac{R}{(x_1^U)^2}\right)\left(v'' + \frac{R}{(x_2^U)^2}\right)hq\frac{h''}{h'^2}. \tag{49}$$

We can verify that (48) is positive. If we sum up (44) and (49), we have:

$$\text{Eqs. }(44) + (49) = x_2\left(v'' + \frac{R}{(x_2^U)^2}\right)hq\frac{h''}{h'^2}\left[v'' - \frac{1}{2}\left(v'' + \frac{R}{(x_1^U)^2}\right)\right]$$
$$- x_1\left(v'' + \frac{R}{(x_1^U)^2}\right)hq\frac{h''}{h'^2}\left[v'' - \frac{1}{2}\left(v'' + \frac{R}{(x_2)^2}\right)\right]$$
$$= \frac{1}{2}hq\frac{h''}{h'^2}\left[(v'')^2x_2 + v''\frac{R}{x_2} - v''\frac{R}{x_1} + \frac{R}{x_1x_2}\right]$$
$$- \frac{1}{2}hq\frac{h''}{h'^2}\left[(v'')^2x_1 + v''\frac{R}{x_1} - v''\frac{R}{x_1} + \frac{R}{x_1x_2}\right]$$
$$= \frac{1}{2}hq\frac{h''}{h'^2}\left[(v'')^2(x_2 - x_1) + v''(p_2 - p_1)\right]$$
$$> 0. \tag{50}$$

It follows from $\theta_1 = \theta - \varepsilon < \theta + \varepsilon = \theta_2$ that $x_1 > x_2 \rightarrow p_1 < p_2$. Therefore, we conclude that $\frac{dEU}{d\varepsilon} < 0$ if $D_A < 0$.

To show $D_A < 0$, we totally differentiate the FOCs with respect to x_1^u, x_2^u, n, p_1, and divide both sides by dp_1:

$$\begin{pmatrix} v'' & 0 & v''(\theta-\varepsilon)h'(n) \\ 0 & v'' & v''(\theta+\varepsilon)h'(n) \\ \frac{1}{2}v''(\theta-\varepsilon) & \frac{1}{2}v''(\theta+\varepsilon) & v''h'(n)(\theta^2+\varepsilon^2) + q\frac{h''(n)}{h'(n)^2} \end{pmatrix}\begin{pmatrix} \frac{dx_1}{dp_1} \\ \frac{dx_2}{dp_1} \\ \frac{dn}{dp_1} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

Let $\frac{dx_1^u}{d_p1} = \frac{Dx_1}{Du}$, where

$$Dx_1 = det\begin{bmatrix} 1 & 0 & v''(\theta-\varepsilon)h'(n) \\ 0 & v'' & v''(\theta+\varepsilon)h'(n) \\ 0 & \frac{1}{2}v''(\theta+\varepsilon) & v''h'(n)(\theta^2+\varepsilon^2) + q\frac{h''(n)}{h'(n)^2} \end{bmatrix}, Du = det\begin{bmatrix} v'' & 0 & v''(\theta-\varepsilon)h'(n) \\ 0 & v'' & v''(\theta+\varepsilon)h'(n) \\ \frac{1}{2}v''(\theta-\varepsilon) & \frac{1}{2}v''(\theta+\varepsilon) & v''h'(n)(\theta^2+\varepsilon^2) + q\frac{h''(n)}{h'(n)^2} \end{bmatrix}.$$

Solving for Dx_1 yields:

$$Dx_1 = v''\left[v''h'(\theta^2+\varepsilon^2) + q\frac{h''}{(h')^2}\right] - \frac{1}{2}(v'')^2(\theta+\varepsilon)^2h'. \tag{51}$$

As for Du, we have

$$Du = (v'')^2 \left[v''h'(\theta^2+\varepsilon^2) + q\frac{h''}{(h')^2} \right] - \frac{1}{2}(v'')^3(\theta-\varepsilon)^2 h' - \frac{1}{2}(v'')^3(\theta+\varepsilon)^2 h' = (v'')^2 q \frac{h''}{(h')^2}.$$

Thus, we can express $\frac{dx_1^u}{dp_1}$ as:

$$\frac{dx_1^u}{dp_1} = \frac{v'' \left[v''h'(\theta^2+\varepsilon^2) + q\frac{h''}{(h')^2} \right] - \frac{1}{2}(v'')^2(\theta+\varepsilon)^2 h'}{(v'')^2 q \frac{h''}{(h')^2}} = \frac{\frac{1}{2}h'(\theta-\varepsilon)^2}{q\frac{h''}{(h')^2}} + \frac{1}{v''}. \tag{52}$$

Assuming inelastic demand (the empirically relevant case), we know that $\frac{dx_1^u}{dp_1}\frac{p_1}{x_1^u} < 1$. This implies that;

$$\frac{dx_1^u}{dp_1}\frac{p_1}{x_1^u} = \frac{p_1}{x_1^u}\left[\frac{\frac{1}{2}h'(\theta-\varepsilon)^2}{q\frac{h''}{(h')^2}} + \frac{1}{v''} \right] > -1 \Leftrightarrow \frac{p_1}{x_1^u}\left[\frac{v''\frac{1}{2}h'(\theta-\varepsilon)^2 + q\frac{h''}{(h')^2}}{v''x_1^u q\frac{h''}{(h')^2}} \right] > -1. \tag{53}$$

Because $v''x_1^u q\frac{h''}{(h')^2} > 0$, it follows from (53) that

$$p_1 v'' \frac{1}{2}h'(\theta-\varepsilon)^2 + p_1 q\frac{h''}{(h')^2} > -v''x_1^u q\frac{h''}{(h')^2}. \tag{54}$$

We divide both sides by x_1^u, while noting that $p_1 = v'(X_1)$, to obtain

$$\begin{aligned} -\frac{dx_1^u}{dp_1}\frac{p_1}{x_1^u} < 1 &\Leftrightarrow \frac{v'(X_1)}{x_1^u}v''\frac{1}{2}h'(\theta-\varepsilon)^2 + \frac{v'(X_1)}{x_1^u}q\frac{h''}{(h')^2} + v''q\frac{h''}{(h')^2} > 0 \\ &\Leftrightarrow \frac{R}{(x_1^u)^2}v''\frac{1}{2}h'(\theta-\varepsilon)^2 + \left(\frac{R}{x_1^u} + v''\right) q\frac{h''}{(h')^2} > 0. \end{aligned} \tag{55}$$

Similarly, we can have

$$\begin{aligned} -\frac{dx_2^u}{dp_2}\frac{p_2}{x_2^u} < 1 &\Leftrightarrow \frac{v'(X_2)}{x_2^u}v''\frac{1}{2}h'(\theta+\varepsilon)^2 + \frac{v'(X_2)}{x_2^u}q\frac{h''}{(h')^2} + v''q\frac{h''}{(h')^2} > 0 \\ &\Leftrightarrow \frac{R}{(x_2^u)^2}v''\frac{1}{2}h'(\theta+\varepsilon)^2 + \left(\frac{R}{x_2^u} + v''\right) q\frac{h''}{(h')^2} > 0. \end{aligned} \tag{56}$$

Recall that:

$$\begin{aligned} D_A = &\left(v'' + \frac{}{(x_1)^2}\right)\left[0.5\frac{}{(x_2)^2}v''h'(\theta+\varepsilon)^2 + 0.5\left(v'' + \frac{}{(x_2)^2}\right) q\frac{h''}{(h')^2}\right] \\ &+ \left(v'' + \frac{}{(x_2)^2}\right)\left[0.5\frac{}{(x_1)^2}v''h'(\theta-\varepsilon)^2 + 0.5\left(v'' + \frac{}{(x_1)^2}\right) q\frac{h''}{(h')^2}\right]. \end{aligned} \tag{57}$$

If the demand for $x_s, s \in S$ is inelastic, then we have the following conditions:

$$\left(v'' + \frac{}{(x_s)^2}\right) < 0 \quad \text{for all } s \in S; \tag{58}$$

$$\left[0.5\frac{}{(x_1)^2}v''h'(\theta-\varepsilon)^2 + \left(v'' + \frac{}{(x_1)^2}\right) q\frac{h''}{(h')^2}\right] > 0; \tag{59}$$

$$\left[0.5\frac{}{(x_2)^2}v''h'(\theta+\varepsilon)^2 + \left(v'' + \frac{}{(x_2)^2}\right) q\frac{h''}{(h')^2}\right] > 0. \tag{60}$$

Taken together, the results in this subsection imply that the risk burden shifts from the utility to the consumers under revenue decoupling. ■