Consumption, Risk and Prioritarianism*

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Abstract

In this paper, we study consumption decisions under risk assuming a "prioritarian" social welfare function. This leads to a distinction between ex ante and ex post prioritarian consumption rules. Under standard assumptions, there is always *more* current consumption under ex ante prioritarianism than under utilitarianism. Thus, a concern for equity (in the ex ante prioritarian sense) means less concern for the risky future. In contrast, there is usually *less* current consumption under ex post prioritarianism than under utilitarianism. We discuss the robustness of these optimal consumption rules to learning, and to more general forms of prioritarianism.

Key words: Cake-eating, precautionary savings, utilitarianism, prioritarianism, discounting, climate change.
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1 Introduction

In this paper, we consider a standard precautionary savings model (Leland 1968, Sandmo 1970, Drèze and Modigliani 1972, Kimball 1990, Deaton 1992). This model is a common device in economics for exploring how optimal intertemporal resource allocation is affected by the interaction between risk and time. It has been used recently to think about climate discounting under growth risk (Gollier 2003, Weitzman 2009, Karp 2009, Millner 2013). The novelty in our precautionary savings model is that we consider a prioritarian social welfare function (SWF): one that takes the form $\sum_i g(u_i)$, with u_i the utility of agent i (a measure of her well-being), and $g(.)$ a strictly increasing and concave function.

The concept of "prioritarianism" originates in contemporary philosophy (Parfit 1991, Nagel 1995). The key idea is to give greater weight to well-being changes affecting worse-off agents. This idea is captured, axiomatically, in the Pigou-Dalton principle, otherwise known as the "principle of transfers": If agent j is better off than agent k, then a pure, non-rank switching transfer of well-being between them (so that j goes from u_j to $u_j - \Delta u$, k from u_k to $u_k + \Delta u$, with $\Delta u > 0, u_j - \Delta u \geq u_k + \Delta u$), with everyone else unaffected, is an ethical improvement.

The Pigou-Dalton principle is the critical axiomatic difference between prioritarianism and the much better known concept of "utilitarianism," captured by a simple additive social welfare function of the form $\sum_i u_i$. Both SWFs satisfy axioms of Pareto superiority, anonymity, separability, and continuity (Adler 2012);[1] but utilitarianism violates the Pigou-Dalton principle since a pure transfer of Δu leaves unchanged the sum total of well-being. In economics, prioritarian SWFs have been used in social choice and optimal taxation literatures (Sen 1970, Kaplow 2008) as well as in policy evaluation when "distributional weights" capture the nonlinearity in both the utility and the social welfare functions (Drèze and Stern 1987, Johansson-Stenman 2005, Adler 2013).

Intuitively, the difference between utilitarianism and prioritarianism is that the latter cares both about total well-being and about the distribution of well-being. Although utilitarianism does take account of the distribution

[1] Note that there also exists an axiomatic foundation to prioritarianism based on an extension of Harsanyi's utilitarian impartial observer theorem, and coined "generalized utilitarianism" (Grant et al. 2010). For a criticism of prioritarianism, see for instance Harsanyi (1975) and Broome (1991).

of income (given the declining marginal utility of income), it is insensitive to the distribution of well-being itself. And many have criticized utilitarianism on these grounds (Parfit 1991, Tungodden 2003, Holtug 2010, Porter 2012). Prioritarianism is a parsimonious and analytically tractable refinement to utilitarianism that takes account of equity concerns: the four common axioms mentioned above are left intact, but the axiom of Pigou-Dalton is added, and this simply reduces to summing a concave function of utilities.

The concept of prioritarianism is now well understood not only in philosophy, but in social choice theory and theoretical welfare economics. Indeed, the axiomatic characterization of prioritarianism comes from economic theory. However, the vast majority of work that uses the SWF construct implements a utilitarian SWF. For example, debates about climate policy have largely assumed a utilitarian SWF (Stern 2007, Dasgupta 2008, Nordhaus 2008, Weitzman 2008, Gollier 2012),[2] disagreeing about parameters of the utility function and the appropriateness of adding a time discount factor, but not about the basic utilitarian formula.

For real-world policy applications like climate change, it is important to consider (at least) the environment of risk—where the decisionmaker does not know for certain what distribution of well-being will result from the various policy choices available to him. In the context of normative risky choice, an important distinction is between ex ante and ex post prioritarianism. The ex ante prioritarian social planner applies the $g(.)$ function to individuals' expected utilities, and maximizes the sum of concavely transformed expected utilities; while the ex post prioritarian social planner applies the $g(.)$ function to individuals' final utilities, and maximizes the expected sum of concavely transformed final (realized) utilities. As a result, the ex post prioritarian social planner cares about the difference in realized utilities ex post, once the risk is resolved; while the ex ante prioritarian social planner cares about the difference in expected utilities ex ante, before the risk is resolved.

While the choice between the ex ante and ex post criteria under risk has been extensively discussed in social choice (Diamond 1967, Broome 1984, Fleurbaey 2010, Adler 2012, Fleurbaey and Bovens 2012), its policy implications have rarely been explored.[3] In a related paper (Adler and Treich 2015),

[2]Exceptions include Ha Duong and Treich (2004), Bommier and Zuber (2008), Roemer (2008), Traeger (2012), Dietz and Asheim (2012), Jensen and Traeger (2014), Fleurbaey and Zuber (2015) and Bommier, Lanz and Zuber (2015).

[3]Exceptions include Ulph (1982), Fleurbaey and Bovens (2012) and Adler, Hammitt and Treich (2014), all in the context of mortality risk policies.

we review some distinctive normative choices that are required for the prioritarian approach, including the specification of a ratio scale for well-being (if the prioritarian SWF takes the standard "Atkinson" form), and the determination of the degree of concavity of the $g(.)$ function (i.e., the degree of inequality aversion).

In this paper, we want to stress the importance and richness of the ex ante/ex post distinction for economic problems that combine risk and equity dimensions. To do so, we consider a basic precautionary savings model (see section 2), which has the simple following interpretation. A social planner must split a cake among agents who arrive sequentially (e.g., among several generations). Under certainty, the problem is trivial, and the cake is equally shared (because the agents are identical). But the problem is that the size of the cake is unknown. If the social planner is prioritarian rather than utilitarian, should he give more or less of the cake to the first agent, given that the remaining portion of the cake is unknown?

The main result of the paper is that the answer depends sensitively on whether the social planner uses an ex ante or an ex post prioritarian approach. The social planner should give *more* to the first agent under ex ante prioritarianism than under utilitarianism (see section 3), but *less* under ex post prioritarianism than under utilitarianism (see section 4). We show that this result is robust to a situation in which the social planner learns the size of the cake after the first decision has been made (see section 5). Finally, we discuss whether this result is robust to more general forms of prioritarianism (see section 6). In particular, we derive a simple condition so that there is always less consumption under "transformed" ex post prioritarianism than under utilitarianism. We also consider Fleurbaey (2010)'s equally distributed equivalent function, an important class of transformed ex post criteria, and exhibit a case where consumption is equal to that under utilitarianism in this setting.

Although the set-up for the precautionary savings model is quite simple, it is—we believe—a fruitful model for exploring the differences between ex ante and ex post prioritarianism. This model, as already mentioned, is a standard device to explore the interaction between risk and time. We now add the third dimension of equity, and consider how the optimal consumption allocations—from an ex ante prioritarian or ex post prioritarian perspective—are affected by the interaction of equity, risk and time. This three-way interaction depends, in subtle ways that we describe, on the functional form of both $u(.)$, the shape of the individual utility function, and $g(.)$, the shape of the SWF. We will assume throughout that the utility function $u(.)$ is the same for every agent—thereby avoiding the thorny question of how different well-being measures might be compared across agents (Arrow 1951, Dhillon and Mertens 1999). This assumption is prevalent in the applied literature using SWFs, and provides a natural starting point for rigorous analysis.

2 A simple precautionary savings model

It will be convenient in the rest of the paper to use the terminology of the precautionary savings literature. We consider two periods, and assume that the utility function u is identical across the two periods, with u strictly increasing, strictly concave and thrice differentiable. The main objective of our analysis is to compare levels of consumption in the first period across three different objectives, namely utilitarianism, ex ante and ex post prioritarianism. Under utilitarianism, the optimal consumption in the first period, denoted c^U, is defined by

$$c^U = \arg\max_c u(c) + Eu(\widetilde{w} - c), \tag{1}$$

where $\widetilde{w}$ is a random variable representing risk over the size of the "cake", namely the risk over lifetime wealth in the standard precautionary savings model. We denote $w_{\inf} > 0$ the smallest realization of $\widetilde{w}$.[4] Since utility is strictly increasing, we have directly introduced into the optimization program the fact that the cake will be fully consumed.

The first order condition (FOC) of this program gives[5]

$$u'(c^U) - Eu'(\widetilde{w} - c^U) = 0. \tag{2}$$

Observe that we assume that the utility function $u(.)$ is the same in both periods, and that there is no discounting. Thus, the only source of heterogeneity across the two periods comes from the risk over the size of the

[4] In this model, it is standard to make sure that the cake cannot be fully consumed before the final period, i.e. $c^U < w_{\inf}$. For instance, in a precautionary savings model, it is typically assumed that the constraint that the agent cannot borrow against more than the minimum value of lifetime wealth is never binding (Kimball 1990, Gollier 2001). Note that the assumption $u'(0) = +\infty$ is sufficient to ensure that this is always the case.

[5] Second order conditions will be satisfied throughout the paper, except when explicitly mentioned.

cake which makes second-period consumption risky. Assuming no discounting throughout will ensure that the utilitarian and prioritarian SWFs respect anonymity.

It is well known from the precautionary savings literature that current consumption is reduced under risk, i.e. $c^U \leq \frac{E\widetilde{w}}{2}$, if and only if (iff) the decision maker is "prudent" $u''' \geq 0$ (Leland 1968, Kimball 1990).[6] Indeed, under prudence, the marginal utility of wealth is higher under risk, and thus it makes sense to transfer more wealth into the future when the risk will be faced. Note that the restriction $u''' > 0$ is necessary for the common decreasing absolute risk aversion (DARA) hypothesis, and is usually accepted in the risk theory literature (Gollier 2001).

3 Ex ante prioritarianism

Under ex ante prioritarianism (EAP), optimal consumption is defined by

$$c^{EAP} = \arg\max_{c} g(u(c)) + g(Eu(\widetilde{w} - c)), \tag{3}$$

where g is strictly increasing, strictly concave and thrice differentiable. Note that the decision maker maximizes the sum of transformed expected utilities, consistent with an ex ante approach. The optimal level of consumption is characterized by the following FOC:

$$f(c^{EAP}) \equiv g'(u(c^{EAP}))u'(c^{EAP}) - g'(Eu(\widetilde{w} - c^{EAP}))Eu'(\widetilde{w} - c^{EAP}) = 0. \tag{4}$$

There is more consumption under EAP than under utilitarianism, i.e. $c^{EAP} \geq c^U$, or equivalently $f(c^U) \geq 0$, which by using (2) holds iff

$$u(c^U) \leq Eu(\widetilde{w} - c^U). \tag{5}$$

This leads to the following result. (All the Propositions in this paper should be understood as stating results which hold true for every level of wealth w. This is a key assumption for the necessary conditions stated in the Propositions.)

[6] Technically, the result holds iff $Eu'(\widetilde{w} - c) \geq u'(E\widetilde{w} - c)$ for all $\widetilde{w}$, namely iff marginal utility is convex by Jensen inequality.

Proposition 1 *There is more consumption under EAP than under utilitarianism iff u is DARA, i.e.*

$$\frac{u'''(w)}{-u''(w)} \geq \frac{-u''(w)}{u'(w)}.$$

Proof: Under DARA, $-u'$ is more risk averse than u. Namely, we have $u = \phi(-u')$ with ϕ strictly increasing and convex. This leads to

$$\begin{aligned} Eu(\widetilde{w} - c^U) &= E\phi(-u'(\widetilde{w} - c^U)) \\ &\geq \phi(-Eu'(\widetilde{w} - c^U)) \\ &= \phi(-u'(c^U)) \\ &= u(c^U), \end{aligned}$$

which proves the inequality (5) above. We now show the necessity. If u is not DARA, then ϕ is locally concave, and the above inequality can be reversed for a well chosen $\widetilde{w}$. Therefore, consumption under EAP can be lower than under utilitarianism. Q.E.D

The intuition for this result may be presented as follows. Assuming utilitarianism, under DARA (and thus under prudence) the reduction in current consumption due to risk implies that the future expected utility is greater than the current utility (see (5)). This in turn gives the ex ante prioritarian decision maker an incentive to increase first period consumption, thereby reducing the difference between current and future expected utility. Hence, this result shows that under a standard assumption on the utility function, prioritarianism leads to more, and not less, current consumption.[7] In other words, this result indicates that a concern for equity (in the EAP sense) means less concern for the risky future. Notice here that ex ante fairness is often viewed as socially desirable in the literature (Diamond 1967, Epstein and Segal 1991). Our model thus maybe illustrates a surprising implication of this view.

Note also that under constant absolute risk aversion (CARA), we have $u(c^U) = Eu(\widetilde{w} - c^U)$ leading to $c^U = c^{EAP}$ and thus to $u(c^{EAP}) = Eu(\widetilde{w} -$

[7] Since there is more consumption under EAP than under utilitarianism, one may wonder whether it is possible that there is more consumption under EAP than under certainty (under either utilitarianism or prioritarianism). It is straightforward to show that this is never the case under $u''' \geq 0$, and that there is thus always precautionary savings.

c^{EAP}). Namely, under the common CARA utility function, the (expected) utilities are equal across the two periods both under utilitarianism and under prioritarianism.

We finally add a comment about the scaling of the vNM utility function $u(.)$. Under expected utility, it is well known that the utility function is unique up to a positive affine transformation. In the standard precautionary savings model under utilitarianism (1), a change from $u(.)$ to some other $u^*(.) = au(.) + b$ with $a > 0$ does not change consumption. However, under prioritarianism, this is no longer true. For a given $g(.)$ in the SWF, consumption under EAP may differ after such an affine transformation. Yet, since DARA is preserved under any affine transformation, the result of Proposition 1 is also preserved. Namely, for any given $g(.)$, and for any given $u(.)$ that is DARA, first-period consumption under EAP is greater than under utilitarianism both under $u(.)$ and for every positive affine rescaling of $u(.)$.

4 Ex post prioritarianism

Under ex post prioritarianism (EPP), optimal consumption is defined by

$$c^{EPP} = \arg\max_c g(u(c)) + Eg(u(\widetilde{w} - c)). \tag{6}$$

Note that the decision maker now maximizes the expectation of transformed utilities, consistent with an ex post approach. The FOC is given by

$$k(c^{EPP}) \equiv g'(u(c^{EPP}))u'(c^{EPP}) - Eg'(u(\widetilde{w} - c^{EPP}))u'(\widetilde{w} - c^{EPP}) = 0. \tag{7}$$

There is less consumption under EPP than under EAP iff $k(c^{EAP}) \leq 0$. In the following Proposition, we derive a sufficient condition for this inequality.

Proposition 2 *There is less consumption under EPP than under EAP when $g''' \geq 0$.*

Proof: Let $\widetilde{w} - c^{EAP} \equiv \widetilde{z}$. Then observe that $k(c^{EAP}) \leq 0$ iff

$$g'(Eu(\widetilde{z}))Eu'(\widetilde{z}) \leq Eg'(u(\widetilde{z}))u'(\widetilde{z}).$$

Observe now that $Eg'(u(\widetilde{z}))u'(\widetilde{z}) = Eg'(u(\widetilde{z}))Eu'(\widetilde{z}) + Cov(g'(u(\widetilde{z})), u'(\widetilde{z}))$, and since $g'(u(z))$ and $u'(z)$ are both decreasing in z, the covariance term is positive. Therefore the result holds if $g'(Eu(\widetilde{z})) \leq Eg'(u(\widetilde{z}))$, which is the case iff $g''' \geq 0$ by Jensen inequality. Q.E.D.

Note that the result only requires a restriction of the third derivative of the function $g(.)$, with no restriction on $u(.)$. Thus, this result is not affected by the scaling of the utility function.

Is the restriction $g''' \geq 0$ plausible? At a minimum, this restriction is surely more plausible than the opposite $g''' < 0$. In fact, it can be shown that if $g'(u) > 0$ and $g''(u) < 0$ and if $g'''(u)$ has the same sign for all $u > 0$, then it must be that $g'''(u) > 0$ (Menegatti 2001). Indeed a positive, decreasing and concave g' would have to cross the origin at some point (thus contradicting $g' > 0$), as illustrated in Figure 1. Note that the standard Atkinsonian function, i.e. $g(u) = (1-m)^{-1}u^{1-m}$ with $u > 0$ and $m > 0$ (and $g(u) = \log u$ for $m = 1$), displays $g''' > 0$. Another standard prioritarian transformation function is the negative exponential function, i.e. $g(u) = -e^{-u}$, which also displays $g''' > 0$.

< INSERT FIGURE 1 >

Thus, under commonly used SWFs, there is less consumption under EPP than under EAP. The next objective is to examine whether there could be less consumption under EPP than under utilitarianism. We know the answer in the CARA case. We saw above that if u has the CARA form, consumption under EAP is equal to that under utilitarianism. Therefore Proposition 2 indicates that consumption under EPP is also lower than under utilitarianism under CARA when $g''' \geq 0$. But we would want to sign the comparison between EAP and utilitarianism in the general case. The answer is given in the following Proposition.

Proposition 3 *There is less consumption under EPP than under utilitarianism iff*

$$\frac{u'''(w)}{-u''(w)} \leq 3\frac{-u''(w)}{u'(w)} + \{\frac{g'''(u(w))}{-g''(u(w))}\}u'(w). \quad (8)$$

Proof: Let us define $v(.) = g(u(.))$. We want to examine under which conditions we have: $u'(c^U) - Eu'(\widetilde{w} - c^U) = 0$ implies $v'(c^U) - Ev'(\widetilde{w} - c^U) \leq 0$.

Now let $-v'(.) = \varphi(-u'(.))$ with φ strictly increasing and concave. Then

$$\begin{aligned} -Ev'(\widetilde{w} - c^U) &= E\varphi(-u'(\widetilde{w} - c^U)) \\ &\leq \varphi(-Eu'(\widetilde{w} - c^U)) \\ &= \varphi(-u'(c^U)) \\ &= -v'(c^U). \end{aligned}$$

Conversely, if φ is locally convex, then it is possible to find a well chosen $\widetilde{w}$ so that the inequality above is reversed. Therefore the necessary and sufficient condition is that $-v'$ is more concave than $-u'$, or $\frac{v'''}{-v''} \geq \frac{u'''}{-u''}$ given that v is itself more concave than u. This condition is provided in the theorem 3.4 in Eeckhoudt and Schlesinger (1994), which yields (8). Q.E.D

As shown in Table 1, our analysis so far has permitted the comparison of first-period consumption levels under utilitarianism, EAP and EPP. Note that the comparison is unambiguous under $g''' \geq 0$, except for the pair (c^U, c^{EPP}) which depends on a complex condition (8).

u CARA	u DARA
$c^{EAP} = c^U \geq c^{EPP}$ if $g''' \geq 0$	$c^{EAP} \geq c^U$, $c^{EAP} \geq c^{EPP}$ if $g''' \geq 0$, $c^U \geq c^{EPP}$ iff (8)

Table 1: Consumption levels under utilitarianism, EAP and EPP under different assumptions regarding $u(.)$ and $g(.)$.

Why is condition (8) so complex? Denoting $v(.) = g(u(.))$, the proof shows that the comparison between EPP and utilitarianism depends on how a change in preference from u to v affects precautionary savings. More precisely, it depends on whether more risk aversion, i.e. $\frac{-u''}{u'} \leq \frac{-v''}{v'}$, leads to more prudence, i.e. $\frac{u'''}{-u''} \leq \frac{v'''}{-v''}$. This last implication explains why condition (8) involves the third derivatives of both u and g as one needs to compute v'''. Although it sounds intuitive that a more risk averse agent should be more prudent, this is not always the case (Eeckhoudt and Schlesinger 1994). For instance, one may change the degree of risk aversion of a quadratic utility function without affecting the degree of prudence.[8]

[8]Moreover, here is an example where the condition (8) does not hold for some wealth levels. Take $g(u) = -e^{-u}$ and $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$, then the condition is violated iff wealth is below $\widehat{w} = (1-2\gamma)^{\frac{1}{1-\gamma}}$.

The last observation illustrates that comparing consumption under EPP and utilitarianism is formally similar to analyzing the effect of more risk aversion in a precautionary savings model. However, this comparison has little meaning in the single individual precautionary savings model. Indeed, it is well known that changing $g(.)$ in (6) is not a proper way to study the effect of individual risk preferences in that model (Bommier, Chassagnon and Le Grand 2012). Indeed, this change affects not only risk aversion, but also the elasticity of intertemporal substitution. In other words, this change affects ordinal preferences over certain prospects, and is not meaningful to capture a "pure" change in risk preferences in intertemporal expected utility models.[9]

We now investigate whether the condition (8) is satisfied for most commonly used utility functions and SWFs. We consider the prevalent Atkinsonian SWF, i.e. $g(u) = (1-m)^{-1}u^{1-m}$ with $u > 0$ and $m > 0$. In that case, the inequality (8) reduces to

$$\frac{u'''(w)}{-u''(w)} \leq 3\frac{-u''(w)}{u'(w)} + (1+m)\frac{u'(w)}{u(w)}. \tag{9}$$

Interestingly, this last inequality exhibits three different utility curvature coefficients, namely the familiar degrees of risk aversion and of prudence, as well as the reciprocal of the degree of fear of ruin $\frac{u}{u'}$ (Foncel and Treich 2005). Take for instance a constant relative risk aversion (CRRA) utility function $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$ with $\gamma \in (0, 1)$. Then the condition (8) is equivalent to $\frac{m(1-\gamma)+\gamma}{w} \geq 0$, and is always satisfied under our parametric assumptions.[10]

We next discuss whether the comparison in condition (8) is robust to a change in the scaling of the utility function $u(.)$. This cannot be true

[9]An early approach to examine the effect of risk aversion in intertemporal models is proposed by Khilstrom and Mirman (1974). They consider a model of the form $\max_c g^{-1}(Eg[u(c) + u(\widetilde{w} - c))])$. In this model, more risk aversion, through a more concave g, decreases first period consumption (for "small risks", see Drèze and Modigliani 1972, Bommier, Chassagnon and Le Grand 2012). A well known alternative is based on so-called "recursive" preferences (Selden 1978, Epstein and Zin 1989), leading to the objective: $\max_c u(c) + u(g^{-1}(Eg((\widetilde{w} - c)))$. A more concave $g(.)$ is interpreted as more risk aversion and reduces current consumption given some specific technical restrictions on $u(.)$ and $g(.)$ (Kimball and Weil 2009).

[10]Observe that under our parameterization we get $u(0) = 0$ so that $w = 0$ can be interpreted as a minimal subsistence level of wealth. This "zeroing out" assumption (Adler 2012) is not innocuous because the scaling of utility matters under prioritarianism.

generically. To see this, observe that the right hand side term of (8) depends directly on the function u through $\frac{g'''(u(w))}{-g''(u(w))}$. As a result, this side of the equation can be arbitrarily affected by an additive change from $u(.)$ to $u^*(.) = u(.) + b$ for some functions $g(.)$, thus possibly modifying the sign of the inequality depending on the value of b. Note however that under a negative exponential SWF, i.e. $g(u) = -e^{-u}$, the term $\frac{g'''(u(w))}{-g''(u(w))}$ becomes a constant. Hence, our comparative statics results are not affected by an additive change in this case. This is not a surprise since it is well known that the ranking of prospects is not affected by such additive re-scaling under exponential SWFs (Bossert and Weymark 2004, Adler 2012). Consider alternatively an Atkinsonian SWF. Then the comparative statics analysis would not be affected by a "ratio-rescaling" of the utility function, namely by a multiplicative change from $u(.)$ to $u^*(.) = au(.)$ with $a > 0$. To see this, observe that none of the curvature coefficients in (9) would be affected by such a multiplicative change. Again, this result is not surprising since the Atkinsonian function is known to be the only prioritarian SWF to display the ratio-rescaling invariance property (Bossert and Weymark 2004, Adler 2012).

Observe finally that (9) is more likely to be satisfied when the "inequity aversion" parameter m increases. At the limit when m tends to infinity, i.e. for a Rawlsian-type SWF, the inequality (8) is always satisfied. This observation provides an intuition for the result. Indeed, under EPP and a Rawlsian-type SWF, the decision maker's objective is to increase consumption in the worst state ex post (i.e., when $\widetilde{w} = w_{\inf}$), as soon as the utility reached in that state is not higher than current utility. He thus essentially chooses consumption such that $u(c) \approx u(w_{\inf} - c)$. This tends to yield less current consumption than under utilitarianism, given by $u'(c) = Eu'(\widetilde{w} - c)$, and to even less current consumption than under EAP (under a Rawlsian-type SWF), given by $u(c) \approx Eu(\widetilde{w} - c)$.

5 A simple model with learning

In this section, we consider a specific multi-periodic model, and we will allow for the possibility of learning. In a three-period model, the objective under utilitarianism becomes

$$\max_{c_1, c_2} u(c_1) + u(c_2) + Eu(\widetilde{w} - c_1 - c_2).$$

Note that perfect smoothing is optimal in the early periods $c_1 = c_2 = c$. The problem of finding optimal current consumption c then becomes

$$c^U = \arg\max_c 2u(c) + Eu(\widetilde{w} - 2c). \tag{10}$$

Similarly, optimal consumptions under EAP and EPP are defined by

$$\begin{aligned} c^{EAP} &= \arg\max_c 2g(u(c)) + g(Eu(\widetilde{w} - 2c)), && (11) \\ c^{EPP} &= \arg\max_c 2g(u(c)) + Eg(u(\widetilde{w} - 2c)). && (12) \end{aligned}$$

It is easy to see then the comparison of c^U, c^{EAP} and c^{EPP} leads to the same results as in the Propositions before. Considering more (than 3) periods would not affect these results provided that perfect smoothing remains optimal in the early periods. However, the situation becomes more complex if learning is allowed, as we now show.[11]

For simplicity, we assume perfect learning between periods 2 and 3. That is, in period 2, the realization of $\widetilde{w}$ is known. Then the period 2 problem is made under certainty, and perfect smoothing is optimal in the future either under prioritarianism or under utilitarianism. Hence, viewed from the first period, optimal future (risky) consumption equals

$$c_2^* = \frac{\widetilde{w} - c_1}{2}.$$

Using obvious notations, the optimal consumption in the first period for utilitarianism and EPP are then defined as follows (the EAP case is treated later)

$$\begin{aligned} c^{UL} &= \arg\max_c u(c) + 2Eu(\frac{\widetilde{w} - c}{2}), \\ c^{EPPL} &= \arg\max_c g(u(c)) + 2Eg(u(\frac{\widetilde{w} - c}{2})). \end{aligned}$$

Note that the effect of learning under utilitarianism, and under prioritarianism, is given by comparing c^{UL} to c^U in (10) and c^{EPPL} to c^{EPP} in (12). It is not very difficult to show that learning usually increases consumption under

[11] Learning is an important factor affecting risk policies in general, especially for long term problems. See for example the literature on climate change (Ulph and Ulph 1997 and Gollier, Jullien and Treich 2000).

utilitarianism and EPP compared to the no learning (i.e., "risk") case.[12] The intuition is that learning allows to better smooth consumption in the future, which thus increases future expected utility. As a result, there is more early consumption under learning because there is less need to worry about the future (Epstein 1980, Eeckhoudt, Gollier and Treich 2005).

Assuming learning, we now want to compare consumption under utilitarianism and EPP, i.e. c^{UL} and c^{EPPL}. This amounts to compare precautionary savings under $u(.)$ and $v(.) = g(u(.))$, and this comparison is direct from previous Proposition 3. Indeed we can show that under learning, there is less consumption under EPP than under utilitarianism iff (8) holds.

The case of EAP is more difficult. Indeed, viewed from the first period, future utility equals $u(\frac{\widetilde{w}-c}{2})$ and is risky, which matters under EAP. Optimal consumption is given by

$$c^{EAPL} = \arg\max_{c} g(u(c)) + 2g(Eu(\frac{\widetilde{w}-c}{2})). \tag{13}$$

The problem here is that the EAP criterion is time-inconsistent (Broome 1984, Adler and Sanchirico 2006). Technically, this relates to the fact that the intertemporal utility function in (13) is not linear in probabilities (Hammond 1983, Epstein and Le Breton 1992).[13] This means that, by contrast with utilitarianism or EPP, the optimization problem over the three periods cannot be formulated recursively under EAP. To see that, consider the second period problem. After the resolution of uncertainty, i.e. $\widetilde{w} = w$, the EAP decision maker evaluates future social welfare in period 2 and 3 by $2g(u(\frac{w-c}{2}))$. Therefore, before the resolution of uncertainty, this decision

[12]Let us first compare consumption under learning and under risk assuming utilitarianism. Under learning, the FOC is given by $u'(c) - Eu'(\frac{\widetilde{w}-c}{2}) = 0$, while under risk it is given by $u'(c) - Eu'(\widetilde{w} - 2c) = 0$. Thus there is more current consumption under learning iff $Eu'(\widetilde{w} - 2c) \geq Eu'(\frac{\widetilde{w}-c}{2})$ given that $u'(c) - Eu'(\widetilde{w} - 2c) = 0$. Observe now $Eu'(\widetilde{w} - 2c) = \frac{1}{2}u'(c) + \frac{1}{2}Eu'(\widetilde{w} - 2c) \geq Eu'(\frac{\widetilde{w}-c}{2})$ by Jensen inequality and $u''' \geq 0$. This leads to the result that under prudence learning increases early consumption under utilitarianism. Note then that the role of learning under EPP is similar by just replacing u by $v = g(u)$ in the previous reasoning. But then observe that $g''' \geq 0$ ensures $v''' \geq 0$ so the result also carries over under $u''' \geq 0$ and $g''' \geq 0$.

[13]Note that this nonlinearity might also imply a negative value of information (Wakker 1988). But we can show that this is not the case here. Indeed the utility reached under learning (see (13)) is always higher than the one reached under no learning (see (11)) iff $g(Eu(\frac{\widetilde{w}-c}{2})) \geq \frac{1}{2}(g(u(c)) + g(Eu(\widetilde{w} - 2c))$. This inequality always holds under g and u concave by simply applying twice the Jensen inequality.

maker would be time-consistent by averaging future values of social welfare across the possible states of the world, namely by considering $2Eg(u(\frac{\widetilde{w}-c}{2}))$. But this does not correspond to the ex ante objective of the EAP social planner under learning, as defined in (13).[14] This form of time-inconsistency can be seen as an important drawback of EAP approach. On the other hand, EAP respects the Pareto principle in terms of individuals' expected utilities, unlike EPP.

Observe now that comparing, under learning, consumption under utilitarianism and under (time-inconsistent) EAP is similar as this comparison under no learning. DARA is, again, the instrumental condition on the utility function that drives the analysis. A sketch of the proof of this result follows. We want to compare c^{EAPL} and c^{UL}. Respective FOCs equal $g'(u(c))u'(c)-g'(Eu(\frac{\widetilde{w}-c}{2}))Eu'(\frac{\widetilde{w}-c}{2})=0$ and $u'(c)-Eu'(\frac{\widetilde{w}-c}{2})=0$. Therefore we are done if we can show $u(c)\leq Eu(\frac{\widetilde{w}-c}{2})$ with $u'(c)=Eu'(\frac{\widetilde{w}-c}{2})$. By a similar reasoning as in the proof of Proposition 1, this holds iff DARA. We thus find that, under learning, there is more consumption under EAP than under utilitarianism iff $u(.)$ is DARA.

Overall, these results under perfect learning indicate there is less consumption under EPP, and more consumption under EAP, than under utilitarianism. To obtain these results, we need similar conditions as in the simple two-period model without learning. Thus, we conclude that the results obtained in previous sections are robust to the introduction of perfect learning.

6 Other prioritarian SWFs

In this section, we widen the analysis by considering two additional families of SWFs: "transformed utilitarianism" and "transformed EPP" (Adler,

[14]This observation is reminiscent of Myerson (1981)'s egalitarian-father example that the evaluation of social welfare may depend on the timing of the resolution of uncertainty. The example goes as follows. An egalitarian father has two children, who can become clerk, teacher or doctor depending wether they go to college for respectively 0, 4 or 8 years. The problem is that the father can afford only 8 years of college. Moreover, the father prefers that the two children have the same situation, whereas the children prefer a 50% chance of being clerk/doctor to being teacher for sure. Suppose that a fair coin decides who will go to the medical school. Before the coin toss, this randomization device Pareto-dominates the "both teachers" plan. Yet, after the coin toss, this device will seem unegalitarian and the father prefers the "both teachers" plan.

Hammitt and Treich 2014).[15] An important property is that the SWFs corresponding to these two cases are no longer separable in general.

We consider again the basic two-period model. Under transformed utilitarianism, optimal consumption in the first period is defined by

$$c^{TU} = \arg\max_c Eh[u(c) + u(\widetilde{w} - c)],$$

in which h is assumed to be increasing and twice differentiable. Note that if h is nonlinear, c^{TU} does not coincide in general with the optimal consumption under plain utilitarianism, namely with c^U.[16] In particular, we have $c^{TU} \leq c^U$ iff

$$E\{h'[u(c^U) + u(\widetilde{w} - c^U)](u'(c^U) - u'(\widetilde{w} - c^U))\} \leq 0, \tag{14}$$

or equivalently iff

$$Cov(h'[u(c^U) + u(\widetilde{w} - c^U)], u'(c^U) - u'(\widetilde{w} - c^U)) \leq 0,$$

since $E(u'(c^U) - u'(\widetilde{w} - c^U)) = 0$ by the definition of c^U. Notice that the term $(u'(c^U) - u'(w - c^U))$ is always increasing in w while the term $h'[u(c^U) + u(w - c^U)]$ is always decreasing in w iff h' is decreasing. Therefore the concavity of the transformation function h is a necessary and sufficient condition for transformed utilitarianism to reduce first-period consumption compared to plain utilitarianism, i.e. $c^{TU} \leq c^U$.

Under transformed EPP, optimal consumption in the first period is defined by

$$c^{TEPP} = \arg\max_c Eh[g(u(c)) + g(u(\widetilde{w} - c))].$$

Simply observe now that the function $v(.) = g(u(.))$ is also increasing and concave under our assumptions on g. As a result, we can straightforwardly use the above reasoning to conclude that the concavity of the transformation function h is also a necessary and sufficient condition for transformed EPP to reduce current consumption compared to plain EPP, i.e. $c^{TEPP} \leq c^{EPP}$. Moreover, we know from the analysis above that current consumption is lower under EPP than under utilitarianism iff the condition (8) is satisfied. We can therefore conclude that EPP, either under the plain version or under the transformed version with h concave, leads to less current consumption compared to utilitarianism under that same condition (8), i.e. $c^{TEPP} \leq c^U$.

[15] The "transformed EAP" case is equivalent to EAP.

[16] In the following, we assume that the second order condition is always satisfied. Note that this need not be the case if h is "sufficiently" convex.

The previous observation suggests however that the comparison between current consumption under utilitarianism and under transformed EPP is not clear when h is convex. A case in point is the prioritarian case of Fleurbaey (2010)'s equally distributed equivalent (EDE). Optimal consumption under EDE is defined by

$$c^{EDE} = \arg\max_{c} Eg^{-1}(\frac{1}{2}g(u(c)) + \frac{1}{2}g(u(\widetilde{w} - c))),$$

so that $h(x) = g^{-1}(x/2)$ is convex since g is concave. Following the above observations, we know that c^{EDE} is greater than c^{EPP}, which is also greater than c^{TEPP} with any h concave. However, we cannot use previous results to directly compare c^{EDE} to c^{U}. Under EDE, the FOC is given by

$$E\frac{g'(u(c))u'(c) - g'(u(\widetilde{w} - c))u'(\widetilde{w} - c)}{g'(g^{-1}(\frac{1}{2}g(u(c)) + \frac{1}{2}g(u(\widetilde{w} - c))))} = 0. \tag{15}$$

Using (15), we derive in the appendix the necessary and sufficient condition to compare c^{EDE} and c^{U} when $\widetilde{w}$ is "small" in the sense of a second order approximation. This condition takes the form of an inequality which is always equal to zero under Atkinsonian and CRRA utility functions. In other words, we have $c^{EDE} = c^{U}$ under this set of assumptions. This suggests that this special type of ex post approach introduced by Fleurbaey (2010) can be viewed as a limit case of "transformed EPP" leading to the same consumption level as under utilitarianism.

7 Conclusion

In this paper, we have examined a simple precautionary savings model with a prioritarian social welfare function. We have shown that, under standard assumptions on utility and social welfare functions, prioritarianism always leads to more current consumption under an ex ante approach, but to less current consumption under an ex post approach, than under utilitarianism. These standard assumptions include the familiar constant relative risk aversion utility and Atkinsonian social welfare functions.

Why is this result interesting? Many economic problems combine a risk, time and equity dimension. Consider the general idea that the risk of future climate change justifies less consumption of energy today. The traditional argument in the economics of discounting under utilitarianism relies on a precautionary savings motive. Our paper shows that this argument is reinforced by a moral prioritarian argument only under the ex post approach, but is weakened under the ex ante approach.

We conclude by presenting a few possible extensions. Our precautionary savings model is parsimonious, but too restrictive to capture some essential features of real-world policy applications like climate change. In particular, the model does not capture stock pollutant effects. Moreover, our model considers an additive risk. Yet, in many applications the future risk grows with the level of consumption today, and is thus multiplicative. Another research direction relates to our assumption that individual preferences are homogeneous: individuals have the same vNM utility function. Although controversial in the social choice literature, this assumption is prevalent in applied welfare economics. For instance, essentially all the literature on climate change that we are aware of assumes homogeneous utilities. Yet, it seems reasonable to allow for the possibility that the preferences of future generations may differ from ours, and it will be important to explore the impact of this difference on today's precautionary actions.

Appendix: The EDE case

In this appendix, we derive conditions so that consumption under Fleurbaey (2010)'s equally distributed equivalent (EDE) is lower than under utilitarianism. Formally, using the FOCs (2) and (15), we want to show

$$u'(c)-Eu'(\widetilde{w}-c)=0 \Longrightarrow E\frac{g'(u(c))u'(c)-g'(u(\widetilde{w}-c))u'(\widetilde{w}-c)}{g'(g^{-1}(\frac{1}{2}g(u(c))+\frac{1}{2}g(u(\widetilde{w}-c))))}\leq 0. \quad (16)$$

We use the diffidence theorem (Gollier 2001, page 86-87). More precisely, we use a Lemma which is directly based on a necessity part of the diffidence theorem when applied to "small risks" in the sense of a second-order approximation.

Lemma. *(Gollier 2001) Let the problem:*

$$\text{for all } \widetilde{w},\ Ef_1(\widetilde{w})=0 \Longrightarrow Ef_2(\widetilde{w})\leq 0. \quad (17)$$

Assume that there exists a scalar w_0 such that $f_1(w_0)=f_2(w_0)=0$ with $f_1'(w_0)\neq 0$. Then, a necessary and sufficient for (17) for any "small" $\widetilde{w}$ around w_0 is

$$f_2''(w_0)\leq \frac{f_2'(w_0)}{f_1'(w_0)}f_1''(w_0). \quad (18)$$

To study condition (16), we now simply apply this Lemma with $f_1(w) = u'(c) - u'(w-c)$ and

$$f_2(w) = \frac{g'(u(c))u'(c) - g'(u(w-c))u'(w-c)}{g'(g^{-1}(\frac{1}{2}g(u(c)) + \frac{1}{2}g(u(w-c))))}.$$

Observe that under $w_0 = 2c$, we have $f_1(w_0) = f_2(w_0) = 0$. We then easily obtain $f_1'(2c) = -u''(c) \neq 0$ and $f_1''(2c) = -u'''(c)$. Moreover, we can compute

$$f_2'(2c) = -\frac{u'(c)^2 g''(u(c))}{g'(u(c))} - u''(c) > 0,$$

and

$$\begin{aligned} f_2''(2c) &= g'(u(c))^{-2}[-2g'(u(c))u'(c)g''(u(c))u''(c) + u'(c)^3(g''(u(c))^2 \\ &\quad -g'(u(c))g'''(u(c))) - g'(u(c))^2 u'''(c)]. \end{aligned}$$

Assuming $g(u) = (1-m)^{-1}u^{1-m}$ with $u > 0$ and $m > 0$, we obtain

$$\frac{f_2''(2c)}{f_2'(2c)} - \frac{f_1''(2c)}{f_1'(2c)} = \frac{mu'(c)^4}{u(c)u''(c)(-mu'(c)^2 + u(c)u''(c))}[\frac{u''(c)}{u'(c)} - 2\frac{u(c)}{u'(c)}\frac{u''(c)^2}{u'(c)^2} + \frac{u(c)}{u'(c)}\frac{u'''(c)}{u'(c)}].$$

Note that the sign of this last expression only depends on the utility function $u(.)$ through the term into brackets. This term can be respectively positive or negative for some utility functions (e.g., take respectively $u(c) = c + \sqrt{c}$ and $u(c) = 1 - 1/(1+c)$). However, if we assume $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$ with $\gamma \in (0,1)$, it is immediate that the term into bracket is always equal to zero. Therefore, the necessary and sufficient condition (18) provided by the Lemma above is always satisfied "just" under Atkinsonian and CRRA functions.

Using the same reasoning as above, one can now simply compare c^{EPP} to c^U. To do so, it is enough to use the Lemma above with $f_1(w) = u'(c) - u'(w-c)$ and $f_2(w) = g'(u(c))u'(c) - g'(u(w-c))u'(w-c)$, consistent with (7). Assuming Atkinsonian and CRRA functions, $\frac{f_2''(2c)}{f_2'(2c)} - \frac{f_1''(2c)}{f_1'(2c)}$ then simplifies to $\frac{m(\gamma-1)}{c}$, which is always strictly negative. This shows that under "small" risks, and under Atkinsonian and CRRA functions, while we have $c^{EDE} = c^U$, we nevertheless still have $c^{EPP} < c^U$. This is consistent with the idea that EDE can be viewed as a limit case of "transformed EPP" leading to the same consumption level as under utilitarianism.

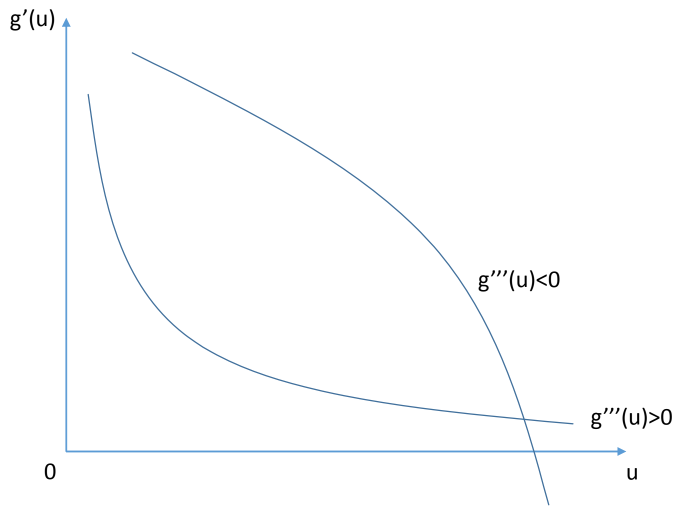

Figure 1:

This figure illustrates that the conditions $g'(u) > 0$, $g''(u) < 0$ together with $g'''(u) < 0$ for all $u > 0$ are mutually inconsistent.

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