On Climate Jumps and Fat Tails*

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November 10, 2011

Abstract

We investigate two assumptions that theoretical economic analyses of climate policies commonly impose, either explicitly or implicitly: that damages are linked directly to carbon stocks, and that climate evolves either deterministically or via a relatively simple stochastic process. It has recently been argued that climate, as proxied by temperature, is related to carbon stocks via a differential equation (so that climate is not directly linked to carbon stocks). Since changes in carbon stocks are linked to emission levels, current emissions map into temperatures via a second-order differential equation; this yields subtler, but ultimately more pronounced, impacts over time. With respect to the evolution of climate, we present evidence that a more complicated process than Brownian motion is appropriate. Implications for optimal policy are discussed, with particular relation to the recent interest within the literature regarding "fat tails" in the distribution over climate profiles.

Keywords: Climate change, stochastic processes, fat tails

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*We thank, without implicating, participants at the 12^{th} Occasional California Workshop on Environmental and Resource Economics, 2010 Colorado University Workshop on Environmental and Resource Economics, 2011 conference of the Association of Environmental and Resource Economists in Seattle, 2011 conference of the European Association of Environmental and Resource Economists in Rome, 2011 Heartland Environmental and Resource Economics Workshop, 2011 conference of Heartland Environmental and Resource Economics Environmental and Resource Economists, and seminar participants at the University of California-Berkeley, Colorado State University, University of Hawaii, Oregon State University, Queen's University Belfast, and University of Wyoming. Special thanks are due to John Antle, Joe Herriges, Gerard Kuper, Anthony Millner and Michael Moore.

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1 Introduction

Perhaps the most pressing issue currently facing environmental economists is the potential large-scale damages that might obtain from climate change. This problem has been investigated from a variety of perspectives, from purely theoretical analyses to detailed and sophisticated econometric investigations highly stylized numerical simulations. Many of these inquiries employ two important simplifying assumptions, namely that carbon stocks map directly into damages and that the system evolves more or less deterministically. Recent work in both the physical sciences and in economics calls these assumptions into question.

With respect to modeling damages, it seems clear that the principal causal factor will be the climate itself, perhaps proxied by some measure of temperature (e.g., global mean temperature). Carbon stocks only matter to the extent they influence the evolution of climate. But the science here also seems clear: *changes in*, not levels of, global temperatures are impacted by radiative forcing, which in turn is linked to carbon stock. In turn, changes in carbon stocks are linked to current emissions. In fact, the relation here is fairly complex, with impacts from current emissions potentially showing up in decades or centuries. Recognizing these facts, one must go beyond a simple one-state-variable model, allowing instead for (at least) two state variables—temperature and carbon stocks. Related to this concern, the vast majority of economic analyses assume a simple exponential decay of carbon stocks. Such decay is motivated by natural uptake into carbon sinks, such as land, forests and oceans. It seems worth noting the analogy between the exponential decay model and the exponential growth scenario employed by fisheries economists many decades ago. In both cases, there is the potential for virtually limitless growth (of the fish stock in the one case, and of natural sequestration in the other). Some time ago, fisheries economists recognized there were limits to natural stock growth; these limits were linked to the so-called carrying capacity of the resource. Likewise, there is finiteness to the potential uptake of carbon into natural sinks such as the oceans. An implication of this finiteness is that one cannot expect uptake to be linearly related to current stocks; either uptake is a non-linear (presumably concave) function, or else carbon stocks must be split into multiple categories, with one category decaying from the atmosphere so slowly as to be essentially non-decaying (at least from the temporal perspective of humankind).

The second important point is that uncertainty abounds. Recognizing this point, Weitzman (2009b) has argued that we need to pay close attention to the relative size of the tails in whatever probability distribution is deemed to characterize the salient uncertainty. While his central logic is sound, the underlying casual empiricism that he uses to sketch out this uncertainty is less so. In particular, the "distribution" he presents is based on a sample of 22 published studies, and it is not clear these represent independent draws. How one might try to organize this data, taking into account the possible correlation, is hard to see.

We suggest an alternative approach. Readily available data on carbon stocks, global emissions, and global temperatures can be used to evaluate the relations underpinnings economic analyses. These data allow one to ask such questions as: Do carbon stocks decay in a relatively simple fashion? and What is the stochastic nature of temperatures? The data we employ is relatively recent, having been tabulated only since the 1950s, but it does shed some light. Based on this data we proposed an econometric methodology that asks whether temperature changes follow a simple stochastic process such as Brownian motion (yielding a Normal distribution over temperatures), or something more involved, such as geometric Brownian motion (yielding a log-Normal distribution) or perhaps a distribution including "jumps."

One might imagine jumps resulting from sun flares, minor incongruities in the Earth's orbit or the like. Such effects are transitory in nature, in the sense that they come and go in a relatively short period of time. But to the extent there is inertia in the global climactic system these impacts might influence temperatures, and hence damages, for a rather longer period of time. While not likely to lead to runaway climactic disasters, they provide a vignette of the potential impacts of climate change. And they also contribute to the potential for "fat tails" in temperature distributions.

Weitzman (2010a) recently conducted a simple analysis based on three 'prototype' distributions: the Normal (which he regards as thin-tailed), the Pareto (which he regards as fat-tailed) and the log-Normal (which he regards as intermediate). Analogously, one can think of our analysis as looking at varying degrees of tail-fatness, with Brownian motion corresponding to thin tails, geometric Brownian motion as relating to the intermediate case, and geometric Brownian motion with jumps as relating to fat tails. Our results are somewhat mixed: on the one hand, it seems clear that the thin-tailed Brownian motion model does not do well in organizing the data. Indeed, the geometric Brownian motion model appears to perform best, in a classical statistical sense. But geometric Brownian motion with jumps also performs better than simple Brownian motion. If one believes the costs associated with forming policy based on a climate model that underestimates the significance of jumps are larger than the costs associated with forming policy based on a climate model that overestimates the significance of jumps, the case for employing the jump model would be fairly strong. Given the highly non-linear damages that have been ascribed to climatological impacts, such an asymmetric ”loss function” seems reasonable.

2 Deterministic Underpinnings

A number of authors have investigated the dynamic optimization problem associated with climate change. By and large, the typical investigation models damages as based on carbon stocks, and often assumes these damages are quadratic. While such a model is certainly more easier to analyze, and relatively transparent, it incorporates two essential elements that are clearly wrong: damages are linked to temperatures, not carbon stocks (Weitzman, 2009a, 2010b), and marginal damages are not likely to be linear (Schlenker and Roberts, 2009). The linkage to carbon stocks would make sense if temperatures were directly related to carbon stocks, but they are not: carbon stocks induce *changes* in temperatures, not levels (Allen et al., 2009; Weitzman, 2010b). Taking note of this fact, the dynamic optimization problem associated with framing optimal climate policy would need to include at least two state variable, temperatures and carbon stocks.

A model that pays attention to these facts can be constructed starting from Weitzman

(2010b). Let $T(t)$ represent a measure of temperature – such as global mean temperature – at time t, and denote the stock of atmospheric carbon by $C(t)$. The equation of motion for temperature can then be written as (Allen et al., 2009; Weitzman, 2010b)

$$\dot{T} = \alpha \ln(C/C_0) - \beta T, \tag{1}$$

where C_0 measures pre-industrial atmospheric levels of CO_2. To complete this part of the model, one then needs to describe the evolution of carbon stocks; this evolution will be linked to anthropogenic contributions.

A fully detailed physical model, such as the one described in Allen et al. (2009), is based on multiple carbon stocks, relating different time scales. For example, writing long-term equilibrium carbon stocks as C_3 and deviations away from those stocks as C_2, the system satisfies

$$\begin{aligned} \dot{C}_2 &= a_0 E - b_2 C_2; \\ \dot{C}_3 &= b_3 C_3; \end{aligned}$$

where E is unabated (e.g., unsequestered) emissions. The important point is that some carbon stays in the atmosphere for a very long time (Solomon et al., n.d.), so assuming exponential decay of the (complete) carbon stock is not appropriate. An alternative scenario, that does not require the introduction of an extra state variable, would be to allow for non-linear decay.[1] Such a construct can be regarded as analogous to the logistic growth component in modern fisheries models. Following this tack, we write

$$\dot{C} = a_1 E - C(b_0 - b_1 C); \tag{2}$$

At a point in time t payoffs are $(\pi(E) - D(T))e^{-\rho t}$, where π is interpreted as net benefits from unabated emissions, D is temperature-related damages, and ρ is the discount rate (i.e., payoffs are denominated in terms of period 0). One could think of π as representing GDP, net of sequestration and abatement costs. It is natural to assume this payoff function is increasing at small levels of E and globally concave; one particular example that has received significant attention in the literature is the iso-elastic form

$$\pi(E) = AE^{\theta+1},$$

where θ is the elasticity of marginal net benefits with respect to net emissions, $\frac{\pi'(E)}{E\pi''(E)}$.

The solution to this problem is described by Pontryagin's maximum principle: we first define the current-value Hamiltonian

$$\mathcal{H} = \pi(E) - D(T) + \mu[a_1 E - C(b_0 - b_1 C)] + \nu[\alpha \ln(C/C_0) - \beta T],$$

where μ and ν refer to the co-state variables (i.e., shadow values) associated with the state

[1] There is the additional point that empirically identifying the different carbon stocks is not feasible, more or less by definition.

variables C and T, respectively. The optimal values of the controls maximize the Hamiltonian at each point in time; assuming interior solutions, we therefore have

$$\pi'(E^*) + a_1\mu = 0, \tag{3}$$

where the asterisk signifies that the net emissions level has been chosen optimally. The solution also requires the co-state variables follow the equations of motion

$$\begin{aligned} \dot{\mu} &= \rho\mu - \partial\mathcal{H}/\partial C; \\ \dot{\nu} &= \rho\nu - \partial\mathcal{H}/\partial T. \end{aligned}$$

Upon inspecting of the current-value Hamiltonian, it is evident that these last equations may be written as

$$\begin{aligned} \dot{\mu} &= (\rho + b_0)\mu - 2b_1 C\mu - \frac{\alpha}{C}\nu; && (4) \\ \dot{\nu} &= (\rho + \beta)\nu + D'(T). && (5) \end{aligned}$$

To make additional headway, time-differentiate equation (3) to obtain

$$\begin{aligned} &\pi''(E^*)\dot{E}^* + a_1\dot{\mu} = 0, \quad \text{or} \\ &\pi''(E^*)\dot{E}^* + a_1[(\rho + b_0)\mu - 2b_1 C\mu - \frac{\alpha}{C}\nu] = 0. && (6) \end{aligned}$$

Combining terms and invoking equation (3) then yields

$$\dot{E}^* = (\rho + b_0 - 2b_1 C)\frac{\pi'(E^*)}{\pi''(E^*)} - \frac{a_1 \alpha}{\pi''(E^*)C}\nu. \tag{7}$$

The key points to be gleaned from equations (3) and (7) is that damages only exert an indirect effect on optimal emissions, in that they influence changes in – but not levels of – E^*. That is, the role played by climate change in determining optimal policy at a point in time is far subtler than would be suggested by a simpler model that regards damages as the result of the stock of greenhouse gases.

Before moving on to a discussion of the empirical evidence, we detour briefly for a discussion of a more traditional version of the model, in which damages are tied to carbon stocks. For purposes of comparison, we retain the representation of damages as based on temperatures, but replace the state equation for temperatures with a simplified form:

$$T(t) = \phi(C(t)).$$

Based on this representation, damages can be expressed as $d(C) \equiv D(\phi(C))$. Using such an approach, the state equation on temperatures becomes irrelevant to the dynamic optimization problem. The optimality condition governing emissions is still given by eq. (3),but now the equation of motion for the (lone remaining) co-state variable is

$$\dot{\mu} = (\rho + b_0)\mu - 2b_1 C\mu + d'(C). \tag{8}$$

With this version of the problem, then, the differential equation governing the path of optimal emissions would be

$$\dot{E}^* = (\rho + b_0 - 2b_1 C)\frac{\pi'(E^*)}{\pi''(E^*)} - \frac{a_1 d'(C)}{\pi''(E^*)}. \tag{9}$$

Comparing against eq. (7), we see that the implication of assuming that carbon stocks directly impact temperatures, as opposed to impacting the change in temperatures, is to replace the component $\alpha\nu/C$ with $d'(C)$. In light of the evolution of the shadow value ν, this substitution is likely to lead to mis-adjustments that change over time.

3 Stochastic Temperatures

While instructive, this deterministic model omits a key ingredient. To the extent that marginal damages are non-linear, and there is uncertainty related to temperatures, the nature of this uncertainty becomes important. In particular, tail-thickness becomes a relevant concern.[2] In this section, we explore the empirical evidence to see what can be learned from the available data.

Data on global mean temperatures and atmospheric carbon is readily available on the Internet. [3] Carbon stocks are measured at a number of locations. We use observations from Mauna Loa, which represent the longest time series of atmospheric carbon observations.

[2] Compare the results in Weitzman (2010a), who finds little difference in the impact of tail thickness when damages are quadratic, but potentially large differences when marginal damage is highly non-linear.

[3] Atmospheric carbon data are available at http://cdiac.esd.ornl.gov/trends/co2/sio-keel.html. Global mean temperature data are available at http://data.giss.nasa.gov/gistemp/tabledata/GLB.Ts+dSST.txt.

Data on both global mean temperature and carbon stocks are available at the monthly level from March 1958 to March 2011, which yields 642 observations.

Using this data, we investigate eq. (1). The regression equation of interest could equally well be viewed as one where the left-side variable is current temperature, in which case one of the regressors is the lagged value of the left-side variable. This fact poses no difficulties if the residual term is serially uncorrelated, but that seems suspect in the case at hand. Because temperatures evolve slowly, the potential for shocks in one month to exert an influence on temperatures in the next month would seem to be very real. Accordingly, we proceed on the assumption that we have serial correlation as well as a lagged left-side variable in the regression equation. Addressing this complication requires the procurement of a consistent estimate of the serial correlation parameter; we follow the approach suggested by Hamilton (1994, p. 226).[4]

Results from this approach are presented in the second column of Table 1 (labeled 'Regression 1'). We note that both lagged temperature (measured in degrees Centigrade) and the natural log of atmospheric carbon stocks (measured in parts per million) are strongly statistically significant, and take the anticipated sign. We also note that there is much residual uncertainty, as indicated by the modest R-squared value. An alternative regression, that allows for the possible influence of the change in carbon stocks upon the change in tem-

[4] The idea is to run a regression that adds lagged values of the original regressors, and then form the negative of the ratio of the point estimates on current and lagged values of the exogenous variable (the natural log of carbon stock). This point estimate is a consistent estimator of the true underlying serial correlation parameter. We then transform the variables by subtracting the product of this estimate with teh lagged value of the variable, for each of our variables. The point estimates reported below are derived from such a transformation. Because this approach requires dropping the first two observations, the regressions emply 640 observations.

peratures, is reported in the third column ('Regression 2'). We note that the coefficients on the three variables form Regression 1 (lagged temperature, the natural log of atmospheric carbon stocks, and a constant) change very little, while the coefficient on the new variable appears insignificant, both numerically and statistically. By contrast, dropping the natural log of carbon stocks, as we do in the fourth column (1Regression 3') yields a noticeable drop in explanatory power (as measured by the sharp reduction in the R^2 statistic), a noticeable change in the coefficient on lagged temperature, and a sign change in the constant. This third regression corresponds to a model in which there is a direct relation between levels of carbon stock and levels of temperatures, as is implied in a number of extant theoretical analyses. In our view, the regression results reported in Table 1 strongly support the physical model described in eq. (1) — where carbon stocks influence changes in temperatures — and cast serious doubt on a model that posits a direct relation between carbon stocks and temperatures.

Proceeding on the basis of Regression 1, we formed the fitted residuals, which are plotted in Figure 1. Our primary interest here regards the distribution of these residuals, particularly the relative thickness of the tails associated with these residuals. If the data were Normally distributed, one would expect .5% of the observations to be further than 2.575 standard deviations from the mean; with 640 observations, that would yield roughly 3 observations in either tail. By contrast, we see 6 observations in the lower tail and 8 observations in the upper tail. Similarly, 4 observations lie more than 2.81 standard deviations below the mean and 4 lie more than 2.81 standard deviations above the mean; were these residuals Normally distributed, these numbers would be each be roughly 1.5. On balance, then, it seems the tails are roughly twice as heavy as would be predicted. This sense is corroborated by Figure 2, which compares the cumulative distribution against a cumulative normal (using a logarithmic scale). Where the residuals normally distributed this comparison would yield a straight line; instead, we see marked departures in both tails. Moreover, the positive outliers appear to be somewhat more important. This visual evidence suggests the presence of relatively fat tails.[5]

If outliers are important, as the previous discussion suggests, then one would expect to pick this up by including a dummy variable that captures any observations associated with outliers. To address this possibility, we included two dummy variables, one for observations that were more than 2.575 standard deviations below the mean (Djump2_50) and one for observations that were more than 2.575 standard deviations above the mean (Djump2_51). The results in the fifth column of Table 1 provide such a regression. The key point here is that including dummy variables for these unusual observations generates statistical significant estimates for both dummies, with a marked improvement in the regression results. Moreover, the average effect associated with these anomalies yields a change of roughly .1 degrees Centigrade. To put this in context, the average monthly change in temperatures, measured in degrees Celsius, is .0007; evidently, these observations yield important effects. Since temperatures are subject to considerable inertia, the implication is that effects tied to these observations tend to last for several months. Ultimately, it appears that these outliers

[5] The sample estimate of kurtosis is 4.285; if the residuals were normally distributed we would expect an estimated value of 3. The probability that a value of 4.285 would obtain if the data were truly normally distributed is less than 1%, reinforcing the conjectured presence of fat tails.

exert a meaningful effect in this data, and seem likely to result in abnormally fat tails.

One concern within this setting is the potential for temperatures to influence carbon stocks. While such a result seems remote in principle, the fact we are using a measurement at a specific location to measure global carbon stocks raises the specter of errors-in-variables. To address this concern, we ran an instrumental variables regression using obervation on carbon stocks fromthe preceding year as an instrument for current readings; the one-year lagged stock is certainly not caused by current temperatures, while the slow evolution of carbon stocks suggests a reasoably strong correlation with current stocks. We therefore expect one-year lagged stocks to perform well as an instrument for current stock. The results from this instrumental variables regression are reported in the sixth column. We see here that the coefficients on lagged temperature and the natural log of carbon stocks are quiote similar to those described above, suggesting the potential endogeneity of the carbon stock variable is likley of minimal concern.

If temperature variations are subject to relatively fat tails, what might be the cause? One possibility we explore is related to the notion of a jump stochastic process. Jump processes are usually thought of as the result of exogenous effects; in the present case, that might be because of sunspots or sun flares, or variations in the Earth's orbit. Indeed, the observations far from the mean observed in Figure 1 do seem indicative of jumps.

To allow for the presence for jumps, we assume jumps arrive with probability λ in any particular period. The jump component is modeled as a Poisson-driven process denoted q; the size of a jump, if it occurs, is distributed with mean θ and variance δ^2. Estimation results based on such a model are given in Table 2. This table lists parameter estimates for mean and variance of the residuals, along with the three key parameters associated with the jump process:the probability of a jump (λ), the mean value of a jump (θ) and the standard deviation of the magnitude of a jump (δ). The estimated coefficients from the jump diffusion model are all statistically important. On the other hand, the restricted model produces a relatively poor fit of the data; indeed, the likelihood ratio statistic is very large, and is signifcant at better than the .01% level.

4 Conclusion

In this paper, we consider extensions to the traditional theoretical growth model associated with climate policy, and consider related empirical evidence. There is evidence, albeit weaker than one would like, based on conventional hypothesis tests, for relatively fat tails associated with temperature changes. To the extent that temperature changes do exhibit relatively heavy tails, the potential impact on optimal climate policy is non-trivial.

Using a stylized model based on a stripped down version of the underlying dynamic system governing the Earth's climate, Weitzman (2010a) argues that it could be worth spending a considerable fraction of current global GDP to combat climate change. The scenarios he investigates are, however,somewhat *ad hoc*, in that they contemplate quite large levels of warming. That point noted, his results do indicate the potentially important role that tail thickness could play: the fraction of global wealth that might optimally be allocated is markedly larger with the thickest tailed distribution, and considerably smaller with the Normal distribution. Our results cast doubt on the empirical saliency of Normally distributed temperature variations, which in turn suggests it could be optimal to allocate meaningful amounts of resources to insure against substantial temperature increases. If temeratures are leptokurtotic, as our results indicate, then idiosyncratic shocks are likely to map into meaningful changes in temperatures that persist for a period of time. In light of the observation that the marginal damage associated with temperature increases is non-linear (Schlenker and Roberts, 2009), the cost associated with falsely deciding such jumps are insignificant has the potential to be considerably larger than the cost associated with falsely deciding jumps are important.

On the other side of the coin, one could argue that taking actions in the near term imposes a different sort of cost, and that before the major economies of the world decide to undertake substantial climate policies (e.g., by devoting significant resource levels or by imposing large carbon taxes) they need more compelling evidence than the potential for the short-term results we resent.

While the philosophy that should win the day is something of a matter of interpretation, we believe our results do point to the possibility for jumps to manifest themselves over what climate scientists would regard as a fairly short time horizon; in turn, this would seem to suggest a real potential for meaningfully fat tails in the distribution of temperature changes going forward. At a minimum, this possibility would seem to suggest a potentially important role for some sort of climate insurance policy.

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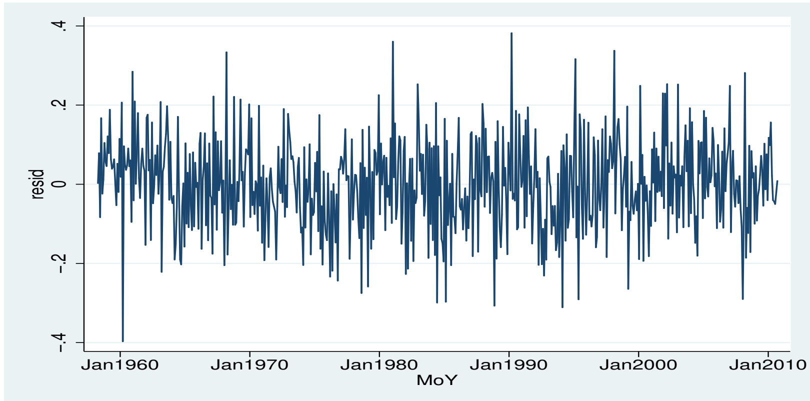

Figure 1: Residuals from climate regression

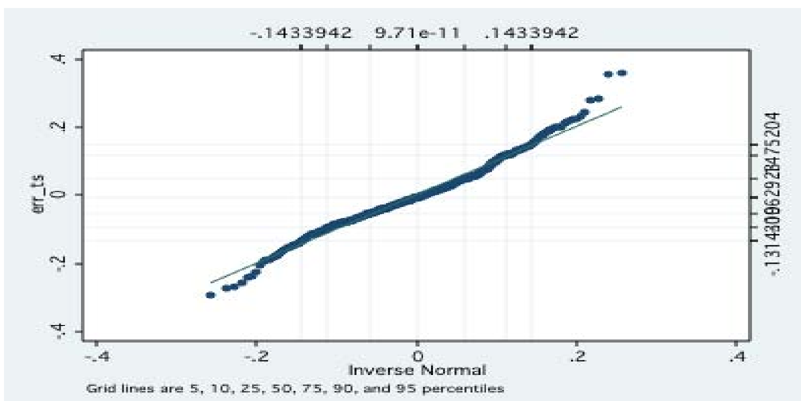

Figure 2: logarithmic plot of residuals vs. Normal distribution

Table 1: Regression results: global mean temperatures

variable	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5
T_{t-1}	-.1634**	-.1639**	-.0403**	-.1587**	-.1647**
	(.0217)	(.0219)	(.0114)	(.0196)	(.0223)
$\ln(C_t)$	.5167**	.5167**	—	.5058**	.5361**
	(.0787)	(.0789)	—	(.0711)	(.0834)
$C_t - C_{t-1}$	—-	.0029	.0308*	—	—
	—	(.0167)	(.0135)	—	—
D_{out0}	—	—	—	-.0957**	—
	—	—	—	(.0019)	—
D_{out1}	—	—	—	.1075**	—
	—	—	—	(.0019)	—
constant	-.9647**	-.9560	.7767**	9699**	-1.0943**
	(.3368)	(.3394)	(.2198)	(.3038)	(.3604)
R-squared	.082	.082	.020	.256	.082

Dependent variable: $T_t - T_{t-1}$
number of observations = 640

Table 2: Jump analysis

coefficient	estimate	std. err.	restricted estimate	restricted std.err.
μ	-.1209	.043	-.0001	.040
σ	.4196**	.049	.9206**	.026
λ	.5816	.069	—	—
θ	.2076*	.089	—	—
δ	1.0661**	.060	—	—

Chi-squared test statistic = 54.87
p-value $<$ .0001
*: significant at 5% level
**: significant at 1% level