Title: A simple formula predicts the social costs of carbon with remarkable precision

Summary paragraph:

The social cost of carbon (SCC), commonly referred to as the carbon price, is the monetized cost from emitting one unit of CO_2 to the atmosphere. The SCC is obtained from large-scale computational Integrated Assessment Models (IAMs) that consolidate interdisciplinary climate research inputs to obtain a number relevant for policy-making. [1] However, the climate-economy interactions of IAMs remain inaccessible to scientists in general. Here we develop a closed-form SCC formula linking the physical climate system and the key economic variables. The formula predicts the SCC outcome of a mainstream IAM with a striking precision, explaining more than 99 percent of the variation in the SCC coming from primitive climate system and economic parameter uncertainties. This allows constructing a SCC distribution from primitive uncertainties and decomposing their respective contributions to the SCC, leading to a strongly right-skewed density with median 15 €/tCO_2, mean 31 €/tCO_2, and more than 5 percent probability for higher than 100 €/tCO_2 for year 2015. Our results allow decision-makers to disentangle the subjective and structural determinants of the carbon price. We anticipate that further research can build on our approach to develop even more detailed tractable climate-economy representations including, for example, heat exchanges between climate layers and oceans. [2]

Main text:

The SCC compresses the monetized cost from releasing a ton of CO_2 to the atmosphere today to one number. The IAMs obtain this number by optimizing current emissions in a model for the global carbon cycle with temperature dynamics connected to a global economy description. There are several widely used IAMs. [3,4,5,6,7] Because of its public availability, conciseness, transparent documentation, and middle-of-the-road assumptions, DICE[8] has become the most well-known IAM. While the IAMs overarch the contributions from various disciplines in climate change research, they have become inaccessible to scientists in general; they can contain hundreds or even tens of thousands lines of code. Here we provide one transparent equation that predicts the SCC produced by DICE with a striking precision.

In our SCC formula parameter π measures the temperature increase squared per teraton of CO_2 added to the atmosphere [$K^2/TtCO_2$]. Its value is approximately equal to c^2/m where c is the temperature increase associated with a doubling of the pre-industrial atmospheric carbon stock m. The second parameter, Δ, is the damage, relative to output, per temperature change [K^{-2}]. Together, $\Delta\pi$ is the equilibrium share of the output lost per atmospheric CO_2. A permanent increase of the atmospheric CO_2 by one unit leads to an annual economic loss $\Delta\pi Y(t)$, where $Y(t)$ is the World's Gross Product (WGP) at time t.

The SCC formula builds on a temperature response that we derive from a closed-form representation for the global carbon cycle and temperature adjustment. The SCC formula gives the present-value of total GWP losses over time caused by one-ton increase in the current emissions. We consider a box representation of the carbon cycle. [9] For set $I = \{1, \dots, n\}$ of boxes, $a = (a_i)_{i \in I}$ is the vector of shares of emissions entering each climate box, and $\eta = (\eta_i)_{i \in I}$ are the respective decay rates. Parameter $\varepsilon > 0$ captures adjustment speed of the global mean temperature. Parameter $\sigma > 0$ is the discount rate for transforming the future impacts to present value. We report the SCC for the year $t=2015$, and measure all economic values in 2010 Euros. The formula for the social costs of carbon can be stated as:

$$SCC(t) = \Delta \pi Y(t) \sum_{i \in I} \frac{\varepsilon a_i}{(\sigma + \eta_i)(\sigma + \varepsilon)}$$

The life-path of the future GWP losses depends on the temperature response following from the current emissions. The response is hump-shaped, with the peak impact lagging 60 to 80 years behind the date of emissions (Fig. 1). Directly after the emissions impulse, the temperature quickly rises, but then, as atmospheric CO_2 decays, the temperature response falls back. The SCC formula is based on a response function that captures the essence of the connection between emissions and damages in IAMs, explaining the formula's strong prediction power of the IAM outputs considered next.
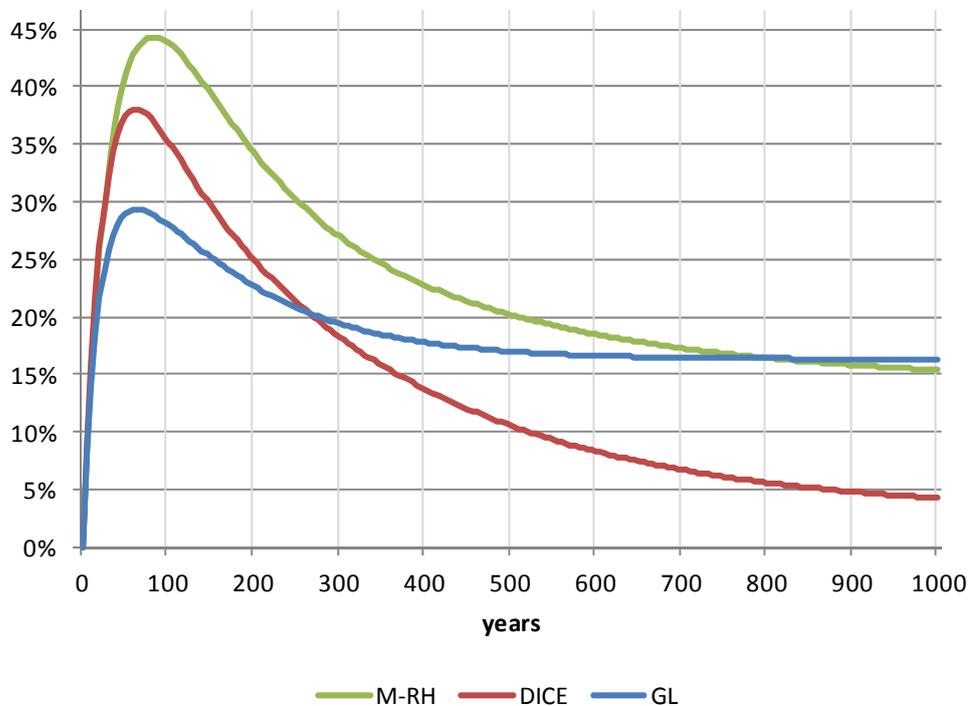
Figure 1: The life-paths of temperature responses following from one unit of emissions, relative to the counterfactual response that assumes that (i) the emitted unit remains in the atmosphere forever and (ii) the emitted unit has immediate full temperature effects. Responses shown for three different carbon cycle models M-RH[9], DICE[8], and GL[10], where parameters of our reduced-form model are set to match the three cited carbon cycle representations. For example, to match DICE we set

We test if the formula predicts the SCC of DICE. We fix the carbon cycle parameters a=(0.029, 0.356, 0.615) and η=(0, 0.0035, 0.0364) to match the DICE carbon cycle, but assume distributions for 12 key climate and economic DICE parameters. We then draw 1000 realizations for the DICE parameter vector, leading to 1000 SCC outputs produced by DICE. Each vector draw defines the parameters for our formula σ, Δ, π, and ε that enter our SCC formula, which allows us to use the formula to predict the SCC for each draw (Fig. 2).

The realizations are clustered tightly along the 45-degree line; regressing the log of the DICE SCC on the log of our formula SCC shows that the formula explains more than 99 percent of the variation (R^2 >.99). The SCC formula, a reduced-form representation of the true model employing only a subset of the parameters in DICE, can thus predict the DICE SCC with a remarkable precision.
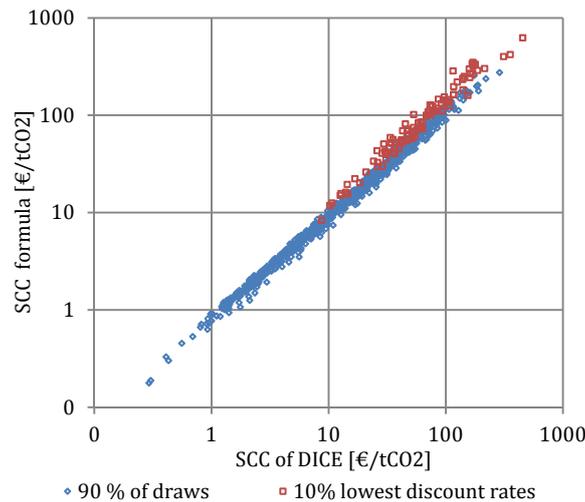
Figure 2: The DICE and formula SCC in a scatter plot where each dot corresponds to one parameter vector realization with the horizontal and vertical co-ordinates presenting the DICE and our formula SCC values, respectively. Both axes present the optimal SCC for the year 2015. Observations corresponding to 10 percent lowest discount rates represent the majority of deviations. This is due to the 600-year time horizon of DICE, while the formula has no such limitation.

The closed-form for the structural determinants of the SCC allows constructing a distribution for the SCC from the primitive uncertainties characterizing the carbon cycle, climate sensitivity, damages, and the economy. For uncertainty regarding the carbon cycle representation, we include the three carbon cycles representations underlying Fig. 1. Then, Fig. 3 depicts the density distribution of the SCC for the year 2015, obtained from the SCC formula.
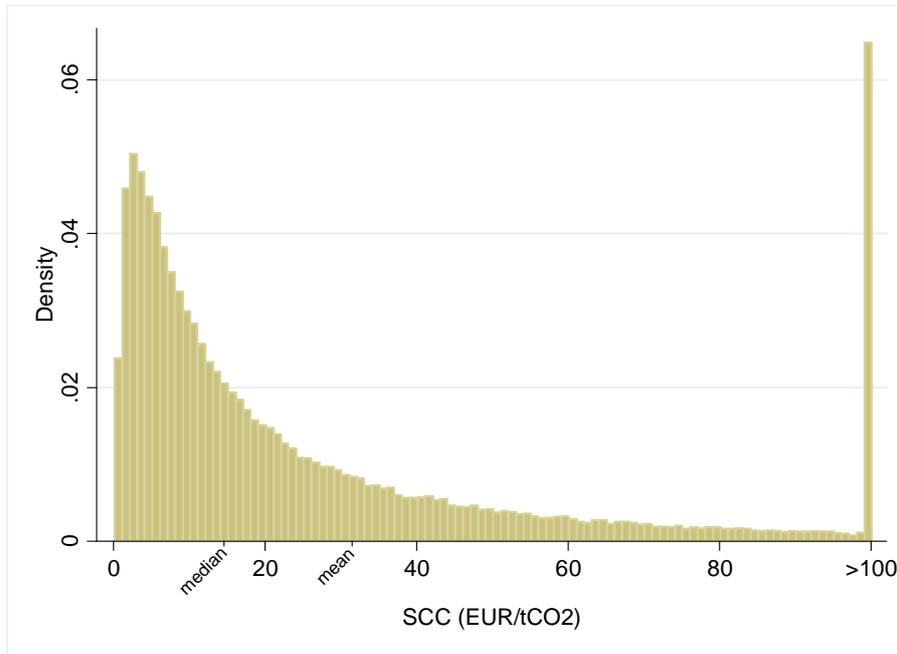
Figure 3: Density distribution of the SCC obtained from 100,000 realizations for the SCC formula parameters, including randomization over the three carbon cycle representations M-RH[9], DICE[8], and GL[10]. The parameter vector includes the climate sensitivity, damage estimate, pure rate of discounting, productivity growth, population growth, elasticity of marginal utility, and costs of carbon-free energy. The parameter distributions for the Monte Carlo draws in DICE are log-normal to avoid negative values, truncated at 2 standard deviations from the median. Further information on the parameters distributions is provided in the supplementary material.

The resulting distribution is strongly right-skewed with median 15 €tCO2, mean 31 €tCO2, and more than 5 percent probability for higher than 100 €tCO2. A distribution from IAM outputs of 232 distinct studies results in very similar median and mean values when the numbers are converted to comparable units and the focus is on studies that use 1 percent discounting.[11] However, survey-based distributions blur the structural and subjective parameter uncertainties. Also, they cannot address the tail of the distribution as these are defined by too few observations. Our SCC formula produces a thick-tailed distribution as the low probability extreme values for climate sensitivity, damage and discounting reinforce each other. Moreover, the formula allows evaluating the contribution of each primitive type of uncertainty to the SCC. Table 1 decomposes the sources of the SCC uncertainty.

Source of variation	Median	Mean	Standard deviation
	€tCO2	€tCO2	€tCO2
None	16.0	16.0	0
Carbon cycle	16.3	15.9	2.0
Climate sensitivity	16.0	19.0	12.0
Damage	15.9	21.3	17.2
Discount rate	16.0	19.7	14.1
All	14.6	31.5	53.8

Table 1: Sources of SCC uncertainty

We fix all but one parameter at their median values to obtain the rows of Table 1. Carbon cycle uncertainty has a minor contribution to the SCC uncertainty but climate sensitivity, damage, discounting all individually introduce considerable spread and right-skewedness to the SCC distribution. The joint interaction of all uncertainties leads to a distribution where the mean is twice as large as the median.

Progress in climate science will improve the precision of estimates for climate sensitivity and damages over time, but very slowly. Discounting is likely to remain partially subjective and uncertain; the views among experts and policy makers on how to weigh the far-future impacts will likely continue to differ. Table 2 describes the sensitivity of the distribution to the annual discount rate. The mean and median approximately double when the pure discount rate σ falls from 3 to 2 percent but they increase more than five-fold when σ falls from 1 to .1 percent. Some climate impacts, due to the non-depreciating climate boxes, are permanent, fattening the tail of the SCC distribution when discounting falls towards zero. For extreme parameter realizations, the social cost of carbon is without bound. This is in sharp contrast with the distributions obtained through survey methods where tails remain bounded.[11]

| Discount rate | Median | Mean | Standard deviation |
	€tCO2	€tCO2	€tCO2
.1 %	175.9	316.0	421.3
1 %	29.2	48.5	58.5
2 %	13.7	22.1	25.5
3 %	8.2	13.0	14.7

Table 2: Discount rate sensitivity of SCC

Methods:

For DICE we use the publicly available source code[8]; the code for Monte Carlo simulations is in the supplementary material. The carbon cycle in DICE has linear diffusion between carbon reservoirs and is transformed through linear algebra to a climate box representation, in a similar vein as in M-RH[9] and GL[10]. All material is available through https://www.dropbox.com/sh/1zz4wvs3h788ooh/RIQCILKBTB.

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Author Contributions: ML and RG jointly developed the SCC formula. LK produced the first Monte Carlo test of the formula using DICE. RG wrote the GAMS source code used for results presented in this manuscript. IvdB assessed the parameter distributions, and wrote the Stata code for the density distribution of the SCC. ML, RG and IvdB all contributed to the final text.

Author Information: Authors should include a set of statements at the end of the paper, in the following order:

Support Method information:

Derivation of the Social Costs of Carbon

The Social Costs of Carbon formula is derived as follows. Consider the following dynamic system of equations for the perturbation of atmospheric carbon stock, *S*, with boxes S_i:

$$S(t) = \sum_i S_i(t)$$

$$\dot{S}_i(t) = a_i E(t) - \eta_i S_i(t)$$

It follows immediately that the dependence of the future perturbation of atmospheric carbon stock on current emissions is given by

$$\frac{dS(t+\tau)}{dE(t)} = \sum_i a_i e^{-\eta_i \tau}$$

Now consider equilibrium global mean temperature perturbations as a concave function of atmospheric CO_2 stock perturbations, $T = \varphi(S)$, and relative damages depending on temperatures through $D = \Delta\psi(T)$, where $\psi(T) = T^2$ and Δ is the damage sensitivity of the economy with respect to temperature change (squared), relative to output Y, so that DY equals gross damages. The equilibrium damages dependence on stock perturbations can be described through

$$D = \Delta\psi\big(\varphi(S)\big)$$

With a concave temperature sensitivity to atmospheric CO_2 and convex damages, the combination can be proxied linearly through $dD = \Delta\pi dS$ with $\pi = \psi'\varphi'$; between 400 and 550 ppmv the value of π is about 5.25 $K^2/TtCO_2$. The global mean temperature as captured by D converges at speed ε and develops through

$$\dot{D}(t) = \varepsilon(\pi S(t) - D(t))$$

Through linear algebra, we can find that the level of relative damages depends on past emissions linearly through

$$\frac{dD(t+\tau)}{dE(t)} = \sum_i a_i \varepsilon\pi \frac{e^{-\eta_i\tau} - e^{-\varepsilon\tau}}{\eta_i - \varepsilon}$$

Finally, consider discounting of future damages at rate σ. This gives the NPV as in the equation in the main text:

$$SCC(t) = \Delta Y(t) \int_0^\infty e^{-\sigma\tau} \frac{dD(t+\tau)}{dE(t)} d\tau$$
$$= \Delta\pi Y(t) \sum_i \frac{\varepsilon a_i}{(\sigma + \eta_i)(\sigma + \varepsilon)}$$

Parameters in DICE used for the Monte Carlo experiment

We included 12 major parameters from DICE[1] in our Monte Carlo sample drawing. These are listed in the table below. For each parameter, we derived a distribution from the literature. The central values are more or less in line with the typical values used for DICE.

Parameter [Units]	Median	Mean*	Standard deviation*	Lower cutoff value*	Upper cutoff value*	Source
Climate sensitivity [K]	3	3.218	1.222	1.3719	6.5601	a
Damage parameter	0.003	0.004	0.0032	0.0006	0.015	b
Pure rate of time preference [yr^{-1}]	0.02	0.0248	0.0171	0.005	0.08	c
Elasticity of marginal utility	1	1.0845	0.4447	0.4305	2.3229	
Asymptotic size of population [mn]	10,000	10,115	1,530	7,300	13,699	d
Productivity growth [dec^{-1}]	0.1537	0.154	0.0093	0.1345	0.1744	e, f

Decline rate of productivity growth [dec^{-1}]	0.001	0.0011	0.0004	0.0005	0.002	[g]
Decarbonization rate [dec^{-1}]	0.073	0.0745	0.0151	0.0479	0.1113	[e,g,h]
Decline rate of decarbonization [dec^{-1}]	0.003	0.0033	0.0013	0.0013	0.007	[e,g,h]
Backstop price [USD/tC]	1,170	1,194	242	768	1783	[g,h]
Ratio initial to final backstop price	2	2.041	0.4142	1.3122	3.0482	[g,h]
Decline rate of backstop price [dec^{-1}]	0.05	0.0521	0.015	0.0275	0.0909	[g,h]

All parameters are lognormally distributed and truncated; *for truncated distribution

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Table SI1: Parameter distributions

Parameters in the formula

The damage parameter in DICE is the same as in our formula. The climate sensitivity in our formula is derived from the climate sensitivity used for DICE. For the output level in our formula, we take the output of DICE in the year 2015 (which mainly depends on labor productivity). The carbon cycle parameters are based on an annual representation of the values that represent DICE as estimated by Gerlagh and Liski.[2] The temperature adjustment parameter is also taken from DICE, but adjusted for the temperature sensitivity. The pure discount rate captures the decrease in the weight given to future damages. Its value equals the pure rate of time preference, plus labor productivity growth times the elasticity of marginal utility minus one, minus population growth. The full source code is available online.

References

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