Carbon Lock-In: The Role of Expectations*

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Abstract

The feasibility, speed, and economy-wide impact of the energy transition from fossil fuels to clean energy sources depend crucially on the rate and nature of technological change. This paper presents a model of directed technical change to study the interaction between innovation and the energy transition from a non-renewable resource to a backstop technology. We find that resource-saving technical change erodes the incentives to implement the backstop technology. Conversely, the anticipation of the backstop being implemented in future diminishes the incentives to invest in resource-saving technology. As a result, two dynamic equilibria may arise, one with a transition to the backstop and low resource efficiency, and one without backstop deployment and fast efficiency improvements. Expectations determine which equilibrium arises in the decentralized market equilibrium. We characterize the transition paths and implications for economic growth.

JEL codes: O30, Q32, Q42, Q55

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1 Introduction

Policy debates during the last decades have witnessed a growing interest in reducing the share of fossil fuels in energy generation. The increasing attention for the sources of energy on which the economy relies is mainly driven by the challenge of combating climate change and the global concern about the sustainability of current living standards. Part of the solution to both the climate change and the sustainability problem may be a phasing out of non-renewable natural resources like fossil fuels and the implementation of backstop technologies that provide renewable substitutes. A more incremental solution would arise from improving resource efficiency and slowing depletion of fossil resources. The question arises how market parties respond to the challenges and which incentives arise over time to invest in resource saving and energy transition. We argue that the energy future of a growing economy is crucially shaped by a two-way interaction between innovation decisions and energy supply decisions. Prospects about future energy generation technologies may affect the time path of fossil fuel consumption, but also the pace and direction of technical progress. Conversely, the speed and direction of technical progress are crucial for the transition from fossil fuels to backstop technologies.

Since our question concerns the structural dynamics in a growing economy, we naturally frame our analysis in a growth model with natural resources and endogenous technical change. Our starting point is the Dasgupta-Heal-Solow-Stiglitz (DHSS) model[1] in which a scarce non-renewable resource (fossil) is an essential input in production. We allow the fossil energy to be replaced by non-scarce energy that can be generated at a constant cost, the so-called backstop technology (cf. Nordhaus, 1973). As is well known, in the DHSS model growth cannot be sustained unless resource-augmenting technological change offsets the negative growth impact of declining availability of the non-renewable resources. At the same time, labor-augmenting technical change fuels growth and boosts the demand for energy. Energy demand thus results from the balance between two types of innovation, resource-augmenting and labour-augmenting technical change. We incorporate both types in our analysis and consider them as endogenous, i.e. allow profit incentives to guide innovators how much and in which direction to innovate. Thus, we merge the DHSS model with a model of directed

[1]See Dasgupta and Heal (1974), Solow (1974a,b), Stiglitz (1974a,b), Van der Ploeg and Withagen (2013), and Benchekroun and Withagen (2011).

technical change.[2]

Our main finding is that the replacement of fossil resources might require a coordination of expectations. If the costs of generating energy with the backstop technology is sufficiently low, it is a viable alternative to fossil fuels in the long run. However, sufficient investment in resource-saving technical change can make fossil effectively cheaper to use than the backstop. Whether in equilibrium fossil is phased out or not then depends on the expectations of fossil suppliers and innovators. A self-fulfilling prophecy arises since when it is expected that the backstop will be implemented, the market for resource-saving inventions will be small and innovations incentives will be eroded; this makes the backstop relatively more attractive and thus justifies the expectation that the backstop will be implemented. Conversely, when no future backstop deployment is expected, resource-saving technical change becomes more profitable, thus making the resource relatively more attractive in the long run. Only when the backstop cost is below a certain threshold, it will always be deployed in the long run.

We also find that different energy transition patterns can emerge that have markedly different impacts on the economy. First, without a transition to the backstop, fossil use typically peaks, i.e. resource use declines over time in later stages of the growth process. However, with a transition to the backstop, resource use is typically rising for a long period. Second, the pattern of innovation differs as well. With backstop resource-augmenting technical change stops well before the backstop is introduced so that the economy displays energy-using technical change in later stages of the transition. This is in contrast with the equilibrium without backstop, in which growth goes together with resource-saving technical change.

Our results imply that it might be hard to shift the economy away from the current dependence on fossil fuels because the economy is "locked into fossil" (cf. Unruh, 2000). Lock-in is studied in the literature in several settings.[3] In the context of energy use, Acemoglu, Aghion, Bursztyn, and Hemous (2012) study lock-in that arises from initial conditions or "history", viz. innovation in pollution/energy-intensive sectors in the past. Our analysis is complementary to theirs in that we focus on lock-in that arises from expectations rather than history.[4] Moreover, we adopt a different view of technical change in which society has to choose

[2]The literature on induced innovations was introduced by Hicks (1932) and more recently formalized in the directed technical change models of Acemoglu (1998; 2002; 2003) and Kiley (1999). We choose for investment in knowledge instead of in physical capital to orient our analysis towards the long run, when technical change rather than capital accumulation is the determinant of output growth.

[3]Arthur (1989) and David (1985) introduced the notion of lock-in into economics.

[4]Krugman (1991) formalized the distinction between history and expectations as driving force of lock-in.

between incremental change that cannot make scarce resource inputs redundant (because of poor substitution) and radical change in the form of the transition to the backstop. Also in the context of energy use, Cheikbossian and Ricci (2013) consider a game between a resource owner and an R&D firm and show that depending on expectations one out of two equilibria is selected, one with high R&D and slow depletion, and one with low R&D and high depletion. Their two-period framework cannot explicitly address the link to economic growth and ignores the possibility of a radical technology change in the form of a backstop, which is the focus of our study. In a growth context, existing studies of self-fulfilling expectations and technology choice are restricted to a one-factor setting and thus abstract from directed technical change (e.g. Chen and Shimomura, 1998; Cozzi, 2007).

Directed technical change has been studied in the context of energy scarcity in several studies, with Smulders and de Nooij (2003) as an early example. A key question in this literature concerns the role of resource-augmenting technical change relative to other types of technical change. With resource inputs growing at a lower rate than other inputs and poor substitution, resource-augmenting technical change dominates along a balanced growth path, as shown in e.g. André and Smulders (2012). With good substitution, however, the resources are not essential for growth and growth can be sustained without technical change in the resource sector, as in Acemoglu, Aghion, Bursztyn, and Hemous (2012). In the model of Di Maria and Valente (2008), in which a non-renewable resource and physical capital are both essential for production, there may be capital-augmenting technical progress in the short run, but technical change will be purely resource-augmenting along any balanced growth path. Pittel and Bretschger (2010) find that technical change is biased towards the resource-intensive sector at the balanced growth equilibrium of their model economy in which sectors are heterogenous with respect to the intensity of natural resource use. We complement these studies by allowing for a regime shift in energy usage after which the value of accumulated knowledge in the resource sector vanishes.

In our endogenous growth model the final output is produced with labor and energy services according to a constant elasticity of substitution (CES) production function. In line with the empirical evidence in Koetse, de Groot, and Florax (2008) and van der Werf (2008), energy and man-made factors of production are poor substitutes, i.e. the elasticity of substitution between them is smaller than unity. Labor services are produced with labor and a set of specific intermediate goods. Energy services are either derived from the resource combined with another set of intermediate goods, or generated by the backstop technology. The economy is endowed with a finite stock of the non-renewable resource, which can be extracted without costs. The production of intermediate goods and energy generation with the backstop technology both use final output. Technological progress is driven by labor allocated to R&D, which is undertaken by the firms in the two intermediate goods sectors to improve the quality of their products. As a result, there are two types of technical change in the model: labor-augmenting and resource-augmenting technical change. Investment in both types of technical change is driven by profit incentives so that both the rate and the direction of technical progress are endogenously determined. Although the model has three predetermined state variables, we can analyze the dynamics and regime switches by using phase diagrams. To quantify the results, we calibrate the model and perform a simulation analysis.

The remainder of the paper is structured as follows. Section 2 describes the model. Subsequently, the solution procedure is provided in Section 3. Section 4 discusses the transitional dynamics and regime shifts. Section 5 determines the initial resource extraction to complete the solution to the model. Section 6 provides a numerical analysis to quantify the results. Finally, Section 7 concludes.

2 The Model

The model describes a closed economy with two primary factors of production, labor and a non-renewable resource. The productivity of these primary factors of production depends on the quality of complementary intermediate goods, as in Acemoglu (1998). By investing in in-house R&D, firms can increase the quality of the intermediates that they produce. Infinitely lived households derive utility from consumption. They own the resource stock and the firms. The remainder of this section describes the different production sectors, energy generation, the process of research and development, and the household sector in more detail. Mathematical derivations can be found in the Appendix.

2.1 Production

2.1.1 Final Output

Final output Y is produced using labor services Y_L and energy services Y_E according to the following constant elasticity of substitution (CES) specification:[5]

$$Y = \left[\gamma Y_L^{\frac{\sigma-1}{\sigma}} + (1-\gamma) Y_E^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where γ regulates the relative productivity of the inputs and $\sigma \in (0,1)$ denotes the elasticity of substitution between labor and energy services. Profit maximization under perfect competition gives rise to the following relative factor demand function:

$$\frac{\gamma}{1-\gamma} \left(\frac{Y_L}{Y_E} \right)^{-\frac{1}{\sigma}} = \frac{p_{YL}}{p_{YE}}, \tag{2}$$

where p_{YL} and p_{YE} are the prices of labor and energy services, respectively.

2.1.2 Energy Generation

Energy can be derived from resource services Y_R or generated by the backstop technology sector Y_H: $Y_E = Y_R + Y_H$. The generation of energy by the backstop technology requires the final good as input, according to $Y_H = \eta H$, where $\eta > 1$ is a productivity parameter and H denotes the input of the final good.

2.1.3 Service Sector

Labor and resource services are produced according to the following Cobb-Douglas specification:

$$Y_i = Z_i^{\beta} \int_0^1 q_{ik} x_{ik}^{1-\beta} dk, \tag{3}$$

where $i = \{L, R\}$, and $Z_L = L$ and $Z_R = R$ denote the inputs of labor and the resource, respectively. The amount and quality of intermediate good variety k used in sector i are indicated by x_{ik} and q_{ik}, respectively, and the mass of different intermediate goods varieties

[5] Time arguments are omitted if there is no possibility of confusion.

in each sector is normalized to unity. The resource can be extracted from the initial resource stock S_0, without extraction costs:

$$\dot{S} = -R, \quad R \geq 0, \quad \int_0^\infty R(t)dt \leq S_0. \tag{4}$$

Producers in the perfectly competitive service sectors take factor remunerations w_i and intermediate goods prices p_{xik} as given. Their resulting demand for primary inputs and intermediate goods follows from

$$p_{Yi}\frac{\partial Y_i}{\partial Z_i} = w_i, \tag{5a}$$

$$p_{Yi}\frac{\partial Y_i}{\partial x_{ik}} = p_{xik}. \tag{5b}$$

2.1.4 Intermediate Goods Sector

Each firm in the monopolistically competitive intermediate goods sector produces a unique variety and faces a demand function from the service sector, according to (5b). Per unit production costs are equal to q_{ik} units of the final good, so that production costs increase proportionally with quality. Firms invest in R&D to increase the quality of their products, according to the following specification:[6]

$$\dot{q_{ik}} = \xi_i Q_i D_{ik}, \tag{6}$$

where ξ_i is a productivity parameter, $Q_i \equiv \int_0^1 q_{ik}dk$ is the aggregate quality level in sector i, and D_{ik} is labor allocated to R&D at unit cost w_D. The producer of each variety chooses how much to produce and how much to spend on in-house R&D in order to maximize the net present value of its profits, giving rise to the following optimality conditions:

$$p_{xik} = \frac{q_{ik}p_Y}{1-\beta}, \tag{7a}$$

$$\lambda_{ik}\xi_i Q_i \leqq w_D \quad \text{with equality if } D_{ik} > 0, \tag{7b}$$

$$\frac{\beta}{1-\beta}x_{ik}p_Y = -\dot{\lambda}_{ik} + r\lambda_{ik}, \tag{7c}$$

[6] Dots above a variable denote time derivatives, i.e. $\dot{x} = dx/dt$, and hats denote growth rates, i.e. $\hat{x} = \frac{dx/dt}{x}$.

where p_Y denotes the price of the final good, r is the nominal interest rate, and the λ_i's are shadow prices of quality in sector i. Price setting equation (7a) shows that firms charge a mark-up over marginal costs. Condition (7b) requires that, at an interior solution, the marginal revenue of improving quality is equal to its marginal costs. Equation (7c) describes the evolution of the shadow prices of quality. We combine the supply function (7a) with the demand for intermediate goods varieties (5b) and the production function (3) to find

$$x_{ik} = x_i = \frac{\theta_i Y(1-\beta)^2}{Q_i}, \tag{8}$$

where $i = \{L, R\}$, and the θ_i's denote the incomes shares of labor and resource services: $\theta_i \equiv p_{Yi}Y_i/(p_Y Y)$. This expression implies that all intermediate goods producers within the same sector produce the same output level x_i. Combining (7b) with (7c) and (8), we get:

$$r = \beta(1-\beta)\xi_i\theta_i \frac{Y p_Y}{w_D} + \hat{w}_D - \hat{Q}_i, \quad \text{if } D_{ik} > 0. \tag{9}$$

Equation (9) can be interpreted as a no-arbitrage condition that requires firms to earn the market interest rate on investment in quality improvements. This return depends positively on the relevant income shares θ_i (price effect: quality improvements of relatively scarce factors are more valuable) and on the rate of change in the cost of quality improvements $\hat{w}_D - \hat{Q}_i$ (capital gain effect: increasing research costs make current improvements more valuable in the future). The transversality conditions associated with the problem of firms in the intermediate goods sector are:

$$\lim_{z\to\infty} \lambda_L(z)Q_L(z)e^{-\int_0^z r(s)ds} = 0, \tag{10a}$$

$$\lambda_R(T^*)Q_R(T^*)e^{-\int_0^{T^*} r(s)ds} = 0 \Rightarrow \lambda_R(T^*) = 0, \tag{10b}$$

where T^* denotes the time at which the economy switches from using the non-renewable resource to using the backstop technology. Transversality condition (10a) requires that the shadow price of quality in the labor service sector vanishes if time goes to infinity, and (10b) requires the shadow price of quality in the resource service sector to be zero at the moment the economy switches from the resource to the backstop.

2.2 Goods and Factor Market Equilibrium

The goods market equilibrium condition is given by:

$$Y = C + \int_0^1 q_{Lk} x_{Lk} dk + \int_0^1 q_{Rk} x_{Rk} dk + H = \frac{C + H}{1 - [1 - \theta_E \omega_H](1 - \beta)^2}, \tag{11}$$

where $\theta_E \equiv p_{YE} Y_E/(p_Y Y)$, $\omega_H \equiv p_{YH} Y_H/(p_{YE} Y_E)$, and the second equality uses (8). Labor market equilibrium requires that labor supply L^S equals labor demand from the labor service sector and from R&D:

$$L^S = Z_L + D, \tag{12}$$

where $D \equiv D_L + D_R$ and $D_i \equiv \int_0^1 D_{ik} dk$ is aggregate research effort in sector i. Labor is perfectly mobile between the production and the research sector, which gives rise to a uniform wage rate in equilibrium: $w_D = w_L$. By using the income share definitions, labor market equilibrium implies:

$$L = \beta \theta_L \frac{Y}{w_L/p_Y} \tag{13}$$

2.3 Households

The representative household lives forever, derives utility from consumption of the final good, and inelastically supplies L^S units of labor at each moment. It owns the resource stock with value $w_R S$ and all equity in intermediate goods firms with value $\lambda_L Q_L + \lambda_R Q_R$. The household maximizes lifetime utility $U(t) = \int_t^\infty \ln C(z) e^{-\rho(z-t)} dz$, subject to its flow budget constraint $\dot{V} = r(V - w_R S) + \dot{w}_R S + w L^S - p_Y C$, and a transversality condition: $\lim_{z\to\infty} \lambda_V(z) V(z) e^{-\rho z} = 0$, where ρ denotes the pure rate of time preference, V total wealth, and λ_V the shadow price of wealth. Optimizing behavior of the households gives rise to the following two familiar conditions:

$$\hat{C} = r - \hat{p}_Y - \rho, \tag{14a}$$

$$\hat{w}_R = r. \tag{14b}$$

Condition (14a) is the Ramsey rule, which relates the growth rate of consumption to the difference between the real interest rate and the pure rate of time preference. Condition (14b) is the Hotelling rule, which requires the resource price to grow at the interest rate so that resource owners are indifferent between extracting and conserving an additional unit of the resource.

3 Solving the Model

In this section, we show that the economy may experience different regimes of technical change and energy generation. Our solution procedure consists of three steps. First, we describe the dynamic behavior of the economy during each regime. Second, we link the three regimes together by using a set of continuity conditions. Finally, we show under which conditions the economy actually shifts from one regime to the other.

3.1 Regime 1: Resource Use, Mixed Technical Change

Definition 1 *Regime 1 is defined as a situation in which energy generation relies completely on the non-renewable resource (i.e., $Y_E = Y_R$) and in which there occurs both labor- and resource-augmenting technical progress (i.e., $D_L > 0$,$D_R > 0$).*

By combining (3), (5a), and (8), and imposing $\omega_H = Y_H = 0$ we rewrite the relative factor demand from the final good sector (2) in regime 1 as

$$\frac{\theta_E}{1-\theta_E} = \left(\frac{w_R}{w_L}\frac{Q_L}{Q_R}\right)^{1-\nu}\left(\frac{1-\gamma}{\gamma}\right)^{\sigma}, \tag{15}$$

where $\nu \equiv 1-\beta(1-\sigma)$ so that $\nu \in (0,1)$ because $\sigma \in (0,1)$. Converting (15) into growth rates and using the Hotelling rule (14b), we obtain:

$$\hat{\theta}_E = (1-\nu)(1-\theta_E)\left[r - \hat{w}_L + \hat{Q}_L - \hat{Q}_R\right]. \tag{16}$$

Equation (16) implies that the energy income share increases if, after correcting for relative productivity changes, the natural resource price grows faster than the wage rate. Using (9) for both sectors, we can derive an expression for the endogenous bias in technological change:

$$\hat{Q}_R - \hat{Q}_L = \beta(1-\beta)\frac{Y}{w_L/p_Y}\left[\theta_E\xi_R - \theta_L\xi_L\right]. \tag{17}$$

The bias in technological progress depends on the energy income share: if the resource is scarce and therefore the energy income share is large, technological change will be relatively resource-augmenting and *vice versa*. Aggregating (6) over all firms in the sector, we find $\hat{Q}_i = \xi_i D_i$. Combining this expression with (9) and $D = D_L + D_R$, and using the Ramsey rule (14a), we obtain an expression that relates output growth to aggregate research effort:

$$\hat{Y} - (\hat{w}_L - \hat{p}_Y) = r - \hat{w}_L - \rho = \psi^{-1}\left[\beta(1-\beta)\frac{Y}{w_L/p_Y} - D\right] - \rho, \tag{18}$$

where we have defined $\psi \equiv \xi_R^{-1} + \xi_L^{-1}$. By using (5a) and imposing $\omega_H = 0$, resource extraction growth can be expressed as:

$$\hat{R} = \hat{\theta}_E - \rho. \tag{19}$$

Converting the labor market equilibrium condition (13) into growth rates and using the Ramsey rule (14a), we find:

$$\hat{L} = \hat{\theta}_L + r - \hat{w}_L - \rho, \tag{20}$$

The results in this subsection together give rise to the dynamic system described in Proposition 1.

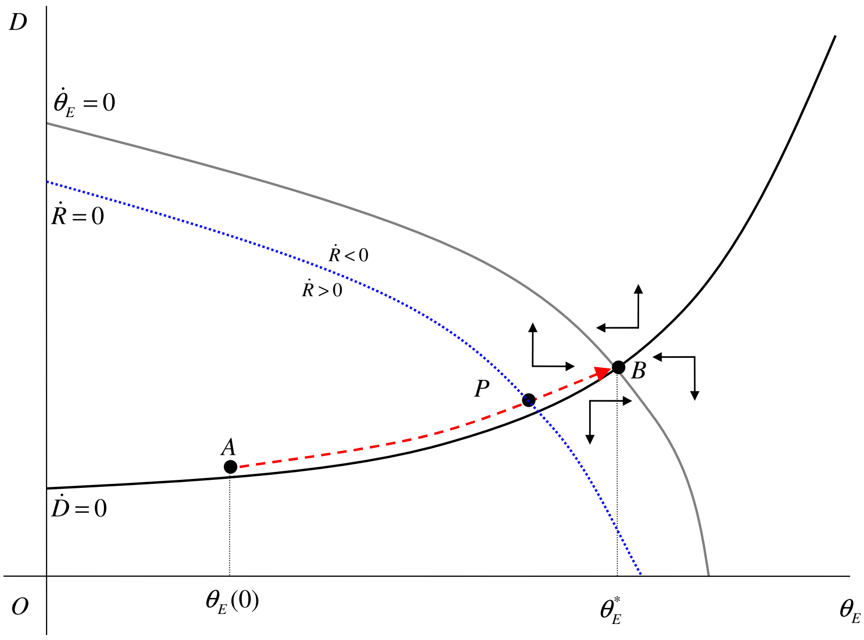
Proposition 1 *The dynamics of regime 1 are described by the following two-dimensional system of first-order nonlinear differential equations in θ_E and D:*

$$\dot{\theta}_E = \theta_E(1-\nu)(1-\beta)\left[\left(\frac{L^S}{\psi} - \frac{D}{\psi}\right)(1 - \psi\left[\theta_E\xi_R - (1-\theta_E)\xi_L\right]) - \frac{1-\theta_E}{1-\beta}\frac{D}{\psi}\right], \tag{21a}$$

$$\dot{D} = \frac{L^S - D}{1-\theta_E}\frac{1}{\psi}\left\{\rho\psi(1-\theta_E) - \left[1-(1-\nu)\theta_E\right]\left[(1-\beta)(L^S - D) - (1-\theta_E)D\right] - \right.$$
$$\left. \theta_E(1-\nu)(1-\beta)\psi[\theta_E\xi_R - (1-\theta_E)\xi_L](L^S - D)\right\}. \tag{21b}$$

Proof. Substitution of (13), (17), and (18) into (16) gives (21a), which proves the first part. The second part of the proof follows immediately from substituting (12), (13), (17), (18), and (21a) into (20). □

Figure 1: Phase diagram in (θ_E, D) space: Regime 1 without backstop



Notes: The solid black and gray lines represent the research and income share isoclines, respectively. The dotted line is the extraction isocline. The dashed arrow represents the saddle path that leads to point B.

Figure 1 shows the phase diagram of regime 1 in (θ_E, D)-space. The figure contains three isoclines that we will discuss in turn. First, the income share isocline $\dot{\theta}_E = 0$, derived from (21a), gives combinations of θ_E and D for which the income shares are constant. Prices of energy and labor services, corrected for productivity changes, grow at the same rate along the income share isocline. For all points below the income share isocline, the relative price of the resource and with it the energy income share increase over time and *vice versa*. The $\dot{\theta}_E = 0$ line is downward sloping, because an increase in θ_E induces technological change to become relatively more resource saving, which puts downward pressure on the energy income share. To counteract this effect, aggregate research must fall to increase the growth rate of the relative price and the income share of the resource. The income share isocline has a vertical asymptote at $\theta_E = \bar{\theta}_E > 0$, to the left of which it tends to minus infinity. The dynamic behavior of θ_E is illustrated by the horizontal arrows in the phase diagram.

Second, the research isocline, derived from (21b), gives combinations of θ_E and D for which aggregate research effort is constant over time. At points below the research isocline, output growth is relatively large compared to the growth rate of the real wage. As a result, labor demand L tends to increase over time, which leads to declining employment in research over time. Although the figure shows a monotonically upward sloping research isocline, this is not necessarily the case. For low values of the energy income share, the locus may be downward-sloping.[7]

Third, the extraction isocline, derived from (19), gives combinations of θ_E and D for which resource extraction is constant over time.[8] At points below the isocline, the energy income share grows relatively fast, so that resource extraction increases over time. The extraction isocline has a negative slope, because an increase in θ_E induces technological change to become relatively more resource-saving, which puts downward pressure on resource extraction. To counteract this effect, aggregate research must fall. Like the income share isocline, the extraction isocline has a vertical asymptote at $\theta_E = \bar{\theta}_E > 0$, to the left of which it tends to minus infinity.

Because it will affect the dynamic behavior of the economy, it is important to determine the relative positions of the isoclines in the phase diagram. In Appendix A.6-A.7, we show that the income share and research isoclines intersect once in the relevant plane with $D \in [0, L^S]$ and $\theta \in [0, 1]$, that the vertical intercept of the income share isocline is located above those of the research and extraction isoclines, and that the vertical intercept of the extraction isocline tends to minus infinity if the elasticity of substitution between labor and resource services, σ, goes to unity.

If the economy would stay in regime 1 forever, it would converge along the stable manifold from point A to point B in Figure 1.[9] Along the stable manifold, two counteracting forces affect the energy income share. On the one hand, increasing physical scarcity of the resource puts upward pressure on the energy income share. On the other hand, the income share

[7] Appendix A.6 shows that the research isocline must be upward sloping at $\theta_E = 1$ and may be downward sloping at $\theta_E = 0$.

[8] By substituting (16), (17), and (18) into (19), one obtains a differential equation for R in terms of θ_E and D.

[9] The determination of point A will be discussed in Section 5.

is negatively affected by induced resource-augmenting technical change.[10] These opposing effects exactly offset each other in the steady state equilibrium, resulting in a constant energy income share. In case the stable manifold starts out below the extraction isocline, the economy necessarily crosses the extraction isocline before the steady state is reached. As a result, resource extraction can only increase temporarily and peaks when the economy crosses point P in Figure 1.

However, the economy does not necessarily stay in regime 1 forever. Section 3.4 discusses under which conditions the economy eventually shifts to another regime. In case of a regime shift, the economy does no longer converge to point B in the phase diagram. Sections 4 and 5 discuss the determination of the begin and end point of regime 1 in this case.

3.2 Regime 2: Resource Use, Labor-Augmenting Technical Change

Definition 2 *Regime 2 is defined as a situation in which energy generation relies completely on the non-renewable resource (i.e., $Y_E = Y_R$), but in which the shadow price of resource-augmenting technology is strictly lower than the marginal cost of investment in quality improvement of resource complementing intermediates. Therefore, regime 2 is characterized by purely labor-augmenting technical progress (i.e., $D_R = 0$, $D_L = D$).*

Expressions for the return to quality improvements and income share growth in regime 2 are easily obtained by imposing $\hat{Q}_R = 0$ in (9) and (16), respectively:

$$r = \beta(1-\beta)\xi_L(1-\theta_E)\frac{Y}{w/p_Y} + \hat{w}_L - \hat{Q}_L, \tag{22a}$$

$$\hat{\theta}_E = (1-\nu)(1-\theta_E)\left[r - \hat{w}_L + \hat{Q}_L\right]. \tag{22b}$$

The growth rate of resource extraction is still described by (19). By using the goods market equilibrium condition (11), the Ramsey rule (14a), and (22a), output growth can be written as

$$\hat{Y} = r - \hat{p}_Y - \rho = \xi_L(1-\beta)(1-\theta_E)(L^S - D) - \rho. \tag{23}$$

[10] We focus on a relatively high initial resource stock, so that the economy is located on the part of the stable manifold below the income share isocline, where the energy income share increases because the scarcity effect dominates the induced technical change effect.

Given that $D_L = D$, (6) implies $\hat{Q}_L = \xi_L D$. Combining this expression with (13) and (22a)-(22b), we obtain the dynamic system described in Proposition 2.

Proposition 2 *The dynamics of regime 2 at an interior solution (i.e., with $D > 0$) are described by the following two-dimensional system of first-order nonlinear differential equations in θ_E and D:*

$$\dot{\theta}_E = \theta_E(1-\nu)(1-\theta_E)\xi_L(1-\beta)(L^S - D), \tag{24a}$$

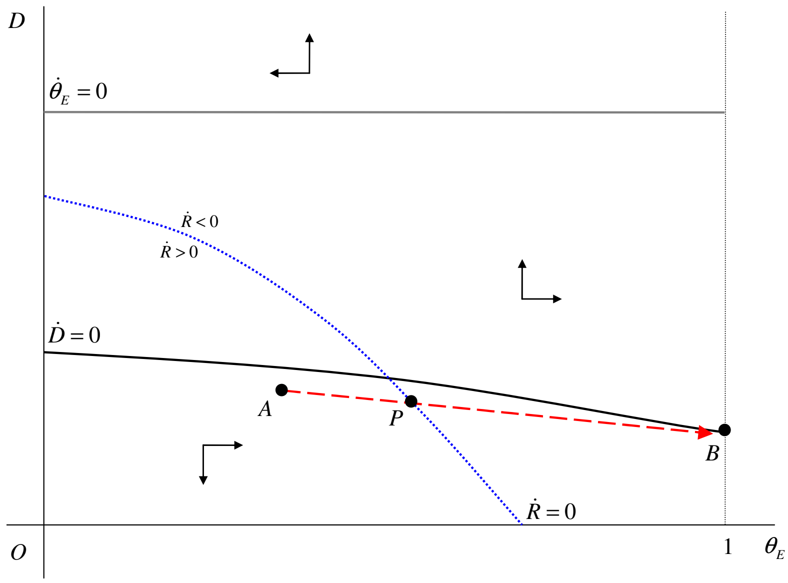
$$\dot{D} = (L^S - D)\left\{\rho - [1-\theta_E(1-\nu)]\,\xi_L(1-\beta)(L^S - D) + \xi_L D\right\}. \tag{24b}$$

Proof. Substitution of (6), (12), (13), and (22a), into (22b) gives (24a). Combining (6), (12), (22a) and (22b) with (20) results in (24b). □

Figure 2 shows the phase diagram of regime 2 in (θ_E, D)-space. We will discuss the income share, research, and extraction isoclines in turn. The income share isocline $\dot{\theta}_E = 0$ is derived from (24a) and gives combinations of θ_E and D for which the income shares are constant over time. There is a unique research level associated with constant income shares, so that the income share isocline is horizontal at this specific value of D. The growth rate of the prices of resource and labor services are equal along the $\dot{\theta}_E = 0$ isocline. At points below the isocline, the price of resource service increases relative to that of labor services, resulting in an increasing energy income share over time and *vice versa*. The dynamic behavior of θ_E is illustrated by the horizontal arrows in the phase diagram.

The research isocline $\dot{D} = 0$ is derived from (24b) and gives combinations of θ_E and D for which research is constant over time. It is represented by a downward sloping line, because an increase in θ_E leads to a lower real interest rate and therefore slower output growth (see (23)). As a result, L tends to decrease over time, which induces a flow of labor from the production to the research sector, causing the innovation rate to rise over time. To counteract this effect, D must decrease thereby increasing the growth rate of labor demand as a result of its combined effect on output growth (through the real interest rate) and the productivity of the factors of production. At points above of the innovation locus, the real interest rate and output growth are lower than in steady state equilibrium, so that L declines and the innovation rate increases over time and *vice versa*. The dynamic behavior of D is illustrated by the vertical arrows in the phase diagram.

Figure 2: Phase diagram in (θ_E, D) space: Regime 2 without backstop



Notes: The solid black and gray lines represent the research and income share isoclines, respectively. The dotted line is the extraction isocline. The dashed arrow represents the saddle path that leads to point B.

The extraction isocline $\dot{R} = 0$ is derived from (19) and gives combinations of θ_E and D for which resource extraction is constant over time.[11] The $\dot{R} = 0$ line is downward sloping, because an increase in θ_E leads to a lower return to research and therefore a lower real interest rate in equilibrium. As a result, the growth rates of output and resource demand go down. To counteract this effect, D must decrease to enhance the growth of resource demand through its combined effect on the real interest rate and the efficiency of resource extraction. At points above the extraction isocline, the real interest rate and therefore output growth are lower than required for constant extraction, so that extraction growth becomes negative and *vice versa*.

In Appendix A.9-A.10, we show that the income share isocline is always located above the research and extraction isoclines, and that the vertical intercept of the extraction isocline tends to minus infinity if the elasticity of substitution between labor and resource services,

[11]By substituting (13), (6), and (22a) into (19), one obtains a differential equation for R in terms of θ_E and D.

σ, goes to unity.

The dynamics in Figure 2 describe the behavior of the economy in regime 2. If the economy would stay in regime 2 forever, it would converge along the dashed equilibrium path towards point B: increasing resource scarcity drives up the energy income share and depresses labor-augmenting technical change over time. However, as shown in Section 3.4, regime 2 cannot last forever and the economy will shift to another regime. The implied begin and end point in the phase diagram of regime 2 are determined in Sections 4 and 5.

3.3 Regime 3: Backstop Use, Labor-Augmenting Technical Change

Definition 3 *Regime 3 is defined as a situation in which the resource stock will be depleted and the backstop technology is used instead (i.e.,* $S = 0$*,* $Y_E = Y_H$*). As a result, only pure labor-augmenting technological progress is possible (i.e.,* $D_R = 0$*,* $D_L = D$*).*

Final good production in regime 3 is given by:

$$Y = \left[\gamma Y_L^{\frac{\sigma-1}{\sigma}} + (1-\gamma) Y_H^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where Y_H denotes energy generation by the backstop technology. Perfect competition implies that the price of energy generated with the backstop technology is equal to its marginal production cost: $p_{YH} = \eta^{-1} p_Y$. Using this equality and the income share definitions in $p_{YH} = p_Y \partial Y / \partial Y_H$, we obtain

$$\theta_E = (1-\gamma)^{\sigma} \eta^{\sigma-1}. \tag{25}$$

Hence, the energy income share is constant over time. Substitution of $\hat{\theta}_E = 0$, (13), and (22a) into (20) gives rise to the following differential equation for research:

$$\dot{D} = (L^S - D)\left\{\rho - \xi_L\left[(1-\beta)(L^S - D) - D\right]\right\}. \tag{26}$$

Proposition 3 summarizes the behavior of the economy in regime 3.

Proposition 3 *The dynamic system of regime 3 gives rise to a constant θ_E and D, given by:*

$$\theta_E = (1-\gamma)^\sigma \eta^{\sigma-1}, \tag{27a}$$

$$D = \frac{(1-\beta)L^S - \frac{\rho}{\xi_L}}{2-\beta}. \tag{27b}$$

Proof. The first part has already been derived in the text. The second part follows because the differential equation (26) is unstable in D, so that the economy immediately settles at its steady state level of research, given by (27b). □

3.4 Sequence of Regimes

Assuming that $D_L > 0$ always holds, the three regimes discussed so far provide an exhaustive list of situations that may occur.[12] Which regime actually prevails at a certain moment in time depends crucially on whether condition (7b) with $i = R$ is satisfied with equality or inequality. In case of inequality, the marginal revenue of quality improvements in the resource sector is lower than its marginal cost, so that there will be no resource-augmenting technical change and the dynamics are described by regime 2. This is either possible if the resource stock is large and therefore the resource income share is small (see (7c) and (5b)) or if the stock is small and a shift from using fossil fuels to the backstop technology is imminent (see (10b)). In between, there may be an interval for which the condition (7b) with $i = R$ holds with equality, so that the dynamics are described by regime 1. We assume that this actually is the case, by choosing ξ_R sufficiently large. Hence, with a sufficiently large resource stock, the economy will start in regime 2 and then move to regime 1. Subsequently, if the backstop will eventually be introduced, the economy shifts back to regime 2 and finally moves to regime 3. However, if the backstop technology will not be implemented, the economy will remain in regime 1 forever. Section 4 discusses whether or not the backstop technology will eventually be implemented.

[12] Appendix A.12 shows that there does not exist a regime of simultaneous use of the resource and the backstop technology. Moreover, $D_L > 0$ can be ensured by choosing xi_L large enough.

3.5 Linking the Regimes

We link the 3 regimes together by imposing 3 continuity conditions. First, optimal behavior of the resource owners ensures that the resource price is equal to the backstop price at the moment that the resource stock is depleted. This implies a continuous energy price at the regime shift where the economy switches from using the resource to using the backstop technology:

$$\lim_{t \to \uparrow T_{23}} p_{YE}(t) = \lim_{t \to \downarrow T_{23}} p_{YE}(t), \tag{28}$$

where T_{23} denotes the time at which the economy shifts from regime 2 to regime 3. Second, the Ramsey rule (14a) requires consumption to be continuous as long as the real interest rate is finite, which constitutes our second continuity condition:

$$\lim_{t \to \uparrow T_{ij}} C(t) = \lim_{t \to \downarrow T_{ij}} C(t), \tag{29}$$

where $i, j \in \mathbb{N}$ indicate the regimes and T_{ij} denotes the time at which the economy shifts from regime i to regime j. Third, the shadow price of quality in the service sectors λ_k should be continuous at the regime shifts:

$$\lim_{t \to \uparrow T_{ij}} \lambda_k(t) = \lim_{t \to \downarrow T_{ij}} \lambda_k(t), \tag{30}$$

where $k = \{R, L\}$. Intuitively, the condition requires that the marginal cost of improving the quality of the intermediate variety at the very end of regime i equals the value of this additional quality at the beginning of the consecutive regime.

4 Transitional Dynamics and Regime Shifts

This section implements the solution method described in Section 3. We first characterize the solution to the model for the scenario in which the backstop technology will necessarily become competitive eventually (Sections 4.1 and 4.2). Subsequently, Section 4.3 discusses scenarios in which the transition to the backstop technology does not occur, or when the eventual introduction of the backstop technology becomes a self-fulfilling prophecy. Which scenario actually prevails, depends crucially on the productivity of the backstop technology.

We use backward induction to determine the equilibrium path in (θ_E, D)-space that starts in regime 1, runs through regime 2, and finally ends as a fixed point in regime 3.[13] Starting with the fixed point in regime 3, we can use the continuity conditions to find the end point of regime 2. Subsequently, we use the transversality condition for the shadow price of quality in the resource service sector to find the starting point of regime 2. Then, the continuity conditions will give us the end point of regime 1, after which we close the model by solving for the initial value of the energy income share that clears the non-renewable resource market.

4.1 Shift from Resource to Backstop

At the shift from regime 2 to regime 3, the economy switches from generating energy with the non-renewable resource to producing energy with the backstop technology. Condition (29) requires that consumption is continuous at this switching instant. Using the goods market equilibrium condition (11), continuity of consumption requires[14]

$$Y_{23}^{-} = (1 - \theta_{E3}^{+})Y_3^{+}, \tag{31}$$

where we have imposed $H = 0$ and $\omega_H = 0$ on the left hand side, and $\omega_H = 1$ and $H = \theta_E Y$ on the right hand side. By using the income share definitions we rewrite output as:

$$Y = Y_L \left[\gamma + (1-\gamma) \left(\frac{\theta_E}{1-\theta_E} \frac{p_{YL}}{p_{YE}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

Using the continuity of prices and income shares, a jump in Y must be proportional to a jump in Y_L. Furthermore, it follows from (3) and (8) that a jump in Y_L is proportional to a jump in L. Combining this with (31), the continuity condition becomes:

$$\frac{L^S - D_3^{+}}{L^S - D_{23}^{-}} = \frac{1}{1-\theta_{E3}^{+}} \Rightarrow D_{23}^{-} = D_{23}^{+}(1-\theta_{E3}^{+}) + \theta_{E3}^{+} L^S. \tag{32}$$

Both aggregate research and the energy income share are constant over time in regime 3, so that D_3^{+} and θ_{E3}^{+} are given by the right-hand-sides of (27a) and (27b), respectively.

[13] We abstract from the first occurrence of regime 2 by choosing the initial resource stock small enough.

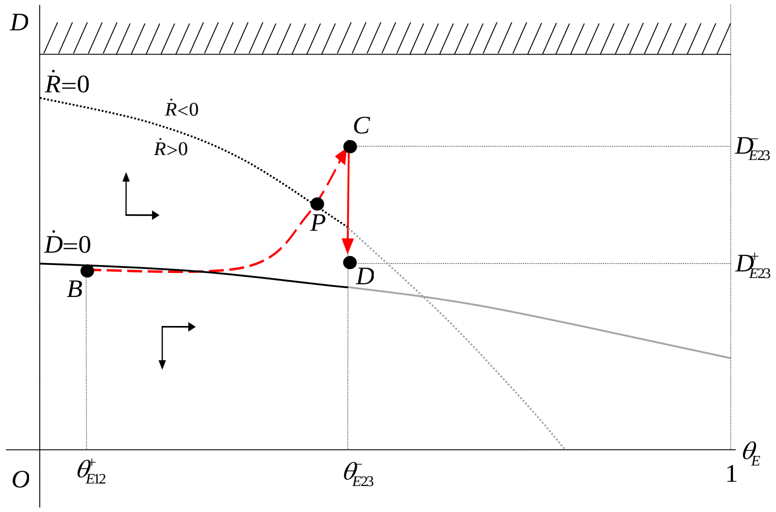
[14] We use the conventional shortcut notation $x_{ij}^{+} \equiv \lim_{t \downarrow T_{ij}} x_j(t)$ and $x_{ij}^{-} \equiv \lim_{t \uparrow T_{ij}} x_i(t)$.

Substitution of (27b) into (32) gives the level of research at the very end of regime 2, such that the corresponding upward jump in output is exactly high enough to keep consumption continuous at the regime shift:

$$D_{23}^{-} = \frac{\xi_L L^S(1-\beta+\theta_{E3}^{+}) - \rho(1-\theta_{E3}^{+})}{\xi_L(2-\beta)}. \tag{33}$$

The energy income share at the end of regime 2, θ_{23}^{-}, is pinned down by equation (27a). Hence, we have determined the point $(\theta_{E23}^{-}, D_{23}^{-})$ to which the economy converges during the second regime. Figure 3 shows the equilibrium path leading to point D. Along the equilibrium

Figure 3: Phase diagram in (θ_E, D) space: Regime 2 and 3



Notes: The solid line and dotted lines represent the research and extraction isoclines in regime 2, respectively. A part of each isocline is plotted in gray, to indicated that they are only valid for $\theta < \theta_{E23}^{-}$.

path, the resource income share is increasing over time. Aggregate research may be declining initially, but it increases during the run-up to the backstop technology until the moment of the regime shift. As soon as the economy hits point C, aggregate research jumps down to point D. The reason is that consumers want to prevent a downward jump in consumption at the regime shift, when energy generation with the backstop technology starts using output. Put differently, by investing relatively more now, consumers effectively shift part of the resource wealth to the backstop era.

In Figure 3, the equilibrium path crosses the extraction isocline at point P, so that resource extraction first increases, peaks at point P and then declines over time until the regime shift. The location of point B in the figure, which marks the regime shift from an era of both resource and labor-augmenting technological progress to a regime of purely labor-augmenting technical change, will be determined below.

4.2 Shift to Purely Labor-Augmenting Technical Change

According to transversality condition (10b), the shadow price of resource-augmenting technical change should be zero at the end of regime 2. Intuitively, after the regime switch at T_{23}, the resource will not be used anymore so that resource-augmenting technology is worthless from that moment onward. Optimality condition (7b) requires that the marginal cost is equal to the marginal value of quality improvement in regime 1. In regime 2, however, this equality does no longer hold: the marginal cost is now larger than the marginal value. We exploit this distinction between the two regimes to determine the time of the shift from regime 1 to regime 2, T_{12}.

First, we define the ratio of marginal value and cost of quality improvements as follows:

$$\mu \equiv \frac{\xi_R Q_R \lambda_R}{w_L}. \tag{34}$$

At the end of regime 1, (7b) holds with equality, so that $\mu(T_{12}^-) = 1$. At the end of regime 2, transversality condition (10b) implies $\mu(T_{23}^-) = 0$. Using the continuity condition for λ_R, (30) with $i = R$ and (7b), we find the value for μ at the beginning of regime 2:[15]

$$\lambda_R(T_{12}^-) = \lambda_R(T_{12}^+) \Rightarrow \mu(T_{12}^-) = \mu(T_{12}^+). \tag{35}$$

As a result, for regime 2 we have the following begin and end condition:

$$\mu(T_{12}^+) = 1, \tag{36a}$$

$$\mu(T_{23}^-) = 0. \tag{36b}$$

[15] The continuity of w_L follows from (9).

Combining (7b), (7c), (8), and (34) we find a differential equation for μ:

$$\dot{\mu} = (r - \hat{w}_L)\mu - \xi_R \beta(1-\beta)\theta_E \frac{Y}{w_L/p_Y}$$

Substitution of (9) and (13) into this expression, we obtain a differential equation for μ in terms of θ_E and D:

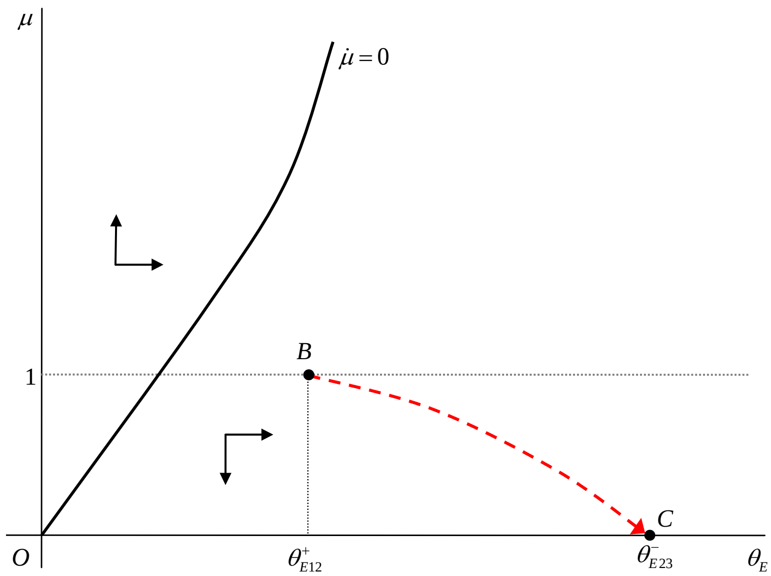
$$\dot{\mu} = \mu\left[(1-\beta)\xi_L(L^S - D) - \xi_L D\right] - \frac{\theta_E}{1-\theta_E}\xi_R(1-\beta)(L^S - D). \tag{37}$$

Because the time paths for θ_E and D are already determined, together with the begin and end conditions (36a)-(36b), equation (37) can be used to find the energy income share at the beginning of regime 2. As a result, this procedure pins down the points $(\theta^+_{E12}, D^+_{12})$ and, because of the continuity conditions, $(\theta^-_{E12}, D^-_{12})$. Figure 4 shows the equilibrium path leading to point C in the (θ_E, μ)-plane. Regime 2 starts at point B, where $\mu = 1$ and investment in resource-augmenting technical progress is still profitable. As θ_E increases further, however, the switch to the backstop technology (when resource-augmenting technology becomes obsolete) is so close that μ falls short of unity, so that D_R jumps down to zero.[16] This event marks the beginning of regime 2. Having determined this point $(\theta^+_{E12}, D^+_{12})$, we have also pinned down the point to which the economy should converge during regime 1.

Figure 5 shows the equilibrium path that leads to point B in the (θ_E, D)-plane. The energy income share is increasing over time along this path. Initially, the resulting price effect induces an increase in profits per unit of quality in the resource sector, leading to an increase in resource-augmenting technical change. This effect is strong enough to outweigh the decreasing amount of labor-augmenting research (as the labor income share declines) so that aggregate research increases. However, as the introduction of the backstop technology comes closer, the remaining time during which quality improvements in the resource sector generate profits becomes smaller. As a result, the increase of resource-augmenting research diminishes and aggregate research start to decline over time until the end of regime 1, when resource-augmenting technical change stops. This decline of aggregate research at the end of regime 1 necessarily occurs if the beginning of regime 2 is located below the research isocline,

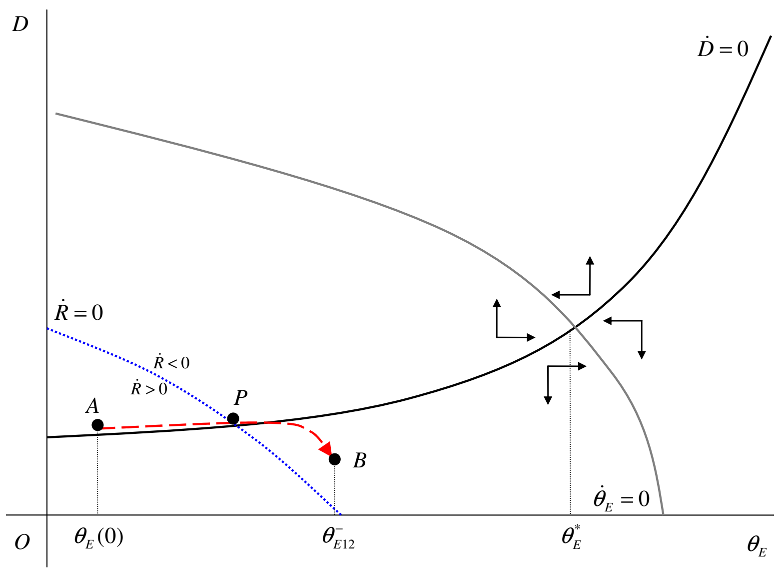
[16] Intuitively, the downward jump in D_R occurs because $\dot{Q}_R$ is linear in D_R (see (6)), so that the marginal cost (in terms of required researchers) of quality improvement does not depend on D_R. The proof for the downward jump in D_R can be found in Appendix A.13.

Figure 4: Phase diagram in (θ_E, μ) space: Regime 2



Notes: The solid black line represents the $\dot{\mu} = 0$ isocline. The gray line corresponds with $\mu = 1$. The dashed arrow represents the equilibrium path that leads to point C.

Figure 5: Phase diagram in (θ_E, D) space: Regime 1



Notes: The solid black and gray lines represent the research and income share isoclines, respectively. The dotted line is the extraction isocline. The fat dots represent the equilibrium path that leads to point B.

as shown Figure 3.

The starting point A on the equilibrium path depends on the initial resource stock. In Section 5, we will derive the resource market clearing condition that determines the initial energy income share and with it, given that we know the equilibrium path, the location of point A.

4.3 Transition Towards Backstop Technology as Self-Fulfilling Prophecy

In this section, we discuss the scenarios in which the backstop technology will never become competitive or when the implementation of the backstop technology is a self-fulfilling prophecy. Proposition 4 summarizes the results of this section.

Proposition 4 *Assuming that $\theta_E(0) < \theta_E^*$, the following three scenarios of backstop technology implementation can be distinguished:*

(i) if $\theta_E^* > \theta_{E3}^+$, the backstop technology will eventually be implemented;

(ii) if $\theta_E^* < \theta_{E3}^+$ and $\theta_{E12}^- > \tilde{\theta}_E$, the backstop technology will never be implemented;

(iii) if $\theta_E^* < \theta_{E3}^+$ and $\theta_{E12}^- < \tilde{\theta}_E$, the implementation of the backstop technology becomes a self-fulfilling prophecy;

where θ_E^* denotes the energy income share at the intersection of the research and income share isoclines in regime 1 and $\tilde{\theta}_E \equiv \theta_E|_{\dot{\theta}_E=0,D=D_{12}^-}$ in regime 1.[17]

Proof. We compare two possible paths in the (θ_E, D) phase diagram: the first one (which we will call 'conservative') is the path in regime 1 that leads to the intersection of the income share isocline and the research isocline in the (θ_E, D)-plane. This corresponds to the saddle path of the model without a backstop technology (see Figure 2). The second one (which we will call 'progressive') is the equilibrium path that ultimately leads to the implementation of the backstop technology, as described in Sections 4.1 and 4.2.

In case (i), the conservative path cannot be an equilibrium, because this path necessarily intersects the line $\theta = \theta_{23}^+$, so that along part of the conservative path the inequality $\theta_E > \theta_{E23}^+$ holds. As a result, the non-renewable resource is relatively more expensive than the backstop

[17] The expression for θ_E^* in terms of parameters can be found in Appendix A.6.

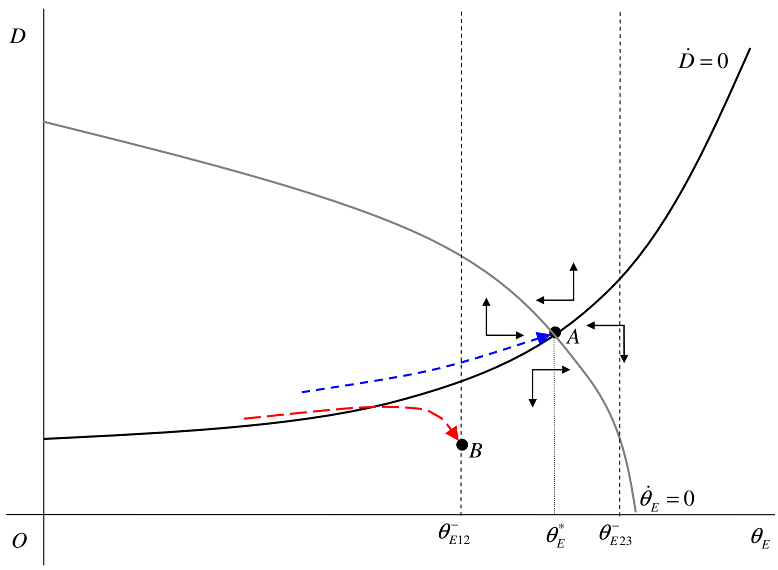
technology. Hence, the resource will not be used anymore and the dynamics of the economy are no longer described by the dynamic system of regime 1. The progressive path is the only remaining equilibrium. This is the scenario described in Sections 4.1 and 4.2.

In case (ii), the progressive path cannot be an equilibrium, because the end point of regime 1 would be located to the right of the income share locus. This point can only be approached if $\theta_E(0) > \tilde{\theta}_E$. However, we have assumed that the initial resource stock is large enough to have $\theta_E(0) < \theta_E^* < \tilde{\theta}_E$. The conservative path can be an equilibrium, because it does not intersect the line $\theta_E = \theta_{E23}^+$, so that along the entire path the inequality $\theta_E < \theta_{E23}^+$ holds. As a result, the non-renewable resource is relatively cheaper than the backstop technology and the dynamics of the economy are accurately described by the dynamic system of regime 1.

In case (iii), both the conservative and the progressive path can be an equilibrium. The conservative path does not intersect the line $\theta_E = \theta_{E23}^+$, so that along the entire path the necessary condition for regime 1, $\theta_E < \theta_{E23}^+$, holds. At the same time, the progressive path may be an equilibrium, as the energy income share increases to θ_{E3}^+ during regime 2, which starts as soon as the economy in regime 1 has converged to $(D_{12}^-, \theta_{E12}^-)$. Hence, in this scenario there are multiple equilibria, as shown in Figure 6. Expectations determine which equilibrium path will be chosen, so that the implementation of the backstop technology is a self-fulfilling prophecy. □

The intuition for the self-fulfilling prophecy in scenario (iii) is that both paths are sensible, given that they are expected by investors. If investors expect that the backstop technology will be too expensive to implement, they foresee that the economy will rely on the non-renewable resource forever. Hence, resource-augmenting technical change is profitable and will occur at a high rate. As a result, the resource indeed remains relatively cheaper than the backstop technology. Conversely, if investors expect that the backstop technology is going to replace the non-renewable resource in the future, they will invest less in resource-augmenting technical change, because their investments will become worthless as soon as the economy shifts to the backstop technology. As a result, resource-augmenting technical change will be low and eventually fall to zero, so that the backstop technology indeed becomes competitive and will be implemented.

Figure 6: Phase diagram in (θ_E, D) space: Scenario (iii)



Notes: The solid black and gray lines represent the research and income share isoclines, respectively. The dashed arrows represent the conservative and progressive equilibrium path that lead to point A and B, respectively

5 Initial Conditions

Although we have constructed the equilibrium path in (θ_E, D)-space that runs through regime 1 and 2 and ends at a fixed point in regime 3, we still have to determine the initial point along this path to complete the solution to the model.[18] We can find this initial point by exploiting the fact that total resource extraction over time should be equal to the initial stock of the resource, or equivalently, that the resource stock should be equal to zero at the moment the economy shifts from using the resource to using the backstop (i.e., at time T_{23}).

In order to do so, we first need a differential equation for the reserve-to-extraction rate $y \equiv S/R$ in terms of y, θ_E, and D. Appendix A.14 derives the expressions for the following differential equations in regime 1 and regime 2, respectively:

$$\dot{y} = -y(1-\theta_E)(1-\nu)\left[\frac{1-\beta}{1-\theta_E}(L^S - D)\left\{\psi^{-1} - (\theta_E(\xi_R + \xi_L) - \xi_L)\right\} - \psi^{-1}D\right], \tag{38a}$$

$$\dot{y} = -y(1-\theta_E)(1-\nu)\xi_L(1-\beta)(L^S - D) + \rho y - 1. \tag{38b}$$

[18]In this section, we only discuss the initial conditions for scenario (i) of Proposition 4, in which the backstop technology will necessarily be implemented. The case in which the economy remains in regime 1 forever is simpler and can be solved in a similar way.

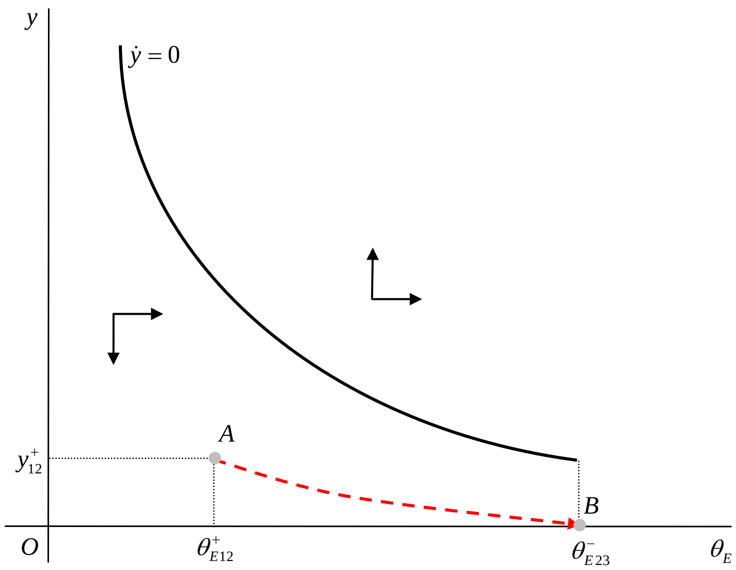
Because we can plug in the already determined time paths for θ_E and D, we can use these differential equations to find a unique equilibrium path in (θ_E, y)-space that leads to a zero reserve-to-extraction rate at the time of the regime shift to the backstop technology. We define this equilibrium path as $y = g(\theta_E)$. Subsequently, by defining the function $D = f(\theta_E)$ as the equilibrium path in (θ_E, D)-space, we can substitute this function $f(\theta_E)$ in the relative factor demand function that follows from the combination of (3) and (15), to derive a relationship between the initial θ_E and y:

$$\frac{\theta_E(0)}{1-\theta_E(0)} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\nu}} \left(\frac{S_0}{y(L^S - f(\theta_E))} \frac{Q_R(0)}{Q_L(0)}\right)^{\frac{\nu-1}{\nu}}. \tag{39}$$

The initial income share $\theta_E(0)$ now follows from the intersection of $g(\theta_E)$ and the initial relative factor demand function in (θ_E, y)-space.

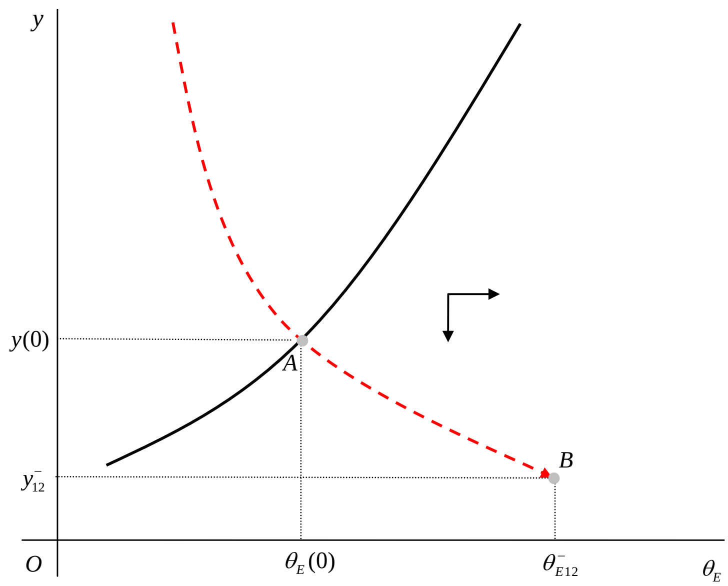
Working backward in time, we first construct the equilibrium path for y in regime 2 and subsequently extend it into regime 1. By imposing $y(T_{23}) = 0$ and using the already determined time paths of θ_E and D, the differential equation (38b) gives a unique equilibrium path in (θ_E, y)-space.

Figure 7: Phase diagram in (θ_E, y) space: Regime 2



Notes: The dashed arrow represents the unique equilibrium path that leads to point B, governed by the dynamic system for θ_E, $D = f(\theta_E)$, and y. The solid line is the $\dot{y} = 0$ locus, which gives combinations of θ_E and y such that y is constant over time.

Figure 8: Phase diagram in (θ_E, y) space: Regime 1



Notes: The dashed arrow represents the unique equilibrium path that leads to point B, governed by the dynamic system for θ_E, D, and y. The solid line gives the relationship between $\theta_E(0)$ and $y(0)$ according to the relative factor demand equation (39). The $\dot{y} = 0$-locus is left out to keep the diagram clear.

Given that we know θ^+_{E12}, we can use the equilibrium path $g(\theta_E)$ to determine y^+_{12}. Figure 7 illustrates this procedure. It shows the phase diagram for the reserve-to-extraction rate in regime 2. The solid line represents the $\dot{y} = 0$-locus and the dashed arrow gives the equilibrium path for y. Hence, during regime 2, the reserve-to-extraction rate moves along the equilibrium path from point A to point B. The dynamic behavior of θ_E and y is illustrated by the solid horizontal and vertical arrows, respectively.

Because of the continuity of the energy income share θ_E and the total research effort D at T_{12} the reserve-to-extraction rate y should also be continuous at T_{12}, i.e. $y^-_{12} = y^+_{12}$. Therefore, having determined the point $(\theta^-_{E12}, y^-_{12}) = (\theta^+_{E12}, y^+_{12})$, we can use the differential equation (38a) together with the already determined time paths of θ_E and D to pin down the equilibrium path of y in regime 1 leading to this end point. The phase diagram in Figure 8 illustrates this. The constructed equilibrium path is represented by the dashed arrow. The solid line in the figure shows the relationship between the initial values of θ_E and y, which is given in (39). The intersection of (39) with the constructed equilibrium path in (θ_E, y)-space, determines the initial point $[\theta_E(0), y(0)]$ that is consistent with factor market equilibrium and with complete depletion of the resource stock. Hence, during regime 1, the reserve-to-extraction rate starts at point A and moves along the equilibrium path to point B in Figure 8. The dynamic behavior of θ_E and y is illustrated by the solid horizontal and vertical arrow, respectively.

6 Numerical Illustration

In this section, we quantify the results of the model by performing a numerical analysis.[19] We first calibrate the model to match data on energy expenditures, reserve-to-extraction rates, and consumption growth in modern industrialized economies. To check the robustness of the model, we also simulate a specification of the model in which the non-renewable resource and the backstop technology are good but imperfect instead of perfect substitutes.[20] Subsequently, we discuss the simulation outcomes of three different scenarios.

6.1 Calibration

In line with empirical evidence, we presume that the elasticity of substitution between labor and energy is smaller than unity. In a meta-analysis Koetse, de Groot, and Florax (2008) find a cross-price elasticity between capital and energy in the United States of 0.383 in the short run and 0.520 in the long run. We take the average of these values and impose $\sigma = 0.45$. The parameter β is the output elasticity of the primary factors, labor and the non-renewable resource, in both service sectors. Our value of 0.80 lies within the range of the labor income shares reported in Gollin (2002) and is in line with the average share of fossil fuel consumption in total energy consumption in the OECD countries, which amounted to 82 percent over the years 2000-2011 (World Bank, 2012). We set the rate of pure time preference ρ to 0.01 and choose $\gamma = 0.50$ for the final good production function parameter. The backstop productivity parameter η is fixed at 10. The initial stocks of quality in both sectors Q_{L0}, Q_{R0} and the labor supply L^S are normalized to unity.

We choose an initial non-renewable resource stock of 1250 to obtain an initial energy income share of 8.8 percent, equal to the average energy expenditure share in GDP over

[19] For the numerical simulation, we use the relaxation algorithm explained in Trimborn, Koch, and Steger (2008).

[20] In the imperfect substitution specification, the elasticity of substitution between the resource and the backstop is set to $\alpha = 50$.

the period 1970-2009 in the United States (U.S. Energy Information Administration, 2011). By choosing the research productivity parameters $\xi_L = 0.165$ and $\xi_R = 0.90$, we get an initial yearly consumption growth rate of 1.7 percent, in line with the average yearly growth rate of GDP per capita in the United States over the period 1970-2011 (The Conference Board, 2011). The reserve-to-production ratios for oil, natural gas, and coal in 2008 were 44, 58, and 127, respectively (U.S. Energy Information Administration, 2012).[21] Our implied initial reserve-to-extraction rate $y(0)$ of 74 lies within this range. The backstop-resource price ratio $p_{YH}(0)/p_{YR}(0)$ is initially equal to 3.8 and gradually declines towards unity. Our calibration implies that $\theta_{23}^{+} < \theta^{*}$, so that the backstop technology will eventually become competitive. The simulated model gives rise to roughly 80 years of resource use before the backstop technology is implemented. Resource-augmenting technical change will disappear after the first quarter of this era.

6.2 Results

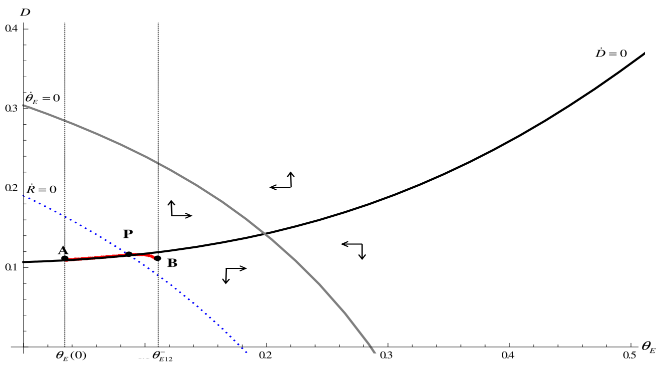
Figure 9 shows the phase diagrams of the calibrated model. Panels (a) and (b) correspond to regime 1 and 2, respectively. Panel (c) shows the phase diagram in (θ_E, μ)-space, which is used to determine the starting point of regime 2. Finally, panel (d) shows the phase diagram for the model without a backstop technology. The equilibrium paths are given by the fat dotted lines in the four panels. The economy starts at point A in panel (a) and moves along the equilibrium path, crosses the extraction isocline at point P and continues until point B, which is also shown in panel (b). After the regime shift, the economy gradually moves along the equilibrium path from this point B in panel (b), passes the extraction isocline at point P, and finally reaches point C. As soon as point C is reached, total research jumps down from point C to point D. Panel (c) illustrates that the energy income share at the time of the first regime shift, θ_{E12}^{+}, can be determined by using the intersection point of the equilibrium path and the horizontal $\mu = 1$ line. Finally, panel (d) shows that an economy without a backstop technology, will move along the indicated equilibrium path from point A to point B, where the income share and research isoclines intersect.

Figure 10 depicts the time paths of six variables of interest. The solid lines represent the benchmark scenario. To illustrate the importance of taking the existence of a backstop

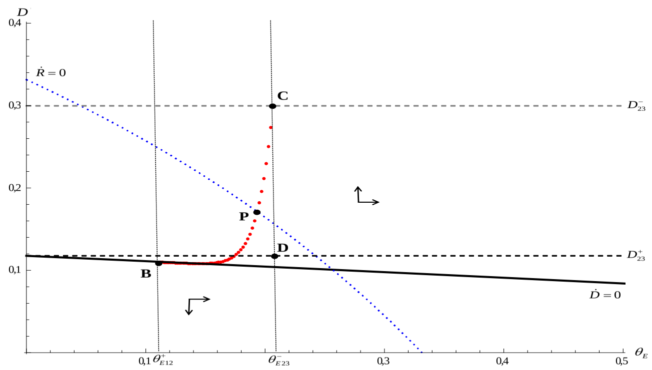
[21] For 2011, BP (2012) reports reserve-to-production rates of 54, 60, and 112 for oil, natural gas, and coal, respectively.

Figure 9: Phase Diagrams of the Calibrated Model

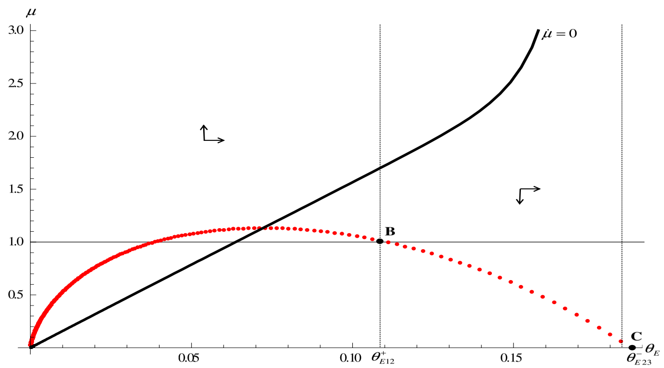
Panel (a): Regime 1



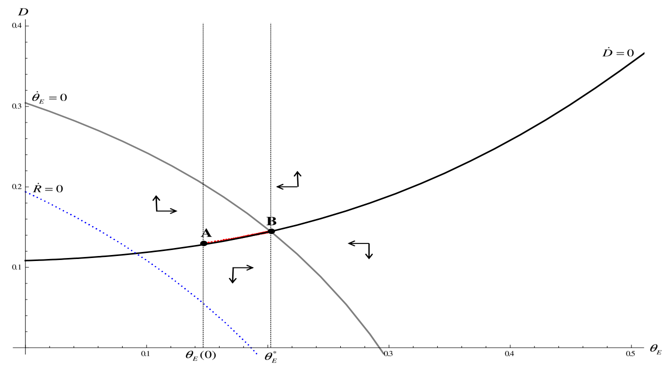
Panel (b): Regime 2



Panel (c): Regime 2: Determination of Starting Point

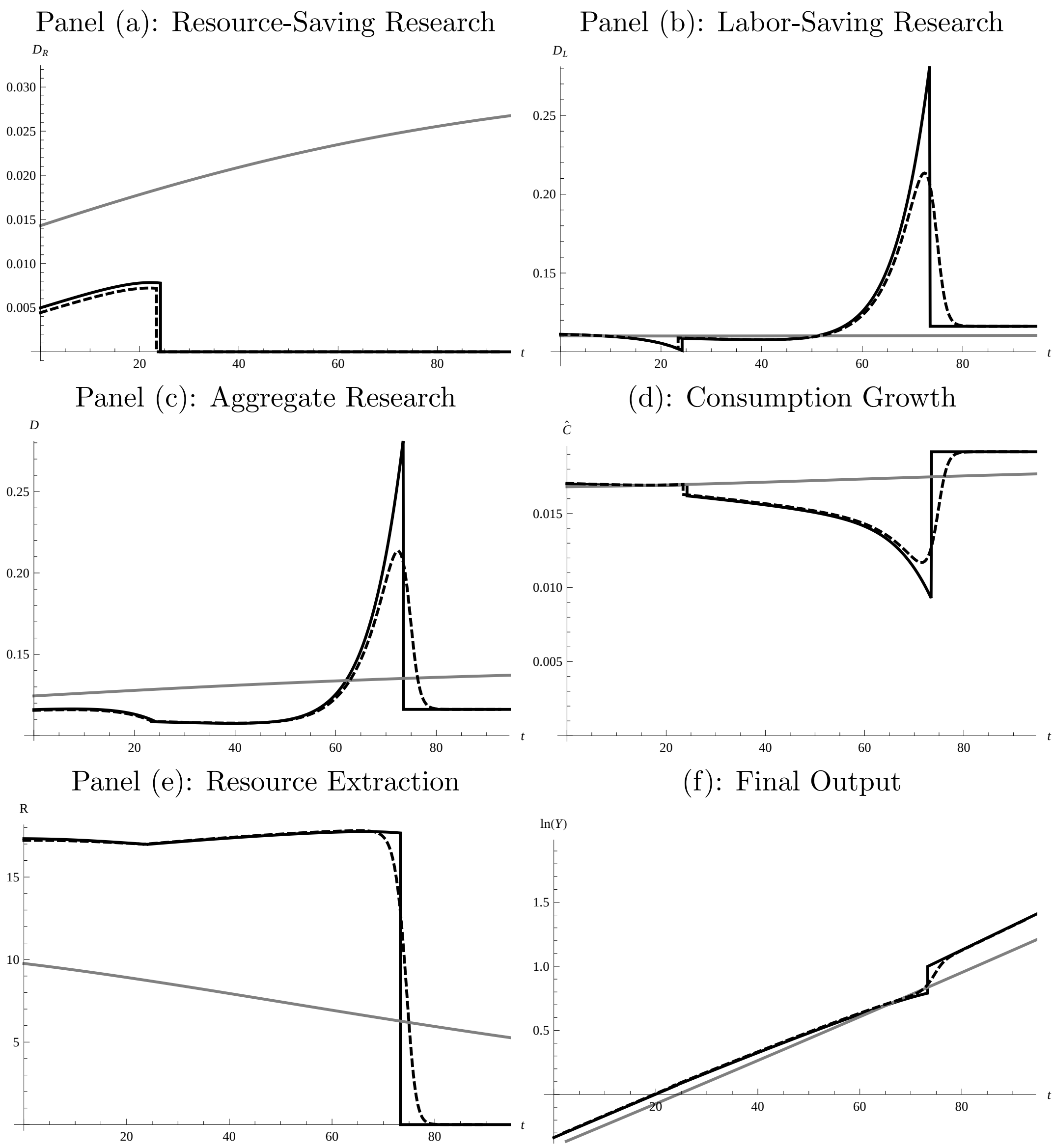


Panel (d): Model without backstop technology



Notes: The dotted lines are the extraction isoclines. The fat dots represent the equilibrium paths. In panels (a), (b), and (d), the solid black line represents the research isocline. In panels (a) and (d), the gray line represents the income share isocline, which is left out in panel (b). In panel (c), the solid black line is the $\dot{\mu} = 0$ isocline. The underlying parameter values are: $L^S = 1$, $\alpha = 50$, $\beta = 0.80$, $\gamma = 0.50$, $\eta = 10$, $\rho = 0.01$, $\sigma = 0.45$, $\xi_L = 0.165$, and $\xi_R = 0.90$. The initial quality levels $Q_L(0)$ and $Q_R(0)$ are equal to 1. The initial resource stock S_0 is set to 1250 in scenarios 1 and 2 to obtain $\theta_E(0) = 0.088$ in scenario 1. In the third scenario, the initial resource stock is chosen such that $\theta_E(0) = 0.088$.

Figure 10: Transitional Dynamics



Notes: The solid line represents scenario 1, in which a backstop technology that provides a perfect substitute for the resource is available. The gray line represents scenario 2, in which there is no backstop technology available. The dashed line represents scenario 3, in which a backstop technology that provides a good, but imperfect substitute for the resource is available. Parameters are set to: $L^S = 1$, $\alpha = 50$, $\beta = 0.80$, $\gamma = 0.50$, $\eta = 10$, $\rho = 0.01$, $\sigma = 0.45$, $\xi_L = 0.165$, and $\xi_R = 0.90$. The initial quality levels $Q_L(0)$ and $Q_R(0)$ are equal to 1. The initial resource stock S_0 is set to 1250 in scenarios 1 and 2 to obtain $\theta_E(0) = 0.088$ in scenario 1. In the third scenario, the initial resource stock is chosen such that $\theta_E(0) = 0.088$.

technology into account, the gray line shows the time paths for an economy without a backstop technology that is similar to the benchmark economy in all other respects. As a robustness check, the dashed lines give the results for a model in which the non-renewable resource and the backstop technology are good, but imperfect instead of perfect substitutes. The time paths generated by the imperfect substitutes model are smoother, but otherwise quite similar to the ones that result from our benchmark model.

Panel (a) of Figure 10 shows that the availability of a backstop technology leads to a smaller amount of research in the resource service sector. Panel (b) indicates that labor saving research jumps up as the economy shifts to the regime without resource-augmenting technical change. Panel (c) delineates the non-monotonic development of aggregate research compared to the monotonically increasing research efforts in the model without a backstop technology. Panel (d) shows the repercussions for consumption growth of the reallocations of labor between the production and the research sector. As illustrated in panel (e), resource extraction is initially declining, starts to increase as soon as the economy shifts to regime 2, then peaks just before the start of regime 3, when the resource stock is depleted and extraction jumps to zero. In the model without a backstop technology, resource extraction is lower initially and decreases monotonically over time. Finally, panel (f) shows the jump in output that materializes at the second regime shift, when energy generation with the backstop technology commences.

7 Conclusion

This paper has investigated the interaction between the existence of backstop technologies (technologies capable of producing renewable substitutes for non-renewable resources) and the rate and direction of technical change. For this purpose, we have constructed a growth model with a non-renewable resource and a backstop technology in which profit incentives determine both the rate and the direction of technical change endogenously. We take into account that natural resources and man-made factors of production are poor substitutes and that energy generation with the backstop technology is costly. The model is solved analytically and we visualize its transitional dynamics and regime shifts in phase portraits of the different regimes. We quantify the results by calibrating the model and performing a simulation analysis. Moreover, we show that the results are robust to relaxing the assumption of perfect substitutability between the non-renewable resource and the backstop technology.

We find that the economy may experience two consecutive regimes of energy generation. Initially, energy generation relies completely on the resource. Depending on the productivity of the available backstop technology, the economy may shift to a regime in which the resource stock is depleted and only the backstop technology will be used to produce energy. In this scenario, short-run resource extraction will be higher than in a model [XXX OR: in an equilibrium I WANT TO COMPARE THE TWO POSSIBLE EQUILIBRIA! XXX] without the backstop technology. Moreover, the transition to a backstop technology reduces resource-saving technical change compared to an economy without a backstop technology available: the increase in energy efficiency even ceases before the backstop technology becomes competitive. Hence, there are also two consecutive regimes of technical change. Initially, both labor and resource-augmenting technical change are taking place. Subsequently, a second regime with purely labor-augmenting technical change commences.

Due to the endogeneity of the direction of technical change, the transition to the backstop technology does not take place in all scenarios. If the productivity of the backstop technology is low enough, the economy remains in the resource regime forever: due to resource-augmenting technical change, the backstop technology will not become competitive. For intermediate values of the backstop technology productivity, the implementation of the backstop technology is a self-fulfilling prophecy: if investors expect energy generation to rely upon the resource forever, investment in resource-augmenting technical change is attractive so that resource-augmenting technical change is high and the resource indeed remains relatively cheaper than the backstop technology. Conversely, if investors expect the backstop technology to be implemented in the future, resource-augmenting technical change becomes unattractive and eventually drops to zero, so that the backstop technology indeed will become competitive in the future.

The existence of expectations-driven multiple equilibria has important implications for policy. As is standard in other models of directed technical change, our model includes externalities that can be addressed by policies: the benefits from research are not fully appropriated and there is monopolistic competition. If the coordination of expectations is difficult, additional temporary policies might be needed to steer the economy into the direction of the optimal path. The representative agent will prefer one of the two equilibria. We leave it to future research to extend the model with externalities associated with fossil use and to analyze policy options.

In our analysis, we have abstracted from stock-dependent extraction costs. To shed light on optimal environmental policy, these should be introduced together with pollution from the combustion of the non-renewable resource. An extension in this direction is especially interesting in the light of the multiple equilibria that may exist if the backstop technology is relatively expensive (i.e., relatively unproductive). Furthermore, the effects of including a separate type of backstop technology improving technical change could be investigated in future work.

Appendix

This appendix contains the derivations of the mathematical results in the paper. It first derives the optimality conditions for firms and households. Second, expressions for the relative income shares and the real interest rate will be derived. Finally, some important properties of the differential equations and the isoclines in the dynamic system will be discussed.

A.1 Final Output

Profit of firms in the final output sector are given by:

$$p_Y Y(Y_L, Y_E) - p_{YL} Y_L - p_{YE} Y_E \tag{A.1}$$

where the function for Y is specified in (1). Profit maximization gives rise to the following first-order conditions:

$$p_{YL} = p_Y \left[\gamma Y_L^{\frac{\sigma-1}{\sigma}} + (1-\gamma) Y_E^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} \gamma Y_L^{-\frac{1}{\sigma}}, \tag{A.2}$$

$$p_{YE} = p_Y \left[\gamma Y_L^{\frac{\sigma-1}{\sigma}} + (1-\gamma) Y_E^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (1-\gamma) Y_E^{-\frac{1}{\sigma}}. \tag{A.3}$$

Dividing both expressions gives (2). Combining (1) with (A.2)-(A.3), we get:

$$p_Y Y = p_{YL} Y_L + p_{YE} Y_E. \tag{A.4}$$

Substitution of (A.2)-(A.3) into the production function (1) and combining the result with (A.4), we obtain an expression for the price index of final output:

$$p_Y = [\gamma(1-\gamma)]^{-1} \left\{ \gamma \left[(1-\gamma) p_{YL}\right]^{1-\sigma} + (1-\gamma) \left[\gamma p_{YE}\right]^{1-\sigma} \right\}^{\frac{1}{1-\sigma}}. \tag{A.5}$$

A.2 Intermediate Goods

The Hamiltonian associated with the optimization problem of firm k in the intermediate good sector is given by:

$$\mathcal{H}_{ik} = p_{Yi}(1-\beta) q_{ik} Z_i^{\beta} x_{ik}^{1-\beta} - q_{ik} p_Y x_{ik} - w_D D_{ik} + \lambda_{ik} \xi_i Q_i D_i, \tag{A.6}$$

where $i = Z_i = \{R, L\}$. The necessary first-order conditions for an optimum are given by:

$$\frac{\partial \mathcal{H}_{ik}}{\partial x_{ik}} = 0 \Rightarrow (1-\beta)^2 p_{Yi} q_{ik} Z_i^{\beta} x_{ik}^{-\beta} = q_{ik} p_Y, \tag{A.7}$$

$$\frac{\partial \mathcal{H}_{ik}}{\partial D_{ik}} \leq 0 \Rightarrow -w_D + \lambda_{ik} \xi_i Q_i \leq 0, \quad \text{with equality if} \quad D_{ik} > 0, \tag{A.8}$$

$$\frac{\partial \mathcal{H}_{ik}}{\partial q_{ik}} = -\dot{\lambda}_{ik} + r\lambda_{ik} \Rightarrow p_{Yi}(1-\beta) Z_i^{\beta} x_{ik}^{1-\beta} - x_{ik} p_Y = -\dot{\lambda}_{ik} + r\lambda_{ik}. \tag{A.9}$$

The transversality conditions are given by (10a)-(10b). Substitution of (5b) in (A.7) gives (7a), (A.8) directly implies (7b), and the combination of (A.7) and (A.9) gives (7c). Combining (5b) with (7a), we obtain:

$$x_{ik} = x_i = \left(\frac{p_{Yi}(1-\beta)^2}{p_Y} \right)^{\frac{1}{\beta}} Z_i = \frac{\theta_i Y (1-\beta)^2}{Q_i}. \tag{A.10}$$

where the second equality uses (3).

A.3 Households

The wealth of households is equal to

$$V = w_R S + \lambda_L Q_L + \lambda_R Q_R, \tag{A.11}$$

so that the change in wealth over time equals:

$$\dot{V} = \dot{w_R}S - w_R R + \dot{\lambda}_L Q_L + \lambda_L \dot{Q}_L + \dot{\lambda}_R Q_R + \lambda_R \dot{Q}_R, \tag{A.12}$$

where we have used (4) to substitute for $\dot{S}$. Defining π_i as profits per unit of quality, total profits in each intermediate goods sector are equal to $Q_i \pi_i = p_{xi} x_i - p_Y Q_i$, so that (7a) and (7c) can be combined to get

$$p_{xi} x_i = Q_i r \lambda_i - Q_i \dot{\lambda}_i + Q_i x_i. \tag{A.13}$$

Combining (A.4), (3), and (5a)-(5b) we obtain:

$$p_Y Y = w_L L + p_{xL} x_L + w_R R + p_{xR} x_R + \eta H. \tag{A.14}$$

Plugging (A.13) in (A.14) and using the resulting expression to substitute for $w_R R$ in (A.12), we get:

$$\dot{V} = p_Y Y - Q_L x_L - Q_R x_R - \eta H + w_L L + r Q_L \lambda_L + r Q_R \lambda_R + \lambda_L \dot{Q}_L + \lambda_R \dot{Q}_R. \tag{A.15}$$

By combining (6) and (A.8) we obtain $\lambda_i \dot{Q}_i = \lambda_i \xi_i Q_i D_i = w_D D_i$. Using this expression together the market equilibrium conditions from Section 2.2 in (A.15), we obtain the flow budget constraint of the households from the main text.

The Hamiltonian associated with the optimization problem of the households reads:

$$\mathcal{H} = \ln(C) + \lambda_V \left[r(V - w_R S) + \dot{w}_R S + w L^S - p_C C \right]. \tag{A.16}$$

The necessary first-order conditions for an optimum are given by:

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \Rightarrow \frac{1}{C} - \lambda_V p_C = 0 \Rightarrow \hat{C} + \hat{p}_C = -\hat{\lambda}_V, \tag{A.17}$$

$$\frac{\partial \mathcal{H}}{\partial S} = 0 \Rightarrow -\lambda_V r w_R + \lambda_V \dot{w}_R = 0 \Rightarrow \hat{p}_R = r, \tag{A.18}$$

$$\frac{\partial \mathcal{H}}{\partial V} = -\dot{\lambda_V} + \rho \lambda_V \Rightarrow \lambda_V r = -\dot{\lambda}_V + \rho \lambda_V. \tag{A.19}$$

Combining (A.17) and (A.19) gives the Ramsey rule (14a). The first-order condition (A.18) is the Hotelling rule (14b).

A.4 Income Shares

This section derives the income shares for $t < T_{23}^{+}$, when $\omega_H = Y_H = 0$ so that $\theta_E = \theta_R$. We substitute (3) into (2) and use (A.10) to get

$$\frac{p_{YR}}{p_{YL}} = \frac{1-\gamma}{\gamma}\left(\frac{Y_R}{Y_L}\right)^{-\frac{1}{\sigma}} = \left(\frac{p_{YR}}{p_{YL}}\right)^{\frac{1}{\beta}}\frac{RQ_R}{LQ_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\beta\sigma}{\nu}}\left(\frac{RQ_R}{LQ_L}\right)^{-\frac{\beta}{\nu}}. \tag{A.20}$$

Using the income share definitions together with (A.20), we find

$$\frac{\theta_R}{1-\theta_R} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\nu}}\left(\frac{RQ_R}{LQ_L}\right)^{\frac{\nu-1}{\nu}} = \left(\frac{w_R}{w_L}\frac{Q_L}{Q_R}\right)^{1-\nu}\left(\frac{1-\gamma}{\gamma}\right)^{\sigma}, \tag{A.21}$$

where the second equality additionally uses (5a) and (A.10) to obtain the price ratio

$$\begin{aligned}\frac{w_R}{w_L} &= \frac{p_{YR}}{p_{YL}}\left(\frac{R}{L}\right)^{\beta-1}\frac{Q_R}{Q_L}\left(\frac{x_R}{x_L}\right)^{1-\beta} = \left(\frac{p_{YR}}{p_{YL}}\right)^{\frac{1}{\beta}}\frac{Q_R}{Q_L}\\ &= \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\nu}}\left(\frac{R}{L}\right)^{-\frac{1}{\nu}}\left(\frac{Q_R}{Q_L}\right)^{\frac{\nu-1}{\nu}}.\end{aligned} \tag{A.22}$$

A.5 Real Interest Rate

If we combine (3) with (5a), and (5b) to find the price index p_{YL} and subsequently convert the expression into growth rates, we get

$$\hat{p}_{YL} = \beta\hat{w}_L + (1-\beta)(\hat{Q}_L + \hat{p}_Y) - \hat{Q}_L. \tag{A.23}$$

Converting the price index (A.5) into growth rates and using (A.2)-(A.3), we obtain

$$\hat{p}_Y = \theta_E\hat{p}_{YE} + (1-\theta_E)\hat{p}_{YL}. \tag{A.24}$$

Combining (A.23) and (A.24), and using (A.22), we find an expression for the real rate of interest:

$$r - \hat{p}_Y = r - \hat{w}_L - (\hat{p}_Y - \hat{w}_D) = (1-\theta_E)\left[r - \hat{w}_D - (\hat{Q}_R - \hat{Q}_L)\right] + \hat{Q}_R. \tag{A.25}$$

The second equalities in (19) and (19) follow from the combination of (A.25) with the first equalities in (19) and (19).

A.6 Research and Income Share Isoclines Regime 1

This section derives some relevant properties of the research and income share isoclines in regime 1.

A.6.1 Properties and First-Order Derivatives

By imposing the steady state (i.e. $\dot{\theta}_E = \dot{D} = 0$ in the dynamic system (21a)-(21b) we find the following isoclines:

$$D|_{\dot{\theta}_E=0} = \frac{L^S(1-\beta)\left[\xi_L(\xi_L+2\xi_R)-\theta(\xi_L+\xi_R)^2\right]}{\xi_L^2(1-\beta)(1-\theta_E)+\xi_L\xi_R(3-2\beta)(1-\theta_E)-\xi_R^2(1-\beta)\theta_E}, \tag{A.26}$$

$$D|_{\dot{D}=0} = \frac{\Xi}{\Omega}, \tag{A.27}$$

where

$$\begin{aligned}
\Xi \equiv\ & L^S(1-\beta)\left[\xi_L\xi_R-(1-\nu)\xi_L(\xi_L+2\xi_R)\theta_E+(1-\nu)(\xi_L+\xi_R)^2\theta_E^2\right] \\
& -(\xi_L+\xi_R)(1-\theta_E)\rho, \\
\Omega \equiv\ & (1-\nu)(1-\beta)\xi_R^2\theta_E^2-(1-\nu)\xi_L^2(1-\beta)(1-\theta_E)\theta_E \\
& +\xi_L\xi_R\left[2-\theta_E(4-3\nu(1-\theta_E)-3\theta_E)-\beta(1-2(1-\nu)(1-\theta_E)\theta_E)\right].
\end{aligned}$$

It follows from the denominator of (A.26) that the $\dot{\theta}_E = 0$ isocline has a vertical asymptote at:

$$\bar{\theta}_E \equiv \frac{\xi_L[\xi_L(1-\beta)+\xi_R(3-2\beta)]}{(1-\beta)(\xi_L+\xi_R)^2+\xi_L\xi_R} > 0. \tag{A.28}$$

The intersection point of the two isoclines, (θ_E^*, D^*) satisfies:

$$\theta_E^* = \frac{\xi_L\left\{L^S\xi_L\xi_R(1-\beta)+[\xi_L(1-\beta)+\xi_R(3-2\beta)]\rho\right\}}{L^S\xi_L\xi_R(\xi_L+\xi_R)(1-\beta)+\left[\xi_L^2+\xi_R^2+3\xi_L\xi_R-(\xi_L+\xi_R)^2\beta\right]\rho} > 0, \tag{A.29}$$

$$D^* = \frac{L^S\xi_L(\xi_L+\xi_R)(1-\beta)-\xi_R\rho}{\xi_L\left[\xi_R(2-\beta)+\xi_L(1-\beta)\right]}. \tag{A.30}$$

The corner properties for the two isoclines are:

$$D|_{\dot{\theta}_E=0,\theta_E=0} = L^S - \frac{L^S \xi_R}{\xi_L(1-\beta)+\xi_R(3-2\beta)}, \tag{A.31}$$

$$D|_{\dot{D}=0,\theta_E=0} = \frac{L^S \xi_L \xi_R (1-\beta) - (\xi_L+\xi_R)\rho}{\xi_L \xi_R (2-\beta)}, \tag{A.32}$$

$$D|_{\dot{\theta}_E=0,\theta_E=1} = D|_{\dot{D}=0,\theta_E=1} = L^S. \tag{A.33}$$

The first-order derivative of the income share isocline with respect to θ_E is given by:

$$\frac{d(D|_{\dot{\theta}_E=0})}{d\theta_E} = -\frac{L^S \xi_L \xi_R^3 (1-\beta)}{\left[\xi_L(1-\beta)(1-\theta_E)+\xi_L\xi_R(3-2\beta)(1-\theta_E)-\xi_R^2(1-\beta)\theta_E\right]^2} < 0.$$

The first-order derivative of the research isocline with respect to θ_E at $\theta_E = 1$ is given by:

$$\left.\frac{d(D|_{\dot{D}=0})}{d\theta_E}\right|_{\theta_E=1} = \frac{(\xi_L+\xi_R)\rho + L^S \xi_L \xi_R [1-\beta(1-\sigma)]}{\xi_R(1-\beta)[\xi_L+\xi_R\beta(1-\sigma)]} > 0. \tag{A.34}$$

The first-order derivative of the research isocline with respect to θ_E at $\theta_E = 0$ is given by:

$$\left.\frac{d(D|_{\dot{D}=0})}{d\theta_E}\right|_{\theta_E=0} = \frac{\Gamma}{\Lambda}, \tag{A.35}$$

where

$$\begin{aligned}\Gamma \equiv\ & L^S \xi_L \xi_R (1-\beta)\left[\xi_R[1-\beta(1-\sigma)] - \xi_L\beta(1-\sigma)\right] \\ & + (\xi_L+\xi_R)\rho\left[\xi_R(1-\beta(4-2\beta(1-\sigma)+3\sigma)) - \xi_L(1-\beta)\beta(1-\sigma)\right], \\ \Lambda \equiv\ & \xi_L\xi_R^2(2-\beta)^2 > 0.\end{aligned}$$

Hence, the sign of (A.35) depends on the parameter values. However, it can be shown that $\Gamma < 0$ if ξ_R is small relative to ξ_L, e.g. if $\xi_R = 0$, we obtain:

$$\Gamma|_{\xi_L=0} = -\xi_L(1-\beta)\beta\rho(1-\sigma) < 0, \tag{A.36}$$

so that the first-order derivative of the research isocline with respect to θ_E is negative at $\theta_E = 0$ if ξ_R is relatively small.

A.6.2 Relative Positions

At $\theta_E = 0$, the difference between the income share isocline and the research isocline is given by:

$$(D|_{\dot{\theta}_E=0} - D|_{\dot{D}=0})|_{\theta_E=0} = \frac{\xi_L + \xi_R}{2-\beta}\left(\frac{\rho}{\xi_L\xi_R} + \frac{L^S(1-\beta)}{\xi_L(1-\beta)+\xi_R(3-2\beta)}\right) > 0.$$

Because $D|_{\dot{D}=0,\theta_E=1} = L^S$ and $\lim_{\theta_E\to\bar{\theta}_E} D|_{\dot{\theta}_E=0} = -\infty$, the two isoclines cross exactly once and the intersection point is located to the left of the vertical asymptote of the research isocline.

A.7 Extraction Isocline Regime 1

This section derives some relevant properties of the extraction isocline in regime 1.

A.7.1 Properties and First-Order Derivative

Substitution of (13), (17) and (18) into (19) and imposing $\dot{R} = 0$, we obtain the extraction isocline, which also has a vertical asymptote at $\bar{\theta}_E$:

$$D|_{\dot{R}=0} = -\frac{(\xi_L+\xi_R)\rho + L^S(1-\beta)\beta(1-\sigma)[(\xi_L+\xi_R)^2\theta_E - \xi_L(\xi_L+2\xi_R)]}{\beta(1-\sigma)[\xi_L^2(1-\beta)(1-\theta_E)+\xi_L\xi_R(3-2\beta)(1-\theta_E)-\xi_R^2(1-\beta)\theta_E]}.$$

The first-order derivative of the extraction isocline with respect to θ_E is given by:

$$\frac{d(D|_{\dot{R}=0})}{d\theta_E} = -\frac{(\xi_L+\xi_R)\left[(\xi_L^2+\xi_R^2)(1-\beta)+\xi_L\xi_R(3-2\beta)\right]\rho + L^S\xi_L\xi_R^3(1-\beta)\beta(1-\sigma)}{\beta[\xi_L^2(1-\beta)(1-\theta_E)+\xi_L\xi_R(3-2\beta)(1-\theta_E)-\xi_R^2(1-\beta)\theta_E](1-\sigma)}.$$

Hence, $d(D|_{\dot{R}=0})/d\theta_E < 0$ if $\theta_E < \bar{\theta}_E$.

A.7.2 Relative Position

The difference between the income share isocline and the extraction isocline is given by:

$$D|_{\dot{\theta}_E=0} - D|_{\dot{R}=0} = \frac{(\xi_L+\xi_R)\rho(1-\sigma)^{-1}}{\beta[\xi_L^2(1-\beta)(1-\theta_E)+\xi_L\xi_R(3-2\beta)(1-\theta_E)-\xi_R^2(1-\beta)\theta_E]}.$$

It follows that $D|_{\dot{\theta}_E=0} - D|_{\dot{R}=0} > 0$ and $\lim_{\sigma\to1}[D|_{\dot{\theta}_E=0} - D|_{\dot{R}=0}] = \infty$ if $\theta_E < \bar{\theta}_E$.

A.8 First-Order Derivatives of Differential Equations in Regime 1

The first-order derivative of the differential equation for θ_E with respect to D is given by:

$$\frac{d\dot{\theta}_E}{dD} = -\frac{\beta\theta_E[\xi_L^2(1-\beta)(1-\theta_E) + \xi_L\xi_R(3-2\beta)(1-\theta_E) - \xi_R^2(1-\beta)\theta_E](1-\sigma)}{(\xi_L+\xi_R)}.$$

Therefore, $d\dot{\theta}_E/dD < 0$ if $\theta_E < \bar{\theta}_E$. The first-order derivative of the differential equation for D with respect to D for combinations of θ_E and D along the D-isocline is given by:

$$\left.\frac{d\dot{D}}{dD}\right|_{\dot{D}=0} = \frac{(\xi_L+\xi_R)\rho + L^S\xi_L\xi_R\left[1-\beta\theta_E(1-\sigma)\right]}{\xi_L+\xi_R} > 0. \tag{A.37}$$

Hence, $d\dot{D}/dD > 0$ in the neighborhood of the research isocline in (θ_E, D)-space. The first-order derivative of the differential equation for R with respect to D is given by:

$$\frac{d\dot{R}}{dD} = -\frac{\beta R[\xi_L^2(1-\beta)(1-\theta_E) + \xi_L\xi_R(3-2\beta)(1-\theta_E) - \xi_R^2(1-\beta)\theta_E](1-\sigma)}{(\xi_L+\xi_R)}.$$

Therefore, $d\dot{R}/dD < 0$ if $\theta_E < \bar{\theta}_E$.

A.9 Research and Income Share Isoclines Regime 2

This section derives some relevant properties of the research and income share isoclines in regime 2.

A.9.1 Properties and First-Order Derivatives

By imposing the steady state (i.e. $\dot{\theta}_E = \dot{D} = 0$) in the dynamic system (24a)-(24b) we find the following isoclines:

$$D|_{\dot{\theta}_E=0} = L^S, \tag{A.38}$$

$$D|_{\dot{D}=0} = L^S - \frac{L^S\xi_L + \rho}{\xi_L\left[2-\beta(1-(1-\beta)\theta_E(1-\sigma))\right]}. \tag{A.39}$$

The corner properties for the two isoclines are:

$$D|_{\dot{\theta}_E=0,\theta_E=0} = D|_{\dot{\theta}_E=0,\theta_E=1} = L^S \tag{A.40}$$

$$D|_{\dot{D}=0,\theta_E=0} = L^S - \frac{L^S\xi_L + \rho}{\xi_L(2-\beta)}, \tag{A.41}$$

$$D|_{\dot{D}=0,\theta_E=1} = L^S - \frac{L^S\xi_L + \rho}{\xi_L\left[2-\beta[1+(1-\beta)(1-\sigma)]\right]}. \tag{A.42}$$

The first-order derivative of the income share isocline with respect to θ_E equals zero. The first-order derivative of the research isocline with respect to θ_E is given by:

$$\frac{d(D|_{\dot{\theta}_E=0})}{d\theta_E} = -\frac{(1-\beta)(1-\sigma)\beta(L^S\xi_L+\rho)}{\xi_L\left[2-\beta(1+(1-\beta)\theta_E(1-\sigma))\right]^2} < 0. \tag{A.43}$$

A.9.2 Relative Positions

Comparing (A.38) and (A.39), we find that $D|_{\dot{\theta}_E=0} > D|_{\dot{D}=0}$.

A.10 Extraction Isocline Regime 1

This section derives some relevant properties of the extraction isocline in regime 1.

A.10.1 Properties and First-Order Derivative

Substitution of (13), (6) and (22a) into (19) and imposing $\dot{R} = 0$, we obtain the extraction isocline

$$D|_{\dot{R}=0} = L^S - \frac{\rho}{\xi_L\beta(1-\beta)(1-\theta_E)(1-\sigma)}. \tag{A.44}$$

The first-order derivative of the extraction isocline with respect to θ_E is given by:

$$\frac{d(D|_{\dot{R}=0})}{d\theta_E} = -\frac{\rho}{\xi_L(1-\beta)(1-\sigma)\beta(1-\theta_E)^2} < 0. \tag{A.45}$$

We have the following corner properties for (A.44):

$$\lim_{\theta_E\to 1} D|_{\dot{R}=0} = -\infty, \tag{A.46}$$

$$D|_{\dot{R}=0,\theta_E=0} < D|_{\dot{\theta}_E=0}. \tag{A.47}$$

It follows that the extraction isocline is located below the income share isocline in (θ_E, D)-space, i.e. $D|_{\dot{R}=0} < D|_{\dot{\theta}_E=0}$.

A.11 First-Order Derivatives of Differential Equations in Regime 2

The first-order derivative of the differential equation for θ_E with respect to D is given by:

$$\frac{d\dot{\theta}_E}{dD} = -\xi_L(1-\beta)(1-\theta_E)(1-\sigma)\beta\theta_E < 0. \tag{A.48}$$

The first-order derivative of the differential equation for D with respect to D for combinations of θ_E and D along the D-isocline is given by:

$$\left.\frac{d\dot{D}}{dD}\right|_{\dot{D}=0} = L^s\psi^{-1}[1-(1-\nu)\theta_E] + \rho > 0. \tag{A.49}$$

Hence, $d\dot{D}/dD > 0$ in the neighborhood of the research isocline in (θ_E, D)-space. The first-order derivative of the differential equation for R with respect to D is given by:

$$\frac{d\dot{R}}{dD} = -\xi_L(1-\beta)(1-\theta_E)(1-\sigma)\beta R < 0. \tag{A.50}$$

A.12 Exclusion of Simultaneous Use

We show that it is not possible to have a regime of simultaneous use of the resource and the backstop technology. Simultaneous use requires equal effective prices of the resource and the backstop technology, so that $p_{YH} = p_{YR} = p_{YE}$. Using $p_{YH} = p_Y/\eta$, this implies

$$\hat{p}_Y = \hat{p}_{YH} = \hat{p}_{YR} = \hat{p}_{YE}. \tag{A.51}$$

If we combine (3) with (5a), and (5b) to find the price indexes p_{YL} and p_{YR}, and subsequently convert the expression into growth rates, we get

$$\hat{p}_{YL} - \hat{p}_Y = \beta(\hat{w}_L - \hat{p}_Y - \hat{Q}_L), \tag{A.52}$$

$$\hat{p}_{YR} - \hat{p}_Y = \beta(\hat{w}_R - \hat{p}_Y - \hat{Q}_R). \tag{A.53}$$

Using (A.24) together with (A.51), we find $\hat{p}_{YL} = \hat{p}_Y$. Substitution of this result into (A.52) and (A.51) into (A.53), and using the Ramsey rule (14a), we obtain

$$r - \hat{w_D} = \hat{Q}_R - \hat{Q}_L \tag{A.54}$$

Substitution of (13) into (9), in a regime with purely labor-augmenting technical change (i.e. $\hat{Q}_L > 0$ and $\hat{Q}_R = 0$) we have

$$r - \hat{w_D} = (1 - \beta)\xi_L(L^S - D) - \hat{Q}_L. \tag{A.55}$$

The conditions (A.54) and (A.55) can only be satisfied jointly if $D = L^S$. However, this implies that $L = Y = 0$, which cannot hold in equilibrium because it implies $\hat{C} = \hat{Y} = 0$, whereas the Ramsey rule (14a) together with (A.53) gives $\hat{C} = -\rho$. Hence, during a regime with purely labor-augmenting technical change, the effective relative price of the resource and the backstop cannot be constant, so that simultaneous use of both energy sources will not occur. As a result, simultaneous use is also impossible in a regime with both resource-augmenting and labor-augmenting technical change. Optimality condition (7b) together with (10b) namely implies that the economy eventually necessarily shifts to a regime without resource-augmenting technical change. Condition (28) requires that θ_E is continuous at this regime shift. However, the beginning of the regime without resource-augmenting technical change, $\theta_E < (1-\gamma)^\sigma \eta^{\sigma-1}$.[22] The jump from a regime with simultaneous use with resource-augmenting and labor-augmenting technical change to a regime with purely labor-augmenting technical change necessarily implies a discontinuity in θ_E. Therefore, a regime of simultaneous use cannot exist.

A.13 Proof Downward Jump in D_R

We proof the downward dump in D_R by contradiction. Suppose that $D^-_{R12} = D^+_{R12} \Rightarrow \hat{Q}^-_{R12} = \hat{Q}^+_{R12}$. From (7b), we get $\hat{\mu}^-_{12} = 0$. The end condition $\mu^+_{23} = 0$ in (36b) and the differential equation for μ, (37), imply that $\hat{\mu}^+_{12} < 0$. Using definition (34), we obtain

$$(\hat{\lambda}_R - \hat{w}_D)^-_{12} > (\hat{\lambda}_R - \hat{w}_D)^+_{12}. \tag{A.56}$$

[22]This inequality follows from the continuity of μ, optimality condition (7b), and (24a).

Rearranging optimality condition (7c), we get

$$r - \hat{w}_D = \frac{\beta}{1-\beta}\frac{\theta_R p_Y Y(1-\beta)^2}{Q_R \lambda_R} + \hat{\lambda}_R - \hat{w}_D. \tag{A.57}$$

Combining (A.56) and (A.57), and using the continuity of Q_R, Y, λ_R, and θ_R, we find $(r - \hat{w}_D)_{12}^{-} > (r - \hat{w}_D)_{12}^{+}$. Substitution of this result into (9) with $i = L$ implies $\hat{Q}_{L12}^{-} > \hat{Q}_{L12}^{+} \Rightarrow D_{L12}^{-} > D_{L12}^{+}$. Using the continuity of D at T_{12} and the identity $D = D_L + D_R$, we obtain $D_{R12}^{-} > D_{R12}^{+}$. This contradicts our initial assumption of a continuous D_R at T_{12}. Hence, D_R jumps down at the end of regime 1. □

A.14 Initial Condition

By combining (3) with (15) and using the definition $y \equiv S/R$, the relative factor demand function can be written as:

$$\frac{\theta_E}{1-\theta_E}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\sigma}{\nu}}\left(\frac{S}{y(L^S - D)}\frac{Q_R}{Q_L}\right)^{\frac{\nu-1}{\nu}}. \tag{A.58}$$

Converting (A.58) into growth rates, we find:

$$\hat{\theta}_E = -(1-\theta_E)\frac{1-\nu}{\nu}\left[\hat{S} - \hat{y} + \frac{D}{L^S - D}\hat{D} + \hat{Q}_R - \hat{Q}_L\right]. \tag{A.59}$$

By using (13), (16), and $\hat{S} = -y^{-1}$, we get a differential equation for y:

$$\dot{y} = -y(1-\nu)(1-\theta_E)\left[r - \hat{w}_L - (\hat{Q}_R - \hat{Q}_L)\right] + \rho y - 1. \tag{A.60}$$

For each regime, the specification of the differential equation in terms of y, θ_E, and D is different. Substitution of (13), (17), and (18) into (A.60) gives (38a), the required differential equation for y in regime 1. By instead substituting $\hat{Q}_R = 0$, (6) and (22a), and using (13) again, we obtain (38b), the required differential equation in regime 2. Expression (39) is obtained by substitution of S_0, $Q_R(0)$, $Q_L(0)$ and the function $D = f(\theta_E)$ into (A.58).

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