HOW MUCH CARBON TO LOCK UP AND THE GREEN PARADOX

A classroom calibration of the optimal carbon tax

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Abstract

A classroom model of global warming, fossil fuel depletion and the optimal carbon tax is formulated and calibrated. It features iso-elastic fossil fuel demand, stock-dependent fossil fuel extraction costs, an exogenous interest rate and no decay of the atmospheric stock of carbon. The optimal carbon tax reduces emissions from burning fossil fuel, both in the short and medium run. Furthermore, it brings forward the date that renewables take over from fossil fuel and encourages the market to keep more fossil fuel locked up. A renewables subsidy brings forward the carbon-free era, locks up more fossil fuel reserves and ultimately curbs global warming, but increases carbon emissions and accelerates global warming. These latter Green Paradox effects are particularly strong for large renewables subsidies in which case social welfare is hurt. We conclude that these subsidies are not a good second-best climate policy.

Keywords: global warming, social cost of carbon, optimal carbon tax, renewables, second best

JEL codes: D81, H20, Q31, Q38

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1. Introduction

Global warming is one of the biggest challenges facing our planet. It is by now accepted that the best way to deal with this is to properly price carbon via either a global carbon tax or a global market for tradable emission permits. The price of carbon should in a first-best world be equal to the social cost of carbon, i.e., the present value of all future marginal global warming damages. As a result of pricing carbon appropriately carbon emissions are curbed by substituting away from fossil fuel to renewable energy and other production factors, more fossil fuel reserves are locked up in the crust of the earth and the carbon-free era is brought forward. Furthermore, pricing carbon will make it attractive to do lots of other things that will mitigate global warming such as carbon capture & sequestration and research & development into renewables.

It is crucial to get a good grasp of the mechanisms underlying the effectiveness of the global carbon tax and also to have a quantitative assessment of the optimal carbon tax. Many empirical integrated assessment models of climate change have been developed for this purpose and yield useful estimates of the social cost of carbon (e.g, Nordhaus (2008), Stern (2007) or Rezai et al. (2012a) and (2012b)). These models are often rather large and are often difficult to understand, so that it is not always clear what the underlying assumptions and the crucial parameters deriving the results are.

Our purpose is therefore to put forward a simple classroom model of climate change and to carefully discuss an illustrative and transparent calibration of this model. We then use this to derive the optimal carbon tax, the optimal amount of fossil fuel to leave untapped, and the optimal time for the commencement of the carbon-free era, and to compare it with the "laissez-faire" outcomes. We also use our model to discuss the adverse effects of subsidizing renewables rather than pricing carbon appropriately. Although this shortens the duration of the fossil fuel era and encourages the market to leave more fossil fuel unexploited, it also leads to faster fossil fuel extraction rates which are known as Green Paradox effects. We show that large renewables subsidies are counterproductive. The optimal carbon tax leads to slower fossil fuel extraction rates than under "laissez faire" and does not suffer from Green Paradox effects. The optimal carbon tax also locks up more fossil fuel in the crust of the earth.

The outline of this paper is as follows. Section 2 discusses and calibrates a simple model of how burning fossil fuel leads to more atmospheric carbon and thus to more global warming. Section 3 discusses various specifications that have been used to capture damages for global warming and puts forward a simple specification which captures that at higher levels of mean global temperature damages increase more than proportionally with temperature. Section 4 puts forward a partial equilibrium model of fossil fuel depletion, renewables and climate change and uses it to discuss and highlight how increases in the carbon tax rate and the renewables subsidy can result in Green Paradox effects. Section 5 discusses social welfare and shows that large renewables subsidies can be counterproductive. Section 6 derives the social optimum and the optimal carbon tax. Section 7 concludes.

2. Burning fossil fuel, atmospheric carbon and global warming

Burning fossil fuel leads to carbon emissions. We suppose that half of these emissions return quickly to the oceans and the surface of the earth and the other half remains forever up in the atmosphere. We thus abstract from atmospheric decay of greenhouse gases, which is less unreasonable for $CO2$ than for methane. We also abstract from positive feedback effects, which may occur at higher temperatures. It follows that the stock of carbon in the atmosphere E (measured in TtC) is given by:

$$(1) \qquad E = E_0 + 0.5\,(S_0 - S) = 1.71 - 0.5\,S,$$

where S is the stock of fossil fuel (measured in TtC) and the subscript 0 indicates current levels. We set the current stock of atmospheric carbon to $E_0 = 0.85$ TtC, which corresponds to 400 ppm CO2 (in May 2013). Cumulative use of fossil fuel from now onwards is given by $S_0 - S$. Initial reserves are to a certain extent arbitrary, since they are largely unknown and depend on market prices and extraction costs for reserves to be not only proven but also economically worthwhile to extract. Specification of the dependence of extraction costs on cumulative fossil fuel use is therefore more important than the specification of S_0. We will set the initial stock of fossil fuel reserves so that it produces the current price of fossil fuel (see section 4 for this calibration), hence $S_0 = 1.72$ TtC[1]. This is more than proven and listed reserves but if the market price is right it will be economically feasible to extract more fossil fuel from the crust of the earth.

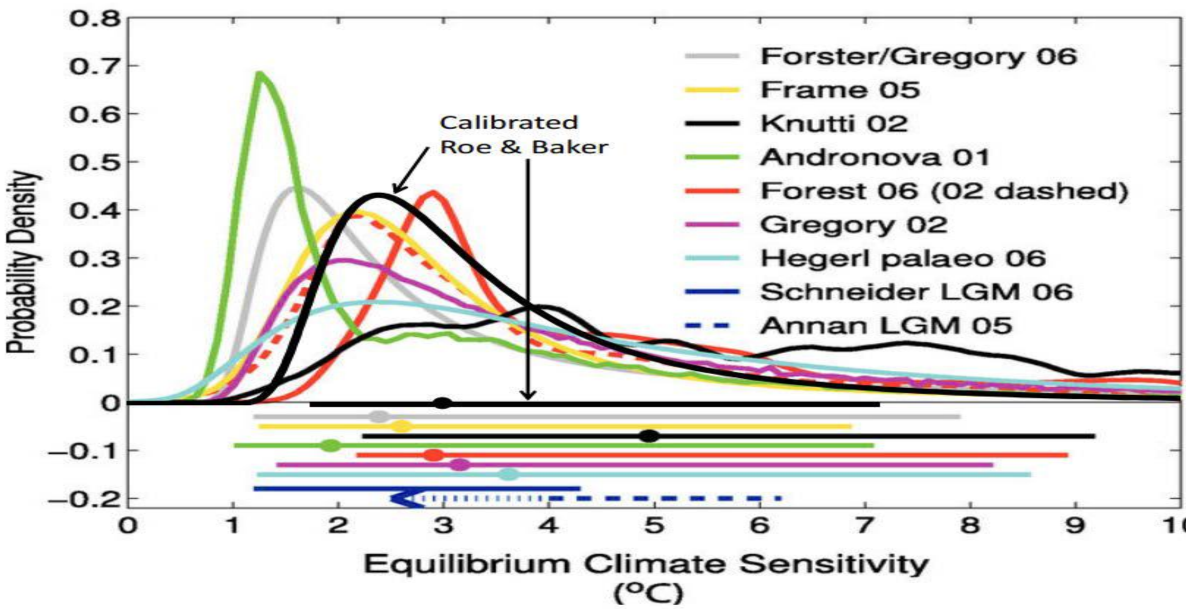
Ensemble simulations of simple climate-carbon-cycle models constrained by observations and projects from more comprehensive models to simulate the temperature response to a broad range of carbon dioxide carbon pathways suggest that limiting cumulative emissions to 1 TtC leads to a mostly likely peak global mean warming of 2^o C above pre-industrial (1900) temperature with a 5-95% confidence interval of 1.3^o to 3.9^o C (Allen et al., 2009b). These results also suggest that the relationship between cumulative emissions and peak warming is remarkably insensitive to the timing of emissions or the peak emission rate. Hence, policy targets based on limiting cumulative emissions of carbon dioxide are likely to be more robust to scientific uncertainty than emission-rate or concentration targets. We use these findings to roughly calibrate the following temperature module:

$$(2) \qquad T = \alpha + \beta \ln(E) = 2 + 3.683 \ln(E),$$

[1] The International Energy Agency suggests in its World Energy Outlook that *proven* international reserves contain 2,860 GtCO2 = 781 GtC, but with higher prices potential reserves can be much higher.

where T denotes peak global mean warming in °C above pre-industrial temperature. Since 1 TtC of carbon leads to a warming of 2° C, we have $\alpha = 2$. The pre-industrial concentration of carbon is 0.581TtC and corresponds to 0° C, hence we have $0 = 2 + \beta \ln(0.581)$ or $\beta = 3.683$. This implies a current temperature of 1.4° C. Although there is a lag between global mean temperature and the carbon stock of about 70 years, we ignore it. This will bias our estimate of the social cost of carbon and optimal carbon tax upwards as can be seen from comparing Liski and Gerlagh (2012) with Golosov et al. (2012).

Figure 2: Estimates of equilibrium climate sensitivity



Source: IPPC (2007)

This simple temperature module implies an equilibrium climate sensitivity (*ECS*) of 2.55, since the global mean temperature increase resulting from a doubling of the atmospheric carbon stock equals $\beta \ln(2) =$ 2.55° C. The IPCC Fourth Assessment Report (2007) estimates the climate sensitivity to be in the range 2 to 4.5° C with a best estimate of about 3° C, and is very unlikely to be less than 1.5° C. Values of the climate sensitivity substantially higher than 4^0 C cannot be excluded, but agreement of models with observations is not as good for those values. The probability density functions for the *ECS* shown in fig. 2 confirms that our climate sensitivity of 2.55° C is within the ballpark range. If anything, it is a bit on the low side in line with recent estimates which suggest that the *ECS* based on the energy budget of the most recent decade is 2° C with a 5-95% confidence interval of 1.2-3.9° C (Otto, et al., 2013).[2]

[2] The transient climate response (*TCR*), defined as the temperature increase at the point of doubling the CO2 concentration following a linear ramp of increasing greenhouse gas forcing, is also sometimes used. Estimates based on the most recent decade suggest a TCR of 1.3° C with a 5-95% confidence interval of 0.9-2.0° C (Otto et al., 2013), which is about 0.3° C lower than the estimates based on the 1990s.

The carbon budget and the risk of unburned fossil fuel

Given E_0 = 0.85, we can burn a total of $S_0 - S$ = 0.3TtC of fossil fuel and limit E to 1 and thus limit global warming to 2° C. If we want to limit the temperature increase to 3° C, we must limit the stock of atmospheric carbon to exp(1/β) = 1.3 TtC and can burn three times as much: 0.9 TtC.

BP (2012) states that in 2011the global economy emitted 34 GtCO2 or 9.3 GtC, so limiting global mean temperature to 2° C means that we can emit for another 300/9.28 = 32 years at this level that before moving fully to a carbon-free economy.[3] If we allow temperature to rise by 3° C, then we can emit like this for almost another century before having to abandon fossil fuel altogether. Of course, if fossil fuel use declines as prices rise over time, fossil fuel reserves can last much longer. But if the climate sensitivity is higher, we have to abandon fossil fuel much more quickly.

3. Production damages from global warming at low and higher temperatures

Nordhaus (2008) assumes for his DICE model that production damages from global warming are 1.7% of GDP at 2.5^0 C and uses it for a particular chosen functional form to calibrate the following ratio of output net of global warming damages to GDP:

$$R^{\text{Nordhaus}}(T) = \frac{1}{1+0.00284T^2} = \frac{1}{1+(T/18.8)^2}.$$

Damages are zero at pre-industrial temperature, so that the net GDP ratio equals one then. Hanemann (2008) supposes production damages are 4.2% of GDP at 2.5° C, which yields the net GDP ratio:

$$R^{\text{Hanemann}}(T) = \frac{1}{1+(T/12.0)^2}.$$

These net GDP ratios imply that half of world GDP is lost at, respectively, 19° C and 12° C, which seems much too small as human mankind let alone economic production will cease at these temperatures. To get higher and more realistic damages at higher temperatures, Weitzman (2010) suggests that output damages should be 50% of world GDP at 6° C and 99% at 12° C. Combining the low-temperature damages of,

[3] The Economist of 4 May 2013 discusses joint research by Carbon Tracker and the Grantham Institute (http://www.carbontracker.org/wastedcapital) which supposes a global carbon budget of 1000 GtCO2 = 273 GtC that can maximally be emitted between now and 2050 for global mean temperatures to be restricted to 2° C (cf. our 0.3 TtC derived above). Listed reserves of energy firms are 421 GtC, which cannot be burnt without jeopardizing the 2° C-target. Hence, governments are either not serious about climate change or energy firms are overvalued as the risk of unburnable fossil fuel is inadequately factored in market prices. Oil and gas companies are betting on a failing climate policy or on a geo-engineering or CCS fix.

respectively, Nordhaus (2008) and Hanemann (2008) with the high-temperature damages of Weitzman (2010), Ackerman and Stanton (2012) thus arrive at the following net GDP ratios:

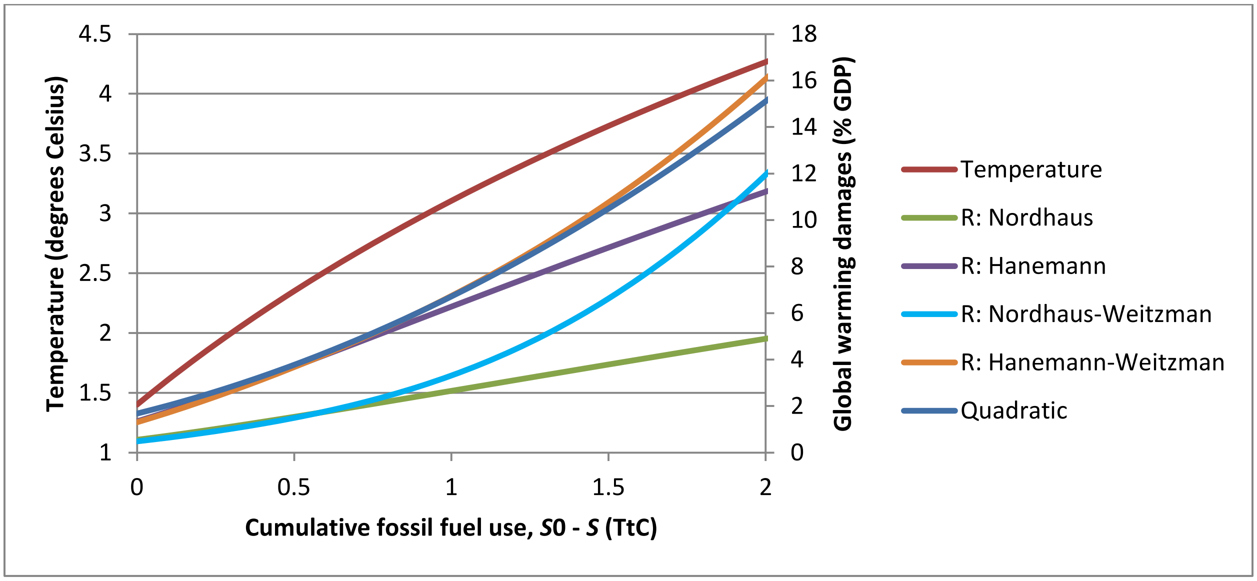
$$R^{\text{Nordhaus-Weitzman}}(T) = \frac{1}{1+(T/20.2)^2+(T/6.08)^{6.76}} \text{ and } R^{\text{Hanemann-Weitzman}}(T) = \frac{1}{1+(T/12.2)^2+(T/6.24)^{7.02}}.$$

Fig. 2 plots on the left-hand side the concave relationship between global mean temperature (T) and cumulative fossil fuel use ($S_0 - S$) and on the right-hand side the percentage of global warming damages ($(1 - R) \times 100\%$) against cumulative fossil fuel use for the various models of global warming damages. The Nordhaus damages can be very well described by the linear function $D^{Nordhaus}(S) = (1 - R) \times 100\% = -2.19\, S + 0.49$ and the Hanemann damages by $D^{Hanemann}(S) = -4.83\, S + 15.89$ in the relevant range, but they are clearly convex for the Nordhaus-Weitzman and Hanemann-specifications. This reflects that large degrees of global warming lead to proportionally much bigger damages. For simplicity, we will use a reduced-form quadratic specification for global warming damages which is close to the Hanemann-Weitzman specifications as long as cumulative fossil use does not exceed 2 TtC (or $E < 1.85$ TtC):

(3) $$D(E) = 0.01(E-0.35)^2\, \bar{p}_0 \quad \text{or} \quad D(S) = 0.0025(1+S_0-S)^2\, \bar{p}_0, \quad \bar{p}_0 = 470 \text{ US\$/tC}.$$

where $\bar{p}_0$ denotes the current market price of fossil fuel. This quadratic specification does not fit too badly at both low and higher levels of global warming.

Figure 2: Temperature and global warming damages



Social cost of carbon

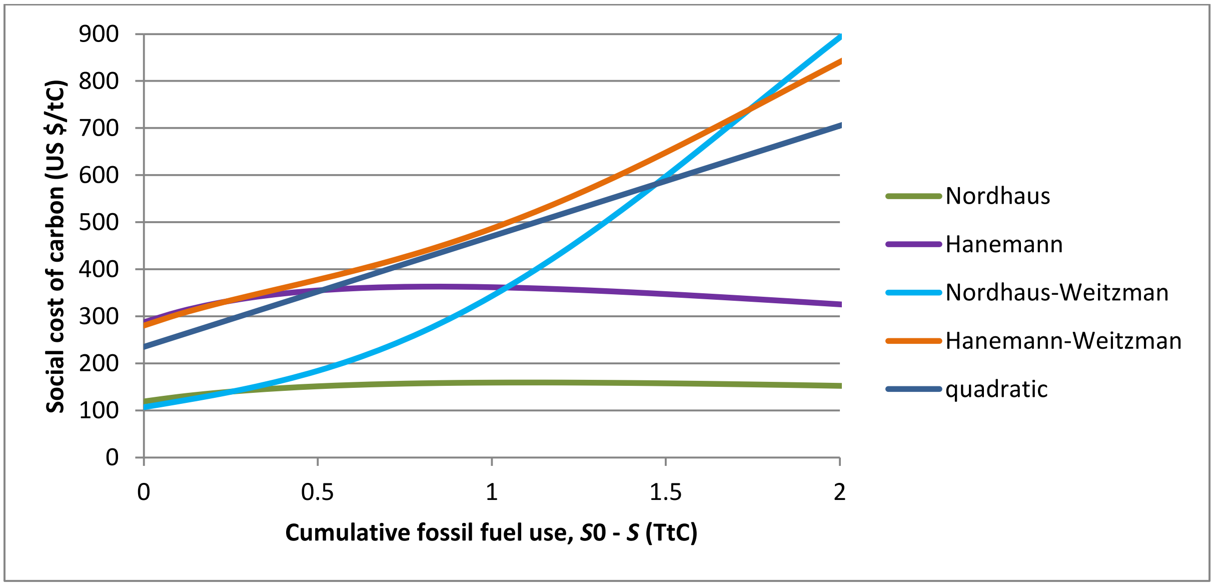
The social cost of carbon equals the present value of marginal damages from global warming. For simplicity, we will use an exogenous and constant interest rate of $r = 0.01$.[4] For the Nordhaus and Hanemann specifications we can use the linear approximations of fig. 2 with constant marginal damages and set world GDP at 70 trillion US$ to get expressions for the social cost of carbon which are more or less constant and independent of cumulative fossil fuel use:

$$SCC^{\text{Nordhaus}} = \frac{2.19 \times 70}{100 \times 0.01} = 153\,\text{US\$/tC} \quad \text{and} \quad SCC^{\text{Hanemann}} = \frac{4.83 \times 70}{100 \times 0.01} = 338\,\text{US\$/tC}.$$

For the other damage functions the social cost of carbon varies with time, since it depends on the time paths of the stock of atmospheric carbon or of cumulative fossil fuel use. However, as we abstract from decay of atmospheric carbon, the social cost of carbon is constant at the end of the fossil fuel era, say time T, and throughout the carbon-free era and only depends on the stock of carbon, $E(T)$ or $S(T)$. For the Nordhaus-Weitzman and Hanemann-Weitzman models we substitute the temperature module (1) and the atmospheric carbon stock equation (2) to get the social cost of carbon during the carbon-free era:

$$SCC = \frac{R'(T) \times GDP \times (\partial T / \partial E) \times (\partial E / \partial S)}{r} = \frac{-R'(T) \times 70 \times (3.683 / E) \times 0.5}{0.01}\,\text{US\$/tC}.$$

Figure 3: The social cost of carbon versus the stock of remaining fossil fuel



[4] The Ramsey rule suggests that the appropriate interest rate to use is $r = 0.01 + \theta g$, where 0.01 is the pure rate of time preference, θ the coefficient of relative intergenerational inequality aversion and g the growth rate in aggregate consumption per capita. With growth richer generations can carry more of the burden and climate policy becomes less ambitious as reflected in a lower social cost of carbon resulting from a higher r, especially if intergenerational inequality aversion is large. We abstract from this and set the interest rate to 1% per annum.

The social cost of carbon for our specification (3) is $SCC^{quadratic} = 0.5D'(E)/r = 2.35(1+S_0-S)/r$.

Fig. 3 plots the social cost of carbon for the various global warming models discussed above. Our specification (3) captures the higher marginal costs of global warming stressed by Weitzman (2010) whereas the Nordhaus (2008) and Hanemann (2008) models do not capture these costs.

4. Effects of carbon taxes and renewables subsidies on market outcomes

We assume that there is a renewable backstop energy source which does not emit carbon, is a perfect substitute for fossil fuel and has an infinitely elastic supply. We suppose that the cost of renewables, *b*, is 50% more expensive than the current market price of fossil fuel to reflect that solar, wind and other forms of renewable energy are not competitive yet, so that $b = 1.5\bar{p}_0 = 705$ US$/etC. These are bold assumptions. For example, wind or solar energy suffer from intermittence problems and require energy storage which oil, gas or coal do not. Renewables are thus not in practice imperfect substitutes for fossil fuel. Furthermore, as the price of energy rises, the supply of renewables will expand. Still, these assumptions about the backstop allow us to get precise insights which will help us to understand the workings of optimal climate policy.

We assume that fossil fuel extraction becomes more costly as more reserves have been depleted, because then the ‘low hanging fruit’ has been taken advantage of and less accessible fields or mines have to be explored. We suppose that the cost of extracting 1 TtC of fossil fuel is given by $G(S) = \bar{p}_0 / S$ US$. This implies that the scarcity rent of fossil fuel equals $100(1-1/S_0) = 42\%$ of the current market price.[5]

The user cost of fossil fuel is defined by $q \equiv p + \tau$, where *p* is the market price of fossil fuel and τ the specific carbon tax. Here we consider exogenous changes in τ; section 6 derives the optimal carbon tax. In both cases, the carbon tax revenues are rebated in lump-sum fashion to the private sector. We also consider subsidies υ on the use of renewables, which are financed by lump-sum taxes.

How much fossil fuel does the market leave untapped in the crust of the earth?

Since the economy currently has fossil fuel prices lower than the cost of renewables, the fossil fuel era comes to an end at the point in time *T* where the user cost of fossil fuel reaches the cost of renewables. Since at the end of the fossil fuel era the scarcity rent on fossil fuel must be zero, the market price of fossil fuel must then be equal to the extraction cost. We thus have the following arbitrage equation:

[5] This will be higher for oil and natural gas and lower for coal.

(4) $$q(T)=G\left(S(T)\right)+\tau(T)=\frac{\overline{p}_0}{S(T)}+\tau(T)=b-\upsilon, \quad b=1.5\overline{p}_0=705 \text{ US\$/etC}.$$

Under “laissez faire” there are no carbon taxes or renewables subsidies, hence (4) indicates that the market outcome locks up 0.67 TtC of fossil fuel in the crust of the earth. Cumulative fossil fuel use is 1.05 TtC which is three to four times bigger than the carbon budget of 0.3 TtC discussed above. The corresponding stock of atmospheric carbon is from (2) 1.37 TtC, which gives from (1) ultimate global warming of 3.2^o C.

Introducing a renewables subsidy of 20% (i.e., $\upsilon = 0.2\,b$) locks up more fossil fuel in the crust of the earth, 0.83 TtC, so that cumulative fossil fuel use is 0.88 TtC and global warming is ultimately curbed to 2.94^o C. Doubling the renewables subsidy to 40% locks up of fossil fuel and thus limits cumulative fossil fuel use to 0.60 TtC and global warming to 2.5^o C.

Demand for fossil fuel and renewables

To understand more about the dynamic adjustment paths and calculate the timing of the advent of the carbon-free era, we need to make additional assumptions about fossil fuel demand. Suppose a constant elasticity of market demand of $\varepsilon = 0.85$, so that demand for fossil is given by $F=(A/q)^{\varepsilon}$ and the inverse demand function by $q=p+\tau=AF^{-1/\varepsilon}$. We calibrate this demand function given a current market price of fossil fuel of 470 US\$/tC and fossil fuel use of 9.3 GtC in 2011, hence $A=470\times\left(\frac{9.3}{1000}\right)^{1/0.85}=1.9.$ If renewables are used, the rates of energy use are:

(5) $$F(t)=0 \quad \text{and} \quad R(t)=\left[A/(b-\upsilon)\right]^{\varepsilon}=6.6 \text{ eGtC} = 340 \text{ T BTU}, \quad \forall t\geq T.$$

Calibration of initial fossil fuel reserves and “laissez faire” outcome

The fossil fuel phase of the market outcome follows from the two additional equations:

(6) $$\dot{S}=-F=-(A/q)^{\varepsilon}, \quad S_0=1.72, \quad A=1.9, \quad \varepsilon=0.85,$$

(7) $$\dot{q}=\left[r-G(S)\right]q+\dot{\tau}-r\tau=(r-\overline{p}_0/S)q+\dot{\tau}-r\tau, \quad q(T)=b=1.5\overline{p}_0=705 \text{ US\$/etC},$$

$R(t)=0,\ 0\leq t\leq T.$ Equation (6) is the fossil fuel depletion equation. Integrating (7) indicates that cumulative fossil fuel use cannot exceed initial reserves: $\int_0^{\infty}F(t)dt\leq S_0.$ The Hotelling rule for efficient resource extraction states that the return on taking fossil fuel out of the ground, selling it and investing it, $r\left[p-G(S)\right],$ must equal the expected return on leaving it in the ground, $\dot{p}$. This arbitrage equation can be rewritten as equation (7). Equations (6) and (7) can be solved together with the terminal condition (4) as a two-point-boundary-value problem.[6] We first solved this system under "laissez faire" to calibrate the initial stock of fossil fuel reserves. We did this by choosing S_0 and T given $q(0) = p(0) = 470$ US$/tC, $q(T) = 705$ US$/etC and $S(T) = 0.67$ TtC. This yielded $S_0 = 1.72$ TtC, which we then used as our calibrated estimate of initial reserves. It also yielded a length of the fossil fuel era of 138 years.

The solid brown lines in fig. 4 below give the dynamic adjustment paths under "laissez faire". As fossil fuel prices rise, fossil fuel use and carbon emissions fall, global warming increases less rapidly from the current level of 1.4^o C. At the end of the fossil fuel era, renewables take over and the global mean temperature stays at 3.2^o C. Of course, in practice the stock of carbon will decay slowly and the global mean temperature will slowly fall again but we abstract from this.

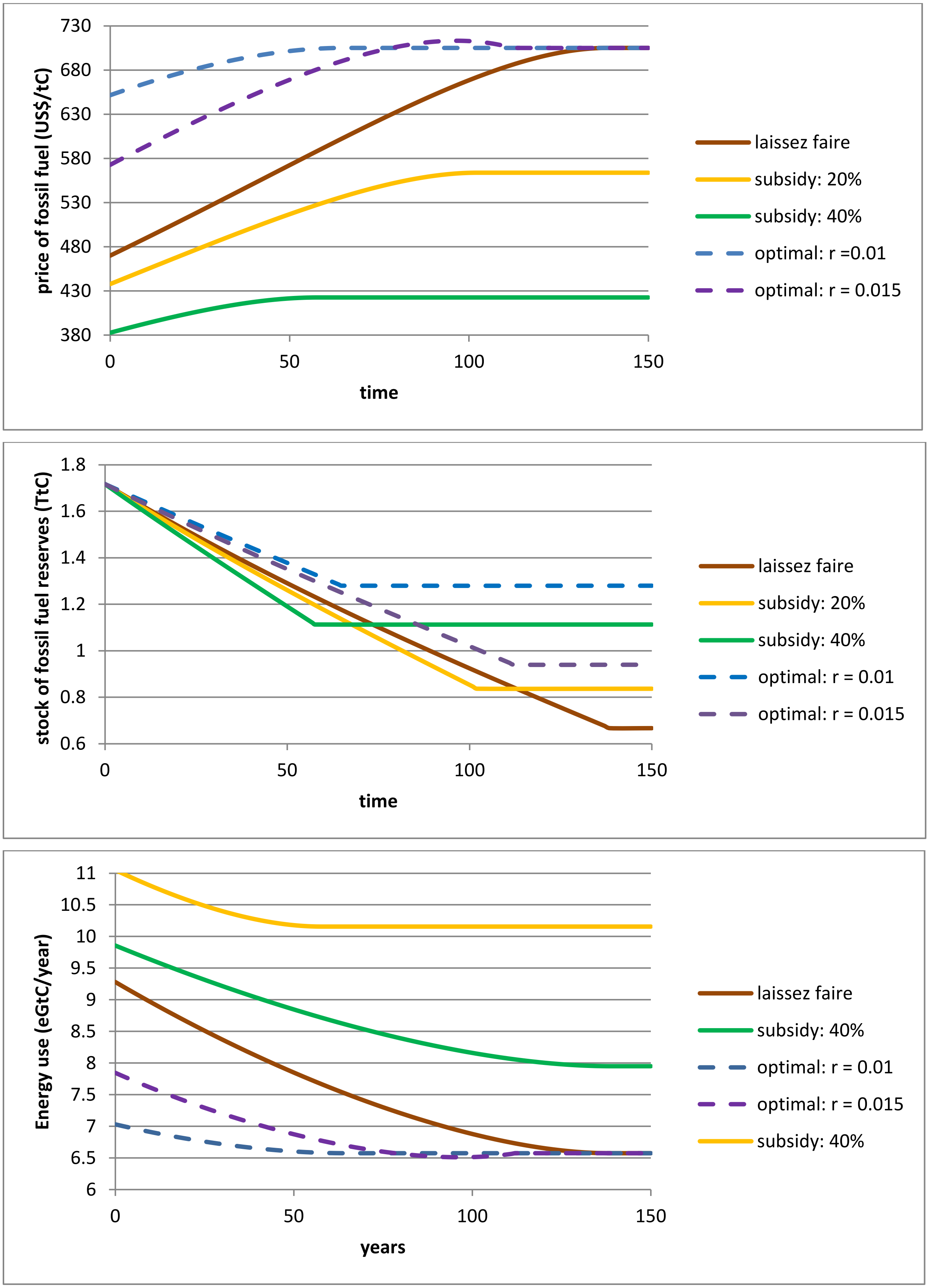
Anticipated carbon taxes, renewables subsidies and Green Paradox effects

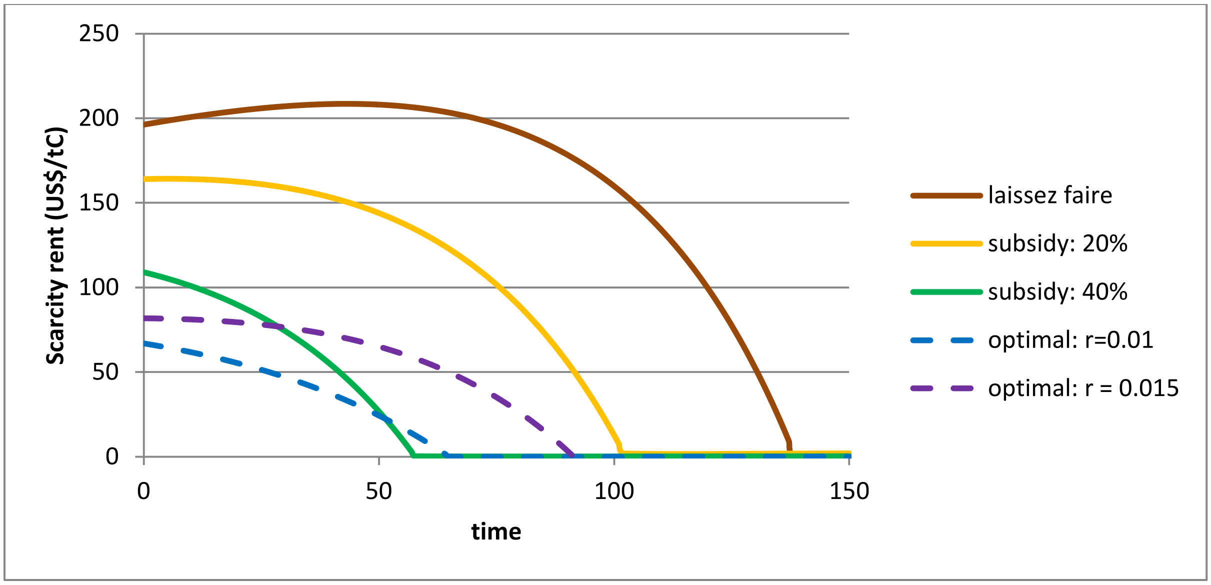
Introducing a specific carbon tax τ that rises at the rate of interest r leaves fossil fuel extraction and reserves paths unaffected. A future carbon or a carbon tax that rises at a faster rate than the interest rate induces fossil fuel owners to extract fossil fuel more rapidly than under "laissez faire". This is known as the Green Paradox, but for the sake of brevity we will illustrate such effects with a renewables subsidy.

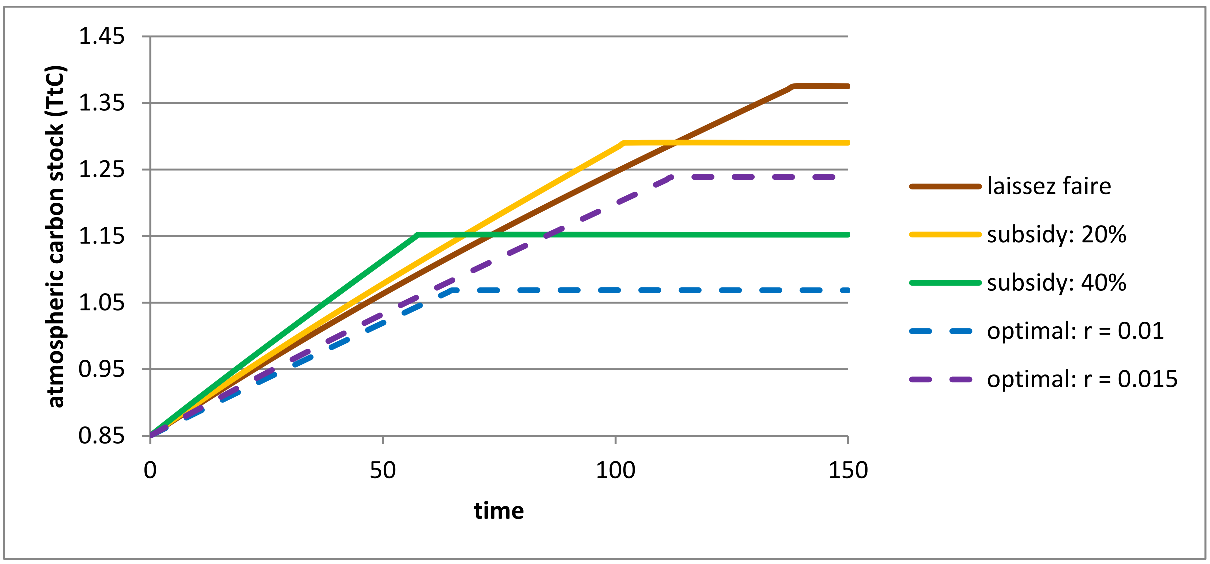
The solid yellow lines in fig. 4 give the time paths if the cost of renewables is reduced from 705 to 564 US$/etC with a 20% subsidy ($\upsilon = 0.2$). We notice five effects. First, the time it takes to transition to the carbon-free era shortens from 138.5 to 102 years. Second, as mentioned above, the stock of fossil fuel that is left forever locked up in the crust of the earth increases from 0.67 to 0.82 TtC. This curbs cumulative the long-run stock of atmospheric carbon from 1.375 to 1.292 TtC, so long-run global warming is reduced from 3.17^o to 2.94^o C. Third, despite curbing global warming in the long run, fossil fuel use is ramped up during the fossil fuel phase which accelerates global warming before the carbon-free era commences. This effect is known as the Green Paradox: owners of fossil fuel reserves pump their fossil fuel up more quickly for fear of their reserves becoming less worth as a result of the cheaper renewables. Fossil fuel prices are lower due to the induced fossil fuel glut. The price of fossil fuel jumps down on impact from 470 to 438 US$/tC and converges at the end of the fossil fuel era to 564 US$/tC instead of 705 US$/tC. Fourth, the scarcity rent on fossil fuel is obviously lower as a result of the renewables subsidy and the induced faster pumping of fossil fuel and ends up being zero at the end of the fossil-fuel era. Finally, fossil fuel use at the end of the fossil fuel must equal renewables use (from the continuity of the path for energy prices). As a result of the subsidy, final energy use jumps from up 6.76 to 7.95 eGtC.

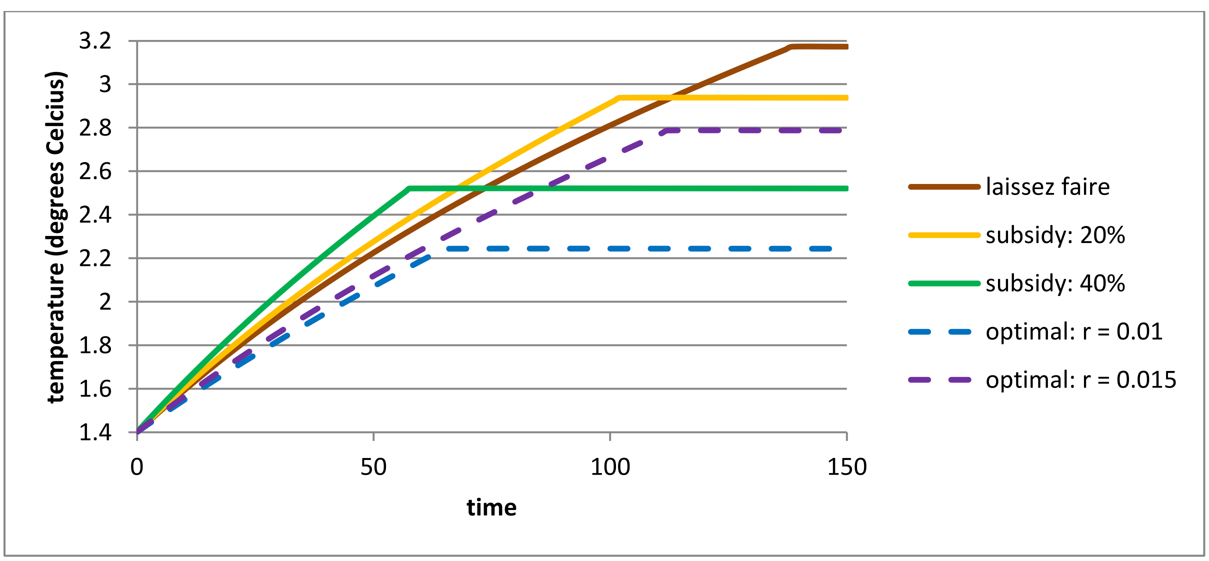
[6] We do this computationally by nesting a 4th-order Runge-Kutta algorithm for solving the differential equations (6) and (7) into a Gauss-Newton algorithm for solving for T and $q(0)$ (or for T and $S(0)$ for the calibration simulation) to ensure that the boundary conditions are satisfied.

Figure 4: Renewables subsidies and the Green Paradox versus optimal policies (S_0 = 1.72 TtC)




Scarcity rent (US$/tC)
250
200
150
100
50
0
0
50
100
150
time
laissez faire
subsidy: 20%
subsidy: 40%
optimal: r=0.01
optimal: r = 0.015


atmospheric carbon stock (TtC)
1.45
1.35
1.25
1.15
1.05
0.95
0.85
0
50
100
150
time
laissez faire
subsidy: 20%
subsidy: 40%
optimal: r = 0.01
optimal: r = 0.015


temperature (degrees Celcius)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
0
50
100
150
time
laissez faire
subsidy: 20%
subsidy: 40%
optimal: r = 0.01
optimal: r = 0.015

The effects of doubling the renewables subsidy to 40% can be seen from the green lines in fig. 4. Green Paradox effects are amplified, but cumulative fossil fuel use and global warming are curbed by more.

A renewables subsidy is often advocated as a second-best alternative to an optimal carbon tax, because electorates prefer the 'carrot' to the 'stick'. It corresponds to 'reculer pour mieux sauter', since global warming first gets worse before it gets better. There are two problems with such a strategy. First, although green welfare may increase, overall welfare may fall (see section 5). Second, if electorates notice that global warming may worsen during the fossil fuel phase, they might undermine the credible announcement of offering renewables subsidies and thereby destroy climate policy altogether.

5. Welfare: Are large renewables subsidies counterproductive?

The global planner maximizes welfare, which is given by the present discounted value of the difference between, on the one hand, the consumer surplus, and, on the other hand, the sum of fossil fuel extraction costs, renewables costs and global warming damages:

(8)
$$W(0) \equiv \int_0^\infty \left[U\left(F(t)+R(t)\right) - G\left(S(t)\right) - bR(t) - D\left(E(t)\right) \right] e^{-rt} dt =$$
$$\int_0^\infty \left\{ \frac{A\left(F(t)+R(t)\right)^{1-1/\varepsilon}}{1-1/\varepsilon} - \left[\frac{F(t)}{S(t)} + bR(t) + 0.005\left(1+S_0-S(t)\right)^2 \right] \bar{p}_0 \right\} e^{-rt} dt \text{ US\$ T.}$$

Notice that the consumer surplus is the area under the demand curve, so $U'(F+R) = q = A(F+R)^{-1/\varepsilon}$. Since carbon taxes are rebated in lump-sum fashion, they do not appear in this expression. The same is true for renewables subsidies, which are financed by lump-sum taxes. Substituting the demand curve, the carbon accumulation equation (1) and the expression for renewables use (5), we rewrite welfare (8) as:

(8′)
$$W(0) = \int_0^T \left\{ \frac{A^\varepsilon q(t)^{1-\varepsilon}}{1-1/\varepsilon} - \left[\frac{\left(A/q(t)\right)^\varepsilon}{S} + 0.0025\left(1+S_0-S(t)\right)^2 \right] \bar{p}_0 \right\} e^{-rt} dt$$
$$+ \left[\frac{A^\varepsilon (b-\upsilon)^{1-\varepsilon}}{\varepsilon - 1} - 0.0025\left(1+S_0-S(T)\right)^2 \bar{p}_0 \right] \frac{e^{-rT}}{r} \text{ US\$ T.}$$

Does a renewables subsidy hurt social welfare?

To convert the change in welfare under the 20% subsidy from the "laissez-faire" outcome into monetary units, we divide the welfare change by the marginal utility of initial consumption of fossil fuel (which is unity under quasi-linear preferences) and express it as a percentage of initial world GDP. This gives a present value welfare gain for the 20% renewables subsidy of 7.25% of world GDP, so that the welfare gains from the ultimate reduction in global warming from 3.17° to 2.94° C dominate the welfare losses of the Green Paradox effects.

However, the benefits of larger renewables subsidies taper off quickly. Indeed, doubling of the renewables subsidy to 40% is counterproductive. Compared with "laissez faire" there is now a welfare *loss* of 21.0% of world GDP. The negative short-run welfare implications of Green Paradox effects now dominate the positive long-run welfare implications of a lower carbon stock (1.24 TtC and 1.07 TtC instead of 1.37 TtC) and less global warming (2.94° and 2.52° C instead of 3.17° C). The negative welfare effects with faster running down of fossil fuel reserves thus outweighs the positive welfare effects of a shorter length of the fossil fuel era (102 and 58 years instead of 138.5 years) and locking up more fossil fuel forever in the crust of the earth (1.11 and 0.84 TtC instead of 0.67 TtC). Hence, renewables subsidies need not improve welfare, especially for large subsidies. It is also easy to see that these subsidies are more likely to be counterproductive if the interest rate is larger, because then the welfare gains from the ultimate curbing of global warming are discounted more heavily. Table 1 below summarizes the impact and long-run effects under "laissez faire" and under a 20% and 40% renewables subsidy. We now consider the social optimum.

Table 1: Effects of renewables subsidies and optimal carbon tax (S_0 = 1.72 TtC)

	"laissez faire"	20% subsidy	40% subsidy	optimum r = 0.01	optimum r = 0.015
Switch time, T	138	102	58	65	112
$S_0 - S(T)$ (GtC)	1050	883	604	437	777
$[S_0 - S(T)]/T$ (GtC)	7.6	8.7	10.5	6.7	6.9
Global warming (° C)	3.17	2.94	2.52	2.24	2.79
Welfare gain (% GDP)	-	7.3	–21.0	71.7	-
$p(0)$ (US$/tC)	470	435	383	341	356
$p(T)$ (US$/tC)	705	564	423	367	427

6. Social optimum and the optimal carbon tax

The social optimum follows from maximizing welfare $W(0)$ subject to the fossil depletion equation (6) and the carbon accumulation equation $\dot{E} = 0.5F,\ E(0) = 0.85$ TtC (see van der Ploeg and Withagen (2012) and the appendix). The optimum has an initial phase where fossil fuel is used exclusively followed from time T onwards by a final phase where renewables are used exclusively. This result derives from the assumptions that fossil fuel and renewables are perfect substitutes and renewables supply is infinitely elastic. Furthermore, it can be established that the time path of the optimal carbon tax always slopes upwards and is concave, which requires no decay of atmospheric carbon. Intuitively, individual owners of fossil fuel reserves internalize that further depletion of their reserves forces them to go to less accessible fields in their region and therefore as the fossil fuel phase continues their extraction costs rise and their use of fossil fuel is curbed. This is why the rise in the carbon tax flattens of as reserves diminish. Finally, the social optimum can be realized in a market economy by setting the specific carbon tax equal to the social cost of carbon. Since there is no learning by doing effects in renewables use, there is no role for a renewables subsidy in the social optimum.

At the switch from the fossil fuel phase to the renewable phase, we have $R(T) = 6.6$ eTtC from (6) and thus also $F(T) = 6.6$ GtC. Hence, the social price of fossil fuel at the time of the switch must equal $q(T) = b = 705$ US$/tC (from continuity in the time path of the social price of energy). The optimal amount of fossil fuel to lock up in the earth at the start of the carbon-free era follows from:

$$(4') \qquad q(T) = G\big(S(T)\big) + \frac{0.5D'\big(E(T)\big)}{r} = \frac{\bar{p}_0}{S(T)} + \frac{0.005\big[1 + S_0 - 0.5S(T)\big]\bar{p}_0}{r} = b, \quad b = 705 \text{ US\$/etC.}$$

The solution to (9) is $S(T) = 1.28$ TtC if $r = 0.01$ and $S_0 = 1.72$. Hence, the optimal ultimate stock of atmospheric carbon is $E(T) = 1.24$ TtC and global warming 2.24^{o} C. The social cost of carbon when the carbon-free era commences is $\theta(T) = 0.5D'\big(E(T)\big)/r = 338$ US$/tC, which is constant from then on. Since the scarcity rent of fossil fuel is zero at the time of the switch, the market price of oil excluding the carbon tax at the time of the switch must equal extraction costs, $p(T) = 470/S(T) = 367$ US$/tC. Fossil fuel reserves, the social price of fossil fuel and the social cost of carbon for the carbon phase and the optimal switch time follow from solving for $t \in [0, T]$ the TBPVP defined by (6) with $S(T)$ from (4′) and

$$(7') \quad \dot{q} = \big[r - G(S)\big]q - 0.5D'\big(E_0 + 0.5(S_0 - S)\big) = r\big[q - \bar{p}_0 / S\big] - 0.005(1 + S_0 - S)\bar{p}_0,\ q(T) = 705 \text{ US\$/etC,}$$

where $q \equiv p + \tau$ is the social price of fossil fuel and $\lambda = p - G(S)$ the scarcity rent on fossil fuel (see appendix).[7] This results in an initial user cost of energy of 652 US$/tC and a duration of the fossil fuel phase of $T = 65$ years. The social cost of carbon θ follows from:

$$(9) \qquad \dot{\theta} = r\theta - 0.5D'\big(E_0 + 0.5(S_0 - S)\big) = r\theta - 0.005(1 + S_0 - S)\bar{p}_0, 0 \le t \le T, \quad \theta(T) = 338 \text{ US\$/tC.}$$

If the specific carbon tax τ is set to the social cost of carbon given by (9), the Hotelling rule for the market economy (7) becomes the Hotelling rule for the social optimum (7′). Hence, with this carbon tax the

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[7] We solve this again by nesting a Runge-Kutta for integrating the ODE's(6) and (7′) into a Gauss-Newton algorithm for solving the three terminal boundary conditions for the appropriate values of $q(0)$, $\tau(0)$ and T.

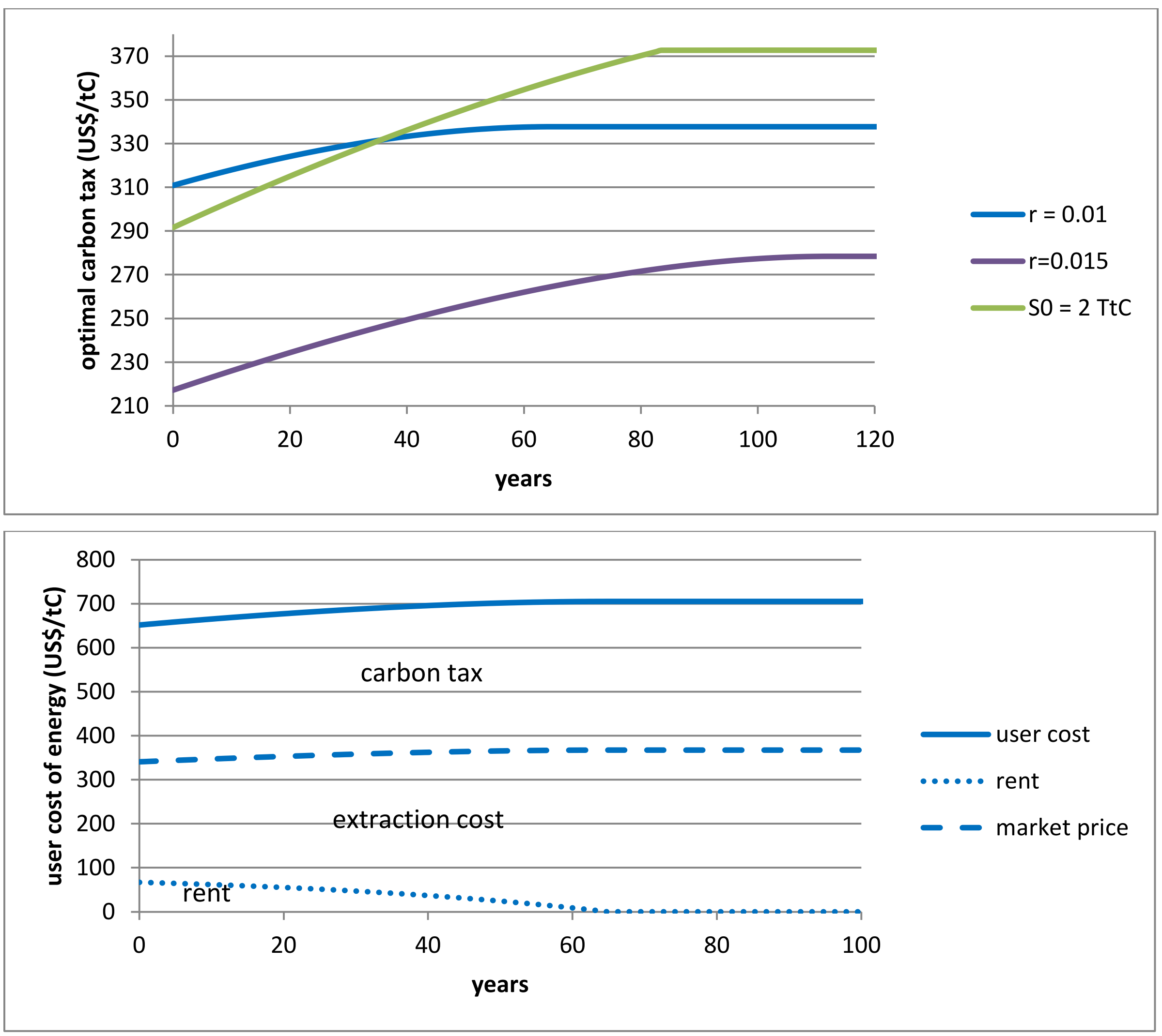
market economy replicates the social optimum. Using this result and integrating (9) with (4′) forwards in time, we get the optimal carbon tax as the present value of all future marginal global warming damages:

$$(10) \qquad \tau(0) = \theta(0) = \int_0^T 0.005\left[1 + S_0 - S(t)\right]\overline{p}_0 e^{-rt} dt + 0.005\left[1 + S_0 - S(T)\right]\overline{p}_0 \frac{e^{-rT}}{r} \text{ US\$/tC.}$$

This yields an initial carbon tax of 311 US$/tC which rises monotonically at a decreasing rate towards a tax of 338 US$/tC at the switch time 65 years later. It does so at a decreasing rate, since fossil fuel producers curb their rates of extraction as reserves are depleted and become less accessible. This carbon tax corresponds to roughly 85 $-cents on a gallon of gasoline or 17 Euro-cents on a litre of petrol.

The dotted blue lines in fig. 4 plot the resulting time paths and the optimal carbon tax (i.e., the social cost of carbon) and the components of the optimal user cost of energy are plotted in fig. 5. Table 1 above also summarizes the impact and long-run effects under the optimal carbon tax.

Figure 5: Optimal cost of carbon and components of user cost of fossil fuel (S_0 = 1.72 TtC)



The social optimum leads compared with "laissez faire" to a present-value welfare gain of 71.7% of world GDP. In annuity terms this is a gain of 0.7% of world GDP. The duration of the optimal fossil fuel phase (65 years) is substantially shorter than under "laissez faire" (138 years). The relatively flat time path for the optimal carbon tax manages to curb cumulative fossil use from 1050 to 437 GtC which corresponds to an average annual use of 6.7 GtC instead of 7.6 GtC under "laissez faire". The social optimum thus does not suffer from the Green Paradox as can be seen from the time paths for fossil fuel depletion in the third panel of fig. 4.

If policy makers are less precautionary and adopt a higher interest rate (say, $r = 0.015$), the time path for the optimal carbon tax is lowered throughout. The carbon tax thus rises from 217 to 278 US$/TC as can be seen from the purple line in fig. 5. Hence, carbon emissions are higher during the fossil fuel phase. Furthermore, the duration of the fossil fuel phase increases from 65 to 112 years. Cumulative carbon emissions are therefore much higher, namely 777 TtC instead of 437 TtC. Average yearly emissions are only a bit higher, namely 6.9 GtC instead of 6.7 GtC. Ultimate global warming is higher also.

If initial reserves were 10% higher which is probably most of what can be expected from the shale gas revolution, the optimal policy is to have initially a lower optimal carbon tax (292 US$/tC instead of 311 US$/tC) but end up with a larger carbon tax (373 US$/tC instead of 278 US$/tC). The time it takes before the fossil fuel era is taken over by the renewables are is longer (84 instead of 65 years). Due to the abundance of fossil fuel reserves, global warming rises to 2.49 C which is larger than the 2.24^o C under our benchmark estimate for S_0 of 1.72TtC. It would have been even more if it were not for the fact that more fossil fuel is locked up at the start of the carbon-free era (1.41 instead of 1.28 TtC).

Renewables subsidies of 20 and 40% lead to higher cumulative fossil fuel use than the social optimum but less than under "laissez faire", namely 883 and 604 GtC, respectively. However, these renewables subsidies induce higher average fossil fuel use than under "laissez faire", namely 8.7 and 10.5 GtC per year, respectively. This confirms that renewables subsidies suffer, in contrast to the optimal carbon tax, from Green Paradox effects. Global warming is curbed more under the optimal carbon tax (2.2^o C) than with renewables subsidies (2.9^o Cand 2.5^o C) and thus more than under "laissez faire" (3.2^o C).

7. Concluding remarks

We have offered an illustrative calibration of a simple model of global warming and of what might be done about it. We suppose that to limit global warming to 2^o C, we must keep the stock of atmospheric carbon below 1 TtC. This corresponds to a climate sensitivity of 2.55^o C for doubling of the atmospheric carbon stock. Half of emitted carbon stays in the atmosphere forever and the other half returns to the surface of the oceans and the earth. We thus abstract from natural decay of atmospheric carbon and from positive feedback effects in the carbon cycle. We let global warming damages increase steeply when cumulative fossil use increases, which is roughly calibrated from a loss of 4.2% of world GDP at 2.5^o C, a loss of 50% of world GDP at 6^o C and a loss of 99% of world GDP at 12^o C. We calibrate fossil fuel reserves to be 1.72 TtC. This ensures that the "laissez-faire" outcome of the model corresponds to the current market price of fossil fuel (470 US$/etC). To limit global warming to 2^o C, we have a carbon budget of fossil fuel that we can burn of 0.3 TtC. Fossil fuel and renewables are perfect substitutes in consumption and production. Fossil fuel extraction costs are currently 58% of the market price and we suppose that they vary inversely with the stock of remaining fossil fuel. Renewables are not competitive yet, since we suppose that their costs are 50% higher than the current market price. Finally, we adopt a partial equilibrium framework by supposing a constant world interest rate of 1% per annum in the benchmark and abstracting from growth and development. This may be justified if there is zero substitution between energy and other factors of production.

The first best policy is to have a carbon tax rising from 311 US$/tC to 338 US$/tC. This amounts to about 140 US$/month per US family or 80 US$/month per UK family.[8] The policy of optimally pricing carbon brings forward the carbon-free era by 74 years and locks up 613 GtC more carbon in the crust of the earth than under "laissez faire". As a result of this and of lower energy use during the fossil fuel phase, global warming is ultimately curbed from 3.17^o C to 2.24^o C. The annuity gain in welfare is 0.72% of world GDP, which seems quite significant. A smaller climate sensitivity and a lag between global mean temperature and the stock of atmospheric carbon would reduce the social cost of carbon and the optimal carbon tax. A more short-sighted policy with a lower interest rate postpones the advent of the carbon-free era and leaves less fossil fuel locked up in the earth, hence leads ultimately to more global warming. The required carbon tax is lower in the short and higher in the long run.

A renewables subsidy 20% shortens the fossil fuel phase by 36 years and locks up 167 GtC more fossil than under "laissez faire". This contributes to mitigating global warming, but the subsidy also elicits the market to extract fossil fuel more rapidly which causes acceleration of global warming during the fossil fuel phase. These Green Paradox effects worsen welfare. But with a 20% subsidy the overall effect is a small annuity gain in welfare of 0.07% of world GDP. However, with larger renewables subsidies the negative welfare effect associated with the Green Paradox increase by more than the potential green welfare gains of bringing the carbon-free era forward and locking up more fossil fuel. For example, with a subsidy of 40% limits global warming to 2.52^o C but nevertheless leads to an annuity welfare loss of

[8] Of course, the revenue from the carbon tax is rebated in lump-sum fashion. Figures are based on an annual footprint of 19 tCO2 per US family and 11 tCO2 per UK family (see UK Carbon Trust, www.carbontrust.com). Poorer countries typically have a smaller CO2 footprint and thus face smaller carbon tax payments.

0.02% of world GDP as the Green Paradox effects have started to dominate. Renewables subsidies thus reduce global warming damages in the long run, but they might harm overall welfare if they are large enough. This is more likely for higher interest rates. In general, renewables subsidies need not be a helpful second-best climate policy. The optimal carbon tax does a better job at flattening the time path for the market price of fossil fuel whilst at the same time not lowering the entire price path and thereby causing Green Paradox effects.

Of course, our calibrations and calculations of the effects of the optimal carbon tax and various renewables subsides are purely illustrative and designed to be used in the classroom. They are meant to highlight the various effects at play as clearly as possible in the simplest possible model with endogenous timing of the advent of the carbon-free era and the optimal amount of fossil fuel reserves to leave untapped. More realistic models of the optimal carbon tax have to allow for general equilibrium and an endogenous interest rate within the context of a Ramsey growth model as, for example, in Golosov et al. (2012), van der Ploeg and Withagen (2013), Gerlagh and Liski (2012), and Rezai et al. (2012b) or within the context of an endogenous growth model of directed technical change as in Acemoglu et al. (2012). They also have to allow in such growth models for different types of fossil fuel (oil, natural gas, coal and unconventional sources such as shale gas and tar sands) and renewables and allow them to be imperfect substitutes as, for example, in Hassler et al. (2011). Finally, it is important to allow for different national jurisdictions and to consider the conflict and cooperation that might evolve between fossil fuel producers and the countries importing fossil fuel as, for example, in Hassler and Krusell (2012).

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Appendix: Social optimum and optimal carbon tax trajectory

The social optimum follows from choosing the time paths of *F* and *R* to maximize social welfare,

(A1) $$\int_0^\infty \left[U(F+R) - G(S)F - bR - D(E) \right] e^{-rt} dt,$$

subject to

(A2) $\dot{S} = -F$ and $\dot{E} = 0.5F$.

This yields the following optimality conditions:

(A3) $\left.\begin{array}{c} U'(F+R) = A(F+R)^{-1/\varepsilon} \le b \\ R \ge 0 \end{array}\right\}$ c.s., $\left.\begin{array}{c} U'(F+R) = A(F+R)^{-1/\varepsilon} \le G(S) + \lambda + 0.5\mu \\ F \ge 0 \end{array}\right\}$ c.s.,

(A4) $\dot{\lambda} = r\lambda + G'(S)F$,

(A5) $\dot{\mu} = r\mu - D'(E)$,

where λ is the scarcity rent of an extra TC of fossil fuel in the ground and μ is the marginal cost of having an extra TC in the atmosphere. The optimum has a distinct fossil-fuel phase and a carbon-free phase:

(A6) $\dot{S} = -(A/q)^{\varepsilon}, \quad \dot{q} = r\left[q - G(S)\right] - D'\left(E_0 + 0.5(S_0 - S)\right), \quad \dot{\tau} = r\tau - 0.5D'(E), \quad 0 \le t \le T,$

(A7) $F(t) = 0$ and $R(t) = (A/b)^{\varepsilon}, \forall t > T.$

where $q = p + \theta$ is the social price of carbon, $\theta = 0.5\ \mu$ is the social cost of carbon, and p is the market price of fossil fuel. The scarcity rent of fossil fuel follows from $\lambda = q - G(S) - \theta = p - G(S).$ The fossil fuel phase is described by a three-dimensional TBVP with the following boundary conditions:

(A8) $S(0) = S_0, \quad q(T) = b, \quad \theta(T) = \dfrac{D'\left(E_0 + 0.5(S_0 - S(T))\right)}{2r}, \quad G\left(S(T)\right) + \dfrac{D'\left(E_0 + 0.5(S_0 - S(T))\right)}{2r} = b.$

We use the first three boundary conditions to simulate the TBPV by nesting a 4th-order Runge-Kutta algorithm within a Gauss-Newton algorithm for ensuring that these boundary conditions are satisfied by varying $q(0)$ and $\tau(0)$. We then determine the switch time T so that the simulated value of $S(T)$ satisfies the fourth boundary condition. To calculate welfare $W(0)$, we solve:

$$\dot{W} = rW - \left[U(F+R) - G(S)F - bR - D(E)\right], \quad 0 \le t \le T, \quad W(T) = \frac{U\left(\Psi(b)\right) - b\Psi(b) - D\left(E_0 + 0.5(S_0 - S(T))\right)}{r}.$$