The Grey Paradox: How Owners Of Carbon-emitting Resources Can Benefit From Carbon Regulation.

Renaud Coulomb *   Fanny Henriet †

October 22, 2013

Abstract

This paper studies how owners of carbon-emitting resources can benefit from carbon taxation. We build a Hotelling-like model with three energy resources: an exhaustible polluting resource, a polluting backstop (not exhaustible) and a clean backstop (not exhaustible). The CO_2 concentration must be kept under a carbon ceiling. The optimal extraction path is decentralized by a tax on emissions, and tax revenues are not redistributed. We consider the cases where the exhaustible resource gets exhausted and the polluting backstop is used at some point and find that, under some conditions, tightening the carbon regulation increases the profits of owners of the exhaustible resource. When this resource is cheaper to extract than the dirty backstop, tightening carbon regulation increases the profits of the exhaustible resource owners : *(i)* if its demand elasticity is low enough or *(ii)* if its extraction cost is close enough to that of the dirty backstop, or *(iii)* if its pollution content is low enough (compared to that of the dirty backstop), or, *(iv)* if its initial stock is low enough. When the exhaustible resource is more expensive to extract than the dirty backstop, tightening the carbon regulation increases the profits of its owners. We extend our results in a two-sector economy where the exhaustible resource has a comparative advantage in one of the two sectors, and to the case with an extra exhaustible polluting resource. In this case, we show that owners of the more polluting of the two exhaustible resources may win more or loose less than those of the other exhaustible resource.

*Paris School of Economics (PSE), 48 boulevard Jourdan 75014 Paris, France. Email: renaud.coulomb@gmail.com,

†Paris School of Economics (PSE), 48 boulevard Jourdan 75014 Paris, France and Banque de France, 31 rue Croix des petits champs 75001 Paris, France.

Keywords: Carbon Taxation, Global Warming, Nonrenewable Resources, OPEC, Fossil Fuels.

JEL Classification: Q31, Q38, Q41, Q48, Q54, Q58, H21, H23

1 Introduction

In 2009, CO_2 from energy production represented 65% of greenhouse gas emissions and fossil fuels accounted for 81% of the world energy supply IEA (2011). Taxing carbon emissions have different effects on the profits of fossil fuels owners depending on the characteristics of the fossil fuels they own. Main fossil fuels – coal, gas and oil – are marked out by their recoverable reserves, their pollution content, as shown in Table 1, and their delivery cost that varies depending on their use (sector, location, technology) and their extraction cost, see Table 1. This paper studies the impact of tightening a carbon cap over the CO_2 concentration on profits of owners of exhaustible polluting resources when energy resources are optimally extracted and the optimum is decentralized by a tax on CO_2 emissions. We find that the profits of owners of not-too-polluting exhaustible resources may increase thanks to (optimal) carbon taxation if a dirtier abundant resource is also used, even if tax revenues are not redistributed (the Grey Paradox).

At the COP6 of the UNFCCC[1] , Dr Rilwanu Lukman, OPEC Secretary General declared *"Especially vulnerable are the oil producing developing countries, which are mainly OPEC member countries, [...], their principal revenue-earner, petroleum, is inextricably associated with the downside of the negotiations. It is important to ensure that measures taken to combat climate change do not place an unfair burden on oil."* Two reasons may explain why tightening carbon taxation would negatively affect oil revenues and more generally profits of carbon-emitting resources owners: first, the demand of these resources may decrease in carbon-regulated countries, second, for a given after-tax resource price, carbon taxation transfers rents from resources owners countries to carbon-regulated countries if tax revenues are not redistributed to resources owners countries.

However, we show that profits of owners of carbon-emitting exhaustible resources may increase with optimal carbon taxation even if tax revenues are not redistributed to them. Two characteristics of these resources lay behind this result: their exhaustibility and their relatively low pollution content compared to very abundant fossil fuels, e.g coal. First, exhaustible resources like oil are likely to be exhausted (unless the carbon regulation is very stringent) so that their cumulative consumption will not depend on carbon regulation. Second, since most of the rare fossil fuels (oil, natural gas) are less

[1]The 6th Conference of the Parties to the UN Framework Convention on Climate Change - The Hague, November 2000

polluting than very abundant fossil fuels[2] their after-tax price may increase more than the carbon tax, that will rise up profits of owners of these resources. The basic idea is that a not-too-polluting exhaustible resource (e.g. oil) will be in competition with more-polluting resources (e.g. coal) at some point ; the carbon tax increases competitive advantage of this not-too-polluting resource vis-ï¿œ-vis very polluting resources.

Our paper casts light on redistribution of wealth amongst countries driven by energy resources scarcity rents. Fossil fuels reserves are spread very unequally in the world as shown in Table 2. More than 57% of conventional proved gas reserves are located in Russia, Iran or Qatar. Almost 70% of crude oil proven reserves are located in only six countries: Canada, Saudi Arabia, Iran, Iraq, Kuwait, and United Arab Emirates. Around 75% of recoverable coal reserves are located in only five countries: the USA, Russia, China, Australia and India. To the noticeable exception of Russia, countries having the largest endowments in oil or gas have low endowments in coal. Carbon taxation raises important issues concerning redistribution of wealth among countries. If carbon taxation lowers profits of coal owners but increases profits of oil owners, some countries like OPEC countries may see their wealth increased by carbon taxation contrary to countries with large coal endowments. We focus only on the redistributional aspects driven by changes in energy prices due to carbon taxation.[3]

The empirical literature has made several attempts to evaluate the impacts of long-term carbon regulation or the impacts of Kyoto Protocol on fossil fuels prices, oil and gas revenues. There is no consensus about the size of the increase of fossil fuels price due to carbon regulation and its effect on revenue of major fossil fuels exporters. For an overview of the models about how Kyoto Protocol impacts OPEC countries, see Barnett et al. (2004). Persson et al. (2007) use an empirical model to quantify the impact of carbon taxation on oil profits assuming that oil producers are in perfect competition and maximize their profits over an intertemporal horizon. Johansson et al. (2009) assume that OPEC producers play strategically as a dominant firm in the transport sector facing only fringe

[2] Oil can be divided into two categories: unconventional and conventional oil. Unconventional oil is petroleum produced or extracted using techniques other than the conventional (oil well) method and includes oil shales, oil sands, based synthetic crudes and derivative products, coal-based liquid supplies, biomass-based liquid supplies and liquids arising from chemical processing of natural gas (the IEA's oil Market Report unconventional oil). Looking only at the upstream process, bituminous sand are 100% more polluting than crude oil. Unconventional is thus far more polluting than conventional oil. In addition, sources for producing unconventional oil are abundant, so that unconventional oil can be considered as a dirty backstop in the transportation sector for instance.

[3] However, benefits or losses from reducing emissions for each country depend also on issues such that country-specific vulnerability to climate change, country-specific mitigation costs, the global mitigation effort and country-specific contribution to it. We do not consider these issues in this paper.

producers. Contrary to the rest of the literature, Persson et al. (2007) and Johansson et al. (2009) found that oil profits increase due to carbon regulation. However, no exlicit analytical results are derived.

We build a Hotelling-like model where the CO_2 concentration must be kept under a carbon ceiling. This paper follows modelization introduced by Chakravorty et al. (2006). The social planner model has the following features. Utility comes from three perfect substitute sources of energy: an exhaustible polluting resource, a non-exhaustible strongly polluting resource (the dirty backstop) and a non-exhaustible clean resource (the clean backstop). Each resource is distinguishable according to its carbon content and its extraction cost. Resources owners are in perfect competition. The regulation takes the form of a carbon cap over the atmospheric carbon stock. This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol, or as the first-best carbon policy if the damage function can be approximated by a binary damage function with nil marginal damage when the CO_2 concentration is kept under the threshold and infinite otherwise. We assume no natural decay of carbon. The social planner seeks to maximize the total surplus taking account of the scarcity constraint and the carbon cap constraint. To implement his optimal policy, the social planner has a soft power: he can put a carbon tax on CO_2 emissions but cannot forbid the use of a particular resource or set a specific tax or a quota for each different resource. When both the exhaustible resource gets effectively exhausted and the dirty backstop is used – relevant case to study the Grey Paradox –, we show that a unique carbon tax path allows to decentralize the equilibrium. We consider the effect of optimal taxation on resources owners profits and tax revenues are not redistributed. The optimal extraction path and the optimal tax path are the same if the social planner includes profits of resource owners in its objective function or not.

Our framework is close to Chakravorty et al. (2006, 2008): a social planner seeks to extract optimally an exhaustible polluting resource and a clean resource and the environmental regulation takes the form of carbon cap over CO_2 concentration. However, questions addressed in this paper are very different. Our paper studies the optimal extraction of polluting resources in different setting and the consequence of optimal taxation of $\mathrm{CO_2}$ emissions on profits. Contrary to Chakravorty et al. (2006) our model includes several polluting resources to be able to point out the increase of profits of owners exhaustible resources due to carbon taxation. Like Chakravorty et al. (2008), we assume the existence of different polluting resources.

Our paper focuses on the impact of carbon regulation on resources owners profits. Because the Grey Paradox may only occur when the exhaustible polluting resources is used before or after a dirtier resource, characterization of the different extraction paths is an important part of the analysis, even if not the main interest of that paper. We simplify the characterization of the different extraction paths by assuming that natural dilution is negligible. In our framework, a more expensive before-tax resource cannot be extracted before a less expensive before-tax resource following Herfindahl principle. However, if the cheaper resource is more polluting, it is not necessarily extracted along the optimal extraction paths even if the more expensive one is extracted at some point.

As in Chakravorty et al. (2006, 2008), fossil fuels owners are in perfect competition. Thus profits are not due to market-power of resources owners but come from the scarcity of the resource. Energy markets, and in particular oil market have been modeled in various ways throughout economic literature. A large empirical literature has tried to determine which market structure explains the best the dynamics of the oil price. Griffin (1985) estimates that market-sharing cartel model explains relatively well OPEC behavior. Jones (1990) and Dahl & Yucel (1991) also support the cartel hypothesis. Other papers find that OPEC manipulates prices only over some periods of time, and that increasing prices of 1974-1980 are not due to an OPEC price manipulation (Loderer 1985). Several studies (Ezzati 1976, MacAvoy 1982 and Verleger 1982) explain oil prices changes in a competitive model.

The optimum is decentralized by a tax on CO_2 emissions. A particular path of emissions can be decentralized either by a carbon tax or by carbon quotas, as long as the price dynamics of carbon-emitting resources is well understood. In our setting, a price instrument and a quantity instrument are similar from the efficiency point of view since benefits and costs of mitigation actions are known without uncertainty[4]. However, the chosen instrument and its modalities of use impact differently profits of fossil fuels owners on energy markets. A carbon tax without abatement or auctioned exchangeable quotas without redistributing carbon revenues are commonly presented as the worst instruments for the profits of fossil fuels owners. Redistributional aspects of the instruments have received few attention in the debate over climate change regulation. The literature has focused on capturing rents from fossil fuels producers thanks to taxation of externalities (Liski & Tahvonen 2004, Bergstrom 1982)

[4]For a comparison of both price and quantity instruments with uncertainty in a general framework, one can refer to Weitzman (1974) and its application to the climate change problem is Pizer (1997).

or using tariffs (Brander & Djajic 1983). Our paper shows that taxing a negative externalities may increase the profits of the owners of fossil fuels that generate it. Note that if tax revenues would be redistributed, owners of exhaustible polluting resources would see their welfare increased by the carbon regulation if their resources still get exhausted despite the regulation.

The Grey Paradox [5] can be expressed in the following way: the profits of owners of not-too-polluting exhaustible resources may increase thanks to (optimal) carbon taxation. the form of a carbon tax or exchangeable quotas will affect differently resources owners depending on the carbon content of each resource, but also on their delivery costs and their relative abundance. We study how rents associated with exhaustible resources change when the ceiling is lowered. The mechanisms are as described below. When switching from one resource to another, the price of the exhaustible fossil fuel must equal the price of the dirty backstop if both are used along the optimal path of extraction. When tightening the carbon regulation, the carbon tax increases. For a given increase of the carbon tax, the tax paid per unit of the dirty backstop is larger than the tax paid per unit of the less-polluting exhaustible resource, due to the difference in pollution contents. It comes that profits of owners of the exhaustible resource at the date of switch must increase to keep prices equal. However, as the demand decreases at each date, this date of switch is postponed so that the effect of carbon regulation on exhaustible resource owners is ambiguous. When the exhaustible resource is used before the dirty backstop i.e it is cheaper to extract than the dirty backstop, if this resource is sufficiently scarce, or if its extraction cost is close enough to the extraction cost of the dirty backstop, or if its pollution content is low enough, or if the demand elasticity is low enough, the profits of the owners of the exhaustible polluting resource will increase when the carbon regulation is tightened. If the exhaustible resource is used after the dirty backstop (more expensive to extract than the dirty backstop) and exhausted, tightening the carbon regulation reduces the cumulative consumption of the dirty backstop, lets unchanged the cumulative consumption of the exhaustible resource and increases the profits of owners of this resource.

We extend the analysis to a two-sector economy. Sectors are modeled as in Chakravorty & Krulce (1994). The dirty backstop and the exhaustible resource are perfect substitutes in the electricity sector

[5] Let us note that in spite of their close names, the phenomenon we describe has nothing to do with the Green Paradox presented by Sinn (2008). The Green Paradox deals with supply-side adverse effects of carbon regulation on carbon emissions.

but the dirty backstop must be transformed into fuel at a positive cost to be used in transportation sector, contrary to the exhaustible resource that can be used at the same cost in both sectors. Once transformed, fuel from the dirty backstop is a perfect substitute to the exhaustible resource in the transportation sector. This setting is realistic when considering for instance the oil/coal specialization in transportation and power generation. This conversion cost can be seen as the cost of coal-to-liquid process for instance. This setting allows for joint use of the dirty backstop and the exhaustible resource in the economy, the exhaustible one tending to be used in priority in the transportation sector. A first condition for the Grey Paradox to appear is that both polluting resources must be in competition at least in one sector. We show that main results of the one-sector model still hold.

A second extension consists in introducing a second exhaustible polluting resource. In this new setting, there are four energy resources, perfect substitute in demand: two exhaustible polluting resources, a non-exhaustible strongly polluting resource and a non-exhaustible clean resource. We show that if a resource is in direct competition only with a dirtier backstop, results of the main model still hold. When both exhaustible resources are in direct competition, we show that owners of the least polluting of the polluting resources do not necessarily benefit more or loose less from making the carbon regulation more stringent. However if two resources in direct competition have close enough extraction costs, tightening the carbon regulation will always benefit more or harm less the profits of owners of the least polluting resource.

The remainder of this paper is organized as follows. Section 2 presents the main model. Section 3 presents the results over the effects of carbon regulation on the profits of owners of the exhaustible resource. Section 4 extends the results in a two-sector economy. Section 5 adds a second exhaustible polluting resource. Section 6 concludes the paper.

2 The model

2.1 Assumptions and notations

We consider that utility comes from energy consumption. Three different energy resources, perfect substitutes in demand, are available: an exhaustible resource, few polluting, in quantity X_e^0, a non exhaustible strongly polluting resource and a non exhaustible clean resource. Resources

Fossil fuels	Reserves (MBtu)	Pollution content (kgCO2/MBtu)
Coal	2.32606E+13	103.54
Oil	7.37308E+12	70.02
Natural Gas	6.37507E+12	52.03

Table 1: Estimated reserves and pollution contents of fossil fuels

IEA (2008), for oil and gas, reserves reported by *Oil and Gas Journal*, MBtu=million of British thermal units.

e,d,b respectively stand for 'the $e$xhaustible resource', 'the $d$irty backstop' and 'the clean $b$ackstop': $u(x_e(t) + x_d(t) + x_b(t))$. Resources are labeled R_e, R_d and R_b. We write $D(.)$, the decreasing energy demand function. We define θ_i, $i = \{e, d, b\}$, the pollution content of resource i: the use of one unit of resource i leads to θ_i units of CO_2. We assume that $\theta_e < \theta_d$ and $\theta_b = 0$. Writing c_i, for $i = \{e, d, b\}$, the extraction cost of resource i, we assume that $c_e < c_b$ and $c_d < c_b$. The dirty backstop is non exhaustible (or equivalently the dirtier resource is abundant enough not to be exhausted for the ceiling regulation we choose). The clean resource is available in infinite quantity at cost c_b. The initial amount of R_e is written X_e^0, and the variation of its current stock writes:

$$\dot{X}_e(t) = -x_e(t)$$

We assume that the social discount rate is constant and equals r. The carbon stock, written $Z(t)$, must be kept under a threshold $\overline{Z}$. This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol. This type of carbon regulation is closer to first-best carbon regulation that constant tax policy due to the fact that marginal damage are steeply increasing with the carbon stock.[6]. Since the dirty backstop is available in infinite quantity, the ceiling is binding

[6] All results found below hold with a constant carbon tax. With a constant carbon tax rather than a carbon ceiling, ordering the extraction would become simpler. Writing μ, the unitary tax, the prices of the exhaustible resource and the dirty backstops respectively become: $c_e + \lambda_e^0 e^{rt} + \theta_e \mu$ and $c_d + \theta_d \mu$. Because, the backstop prices are constant through time, for a given set of parameters, only one backstop – the clean or the dirty one – is used through time. Thus, the condition to get both polluting resources used (and R_e exhausted) writes $c_e + \theta_e \mu < c_d + \theta_d \mu < c_b$ and $\theta_e X_e^0 < \overline{Z} - Z^0$. The solution must satisfy:

$$\begin{aligned} \int_0^T D(c_e + \lambda_e^0 e^{rt} + \theta_e \mu) dt &= X_e^0 \\ c_e + \lambda_e^0 e^{rT} + \theta_e \mu &= Min(c_b; c_d + \theta_d \mu). \end{aligned}$$

	Coal	Crude Oil	Natural Gas
North America	**28.4**	**15.7**	**5.1**
Canada	0.8	13.4	0.9
United States	27.5	1.4	3.9
Central & South America	**1.5**	**8.3**	**4.2**
Venezuela	0.1	6.5	2.7
Europe	**8.9**	**1.1**	**2.8**
Eurasia	**26.5**	**7.4**	**32.4**
Russia	18.3	4.5	27.0
Middle East	**0.1**	**56.3**	**41.0**
Iran	0.1	10.4	15.2
Iraq	.	8.6	1.8
Kuwait	.	7.8	0.9
Qatar	.	1.1	14.6
Saudi Arabia	.	20.1	4.1
United Arab Emirates	.	7.4	3.4
Africa	**3.7**	**8.6**	**7.9**
Asia & Oceania	**30.9**	**2.6**	**6.7**
Australia	8.9	0.1	0.5
China	13.3	1.2	1.3
India	7.0	0.4	0.6
World	**100.0**	**100.0**	**100.0**

Table 2: Share of fossil fuels reserves by country and world region in 2008.

Calculus based on the Energy Information Administration data over recoverable coal reserves, proven crude oil reserves and proven natural gas reserves. Estimates of coal reserves for Iraq, Kuwait, Qatar, Saudi Arabia and United Arab Emirates are missing. Only countries having more than 5% of world reserves of at least one fossil fuel are included in this table.

	Lifting costs	Finding costs	Upstream costs
United States	12.18	21.58	33.76
On-shore	12.73	18.65	31.38
Off-shore	10.09	41.51	51.60
All Other Countries	9.95	15.13	25.08
Canada	12.69	12.07	24.76
Africa	10.31	35.01	45.32
Middle East	9.89	6.99	16.88
Central & South America	6.21	20.43	26.64

2009 dollars per barrel of oil equivalent. Values represent average costs. Upstream costs represent the sum of finding and lifting costs.

Table 3: Costs for producing crude oil and natural gas, 2007–2009.

for any value of $\overline{Z}$. We assume that $\overline{Z} > Z^0$. There is no natural decay of carbon. Thus the variation of the carbon stock through time is simply given by:

$$\dot{Z}(t) = \theta_e x_e(t) + \theta_d x_d(t)$$

To implement his optimal policy, the social planner has a soft power: he can put a carbon tax on CO_2 emissions but cannot forbid the use of a particular resource or set a specific tax, a quota for each different resource. This carbon tax can be paid by consumers (demand side) or by fossil fuels providers (extraction side). Resources owners are in perfect competition.

2.2 The Welfare maximization program

The social planner seeks to find the extraction $\{x_e(t), x_d(t), x_b(t)\}$ which maximizes the net discounted social surplus under the environmental constraint:

$$\int_0^{\infty} e^{-rt}\Big(u(x_e(t)+x_d(t)+x_b(t)) - c_e x_e(t) - c_d x_d(t) - c_b x_b(t)\Big)dt$$

s.t.

$$\begin{aligned}
\dot{X}_e(t) &= -x_e(t) \\
\dot{Z}(t) &= \theta_e x_e(t) + \theta_d x_d(t) \\
Z(t) &\leq \overline{Z} \\
X_e(t), x_i(t) &\geq 0
\end{aligned}$$

with Z^0, X_e^0 given.

Writing $\lambda_e(t)$, the shadow value of the remaining stock of the exhaustible resource, R_e $X_e(t)$ and $\mu(t)$ the shadow cost of the pollution stock $Z(t)$, Transversality conditions are given by:

$$\lim_{t\to\infty} \lambda_e(t)e^{-rt}X_e(t) = 0 \tag{2.1}$$

$$\lim_{t\to\infty} \mu(t)e^{-rt}Z(t) = 0 \tag{2.2}$$

Equation 2.1 simply states that the exhaustible resource must be exhausted in the long run if the scarcity rent is positive.

2.3 First Order Conditions

We define the current value Hamiltonian:

$$\begin{aligned}
H(t) = u(x_e(t) + x_d(t)) - c_e x_e(t) - c_d x_d(t) \\
-\lambda_e(t)x_e(t) \\
-\mu(t)(\theta_e x_e(t) + \theta_d x_d(t))
\end{aligned}$$

with the following slackness conditions:

$$\nu(t) \geq 0 \quad \text{and} \quad \nu(t)(\overline{Z} - Z(t)) = 0 \tag{2.3}$$

$$\beta(t) \geq 0 \quad \text{and} \quad \beta(t)X_e(t) = 0 \tag{2.4}$$

$$\epsilon(t) \geq 0 \quad \text{and} \quad \epsilon(t)x_e(t) = 0 \tag{2.5}$$

For any control $\{x_e(t), x_d(t)\}$ there exist co-state variables $\lambda_e(t)$, $\mu(t)$, that must satisfy following conditions together with Transversality conditions and slackness conditions:

$$\dot{\lambda}_e(t) = r\lambda_e(t) - \frac{\partial H(t)}{\partial X_e(t)} \iff \dot{\lambda}_e(t) = r\lambda_e(t) + \beta(t) \tag{2.6}$$

$$\dot{\mu}(t) = r\mu(t) - \frac{\partial H(t)}{\partial Z(t)} \iff \dot{\mu}(t) = r\mu(t) + \nu(t) \tag{2.7}$$

$$\frac{\partial H(t)}{\partial x_e(t)} = 0 \iff p_e(t) = c_e + \lambda_e(t) + \theta_e\mu(t) \tag{2.8}$$

$$\frac{\partial H(t)}{\partial x_d(t)} = 0 \iff p_d(t) = c_d + \theta_d\mu(t) \tag{2.9}$$

$$\frac{\partial H(t)}{\partial x_b(t)} = 0 \iff p_b(t) = c_b \tag{2.10}$$

The co-state variable $\lambda_e(t)$ represents the current value of the scarcity rent of the exhaustible resource. As shown in Hotelling 1931, it increases at rate r: the discounted net marginal surplus of extraction must be constant. Along the optimal path, extracting a supplementary unit must be equivalent to saving it for a latter use. Writing $\lambda_e^0 \equiv \lambda_e(0)$, it comes that:

$$\lambda_e(t) = \lambda_e^0 e^{rt}.$$

The co-state variable $\mu(t)$ represents the current value of the shadow cost of marginal pollution. It exhibits a familiar pattern driven by the ceiling-shaped carbon regulation. If the ceiling does not bind but will bind, the pollution cost increases at the rate of the discount rate. The intuition behind this result is similar to Hotelling rule since emitting CO_2 can be seen as extracting clean air from a reservoir with an initial stock of defined by $\overline{Z} - Z^0$. With positive constant natural dilution, this result is unchanged. Writing $\mu^0 \equiv \mu(0)$, it comes that:

$$\mu(t) = \mu^0 e^{rt}.$$

The optimal price of R_e is simply the sum of its extraction cost, its pollution cost and its scarcity rent by equation 2.8. The optimal price of the dirty backstop is simply the sum of its extraction cost and its pollution cost by equation 2.9. The unitary pollution cost of carbon is independent from the source it comes from, but the pollution costs per unit of energy differ due to the difference in pollution contents.

Finding the optimal extraction path requires to determine the initial scarcity rent, λ_e^0, the initial shadow cost of pollution μ^0, the date of switch from one fossil fuel to another, t_1, and the date the ceiling is reached, $\underline{t}$, such that taking account of the dynamics of the prices, of the scarcity rent and of the carbon tax (expressed by equations 2.6-2.9), the solution verifies that: (i) the energy price is continuous through time (this implies that price is continuous at dates t_1 and $\underline{t}$); (ii) if R_e is exhausted, it gets exhausted when its price reaches the price of the following resource in the extraction order (if R_e is used in first position, the dirty backstop succeeds to R_e at time t_1, if R_e is used in second position, the clean backstop succeeds to R_e, at time $\underline{t}$); (iii) the backstop starts to be used exactly when the carbon ceiling starts to bind; (iv) when the ceiling starts to bind, at time $\underline{t}$ cumulated emissions equal $\overline{Z} - Z^0$.

Decentralizing the optimal extraction path requires to implement a carbon tax. We show below that when both both polluting resources are used and R_e gets exhausted, a unique carbon tax exists that allows to decentralize the equilibrium (Lemma 1). This tax must equal the shadow cost of pollution, $\mu(t)$, and current marginal profit writes $\lambda_e(t)$.

2.4 Ordering resources extraction: the "least cost first" principle.

Without natural absorption and carbon sequestration, we stop using fossil fuels once the CO_2 concentration reaches the ceiling, thus the date the ceiling binds corresponds to the date of switch to the clean backstop. The maximum amount of pollution put in the atmosphere is fixed and equals $\overline{Z} - Z^0$.

With equal extraction costs, only the least polluting resource is used or this resource is exhausted (and the dirty backstop is used). If this resource gets exhausted and the dirty backstop is used, there is no particular order of extraction, energy prices are equal through the whole path if both resources are used. Hereafter, it is assumed that extraction costs are different. From price equations 2.8 and 2.9, it comes than there is no stop-and-go in the use of a polluting resource and no joint use. A resource whose extraction cost is higher than another resource cannot be used before that resource following the Herfindahl principle (Herfindahl 1967). Indeed, without natural dilution of CO_2, the scarcity rent and the pollution cost increase at the same rate, r. If $c_e < c_d$ and $\lambda_e^0 + \theta_e\mu^0 > \theta_d\mu^0$, it would imply that $\forall t$, $p_d(t) < p_e(t)$ and only the dirty backstop would be used whereas R_e would be cheaper and less polluting.[7]

Finally, among the different extraction paths described in Table 4, only cases where both resources are used and R_e gets exhausted (Cases A1 and B1) are relevant to study the Grey Paradox. It follows that the analysis only focuses on these cases. However, we fully characterize the different extraction paths in order to determine the conditions over parameters to get the relevant cases.

Case	Extraction costs	The exhaustible resource	The dirty backstop
Case A1	$c_d > c_e$	used, exh.	used
Case A2	...	used, not exh.	not used
Case B1	$c_d < c_e$	used, exh.	used
Case B2	...	used, not exh.	used
Case B4	...	used, not exh.	not used
Case B3	...	not used	used

"..." means that the value of that cell is the same than the value of the cell right above.

Table 4: The different extraction paths with two polluting resources.

2.5 Decentralization of the optimal extraction path by using a tax on CO2 emissions

We assume that fossil fuels owners face no threat concerning their property rights over their fossil fuels reserves (See Strand 2010 for a study of the impact of unsecured property rights over extraction). We assume that the tax scheme of the social planner is credible for perfectly foresighted individuals and fossil fuels owners are in perfect competition. The carbon tax can be paid by consumers (demand side) or by fossil fuels providers (extraction side). Both options lead to the same results in this framework.

Lemma 1. *If both polluting resources are used and R_e gets exhausted, setting the carbon tax to the value of the shadow cost of pollution, $\mu(t)$, is the only way to decentralize the optimum.*

In other words, the dynamics of the carbon tax in such models is always given, but when both

[7] Note that if natural dilution is positive but constant, the Herfindahl principle still holds.

polluting resources are used, the value of the tax when extraction goes from one polluting resource to the other polluting resource is fixed thus the entire tax path is determined. In a decentralized economy, writing $\pi(t)$, the unitary profits of owners or R_e and $\tau(t)$, the carbon tax, optimal prices must write:

$p_e(t) = c_e + \lambda_e(t) + \theta_e\mu(t) = c_e + \pi_e(t) + \theta_e\tau(t)$ when $p_e(t) < p_d(t)$

$p_d(t) = c_d + \theta_d\mu(t) = c_d + \theta_d\tau(t)$ when $p_e(t) > p_d(t)$

where $\lambda_e(t)$ and $\mu(t)$ are defined by the set of necessary conditions over the continuity of the energy price, the exhaustion of R_e, and cumulative emissions.

A necessary condition to decentralize the optimum is that profits increase at the interest rate, otherwise fossil fuels owners would have an incentive to reallocate the resource extraction to increase their profits. It is clear that setting the carbon tax at $\mu(t)$ allows to decentralize the optimum. We show that if both resources are used and R_e gets exhausted, $\mu(t)$ is the only carbon tax that allows to decentralize the equilibrium. Since the energy price path is fully determined and unique and profits must increase at rate r, the carbon tax has specific dynamics. In our model, fossil fuels prices net of extraction cost increase at the rate of the social discount rate, thus the tax must also increase at the social discount rate. We assume that both resources are used, and R_e gets exhausted. It is clear that the tax cannot be discontinuous when the dirty backstop is used or when R_e is used. When the dirty backstop is used, by equation 2.9, the carbon tax equals $\mu(t)$ and increases at rate r. If the tax is different that $\mu(t)$ when R_e is used, the carbon tax must be discontinuous at the date of switch from one fossil fuel to another. However, there is no downward jump of the carbon tax at this date, since otherwise owners of the resource used in position 1 would have an incentive to postpone their extraction to increase their profits, and similarly there is no upward jump since owners of resource used in position 2 could increase their profits by bringing forward their extraction. It comes that the tax is continuous, and thus there is a unique initial value of the tax that allows to decentralize the optimum, μ^0.

Corollary 2. *When both resources are used and R_e gets exhausted, the marginal profit at time t is given by $\lambda_e(t)$. Cumulative discounted profits are proportional to the initial scarcity rent, λ_e^0 and write:*

$$\Pi = \int_0^\infty e^{-rt} D(p(t))\lambda_e(t)dt = \lambda_e^0 {X_e}^0.$$

If R_e is not exhausted, perfect competition annihilates profits of owners of R_e. Profits are not due to market-power of resources owners but come from the scarcity of the resource. When only one polluting resource is used (Chakravorty et al. 2006), even if the energy price is well defined through time, there is an infinite number of ways of setting the carbon tax and the scarcity rent to implement the optimal extraction. The social planner can always capture the profits of resources owners by setting a tax. To fully capture profits, he can set the tax equal to the optimal price net of extraction cost.

If tax revenues are redistributed, owners of the exhaustible resources will see their welfare increase if their resource still gets exhausted. This due to the fact that the price of the resource must increase, thus the sum of the profits and tax must increase at each date. Owners of a resource that is not exhausted may win or loose depending on the increase in price compensates in their revenues the decrease of consumption or not.

3 Results

3.1 The exhaustible resource is cheaper to extract than the dirty backstop, $c_e < c_d$

If $c_e < c_d$, the exhaustible resource R_e is necessarily used since it is less polluting that the dirty backstop R_d. Two cases exist depending on whether R_e gets exhausted (the dirty backstop is used) or not (the dirty backstop is not used). R_e gets exhausted and both resources are used if and only if $X_e^0 < \frac{\overline{Z}-Z^0}{\theta_e}$.

We consider Case $A1$, both polluting resources are used to get to the celling ($X_e^0 < \frac{\overline{Z}-Z^0}{\theta_e}$). The social planer must set the carbon tax such that the exhaustible resource gets exhausted when its price equals the dirty backstop price, given consumption of both resources until the dirty backstop price equals the clean backstop price exactly when the carbon stock reaches the carbon ceiling. The solution

$\{\lambda_e^0, \mu^0, t_1, \underline{t}\}$[8] must satisfy:

$$
\begin{aligned}
c_e + \lambda_e^0 e^{rt_1} + \theta_e \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} && (3.1)\\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b && (3.2)\\
\int_0^{t_1} D(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt &= X_e^0 && (3.3)\\
\theta_e X_e^0 + \int_{t_1}^{\underline{t}} \theta_d D(c_d + \theta_d \mu^0 e^{rt}) dt &= \overline{Z} - Z^0 && (3.4)
\end{aligned}
$$

Case $A1$ is described in Figure 3.1. Bold curves represent the dirty backstop and the exhaustible resource prices and the medium curve represents the scarcity rent. The initial prices paths are described by the plain curves. Dotted curves represent these prices and the scarcity after a decrease of the ceiling. Vertical bold black lines represent the tax by unit of resource and vertical bold grey lines represent the profits at the date of switch from one resource to another.

Reducing the carbon ceiling increases the carbon tax (effect 1 in Figure 3.1). Indeed, let assume that this is not the case, it comes that the dirty backstop price would be lower over the whole path. The exhaustible resource price must also decrease otherwise, its global consumption would decrease whereas the dirty backstop consumption would increase, and R_e would not be exhausted. However, if its price decreases, the exhaustible resource gets exhausted earlier. At the new switch date from R_e to the dirty backstop, the dirty backstop price is thus lower than the switch price before the carbon ceiling was tightened, thus the dirty backstop consumption increases. Contradiction. The price at which the dirty backstop starts to be used increase (effect 2). Indeed let assume that this is the opposite. If the switch date t_1 decreases, the initial energy price must decrease to keep R_e exhausted, thus the price at which this resource gets exhausted – and the dirty backstop starts to be used –, must decrease (effect 3). However, by equation 3.4 when tightening the carbon constraint, the dirty backstop consumption must decrease thus the price at which it starts to be used must increase. Contradiction. The date the ceiling binds is brought forward when the carbon ceiling is tightened (effect 4).

Tightening the ceiling has two effects on profits of owners of R_e. The price at which the dirty

[8] If $X_e^0 > \frac{\overline{Z} - Z^0}{\theta_e}$, R_e is not fully exhausted and the solution is described by a similar set of equations, excluding the dirty backstop, and setting R_e rent to zero, excluding equation of exhaustion of R_e (equation 3.3) and equaling final price of R_e to the clean backstop price. The level of the carbon tax and the scarcity rent are undetermined. The social planner will take the whole carbon tax revenues for itself, it comes that profits of R_e producers are discontinuous and undefined when $\overline{Z}$ goes lower $Z^0 + \theta_e X_e^0$

backstop starts to be used increases (effect 1), so that value of the tax at the date of switch (long vertical bold black lines), $\mu^0 e^{rt_1}$ increases. The rent at date t_1 is exhausted (vertical bold grey lines) increases as well (effect 3), as it satisfies $\lambda_e^0 e^{rt_1} = (c_d - c_e) + (\theta_d - \theta_e)\mu^0 e^{rt_1}$.[9] On the other hand, as the tax increases, the after-tax price of the exhaustible resource increases, the demand at each date decreases so that the date t_1 at which it is exhausted, is postponed. Its final price is higher, thus the current profits when it gets exhausted are higher. However, it gets exhausted over a longer period of time. It is not straightforward to see which effect is the larger (effect 5). Let look at a (too) simple case and assume that the demand is totally inelastic. Then, the only effect of a carbon tax is not to postpone extraction (the demand remains the same at each date during extraction) but to shorten the length of extraction, that is to say to bring forward the date at which we switch from the dirty backstop to the clean backstop. The date t_1 of switch from the exhaustible resource to the dirty backstop, on the other hand, cannot be moved (as long as $\overline{Z} - Z^0 > \theta_e X_e^0$). So that the second effect described earlier vanishes: when the tax increases, the date t_1 at which R_e is exhausted is not postponed. On the other hand, the first effect does not disappear: the date at which the dirty backstop extraction stops must be brought forward, so that the tax must increase at each date in order to make the price of the dirty backstop $c_d + \theta_d \mu^0 e^{rt}$ reach sooner the clean backstop price, c_b. As a result, the final before tax price of the exhaustible resource is increased ($\lambda_e^0 e^{rt_1} = (c_d - c_e) + (\theta_d - \theta_e)\mu^0 e^{rt_1}$), as t_1 remains the same, the scarcity rent increases.

Lemma 3. $\forall \overline{Z} > \theta_e X_e^0 + Z^0$,

$$\frac{d\lambda_e^0}{d\overline{Z}} < 0$$

iff.

$$\frac{D(p(t_1))}{D(p(0))}\frac{\theta_d}{\theta_e} + (\theta_d - \theta_e)\frac{\mu^0}{\lambda_e^0} > 1.$$

Proof. Straightforward from the differentiation of Eqs 3.1-3.4 with respect to $\overline{Z}$, see 7.1. □

Remark that $(\theta_d - \theta_e)\frac{\mu^0}{\lambda_e^0} > 0$, then a sufficient condition is that $\frac{D(p(t_1))}{D(p(0))}\frac{\theta_d}{\theta_e} > 1$. When $D(p(t_1)) = D(p(0))$, it is always the case that $\frac{D(p(t_1))}{D(p(0))}\frac{\theta_d}{\theta_e} > 1$. This very simple example shows the role of the

[9] It is straightforward, there that if the polluting backstop was less polluting than the exhaustible resource, the profits of owners of the exhaustible resources will decrease if the carbon ceiling is tightened.

elasticity of demand on the outcome. Without any more information on this elasticity, one can, however, shows that following propositions hold:

Proposition 4. *If R_e gets exhausted and R_d is used, and if the elasticity of demand is small enough, tightening the carbon ceiling increases the profit of R_e owners.*

$\exists \epsilon^*$ *such that:*

$$\left\{ \forall p, -\frac{D^{'}(p)p}{D(p)} \leq \epsilon^* \, and \, \overline{Z} > \theta_e X_e^0 + Z^0 \right\} \Longrightarrow \frac{d\lambda_e^0}{d\overline{Z}} < 0.$$

Proof. We know that $\frac{d\lambda_e^0}{d\overline{Z}} < 0$ iff.

$$\frac{D(p(t_1))}{D(p(0))} \frac{\theta_d}{\theta_e} + (\theta_d - \theta_e) \frac{\mu^0}{\lambda_e^0} > 1.$$

By the mean value theorem, there exists a date t_i, satisfying $0 \leq t_i \leq t_1$ such that:

$$\begin{aligned} \frac{D(p(t_1))}{D(p(0))} &= 1 + \frac{D^{'}(p(t_i))}{D(p(0))}(p(t_1) - p(0)) \\ &\geq 1 - \left(-\frac{D^{'}(p(t_i))}{D(p(t_i))} p(t_i) \right) \frac{p(t_1) - p(0)}{p(t_i)} \\ &\geq 1 - \left(-\frac{D^{'}(p(t_i))}{D(p(t_i))} p(t_i)\right) \frac{c_b - c_e}{c_e} \end{aligned}$$

If $\forall p$, $-\frac{D^{'}(p)}{D(p)} p \leq \frac{c_e(1-\theta_e/\theta_d)}{c_b - c_e} \equiv \epsilon^*$, then tightening the carbon ceiling increases R_e profits. □

Proposition 5. *If R_e gets exhausted and R_d is used, and if the pollution content of R_e is small enough compared to that of R_d, tightening the carbon ceiling increases the profit of R_e producers.*
$\exists \eta^*$*, with* $0 < \eta^* < 1$*, such that:*

$$\left\{ \frac{\theta_e}{\theta_d} \leq 1 - \eta^* \, and \, \overline{Z} > \theta_e X_e^0 + Z^0 \right\} \Longrightarrow \frac{d\lambda_e^0}{d\overline{Z}} < 0.$$

Proof. Straightforward from Lemma 3. □

If the pollution contents of R_e is low enough compared to that of R_d, then the comparative advantage of R_e is high with respect to its direct competitor (R_d), and the owners of R_e benefit from carbon regulation.

Consider now the extreme case in which $c_e = c_d$. In this case, the scarcity rent of R_e is initially nil. With carbon regulation, the two resources are extracted simultaneously, so that $\lambda_e^0 = (\theta_d - \theta_e)\mu^0$. The initial scarcity rent is no longer nil ant it increases with carbon regulation. We show in the next proposition that this result remains true even with different extraction costs, if they are close enough.

Proposition 6. *If R_e gets exhausted and R_d is used, and the extraction cost of the exhaustible resource is close enough from that of the dirty backstop, tightening the carbon ceiling increases the profits of owners of R_e.*

$\forall \overline{Z} > \theta_e X_e^0 + Z^0$, $\exists c^* < c_d$ such that:

$$c_d \geq c_e \geq c^* \quad \Longrightarrow \quad \frac{d\lambda_e^0}{d\overline{Z}} < 0$$

Proof. Using Lemma 3, and replacing $(\theta_d - \theta_e)\frac{\mu^0}{\lambda_e^0}$ by $1 - \frac{c_d - c_e}{\lambda_e e^{rt_1}}$, it comes that $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$-\frac{D(p(t_1))}{D(p(0))}\frac{\theta_d}{\theta_e} + \frac{c_d - c_e}{\lambda_e e^{rt_1}}.$$

But $\lambda_e^0 e^{rt_1} > (\theta_d - \theta_e)\mu^0 e^{rt_1}$ But $\mu^0 e^{rt_1}$ does not depend on c_e as $(\mu^0 e^{rt_1}, \underline{t} - t_1)$ are defined by:

$$\begin{aligned} \theta_d \int_0^{\underline{t}-t_1} D(c_d + \mu^0 e^{rt_1 + ru})du &= \bar{Z} - Z^0 - \theta_e X_e \\ \mu^0 e^{rt_1} e^{r(\underline{t}-t_1)} &= c_b - c_d \end{aligned}$$

and $\mu^0 e^{rt_1}$ is strictly positive for any $\bar{Z}$. At $\mu_0 e^{rt_1}$ given, as $-\frac{D(c_b)}{D(c_e)}\frac{\theta_d}{\theta_e} + \frac{c_d - c_e}{(\theta_d - \theta_e)\mu^0 e^{rt_1}}$ is continuous with c_e and decreases with c_e and is strictly negative for $c_e = c_d$, then there exists c^* such that Proposition 6 holds. □

Proposition 7. *If R_e gets exhausted and the backstop used, and R_e is scarce enough, tightening the carbon ceiling increases the profits of owners of R_e.*

$\forall \overline{Z} > Z^0$, there exists X^* such that:

$$X_e^0 < X^* \Longrightarrow \frac{d\lambda_e^0}{d\overline{Z}} < 0.$$

Proof. Demands $D(p(t_1))$ and $D(p(0))$ are continuous functions of the initial stock X_e^0. Moreover, $lim_{X\to 0}\frac{D(p(t_1))}{D(p(0))} = 1$, as a result, $\forall\epsilon$, $\exists X^*$ such that $X_e^0 < X^* \implies \left\{\frac{D(p(t_1))}{D(p(0))} \geq 1-\epsilon \text{and } \theta_e X_e^0 + Z^0 < \bar{Z}\right\}$. Take $\epsilon < \frac{\theta_d - \theta_e}{\theta_d}$ and the corresponding X^*, then it is the case that for $X_e^0 < X^*$, $\frac{d\lambda_e^0}{d\overline{Z}} < 0$ ☐

Remark 8. $\forall X_e$

$$\lim_{\overline{Z}\to(Z^0+\theta_e X_e)^+} \lambda_e^0 > 0.$$

Proof. Using equation 3.1, it comes that $\lambda_e^0 = e^{-rt_1}(c_d - c_e) + (\theta_d - \theta_e)\mu^0$. Rewriting equation 3.4 into $X_e^0\theta_e + \int_{t_1}^{\underline{t}} \theta_d D(c_d + \theta_d\mu^0 e^{rt})dt = \overline{Z} - Z^0$, it comes that, $\forall\bar{Z}$, writing $\bar{Z} = Z^0 + \theta_e X_e + \epsilon$, we get $\underline{t} - t_1 < \frac{\epsilon}{\theta_d D(c_b)}$. Using Equation 3.2, $c_d + \theta_d\mu^0 e^{r\underline{t}} = c_b$ we get $\lambda_e^0 = e^{-rt_1}(c_d - c_e) + (\theta_d - \theta_e)\frac{c_b - c_d}{\theta_d e^{r\underline{t}}}$. But $t_1 < \frac{\theta_e X_e}{D(c_b)}$, so that $\lambda_e^0 > e^{-r\frac{\theta_e X_e}{D(c_b)}}(c_d - c_e) + (\theta_d - \theta_e)\frac{c_b - c_d}{\theta_d} e^{-r(\frac{\theta_e X_e}{D(c_b)} + \frac{\epsilon}{\theta_c} D(c_b))} > e^{-r\frac{\theta_e X_e}{D(c_b)}}(c_d - c_e) > 0$ ☐

Proposition 9. *For a concave or linear demand function, oil profits cannot exhibit a U-shape when the carbon ceiling is tightened i.e the Grey Paradox cannot occur when making the carbon regulation more stringent, if it does no occur for a less strict carbon regulation.*

$\forall Z^1, Z^2$ *such that* $Z^1 > \theta_e X_e^0 + Z^0$, $Z^2 > \theta_e X_e^0 + Z^0$ *and* $Z^1 > Z^2$, *if* $D'' < 0$,

$$\frac{d\lambda_e^0}{d\overline{Z}}_{|\overline{Z}=Z^1} > 0 \Longrightarrow \frac{d\lambda_e^0}{d\overline{Z}}_{|\overline{Z}=Z^2} > 0$$

Proof. From Lemma 3, $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of $N = -\theta_d e^{rt_1} D(p(t_1)) + \theta_e e^{rt_1} D(p(0)) + r\theta_d\mu^0 e^{rt_1}\theta_e \int_0^{t_1} D'(p(t))e^{rt}dt$. Thus, $\frac{d^2\lambda_e}{d\overline{Z}d\overline{Z}}$ has the sign:

$$-\theta_d D'(p(t_1))\lambda_e^0 r e^{rt_1}\frac{dt_1}{d\overline{Z}} + (\theta_e - \theta_d)D'(p(0))\theta_e\frac{d\mu^0}{d\overline{Z}} - \theta_d\frac{d\mu^0}{d\overline{Z}}\theta_e\int_0^{t_1} D''(p(t))re^{rt}\lambda_e^0 e^{rt}dt$$

If $D'' \leq 0$, $-\theta_d\frac{d\mu^0}{d\overline{Z}}\theta_e\int_0^{t_1} D''(p(t))re^{rt}\lambda_e^0 e^{rt}dt < 0$ thus $\frac{d^2\lambda_e}{d\overline{Z}d\overline{Z}}|_{\frac{d\lambda_e^0}{d\overline{Z}}=0} < 0$. ☐

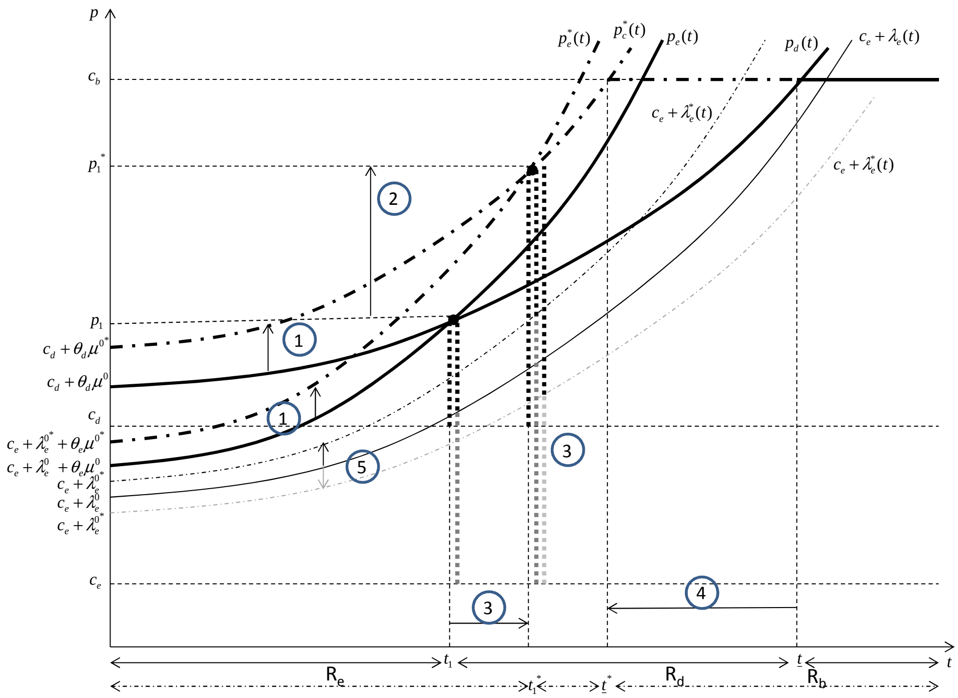

Figure 3.1: Profits, the carbon tax and the price paths when the carbon ceiling is lowered, $c_e < c_d$

3.2 The exhaustible resource is more expensive to extract than the dirty backstop, $c_e > c_d$.

If $c_e > c_d$, without carbon regulation, the exhaustible resource is not be used[10]. The carbon regulation may help the exhaustible resource to be used if its price becomes cheaper than the dirty backstop price.

As shown in Table 4, four different cases exist. Both the dirty backstop and the exhaustible resource are used to get to the ceiling, the dirty backstop is used first, the exhaustible resource gets exhausted (Case $B1$). Both polluting resources are used to get to the ceiling, the dirty backstop first, the exhaustible resource is not exhausted (Case $B2$). Only the exhaustible resource is used to get to the ceiling (Case $B4$). Only the dirty backstop is used to get to the ceiling (Case $B3$). The relevant case to study is Case $B1$ where both resources are used, and the exhaustible resource gets exhausted.

[10]Note that if the dirty backstop was less polluting than the exhaustible resources and $c_d < c_e$, the exhaustible resource would never be used, even with carbon regulation.

We show the conditions over parameters to get this case below in Lemma 10.

When the exhaustible resource is used after the dirty backstop and exhausted, the solution $\{\lambda_e^0, \mu^0, t_1, \underline{t}\}$ [11] satisfies:

$$c_e + \lambda_e e^{rt_1} + \theta_e \mu^0 e^{rt_1} = c_d + \theta_d \mu^0 e^{rt_1} \quad (3.5)$$

$$c_e + \lambda_e e^{r\underline{t}} + \theta_e \mu^0 e^{r\underline{t}} = c_b \quad (3.6)$$

$$\int_{t_1}^{\underline{t}} D(p(t))dt = X_e^0 \quad (3.7)$$

$$\int_0^{t_1} \theta_d D(p(t))dt + \theta_e X_e^0 = \overline{Z} - Z^0. \quad (3.8)$$

We assume that resource extraction is as described by Case $B1$ and that after a marginal decrease of the carbon ceiling we still stay in that case. Case $B1$ is described in Figure 3.3. Bold curves represent the fossil fuels prices and the medium curve represents the scarcity rent. The initial prices paths are described by the plain curves. Dotted curves represent these prices and the scarcity rent after a decrease of the ceiling. Vertical bold black lines represent the tax by unit of each resource and vertical bold grey lines represent the profits of R_e at the date of switch from one resource to another. In that case, tightening the carbon regulation will increase the carbon tax, bring forward the date of switch to R_e and the date the ceiling binds, and increase the profits of owners of R_e. Indeed, first let us remark that if the case is stable, the consumption of R_e is unchanged, thus its price when it starts to be used is unchanged and noted p_1. If the carbon tax is reduced, thus the date of switch must be postponed, and thus the global consumption of the dirty backstop increases, that is not possible. Thus the carbon tax must increase (effect 1 in Figure 3.3) and the date of switch is brought forward (effect 2). The date the ceiling is reached is also brought forward (effect 4), and the length of the period of consumption of R_e is unchanged since the sum of the scarcity rent and the carbon tax at the switch date is unchanged. Since the switch price is unchanged, the value of the tax at the date of switch is unchanged (vertical bold black lines), thus profits at that date must be unchanged (vertical bold grey lines). Since the date of switch is brought forward, the initial scarcity rent must increase (effect 5) to keep the profit of time t_1 unchanged.

Lemma 10. *If $c_d < c_e$, different cases can arise.*

[11] Sets of equations describing the solution in the other cases are straightforward. For Case $B1$, the equation set is similar, except that equation 3.7 must be dropped and the scarcity rent is set to 0. For cases $B4$ and $B3$, the solution is as described by the solution of the one-resource case, as shown above.

- *If* $c_b < \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$*, only the dirty backstop is used to get to the ceiling (Case* $B3$*) and the exhaustible resource is never used.*

- *If* $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$*, then the exhaustible resource is used when the ceiling is about to bind and there exists* Z^* *such that:*

  1. *If* $\overline{Z} < Z^*$ *and* $X_e^0 > \frac{\bar{Z} - Z^0}{\theta_e}$*, only the exhaustible resource is used to get to the ceiling and the dirty backstop is never used (Case B4);*
  2. *If* $\overline{Z} > Z^*$ *and* $X_e^0 > \frac{Z^* - Z^0}{\theta_e}$*, the dirty backstop is used, then the exhaustible resource is used to get to the ceiling but not exhausted (Case* $B2$*);*
  3. *Otherwise, if* $X_e^0 \leq \min(\frac{Z^* - Z^0}{\theta_e}, \frac{\bar{Z} - Z^0}{\theta_e})$ *the dirty backstop is used at the beginning, then the exhaustible resource is used to get to the ceiling and is exhausted (Case* $B1$*).*

Proof. First, remark that if the dirty backstop is used at the date of switch with the clean backstop (the date the ceiling is reached), then the exhaustible resource is never used. Indeed, assume that this resource is used first and then the dirty backstop is used. If the latest resource is used at the switch date with the clean backstop $\underline{t}$, then it must be the case that $c_e + (\theta_e \mu^0 + \lambda_e^0) e^{r\underline{t}} > c_d + \theta_d \mu^0 e^{r\underline{t}}$, but then $\forall t \leq \underline{t}$, $c_e + (\theta_e \mu^0 + \lambda_e^0) e^{rt} > c_d + \theta_d \mu^0 e^{rt}$, so that the exhaustible resource is never used. If the dirty backstop is used alone until the date of switch with the clean backstop, then the carbon tax at this date, $\mu^0 e^{r\underline{t}}$, satisfies:

$$c_d + \theta_d \mu^0 e^{r\underline{t}} = c_b.$$

A necessary condition for the dirty backstop to be used alone until the ceiling is reached is:

$$c_e + \theta_e \mu^0 e^{r\underline{t}} > c_d + \theta_d \mu^0 e^{r\underline{t}}.$$

Using that $c_d + \theta_d \mu^0 e^{r\underline{t}} = c_b$, a necessary condition writes:

$$c_e - c_d \geq \frac{\theta_d - \theta_e}{\theta_e}(c_b - c_d),$$

which can be rewritten:

$$c_b \leq \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}.$$

Similarly, it is easy to show that a necessary condition for the exhaustible resource to be used at the binding date is $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$. So that the exhaustible resource is used at the date the ceiling is reached if and only if $c_b \geq \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$.

Assume now that $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, then the exhaustible resource is used at the binding date. The carbon tax τ at the date the exhaustible resource starts to be used is such that: $c_e + \lambda_e^0 e^{rt} + \theta_e \tau \leq c_d + \theta_d \tau$, that implies:

$$\tau \geq \frac{c_e - c_d}{\theta_d - \theta_e}.$$

The lowest possible price path of the exhaustible resource is $p(t) = c_e + \frac{c_e - c_d}{\theta_d - \theta_e} e^{rt}$. Call T^* the date such that:

$$c_e + \frac{c_e - c_d}{\theta_d - \theta_e} e^{rT^*} = c_b.$$

Then the maximum amount of R_e that can be consumed, if $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$ is:

$$X^* = \int_0^{T^*} D(c_e + \frac{c_e - c_d}{\theta_d - \theta_e} e^{rt}) dt.$$

Note that X_e^0 does not depend on $\overline{Z}$. If $X_e^0 > X^*$, R_e is not exhausted. If $X_e^0 > X^*$ and $\bar{Z} > Z^0 + \theta_e X^* \equiv Z^*$, then the dirty backstop is used first, then an amount X^* of R_e is used to get to the ceiling, R_e is not exhausted (Case $B2$). If $X_e^0 < X^*$ and $\bar{Z} < Z^0 + \theta_e X$, then only R_e is used to get to the ceiling and R_e is not exhausted (Case $B4$). If $X_e^0 < X^*$ and and $\bar{Z} > Z^0 + \theta_e X_e^0$, then the dirty backstop is used first, then R_e is used to get to the ceiling and is exhausted (Case $B1$). The different cases when R_e is exhausted are indicated in Figure 3.2. □

Proposition 11. *As long as both resources are used and R_e is fully used, tightening the ceiling constraint increases the scarcity rent of R_e.*
If $c_b > \frac{\theta_e c_d - \theta_d c_e}{\theta_e - \theta_d}$, and $X_e^0 < \min(X^*, \frac{\bar{Z} - Z^0}{\theta_e})$, then:

$$\frac{d\lambda_e^0}{d\overline{Z}} < 0.$$

Proof. See supra. □

Remark that if $X_e^0 = \min(X^*, \frac{\bar{Z} - Z^0}{\theta_e})$, the scarcity rent and the carbon tax are undefined, only the sum of the carbon tax and the rent is defined.

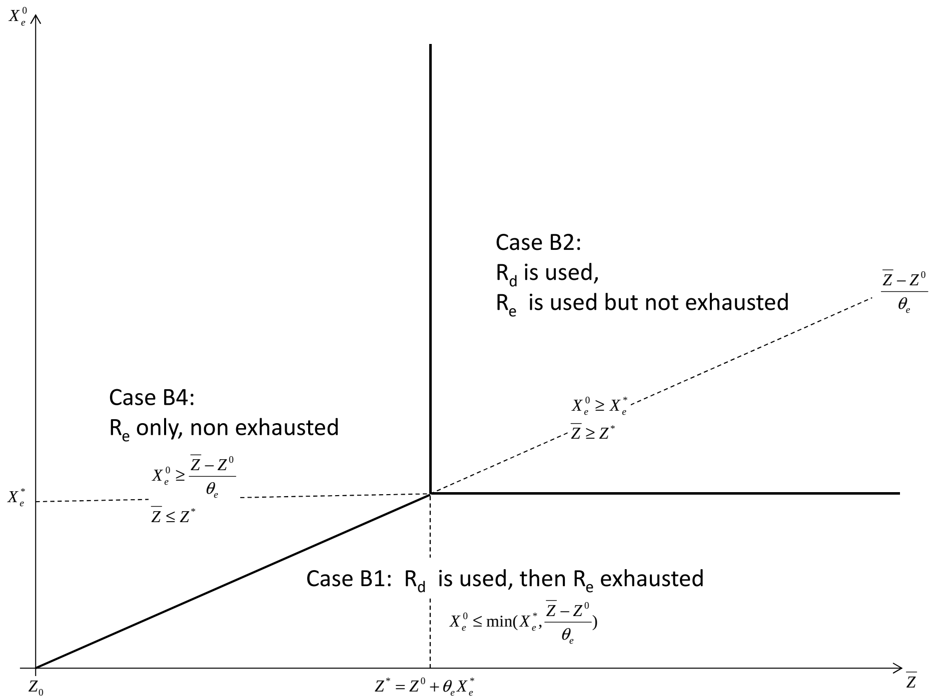

Figure 3.2: Characterization of the different cases when $c_e > c_d$

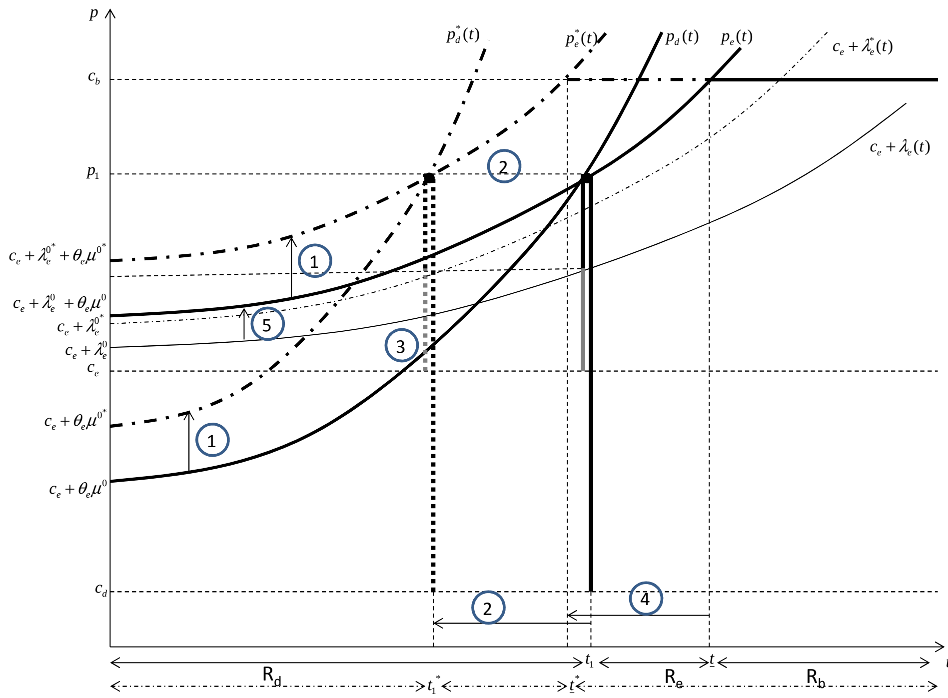

Figure 3.3: Profits, the carbon tax and the price paths when the carbon ceiling is lowered, $c_e > c_d$

4 Extension: the three-resource and two-sector model

The economy is divided into two sectors: for instance electricity production (power) and transportation sectors. The dirty backstop R_d and the exhaustible resource R_e are perfect substitute in the power sector. In transportation sector, R_d needs to be transformed at a unitary cost of z, representing for instance the cost of "coal-to-liquid" process. Once transformed, R_d and R_e are perfect substitute in the transportation sector. An infinite backstop (for instance solar) is available at a constant cost, similar in both sectors.[12] We assume no natural dilution.[13] We assume that the utility function is separable in transport and energy. We write u_E (resp. u_T) the utility associated with energy consumption in the electricity (resp. transports) sector. We assume that this utility function satisfies the standard regularity conditions as defined above. We write $x_{j,i}(t)$ the consumption of resource i in sector j at

[12] Following results still hold when considering heterogeneous backstop price

[13] However, introducing a light constant natural dilution in the case where R_d is used before the ceiling binds in both sectors does not change our results.

time t, $x_i(t)$ the global consumption of resource i at time t, $x_j(t)$ the global consumption in sector j at time t. Notation D_j stands for the energy demand in sector j, $D_j(p(t)) = x_j(t)$. $p_{j,i}(t)$ represents the price of resource i in the sector j. Others assumptions from section 2 are unchanged.

4.1 The social planner model and first order conditions

The social planner seeks to find the extraction path $\{x^*_{E,e}(t), x^*_{T,e}(t), x^*_{E,d}(t), x^*_{T,d}(t),\ x^*_{E,b}(t), x^*_{T,b}(t)\}$ that maximizes his welfare function given by:

$$\int_0^{\infty} e^{-rt}\bigg(u_E(x_E(t)) + u_T(x_T(t)) - c_e x_e(t) - c_d x_{E,d}(t) - (c_d+z)x_{T,d} - c_b x_b(t)\bigg)dt$$

where

$$x_E(t) = x_{E,e}(t) + x_{E,d}(t) + x_{E,b}(t)$$

and

$$x_T(t) = x_{T,e}(t) + x_{T,d}(t) + x_{T,b}(t)$$

s.t. $\forall t$, $\forall i$,

$$\begin{aligned}
\dot{X}_e(t) &= -x_{E,e}(t) - x_{T,e}(t) \\
\dot{Z}(t) &= \theta_e(x_{E,e}(t) + x_{T,e}(t)) + \theta_d(x_{E,d}(t) + x_{T,d}(t)) \\
Z(t) &\leq \overline{Z} \\
0 &\leq x_e(t), x_{E,i}, x_{T,i}(t)
\end{aligned}$$

with Z^0, x_e^0 given.

We get the following optimal energy prices:

$$\begin{aligned}
p_{E,e}(t) = \quad p_{T,o}(t) &= c_e + \lambda_e(t) + \theta_e\mu(t) && (4.1)\\
p_{E,d}(t) &= c_d + \theta_e\mu(t) && (4.2)\\
p_{T,c}(t) &= c_d + \theta_e\mu(t) + z && (4.3)\\
p_{E,b}(t) = \quad p_{T,s}(t) &= c_b && (4.4)
\end{aligned}$$

Dynamics of the scarcity rent and the shadow cost of pollution are unchanged. In each sector, energy demand is satisfied by the cheapest resource the sectoral use, thus:

$$p_E(t) = min\{p_{E,e}(t), p_{E,d}(t), p_{E,b}(t)\} \tag{4.5}$$

$$p_T(t) = min\{p_{T,e}(t), p_{T,d}(t), p_{T,b}(t)\} \tag{4.6}$$

4.2 Ordering the extraction of resources

As in previous section, the "least cost first" principle holds. A more expensive resource for a specific sectoral use cannot be used prior to a cheaper resource for that specific use. However, a cheaper resource in a sector may not be used if this resource is the most polluting, and obviously a more expensive resource in a sector is not necessarily used in that sector.

Defining conditions over parameters such that one specific-case is obtained, is more complicated than in the one-sector model. The initial quantity of the exhaustible resource affects its use. For instance, if $c_e < c_d$ a relatively scarce exhaustible resource R_e leads to specialize R_e for the transport sector and leads to use only R_d in power sector.

All the possible cases are described in table 5

4.2.1 Characterization of cases D: R_d cheaper then R_e only in the power sector: $c_d < c_e < c_d + z$

For any constraint on the stock of pollution $\bar{Z} < \infty$, the dirty backstop price reaches the clean backstop price at some date. The carbon tax at this date is equal to $\frac{c_b - c_d}{\theta_d}$. The price of the exhaustible resource at this date is thus, if it is not exhausted: $c_e + \theta_e \frac{c_b - c_d}{\theta_d}$. If $c_e + \theta_e \frac{c_b - c_d}{\theta_d} \leq c_b$ (or, equivalently $c_b \geq \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$), if R_e is not exhausted, then it is being used in both sectors as the energy price reaches c_b. We assume here that this is the case, we characterize next the case when $c_b < \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$.

Assume that R_d price in the power sector and R_e price in the power sector cross at some date t_1. Then from this date on, R_e is used in both sectors. Assume that on this price path, R_e is not exhausted. The quantity of R_e consumed from this date t_1 is called X^*, satisfying:

$$X^* = \int_0^{1/r \ln(\frac{(c_b - c_e)(\theta_d - \theta_e)}{\theta_e (c_e - c_d)})} (D_E(c_e + \theta_e \frac{c_e - c_d}{\theta_d - \theta_e} e^r dt) + D_T(c_d + \theta_d \frac{c_e - c_d}{\theta_d - \theta_e} e^{rt}))dt$$

Case	Extraction costs	Transports sector	Power sector	R_e exhausted?
Case C1	$c_e < c_d$	R_e	R_e	No
Case C2	...	R_e	R_e,R_d	Yes
Case C3	...	R_e	R_d	Yes
Case C4	...	R_e,R_d	R_e,R_d	Yes
Case C5	...	R_e,R_d	R_d	Yes
Case D1	$c_d + z > c_e > c_d$	R_e	R_e	No
Case D2	...	R_e	R_d,R_e	No
Case D3	...	R_e	R_d	No
Case D4	...	R_e	R_d,R_e	Yes
Case D5	...	R_e	R_d	Yes
Case D6	...	R_e,R_d	R_d	Yes
Case E1	$c_e > c_d + z$	R_e	R_e	No
Case E2	...	R_e	R_d,R_e	No
Case E3	...	R_d,R_e	R_d,R_e	No
Case E4	...	R_e	R_d,R_e	Yes
Case E5	...	R_d,R_e	R_d,R_e	Yes
Case E6	...	R_e	R_d	Yes
Case E7	...	R_d,R_e	R_d	Yes

"..." means that the value of that cell is the same than the value of the cell right above.

Table 5: The different extraction paths in the two-sector economy

Remark that X^* does not depend on $\bar{Z}$. The quantity X^* is the maximum quantity of R_e consumed while R_e is consumed in both sectors at the same time. Call $Z^* = Z^0 + \theta_e X^*$ the corresponding CO_2 emissions. Assume that $\bar{Z} > Z^*$, then there is a switch in at least one sector (as X^* is the maximum quantity of R_e consumed when both sectors use R_e simultaneously). If $\bar{Z} > Z^*$ and R_e is not exhausted, it must be the case that there is a switch in the energy sector, and no switch in the transport sector(case D2). Indeed, as $c_e < c_d + z$, if R_e is not exhausted, R_e price in the transport sector is always below R_d price in the transport sector. So that, only R_e is used in the transport sector and R_d then R_e are used in the energy sector. The quantity of R_e consumed on this price path ($\bar{Z} > Z^*$ and R_e is not exhausted) is thus equal to $f(\bar{Z})$ defined, for $\bar{Z} > Z^*$, by:

$$\begin{aligned} c_e + \theta_e \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} \\ c_e + \theta_e \mu^0 e^{rT} &= c_b \\ \theta_e \int_{t_1}^{T} (D_T + D_E)(c_e + \theta_e \mu^0 e^{rt}) dt + \theta_e \int_0^{t_1} D_T(c_e + \theta_e \mu^0 e^{rt}) dt & \\ + \theta_d \int_0^{t_1} D_E(c_d + \theta_d \mu^0 e^{rt}) dt &= \bar{Z} - Z_0 \\ \int_{t_1}^{T} (D_T + D_E)(c_e + \theta_e \mu^0 e^{rt}) dt + \int_0^{t_1} D_T(c_e + \theta_e \mu^0 e^{rt}) dt &= f(\bar{Z}) \end{aligned}$$

It is straightforward that, for $\bar{Z} > Z^*$, $f(\bar{Z}) > X^*$ and $f(Z^*) = X^*$. Moreover, $f(\bar{Z}) \leq \frac{\bar{Z} - Z_0}{\theta_e}$ because as $\bar{Z}$ increases, more R_e but also more R_d are used. The price path for $X = f(\bar{Z})$, with $\bar{Z} > Z^*$ is illustrated on the upper left hand graph of Figure 4.1.

If $\bar{Z} < Z^*$, and R_e is not exhausted then only R_e is used in both sector (case D1). The quantity of R_e consumed on this price path ($\bar{Z} < Z^*$ and R_e is not exhausted) is thus equal to $f(\bar{Z})$ defined, for $\bar{Z} < Z^*$, by $f(\bar{Z}) = \frac{\bar{Z} - Z_0}{\theta_e}$.

If $X \leq f(\bar{Z})$, the exhaustible resource is exhausted. Keeping $\bar{Z}$ constant and decreasing X, μ^0 decreases. As a result the date of switch, in the power sector, from R_d to R_e is postponed (case D4) as illustrated in the upper right hand graph of Figure 4.1) until this date coincides with R_e exhaustion date, that is to say until $X \geq g(\bar{Z})$, where $g(\bar{Z})$ satisfies:

$$\begin{aligned}
c_e + (\lambda_o + \theta_e \mu^0) e^{rT} &= c_d + \theta_d \mu^0 e^{rT} \\
c_e + (\lambda_o + \theta_e \mu^0) e^{rT} &= c_b \\
\theta_e \int_0^T D_T(c_e + (\lambda_o + \theta_e \mu^0) e^{rt}) dt + \theta_d \int_0^T D_E(c_d + \theta_d \mu^0 e^{rt}) dt &= \bar{Z} - Z_0 \\
\int_0^T D_T(c_e + (\lambda_o + \theta_e \mu^0) e^{rt}) dt &= g(\bar{Z})
\end{aligned}$$

The lower left hand graph of Figure 4.1 illustrates the price path when $X = g(\bar{Z})$. As X decreases below $g(\bar{Z})$, R_d price remains below R_e price in the power sector and R_e is used only in the power sector (case D5). Only R_e is used in the transport sector until the final R_e price in the transport sector is above the final R_d price in the transport sector, that is to say as long as $X > h(\bar{Z})$ with $h(\bar{Z})$ defined by:

$$\begin{aligned}
c_e + (\lambda_o + \theta_e \mu^0) e^{rT} &= c_d + z + \theta_d \mu^0 e^{rT} \\
c_e + (\lambda_o + \theta_e \mu^0) e^{rT} &= c_b \\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b \\
\theta_e \int_0^T D_T(c_e + (\lambda_o + \theta_e \mu^0) e^{rt}) dt + \theta_d \int_0^{\underline{t}} D_E(c_d + \theta_d \mu^0 e^{rt}) dt &= \bar{Z} - Z_0 \\
\int_0^T D_T(c_e + (\lambda_o + \theta_e \mu^0) e^{rt}) dt &= h(\bar{Z})
\end{aligned}$$

for $\bar{Z} > Z_2$ where Z_2 is such that:

$$Z_2 = Z_0 + \theta_d \int_0^{1/r \ln(\frac{c_b - c_d}{c_b - (c_d + z)})} D_E(c_d + (c_b - (c_d + z)) e^{rt}) dt.$$

The price path for $X = h(\bar{Z})$ is illustrated on the lower right hand graph of Fig.4.1.
If $X \leq h(\bar{Z})$, then the exhaustible resource is exhausted, only R_d is used in the power sector, the exhaustible resource then R_d are used in the transport sector (case D6).

If $c_b < \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$ (and $c_d < c_e < c_d + z$) then R_d price in the power sector always remain below R_e price in the power sector (prices in the power sector never cross). If R_e is abundant enough, it is not exhausted and it is used in the transport sector, while R_e is used in the power sector (case D3).

Define $\bar{Z}_1 = Z_0 + \theta_d \int_0^{1/r \ln(\frac{(c_b-c_d)\theta_e}{(c_b-c_e)\theta_d})} D_E(c_d + \theta_d \frac{c_b-c_e}{\theta_e})e^{rt}dt$.

For $\bar{Z} > \bar{Z}_1$, define $f(\bar{Z})$ by:

$$\begin{aligned} c_e + \theta_e \mu^0 e^{rT_1} &= c_b \\ c_d + \theta_d \mu^0 e^{rT_2} &= c_b \\ \theta_e \int_0^{T_1} D_T(c_e + \theta_e \mu^0 e^{rt})dt + \theta_d \int_0^{T_2} D_E(c_d + \theta_d \mu^0 e^{rt})dt &= \bar{Z} - Z_0 \\ \int_0^{T} D_T(c_e + \theta_e \mu^0 e^{rt})dt &= f(\bar{Z}) \end{aligned}$$

If $X \leq f(\bar{Z})$, then the exhaustible resource is exhausted, only R_d is used in the power sector, only the exhaustible resource is used in the transport sector (case D5) as long as $X > h(\bar{Z})$, where $h(\bar{Z})$ is defined, for $\bar{Z} > \bar{Z}_2$, with $\bar{Z}_2 = Z_0 + \theta_d \int_0^{1/r \ln(\frac{c_b-c_d}{c_b-(c_d+z)})} D_E(c_d + (c_b - (c_d + z))e^{rt})dt$ by:

$$\begin{aligned} c_e + (\lambda_o + \theta_e \mu^0)e^{rT} &= c_d + z + \theta_d \mu^0 e^{rT} \\ c_e + (\lambda_o + \theta_e \mu^0)e^{rT} &= c_b \\ c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b \\ \theta_e \int_0^{T} D_T(c_e + (\lambda_o + \theta_e \mu^0)e^{rt})dt + \theta_d \int_0^{\underline{t}} D_E(c_d + \theta_d \mu^0 e^{rt})dt &= \bar{Z} - Z_0 \\ \int_0^{T} D_T(c_e + (\lambda_o + \theta_e \mu^0)e^{rt})dt &= h(\bar{Z}) \end{aligned}$$

If $X \leq h(\bar{Z})$, then the exhaustible resource is exhausted, only R_d is used in the power sector, the exhaustible resource then R_d are used in the transport sector (case D6).

4.2.2 Characterization of all the cases

The following lemma gives the order of extraction for all parameters value. The characterizations are detailed in Appendix 8.1.

Lemma 12. *The order of use of the two resources in the two sectors follows the following pattern:*

- *For all $c_e < c_d < c_d + z$, there exist X^* and $\bar{Z}_2$ such that there exists a continuous increasing function $f(\bar{Z})$, defined for all values of the ceiling $\bar{Z}$, and there exists a continuous increasing function $h(\bar{Z})$ for all $\bar{Z} > \bar{Z}_2$, such that the ordering of the resources is as described on Fig.4.2.*

- *For all $c_d < c_e < c_d + z$ and $\frac{c_b-c_d}{\theta_d} \leq \frac{c_b-c_e}{\theta_e}$, there exist X^* and $\bar{Z}_2$ such that there exist two*

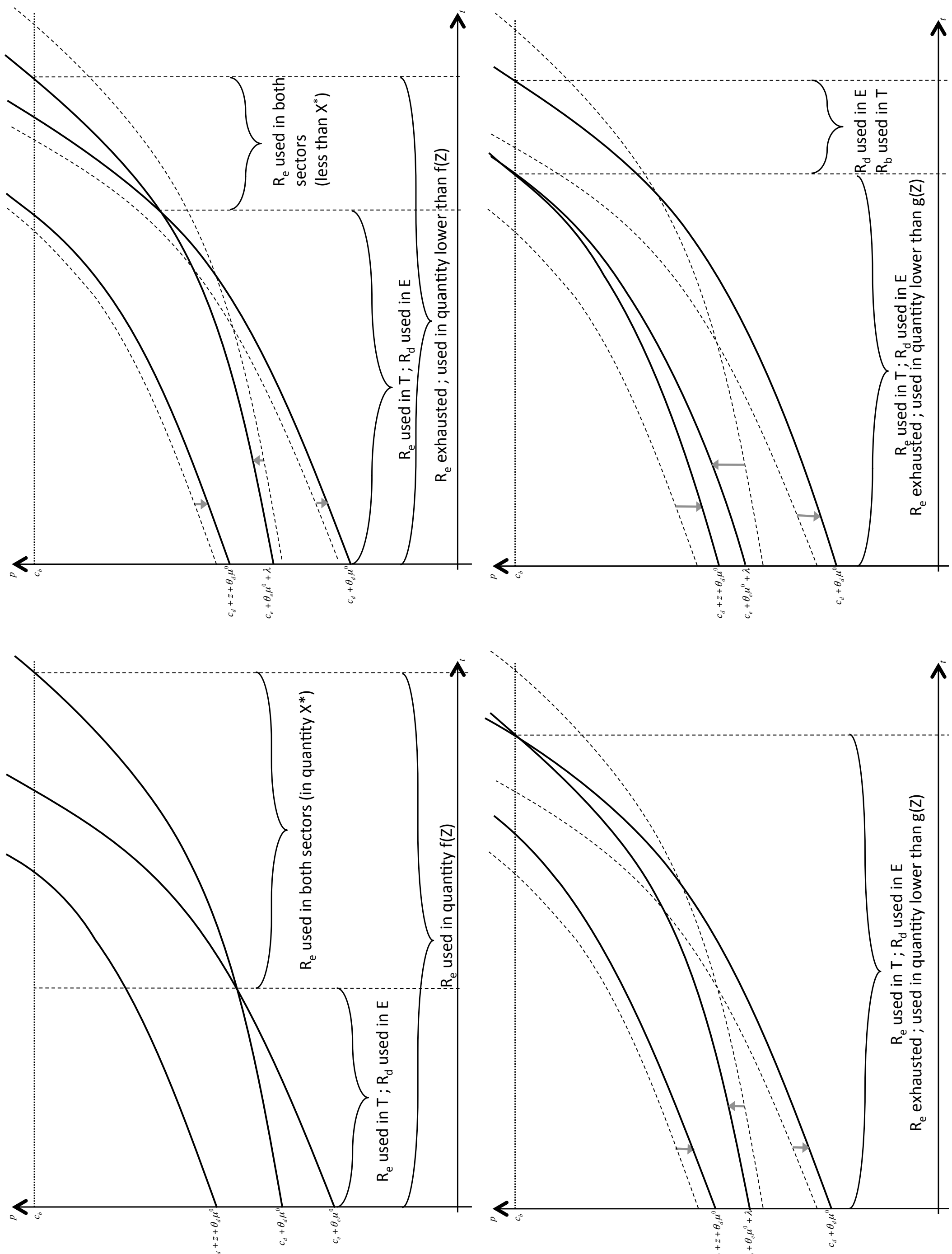

Figure 4.1: Price paths when $c_d < c_e < c_d + z$ with different values of X_e

continuous increasing function $f(\bar{Z})$, $g(\bar{Z})$ defined for all values of the ceiling $\bar{Z}$, and there exists a continuous increasing function $h(\bar{Z})$ for all $\bar{Z} > \bar{Z}_2$ such that the ordering of the resources is as described on Fig.4.3.

- *For all $c_d < c_e < c_d + z$ and $\frac{c_b - c_d}{\theta_d} > \frac{c_b - c_e}{\theta_e}$, there exist $\bar{Z}_1$ and $\bar{Z}_2$ such that there exists a continuous increasing function $f(\bar{Z})$ defined for all values of the ceiling $\bar{Z} > \bar{Z}_1$, and there exists a continuous increasing function $h(\bar{Z})$ for all $\bar{Z} > \bar{Z}_2$ such that the ordering of the resources is as described on Fig.4.4.*

- *For all $c_d + z < c_e$ and $\frac{c_b - c_d}{\theta_d} \leq \frac{c_b - c_e}{\theta_e}$, there exists $\bar{Z}_2$ such that there exist two continuous increasing function $f(\bar{Z})$, $g(\bar{Z})$ defined for all values of the ceiling $\bar{Z}$, and there exists a continuous increasing function $h(\bar{Z})$ for all $\bar{Z} > \bar{Z}_2$ such that the ordering of the resources is as described on Fig.4.5.*

- *For all $c_d + z < c_e$ and $\frac{c_b - c_d}{\theta_d} > \frac{c_b - c_e}{\theta_e} > \frac{c_b - (c_d + z)}{\theta_d}$, there exists $\bar{Z}_1$ such that there exists a continuous increasing function $h(\bar{Z})$ for all $\bar{Z} > \bar{Z}_1$ such that the ordering of the resources is as described on Fig.4.6.*

4.3 General results

The way the scarcity rent of R_e varies with the value of the ceiling is given by the following lemma[14], in which we restrict our attention to the case when R_e is exhausted (otherwise the scarcity in zero):

Lemma 13. *The derivative of the scarcity rent λ_e^0 with respect to $\bar{Z}$ satisfies:*

- *In cases C2 and E4 $\frac{d\lambda_e^0}{d\bar{Z}}$ has the sign of:*

$$1 - (\theta_d - \theta_e)\frac{\mu^0}{\lambda_e^0} - \frac{\theta_d}{\theta_e}\frac{D_E(p(t_1))}{D_T(p(0)) + D_E(p(0))}$$

- *In cases C3, D5 and E6. $\frac{d\lambda_e^0}{d\bar{Z}} > 0$*

- *In case C4, $\frac{d\lambda_e^0}{d\bar{Z}}$ has the sign of:*

$$1 - \frac{\theta_d}{\theta_e}\frac{D_E(t_1) + D_T(t_2)}{D_E(0) + D_T(0)} - (\theta_d - \theta_e)\frac{\mu^0}{\lambda_o^0}$$

[14] Remark that the value of $d\lambda/d\bar{Z}$ is not continuous when crossing the frontier between two cases. This comes from the fact that the demand addressed to one specific resource at a switch date is not continuous when crossing a frontier

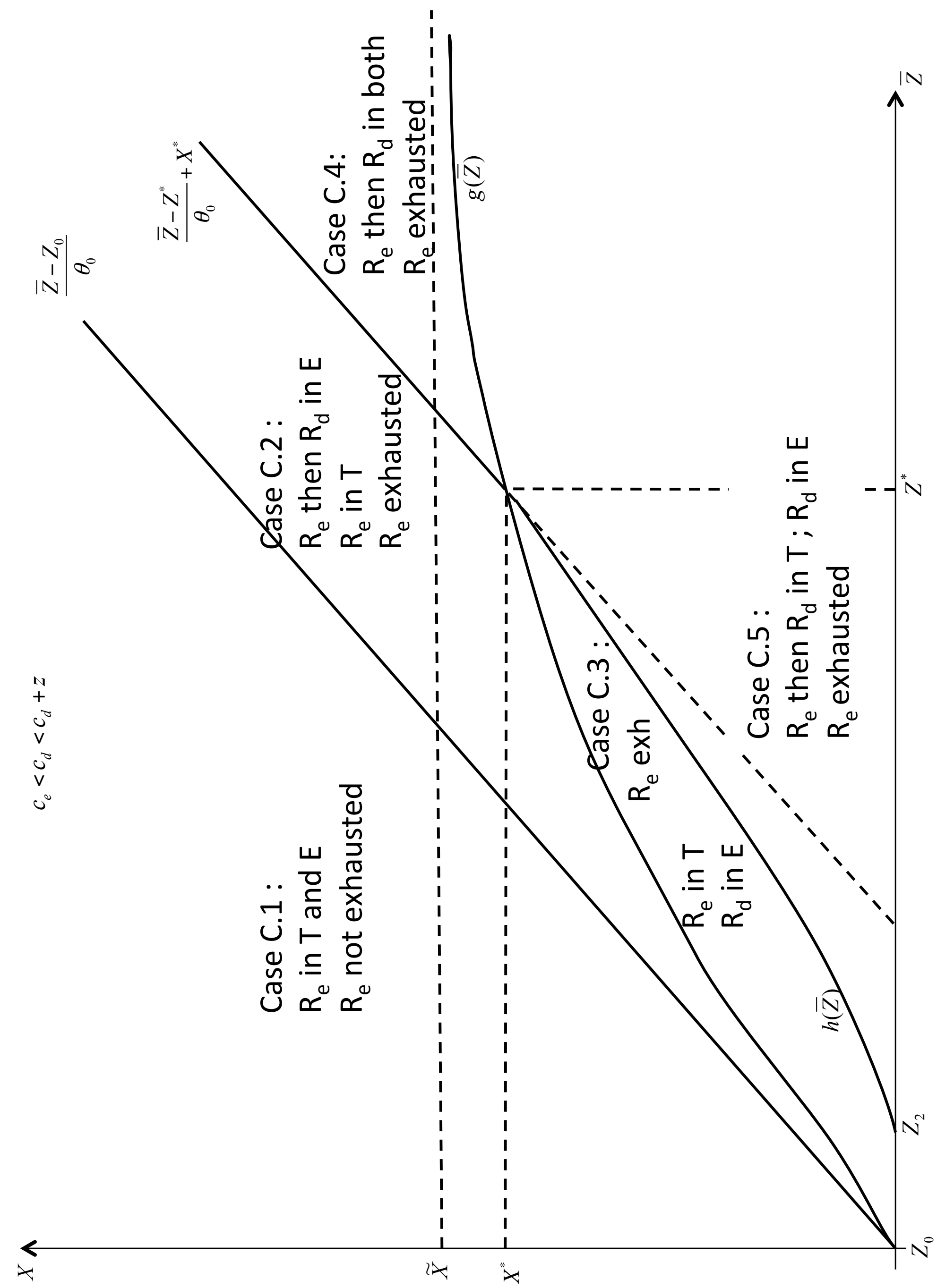

Figure 4.2: Order of extraction of the resources in both sectors when $c_e < c_d$

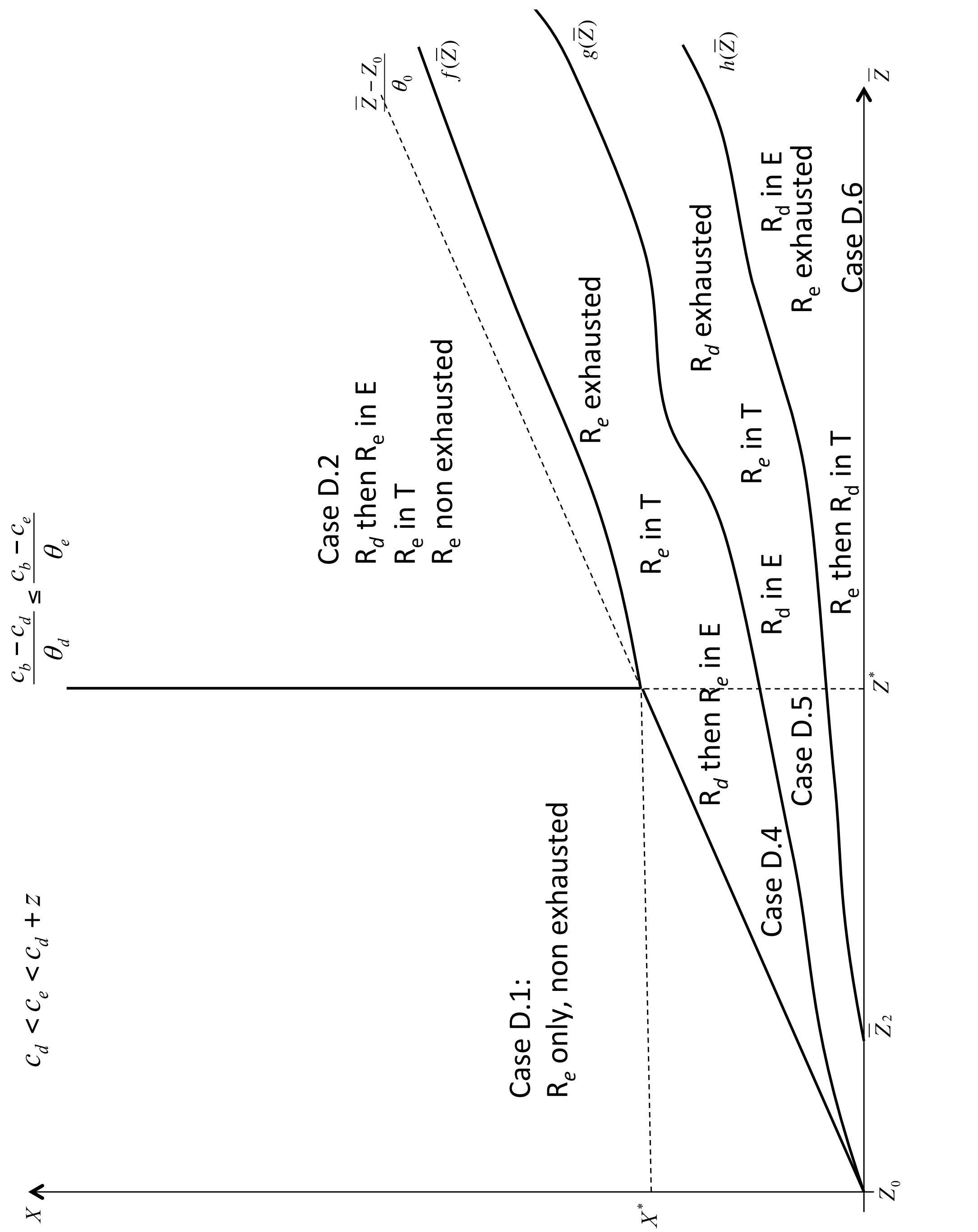

Figure 4.3: Order of extraction of the resources in both sectors when $c_d < c_e < c_d + z$ and $\frac{c_b - c_d}{\theta_d} \le \frac{c_b - c_e}{\theta_e}$

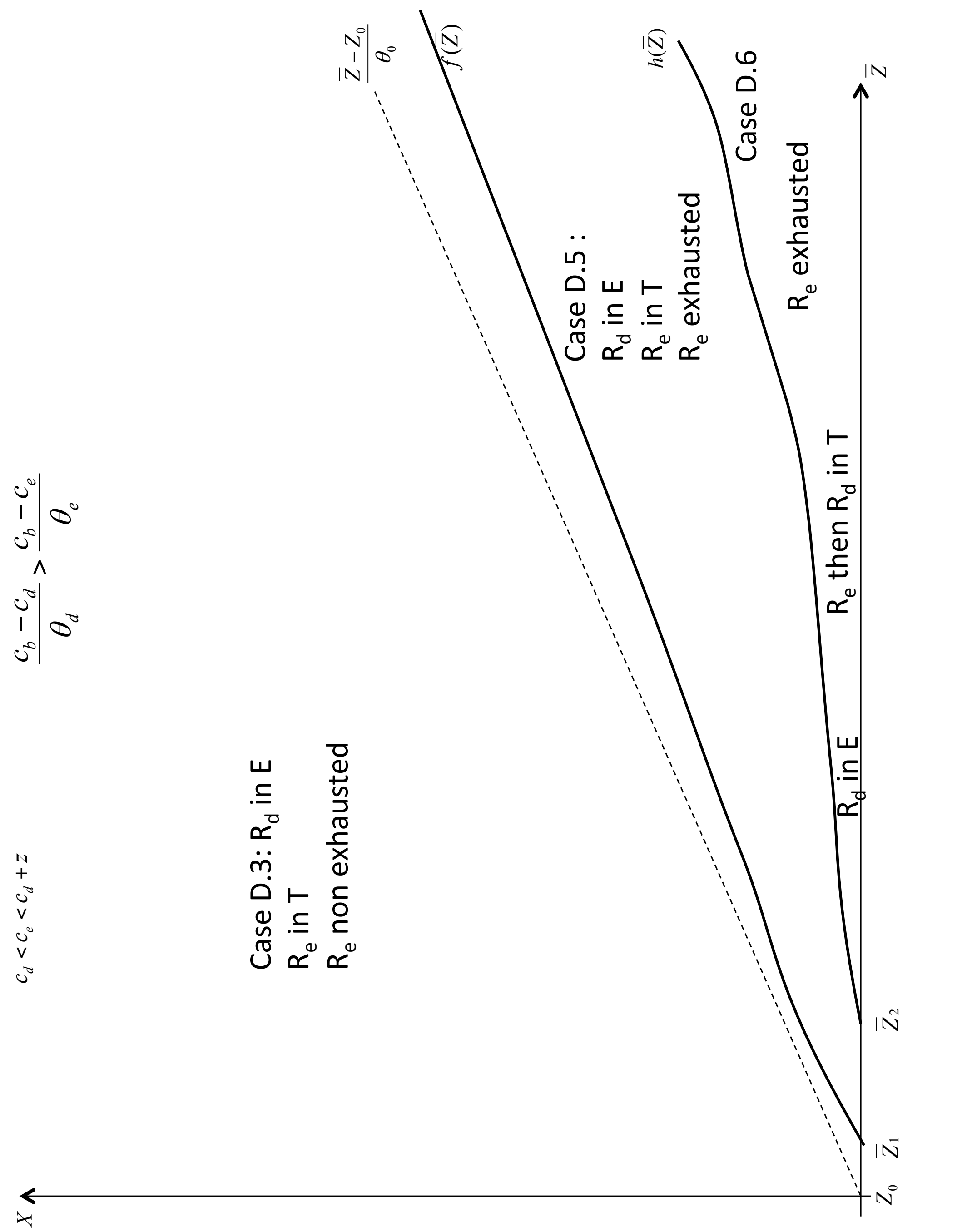

Figure 4.4: Order of extraction of the resources in both sectors when $c_d < c_e < c_d{+}z$ and $\frac{c_b-c_d}{\theta_d} > \frac{c_b-c_e}{\theta_e}$

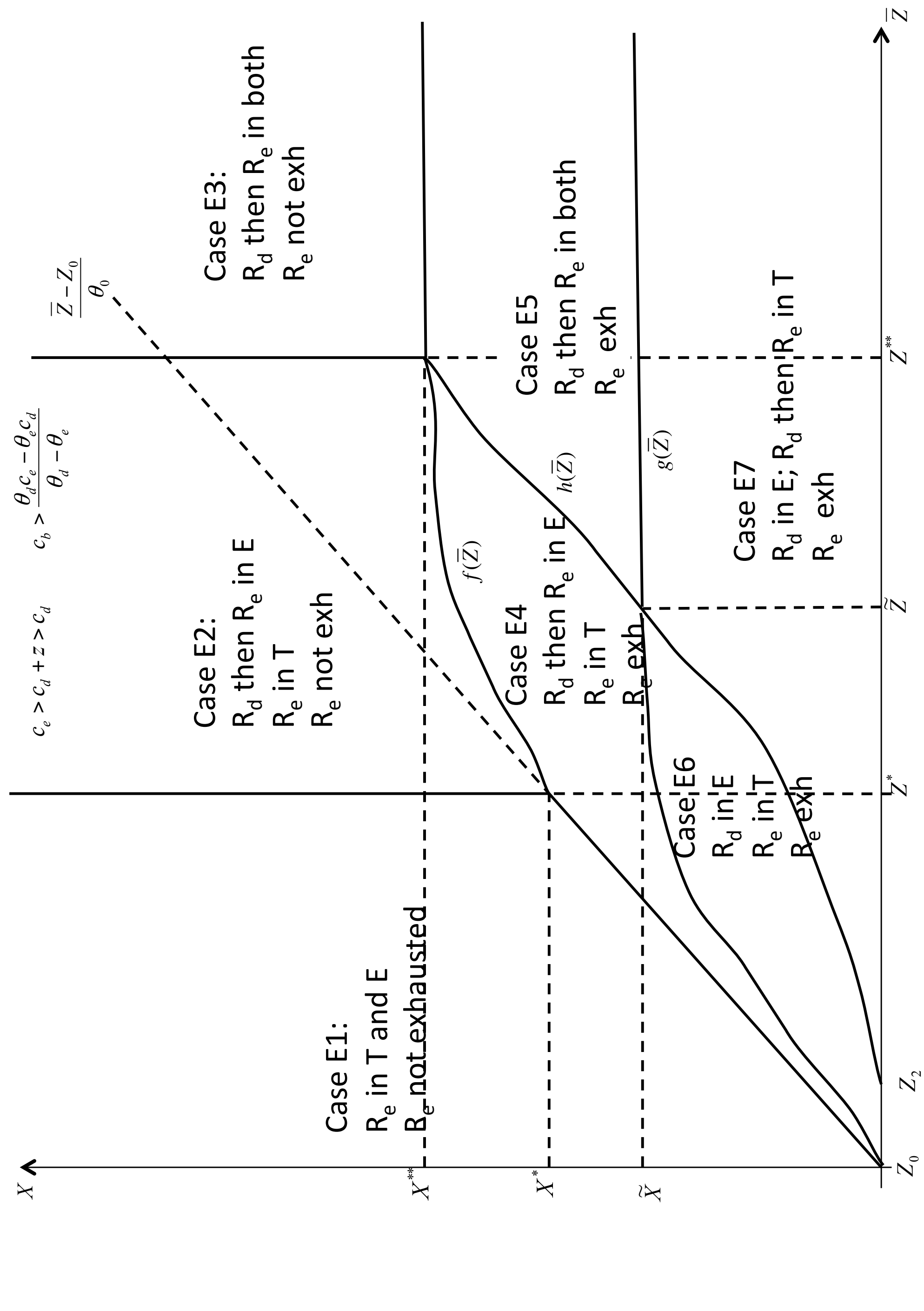

Figure 4.5: Order of extraction of the resources in both sectors when $c_d + z < c_e$ and $\frac{c_b - c_d}{\theta_d} \leq \frac{c_b - c_e}{\theta_e}$

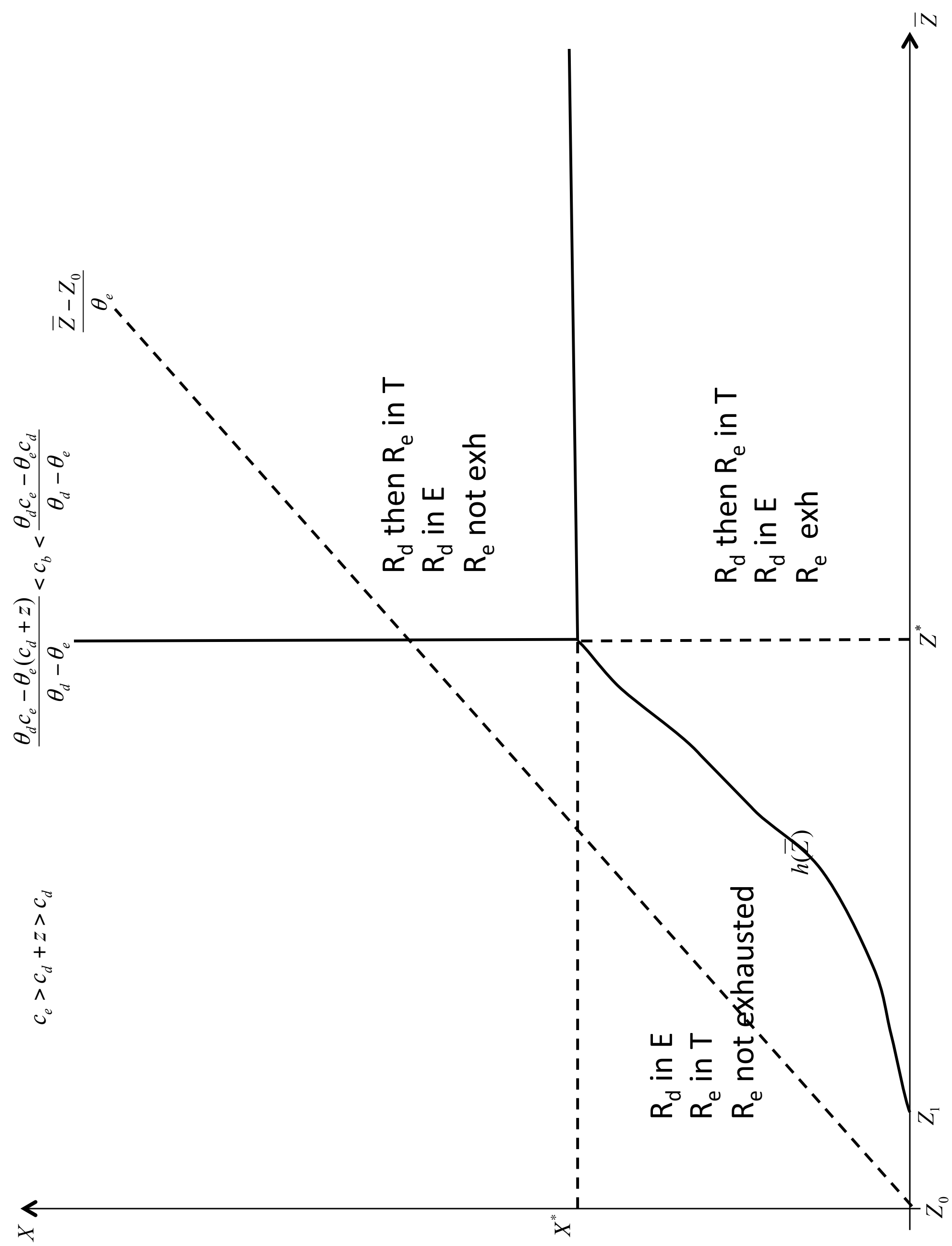

Figure 4.6: Order of extraction of the resources in both sectors when $c_d + z < c_e$ and $\frac{c_b - c_d}{\theta_d} > \frac{c_b - c_e}{\theta_e} > \frac{c_b - (c_d + z)}{\theta_d}$

- *In cases C5 and D6, $\frac{d\lambda_e^0}{d\bar{Z}}$ has the sign of:*

$$1-(\theta_d-\theta_e)\frac{\mu_o}{\lambda_e}-\frac{\theta_d}{\theta_e}\frac{D_T(t_1)}{D_T(0)}$$

- *In cases D4, E4 and E5 $\frac{d\lambda_e^0}{d\bar{Z}}>0$*

Proposition 14. *If there is a switch from the exhaustible resource to the dirty backstop in at least one sector, if R_e is exhausted strictly before the clean backstop starts to be used, and if the elasticity of demand in the sector(s) in which there is a switch is small enough ; then tightening the carbon ceiling increases the profit of the exhaustible resource producers:*

- $\forall c_e \leq c_d+z$, *$\exists \epsilon^*$ and $\exists(h(\bar{Z}),g(\bar{Z}))$ such that:*

$$\left\{\forall p, -\frac{D_T^{'}(p)p}{D_T(p)}\leq \epsilon^* \textit{and } X<\min(h(\bar{Z}),g(\bar{Z}))\right\} \Longrightarrow \frac{d\lambda_e^0}{d\bar{Z}}\leq 0$$

- $\forall c_e \leq c_d$, *$\exists(\epsilon_T^*,\epsilon_E^*)$ and $\exists(h(\bar{Z}),g(\bar{Z}))$ such that::*

$$\left\{\forall p, -\frac{D_T^{'}(p)p}{D_T(p)}\leq \epsilon_T^* \textit{and } -\frac{D_E^{'}(p)p}{D_E(p)})\leq \epsilon_E^* \textit{and } g(\bar{Z})<X<h(\bar{Z})\right\} \Longrightarrow \frac{d\lambda_e^0}{d\bar{Z}}\leq 0.$$

Proposition 15. *If there is a switch from the exhaustible resource to the dirty backstop along the optimal path of extraction, and if the pollution content of the exhaustible resource is low enough, then tightening the carbon ceiling increases the profit of the exhaustible resource producers:*

- $\forall c_e < c_d+z$, *$\exists \eta^*$, with $0<\eta^*<1$, such that:*

$$\left\{\frac{\theta_e}{\theta_d}\leq 1-\eta^* \textit{and } X<\min(h(\bar{Z}),g(\bar{Z}))\right\} \Longrightarrow \frac{d\lambda_e^0}{d\bar{Z}}<0$$

- $\forall c_e \leq c_d$, *$\exists \eta^{**}$, with $0<\eta^*<1$, such that:*

$$\left\{\frac{\theta_e}{\theta_d}\leq 1-\eta^{**} \textit{andg}(\bar{Z})<X<h(\bar{Z})\right\} \Longrightarrow \frac{d\lambda_e^0}{d\bar{Z}}<0.$$

Proof. See infra. □

Proposition 16. *If there is no switch between fossil fuel in both sectors along the optimal path of extraction, $\frac{d\lambda_e^0}{d\bar{Z}}>0$ i.e tightening the carbon ceiling will reduce the profits of oil owners.*

Proof. See infra. □

If there is no switch between fossil fuel in both sectors along the optimal path of extraction i.e if resources are fully sector-specialized, resources are no more in direct competition. Tightening the carbon regulation leads to a reduction of consumption in both sectors. In the sector(s) where only the exhaustible resource is used, the situation is similar to the one-sector case when only the exhaustible resource is used: profits of the exhaustible resource owners will decrease when tightening the carbon regulation.

Proposition 17. *For all $\overline{Z}$, if R_e is exhausted and if there is a switch from R_e to the dirty backstop in at least one sector, and if the delivery cost of R_e in the sector in which there is the first switch is close enough from that of the dirty backstop, tightening the carbon ceiling increases the profits of owners of R_e. Writing c_d^d the delivery cost of R_d in that sector, $\exists 0 < c^* < 1$, such that:*

$$c^* \leq c_e \leq c_d^d \Longrightarrow \frac{d\lambda_e^0}{d\overline{Z}} < 0.$$

If the delivery costs of both resources are close enough in the sector where they are in direct competition, tightening the ceiling constraint will increase the profits of the exhaustible resource owners.

Proof. Straightforward from Section 3. □

Proposition 18. *If the exhaustible resource gets exhausted, but is not used at the beginning in any sector, $\frac{d\lambda_e^0}{d\overline{Z}} < 0$ i.e tightening the ceiling constraint will increase the profits of the exhaustible resource owners.*

Proof. Straightforward from Section 3. □

In the one-sector case of Section 3, if the exhaustible resource is used latter than R_d, but fully used tightening the carbon ceiling increases the profit of the exhaustible resource owners. The reason is that without a carbon regulation the exhaustible resource will not be used at all due to its expensiveness. Tightening the carbon ceiling would reinforce the use of the exhaustible resource and increase its scarcity rent. In the two-sector case one must add the condition than there is no sector where only the exhaustible resource is used. Indeed, if the exhaustible resource consumption is largely favored in the sector where the exhaustible resource is in competition with R_d but used after R_d, in the sector where only the exhaustible resource is used this is obviously not the case. The effect depends on the sectoral demands.

Proposition 19. *$\forall c_d, c_e$, there exists $\bar{Z}_{\min}$ such that, $\forall\ \bar{Z} > \bar{Z}_{\min}$, $\exists X^*$:*

$$X_e^0 < X^* \Longrightarrow \frac{d\lambda_e^0}{d\overline{Z}} < 0.$$

As long as R_d and R_e are cheaper to extract than solar, if the exhaustible resource resource is scarce enough, tightening the ceiling constraint will increase the profits of the exhaustible resource owners.

Proof. Straightforward from Section 3. □

5 Extension: the four-resource and one-sector economy

In this section, we assume that two polluting exhaustible resources (resources 1 and 2) are available in addition to the dirty and the clean backstops. The variables λ_i^0, c_i and θ_i, respectively stand for the initial scarcity of the scarce resource i, $i \in \{0; 1\}$, its extraction cost and its pollution content. We call resource 1, the exhaustible resource that is the cheaper to extract of the two exhaustible resources, and resource 2 the other one, thus by definition $c_1 < c_2$. The non-exhaustible polluting resource is the most polluting resource, $\theta_d > max\{\theta_1; \theta_2\}$ and the clean backstop is the most expensive without carbon taxation, $max\{c_1; c_2; c_d\} < c_b$. Others assumptions from Section 2 are unchanged. If two resources are extracted contiguously in time, they are said to be in "direct competition".

Extension of the maximization program to the four-resource case is straightforward. Previous optimal pricing rules still hold and for any scarce resource i, $i \in \{0; 1\}$ we get: $c_i + \lambda_i^0 e^{rt} + \theta_i \mu^0 e^{rt}$.

Without natural dilution, as shown in Section 3, the Herfindahl principle holds (Herfindahl 1967): a more expensive resource cannot be used after a cheaper resource. We study two different cases. First, the case where the extraction cost of the dirty backstop is larger than the extraction costs of the exhaustible resources, $c_d > c_2 > c_1$. Second, the case where it lays in-between, $c_2 > c_d > c_1$.

5.1 Ordering the extraction of resources

5.1.1 The dirty backstop is the most expensive to extract, $c_d > c_2 > c_1$

Different extraction paths can exist. Labels of the different cases are given in Table 6. The full characterization of the different extraction paths when $c_d > c_2 > c_1$ is given in Lemma 20. We study here the case where resource 1 and 2 and the dirty backstop are used along the optimal path. For the other cases, one must refer to Section 2. If the dirty backstop is used, thus necessarily both exhaustible resources get exhausted since they are cheaper to extract and less polluting than the dirty backstop. As the extraction cost of the dirty backstop is lower than that of the clean backstop ($c_d < c_b$), a sufficient condition to get the dirty backstop used writes $\theta_1 X_1^0 + \theta_2 X_2^0 < \overline{Z} - Z^0$. Thus, both resources 1 and 2 are exhausted and the dirty backstop is used iff. $\theta_1 X_1^0 + \theta_2 X_2^0 < \overline{Z} - Z^0$.

Table 6: The different extraction paths with three polluting resources when $c_d > c_2 > c_1$.

Case	**Extraction costs**	**Resource** 1	**Resource** 2	**Dirty backstop***
Case F1	$c_d > c_2 > c_1$	used, exh.	used, exh.	used
Case F2	...	used, exh.	used, not exh.	not used
Case F3	...	used, not exh.	used, exh.	not used
Case F4	...	used, not exh.	used, not exh.	not used
Case F5	...	used, not exh.	not used	not used
Case F6	...	not used	used, not exh.	not used

"..." means that the value of that cell is the same than the value of the cell right above. *Recall that the dirty backstop is not exhaustible.

We can state the following Lemma:

Lemma 20.

- *If* $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$ *and* $\theta_1 > \theta_2$, *or if* $\theta_1 \leq \theta_2$ *(See Figure 5.1), then*
  - *Only resource* 1 *is used (case* F5*) iff.* $\theta_1 X_1^0 \geq \overline{Z} - Z^0$*;*
  - *Resource* 1 *is exhausted, resource* 2 *is used but not exhausted, the dirty backstop is not used (case F*2*) iff.* $\theta_1 X_1^0 < \overline{Z} - Z^0 < \theta_1 X_1^0 + \theta_2 X_2^0$*;*
  - *Both resources* 1 *and* 2 *are exhausted and the dirty backstop is used (case F*1*) iff.* $\theta_1 X_1^0 + \theta_2 X_2^0 < \overline{Z} - Z^0$.

- *If* $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$ *and* $\theta_1 > \theta_2$ *(See Figure 5.2), then* $\exists Z^*, Z^{**}$, $Z^* < Z^{**}$, *such that:*
  - *If* $\overline{Z} < Z^*$ *and* $X_2^0 > \frac{\overline{Z} - Z^0}{\theta_1}$, *only resource* 2 *is used to get to the ceiling (case F*6*);*
  - *If* $\theta_2 X_2^0 + Z^0 < \overline{Z} < \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$, *and* $X_2^0 < \frac{Z^* - Z^0}{\theta_2}$, *resource* 1 *is used but not exhausted, and resource* 2 *is exhausted, the dirty backstop is not used (case F*3*);*

– *If* $Z^{**} > \overline{Z} > Z^*$ *and* $X_2^0 > \frac{Z^*-Z^0}{\theta_2}$, *resource* 1 *and* 2 *are used but not exhausted, the dirty backstop is not used (case* $F4$*);*

– *If* $Z^{**} < \overline{Z}$ *and* $X_2^0 > max(\frac{Z^*-Z^0}{\theta_2}, \frac{\overline{Z}-Z^0-\theta_1 X_1}{\theta_2})$, *resource* 1 *is exhausted and resource* 2 *is used but not exhausted, the dirty backstop is not used (case* $F2$*);*

– *If* $\overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$, *both resources* 1 *and* 2 *are exhausted, the dirty backstop is used (case* $F1$*).*

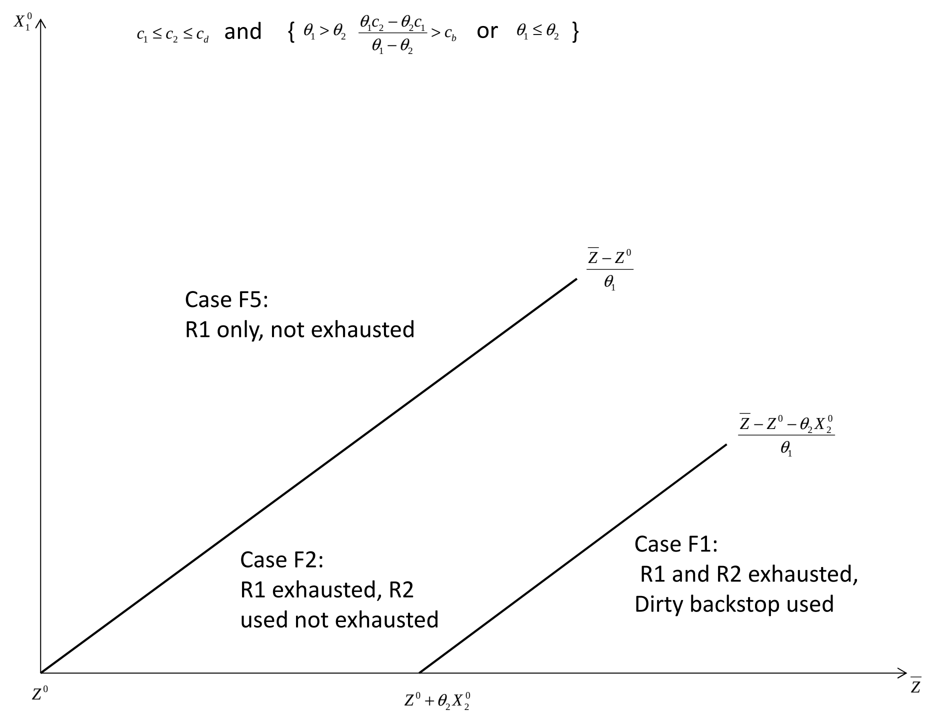

Figure 5.1: The different extraction path when $c_1 < c_2 < c_d$ and only resource 2 is used for a very low $\overline{Z}$

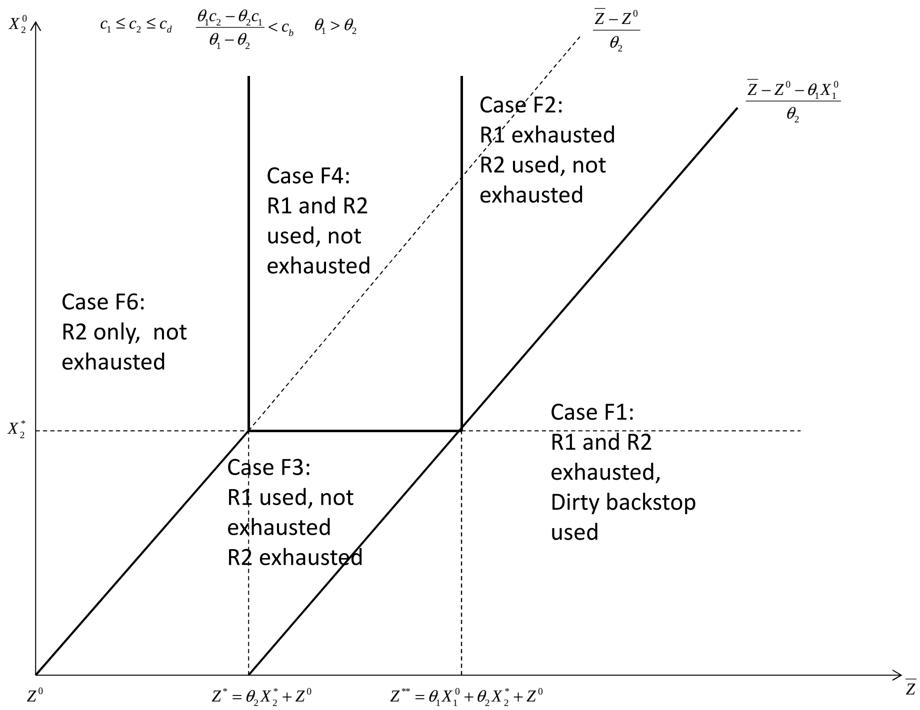

Figure 5.2: The different extraction path when $c_1 < c_2 < c_d$ and only resource 1 is used for a very low $\overline{Z}$

See Appendix for discussion of Lemma 20.

5.1.2 Intermediate cost for the dirty backstop, $c_1 < c_d < c_2$

Different extraction paths can exist. Labels of the different cases are presented in Table 7. The full characterization of the different extraction paths when $c_d > c_2 > c_1$ is given in Lemma 7.

Resource 1 is cheaper and less polluting to extract that the dirty backstop, thus using the dirty backstop implies to exhaust resource 1 first. If resource 1 is less polluting than resource 2, resource 1 gets exhausted iff. $\theta_1 X_1^0 < \overline{Z} - Z^0$. Two cases where the three polluting resources are used, and at least one resource is exhausted can occur: resources 1 and 2 are exhausted and the dirty backstop is used (case $G1$), or resource 1 is exhausted, the dirty backstop used, resource 2 is used but not exhausted (case $G2$). Other cases are either not relevant (cases where only one polluting resource is used) or already studied in Section 3 (cases where only two polluting resources are used). [15]

Case	Extraction costs	Resource 1	Resource 2	Dirty backstop*
Case G1	$c_2 > c_d > c_1$	used, exh.	used, exh.	used
Case G2	...	used, exh.	used, not exh.	used
Case G3	...	used, exh.	used, not exh.	not used
Case G4	...	used, exh.	not used	used
Case G5	...	used, not exh.	used, exh.	not used
Case G6	...	used, not exh.	used, not exh.	not used
Case G7	...	used, not exh.	not used	not used
Case G8	...	not used	used, not exh.	not used

"..." means that the value of that cell is the same than the value of the cell right above. *Recall that the dirty backstop is not exhaustible.

Table 7: The different extraction paths with three polluting resources when $c_2 > c_d > c_1$.

We can state the following Lemma:

Lemma 21. • *If* $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} > c_b$ *(*$\theta_1 >< \theta_2$*) (See Figure 3.2), then:*

- *Only resource 1 is used (case G7) iff.* $\theta_1 X_1^0 \geq \overline{Z} - Z^0$*;*
- *Resource 1 is exhausted and the dirty backstop is used (case G4) iff.* $\theta_1 f X_1^0 < \overline{Z} - Z^0$.

• *If* $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ *and [*$\theta_1 < \theta_2$ *or (*$\theta_1 > \theta_2$ *and* $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$ *)] (See Figure 5.4)), then* $\exists \tilde{Z}, Z^{**}, Z^{***}$, $\tilde{Z} < Z^{**} < Z^{***}$, *such that:*

- *If* $\overline{Z} < \tilde{Z}$*, only resource 1 is used to get to the ceiling (case G7);*
- *If* $Z^{**} > \overline{Z} > \tilde{Z}$*, and* $X_2^0 > \frac{\overline{Z} - Z^0 - \theta_1 X_1}{\theta_2}$*, resource 1 is exhausted and resource 2 is used but not exhausted (case G3);*
- *If* $\overline{Z} > \tilde{Z}$*, and* $X_2^0 < min(\frac{\overline{Z} - Z^0 - \theta_1 X_1}{\theta_2}, \frac{Z^{**} - Z^0 - \theta_1 X_1}{\theta_2})$*, resources 1 and 2 are exhausted and the dirty backstop is used (case G1);*

[15] Note that if the polluting backstop was less polluting than the second exhaustible resource, this resource would never be used, and the situation will be as described in Section 3.

- *If* $\overline{Z} > Z^{**}$ *and* $X_2^0 > \frac{Z^{**}-Z^0-\theta_1 X_1}{\theta_2}$*, resource* 1 *is exhausted, resource* 2 *is used but not exhausted, and the dirty backstop is used (case G*2*).*

- *If* $\frac{\theta_d c_2-\theta_2 c_d}{\theta_d-\theta_2} < c_b$ *and* $\theta_1 > \theta_2$ *and* $\frac{\theta_1 c_2-\theta_2 c_1}{\theta_1-\theta_2} < c_b$ *(See Figure 5.3), then* $\exists Z^*, Z^{**}, Z^{***}$*,* $Z^* < Z^{**} < Z^{***}$*, such that:*

  - *If* $\overline{Z} < Z^*$ *and* $X_2^0 > \frac{\overline{Z}-Z^0}{\theta_2}$*, only resource* 2 *is used to get to the ceiling (case G*8*);*
  - *If* $X_2^0 < \frac{Z^*-Z^0}{\theta_2}$ *and* $\frac{\overline{Z}-Z^0}{\theta_2} < X_2^0 < \frac{\overline{Z}-Z^0-\theta_1 X_1}{\theta_2}$*, resource* 1 *is exhausted, resource* 2 *is used but not exhausted, the dirty backstop is unused (case G*5*);*
  - *If* $Z^{**} > \overline{Z} > Z^*$ *and* $X_2^0 > \frac{Z^*-Z^0}{\theta_2}$*, resource* 1 *is exhausted, resource* 2 *is used but not exhausted, the dirty backstop is unused (case G*6*);*
  - *If* $Z^{***} > \overline{Z} > Z^{**}$ *and* $X_2^0 > \frac{\overline{Z}-Z^0-\theta_1 X_1}{\theta_2}$*, resource* 1 *is exhausted, resource* 2 *is used but not exhausted, the dirty backstop is unused (case G*3*);*
  - *If* $Z^{***} < \overline{Z}$ *and* $X_2^0 > \frac{Z^{***}-Z^0-\theta_1 X_1}{\theta_2}$*, resource* 1 *is exhausted, resource* 2 *is used but not exhausted, the dirty backstop is used (case G*2*);*
  - *If* $Z^* < \overline{Z}$ *and* $X_2^0 < min(\frac{\overline{Z}-Z^0-\theta_1 X_1}{\theta_2}, \frac{Z^{***}-Z^0-\theta_1 X_1}{\theta_2})$*, resources* 1 *and* 2 *are exhausted, the dirty backstop is used (case G*1*).*

See Appendix for discussion of Lemma 21

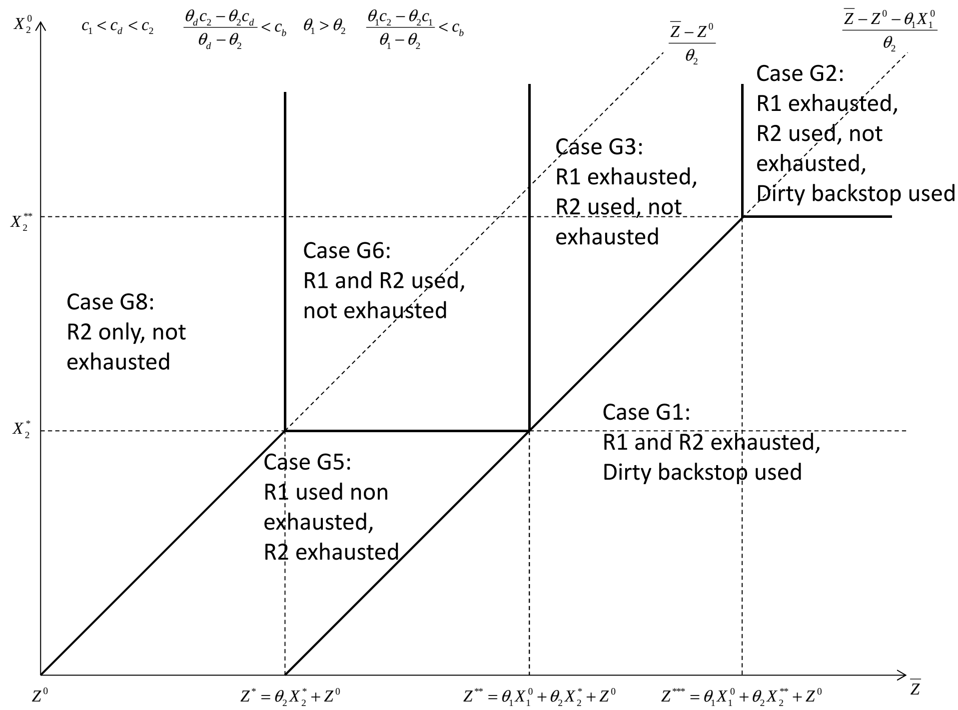

Figure 5.3: The different extraction path when $c_1 < c_d < c_2$ and only resource 2 is used for a very low $\overline{Z}$

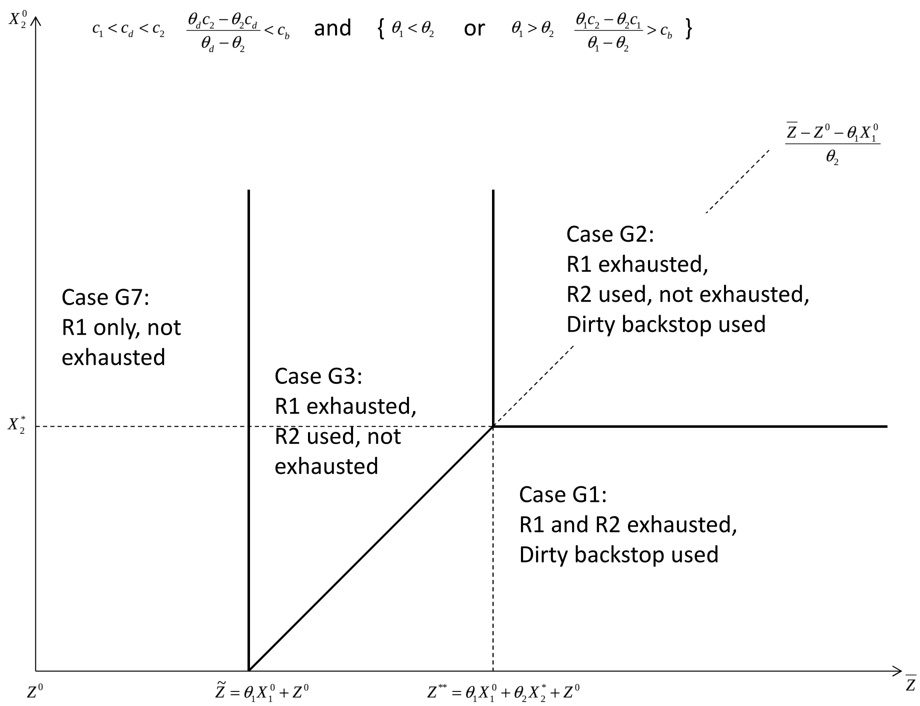

Figure 5.4: The different extraction path when $c_1 < c_d < c_2$ and only resource 1 is used for a very low $\overline{Z}$

5.2 General results

5.2.1 Preliminary remarks over profits of resources 1 and 2 when $c_1 < c_2 < c_d$

If both resources 1 and 2 are exhausted and the dirty backstop is used (case $F1$), thus writing t_1, the date of switch from resource 1 to resource 2, t_2 the date of switch from resource 2 to the dirty backstop, and finally $\underline{t}$, the date of switch to the clean backstop. The solution $\{\lambda_1^0, \lambda_2^0, \mu^0, t_1, t_2, \underline{t}\}$

must satisfy:

$$
\begin{aligned}
c_1 + \lambda_1^0 e^{rt_1} + \theta_1 \mu^0 e^{rt_1} &= c_2 + \lambda_2^0 e^{rt_1} + \theta_2 \mu^0 e^{rt_1} && (5.1)\\
c_2 + \lambda_2^0 e^{rt_2} + \theta_2 \mu^0 e^{rt_2} &= c_d + \theta_d \mu^0 e^{rt_2} && (5.2)\\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= q && (5.3)\\
\int_0^{t_1} D(c_1 + \lambda_1^0 e^{rt} + \theta_1 \mu^0 e^{rt})dt &= X_1^0 && (5.4)\\
\int_{t_1}^{t_2} D(c_2 + \lambda_2^0 e^{rt} + \theta_2 \mu^0 e^{rt})dt &= X_2^0 && (5.5)\\
\theta_1 X_1^0 + \theta_2 X_2^0 + \theta_d \int_{t_2}^{\underline{t}} D(c_d + \theta_d \mu^0 e^{rt})dt &= \overline{Z} - Z^0 && (5.6)
\end{aligned}
$$

Profits of resource 2 **'s owners** $(c_1 < c_2 < c_d)$. Tightening the ceiling has two effects on resource 2 profits. Because resource 2 is in direct competition with the dirty backstop, these effects are similar to those on profits of owners of the exhaustible resource described in Section 2 when $c_e < c_d$. The price at which the dirty backstop starts to be used increases, so that the value of the tax at the date of switch to the dirty backstop, $\mu^0 e^{rt_2}$, increases. The rent of resource 2 when it gets exhausted increases as well, as it satisfies $\lambda_2^0 e^{rt_2} = (c_d - c_2) + (\theta_d - \theta_2)\mu^0 e^{rt_2}$. On the other hand, the after-tax price of resource 2 increases, the demand at each date decreases so that resource 2 gets exhausted over a longer period of time, and its price when it starts to be used, must therefore increase. It follows that the price of resource 1 must increase and the date of switch from resource 1 to resource 2 is postponed. Because the period over which resource 2 is used is extended, it follows that the date of switch from resource 2 to the dirty backstop is postponed. Finally, the current profits of resource 2 owners when it gets exhausted are higher but the date of exhaustion of resource 2 is postponed, so that the resulting effect is ambiguous. However, we can state the following Lemma:

Lemma 22. $\forall c_1, c_2, c_d$ *such that* $c_1 < c_2 < c_d$, $\forall \overline{Z} > Z^0 + \theta_1 X_1^0 + \theta_2 X_2^0$,

$\frac{d\lambda_2^0}{d\overline{Z}} < 0$

iff.

$$1 - (\theta_d - \theta_2)\frac{\mu^0}{\lambda_2^0} - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)} < 0.$$

Proof. See Appendix □

Profits of resource 1 's owners ($c_1 < c_2 < c_d$). Contrary to R_2, R_1 is not in direct competition with the dirty backstop. The current rent at that date may decrease or increase depending on the ordering of pollution contents, θ_1 and θ_2 and on the dynamics of the scarcity rent of resource 2. Indeed, by equation 5.1, we get that: $\lambda_1^0 = (c_2 - c_1)e^{-rt_1} + \lambda_2^0 + (\theta_2 - \theta_1)\mu^0$. When tightening the carbon ceiling, the date of switch to resource 2 is postponed. In the general case, the effect of tightening the carbon ceiling on profits of owners of R_1 is thus undetermined. However, we can state the following Lemma:

Lemma 23. $\forall c_1, c_2, c_d$ *such that* $c_1 < c_2 < c_d$, $\forall \overline{Z} > Z^0 + \theta_2 X_2^0 + \theta_1 X_1^0$,

$\frac{d\lambda_1^0}{d\overline{Z}} < 0$

iff.

$$\frac{\theta_d}{\theta_1}\frac{D(t_2)}{D(0)} + \frac{\theta_d\mu^0}{\lambda_1^0 + \theta_1\mu^0}(1 - \frac{D(t_1)}{D(0)}) + \frac{\theta_d\mu^0}{\lambda_2^0 + \theta_2\mu^0}(1 - \frac{D(t_2)}{D(t_1)}) > 1.$$

Proof. See Appendix. □

5.2.2 Preliminary remarks over profits of resources 1 and 2 when $c_1 < c_d < c_2$

If the extraction path is as described by case $G1$, thus writing t_1, the date of switch from resource 1 to the dirty backstop, t_2, the date of switch from the dirty backstop to resource 2, and finally $\underline{t}$, the date of switch to the clean backstop, the solution $(\lambda_1^0, \lambda_2^0, \mu^0, t_1, t_2, \underline{t})$ must satisfy:

$$
\begin{aligned}
c_1 + \lambda_1^0 e^{rt_1} + \theta_1 \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} \\
c_2 + \lambda_2^0 e^{rt_2} + \theta_2 \mu^0 e^{rt_2} &= c_d + \theta_d \mu^0 e^{rt_2} && (5.7) \\
c_2 + \lambda_2^0 e^{r\underline{t}} + \theta_2 \mu^0 e^{r\underline{t}} &= c_b && (5.8) \\
\int_0^{t_1} D(c_1 + \lambda_1^0 e^{rt} + \theta_1 \mu^0 e^{rt}) dt &= X_1^0 && (5.9) \\
\int_{t_2}^{\underline{t}} D(c_2 + \lambda_2^0 e^{rt} + \theta_2 \mu^0 e^{rt}) dt &= X_2^0 && (5.10) \\
\theta_1 X_1^0 + \theta_2 X_2^0 + \int_{t_1}^{t_2} \theta_d D(c_d + \theta_d \mu^0 e^{rt}) dt &= \overline{Z} - Z^0 && (5.11)
\end{aligned}
$$

If both resources 1 and 2 are exhausted and the dirty backstop is used, and that after a marginal decrease of the carbon ceiling we still stay in that case, tightening the carbon regulation will increase the carbon tax, postpone the date of switch to the dirty backstop, t_1, bring forward the date of switch to resource 2 and the date the ceiling binds, and increase profits of owners of resource 2. The price of resource 2 when it starts to be used, $p(t_2)$, is independent of $\overline{Z}$. Indeed, from equations 5.8 and 5.10, if resource 2 is exhausted, its price when it starts to be used, is defined by $\int_{p(t_2)}^{c_b} D(c_2 + (p(t_2) - c_2)e^{rt})dt = X_2^0$. If resource 2 is not exhausted, using equations 5.8 and setting λ_2^0 to zero, it comes that $p(t_2) = c_2 + \theta_2 \frac{c_d - c_2}{\theta_d - \theta_2}$.

The consumption of resource 2 equals X_2^0 if resource 2 is exhausted i.e if $X_2^0 < X_2^*$, or X_2^* otherwise. Thus the consumption of resource 2 equals $min(X_2^0, X_2^*)$. The length of period during which resource 2 is used is unchanged, since the starting price and the final price of resource 2 is unchanged.[16]

Profits of owners of resource 2 **($c_1 < c_d < c_2$).** The reasoning is similar than the one exposed in the case with two polluting resources where the exhaustible resource is the most expensive but gets exhausted (Section 3, subsection 3.2, $c_e > c_d$). Since the starting price of resource 2, $p(t_2)$, is

[16]Indeed, with a marginal decrease of $\overline{Z}$, the price at which resource 2 starts to be used is unchanged. If the carbon tax was reduced, the dirty backstop price path would be put downward, and thus the date of switch to resource 2 would be postponed to get the starting price of R_2 unchanged. If the date of switch from resource 1 to the dirty backstop is brought forward, the global consumption of the dirty backstop increases since it would be used over a longer period at a lower price, that is not possible due to the ceiling constraint and the assumption that both other resources 1 and 2 are exhausted. If the date of switch from resource 1 to the dirty backstop is postponed, the price of resource 1 must be put downward, so that the dirty backstop price and resource 1 price gets equal at a later date, but in that case, the consumption of resource 1 must increase since it would be used over a longer period of time at a lower price, that is not possible. It follows that when decreasing $\overline{Z}$, if both resources still get exhausted and the dirty backstop used, the carbon tax must increase and the date of switch from the dirty backstop to resource 2 is brought forward. The date the ceiling is reached is also brought forward.

unchanged, thus the sum of the scarcity rent and the carbon tax of resource 2. Note that $p(t_2)$ is also the price at which the dirty backstop stops to be used. So, the dirty backstop price and thus the value of the tax at that time, $\mu^0 e^{rt_2}$, are unchanged. It follows that profits at the date of switch from the dirty backstop to resource 2 must be unchanged. Since the date of switch to the dirty backstop is brought forward, the initial scarcity rent of resource 2 must increase to keep the current value rent at the switch date to resource 2 unchanged. but less polluting than the dirty backstop increase when the carbon ceiling is tightened, as long as this resource is exhausted. We call N, the set of parameters such that R_2 is used after the dirty backstop and gets exhausted.[17]

Proposition 24. *If resource 2 is used after the dirty backstop and exhausted, tightening the carbon ceiling increases profits of resource 2 owners.*

If parameters belong to N, then $\frac{d\lambda_2^0}{d\overline{Z}} < 0$.

Proof. See above

□

Profits of owners of resource 1 ($c_1 < c_d < c_2$). As mentioned above, the starting price of resource 2, $p(t_2)$, is fixed and does not depend on $\overline{Z}$ and the consumption of resource 2 equals $min(X_2^0; X_2^*)$. It follows that $\{\lambda_1^0, \mu^0, t_1, t_2\}$ must satisfy:

$$c_1 + \lambda_1^0 e^{rt_1} + \theta_1 \mu^0 e^{rt_1} = c_d + \theta_d \mu^0 e^{rt_1}$$

$$p(t_2) = c_d + \theta_d \mu^0 e^{rt_2} \tag{5.12}$$

$$\int_0^{t_1} D(c_1 + \lambda_1^0 e^{rt} + \theta_1 \mu^0 e^{rt}) dt = X_1^0 \tag{5.13}$$

$$\theta_1 X_1^0 + \int_{t_1}^{t_2} \theta_d D(c_d + \theta_d \mu^0 e^{rt}) dt = \overline{Z} - Z^0 - \theta_2 min(X_2^0; X_2^*) \tag{5.14}$$

This system is similar to the system describing the solution when only one exhaustible resource,

[17] N is the collection of parameters satisfying:
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $[\theta_1 < \theta_2$ or $(\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b)]$ and $\overline{Z} > \tilde{Z}$, and $X_2^0 < min(\frac{\overline{Z} - Z^0 - \theta_1 X_1}{\theta_2}, \frac{Z^{**} - Z^0 - \theta_1 X_1}{\theta_2})$
or
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$ and $Z^* < \overline{Z}$ and $X_2^0 < min(\frac{\overline{Z} - Z^0 - \theta_1 X_1}{\theta_2}, \frac{Z^{***} - Z^0 - \theta_1 X_1}{\theta_2})$.

Note that for parameters such that $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $[\theta_1 < \theta_2$ or $(\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b)]$ or $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$, we can find $\overline{Z}$ and X_2^0 such that parameters belong to N.

– cheaper than the dirty backstop –, is available in Section 3. The difference lays in the fact that the final price of the dirty backstop equals resource the starting price of R_2 price and not the clean backstop price, c_b. The final date we consider is thus t_2 and not $\underline{t}$. The global amount of pollution allowed from the consumption of resource 1 and the dirty backstop becomes $\overline{Z} - Z^0 - min(X_2^0; X_2^*)$. Recall that in Lemma 3 of Section 3 , the condition over the sign of $\frac{d\lambda_e^0}{d\overline{Z}} < 0$ does not depends on $\underline{t}$ nor Z^0, nor c_b It follows that if resource 1 is exhausted, the dirty backstop and resource 2 used, the condition to get $\frac{d\lambda_1^0}{d\overline{Z}} < 0$ is similar to the condition expressed in Lemma 3 of Section 3 when R_1 is exhausted, the dirty backstop is used but R_2 is not used.

We call M this set of parameters such that resource 1 gets exhausted and the dirty backstop is used after, resource 2 being used or not later, and being exhausted or not if used..[18]

Lemma 25. *For all parameters that belong to* M_1 *,*

$$\frac{d\lambda_1^0}{d\overline{Z}} < 0$$

iff.

$$\frac{D(p(t_1))}{D(p(0))}\frac{\theta_d}{\theta_1} + (\theta_d - \theta_1)\frac{\mu^0}{\lambda_1^0} > 1.$$

It is straightforward to verify that Propositions concerning the sign of $\frac{d\lambda_1^0}{d\overline{Z}}$ (Propositions 14-9) of Section 3 still hold when taking care of rewriting the condition to get resource 1 exhausted and the three polluting resources used.

5.2.3 Results over profits of resources owners when effects are ambiguous

As indicated above, if $c_1 < c_2 < c_d$, the impacts of tightening the ceiling on profits of either resource 1 or 2 is undetermined. Similarly, if $c_1 < c_2 < c_d$, the impacts of tightening the ceiling on profits of either resource 1. Hereafters, we show some proposition concerning the effect in the ambiguous cases.

[18] See Lemma 21, parameters belonging to M satisfy:
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} > c_b$ and $\theta_1 X_1^0 < \overline{Z} - Z^0$
or
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $[\theta_1 < \theta_2$ or $(\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$)] and $\overline{Z} > min(Z^0 + \theta_1 X_1 + \theta_2 X_2^0, Z^0 + \theta_1 X_1 + \theta_2 X_2^*)$
or
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$ and $\overline{Z} > min(Z^0 + \theta_1 X_1 + \theta_2 X_2^0, Z^0 + \theta_1 X_1 + \theta_2 X_2^{**})$.

It is straightforward that for parameters such that $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} > c_b$
or
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $[\theta_1 < \theta_2$ or $(\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$)]
or
$\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$
, it is possible to find $\overline{Z}$, X_2^0 such that the parameters belong to M.

If $c_1 < c_2 < c_d$, tightening the carbon ceiling increases the profits of resource 2 if this resource gets exhausted after the dirty backstop is used, thus this case doe snot require more space in the analysis.

Proposition 26. *If resource i gets exhausted and the dirty backstop is used and the pollution content of resource i is low enough, tightening the carbon ceiling increases the profit of resource i owners.*

- $\forall c_1, c_2, c_d$ *such that* $c_1 < c_2 < c_d$, *for* $i \in \{1,2\}$, $\exists \eta^*$, *with* $0 < \eta^* < 1$, *such that:*

$$\left\{ \frac{\theta_i}{\theta_d} \leq 1 - \eta_i^* \textit{and } \overline{Z} > \theta_{-i} X_{-i}^0 + Z^0 + \theta_i X_i \right\} \Longrightarrow \frac{d\lambda_i^0}{d\overline{Z}} < 0$$

- $\forall c_1, c_2, c_d$ *such that* $c_1 < c_d < c_2$, $\exists 0 < \eta_1^*$ *such that*

$$\left\{ \frac{\theta_1}{\theta_d} \leq 1 - \eta_1^* \textit{and parameters belong to } N_1 \right\} \Longrightarrow \frac{d\lambda_1^0}{d\overline{Z}} < 0.$$

Proof. Straightforward from Lemmas 22 and 23, and previous sections. □

Proposition 27. *If resource i gets exhausted and the dirty backstop is used and the elasticity of demand is small enough, tightening the carbon ceiling increases the profit of the owners of resource i.*

- $\forall c_1, c_2, c_d$ *such that* $c_1 < c_2 < c_d$, , $\exists \epsilon_i^*$ *such that* $\forall \overline{Z} > Z^0 + \theta_1 X_1^0 + \theta_2 X_2^0$*:*

$$\left\{ \forall p, -\frac{D^{'}(p)p}{D(p)} \leq \epsilon_i^* \right\} \Longrightarrow \frac{d\lambda_i^0}{d\overline{Z}} < 0.$$

- $\forall c_1, c_2, c_d$ *such that* $c_1 < c_d < c_2$, $\exists \epsilon^*$ *such that, for all parameters belonging to* N_1*:*

$$\left\{ \forall p, -\frac{D^{'}(p)p}{D(p)} \leq \epsilon_1^* \right\} \Longrightarrow \frac{d\lambda_1^0}{d\overline{Z}} < 0.$$

Proof. Straightforward from Lemmas 22 and 23, and previous sections. □

Remark 28. When X_2^0 is low enough, tightening the carbon ceiling does not necessarily increase the profits of owners of resource 2. Result of Proposition 7 cannot be extended to the case with several exhaustible resources. Demands $D(t_1)$ and $D(t_2)$ are continuous functions of the initial stock of resource 2, and $\underset{X_2^0 \mapsto 0}{lim} D(t_2) = D(t_1)$. Using Lemma 22, for X_2^0 low enough, $\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of: $-(\frac{\theta_d}{\theta_2} - 1) + \frac{\lambda_2^0 - \theta_2 \mu^0}{\lambda_1^0 + \theta_1 \mu^0} \frac{\theta_d}{\theta_2}(1 - \frac{D(t_1)}{D(0)}) + \frac{\theta_d \mu^0}{\lambda_1^0 + \theta_1 \mu^0}(1 - \frac{D(t_1)}{D(0)})$ that can be either negative or positive. However, for X_1^0 and X_2^0 low enough, $\frac{d\lambda_2^0}{d\overline{Z}}$ is negative.

Remark 29. When the carbon ceiling is getting low enough, to avoid exhaustion of resource 2, we find that the scarcity rent associated with this resource is discontinuous when $\overline{Z}$ is getting lower than $\theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$. This result is similar to result found in remark 8. Using equations 5.2 and 5.3, it comes that $\lambda_2^0 = e^{-rt_2}(c_d - c_2) + (\theta_d - \theta_2)\frac{c_b - c_d}{\theta_d} e^{-r\underline{t}}$. We know that $t_2 > \frac{X_1^0 + X_2^0}{D(c_1)}$ and $t_1 > \frac{X_1^0}{D(c_1)}$, it follows that there exists ϵ such that $\forall \overline{Z}\ \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$, $\lambda_2^0 > \epsilon > 0$.

Proposition 30. *If resource i gets exhausted and the dirty backstop is used just after resource i and the extraction cost of resource i is close enough from the dirty backstop extraction cost, tightening the carbon ceiling increases the profit of R_i owners.*

- $\forall c_1, c_2, c_d$ *such that* $c_1 < c_2 < c_d$, $\forall \overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$, $\exists c^* < c_d$ *such that:*

$$c_d \geq c_2 \geq c^* \implies \frac{d\lambda_2^0}{d\overline{Z}} < 0$$

- $\forall c_1, c_2, c_d$ *such that* $c_1 < c_d < c_2$, *for all parameters belonging to* N_1 $\exists c^* < c_d$ *such that:*

$$c_d \geq c_1 \geq c^* \implies \frac{d\lambda_1^0}{d\overline{Z}} < 0$$

Proof. Assume that $c_1 < c_2 < c_d$. In that case, recall that $\frac{d\lambda_2^0}{d\overline{Z}}$ is negative iff. $1 + (\theta_2 - \theta_d)\frac{\mu^0}{\lambda_2^0} - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)}$ is negative. Using equation 5.3, one must replace $(\theta_d - \theta_2)\frac{\mu^0}{\lambda_2^0}$ by $1 - \frac{c_d - c_2}{\lambda_2^0}e^{-rt_2}$, it comes that $\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of: $\frac{c_d - c_2}{\lambda_2^0}e^{-rt_2} - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)}$. It follows that if $\frac{c_d - c_2}{\lambda_2^0}e^{-rt_2} - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2}$ is negative, $\frac{d\lambda_2^0}{d\overline{Z}}$ is negative. We know that $\frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} > \frac{\theta_d}{\theta_2}\frac{D(c_b)}{D(c_2)} > 0$ and $\lambda_2^0 e^{rt_2} > (\theta_d - \theta_2)\mu^0 e^{rt_2} > 0$. Thus, $\frac{d\lambda_2^0}{d\overline{Z}}$ is negative if $-\frac{D(c_b)}{D(c_2)}\frac{\theta_d}{\theta_1} + \frac{c_d - c_2}{(\theta_d - \theta_2)\mu^0 e^{rt_2}}$ is negative. $\mu^0 e^{rt_2}$ does not depend on c_2 (-see proof of Proposition 6) and is strictly positive for any $\bar{Z}$. At $\mu^0 e^{rt_2}$ given, as $-\frac{D(c_b)}{D(c_2)}\frac{\theta_d}{\theta_1} + \frac{c_d - c_2}{(\theta_d - \theta_2)\mu^0 e^{rt_2}}$ is continuous with c_2 and decreases with c_2 and is strictly negative for $c_2 = c_d$, then there exists c^* such that Proposition 30 holds.

For the case where $c_1 < c_d < c_2$, see discussion over profits of R_1. □

Remark 31. Contrary to Proposition 7, when X_1^0 is low enough, tightening the carbon ceiling does not necessarily increase the profits of owners of resource 1. Demands $D(0)$ and $D(t_1)$ are continuous functions of the initial stock of resource 1, and $\underset{X_1^0 \mapsto 0}{lim} D(0) = D(t_1)$. $\frac{d\lambda_1^0}{d\overline{Z}}$ has the sign of $1 - \frac{\theta_d}{\theta_1}\frac{D(t_2)}{D(0)} - \frac{\theta_d \mu^0}{\lambda_2^0 + \theta_2 \mu^0} + \frac{\theta_d \mu^0}{\lambda_2^0 + \theta_2 \mu^0}\frac{D(t_2)}{D(0)}$, that can be either positive or negative.

Comparison of the profits of owners of resources 1 and 2

We now turn to investigate how a change in $\overline{Z}$ impacts the difference of rents $\lambda_1^0 - \lambda_2^0$ to determine who benefits the most or looses the least from tightening the carbon regulation.

If $c_1 < c_d < c_2$, the exhaustible resources are not in direct competition if the dirty backstop is used. it follows that there is no particular rule concerning the effect of $\overline{Z}$ on $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}$.

Assume hereafters that If $c_1 < c_2 < c_d$. In this case, the exhaustible resources are not in direct competition. Rewriting equation 5.1, it comes that $\lambda_1^0 - \lambda_2^0 = (c_2 - c_1)e^{-rt_1} + (\theta_2 - \theta_1)\mu^0$. Lowering $\overline{Z}$ has an ambiguous effect on $\lambda_1^0 - \lambda_2^0$ if $\theta_2 - \theta_1 > 0$. However if $\theta_2 < \theta_1$, lowering $\overline{Z}$ decreases $\lambda_1^0 - \lambda_2^0$, i.e benefits more or harms less profits of resource 2 owners than profits of resource 1 owners. We can state the following Lemma:

Lemma 32. $\forall c_1, c_2, c_d,\ c_1 < c_2 < c_d,\ \forall \overline{Z} > Z^0 + \theta_1 X_1^0 + \theta_2 X_2^0,$

$\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}} < 0$

iff.

$\theta_d(1 - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0})(1 - \frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)} + (\theta_1 - \theta_2)\left(1 - \frac{\theta_d \mu^0}{\lambda_2^0 + \theta_2\mu^0}(1 - \frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}\right) < 0$

Proof. See Appendix. □

Proposition 33. *If the pollution content of resource 2 is lower or is higher – but close enough in this case– than the pollution content of resource 1, tightening the carbon ceiling increases more (or decreases less) the marginal profit of resource 2 owners than that of resource 1 owners. Owners of the least polluting resource are not necessary those who gain the most or loose the least when the carbon regulation is tightened.*

$\forall c_1, c_2, c_d,\ c_1 < c_2 < c_d,\ \forall \overline{Z} > \theta_1 X_1^0 + \theta_1 X_2^0 + Z^0,\ \exists \theta_1 < \theta^* < 1,$

$$\theta_2 \leq \theta^* \Longrightarrow \frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}} > 0.$$

Note that this implies that if $\theta_2 \leq \theta_1$, then $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}} > 0$.

Proof. The result for $\theta_1 > \theta_2$ is straightforward from Lemma 32. Assume now that $\theta_1 < \theta_2$, and $\overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$. We know that $0 < (1 - \frac{\theta_d \mu^0}{\lambda_2^0 + \theta_2\mu^0}(1 - \frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}) < 1$, it follows that

$\underset{\theta_2 \to \theta_1}{lim}(\theta_1-\theta_2)\left(1-\frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}\right)=0$. We also know that $\frac{D(t_2)}{D(t_1)} > \frac{D(c_b)}{D(c_2)}$. Using equation 5.4, and the First Mean Value Theorem for Integrals there exists p^*, such that: $p(0) < p^* < p(t_1)$, and $(p(t_1)-p(0))D(p^*) = X_1^0$. Note that $D(c_b) < D(p^*) < D(c_1)$. It follows that $\underset{\theta_2 \to \theta_1}{lim}(p(t_1) - p(0)) > 0$. Since D is strictly decreasing with p, we get that $\exists \beta$, $(1-\frac{D(t_1)}{D(0)}) > \beta > 0$. Using equation 5.1, we get that: $\lambda_1^0 - \lambda_2^0 + (\theta_1-\theta_2)\mu^0 = (c_2-c_1)e^{rt_1}$. As $t_1 \leq \frac{X_1^0}{D(c_b)}$, it follows that $\lambda_1^0 - \lambda_2^0 + (\theta_1-\theta_2)\mu^0 > (c_2-c_1)e^{r\frac{X_1^0}{D(c_b)}} > 0$, thus $\exists\gamma$, such that $(1-\frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0}) > \gamma > 0$. Finally, $\theta_d(1-\frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0})(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)} > \theta_d\beta\gamma\frac{D(c_b)}{D(c_2)} > 0$. It follows that there exists θ^* such that Proposition 33 holds.

□

Remark 34. If θ_1 low enough, no particular result.

Proposition 35. *If both resources* 1 *and* 2 *are exhausted and the dirty backstop is used, and resource* 1 *is scare enough then tightening the carbon ceiling will benefit more or harm less the profits of owners of the least polluting resource.*

$\forall c_1, c_2, c_d$, $c_1 < c_2 < c_d$, $\forall \overline{Z} > \theta_2 X_2^0 + Z^0$, $\exists X_1^*$ *such that* $\forall X_1^0$:

$$X_1^0 \leq X_1^* \Longrightarrow (\theta_1-\theta_2)(\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}) > 0$$

Proof. Demands $D(0)$ and $D(t_1)$ are continuous functions of the initial stock of resource 1, and $\underset{X_1 \mapsto 0}{lim} D(0) = D(t_1)$. As show above, $\exists\delta$ such that $0 < \theta_d(1-\frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1+\theta_1\mu^0})\frac{D(t_2)}{D(t_1)} < \delta$, similarly we can show that $\exists\zeta$ such that $0 < \frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}\frac{D(t_2)}{D(t_1)} < \zeta$, for X_1 is low enough, $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of $\theta_1-\theta_2$ and we can find X_1^* such that Proposition 35 holds.

□

Remark 36. If X_2 is low enough, the sign of $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}$ is undetermined.

Proposition 37. *If both resources* 1 *and* 2 *are exhausted and the dirty backstop is used, and the extraction cost of resource* 2 *is close enough to that one of resource* 1*, tightening the carbon ceiling will benefit more or harm less the profits of owners of the least polluting resource.*

$\forall c_1, c_2, c_d$, $c_1 < c_2 < c_d$, $\forall \overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$, $\forall c_d$, $\exists c^* < c_d$ *such that* $\forall c_2$:

$$c_1 < c_2 \leq c^* \implies (\theta_1 - \theta_2)(\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}) > 0$$

Proof. We replace $\theta_1 - \theta_2$ by $\frac{c_2 - c_1}{e^{rt_2}\mu^0} - \frac{\lambda_1 - \lambda_2^0}{\mu^0}$, it comes that when c_2 tends towards c_1, $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of $(\theta_2 - \theta_1)(-\theta_d \frac{\mu^0}{\lambda_1 + \theta_1\mu^0}\frac{D(t_2)}{D(0)} - 1 + \mu^0 \frac{\theta_d}{\lambda_1 + \theta_1\mu^0})$. Since $-\theta_d \frac{\mu^0}{\lambda_1 + \theta_1\mu^0}\frac{D(t_2)}{D(0)} - 1 + \mu^0 \frac{\theta_d}{\lambda_1 + \theta_1\mu^0} < 0$, it follows that $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of $\theta_1 - \theta_2$. Thus there exists c^* such that Proposition 37 holds. □

Proposition 38. *If the elasticity of demand is small enough, tightening the carbon ceiling will benefit more or harm less the profits of owners of the least polluting resource.*

$\forall c_1, c_2, c_d$, $c_1 < c_2 < c_d$, $\forall \overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$, $\exists \epsilon^*$ *such that:*

$$\left\{\forall p, -\frac{D'(p)p}{D(p)} \leq \epsilon^*\right\} \implies (\theta_1 - \theta_2)(\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}) > 0.$$

Proof. From Lemma 32, note that $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}} < 0$ iff. $(\theta_1 - \theta_2) + \left(\theta_d(1 - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0}) - \frac{\theta_d\mu^0}{\lambda_2^0 + \theta_2\mu^0}\right)(1 - \frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)} < 0$. Recall that $|1 - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0} - \frac{\theta_d\mu^0}{\lambda_2^0 + \theta_2\mu^0}| < 1$ and $\frac{D(t_2)}{D(t_1)} < \frac{D(c_1)}{D(c_s)}$, it comes that $|\left(\theta_d(1 - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0}) - \frac{\theta_d\mu^0}{\lambda_2^0 + \theta_2\mu^0}\right)\frac{D(t_2)}{D(t_1)}| < \theta_d \frac{D(c_1)}{D(c_s)}$. As done above, using the mean value theorem, there exists $t_i \in [0; t_1]$, such that: $\frac{D(t_1)}{D(0)} \geq 1 - (\frac{-D'(p(t_i))}{D(p(t_i))}p(t_i))\frac{p(t_1) - p(0)}{p(t_i)}$. We also have $\frac{p(t_1) - p(0)}{p(t_i)} \leq \frac{p(t_1) - p(0)}{p(0)} < \frac{c_2 - c_1}{c_1}$. Thus, if $\frac{-D'(p(t_i))}{D(p(t_i))}p(t_i)\frac{c_2 - c_1}{c_1} \leq \frac{|\theta_1 - \theta_2|}{\theta_d \frac{D(c_1)}{D(c_s)}}$, then

$$|\theta_1 - \theta_2| > |[\theta_d(1 - \frac{\lambda_2^0 + \theta_2\mu^0}{\lambda_1^0 + \theta_1\mu^0}) - \frac{\theta_d\mu^0}{\lambda_2^0 + \theta_2\mu^0}](1 - \frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}|.$$

Finally, if $\frac{-D'(p(t_i))}{D(p(t_i))}p(t_i) \leq \frac{|\theta_1 - \theta_2|}{\theta_d \frac{D(c_1)}{D(c_s)}\frac{c_2 - c_1}{c_1}} = \epsilon$, then $\frac{d\lambda_1^0 - d\lambda_2^0}{d\overline{Z}}$ has the sign of $\theta_1 - \theta_2$. □

6 Conclusion

This paper casts some light on redistributional effects of carbon taxation and shows that carbon-emitting resources owners can benefit from carbon taxation if a dirtier abundant resource is also used, even if tax revenues are not redistributed.

Crude oil is likely to be in competition through time with unconventional oil, that is abundant (oil shales, oil sands, based synthetic crudes and derivative products, coal-based liquid supplies, biomass-based liquid supplies and liquids arising from chemical processing of natural gas) and more polluting. Following our model, oil owners may benefit from carbon taxation. The same remark holds for natural gas producers since gas is in competition with more polluting resources like coal.

Our results lead to reconsider the debate over compensations for losses in oil export revenues induced by carbon taxation, claimed for instance by OPEC countries. Major coal exporters are likely to be durably not sensitive to pro-mitigation arguments as long as their losses are not at least partially compensated. Oil and gas exporters may be more easily convinced about the necessity of carbon regulation since they may take direct advantage of carbon taxation.

However, we have shown in the last section of this paper that owners of a polluting resource – in direct competition with the dirtiest resource – may benefit more or loose less from carbon taxation than those of a less polluting resource that is not in direct competition with the dirtiest resource.Applying this result to the transportation sector where inshore oil competes with offshore oil (more polluting and more expensive) and unconventional oil (the most polluting and the most expensive type of oil), it comes that owners of offshore oil may benefit more or loose less from a tighter carbon regulation than owners of inshore oil, despite the fact that offshore oil is more polluting than inshore oil.

References

Barnett, J., Dessai, S. & Webber, M. (2004), 'Will opec lose from the kyoto protocol?', *Energy Policy* **32**(18), 2077–2088.

Bergstrom, T. C. (1982), 'On capturing oil rents with a national excise tax', *American Economic Review* **72**(1), 194–201.

Brander, J. & Djajic, S. (1983), 'Rent-extracting tariffs and the management of exhaustible resources', *Canadian Journal of Economics* **16**(2), 288–98.

Chakravorty, U. & Krulce, D. L. (1994), 'Heterogeneous demand and order of resource extraction', *Econometrica* **62**(6), 1445–52.

Chakravorty, U., Magné, B. & Moreaux, M. (2006), 'A hotelling model with a ceiling on the stock of pollution', *Journal of Economic Dynamics and Control* **30**(12), 2875–2904.

Chakravorty, U., Moreaux, M. & Tidball, M. (2008), 'Ordering the extraction of polluting nonrenewable resources', *American Economic Review* **98**(3), 1128–44.

Dahl, C. & Yucel, M. (1991), 'Testing alternative hypotheses of oil producer behavior', *The Energy Journal* **0**(4), 117–138.

Ezzati, A. (1976), 'Future opec price and production strategies as affected by its capacity to absorb oil revenues', *European Economic Review* **8**(2), 107–138.

Griffin, J. M. (1985), 'Opec behavior: A test of alternative hypotheses', *American Economic Review* **75**(5), 954–63.

Herfindahl, O. C. (1967), *"Depletion and Economic Theory" In Extractive Resources and Taxation ed. Mason Gaffney, 63-90*, University of Wisconsin Press.

Hotelling, H. (1931), 'The economics of exhaustible resources', *Journal of Political Economy* **39**, 137.

IEA (2008), 'International energy annual: Deploying renewable', *IEA Papers* .

IEA (2011), 'co_2 emissions from fuel combustion'.

Johansson, D. J., Azar, C., Lindgren, K. & Persson, T. A. (2009), 'Opec strategies and oil rent in a climate conscious world', *The Energy Journal* **0**(Number 3), 23–50.

Jones, C. T. (1990), 'Opec behaviour under falling prices: Implications for cartel stability', *The Energy Journal* **11**(Number 3), 117–130.

Liski, M. & Tahvonen, O. (2004), 'Can carbon tax eat opec's rents?', *Journal of Environmental Economics and Management* **47**(1), 1–12.

Loderer, C. (1985), 'A test of the opec cartel hypothesis: 1974–1983', *The Journal of Finance* **40**(3), 991–1006.

MacAvoy, P. W. (1982), *Crude Oil Prices as Determined by OPEC and Market Fundamentals.*, Ballinger Publishing Company, Cambridge, MA.

Persson, T. A., Azar, C., Johansson, D. & Lindgren, K. (2007), 'Major oil exporters may profit rather than lose, in a carbon-constrained world', *Energy Policy* **35**(12), 6346–6353.

Pizer, W. (1997), Prices vs. quantities revisited: The case of climate change, Discussion Papers dp-98-02, Resources For the Future.

Sinn, H.-W. (2008), 'Public policies against global warming: a supply side approach', *International Tax and Public Finance* **15**(4), 360–394.

Strand, J. (2010), 'Optimal fossil fuel taxation with backstop technologies and tenure risk', *Energy Economics* **32**(2), 418 – 422.

Verleger, P. K. (1982), 'The determinants of official opec crude prices', *The Review of Economics and Statistics* **64**(2), 177–82.

Weitzman, M. L. (1974), 'Prices vs. quantities', *Review of Economic Studies* **41**(4), 477–91.

7 Appendix of Section 2

7.1 Proof of Lemma 3

Differentiating Eqs 3.1-3.4 with respect to $\overline{Z}$, it comes that

$\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$-\theta_d D(p(t_1)) + \theta_e D(p(0)) + r\theta_d \mu^0 \theta_e \int_0^{t_1} D'(p(t))e^{rt}dt$$

that can be rewritten, using that $\dot{p}(t) = r(\lambda_e^0 + \theta_e \mu^0)e^{rt}$,

$$-\theta_d D(p(t_1)) + \theta_e D(p(0)) + \theta_d \mu^0 \theta_e \frac{D(p(t_1)) - D(p(0))}{\lambda_e^0 + \theta_e \mu^0}$$

thus finally, $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$1 - \frac{D(p(t_1))}{D(p(0))}\frac{\theta_d}{\theta_e} - (\theta_d - \theta_e)\frac{\mu^0}{\lambda_e^0}$$

8 Appendix of Section 4

8.1 Charaterization of the different cases

8.1.1 Cases C: R_d cheaper than R_e in both sectors: $c_e < c_d < c_d + z$

If $c_e < c_d < c_d + z$, then define X^*, Z^* such that the initial prices of R_e and R_d in the power sector are equal and the final prices of R_e and R_d in the transport sectors are equal (i.e they reach c_b at the same date). Then one can verify that the quantity of R_e used on this price path is equal to:

$$X^* = \int_0^{1/r \ln(\frac{c_d + z - c_e}{c_d - c_e})} D_T(c_e + \frac{(c_d - c_e)(c_b - c_e)}{c_d + z - c_e}e^{rt})dt$$

and the CO_2 emissions on this price path are equal to:

$$Z^* = Z_0 + \theta_e X^* + \theta_d \int_0^{1/r \ln(\frac{(c_b - c_d)(c_d + z - c_e)}{(c_d - c_e)(c_b - (c_d + z))})} D_E(c_d + \frac{(c_d - c_e)(c_b - (c_d + z))}{c_d + z - c_e}e^{rt})dt$$

And we define $\tilde{X}$ such that:

$$\tilde{X} = \int_0^{1/r \ln(\frac{c_d+z-c_e}{c_d-c_e})} D_T(c_e + (c_d - c_e)e^{rt})dt$$

The quantity $\tilde{X}$ is the quantity of R_e that would be used if the initial price of R_e (including the scarcity rent) was c_d and its final price $c_d + z$.

We define $h(\bar{Z})$, for $\bar{Z} \leq Z^*$, such that final R_e price and final R_d price in the transport sector are equal (i.e. they reach c_b at the same date):

$$\begin{aligned}
c_e + (\lambda_o + \theta_e \mu^0)e^{rT} &= c_b \\
c_e + (\lambda_o + \theta_e \mu^0)e^{rT} &= c_d + z + \theta_d e^{rT} \\
c_d + \theta_d \mu^0 e^{rT_2} &= c_b \\
h(\bar{Z}) &= \int_0^T D_T(c_e + (\lambda_o + \theta_e \mu^0)e^{rt})dt \\
\theta_e f(\bar{Z}) + \theta_d \int_0^{T_2} D_E(c_d + \theta_d e^{rT})dt &= \bar{Z} - Z_0
\end{aligned}$$

for $\bar{Z} > Z_2$ with

$$Z_2 = Z_0 + \theta_d \int_0^{1/r \ln(\frac{c_b - c_d}{c_b - (c_d + z)})} D_E(c_d + (c_b - (c_d + z))e^{rt})dt$$

and for $\bar{Z} > Z^*$, $h(\bar{Z}) = \frac{\bar{Z} - Z_0}{\theta_e} + X^*$. It is straightforward that $h(\bar{Z}) \geq \frac{\bar{Z} - Z_0}{\theta_e} + X^*$. And we define $g(\bar{Z})$, such that for $\bar{Z}_1 < \bar{Z} \leq Z^*$, with $\bar{Z}_1 = Z_0 + \theta_d \int_0^{1/r \ln(\frac{(c_b - c_d)\theta_e}{((c_b - c_e)\theta_d)})} D_E(c_d + \theta_d \frac{c_b - c_e}{\theta_e} e^{rt})dt$:

$$\begin{aligned}
c_e + \lambda_o + \theta_e \mu^0 &= c_d + \theta_d \mu^0 \\
c_e + (\lambda_o + \theta_e \mu^0)e^{rT_1} &= c_b \\
c_d + \theta_d e^{rT_2} &= c_b \\
g(\bar{Z}) &= \int_0^{T_1} D_T(c_e + (\lambda_o + \theta_e \mu^0)e^{rt})dt \\
\theta_e g(\bar{Z}) + \theta_d \int_0^{T_2} D_E(c_d + \theta_d e^{rt})dt &= \bar{Z} - Z_0
\end{aligned}$$

and for $\bar{Z} > Z^*$:

$$\begin{aligned}
c_e + \lambda_o + \theta_e \mu^0 &= c_d + \theta_d \mu^0 \\
c_e + (\lambda_o + \theta_e \mu^0) e^{rt_1} &= c_d + z + \theta_d \mu^0 e^{rt_1} \\
c_d + z + \theta_d e^{rt_2} &= c_b \\
c_d + \theta_d e^{rt_3} &= c_b \\
g(\bar{Z}) &= \int_0^{t_1} D_T(c_e + (\lambda_o + \theta_e \mu^0) e^{rt}) dt \\
\theta_e g(\bar{Z}) + \theta_d (\int_0^{t_3} D_E(c_d + \theta_d e^{rt}) dt + \int_{t_1}^{t_2} D_T(c_d + z + \theta_d e^{rt}) dt) &= \bar{Z} - Z_0
\end{aligned}$$

It is straightforward that $g(\bar{Z}) \leq \tilde{X}$.

8.1.2 Cases E: R_e more expensive than R_d in both sectors: $c_e > c_d + z > c_d$

If $c_e > c_d + z > c_d$ and $c_b > \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, then if the exhaustible resource is not exhausted, R_e is being used just before the switch date with solar in the energy sector. Let define Z^* and X^* such that R_e is not exhausted and the initial prices of R_e and R_d in the energy sector are equal:

$$\begin{aligned}
c_e + \theta_e \mu_o &= c_d + \theta_d \mu_o \\
c_e + \theta_e \mu_o e^{rT} &= c_b \\
X^* &= \int_0^T D_E(c_e + \theta_e \mu_o e^{rt}) dt + \int_0^T D_T(c_e + \theta_e \mu_o e^{rt}) dt \\
Z^* &= Z_0 + \theta_e X^*
\end{aligned}$$

Let define Z^{**} and X^{**} such that R_e is not exhausted and the initial prices of R_e and R_d in the transport sector are equal:

$$\begin{aligned}
c_e + \theta_e \mu_o &= c_d + \theta_d \mu_o + z \\
c_e + \theta_e \mu_o e^{rT} &= c_b \\
c_e + \theta_e \mu_o e^{rt_1} &= c_d + \theta_d \mu_o e^{rt_1} \\
X^{**} &= \int_{t_1}^T D_E(c_e + \theta_e \mu_o e^{rt}) dt + \int_0^T D_T(c_e + \theta_e \mu_o e^{rt}) dt \\
Z^{**} &= Z_0 + \theta_e X^{**} + \theta_d \int_0^{t_1} D_E(c_d + \theta_d \mu_o e^{rt}) dt
\end{aligned}$$

Define $f(\bar{Z})$, such that for $\bar{Z} \leq Z^*$, $f(\bar{Z}) = \frac{\bar{Z}-Z_0}{\theta_0}$ and for $Z^{**} > \bar{Z} > Z^*$:

$$\begin{aligned}
c_e + \theta_e \mu_o e^{rt_1} &= c_d + \theta_d \mu_o e^{rt_1} \\
c_e + \theta_e \mu_o e^{rT} &= c_b \\
f(\bar{Z}) &= \int_{t_1}^{T} D_E(c_e + \theta_e \mu_o e^{rt})dt + \int_0^T D_T(c_e + \theta_e \mu_o e^{rt})dt \\
\theta_e f(\bar{Z}) + \theta_d \int_0^{t_1} D_E(c_d + \theta_d \mu_o e^{rt})dt &= \bar{Z} - Z_0
\end{aligned}$$

and for $\bar{Z} > Z^{**}$, $f(\bar{Z}) = Z^{**}$ Define $\tilde{Z}$ such that lR_e is exhausted ; and the initial prices of R_e and R_d in the transport sector are equal ; and the final prices of R_e and R_d in the energy sector are equal:

$$\begin{aligned}
c_e + \lambda_o + \theta_e \mu_o &= c_d + z + \theta_d \mu_o \\
c_e + (\lambda_o + \theta_e \mu_o) e^{rT} &= c_d + \theta_d \mu_o e^{rT} \\
c_e + (\lambda_o + \theta_e \mu_o) e^{rT} &= c_b \\
\tilde{X} &= \int_0^T D_T(c_e + (\lambda_o + \theta_e \mu_o) e^{rt})dt \\
\tilde{Z} &= \theta_e X_1 + \theta_d \int_0^T D_E(c_d + \theta_d \mu_o e^{rt})dt
\end{aligned}$$

For $\bar{Z} < \tilde{Z}$ define $g(\bar{Z})$:

$$\begin{aligned}
c_e + (\lambda_o + \theta_e \mu_o) e^{rT} &= c_d + \theta_d \mu_o e^{rT} \\
c_e + (\lambda_o + \theta_e \mu_o) e^{rT} &= c_b \\
\bar{Z} &= Z_0 + \theta_e \int_0^T D_T(c_e + (\lambda_o + \theta_e \mu_o) e^{rt})dt + \theta_d \int_0^T D_E(c_d + \theta_d \mu_o e^{rt})dt \\
g(\bar{Z}) &= \int_0^T D_T(c_e + (\lambda_o + \theta_e \mu_o) e^{rt})dt
\end{aligned}$$

and for $\bar{Z} > \tilde{Z}$, then $g(\bar{Z}) = \tilde{Z}$. We define $\bar{Z}_2$ by:

$$\bar{Z}_2 = Z_0 + \theta_d \int_0^{1/r \ln(\frac{c_b - c_d}{c_b - (c_d + z)})} D_E(c_d + (c_b - (c_d + z)) e^{rt})dt$$

Define $h(\bar{Z})$, such that for $\bar{Z} < \tilde{Z}$

$$
\begin{aligned}
c_e + \lambda_o + \theta_e \mu_o &= c_d + z + \theta_d \mu_o \\
c_e + (\lambda_o + \theta_e \mu_o) e^{rT_1} &= c_b \\
c_d + \theta_d \mu_o e^{rT_2} &= c_b \\
\theta_e \int_0^{T_1} D_T(c_e + (\lambda_o + \theta_e \mu_o) e^{rt}) dt + \theta_d \int_0^{T_2} D_E(c_d + \theta_d \mu_o e^{rt}) dt &= \bar{Z} - Z_0 \\
h(\bar{Z}) &= \int_0^{T_1} D_T(c_e + (\lambda_o + \theta_e \mu_o) e^{rt}) dt
\end{aligned}
$$

and for $\bar{Z} > \tilde{Z}$

$$
\begin{aligned}
c_e + \lambda_o + \theta_e \mu_o &= c_d + z + \theta_d \mu_o \\
c_e + (\lambda_o + \theta_e \mu_o) e^{rt_1} &= c_d + \theta_d \mu_o e^{rt_1} \\
c_e + (\lambda_o + \theta_e \mu_o) e^{rT_2} &= c_b \\
c_d + \theta_d \mu_o e^{rT_1} &= c_b \\
h(\bar{Z}) &= \int_0^{T_2} D_T(c_e + (\lambda_o + \theta_e \mu_o) e^{rt}) dt + \int_{t_1}^{T_1} D_E(c_e + (\lambda_o + \theta_e \mu_o) e^{rt}) dt \\
\theta_e h(\bar{Z}) + \theta_d \int_0^{t_1} D_E(c_d + \theta_d \mu_o e^{rt}) dt &= \bar{Z} - Z_0
\end{aligned}
$$

We can show that $h(Z^{**}) = X^{**}$

If $c_e > c_d + z > c_d$ and $\frac{\theta_d c_e - \theta_e (c_d + z)}{\theta_d - \theta_e} < c_b < \frac{\theta_d c_e - \theta_e c_d}{\theta_d - \theta_e}$, then define X^* and Z^* such that:

$$
\begin{aligned}
c_e + \theta_e \mu^0 &= c_d + z + \theta_d \mu^0 \\
c_e + \theta_e \mu^0 e^{rt_1} &= c_b \\
c_d + \theta_d \mu^0 e^{rt_2} &= c_b \\
X^* &= \int_0^{t_1} D_T(c_e + \theta_e \mu^0 e^{rt}) dt \\
Z^* &= Z_0 + \theta_e X^* + \theta_d \int_0^{t_2} D_E(c_d + \theta_d \mu^0 e^{rt}) dt
\end{aligned}
$$

And define $h(\bar{Z})$, for $\bar{Z} < Z^*$ and $\bar{Z} > Z_1$, with $Z_1 = Z_0 + \int_0^{1/r \ln(\frac{(c_b - c_b)\theta_e}{(c_b - c_e)\theta_d})} D_E(c_d + \frac{\theta_d (c_b - c_e)}{\theta_e} e^{rt}) dt$

by:

$$\begin{aligned}
c_e + \theta_e \mu^0 e^{rt_1} &= c_b \\
c_d + \theta_d \mu^0 e^{rt_2} &= c_b \\
h(\bar{Z}) &= \int_0^{t_1} D_T(c_e + \theta_e \mu^0 e^{rt})dt \\
\bar{Z} &= Z_0 + \theta_e h(\bar{Z}) + \theta_d \int_0^{t_2} D_E(c_d + \theta_d \mu^0 e^{rt})dt
\end{aligned}$$

And for $\bar{Z} > Z^*$, $h(\bar{Z}) = X^*$ Then if $X \geq h(\bar{Z})$, the exhaustible resource is not exhausted. If $\bar{Z} < Z^*$ and $X \geq h(\bar{Z})$, the exhaustible resource is used in the transport sector and R_d in the energy sector. If $\bar{Z} > Z^*$ and $X \geq h(\bar{Z})$, then only R_d is used in the energy sector and R_d then the exhaustible resource is used in the transport sector. If $X < h(\bar{Z})$ then the exhaustible resource is exhausted and the exhaustible resource the R_d is used in the transport sector, only R_d is used in the energy sector.

If $c_e > c_d + z > c_d$ and $\frac{\theta_d c_e - \theta_e(c_d+z)}{\theta_d - \theta_e} > c_b$, then the exhaustible resource, is never used.

8.2 Proof of Results

8.2.1 Case C2 and E4. Only R_e is used in transport sector; R_e then R_d are used in power sector

The solution is given by:

$$\begin{aligned}
c_e + \lambda_e^0 e^{rt_1} + \theta_e \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} && (8.1) \\
c_e + \lambda_e^0 e^{rt_2} + \theta_e \mu^0 e^{rt_2} &= c_b && (8.2) \\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b && (8.3) \\
\int_0^{t_2} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt})dt & && (8.4) \\
+ \int_0^{t_1} D_E(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt})dt &= x_e^0 && (8.5) \\
\theta_e x_e + \theta_d \int_{t_1}^{\underline{t}} D_E(c_d + \theta_d \mu^0 e^{rt})dt &= \overline{Z} - Z^0 && (8.6)
\end{aligned}$$

Differentiating the previous system with respect to $\overline{Z}$,we get that $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$1 - (\theta_d - \theta_e)\frac{\mu^0}{\lambda_o^0} - \frac{\theta_d}{\theta_e}\frac{D_E(p(t_1))}{D_T(p(0)) + D_E(p(0))} \tag{8.7}$$

It is the case that $\frac{D_E(p(t_1))}{D_T(p(0))+D_E(p(0))} > \frac{D_E(c_b)}{D_T(c_e)+D_E(c_e)}$, so that $\exists\theta^* > 1$ such that if $\frac{\theta_d}{\theta_e} > \theta^*$, then $\frac{d\lambda_e^0}{d\overline{Z}} < 0$.

Using that $(\theta_d - \theta_e)\frac{\mu_o^0}{\lambda^0} = 1 - \frac{c_d - c_e}{\lambda_o e^{rt_1}}$, we get that $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$\frac{c_d - c_e}{\lambda_o e^{rt_1}} - \frac{\theta_d}{\theta_e}\frac{D_E(p(t_1))}{D_T(p(0)) + D_E(p(0))} \tag{8.8}$$

But $\lambda_o e^{rt_1} > (\theta_d - \theta_e)\mu_o e^{rt_1}$ And $\underline{t} - t_1$ and $\mu^0 e^{rt_1}$ are defined by

$$\begin{aligned} \theta_d\mu^0 e^{rt_1} e^{r(\underline{t}-t_1)} &= c_b - c_d & (8.9)\\ \theta_e X_e + \theta_d \int_{\underline{t}-t_1}^{\underline{t}} D_E(c_d + \theta_d\mu^0 e^{rt_1} e^{ru})du &= \overline{Z} - Z^0 & (8.10)\end{aligned}$$

we have the results on extraction costs.

8.2.2 Case C3, D5 and E6. Only R_e is used in transport sector; R_d in power sector.

The solution is given by:

$$\begin{aligned} c_e + \lambda_e^0 e^{rt_1} + \theta_e\mu^0 e^{rt_1} &= c_b & (8.11)\\ c_d + \theta_d\mu^0 e^{r\underline{t}} &= c_b & (8.12)\\ \int_0^{t_1} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e\mu^0 e^{rt})dt &= x_e^0 & (8.13)\\ \theta_e x_e + \theta_d \int_0^{\underline{t}} D_E(c_d + \theta_d\mu^0 e^{rt})dt &= \overline{Z} - Z^0 & (8.14)\end{aligned}$$

Differentiating the previous system with respect to $\overline{Z}$, it comes that:

$$\frac{d\lambda_e^0}{d\overline{Z}} > 0.$$

8.2.3 Case C5 and D6: R_e then R_d in transport sector; only R_d is used in power sector (this case implies that the exhaustible resource is exhausted).

For X_e^0 low enough, extraction is necessarily as described by this case. The solution is given by:

$$\begin{aligned}
c_e + \lambda_e^0 e^{rt_1} + \theta_e \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} + z && (8.15)\\
c_d + \theta_d \mu^0 e^{rt_2} + z &= c_b && (8.16)\\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b && (8.17)\\
\int_0^{t_1} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt})dt &= X_e^0 && (8.18)\\
\theta_e X_e + \theta_d \int_{t_1}^{t_2} D_T(c_d + \theta_d \mu^0 e^{rt} + z)dt & && (8.19)\\
+\theta_d \int_0^{\underline{t}} D_E(c_d + \theta_d \mu^0 e^{rt})dt &= \overline{Z} - Z^0 && (8.20)
\end{aligned}$$

Differentiating the previous system with respect to $\overline{Z}$ we get that $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$1 - (\theta_d - \theta_e)\frac{\mu_o}{\lambda_o} - \frac{\theta_d}{\theta_e}\frac{D_T(t_1)}{D_T(0)}$$

So that the propositions of the first section continue to hold.

8.2.4 Case C4 ; R_e then R_d in both sectors

The solution is given by:

$$\begin{aligned}
c_e + \lambda_e^0 e^{rt_1} + \theta_e \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} && (8.21)\\
c_e + \lambda_e^0 e^{rt_2} + \theta_e \mu^0 e^{rt_2} &= c_d + \theta_d \mu^0 e^{rt_2} + z && (8.22)\\
c_d + \theta_d \mu^0 e^{rt_3} + z &= c_b && (8.23)\\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b && (8.24)\\
\int_0^{t_1} D_E(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) & && (8.25)\\
+\int_0^{t_2} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt})dt &= x_e^0 && (8.26)\\
\theta_e x_e + \theta_d \int_{t_1}^{\underline{t}} D_E(c_d + \theta_d \mu^0 e^{rt})dt & && (8.27)\\
+\theta_d \int_{t_2}^{t_3} D_T(c_d + \theta_d \mu^0 e^{rt} + z)dt &= \overline{Z} - Z^0 && (8.28)
\end{aligned}$$

Differentiating the previous system with respect to $\overline{Z}$ we get that $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$1 - \frac{\theta_d}{\theta_e}\frac{D_E(t_1) + D_T(t_2)}{D_E(0) + D_T(0)} - (\theta_d - \theta_e)\frac{\mu^0}{\lambda_o^0}$$

Remark that : $\frac{d\lambda_e^0}{d\overline{Z}}$ has then the sign of:

$$-\frac{\theta_d}{\theta_e}\frac{D_E(t_1) + D_T(t_2)}{D_E(0) + D_T(0)} - \frac{c_d - c_e}{\lambda_e e^{rt_1}}$$

But $\lambda_e e^{rt_1} > (\theta_d - \theta_e)\mu_0 e^{rt_1}$. And and it is straightfoward that $\mu^0 e^{rt_1} > \mu_*^0 e^{rt_1}$, with $\mu_*^0 e^{rt_1}$ solution of:

$$\begin{aligned} \theta_d \mu_*^0 e^{rt_1} e^{r(\underline{t} - t_1)} &= c_b - c_d \\ \theta_e X_e + \theta_d \int_{\underline{t}-t_1}^{\underline{t}} D_E(c_d + \theta_d \mu_*^0 e^{rt_1} e^{ru}) du &= \overline{Z} - Z^0 \end{aligned}$$

which do not depend on c_e.

8.2.5 Cases D4 and E4: only R_e is used in transport sector; R_d then R_e are used in power sector.

The solution is given by:

$$c_e + \lambda_e^0 e^{rt_1} + \theta_e \mu^0 e^{rt_1} = c_d + \theta_d \mu^0 e^{rt_1} \tag{8.29}$$

$$c_e + \lambda_e^0 e^{r\underline{t}} + \theta_e \mu^0 e^{r\underline{t}} = c_b \tag{8.30}$$

$$\int_{t_1}^{\underline{t}} D_E(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt \tag{8.31}$$

$$+ \int_0^{\underline{t}} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt = x_e^0 \tag{8.32}$$

$$\theta_e x_e + \theta_d \int_0^{t_1} D_E(c_d + \theta_d \mu^0 e^{rt}) dt = \overline{Z} - Z^0 \tag{8.33}$$

Differentiating the previous system with respect to $\overline{Z}$ $\frac{d\lambda_e^0}{d\overline{Z}}$ has the sign of:

$$\frac{c_e - c_d}{\lambda_o e^{rt_1}} - \frac{\theta_d}{\theta_e}\frac{D_E(t_1)}{D_T(0)}$$

8.2.6 Case E5. R_d then R_e in transport sector, R_d then R_e in power sector.

Writing t^* is the switch date from R_d to the exhaustible resource in the transport sector, the solution is given by:

$$
\begin{aligned}
c_e + \lambda_e^0 e^{rt^*} + \theta_e \mu^0 e^{rt^*} &= c_d + \theta_d \mu^0 e^{rt^*} + z && (8.34)\\
c_e + \lambda_e^0 e^{rt_1} + \theta_e \mu^0 e^{rt_1} &= c_d + \theta_d \mu^0 e^{rt_1} && (8.35)\\
c_e + \lambda_e^0 e^{r\underline{t}} + \theta_e \mu^0 e^{r\underline{t}} &= c_b && (8.36)\\
\int_{t_1}^{\underline{t}} D_E(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) & && (8.37)\\
+ \int_{t^*}^{\underline{t}} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt &= x_e^0 && (8.38)\\
\theta_e x_e + \theta_d \int_0^{t_1} D_E(c_d + \theta_d \mu^0 e^{rt}) dt & && (8.39)\\
+\theta_d \int_0^{t^*} D_T(c_d + \theta_d \mu^0 e^{rt}) dt &= \overline{Z} - Z^0 && (8.40)
\end{aligned}
$$

Differentiating the previous system with respect to $\overline{Z}$ $\frac{d\lambda_e^0}{d\overline{Z}} < 0$, tightening the carbon regulation will increase profit of the exhaustible resource owners.

8.2.7 Case E7. R_d then R_e in transport sector, R_d in power sector.

The solution is given by:

$$
\begin{aligned}
c_e + \lambda_e^0 e^{rt^*} + \theta_e \mu^0 e^{rt^*} &= c_d + \theta_d \mu^0 e^{rt^*} + z && (8.41)\\
c_e + \lambda_e^0 e^{rt_2} + \theta_e \mu^0 e^{rt_2} &= c_b && (8.42)\\
c_d + \theta_d \mu^0 e^{r\underline{t}} &= c_b && (8.43)\\
\int_{t^*}^{t_2} D_T(c_e + \lambda_e^0 e^{rt} + \theta_e \mu^0 e^{rt}) dt &= x_e^0 && (8.44)\\
\theta_e x_e + \theta_d \int_0^{\underline{t}} D_E(c_d + \theta_d \mu^0 e^{rt}) dt & && (8.45)\\
+\theta_d \int_0^{t^*} D_T(c_d + \theta_d \mu^0 e^{rt}) dt &= \overline{Z} - Z^0 && (8.46)
\end{aligned}
$$

Differentiating the previous system with respect to $\overline{Z}$ we get that $\frac{d\lambda_e^0}{d\overline{Z}} < 0$.

9 Appendix of Section 5

9.1 Ordering the extraction of resources

9.1.1 The dirty backstop is the most expensive to extract

Discussion of Lemma 20

If $\theta_1 \leq \theta_2$ and if resource 1 is not exhausted, the price of resource 1 is strictly lower than the prices of resource 2 and the dirty backstop over the whole extraction path. Thus, before using resource 2 or the dirty backstop, one must first exhaust resource 1. It comes that resource 1 is necessarily used, and is exhausted iff. $\overline{Z} \geq \theta_1 X_1^0 + Z^0$. Assume now that $\theta_1 > \theta_2$ but $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$. Condition $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$ is equivalent to $p_1(\underline{t}) < p_2(\underline{t})$ with $p_1(\underline{t}) = c_b$ if both resources 1 and 2 are not exhausted. It follows that if $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$ and $\theta_1 > \theta_2$, for $\overline{Z}$ close enough to Z^0, only resource 1 is used. Increasing $\overline{Z}$ allows to consume more of resource 1, and keeps its price lower than resource 2 price, thus if resource 2 is used, resource 1 must be exhausted. Finally, if $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$ and $\theta_1 > \theta_2$, or if $\theta_1 \leq \theta_2$, and $\overline{Z} < \theta_1 X_1^0 + Z^0$, only resource 1 is used (case $F5$). Since resource 2 is less polluting than the dirty backstop and cheaper to extract, before turning to the extraction of the dirty backstop, one must exhaust first resource 2, and we get directly that if $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$ and $\theta_1 > \theta_2$, or if $\theta_1 \leq \theta_2$, resource 1 is exhausted and resource 2 is used but not exhausted (case $F2$) iff. $\theta_1 X_1^0 < \overline{Z} - Z^0 < \theta_1 X_1^0 + \theta_2 X_2^0$.

Assume now that $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$ and $\theta_1 > \theta_2$. For $\overline{Z}$ close enough to Z^0, only resource 2 is used. Increasing $\overline{Z}$ leads to start extracting resource 1 at some point. Using similar reasoning as in proof of Lemma 11 in Section 3, there exists X_2^0 defined by $X_2^* = \int_{p^*}^{c_b} D(c_2 + (p^* - c_2)e^{rt})dp$ such that p^* is the switch price from resource 2 to resource 1 if both resources are not exhausted. If $X_2^0 < X_2^*$, resource 2 is exhausted before using resource 1 when $\overline{Z}$ is increased. We call $Z^* = \theta_2 X_2^* + Z^0$. If $X_2^0 < X_2^*$, for $\overline{Z} > Z^*$, resource 2 gets exhausted iff. $\overline{Z} > \theta_2 X_2^0 + Z^0$ and both resource 1 and 2 get exhausted iff. $\overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$. If $X_2^0 > X_2^*$, resource 2 is not exhausted when resource 1 is used but not exhausted. Both resources are used and not exhausted iff. $\theta_2 X_2^* + Z^0 < \overline{Z} < \theta_1 X_1^0 + \theta_2 X_2^* + Z^0 = Z^{**}$. Resource 1 gets exhausted iff. $\overline{Z} > \theta_1 X_1^0 + \theta_2 X_2^* + Z^0 = Z^{**}$. Increasing $\overline{Z}$ over Z^{**}, leads to extract more of resource 2, until getting it exhausted for $\overline{Z} = \theta_1 X_1^0 + \theta_2 X_2^0 + Z^0$. For a larger $\overline{Z}$, the dirty backstop starts to be used (case $F1$).

9.1.2 Intermediate cost for the dirty backstop, $c_1 < c_d < c_2$

Discussion over Lemma 21

Condition $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} > c_b$ implies that the dirty backstop is preferred to resource 2 on the whole extraction path, thus resource 2 is never used. The characterization of the paths is thus similar to the case of Section 3 where only one exhaustible resource is available, and is more expensive but less polluting than the dirty backstop.

If $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$, if resource 2 was abundant, its final price would be lower than the final price of the dirty backstop, thus for a relatively low carbon ceiling, resource 2 is necessarily used if the dirty backstop is used. If $\theta_1 < \theta_2$ or ($\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} > c_b$), for $\overline{Z}$ close enough to Z^0, resource 1 is used, and resource 1 must be exhausted before turning to extract resource 2 when $\overline{Z}$ gets larger than $Z^* = \theta_1 X_1^0 + Z^0$. If resource 1 is used, its consumption increases when $\overline{Z}$ increases until being exhausted. This is due to the fact that this resource is the cheapest to extract. In that case, for $\overline{Z} \leq \theta_1 X_1^0 + Z^0$, only resource 1 is used, and if $\overline{Z} > \theta_1 X_1^0 + Z^0$, resource 2 is used after resource 1 gets exhausted. When increasing $\overline{Z}$, at some point the dirty backstop starts to be extracted. Resource 2 can be exhausted or not before turning to the dirty backstop extraction cost. To determine the conditions to get resource 2 exhausted after using the dirty backstop through time, let us remark that the situation is very close to situation described in Section 3 when the exhaustible resource is more expensive than the dirty backstop. The difference is simply that a quantity $\theta_1 X_1^0$of pollution is necessarily added to the initial carbon stock Z^0 before using resource 2. Considering the value of the carbon stock when the backstop starts to be used, $\theta_1 X_1^0 + Z^0$ as an initial carbon stock of a new extraction problem starting at date t_1, following reasoning exposed in Section 3, proof of Lemma 11, we get that $\exists Z^{**}, Z^{***}$, $Z^{**} < Z^{***}$, such that if $Z^{**} > \overline{Z} > Z^*$, and $X_2^0 > \frac{\overline{Z} - Z^0 - \theta_1 X_1}{\theta_2}$, resource 1 is exhausted and resource 2 is used but not exhausted (case $G3$); if $\overline{Z} > Z^*$, and $X_2^0 < min(\frac{\overline{Z} - Z^0 - \theta_1 X_1}{\theta_2}, \frac{Z^{**} - Z^0 - \theta_1 X_1}{\theta_2})$, resource 1 and 2 are exhausted and the dirty backstop is used (case $G1$); if $\overline{Z} > Z^{**}$ and $X_2^0 > \frac{Z^{**} - Z^0 - \theta_1 X_1}{\theta_2}$, resource 1 is exhausted, and resource 2 is used but not exhausted, the dirty backstop is used (case $G2$).

Now let assume that $\frac{\theta_d c_2 - \theta_2 c_d}{\theta_d - \theta_2} < c_b$ and $\theta_1 > \theta_2$ and $\frac{\theta_1 c_2 - \theta_2 c_1}{\theta_1 - \theta_2} < c_b$. For $\overline{Z}$ close enough to Z^0, only resource 2 is used. Increasing $\overline{Z}$ from Z^0, it comes that resource 1 must be used at some point. Resource 2 is exhausted before starting to use resource 1 when $\overline{Z}$ increases iff. $X_2^0 < X_2^*$, where X_2^* represents the maximum quantity of resource 2 that can be used before its initial price equals the price of resource 1 if both resources were abundant. $X_2^* = \int_{p^*}^{c_b} D()dp$ where p^* would be the switch price from resource 1 to 2 if both were abundant. It follows that, if $X_2^0 < X_2^*$: resource 2 is exhausted iff. $\overline{Z} \geq \theta_2 X_2^0 + Z^0$; resource 1 is used and not exhausted and resource 2 is exhausted iff.

$\theta_1 X_1^0 + \theta_2 X_2^0 + Z^0 > \overline{Z} > \theta_2 X_2^0 + Z^0$; both resources 1 and 2 are exhausted and the dirty backstop is used iff. $\theta_1 X_1^0 + \theta_2 X_2^0 + Z^0 < \overline{Z}$. If $X_2^0 > X_2^*$, resource 2 is not exhausted when $\overline{Z}$ is large enough to start using resource 1: only resource 2 is used iff. $\overline{Z} < \theta_2 X_2^* + Z^0 \equiv Z^*$; resource 1 is used but not exhausted and resource 2 is used but not exhausted iff. $Z^{**} \equiv \theta_1 X_1^0 + \theta_2 X_2^* + Z^0 > \overline{Z} > \theta_2 X_2^* + Z^0 \equiv Z^*$. For $\overline{Z} \geq Z^{**}$, resource 1 is exhausted. Let us define X_2^{**}, the maximum quantity of resource 2 that could be used if this resource was abundant. X_2^{**} writes $X_2^{**} = \int_{p^{**}}^{c_b} D()dp$ where $p^{*}*$ would be the switch price from resource 2 to the dirty backstop if resource 2 was abundant.. For $Z^{**} \equiv \theta_1 X_1^0 + \theta_2 X_2^* + Z^0 < \overline{Z} < \theta_1 X_1^0 + \theta_2 X_2^{**} + Z^0 \equiv Z^{***}$ and $\overline{Z} < \theta_1 X_1^0 + \theta_2 X_2 + Z^0$, resource 1 is exhausted and resource 2 is used but not exhausted, the dirty backstop unused. Let assume that $X_2^{**} > X_2^0 > X_2^*$, we get that both resources 1 and 2 are exhausted and the dirty backstop used iff. $Z^{**} \equiv \theta_1 X_1^0 + \theta_2 X_2 + Z^0 < \overline{Z}$. Let assume that $X_2^{**} < X_2^0$, resource 2 cannot be exhausted, and resource 1 gets exhausted and resource 2 is used but not exhausted and the dirty backstop used iff. $\theta_1 X_1^0 + \theta_2 X_2^{**} + Z^0 < \overline{Z}$.

9.2 General results

9.2.1 Preliminary remarks over profits of resources 1 and 2 when $c_1 < c_2 < c_d$

Detailed calculus to get $\frac{d\lambda_1^0}{d\overline{Z}}$, $\frac{d\lambda_2^0}{d\overline{Z}}$ and $\frac{d\lambda_1^0}{d\overline{Z}} - \frac{d\lambda_2^0}{d\overline{Z}}$:

Proof of Lemma 22

Proof. $\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of:

$$X = -r(\lambda_1^0 - \lambda_2^0 + (\theta_1 - \theta_2)\mu^0)\theta_d(-\textstyle\int_0^{t_1} D'()e^{rt}dt)D(t_2) - \theta_d D(t_2)D(t_1)$$
$$+\theta_2\left(r(\lambda_1^0 - \lambda_2^0 + (\theta_1 - \theta_2)\mu^0)(-\textstyle\int_0^{t_1} D'()e^{rt}dt)D(t_2)\right)$$
$$+\theta_2\left(r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)(-\textstyle\int_0^{t_1} D'()e^{rt}dt)D(t_1) + D(t_2)D(t_1)\right)$$
$$+\theta_2\left(r(\lambda_1^0 - \lambda_2^0 + (\theta_1 - \theta_2)\mu^0)r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)(-\textstyle\int_0^{t_1} D'()e^{rt}dt)(-\int_{t_1}^{t_2} D'()e^{rt}dt)\right)$$
$$+\theta_2\left(r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)D(t_1)(-\textstyle\int_{t_1}^{t_2} D'()e^{rt}dt)\right)$$

Remarking that $\dot{p} = r(\lambda_1^0 + \theta_1)\mu^0 e^{rt}$ over $[0; t_1]$ and $\dot{p} = r(\lambda_2^0 + \theta_2)\mu^0 e^{rt}$ over $[t_1; t^1]$, it comes that $\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of:

$$X = r(\lambda_2^0 + \theta_2\mu^0)(\theta_d - \theta_2)(-\textstyle\int_0^{t_1} D'(p(t))e^{rt}dt)D(t_2) - D(t_2)(\theta_d - \theta_2)D(0)$$
$$+\theta_2\left(-D(t_2)D(t_1) + r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)(-\textstyle\int_0^{t_1} D'(p(t))e^{rt}dt)D(t_1)\right)$$
$$+\theta_2\left(Y - r\theta_d\mu^0 D(t_1)(-\textstyle\int_{t_1}^{t_2} D'(p(t))e^{rt}dt) + D(t_1)D(t_1)\right)$$

where

$Y = r(\lambda_1^0 - \lambda_2^0 + (\theta_1 - \theta_2)\mu^0)r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)(-\int_0^{t_1} D'(p(t))e^{rt}dt)(-\int_{t_1}^{t_2} D'(p(t))e^{rt}dt)$

We get

$Y = (D(0) - D(t_1))(D(t_1) - D(t_2))$

$-r\theta_d\mu^0(D(0) - D(t_1))(-\int_{t_1}^{t_2} D'()e^{rt}dt)$

$-r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)(-\int_0^{t_1} D'()e^{rt}dt)(D(t_1) - D(t_2))$

we thus get:

$X = r(\lambda_2^0 + \theta_2\mu^0)(\theta_d - \theta_2)(-\int_0^{t_1} D'(p(t))e^{rt}dt)D(t_2)$

$-D(t_2)\theta_d D(0) + \theta_2 D(0)(D(t_1))$

$-\theta_2 r\theta_d\mu^0 D(0)(-\int_{t_1}^{t_2} D'()e^{rt}dt)$

$+\theta_2 r(\lambda_2^0 + (\theta_2 - \theta_d)\mu^0)(-\int_0^{t_1} D'()e^{rt}dt)D(t_2)$

Finally,

$X = D(0)(-D(t_2)\theta_d + \theta_2 D(t_1)) + r\lambda_2^0\theta_d(-\int_0^{t_1} D'()e^{rt}dt)D(t_2)$

$-\theta_2 r\theta_d\mu^0 D(0)(-\int_{t_1}^{t_2} D'()e^{rt}dt)$

$\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of:

$1 - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} - \frac{\lambda_2^0}{\lambda_1^0+\theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)} - \frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1 - \frac{D(t_2)}{D(t_1)})$

that we can also rewrite into

$1 - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} - \frac{\lambda_2^0}{\lambda_1^0+\theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)} - \frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1 - \frac{D(t_2)}{D(t_1)})$

$1 + \theta_2\frac{\mu^0}{\lambda_2^0} - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2}\frac{(\lambda_2^0+\theta_2\mu^0)}{\lambda_2^0} - \frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)} - \frac{\theta_d\mu^0}{\lambda_2^0}(1 - \frac{D(t_2)}{D(t_1)})$

Finally, $\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of: $1 + (\theta_2 - \theta_d)\frac{\mu^0}{\lambda_2^0} - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2} - \frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0}\frac{\theta_d}{\theta_2}(1 - \frac{D(0)}{D(t_1)})\frac{D(t_2)}{D(0)}$

and $1 - \frac{D(t_2)}{D(t_1)}\frac{\theta_d}{\theta_2}$ negative is still a sufficient condition to get $\frac{d\lambda_2^0}{d\overline{Z}}$ negative.

□

Proof of Lemma 23

Proof. Remarking that $\dot{p} = r(\lambda_1^0 + \theta_1)\mu^0 e^{rt}$ over $[0; t_1]$ and $\dot{p} = r(\lambda_2^0 + \theta_2)\mu^0 e^{rt}$ over $[t_1; t^1]$, it comes that $\frac{d\lambda_1^0}{d\overline{Z}}$ has the sign of:

$-D(t_1)(\theta_d D(t_2) - \theta_1 D(0)) - r\theta_d\theta_1\mu^0(-\int_0^{t_1} D'()e^{rt}dt)D(t_2) - r\theta_1\theta_d\mu^0 D(0)(-\int_{t_1}^{t_2} D'(p(t))e^{rt}dt)$

and finally, $\frac{d\lambda_1^0}{d\overline{Z}}$ has the sign of:

$1 - \frac{\theta_d D(t_2)}{\theta_1 D(0)} - \frac{\theta_d\mu^0}{\lambda_1^0+\theta_1\mu^0}(1 - \frac{D(t_1)}{D(0)}) - \frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1 - \frac{D(t_2)}{D(t_1)})$

Detailed calculus:

$\frac{d\lambda_1^0}{d\overline{Z}}$ has the sign of:

$W = -e^{rt_1}e^{rt_2}\theta_d D(t_2)D(t_1)$

$+\theta_1\left(r(\lambda_1^0-\lambda_2^0+(\theta_1-\theta_2)\mu^0)e^{rt_1}e^{rt_2}(-\int_0^{t_1}D'()e^{rt}dt)D(t_2)\right)$

$+\theta_1\left(e^{rt_1}r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)e^{rt_2}(-\int_0^{t_1}D'()e^{rt}dt)D(t_1)+e^{rt_1}e^{rt_2}D(t_2)D(t_1)\right)$

$+\theta_1\left(r(\lambda_1^0-\lambda_2^0+(\theta_1-\theta_2)\mu^0)e^{rt_1}r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)e^{rt_2}(-\int_0^{t_1}D'()e^{rt}dt)(-\int_{t_1}^{t_2}D'()e^{rt}dt)\right)$

$+\theta_1\left(e^{rt_1}r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)e^{rt_2}D(t_1)(-\int_{t_1}^{t_2}D'()e^{rt}dt)\right)$

Remarking that $\dot{p}=r(\lambda_1^0+\theta_1)\mu^0e^{rt}$ over $[0;t_1]$ and $\dot{p}=r(\lambda_2^0+\theta_2)\mu^0e^{rt}$ over $[t_1;t^1]$, it comes that $\frac{d\lambda_1^0}{d\overline{Z}}$ has the sign of:

$X=-\theta_dD(t_2)D(t_1)$

$+\theta_1\left(-r(\lambda_2^0+\theta_2\mu^0)(-\int_0^{t_1}D'()e^{rt}dt)D(t_2)\right)$

$+\theta_1\left((D(0)-D(t_1))D(t_2)+r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)(-\int_0^{t_1}D'()e^{rt}dt)D(t_1)+D(t_2)D(t_1)\right)$

$+\theta_1\left(Y+D(t_1)(D(t_1)-D(t_2))-r\theta_d\mu^0D(t_1)(-\int_{t_1}^{t_2}D'()e^{rt}dt)\right)$

where

$Y=r(\lambda_1^0-\lambda_2^0+(\theta_1-\theta_2)\mu^0)r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)(-\int_0^{t_1}D'()e^{rt}dt)(-\int_{t_1}^{t_2}D'()e^{rt}dt)$

We get $Y=(D(0)-D(t_1))(D(t_1)-D(t_2))$

$-r(\lambda_1^0+\theta_1\mu^0)r\theta_d\mu^0(-\int_0^{t_1}D'()e^{rt}dt)(-\int_{t_1}^{t_2}D'()e^{rt}dt)$

$-r(\lambda_2^0+\theta_2\mu^0)r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)(-\int_0^{t_1}D'()e^{rt}dt)(-\int_{t_1}^{t_2}D'()e^{rt}dt)$

then,

$Y=(D(0)-D(t_1))(D(t_1)-D(t_2))$

$-r\theta_d\mu^0(D(0)-D(t_1))(-\int_{t_1}^{t_2}D'()e^{rt}dt)$

$-r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)(-\int_0^{t_1}D'()e^{rt}dt)(D(t_1)-D(t_2))$

thus

$X=D(t_1)(\theta_1D(0)-\theta_dD(t_2))-\theta_1r\theta_d\mu^0D(0)(-\int_{t_1}^{t_2}D'()e^{rt}dt)$

$-\theta_1r\theta_d\mu^0D(t_2)(-\int_0^{t_1}D'()e^{rt}dt)$

Finally, $\frac{d\lambda_1^0}{d\overline{Z}}$ has the sign:

$1-\frac{\theta_d}{\theta_1}\frac{D(t_2)}{D(0)}-\frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1-\frac{D(t_2)}{D(t_1)})-\frac{\theta_d\mu^0}{\lambda_1^0+\theta_1\mu^0}\frac{D(t_2)}{D(t_1)}(1-\frac{D(t_*)}{D(0)})$

A sufficient condition to get $\frac{d\lambda_1^0}{d\overline{Z}}<0$ is $1<\frac{\theta_d}{\theta_1}\frac{D(t_2)}{D(0)}$

□

Proof of Lemma 32

Proof. $\frac{d\lambda_1^0}{d\overline{Z}}-\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of:

$W=r(\lambda_1^0-\lambda_2^0+(\theta_1-\theta_2)\mu^0)\theta_d(-\int_0^{t_1}D'()e^{rt}dt)D(t_2)$

$+(\theta_1-\theta_2)\left(r(\lambda_1^0-\lambda_2^0+(\theta_1-\theta_2)\mu^0)(-\int_0^{t_1}D'()e^{rt}dt)D(t_2)\right)$

$+(\theta_1-\theta_2)\left(r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)(-\int_0^{t_1} D'()e^{rt}dt)D(t_1)+D(t_2)D(t_1)\right)$
$+(\theta_1-\theta_2)\left(Y+r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)D(t_1)(-\int_{t_1}^{t_2} D'()e^{rt}dt)\right)$
$W=\theta_d(D(0)-D(t_1))D(t_2)$
$-r(\lambda_2^0+\theta_2\mu^0)\theta_d(-\int_0^{t_1} D'()e^{rt}dt)D(t_2)$
$+(\theta_1-\theta_2)\left((D(0)-D(t_1))D(t_2)-r(\lambda_2^0+\theta_2\mu^0)(-\int_0^{t_1} D'()e^{rt}dt)D(t_2)\right)$
$+(\theta_1-\theta_2)\left(r(\lambda_2^0+(\theta_2-\theta_d)\mu^0)(-\int_0^{t_1} D'()e^{rt}dt)D(t_1)+D(t_2)D(t_1)\right)$
$+(\theta_1-\theta_2)\left(Y-r\theta_d\mu^0D(t_1)(-\int_{t_1}^{t_2} D'()e^{rt}dt)+D(t_1)(D(t_1)-D(t_2))\right)$
$W=\theta_d(D(0)-D(t_1))D(t_2)+(\theta_1-\theta_2)D(0)D(t_1)$
$-r(\lambda_2^0+\theta_2\mu^0)\theta_d(-\int_0^{t_1} D'()e^{rt}dt)D(t_2)$
$-(\theta_1-\theta_2)r\theta_d\mu^0D(0)(-\int_{t_1}^{t_2} D'()e^{rt}dt)$
$-(\theta_1-\theta_2)r\theta_d\mu^0(-\int_0^{t_1} D'()e^{rt}dt)D(t_2)$
$W=\theta_d(D(0)-D(t_1))D(t_2)+(\theta_1-\theta_2)D(0)D(t_1)$
$-\frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0}\theta_d(D(0)-D(t_1))D(t_2)$
$-(\theta_1-\theta_2)\frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}D(0)(D(t_1)-D(t_2))$
$-(\theta_1-\theta_2)\frac{\theta_d\mu^0}{\lambda_1^0+\theta_1\mu^0}(D(0)-D(t_1))D(t_2)$
$W=\theta_d(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}+\theta_1-\theta_2$
$-\frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0}\theta_d(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}$
$-(\theta_1-\theta_2)\frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1-\frac{D(t_2)}{D(t_1)})$
$-(\theta_1-\theta_2)\frac{\theta_d\mu^0}{\lambda_1^0+\theta_1\mu^0}(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}$
Finally, $\frac{d\lambda_1^0}{d\overline{Z}}-\frac{d\lambda_2^0}{d\overline{Z}}$ has the sign of:
$\theta_d(1-\frac{\lambda_2^0+\theta_2\mu^0}{\lambda_1^0+\theta_1\mu^0})(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}+(\theta_1-\theta_2)\left(1-\frac{\theta_d\mu^0}{\lambda_2^0+\theta_2\mu^0}(1-\frac{D(t_1)}{D(0)})\frac{D(t_2)}{D(t_1)}\right)$

□