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Asymmetry, optimal transfers and international environmental agreements*

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This paper applies optimal sharing rules to a coalition formation game with positive externalities, demonstrating the effectiveness of well-designed transfer schemes in improving outcomes for participation in International Environmental Agreements. A numerical exercise is conducted, providing proof of the failure of the conventional transfer schemes (Shapley value, Nash bargaining solution, Chander Tulkens transfer scheme) to meet the existence, robustness and optimality conditions set by the optimal sharing rule literature. The core result is derived from a systematic analysis of the effect that the degree of full, mean-preserving asymmetry on the formation of stable self-enforcing coalitions, under two different conditions: transfers and no transfers. The effectiveness of, and participation in, International Environmental Agreements is found to increase with the degree of asymmetry under the optimal sharing rule.

Keywords: Asymmetry, Climate Change, Coalition Games, Externalities, Transfer Schemes, Partition Function.

JEL Classification Numbers: H41, C72, Q50, F53.

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1 Introduction

Addressing Climate change is arguably one of the most complex collective actions problems facing society today. Collective action is necessary to address the challenges associated with climate change because its mitigation is a global public good.

One of the fundamental constraints associated with addressing climate change arises from an institutional failure to cope with the externalities caused by this phenomenon, due to the transboundary nature of the pollutant and the absence of a single world government to internalise the externality. Consequently, international cooperation that results in coordination can significantly improve upon uncoordinated unilateral action. International Environmental Agreements (hereafter IEAs) provide a framework with which to foster cooperation and coordinate the action of members to address climate change. However, the theoretical literature on IEAs has produced pessimistic results. IEAs are typically associated with free-rider incentives: abatement by one nation not only generates costs and benefits for the abater, but also results in a positive externality that benefits all nations, driving the incentive to free ride. When positive externalities generate free-rider incentives, these incentives increase with the size of the coalition. As a result, the payoff from avoided abatement to a nation outside of an agreement increases as the number of members (the size of the coalition) increases. This arises from increasing differences in the payoff between full and no cooperation (Barrett, 1994).

The challenges associated with the free-rider incentives endemic to IEAs are further exacerbated by the principle of sovereignty and the Coasian characteristics of Public International Law, which remove the possibility of third party enforcement. Moreover, the political, economic and cultural diversity across nations further reduces the likelihood of consensus.

This paper intends to attenuate some of the pessimism associated with the literature by analysing a single-coalition, open-membership game with asymmetric nations, supporting the recent work by McGinty (2007, 2011), Eyckmans and Finus (2009), and Weikard (2009), in which they present a class of optimal sharing rules. The main objective is to provide the first systematic analysis of the effect that different degrees of asymmetry can have for on the formation of non-trivial coalitions and the resulting division of coalition worth in situations with transfers and without transfers. From changing the degree of mean-preserving asymmetry for a single-coalition, open-membership game coalition formation game with positive externalities we find that the optimal sharing rules consistently show an increase in the percentage of global welfare and the size of the stabilised coalition as the degree of asymmetry increases.

To begin, we conduct a numerical analysis, which supports the use of optimal sharing rules by demonstrating that the conventional transfer schemes are ineffective for coalition formation games with positive externalities. We demonstrate that there is no guarantee of the existence of a stable equilibrium coalition . Secondly, that the prediction of stable coalition structures is extremely sensitive to the specification of the sharing rules. Finally, that they do not maximise aggregate worth, irrespective of whether the aggregate coalition worth is actually maximised.

Throughout these objectives we employ the use of a linear benefit function and a quadratic cost function, for analytical and computational tractability. We use a single-coalition, open-membership game for a number of reasons. First, it seems counterintuitive that signatories of an agreement would limit participation in an agreement by blocking membership as, in order to maximise global abatement, it is necessary to increase participation. Secondly, single coalitions limit the number of necessary structural assumptions, resulting in the derivation of conclusive results with wide applicability to economic problems with positive externalities. Furthermore, IEAs have historically been characterised by single coalitions such as the Kyoto protocol, not multiple coalitions, which are more often encountered in the international trade literature. Finally, for the purpose of continuity, it is logical to use this framework due to its wide use in this context within the literature (see McGinty, 2007; Eyckmans and Finus, 2009; Weikard, 2009).

The remainder of the paper is structured as follows: section 2 provides a background to the literature on IEAs, Asymmetry and Optimal Transfer Schemes; section 3 introduces the notation and defines the concepts related to the model and stability; section 4 presents the class of optimal sharing rules; section 5 presents the main results relating to non-existence, non-robustness and non-optimality, showing the failure of the conventional transfer schemes to meet the conditions of existence, robustness and optimality set out by Eyckmans and Finus (2009); section 6 presents results of changes to the degree of asymmetry, with and without transfers, demonstrating the effectiveness of the optimal sharing rule under mean-preserving asymmetry. Finally, section 7 presents the main conclusions and give thought to extensions and future research.

2 Background and Literature Review

The classical approach to coalition formation games has been to use the characteristic function (von Neumann and Morgenstern, 1944), whereby each coalition is assigned a worth that represents the total amount of transferable utility that the members of the coalition can receive, whilst ignoring the behaviour of players outside of the coalition. However, this approach is not appropriate to deal with economic problems which contain positive externalities, such as climate change mitigation. Positive externalities change the focus for nations from the value that they might add to an agreement by cooperating to the payoff that they would receive if they were not part of an agreement.

It has been argued that in the presence of externalities, the appropriate framework for examining coalition formation is the partition function form (Thrall and Lucas, 1963; Ray and Vohra, 1999). The partition function assigns a worth to both the coalition and to those outside the coalition which is dependent on the entire coalition structure, i.e. the partition of players. This allows for the examination of the incentives behind joining and leaving a coalition in situations with externalities.

This paper uses the non-cooperative approach to the formation of IEAs, in which valuations emerge as an outcome of a two-stage process where the players decide on membership in the first stage, then on the level of abatement, given the fixed coalition structure, in the second stage. The stable equilibrium coalition structures are then found via backwards induction to find the sub-game perfect Nash equilibria. The set of Nash equilibria is determined using the equilibrium concept developed by d'Aspremont et al. (1983), originally used to examine the stability of cartels. Under this equilibrium condition, no members of a coalition have an incentive to leave and no outsiders have an incentive to join the coalition.

The application of this cartel stability condition to the field of IEAs was originally characterised by the strong assumption of symmetrical nations (Hoel, 1992; Carraro and Siniscalco, 1993; Barrett, 1994; Hoel and Schneider, 1997), which, whilst supplying analytical tractability, is very difficult to justify in most economic problems, especially when examining cooperation at an international level.

More recently, the assumption of symmetry was removed to incorporate asymmetric nations. Indeed, 'greater degrees of cooperation and abatement are possible through relaxing the assumption of constant marginal costs and benefits of abatement across nations' (McGinty, 2007).

The benefits from abatement relate to the damages that a nation will face from climate change. Developing nations face steep damage functions as it is expected that climate change will affect these nations most severely since their economies are highly dependent on environmental production processes, such as forestry, fisheries, grazing lands, irrigation and agriculture (World Bank, 2010). An increase in the mean global temperature, rising sea levels, more volatile weather patterns, increasing water scarcity and direct impacts on human health will more significantly affect those nations.

In contrast, developed nations are likely to be less adversely affected from climate change. Some areas may even benefit slightly from moderate climate change, due to increased agricultural productivity through increases in temperature and availability of arable land (Deschênes and Greenstone, 2007). When thinking about climate change, the main human concern is not related to increased temperature, but ultimately the consequences related to food and water security. A moderate increase in the concentration of carbon dioxide may in fact increase agricultural production and aid water security, as increased productivity of plant processes could lead to increased crop yields and higher water retention. However, this relates only to moderate climate change. It is predicted that global emissions need to be reduced by between 2.5% and 3% per year on average between 2010 and 2050 to have a 50% chance of limiting global mean temperature increase to 2% (Bowen and Ranger, 2009).

As nations face no restrictions when emitting green house gases (GHGs), the marginal abatement cost (MAC) curve for a country begins at the origin, zero. However, the rates of increase vary considerably, reflecting differences in energy efficiency and fuel mix. Ellerman et al. (1998) state that the rate of increase in marginal costs related to climate change mitigation in Japan is more than ten times greater than in the United States and more than 50 times greater than in China. Nations that have high levels of energy efficiency, or use cleaner technology for energy production, face a much greater MAC, c_i, as there are fewer substitutes available, given that the low-hanging fruit (the easiest and cheapest opportunities for abatement) have already been exploited, in addition to issues such as technological lock-in. Consequently, developed nations face steeper abatement cost curves, whilst developing nations have more opportunity for energy efficiency improvements and therefore face flatter abatement cost curves. Arguably, the nations with greater opportunities for energy efficiency could be seen to have a comparative advantage in abatement.

Given the substantial differences between nations relating to the costs and benefits of abatement, the fact that only small self-enforcing coalitions tend to be stable is not unexpected. However, in view of these differences, there may be large gains from cooperation if abatement is allocated efficiently.

This has opened up research in the area of transfer schemes, which are used to increase the incentive to cooperate through altering the division of the gains from cooperation.[1] However, there has been no systematic analysis as to the effect that different degrees of asymmetry can have on the formation of non-trivial coalitions and the resulting division of coalition worth in situations with transfers and without transfers.

The incorporation of asymmetry also opens up questions on equity, which is an issue of particular importance in the context of climate change mitigation. Indeed, the historical differences in emissions between countries is seen as one of the fundamental issues that

[1] See Barrett (2001); Botteon and Carraro (1997, 2001); Eyckmans and Finus (2007, 2009); Weikard (2009); McGinty (2007, 2011).

limits cooperation. The design of a mechanism that could incorporate equity issues without reductions in efficiency considerations would be of great value in mitigating free-rider incentives and increasing participation in IEAs.

The class of optimal sharing rules derived by McGinty (2007, 2011), Eyckmans and Finus (2009), and Weikard (2009) achieve this objective. Optimal sharing rules allocate each member their free-rider payoff plus a weighted share of the coalition surplus, subject to the coalition stability requirements of d'Aspremont et al. (1983). As a result, a larger free-rider payoff results in an increased total payoff. The rules result in a transfer mechanism that guarantees the existence of a non-trivial coalition, is robust to sharing weights (equity considerations) and maximises the size of the stable coalition. McGinty (2011) notes that this differs greatly from the conventional transfer schemes, such as the Nash bargaining solution (1953) and Shapley value (1953), the values of which increase with a nation's contribution to the coalition and, consequently, may allocate the worth in the wrong direction, resulting in the destabilisation of a potentially stable coalition.

These surplus sharing schemes have also been used in the context of the partition function form. For example, Myerson (1977) provides a generalisation of the Shapley value that allows for externalities. However, he assumes full cooperation, which, given the presence of positive externalities, is unlikely to be the equilibrium outcome. Maskin (2003) also extends the Shapley value to games with externalities and shows that only partial cooperation may be sustained. However, in the sequential bidding process, each player's allocation is determined by their marginal contribution, reducing the allocation to the conventional Shapley value absent of externalities (McGinty, 2011). Other papers (Macho-Stadler et al. 2007; de Clippel and Serrano, 2008) incorporate externalities, but assume either symmetric players or full cooperation, in the form of the grand coalition.

In defence of the conventional transfer schemes, McGinty (2007) argues that they work well for orthogonal characteristic function games without externalities (Winter, 2002), as collective rationality requires that the grand coalition form since the core is non-empty. However, in the literature relating to IEAs (Barrett, 1997; Botteon and Carraro, 2001), the Shapley value (1953) and Nash bargaining solution (1953) have not performed as well.

3 The Coalition Formation Game

3.1 Definitions

For a finite set of N nations, in which N=1,...,n we define the coalition formation game for n$\geq$2 nations as $\Gamma\left(N, S, v\right)$. For a single-coalition game, the set of all possible coalitions is denoted by 2^n, which is the power set of N. All coalitions are subsets of N, denoted by S, in which $S \subseteq N$ is the set of coalition members and $j \in N \setminus S$ are classified as singletons, outside of the coalition. The partition function assigns a payoff to the coalition and to every non-member.

Definition 1: The Partition Function: a mapping π that assigns a payoff $\pi(S)$ to S and $\pi_j(S)$ to all $j \in N \setminus S$ for every $S \subseteq N$. i.e.

$$\pi : S \mapsto \pi(S) = (\pi_S(S), \pi_j(S)) \in \mathbb{R}^{1+n-s}$$

$\square$

In order to analyse the incentives of individual nations to form coalitions, we must assign valuations to each nation. This is done through the valuation function, which maps coalition structures into a vector of valuations.

Definition 2: The Valuation Function: a function that assigns each coalition, $S \subseteq N$, a sharing rule,

$$v : 2^n \to \mathbb{R}^n : S \mapsto v(S)$$

such that $\sum_{i \in S} v_i(S) = \pi_S(S)$ and $v_j(S) = \pi_j(S), \qquad \forall j \in N \setminus S$

That is, for every coalition, S, the valuation function, v, specifies how the worth of the coalition will be allocated among its members, in addition to the payoff to those outside of the coalition. $\square$

By definition, collective rationality holds, as the entire worth of the coalition S, $\pi_S(S)$, is allocated among its members. Furthermore, the valuation for every outsider, $v_j(S)$, coincides with the worth, $\pi_j(S)$, assigned to them by the partition function.

It is important, given the applied nature of this work, to be explicit in the way that v is constructed. In the context of IEAs, we define $v = \pi$ such that the valuation function corresponds to the payoff function. Consequently, valuations correspond to payoffs. As a result, the global net benefit of abatement is defined as,

$$\pi(N) = B(Q) - \sum_{i \in N} C_i(q_i, c_i) \tag{1}$$

From this we define the global benefit as $B(Q) = bQ$, where total quantity of abatement,$Q = \sum_{i \in N} q_i$, and total benefits from abatement are, $b = \sum_{i \in N} b_i$. For a given share of global benefits, b_i, the benefit for nation i is $B_i(Q) = b_i q_i$.

The abatement cost functions for a given nation is defined:

$$C_i(q_i, c_i) = \frac{c_i(q_i)^2}{2} \tag{2}$$

By combining the benefit and cost functions, the net benefit for nation i can be obtained. Following Barrett (1994), the objective of each nation is to maximise this net benefit.

$$\pi_i(b_i, c_i, q_i(S)) = b_i \sum_{i \in S} q_i(S) - \frac{c_i q_i(S)^2}{2} \tag{3}$$

under the assumption that, $b_i > 0$ and $c_i > 0$, in which N is the set of nations, b_i is the benefit parameter, c_i is the cost parameter and q_i is the abatement level taking into account the coalition structure. For this model, only the connection between the benefit and cost parameters $\frac{b_i}{c_i}$ is important, therefore $\gamma_i = \frac{b_i}{c_i}$.

Each nation chooses a strategy, $\theta_i \in 0, 1$, where, if $\theta_i = 1$, they will become a member of the coalition and, if $\theta_i = 0$, they will free ride. In the context of an IEA, nations have a strong incentive to free ride as a result of positive externalities.

Definition 3: Positive Externalities: A coalition formation game $\Gamma(N, S, \pi)$ exhibits positive externalities if and only if:

$$\pi_j(S) \geq \pi_j(S \setminus \{i\}) \qquad \forall S \subseteq N, j \neq i \quad \& \quad j \notin S$$

and

$$\pi_k(S) > \pi_k(S \setminus \{i\}) \qquad \forall S \subseteq N, k \neq i, \quad \& \quad k \notin S$$

No free-riders are made worse-off from the enlargement of the coalition and at least one free-rider is made better off. Consequently, the incentive to free ride increases with the size of the coalition. □

Even when a coalition formation game with positive externalities is associated with superadditivity, in which the aggregate payoff to all nations is an increasing function of the size of the coalition, the free-rider problem may still exist. Under these circumstances, the superadditivity effect is smaller than the free-rider effect.

Definition 4: Superadditivity: a coalition formation game $\Gamma(N, S, \pi)$ is superadditive if and only if:

$$\forall i \in S, \forall S \subseteq N : \pi_S(S) \geq \pi_{S \setminus \{i\}}(S \setminus \{i\}) + \pi_i(S \setminus \{i\})$$

Superadditivity implies that by increasing the size of the coalition, it is possible to distribute the gains from cooperation in such a way that there is a Pareto-improvement to all nations involved in cooperation. □

It is evident that the incentive to join a coalition, as well as the stability of any agreement, depends on the magnitude of these two properties defined above. The magnitude of the superadditivity effect determines the payoff from cooperation. The magnitude of the positive externality effect determines the free-rider payoff for those that deviate from cooperation.

3.2 Coalition Stability

In addition to establishing the assumptions underlying a coalition formation game in the context of IEAs, it is also important to explain the stability conditions for a self-enforcing IEA, as defined by d'Aspremont et al. (1983), used in this paper.

Definition 5: Internal and External Stability: for a coalition formation game $\Gamma(N, S, \pi)$ π is a payoff function, which is a vector of payoffs for all nations in N when a coalition S forms, $\pi(S) \in \mathbb{R}^n$. Let $\pi_i(S \setminus i)$ denote the vector of payoffs for the resulting coalition following a deviation by nation i. Let $\pi_j(S \cup j)$ denote the vector of payoffs for the resulting coalition following the accession of nation j. A coalition S is stable for the valuations $\pi(S)$ if and only if:

$$
\begin{aligned}
&\textit{Internal Stability}: \qquad && \pi_i(S) \geq \pi_i(S \setminus i) \qquad && \forall i \in S \\
&\textit{External Stability}: \qquad && \pi_j(S) \geq \pi_j(S \cup j) \qquad && \forall j \in N \setminus S
\end{aligned}
$$

That is, coalition S is stable if no members inside the coalition have an incentive to leave and no free-riders have an incentive to join. □

This definition corresponds to a Nash equilibrium, in which no nations have any incentive to change their strategy. This implies open membership as all nations are free to join the coalition i.e. no members can block the accession of outsiders in to the coalition.

4 Optimal Sharing Rules

The construction of optimal sharing rules begin with the observation that a necessary condition for internal stability is that each nation receives their free-rider payoff. In the context of an IEA, we associate the free-rider payoff with the situation in which an individual nation, i, chooses not to join the agreement, remaining a singleton. These payoffs constitute lower bounds on the claims of individual coalition members, regarding the allocation of coalition surplus necessary to provide the incentive to join the agreement.

Weikard (2009) derives this condition as the 'Claim Rights Condition', in which an agreement will only be stable if the coalition surplus exceeds the sum of the individual claims. This is related to rights-egalitarian sharing, axiomatised by Herrero et al. (1999), in which the emphasis is on the importance of individual rights, resulting in the collective responsibility being to meet the claims of all members. In the context of an IEA, in which stability is self-enforcing, it is a necessary assumption that each member is granted the right to a position no worse than their free-rider payoff if the agreement is to be stable. Formally, the Claim Rights Condition will hold $\forall i \in S$ and all $S \subseteq N, \pi_i(S) \geq \pi_i(S \setminus i)$ if and only if $\pi_S(S) \geq \sum_{i \in S} \pi_i(S \setminus i)$. Weikard (2009) notes that the Claim Rights Condition is weaker than rights-egalitarian sharing as it proposes only a general solution to the sharing problem. However, from a policy perspective, one could argue this to be a benefit of the scheme, providing greater flexibility.

The scheme derived by Eyckmans and Finus (2009) doesn't guarantee the grand coalition, but does maximise aggregate welfare subject to the stability conditions derived by d'Aspremont et al. (1983). Furthermore, they state that as every sharing rule has a corresponding valuation function, referring to a sharing rule is equivalent to referring to a valuation function. Consequently an optimal sharing rule denote the entire class of optimal valuation functions. Consequently, an optimal valuation function is a particular member of the class of optimal sharing rule.

Definition 6: Optimal Sharing Rule: A sharing rule is an optimal sharing rule only if it meets the Claim Rights Condition.

For a coalition formation game $\Gamma(N, S, \pi)$ an optimal sharing rule is a payoff function $\pi^{*(\lambda)}$ that satisfies:

$$\forall S \subseteq N : \begin{cases} \forall i \in S : & \pi_i^{*(\lambda)}(S) = \pi_i(S \setminus \{i\}) + \lambda_i(S)\sigma(S) \\ \forall j \in N \setminus S : & \pi_j^{*(\lambda)}(S) = \pi_j(S) \end{cases}$$

in which $\lambda(S) \in \Delta^{S-1}\left\{\lambda \in \mathbb{R}_+^S \middle| \sum_{j\in S} \lambda_j = 1\right\}$ *and* $\sigma(S) = \pi_S(S) - \sum_{i\in S} \pi(S \setminus \{i\})$, *where* $\lambda(S) \in \Delta^{S-1}$ *denotes the set of all possible sharing weights for a coalition with S players and* $\sigma(S)$ *relates to the Claim Rights Condition denoting the surplus (or deficit) of a coalition* S. □

Consequently, an optimal valuation function allocates each coalition member its free-rider payoff, plus a non-negative share of the coalition surplus, which, when combined, add up to one.

If the Claim Rights Condition is met with a positive surplus ($\sigma(S) \geq 0$), then the coalition S is feasible. This relates to the partition function as only aggregate payoffs matter in order for a coalition to be feasible. This is in contrast to the concepts of internal and external stability, which are properties of the valuation function, as individual payoffs matter.

The three key results derived by Eyckmans and Finus (2009) are the existence, robustness and optimality of their class of optimal sharing rules. The existence of a coalition is the minimum condition for the formation of an IEA. No structural assumptions relating to the underlying economic model are required for the existence of a coalition through the use of the optimal sharing rule, resulting in a wide degree of applicability to coalition formation games, beyond the formation of IEAs.[2]

The second result associated with the optimal sharing rules is an invariance, or robustness, result that shows that the set of stable coalitions is independent of the weights assumed. Again, no structural assumptions relating to the underlying economic model

[2]The assumption of superadditivity is necessary for the existence of a non-trivial coalition.

are required.

The main result, derived by Eyckmans and Finus (2009), relates to optimality. Unlike the first two results, the optimality result requires the assumption of positive externalities. However, this is the only structural assumption relating to the underlying economic model. The optimality result demonstrates that the use of the optimal sharing rules guarantees that the coalition that generates the highest aggregate worth among all coalitions that are feasible will be stable, as in Definition 5.

If we recall the robustness result from above, the value of the optimal sharing rules are revealed. Considering that any optimal valuation function is dependent on the sharing weights $\lambda(S)$, there is a great degree of flexibility as to how the coalition surplus is allocated among its members, without compromising the optimality result. Indeed, any set of sharing weights will stabilise the coalition that generates the highest aggregate payoff. This is of particular interest from a policy perspective, as this result has the potential, if a mechanism were designed to incorporate the features of the optimal sharing rules, to create opportunities for the application of different equity criteria without jeopardising coalition stability.

5 Results

In this section, the results associated with the conventional transfer schemes pertaining to non-existence, non-robustness, and non-optimality are attained. In subsection 5.1, results show the failure of the conventional transfer schemes to stabilise a coalition. In subsection 5.2, results show how sensitive the conventional transfer schemes are at stabilising coalitions. Subsection 5.3 provides the final result, showing the failure of the conventional transfer schemes to stabilise the optimal coalition, with regards to the level of global welfare and coalition size.

5.1 Non-Existence

As mentioned, the existence of a non-trivial coalition is the minimum condition for the formation of an IEA. Consequently, the use of conventional transfer schemes that could result in non-existence is inappropriate.

Whilst Eyckmans and Finus (2009) provide theoretical proof on the existence of the optimal transfer scheme, this paper provides numerical evidence of non-existence through the use of the conventional transfer schemes, whilst demonstrating existence of the optimal sharing rules. The two-stage membership game was conducted using the parameters shown in Table 1, the results of which are expanded upon below.[3]

Table 1: Parameters

	1	2	3	4	5
b_i	80	33	8	88	28
c_i	58	29	83	100	31
γ_i	1.379	1.138	0.096	0.88	0.903

The example below, derived from the parameters in Table 1, assumes 5 nations and a partition function that exhibits positive externalities and superadditivity. The set of internally and externally stable coalitions are exhibited by the no transfers condition (Table A1), for three of the most prominent transfer rules seen in the literature on the formation of IEAs (Shapley value, Table A2; Nash bargaining solution with equal weights, Table A3; Chander Tulkens transfer scheme, Table A4) and by the optimal sharing rules (Table A5) derived by Eyckmans and Finus (2009).

Table 2 proves that the Shapley value, one of the conventional transfer schemes associated with the literature, does not guarantee the existence of either a stable non-trivial coalition or even a trivial stable coalition, whilst the optimal sharing rules (Table 3) provides 7 stable coalitions, attaining 76% of global welfare defined as $\frac{(\pi(S)-\pi(\emptyset))}{(\pi(N)-\pi(\emptyset))}$.[4]

This is the first time that the non-existence of a non-trivial coalition has been calculated numerically using a set of parameters and the partition function. McGinty (2011) shows

[3] These parameters were derived through Monte Carlo Simulations using parameter values between 1 and 100 to determine non-existence, non-robustness, and non-optimality of the Shapley value.

[4] Full tables and calculations can be found in the Appendices.

Table 2: The Shapley value

Coalition Size	Stable Coalitions	Largest Abatement
1	None	0%
2	None	0%
3	None	0%
4	None	0%
5	None	0%

Table 3: The Optimal sharing rule

Coalition Size	Stable Coalitions	Largest Abatement
1	None	0%
2	None	0%
3	6	{1,2,5} 61%
4	1	{2,3,4,5} 76%
5	None	0%

how the Shapley value and Nash bargaining solution perform less effectively than the optimal sharing rules. However, until now, proof that these sharing schemes can fail to stabilise a coalition entirely has not been recorded until now.

5.2 Non-Robustness

Whilst, the optimal sharing rules are shown to be robust, the coalitions stabilised by the conventional transfer schemes are not invariant to changes in the the weights assumed.

Tables 4 and 5 provide numerical evidence as proof to these claims. From the definitions above, it is intuitive that the Nash bargaining solution and Chander Tulkens transfer scheme are conceptually the same, but have different weights associated with the sharing of the surplus. This would be equivalent to the optimal sharing rules being calculated twice, once with equal weights, as shown in Table 6, and once with weights equal to the marginal damages of each player. Under the robustness result proven above, the set of stable coalitions would be the same under both calculations. If the conventional transfer schemes were to perform equivalently, we would expect the set of internally and externally stable, and therefore stable, coalitions to be the same for both the Nash bargaining solution and the Chander Tulkens transfer scheme.

However, as we can see in Tables 4 and 5, this is not the case. Thus we have proven

Table 4: The Nash bargaining solution

Coalition Size	Stable Coalitions	Largest Abatement
1	None	0%
2	None	0%
3	1	{2,3,5} 30%
4	None	0%
5	None	0%

Table 5: The Chander Tulkens transfer scheme

Coalition Size	Stable Coalitions	Largest Abatement
1	None	0%
2	None	0%
3	2	{1,2,4} 49%
4	None	0%
5	None	0%

a non-robustness result for these transfer schemes, further emphasising the effectiveness of the optimal sharing rules in comparison to the conventional transfer schemes. Indeed, neither of the two stable coalitions associated with the Chander Tulkens transfer scheme are the same as the single stable coalition associated with the Nash bargaining solution.

5.3 Non-Optimality

Unlike the optimal sharing rules, the conventional transfer schemes can fail to guarantee that the coalition that generates the highest aggregate worth among all feasible coalitions will be stable.

As with the previous results, the coalition game derived from the parameters in Table 1 (the results of which are available in the Appendix, Tables A1-A5) also provides evidence of non-optimality under the conventional transfer schemes, whilst presenting the optimality result under the optimal sharing rules.

In Table 6 we can see that the optimal sharing rule stabilizes the four-member coalition $\{2, 3, 4, 5\}$. This coalition results in the highest aggregate payoff among all feasible coalitions, attaining 76% of global welfare, whereas the conventional transfer schemes are able to stabilise only three-member coalitions, attaining 30% (Nash bargaining solution) and 49% (Chander Tulkens transfer scheme) of global welfare.

Table 7 emphasises the effectiveness of the optimal sharing rule, illustrating that the conventional transfer schemes were not only unable to maximise the aggregate payoff among all feasible coalitions, but also to maximise the aggregate payoff among all the feasible coalitions within the coalition size that they were able to stabilise. The Chander Tulkens transfer scheme achieved 49% of the potential 61% available from the three-member coalitions, and the Nash bargaining solution achieved only 30% of the potential 61% available from the three-member coalitions. This further emphasises the flexibility associated with the optimality result that the optimal sharing rule achieves in comparison to the conventional transfer schemes, once again illustrating both the non-robustness and non-optimality of the Nash bargaining solution and Chander Tulkens transfer scheme.

Table 6: Optimality and Abatement Levels

Sharing Scheme	Stable Coalitions	Largest Abatement
Nash bargaining	1	{2,3,5} 30%
Chander Tulkens	2	{1,2,4} 49%
Shapley value	None	0%
Optimal sharing rule	1	{2,3,4,5} 76%

Table 7: Feasible Coalitions and Abatement Levels

Coalition Size	Feasible Coalitions	Largest Abatement
1	All	0%
2	All	$1, 2\}, \{2, 4\}$26%
3	All	{1,2,5} 61%
4	{2,3,4,5}	{2,3,4,5} 76%
5	None	100%

In summary, this section has demonstrated the inability of the conventional transfer schemes to meet the conditions of existence, robustness and optimality that characterise optimal sharing rules. This class of optimal sharing rules has improved upon the existing literature on the formation of IEAs and the incorporation of transfers by departing from the overly strong assumption of symmetrical nations, establishing the guaranteed existence of stable coalitions and demonstrating its flexibility in deciding sharing weights and robustness in the prediction of stable coalitions, in addition to the most important property of establishing optimality subject to stability as defined by d'Aspremont et al.

(1983). Particularly, this section has provided a numerical solution to the results derived by Eyckmans and Finus (2009), and non-existence, non-robustness and non-optimality of the conventional transfer scheme. One of the most valuable features of the optimal sharing rules, especially in the context of climate change is it's ability to address equity considerations, without any reduction in efficiency.

6 Asymmetry and International Environmental Agreements

This section intends to understand the role that the differences between countries have by providing the first systematic analysis of the effect that the degree of mean-preserving asymmetry has on the formation of stable self-enforcing IEAs under two conditions: with and without transfers, using the same framework as previously.

Mean-preserving asymmetry is analysed through altering the variance of the benefit and cost parameters associated with the payoff function. The use of mean-preserving asymmetry (see McGinty, 2007), as opposed to using two-type asymmetry (see Barrett, 1997 and Fuentes-Albero and Rubio, 2010), investigates the impact of allowing all nations to be asymmetric, rather than simply of two types.

Fuentes-Albero and Rubio (2010) show that in a two-stage game with two types of nations, asymmetry between nations has no effect on cooperation when there are no transfers and that, with transfers, the effect of asymmetry depends on the type of asymmetry.

This highlights the importance of defining exactly what is meant by asymmetry. In this section, four different cases of mean-preserving asymmetry are analysed. The first case examines the impact of asymmetry on the cost parameter, *ceteris paribus*. The second examines the impact of asymmetry on the benefit parameter, *ceteris paribus*. In both of these cases, the remaining variable is equal to the mean for all nations. The third case examines the impact of asymmetry when nations are on a mean-preserving scale ranging from high costs and low benefits to low costs and high benefits. The final case examines the impact of asymmetry when nations are on a mean-preserving scale ranging from high costs and high benefits to low costs and low benefits. In these final two cases, the mean for both parameters are equal to allow comparison between the different types of asymmetry.

In this analysis, a seven-nation, two-stage game is analyzed (as in Barrett, 1997 and McGinty, 2007). The measure used in this paper to assess the effectiveness of the IEA is the Closing the Gap Index (CGX) defined by Eyckmans and Finus (2006) as:

$$\frac{(\pi(S) - \pi(\emptyset))}{(\pi(N) - \pi(\emptyset))} \tag{5}$$

This is similar to the measurement used by Barrett (1997), which measures the performance of an IEA as the proportion of the difference in abatement between no and full cooperation. The main difference between this measurement and the CGX is its focus on abatement as opposed to payoff.

This work builds on the asymmetry analysis conducted by McGinty (2007), in which the introduction of full asymmetry results in the level of abatement rising considerably and the effectiveness of the IEA more than doubling under the class of optimal sharing rules in comparison to the symmetric models and two-type asymmetry models. However, McGinty (2007) only briefly mentions the potential impact of a slight change in the degree of asymmetry, without providing an extended analysis. This section builds on this work, providing an analysis of the impact of changing the degree of asymmetry to understand how full asymmetry can impact on global cooperation with the use of optimal transfers.

6.1 Results

The initial parameters used to derive the symmetry results in the seven-nation coalition formation game are shown in Table 8. The results from the symmetry analysis are presented in Table 9.

Table 8: Parameters

	b_i	c_i	$q(\emptyset)$	$q(N)$	$\pi(\emptyset)$	$\pi(N)$
i=1,...,7	100	100	1.00	7.00	650	1900
$\sum_{i\in N}$	700	700	7.00	49.00	4550	13300

Table 9: Symmetry

Stable Coalitions	Largest Abatement
56	24% (3)

As expected, even the slightest change from symmetry results in the three-member coalitions becoming unstable under the no transfers condition. As a result, any degree of asymmetry results in only two-member coalitions being stabilised under the no transfers condition.

One of the findings of this analysis, as in Fuentes-Albero and Rubio (2010), is that the impact of the degree of asymmetry on cooperation is only affected by relative changes in the marginal costs and benefits. Fuentes-Albero and Rubio (2010) argue that, as a result, it is not possible to establish any systematic relationship between the gains of cooperation and participation in an agreement. The impact of the degree of asymmetry will also be affected by the number of nations.

Of particular importance is a general result which indicates that, as the degree of asymmetry increases, the difference in coalition worth between the grand coalition and the non-cooperative coalition increases, resulting in greater gains from cooperation.

Another general result is in all cases, as the degree of asymmetry increases, the percentage of global welfare increases under the optimal sharing rule, whilst the no transfers condition results either in a decrease in global welfare or in a percentage of global welfare considerably less than under symmetry.

In the following subsections the results of the analysis are further examined for each of the four cases.

6.1.1 Asymmetry in Costs

Table A6 in the Appendices displays the parameters used at each stage of the analysis. Table 10, below, indicates the results of using these parameters in the two-stage single-coalition, open-membership game. It shows the difference, d, between each nation in the parameter in which there is asymmetry, the maximum size of the stable coalition and the maximum percentage of global welfare attained under each condition.

Table 10: Asymmetry in Costs

d	No Transfers	Optimal sharing rule
1	9% (2)	24% (3)
5	8% (2)	26% (3)
10	7% (2)	29% (3)
15	6% (2)	32% (3)
20	5% (2)	35% (3)
25	4% (2)	39% (3)
30	2% (2)	46% (3)

In case 1, under asymmetry in costs, *ceteris paribus*, and under the condition of no transfers the percentage of global welfare falls considerably as asymmetry is introduced. As the degree of asymmetry increases the percentage of global welfare falls from 9% to 2%. This is due to the relationship between the non-cooperative coalition, in which all nations act as singletons, and the grand coalition. As the degree of asymmetry increases, the worth of both the non-cooperative and the grand coalition increases. With regards to the numerator function of the CGX, presented in equation 19, if the worth of coalition S increases by a higher percentage than the non-cooperative outcome as the degree of asymmetry increases, then the percentage of global welfare attained by coalition S will increase as the degree of asymmetry increases. However, if the worth of coalition S increases at a rate lower than the non-cooperative coalition, then as the degree of asymmetry increases, the percentage of global welfare attained by coalition S will fall. Within this case, the participation of nation 1 (the nation with the highest abatement in this case) results in an increase in global welfare as the degree of asymmetry increases. Nation 1 has a lower marginal abatement cost resulting from easier and cheaper opportunities in abatement. In absence of this nation, the differences in abatement of $\pi(S \setminus 1) \leq \pi(\emptyset)$ drops as the degree of asymmetry increases and the percentage of global welfare decreases.

Recalling the no transfers condition, only one coalition is stabilised as the degree of asymmetry increases. This coalition consists of nation 6 and 7, the two nations that produce the least abatement. This explains why the percentage of global welfare falls as the degree of asymmetry rises. Both of these nations face high marginal abatement costs and so an IEA comprising these nations will have only a limited impact on global abatement and welfare.

Evidence of the effectiveness of the optimal sharing rule is clear when we examine its implications under asymmetry in costs, *ceteris paribus.* Following the introduction of asymmetry, there is no immediate change in the percentage of global welfare (24%), nor the size of the stable coalition (three members). However, as the degree of asymmetry increases, the percentage of global welfare increases above the level recorded under symmetry to 46%, nearly double the level recorded under symmetry and 23 times the level recorded under the no transfers condition. This increase in effectiveness is due to both a higher proportion of abatement and a greater difference between the non-cooperative coalition and the grand coalition, resulting in increased gains from cooperation. This result is of great value, as the conventional wisdom of Barrett (1994) implies that a self-enforcing IEA cannot significantly improve upon the non-cooperative coalition when the gains from cooperation are large.

6.1.2 Asymmetry in Benefits

Table A7 in the Appendices displays the parameters used at each stage of the analysis. Table 11, below, indicates the results of using these parameters in the two-stage single-coalition, open-membership game, displaying the difference, d, between each nation in the parameter in which there is asymmetry, the maximum size of the stable coalition and the maximum percentage of global welfare attained under each condition.

Table 11: Asymmetry in Benefits

d	No Transfers	Optimal sharing rule
1	9% (2)	24% (3)
5	9% (2)	25% (3)
10	10% (2)	26% (3)
15	11% (2)	27% (3)
20	11% (2)	28% (3)
25	12% (2)	28% (3)
30	12% (2)	40% (4)

In case 2, where there is asymmetry in benefits only, we see the same transition from symmetry to asymmetry as in case 1. There is a considerable reduction in the percentage of global welfare under the no transfers condition and under the conventional transfer schemes. However, in contrast to case 1, the percentage of global welfare increases under the no transfer condition to 12% under a difference of 30. As in case 1, the increase in the percentage of global welfare relates to the differences in abatement between coalition S and the non-cooperative coalition. The same coalitions are stabilised under the no transfers condition at every degree of asymmetry, therefore the increase in global welfare results from the increased coalition worth (associated with the increase in asymmetry) relative to the worth of the non-cooperative coalition.

In contrast to case 1, there is no further increase in the coalition worth of the grand coalition after an initial increase following the introduction of asymmetry. However, the coalition worth of the non-cooperative outcome decreases slightly as the degree of asymmetry increases, so that there remains an increase in the difference between the grand coalition and non-cooperative outcome, as in case 1, increasing the potential amount of welfare that could be attained.

This relates to the ratio between the benefits and costs, which quantifies the abatement for each nation. As the benefit parameter is the numerator and the cost parameter is the denominator, a change in the denominator, holding the numerator fixed, results in a much larger variation, whilst a change to the numerator, holding the denominator fixed, results in a smaller variation. As a result, the variation in abatement between the nation that abates the most and the nation that abates the least is much smaller than in case 1. Consequently, the participation of the nation that abates the most is no longer a condition that guarantees an increase in the percentage of global welfare attained. However, coalitions that consist of members that collectively abate more than the average are guaranteed to result in a growth of coalition worth as the degree of asymmetry increases, as long as their abatement is not offset by members that abate less than average by the same proportion or more. This is demonstrated in the no transfers condition, in which the coalition $\{6, 7\}$ is stabilised continuously as the degree of asymmetry increases. Given that, in this case, nations 6 and 7 abate the most (due to their incentive to abate more resulting from greater returns from cooperation), this results in the percentage of global welfare increasing under the no transfers condition.

As with case 1, the optimal sharing rule once again demonstrates its power, starting at 24% before attaining a percentage of global welfare equal to 40% at the highest degree of asymmetry and stabilising a four-member coalition. This once again disputes the conventional wisdom of Barrett (1994) that a self-enforcing IEA will struggle to improve on the non-cooperative level when the gains from cooperation are large. For every degree of asymmetry measured, the optimal sharing rule manages to stabilise coalition {5,6,7}, comprised of all the nations that abate the most. It is only when the optimal sharing rule stabilises the four-member coalition that it stabilises coalition {1,2,4,7}, formed by nations 1 and 2 that abate the least, nation 4 that abates the average and nation 7 that abates the most. Again, at the highest degree of asymmetry, the optimal sharing rule attains nearly four times the percentage of global welfare attained by the no transfers case.

6.1.3 Asymmetry HiLo LoHi

Table A8 in the Appendices displays the parameters used at each stage of the analysis. Table 12, below, indicates the results of using these parameters in the two-stage single-coalition, open-membership game, displaying the difference, d, between each nation in both parameters, the maximum size of the stable coalition and the maximum percentage of global welfare attained under each condition.

Table 12: Asymmetry HiLo LoHi

d	No Transfers	Optimal sharing rule
1	9% (2)	25% (3)
5	11% (2)	28% (3)
10	13% (2)	32% (3)
15	16% (2)	37% (3)
20	13% (2)	43% (3)
25	13% (2)	50% (3)
30	11% (2)	58% (4)

Case 3, which examines asymmetry as an inverse relationship between costs and benefits, presents the most accurate representation of the situation that nations are facing. This is because developing nations face low costs from abatement and stand to gain the most from abatement and developed nations face higher costs from abatement and stand to gain less from abatement, as discussed in section 2. By examining the impact of the degree of asymmetry in this way, we develop a mean-preserving scale between the HiLo and LoHi extremes. As in the previous cases, the difference between the coalition worth of the grand coalition and the non-cooperative coalition increases as the degree of asymmetry increases. Similarly, the percentage of global welfare will fall if the non-cooperative coalition increases at a higher rate than coalition S as the degree of asymmetry increases, and rise if the worth of coalition S increases at a higher rate than the worth of the non-cooperative coalition.

Under the condition of no transfers, the relationship between asymmetry and the percentage of global welfare is non-linear, and non-monotonic. The initial rise in the percentage of global welfare can be explained by the formation of coalition {6,7}, which is comprised of the nations that abate the most. Nation 7 has low marginal abatement costs as well as an increased incentive to abate due to the high returns from abatement. Nation 7 is a key member in order for the percentage of global welfare to rise. In fact, the formation of any two-member coalition of which nation 7 is a member, apart from coalition {1,7} guarantees an increase in the percentage of global welfare attained. In coalition {1,7}, the abatement by nation 1 (the nation that abates the least) counteracts the amount of abatement by nation 7, resulting in the proportion of coalition worth remaining constant as the degree of asymmetry rises. However, as the degree of asymmetry increases to a difference of 20, player 7 is a necessary member in order for the percentage of global welfare to increase. Consequently, the percentage of global welfare attained by coalition {5,6} falls as the degree of asymmetry increases from a difference of 20 onwards. This results from the worth of the non-cooperative coalition increasing at a rate higher than the worth of coalition {5,6}, leading to the decrease in $\pi(S) - \pi(\emptyset)$ and a consequent fall in the percentage of global welfare attained by the coalition.

Once again, the optimal sharing rule, which stabilises coalition {5,6,7}, consisting of the nations that abate the most, until the degree of asymmetry reaches a difference of 30, at which point the optimal sharing rule attains 58% of global welfare and stabilises a four-member coalition (coalition {1,4,6,7}). In this case, the optimal sharing rule performs the most effectively, relative to cases 1, 2, which is encouraging in view of the applicability of this case to reality. The optimal sharing rule attains a percentage of global welfare nearly six times greater than under the no transfers condition and nearly three times greater than under the assumption of symmetry, capturing the fact that asymmetry results in a greater percentage of the greater difference between the grand coalition worth and non-cooperative coalition worth being attained. One important observation from this analysis is that the increase in global welfare created by the introduction of asymmetry is greater when there is asymmetry in both parameters rather than in only one parameter.

6.1.4 Asymmetry HiHi LoLo

Table A9 in the Appendices displays the parameters used at each stage of the analysis. Table 13, below, indicates the results of using these parameters in the two-stage single-coalition, open-membership game, displaying the difference, d, between each nation in both parameters, the maximum size of the stable coalition and the maximum percentage of global welfare attained under each condition.

Table 13: Asymmetry HiHi LoLo

d	No Transfers	Optimal sharing rule
1	9% (2)	24% (3)
5	8% (2)	25% (3)
10	8% (2)	26% (3)
15	7% (2)	28% (3)
20	6% (2)	31% (3)
25	4% (2)	37% (3)
30	2% (2)	59% (4)

Case 4 presents a situation in which high benefit members also face high costs and low benefit members also face low costs, providing a ratio that is constant across all members, despite them facing different costs and benefits. This is the same case used in Barrett (1997). However, this analysis focusses on full asymmetry instead of using only two types of asymmetry (HiHi and LoLo). This case may present one reality in the formation of IEAs. Developing nations observe developed nations as having low costs, or at least lower opportunity costs, whilst observing themselves as having high costs from abatement, or rather higher opportunity costs. As in the previous cases, the difference between the non-cooperative coalition and the grand coalition increases, resulting in an increase in the total worth of global welfare as the degree of asymmetry increases. However, unlike cases 1, 2 and 3, the non-cooperative coalition remains constant. If the worth of coalition S increases as the degree of asymmetry increases, then the percentage of global welfare attained will rise, and if the worth of coalition S falls, then the percentage of global welfare attained will fall.

In this case, the percentage of global welfare under the no transfer condition is equivalent to the case in which there is asymmetry in costs, *ceteris paribus*, resulting in the expected inverse relationship between the degree of asymmetry and percentage of global welfare. The same coalition is stabilised as the degree of asymmetry increases (coalition $\{6,7\}$), indicating that the worth of coalition $\{6,7\}$ decreases as the degree of asymmetry increases. This coalition relates to the two nations with the highest benefits and costs (HiHi). From what we can see of the Kyoto Protocol, perceived LoLo nations such as the US are not members, but more and more developing nations, perceived HiHi nations, are signing and ratifying the agreement. It could be argued that this case reflects what we see most often in reality, as opposed to case 3.

Again, the optimal sharing rule attains a percentage of global welfare almost 30 times greater than the no transfers condition, highlighting the effectiveness of the sharing scheme under the assumption of asymmetry. This captures the fact that asymmetry results in a greater percentage of the greater difference between the worth of the grand coalition and non-cooperative coalition being attained. This shows the potential that transfer schemes could have in increasing cooperation if properly incorporated into IEAs. Indeed, the optimal sharing rule consistently shows an increase in the percentage of global welfare and the size of the stabilised coalition as the degree of asymmetry increases.

7 Conclusions

In this paper, further evidence to support the optimal sharing rules derived by McGinty (2007, 2011), Eyckmans and Finus (2009), and Weikard (2009) has been revealed, demonstrating its absolute advantage compared to the no transfers condition and the conventional transfer schemes in a single-coaltion, open-membership coalition game with positive externalities. The effectiveness of the scheme was demonstrated when used in coalition formation games such as the formation of IEAs, in which the assumption of symmetry is relaxed, and positive externalities exist.

The optimal sharing rules guarantee the existence of a stable coalition irrespective of the weights chosen, and attain the highest aggregate welfare among the members of the set of coalitions that can be stabilised. The grand coalition may or may not be a member of these set of coalitions, therefore Eyckmans and Finus (2009) emphasise the importance of capturing externalities across players, and considering coalitions other than the grand coalition. Consequently, the analysis is based around the partition function and stability is examined using the stability conditions derived by d'Aspremont et al. (1983).

These properties were shown not to hold for the conventional transfer schemes, demonstrating the greater effectiveness of the optimal sharing rules compared to the Nash bargaining solution, the Chander Tulkens transfer scheme and the Shapley value and the value that the transfer scheme can have when there are free-rider incentives.

Given the properties shown to hold only for the optimal sharing rules, there can be no doubt that the optimal sharing rule is the most effective sharing scheme for a coalition formation game with positive externalities, such as the formation of a self-enforcing IEA.

This effectiveness of the optimal sharing rules was further demonstrated in section 5, which analysed the effectiveness of the sharing rules and no transfers condition under different degrees of asymmetry for four different cases, two of which focussed on asymmetry in only one parameter, and two allowing for asymmetry in both parameters. In general, the optimal sharing rules consistently showed an increase in the percentage of global welfare and the size of the stabilised coalition as the degree of asymmetry increased. A key result for policy, is the indication that composition of a coalition, i.e. it's members, is of more concern than the size of the coalition, when the objective is maximising abatement. Indeed, it is possible to have a smaller coalition that results in larger levels of abatement than under larger coalitions.

When both the results of the asymmetry analysis, and the properties shown to hold only for the optimal sharing rules are taken into consideration, there can be no doubt that the optimal sharing rules are the most effective sharing scheme for a coalition formation game with positive externalities, such as the formation of a self-enforcing IEA.

These results are impossible to attain under symmetry, and call attention to the role that optimal transfers can play in increasing cooperation and addressing equity issues, without impinging on efficiency objectives. Furthermore, in contrast to the conventional wisdom of Barrett (1994), when there are large gains to cooperation, asymmetry can result in a significant increase in abatement and payoffs. Consequently, self-enforcing IEAs can markedly improve on the non-cooperative result when the gains to cooperation are large and optimal transfer schemes featuring the properties demonstrated by the optimal sharing rule are incorporated.

These results underline the importance of using optimal sharing rules, which take into consideration free-rider incentives, as opposed to the use of ad hoc sharing rules such as the conventional transfer schemes, which do not consider free-riding.

In the context of the formation of IEAs, the rules regarding individual abatement requirements do not, in reality, incorporate the characteristics of the optimal sharing rules, nor do they address coalition stability. Clearly, the use of a scheme that takes into consideration the free-rider incentives endemic in such agreements could improve upon both participation and the stability of an agreement. In particular, the robustness result would mitigate the equity issues that are inherent throughout the global negotiations. The implementation of such a scheme is outside the scope of this paper, although the use of the Global Environmental Fund (GEF) or Clean Development Mechanism (CDM) provide a framework within which the principles of such a transfer scheme could be applied.

Extensions to this paper may include, as mentioned above, the design of a mechanism to incorporate the features of the optimal sharing rules, resulting in a practical application of the concepts in a policy setting. Alternatively, from a more theoretical perspective, the application of the optimal sharing rule to a multiple-coalition, open-membership game could be explored. However, this would require a much more complicated analysis and new stability criteria to take into account stability between coalitions as well as the decision to free-ride. Finally, it would be interesting to systematically examine the effect of asymmetry and the optimal sharing rule under uncertainty. This would provide a deeper understanding of the role that asymmetry plays in coalition formation games, its implications for policy making and the design of IEAs.

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Appendices

The following tables show the payoffs for each nation under the conditions of transfers and no transfers. Each table shows: the size of the coalition and the members that are part of it (column 1); the payoff to each member (columns 2-6); the aggregate payoff to all members and non-members (column 7); the sum of all internally stable coalitions under that transfer scheme (column 8); the sum of all externally stable coalitions under that transfer scheme (column 9) and the sum of all stable coalitions (column 10).

Tables A-1 - A-5 relate to the analysis of the optimal sharing rule properties. The tables are calculated using the parameters shown in Table 1, in section 4, using the linear-quadratic payoff function (equation 3) in section 2.

Table A-1: Payoff Function with No Transfers

Payoff Function	$\pi_1(S)$	$\pi_2(S)$	$\pi_3(S)$	$\pi_4(S)$	$\pi_5(S)$	$\sum_j \pi_j(S)$	IS	ES	S
Singletons	297	126	35	348	110	916	1	0	0
{1, 2}	508	35	61	641	204	1449	0	0	0
{1, 3}	373	163	-3	445	141	1120	0	0	0
{1, 4}	294	203	53	450	175	1175	0	1	0
{1, 5}	496	227	59	618	21	1422	0	0	0
{2, 3}	350	138	30	407	129	1056	0	0	0
{2, 4}	566	4	62	610	205	1446	0	0	0
{2, 5}	459	148	51	527	120	1305	1	0	0
{3, 4}	388	164	-11	441	142	1124	0	0	0
{3, 5}	344	146	32	401	119	1042	0	0	0
{4, 5}	546	229	60	594	-7	1423	0	0	0
{1, 2, 3}	634	61	-12	797	253	1733	0	0	0
{1, 2, 4}	724	-254	107	978	363	1917	0	1	0
{1, 2, 5}	854	80	102	1089	38	2164	0	0	0
{1, 3, 4}	449	277	-115	633	238	1483	0	1	0
{1, 3, 5}	617	283	-8	767	39	1699	0	0	0
{1, 4, 5}	701	407	103	943	-259	1894	0	1	0
{2, 3, 4}	711	29	-24	759	256	1731	0	0	0
{2, 3, 5}	560	172	33	639	139	1542	0	0	0
{2, 4, 5}	978	43	103	1025	3	2152	0	0	0
{3, 4, 5}	685	287	-19	737	11	1701	0	0	0
{1, 2, 3, 4}	929	-215	-133	1218	444	2244	0	1	0
{1, 2, 3, 5}	1024	118	-12	1298	67	2495	0	0	0
{1, 2, 4, 5}	1277	-191	173	1640	-241	2659	0	1	0
{1, 3, 4, 5}	900	500	-125	1176	-231	2221	0	1	0
{2, 3, 4, 5}	1170	81	-26	1225	31	2481	0	0	0
{1, 2, 3, 4, 5}	1526	-139	-137	1931	-202	2978	0	1	0

Table A-2: Payoff Function for the Shapley value

Payoff Function	$\pi_1^S(S)$	$\pi_2^S(S)$	$\pi_3^S(S)$	$\pi_4^S(S)$	$\pi_5^S(S)$	$\sum_j \pi_j^S(S)$	IS	ES	S
Singletons	297	126	35	348	110	916	1	0	0
{1, 2}	357	186	61	641	204	1449	1	0	0
{1, 3}	316	163	54	445	141	1120	1	0	0
{1, 4}	346	203	53	398	175	1175	1	0	0
{1, 5}	352	227	59	618	165	1422	1	0	0
{2, 3}	350	130	39	407	129	1056	1	0	0
{2, 4}	566	196	62	418	205	1446	1	0	0
{2, 5}	459	142	51	527	126	1305	1	0	0
{3, 4}	388	164	58	372	142	1124	1	0	0
{3, 5}	344	146	38	401	113	1042	1	0	0
{4, 5}	546	229	60	413	175	1423	1	0	0
{1, 2, 3}	396	209	78	797	253	1733	1	0	0
{1, 2, 4}	512	362	107	573	363	1917	0	0	0
{1, 2, 5}	471	261	102	1089	240	2164	1	0	0
{1, 3, 4}	400	277	112	456	238	1483	1	0	0
{1, 3, 5}	388	283	74	767	186	1699	1	0	0
{1, 4, 5}	498	407	103	559	327	1894	0	0	0
{2, 3, 4}	711	220	83	462	255	1731	1	0	0
{2, 3, 5}	560	155	50	639	138	1542	0	1	0
{2, 4, 5}	978	274	103	545	253	2152	1	0	0
{3, 4, 5}	685	287	79	454	196	1701	1	0	0
{1, 2, 3, 4}	586	406	156	652	444	2244	0	0	0
{1, 2, 3, 5}	527	294	107	1298	270	2495	0	1	0
{1, 2, 4, 5}	724	500	173	798	465	2659	0	0	0
{1, 3, 4, 5}	569	500	151	635	365	2221	0	0	0
{2, 3, 4, 5}	1170	308	113	607	283	2481	0	1	0
{1, 2, 3, 4, 5}	814	553	204	895	512	2978	0	1	0

The payoffs for the Shapley value are calculated using the formula:

$$\pi_i^{Shap}(S) = \sum_{T \subseteq S} \frac{[t-1]![s-t]!}{t!} [\pi_T(T) - \pi_{T \setminus \{i\}}(T \setminus \{i\})] \; \forall i \in S \tag{6}$$

Table A-3: Payoff Function for the Nash bargaining solution

Payoff Function	$\pi_1^N(S)$	$\pi_2^N(S)$	$\pi_3^N(S)$	$\pi_4^N(S)$	$\pi_5^N(S)$	$\sum_j \pi_j^N(S)$	IS	ES	S
Singletons	297	126	35	348	110	916	1	0	0
{1, 2}	357	186	61	641	204	1449	1	0	0
{1, 3}	316	163	54	445	141	1120	1	0	0
{1, 4}	346	203	53	398	175	1175	1	0	0
{1, 5}	352	227	59	618	165	1422	1	0	0
{2, 3}	350	130	39	407	129	1056	1	0	0
{2, 4}	566	196	62	418	205	1446	1	0	0
{2, 5}	459	142	51	527	126	1305	1	0	0
{3, 4}	388	164	58	372	142	1124	1	0	0
{3, 5}	344	146	38	401	113	1042	1	0	0
{4, 5}	546	229	60	413	175	1423	1	0	0
{1, 2, 3}	372	201	110	797	253	1733	1	0	0
{1, 2, 4}	522	352	107	574	363	1917	0	0	0
{1, 2, 5}	443	273	102	1089	257	2164	0	0	0
{1, 3, 4}	393	277	131	444	238	1483	0	0	0
{1, 3, 5}	365	283	104	767	179	1699	1	0	0
{1, 4, 5}	506	407	103	558	329	1894	0	0	0
{2, 3, 4}	711	211	120	433	255	1731	1	0	0
{2, 3, 5}	560	150	59	639	134	1542	1	1	1
{2, 4, 5}	978	289	103	510	273	2152	0	0	0
{3, 4, 5}	685	287	114	427	189	1701	1	0	0
{1, 2, 3, 4}	545	375	283	597	444	2244	0	0	0
{1, 2, 3, 5}	454	284	192	1298	268	2495	0	1	0
{1, 2, 4, 5}	698	528	173	749	512	2659	0	0	0
{1, 3, 4, 5}	529	500	268	581	343	2221	0	0	0
{2, 3, 4, 5}	1170	299	208	521	283	2481	0	1	0
{1, 2, 3, 4, 5}	709	539	447	760	523	2978	0	1	0

The payoffs for the Nash bargaining solution are calculated using the formula:

$$\pi_i^{Nash}(S) = \pi_i(\{i\}) + \lambda_i[\pi_S(S) - \sum_{i \in S} \pi_i(\{i\})] \tag{7}$$

in which we assume $\sum_{j \in S} \lambda_j = 1$ and that there are equal weights i.e. $\lambda_i = \frac{1}{S}$.

Table A-4: Payoff Function for the Chander Tulkens transfer scheme

Payoff Function	$\pi_1^C(S)$	$\pi_2^C(S)$	$\pi_3^C(S)$	$\pi_4^C(S)$	$\pi_5^C(S)$	$\sum_j \pi_j^C(S)$	IS	ES	S
Singletons	297	126	35	348	110	916	1	0	0
{1, 2}	382	161	61	641	204	1449	1	0	0
{1, 3}	332	163	38	445	141	1120	1	0	0
{1, 4}	344	203	53	400	175	1175	1	0	0
{1, 5}	378	227	59	618	139	1422	1	0	0
{2, 3}	350	132	36	407	129	1056	1	0	0
{2, 4}	566	164	62	449	205	1446	1	0	0
{2, 5}	459	146	51	527	125	1305	1	0	0
{3, 4}	388	164	39	391	142	1124	1	0	0
{3, 5}	344	146	36	401	115	1042	1	0	0
{4, 5}	546	229	60	446	142	1423	1	0	0
{1, 2, 3}	446	187	50	797	253	1733	0	1	0
{1, 2, 4}	566	237	107	644	363	1917	1	1	1
{1, 2, 5}	546	230	102	1089	198	2164	0	1	0
{1, 3, 4}	428	277	48	492	238	1483	0	0	0
{1, 3, 5}	439	283	49	767	160	1699	0	1	0
{1, 4, 5}	554	407	103	631	200	1894	1	1	1
{2, 3, 4}	711	192	51	522	255	1731	0	1	0
{2, 3, 5}	560	161	43	639	140	1542	0	0	0
{2, 4, 5}	978	234	103	636	202	2152	0	1	0
{3, 4, 5}	685	287	50	516	164	1701	0	1	0
{1, 2, 3, 4}	677	283	73	767	444	2244	0	1	0
{1, 2, 3, 5}	634	266	69	1298	229	2495	0	1	0
{1, 2, 4, 5}	857	358	173	965	307	2659	0	1	0
{1, 3, 4, 5}	662	500	71	750	238	2221	0	1	0
{2, 3, 4, 5}	1170	272	70	736	234	2481	0	1	0
{1, 2, 3, 4, 5}	992	413	104	1114	354	2978	0	1	0

The payoffs for the Chander Tulkens transfer scheme are calculated using the same formula as the Nash bargaining solution:

$$\pi_i^{NASH}(S) = \pi_i(\{i\}) + \lambda_i[\pi_S(S) - \sum_{i \in S} \pi_i(\{i\})] \tag{8}$$

in which we assume $\sum_{j \in S} \lambda_j = 1$. However, in contrast to the Nash bargaining solution, the weights are equal to each members marginal damages i.e. $\lambda_i = (\frac{b_i}{\sum_{i \in S} b_i})$. The values for b_i are taken from Table 1.

Table A-5: Payoff function for the Optimal Sharing Rule

Payoff Function	$\pi_1^*(S)$	$\pi_2^*(S)$	$\pi_3^*(S)$	$\pi_4^*(S)$	$\pi_5^*(S)$	$\sum_j \pi_j^*(S)$	IS	ES	S
Singletons	297	126	35	348	110	916	1	0	0
{1, 2}	357	186	61	641	204	1449	1	0	0
{1, 3}	316	163	54	445	141	1120	1	0	0
{1, 4}	346	203	53	398	175	1175	1	0	0
{1, 5}	352	227	59	618	165	1422	1	0	0
{2, 3}	350	130	39	407	129	1056	1	0	0
{2, 4}	566	196	62	418	205	1446	1	0	0
{2, 5}	459	142	51	527	126	1305	1	0	0
{3, 4}	388	164	58	372	142	1124	1	0	0
{3, 5}	344	146	38	401	113	1042	1	0	0
{4, 5}	546	229	60	413	175	1423	1	0	0
{1, 2, 3}	386	199	97	797	253	1733	1	1	1
{1, 2, 4}	578	216	107	654	363	1917	1	1	1
{1, 2, 5}	486	255	102	1089	231	2164	1	1	1
{1, 3, 4}	415	277	80	472	238	1483	1	1	1
{1, 3, 5}	379	283	94	767	176	1699	1	1	1
{1, 4, 5}	561	407	103	633	191	1894	1	1	1
{2, 3, 4}	711	208	106	451	255	1731	1	0	0
{2, 3, 5}	560	152	57	639	135	1542	1	0	0
{2, 4, 5}	978	266	103	564	242	2152	1	0	0
{3, 4, 5}	685	287	102	443	185	1701	1	0	0
{1, 2, 3, 4}	688	254	84	774	444	2244	0	1	0
{1, 2, 3, 5}	560	283	102	1298	253	2495	0	1	0
{1, 2, 4, 5}	890	319	173	1002	275	2659	0	1	0
{1, 3, 4, 5}	667	500	85	749	220	2221	0	1	0
{2, 3, 4, 5}	1170	293	110	645	262	2481	1	1	1
{1, 2, 3, 4, 5}	1049	379	51	1176	323	2978	0	1	0

The payoffs for the optimal sharing rule is calculated using the formula:

$$\pi_i^{*(\lambda)}(S) = \pi_i(S \setminus \{i\}) + \lambda_i(S)[\pi_S(S) - \sum_{i \in S} \pi(S \setminus \{i\})] \tag{9}$$

in which we assume $\sum_{j \in S} \lambda_j = 1$ and that there are equal weights i.e. $\lambda_i = \frac{1}{S}$.

Tables A-6 - A-7 refer to the parameters used in the asymmetry analysis conducted in section 5. Tables A-6 and A-6 show: the difference in the value of the parameter between each nation (column 1); the sum of the differences between the largest and smallest value of the parameters (column 2); the parameter value given to nations 1-7 (columns 3-9); the value of the parameter held fixed (column 10) and the mean value of the variable parameter (column 11), equal to 100, represented by nation 4.

Table 6: Parameters for Asymmetry in Costs

d	$\sum d$	c_1	c_2	c_3	c_4	c_5	c_6	c_7	b_i	μ
1	6	97	98	99	100	101	102	103	100	100
5	30	85	90	95	100	105	110	115	100	100
10	60	70	80	90	100	110	120	130	100	100
15	90	55	70	85	100	115	130	145	100	100
20	120	40	60	80	100	120	140	160	100	100
25	150	25	50	75	100	125	150	175	100	100
30	180	10	40	70	100	130	160	190	100	100

Table 7: Parameters for Asymmetry in Benefits

d	$\sum d$	b_1	b_2	b_3	b_4	b_5	b_6	b_7	c_i	μ
1	6	97	98	99	100	101	102	103	100	100
5	30	85	90	95	100	105	110	115	100	100
10	60	70	80	90	100	110	120	130	100	100
15	90	55	70	85	100	115	130	145	100	100
20	120	40	60	80	100	120	140	160	100	100
25	150	25	50	75	100	125	150	175	100	100
30	180	10	40	70	100	130	160	190	100	100

Tables A-8 - A-9 present: the difference in the value of the parameter between each nation (column 1); the sum of the differences between the largest and smallest value of the parameters (column 2); the value of the benefit parameter given to nations 1-7 (columns 3-9); the value of the cost parameter given to nations 1-7 (columns 10 - 16); the mean value of the benefit parameter (column 17), equal to 100, represented by nation 4 and the mean value of the cost parameter (column 18), equal to 100, represented by nation 4.

Table 8: Parameters for Asymmetry HiLo LoHi

d	$\sum d$	b_1	b_2	b_3	b_4	b_5	b_6	b_7	c_1	c_2	c_3	c_4	c_5	c_6	c_7	μ_b	μ_c
1	6	97	98	99	100	101	102	103	103	102	101	100	99	98	97	100	100
5	30	85	90	95	100	105	110	115	115	110	105	100	95	90	85	100	100
10	60	70	80	90	100	110	120	130	130	120	110	100	90	80	70	100	100
15	90	55	70	85	100	115	130	145	145	130	115	100	85	70	55	100	100
20	120	40	60	80	100	120	140	160	160	140	120	100	80	60	40	100	100
25	150	25	50	75	100	125	150	175	175	150	125	100	75	50	25	100	100
30	180	10	40	70	100	130	160	190	190	160	130	100	70	40	10	100	100

Table 9: Parameters for Asymmetry HiHi LoLo

d	$\sum d$	b_1	b_2	b_3	b_4	b_5	b_6	b_7	c_1	c_2	c_3	c_4	c_5	c_6	c_7	μ_b	μ_c
1	6	97	98	99	100	101	102	103	97	98	99	100	101	102	103	100	100
5	30	85	90	95	100	105	110	115	85	90	95	100	105	110	115	100	100
10	60	70	80	90	100	110	120	130	70	80	90	100	110	120	130	100	100
15	90	55	70	85	100	115	130	145	55	70	85	100	115	130	145	100	100
20	120	40	60	80	100	120	140	160	40	60	80	100	120	140	160	100	100
25	150	25	50	75	100	125	150	175	25	50	75	100	125	150	175	100	100
30	180	10	40	70	100	130	160	190	10	40	70	100	130	160	190	100	100