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Hotelling Rules: Oscillatory Versus Quadratic Trends in Natural Resource Prices

Antonios Antypas* Phoebe Koundouri^{†‡}
Nikolaos Kourogenis[§]

Abstract. A model is introduced for the description of natural resources' price paths, which, in contrast to the existing literature, captures non-linear trends by means of a simple trigonometric function. This model is then compared by means of a set of model selection criteria with a quadratic trend model and with a more general one that nests both models. All models are estimated on the price series of eleven major natural resources. In most cases, the trigonometric trend model is selected as the one better fitting the data, providing evidence against the long-run increase of the corresponding natural resource real prices, with interesting policy implications.

Key words: Oscillatory trend, quadratic trend, Hotelling rule, natural resource prices, model selection.

JEL classification: E3, C22.

*Department of Banking and Financial Management, University of Piraeus.

[†]Department of International and European Economic Studies, Athens University of Economics and Business.

[‡]Grantham Research Institute on Climate Change and the Environment, London School of Economics.

[§]**Corresponding author:** Nikolaos Kourogenis, Department of Banking and Financial Management, University of Piraeus, 80 Karaoli & Dimitriou St., 185 34, Piraeus, GREECE; email: nkourogenis@yahoo.com ; tel: +302104142142.

1 Introduction.

The implications of increased natural resource scarcity and its effect on economic growth have been discussed since the 18th century. Malthus (1798) and Ricardo (1817) held that agricultural land scarcity implied strict limits on population growth and the development of living standards. Harold Hotelling offered his well-known counterargument in his seminal article of 1931: Competitive firms would manage exhaustible resource stocks to maximize present-value profits; competitive extraction paths would therefore match those chosen by a social planner seeking to maximize intertemporal social surplus; and subject to the caveat of social and private discount rates equality; equivalence between competitive outcome and the work of a rational social planner would be achieved. The Hotelling rule provides the fundamental no-arbitrage condition that every competitive or efficient resource utilization path has to meet. In its basic form it indicates that along such a path the price of an exhaustible resource has to grow with a rate that equals the interest rate.

Hotelling's theory was not empirically tested until the second half of the 20th century. Slade and Thille (1997) categorized the existing empirical tests as (a) price behaviour, (b) shadow price, and (c) Hotelling valuation tests. Extant empirical tests showed mixed support. Barnett and Morse examined trends in the prices and unit costs of extractive goods (including agricultural, mineral, and forest products) in the United States. Their findings suggested that natural resources were becoming less scarce, not scarcer, in an economic sense. Smith (1979) employed an econometric analysis of annual (1900-1973) price data of four aggregate resource groups and concluded that the trend in mineral prices was negative with

the rate of decline decreasing over time in absolute magnitude. These results raised the question if the basic Hotelling model is sufficient to explain the real world and motivated economists to expand it by adopting more realistic assumptions and fit the behaviour of real data.

Among others, Solow and Wan (1976) by assuming increasing extraction costs and Pindyck (1978) by adopting unlimited potential reserves or the presence of uncertainty (1980), demonstrated that Hotelling's model is able to give expectations for falling resource prices. Slade (1982) allowed for the presence of technological progress which reduces the production cost and therefore the price paths for nonrenewable natural resources can be U-shaped. Slade hypothesized that the declining, flat and increasing price trends implicit in U-shaped price paths, come at different points in the life cycle of the exhaustive resource. Berck and Roberts (1996) suggested three cases where the prices can be expected to fall or stagnant, namely "the depletion and progress case, the great abundance case and the environmental constraint case." Their empirical findings suggest that it is more adequate to consider that resource prices exhibit trend over short time of periods, while this trending behaviour is not reflected in the large samples. In a more recent paper Slade and Thille (1997) acknowledge the fact that Slade's model (1982) did not receive much support by the subsequent data: "Since that time however, prices have been increasingly volatile with large run-ups followed by equally large declines but there is little evidence of sustained trends" (1997, pp. 688). In view of this evidence, they developed another theoretical model (different than Slade 1982) which is able to produce substantial periods of falling prices.

More recent studies also deal with the temporal properties of nonrenewable resource prices testing whether prices exhibit deterministic or stochastic trends.

Slade (1988), Berck and Roberts (1996) and Ahrens and Sharma (1997) find evidence that many of natural resource commodity prices have a stochastic trend. However, in a more recent paper, Lee, List and Strazicich (2006) reject stochastic trend behavior under the alternative of a quadratic trend with two breaks.

This paper contributes to the literature reviewed above, by examining whether natural resource prices exhibit oscillatory behavior, that is, periods of falling prices followed by periods of increasing prices, which may again be followed by periods of falling prices. If we show that the natural resource price model exhibits oscillatory trends, then the polynomial trend function suggested in the literature, is not the relevant one. It is worth mentioning that existing Hotelling based models can support the existence of the oscillatory structure by modifying initial assumptions. For example, consider the technological change that occurred in the beginning of the twentieth century. This change may have caused the real prices to fall until they reach the point at which the effects of this change can no longer sustain a relatively low price. From this point onwards, the real price starts increasing thus producing the first U-shaped part in the overall picture of the behaviour of the real price. Then at some later point, for example after the World War II, a new technological change occurs (or a new discovery is made) which forces the real price to start falling again. At this point a second U-shaped pattern starts forming which will be eventually completed when the second technological wave gets exhausted. This means that instead of a single U-shaped pattern in our long series of real prices, we might be able to identify more than one U-shaped patterns, the number of which may be determined by means of statistical criteria. This scenario is related to the criticism that Mueller and Gorin (1985) applied to the single U-shaped pattern of Slade (1982), according to which the technological progress does not

evolve smoothly over time but occurs in discrete jumps. Moreover, these sequential cycles may occur either around a constant mean or a linear or quadratic trend. In the first case, the scarcity property is fully compensated by the technological progress, whereas in the second case (of an upward sloping trend), the ability of (the discrete jumps of) technological progress to alleviate the scarcity feature of the real price is decreasing over time.

In particular, in this paper we analyze the prices of the main fuel and metal resources that have been considered in the literature reviewed above, that is, the prices of aluminum, copper, iron, lead, nickel, silver, tin, zinc, bituminous coal, petroleum and natural gas. Then by using a set of model selection criteria we find that in most cases a trigonometric trend model, which supports oscillatory trends, outperforms Slade's (1982) quadratic trend model, as well as a more general one, that nests both the trigonometric and quadratic models. These results provide empirical support for the observation made by Smith (1979) that the estimated polynomial coefficients are unstable over time. The oscillatory cycles occur at a very low frequency within the sample. This behaviour is manifested as a U-shaped trend in the data whose curvature, however, is sinusoidal rather than quadratic.

Our results have implications for the validity of the natural resources scarcity hypothesis, as well as long-run natural resource optimal pricing and conservation policies. In particular, our results suggest that we should expect that in the long-run, the natural resource real prices will not exhibit a monotonic trend, linear or quadratic, but instead will oscillate around their mean. This result should be internalized in natural resources pricing and conservation policies.

2 Oscillatory Trends in Natural Resource Prices

Let y_t denote the real price of the natural resource commodity i . We assume that y_t is equal to the sum of a deterministic, $g(t; \theta)$ and a stochastic component u_t ,

$$y_t = g(t; \theta) + u_t. \quad (1)$$

The deterministic component is a parametric function of time, t , with θ denoting the parameter(s) in g , whereas the stochastic component, u_t , is a sequence of random variables that may exhibit temporal dependence and heterogeneity. In this set up, the applied researcher has to deal with the following two issues, referred to as *specification* and *estimation* problems. The specification problem concerns the choice of the function g . The estimation problem addresses the issue of conducting asymptotic inferences on θ (estimation and hypothesis testing) in an optimal way. In particular, in this stage, the applied researcher has to choose specific testing procedures for hypothesis testing on θ that retain their optimal properties in the presence of various departures (some of which may be severe) of u_t from the *iid* benchmark. For example, u_t might display a very high degree of persistence together with unconditional and/or conditional heteroscedasticity. Next, the specification and estimation issues are analyzed in detail.

2.1 Trend Specification

Following Slade (1982), the empirical literature has specified $g(t; \theta)$ as a quadratic polynomial of t , that is

$$g(t; \theta) = c_0 + c_1 t + c_2 t^2. \quad (2)$$

In the context of this specification, the following two competing hypotheses have been tested:

Case I: $c_1 > 0$ and $c_2 \geq 0$. In this case, y_t increases continuously with time in either linear or quadratic fashion.

Case II: $c_1 < 0$ and $c_2 > 0$. In this case, y_t initially decreases and then increases with time, following a U-shaped pattern.

The second case allows for a period during which the real commodity price is falling whereas the first case predicts a continuously increasing real price.

However, as mentioned in the Introduction, the polynomial model (2) restricts severely the set of patterns that the trend in the real prices might follow. In particular, this model does not allow the real price to exhibit oscillatory behaviour. These oscillations are likely to arise if the U-shaped pattern, that is periods over which the real price is falling followed by periods of increasing price, occurs repeatedly over time instead of once.

An interesting case that may arise in practice is the case in which the length of the oscillatory cycles is very long. In such a case, there might be only one U-shaped pattern even in a long data series. However, the curvature of this pattern will be different than the quadratic one implied by (2). Indeed, a single U-shaped pattern with quadratic curvature has totally different implications about the long-run behaviour of the real price than a U-shaped pattern with trigonometric curvature. In the former case, the polynomial origin of the pattern implies that after the initial fall, the real price will increase indefinitely at a quadratic rate. On the contrary, if the observed U-shaped pattern is just the two-thirds of a sinusoidal cycle that is being formed, then the real price is expected to start falling in the near future before it starts rising again.

The preceding discussion suggests that the trend specification (2) should be augmented in a way that allows the real price to exhibit more than one full U-shaped cycles. To this end, we suggest the following specification:

$$g(t; \theta) = c_0 + c_1 t + c_2 t^2 + c_3 \left(\sin \left(\frac{t}{d} \right) + 1 \right) + c_4 t \left(\sin \left(\frac{t}{d} \right) + 1 \right), \quad d > 0. \quad (3)$$

The two extra terms, $(\sin(\frac{t}{d}) + 1)$ and $t(\sin(\frac{t}{d}) + 1)$ in (3) capture the potentially oscillating behaviour of the real price. The parameter, d , controls for the number of the sinusoidal cycles that are likely to be present in a sample of T observations. For example, for $T=100$, the values of d equal to 7.5, 10, 15 and 30 corresponds to approximately 2.1, 1.6, 1.06 and 0.5 cycles, respectively. It is worth noting that the term $c_3(\sin(\frac{t}{d}) + 1)$ with $c_3 < 0$, and $d = 30$ produces a U-shaped pattern in a series of 100 observations whose curvature, however, is different than the quadratic curvature of (2) with $c_1 < 0$ and $c_2 > 0$. The second term, $t(\sin(\frac{t}{d}) + 1)$ is included in (3) to allow the amplitude of the oscillations to be time varying. The above specification nests various special cases among which the following three appear to be the most interesting ones:

Case 1: The General Model, in which all the c_i , $i = 0, 1, 2, 3, 4$ coefficients are different from zero. In this case, the trend function contains both polynomial and oscillatory components, while the oscillations occur around a quadratic trend. However, the long-run behaviour of the deterministic part of y_t is governed by c_2 .

Case 2: The Polynomial Model, in which $c_3 = c_4 = 0$. This is the standard case already analyzed in the literature, which is identical to (2). Again a positive estimate of c_2 is interpreted as favorable evidence for the scarcity hypothesis.

Case 3: The Oscillatory Model, in which $c_1 = c_2 = 0$. This case can be further

decomposed into two additional subcases, according to whether the coefficient c_4 is equal to zero. Specifically, if both c_3 and c_4 are different from zero, then $g(t; \theta)$ exhibits trending-like behaviour (despite the fact that $c_1 = 0$) due to the positivity of the term multiplied by c_4 and the changing amplitude of the oscillations over time. We refer to this case as Oscillatory-I model. The second case, which is the only case in which the trending behaviour of the real price is purely oscillatory (with no polynomial or polynomial-like elements) occurs when $c_1 = c_2 = c_4 = 0$ and $c_3 \neq 0$. In such a case, which we refer to as Oscillatory-II model, the amplitude of the oscillations remains constant over time. Moreover, as already mentioned for values of d , which are relatively large with respect to the sample size, and $c_3 < 0$, the Oscillatory-II model produces a single U-shaped pattern in the available data, whose curvature however, is different than the one implied by the Polynomial model. This U-shaped pattern observed in the available data is only the first part of a more general sinusoidal pattern that has started to be formed. In spite of the fact that we have detected a U-shaped pattern within our sample, we are entitled to expect the real price to experience a long period of decrease in the future.

2.2 Implications of Trends Misspecification

In this subsection we investigate the effects of omitting the oscillatory terms, $(\sin(\frac{t}{d}) + 1)$ and $t(\sin(\frac{t}{d}) + 1)$ from the estimated trend equation on the inferences on the coefficients c_1 and c_2 of the polynomial terms t and t^2 , respectively. In other words, under the assumption that the correct trend specification is given by (3), we investigate the reliability of inferences on c_1 and c_2 that are produced when the researcher has erroneously specified the trend function (2). As will be

shown below the inferences on c_1 and c_2 are severely distorted even under the assumption that u_t is an iid process.

To this end we have conducted a small Monte Carlo study as follows: The simulated data are produced according to (1) with $g(t; \theta)$ being given by (3) and $u_t \sim iid N(0, \sigma_u^2)$. As far as the parameter d is concerned, we consider four alternative values, namely $d = 7.5, 10, 15$ and 30 . For each value of d we use the corresponding values obtained by estimating the parameters of (3) for aluminum, as presented in Table 1. The simulated series are then produced by setting the autoregressive parameter, ρ , equal to zero.

(TABLE 1 AROUND HERE)

Next, for each simulated series we estimate the model that employs the trend function $c'_0 + c'_1 t + c'_2 t^2$ instead of the correct one given by (3), thus being a misspecified model. Table 2 reports the rejection rates of the null hypotheses $c'_1 = c_1$ and $c'_2 = c_2$ with c_1 and c_2 as in Table 1. Moreover, we report the empirical size for testing the null hypothesis that u_t is a serially uncorrelated process, that is, for testing $\rho = 0$ in the context of an AR(1) model for the error term, $u_t = \rho u_{t-1} + \nu_t$. The number of replications is equal to 5000 and the size, T , of the sample is set equal to 100 and 200.

(TABLE 2 AROUND HERE)

As expected, severe size distortions are present for all sample sizes under consideration. Moreover, the empirical size of the t-test for testing the hypothesis

$\rho = 0$ is increasing with the sample size, reaching the value of 100% for a sample size of $T=200$. This means that the omitted oscillatory terms from the estimated trend model cause the researcher to erroneously conclude that y_t exhibits a substantial degree of persistence, whereas in fact y_t is serially uncorrelated. In fact, the test misinterprets the deterministic oscillation that characterizes the residuals of the estimated model (which is the result of the omitted terms) as stochastic cycles which are “captured” by estimates of ρ , which appear to be statistically different from zero.

2.3 Identification of the Trend Function

The preceding analysis has shown that the omission of the oscillatory terms may produce misleading inferences concerning the coefficients of the polynomial terms in the trend function. In this subsection, we investigate the extent to which the information criteria suggested by Akaike (1973), Schwarz (1978) and Hannan and Quinn (1979), denoted by AIC, SIC and HQ respectively, are capable of detecting the correct model within a set, \mathcal{M} , of competitive models which consists of the General, the Polynomial, the Oscillatory-I and the Oscillatory-II models defined above.

One feature that distinguishes AIC from SIC or HQ concerns the question of whether the true model is actually included in \mathcal{M} . If it is, the SIC and HQ consistently select the true model, that is, the selection rate of the true model approaches 100% as the sample size increases. On the other hand, if the true model is not included in \mathcal{M} , then AIC tends to select the best approximating model to the true one. Put it differently, AIC, as opposed to SIC and HQ, was not

designed to consistently estimate the true model. Inconsistency of AIC, however, is not always treated as an unpleasant feature of the selection procedure, especially in cases where the true model is not expected to belong to \mathcal{M} . According to Shibata (1983), “Inconsistency does not imply a defect of the selection procedure, but rather the inevitable concomitant of balancing underfitting and overfitting risks.” In the case under study, if oscillations of any form are present in the trend function (even if they are of different parametric form than the one specified in (3)) then AIC is expected to display a tendency towards selecting a model which contains oscillatory terms (that is one among the General, the Oscillatory-I and the Oscillatory-II models) over the polynomial model at a higher frequency than SIC or HQ. On the contrary, if such oscillations are absent and the trend function displays solely polynomial-type behaviour then SIC and HQ are expected to select the true polynomial model (2) at a higher frequency than AIC.

To investigate these issues, we conduct a Monte Carlo study as follows: Concerning the trend function, we examine four alternative scenarios: (i) $g(t; \theta)$ follows the General model, (ii) $g(t; \theta)$ follows the Polynomial model ($c_3 = c_4 = 0$), (iii) $g(t; \theta)$ follows the Oscillatory-I model ($c_1 = c_2 = 0$), and (iv) $g(t; \theta)$ follows the Oscillatory-II model ($c_1 = c_2 = c_4 = 0$). For each scenario, we examine the percentages by which the AIC, SIC and HQ select the correct model between the four models mentioned above. The number of replications is equal to 5000 and the sample size, T , is set 100 and 200. The models are estimated by Generalised Least Squares (GLS), in which the error term is assumed to follow an AR(1) process, $u_t = \rho u_{t-1} + v_t$. In this setting the researcher is assumed to have full information on the true parametric structure of the error. Additional experiments, in which the order, p , of the AR(p) specification in GLS is different than the true lag order

of the autoregressive representation of u_t , have also been conducted, with results similar to those of the “full information” case. Concerning the parameters, θ , of the trend functions, we explored many alternative parameter settings, covering the majority of cases that present either theoretical or empirical interest. For brevity, we report the results for the case in which the parameters of the data generating process equal to these of Table 1. The results, reported in Table 3 may be summarized as follows:

(TABLE 3 AROUND HERE)

(i) When the series is generated by the General model and $T=100$, all criteria seem to be biased towards the Polynomial model. This bias is significantly higher for SIC, reaching a selection of the Polynomial model for 85% of the simulated series when $d=10$. AIC exhibits the best performance in selecting the true General model for all values of d , followed by HQ. However, when $d=7.5$ or $d=10$, both AIC and HQ select the Polynomial model for more than 50% of the simulated series, while the same holds for SIC for all the values of d . When the sample size increases to $T=200$, in which case there are more than one cycles in the data even for $d=30$, all three criteria select the true General model with frequency practically equal to 100%.

(ii) In the case that the true model is the Polynomial, SIC is the best performing criterion followed by HQ and AIC. SIC seems to work well even when the sample is small, with a correct model selection for at least 92% of the simulated series for any value of d . Even AIC, which is the worst performing criterion, selects the correct model at a rate of at least 73%. When $T=200$, the rate of correct model selection

is practically 100% for SIC and at least 93% for HQ. However, AIC remains biased towards the General model, which is selected with a rate of around 20%, while the correct, Polynomial model, is selected at a rate of around 80%. As expected, in this case, the performance of all three criteria is invariant to the value of d .

(iii) When the true model is the Oscillatory-I and $T=100$, the performances of all three criteria are similar, with rates of selecting the correct model of about 50%–55% when $d=7.5$ and $d=10$, at most 23% when $d=15$ and at least 70% when $d=30$. It is worth mentioning that when $d=15$, all criteria select the Oscillatory-II model at a rate of more than 50%. When $T=200$, and $d=7.5$, 10 and 30, SIC emerges again as the best performing criterion with almost 100% rates of correct model selection, while HQ follows with corresponding rates of at least 93%. When $T=200$ and $d=15$, the rate of correct model selection drops to 63% for SIC and 75% for HQ, these criteria being biased towards the Oscillatory-II model. AIC exhibits rates of correct model selection that range from 72%–81%, while it remains biased towards the General model.

(iv) When the series are generated by the Oscillatory-II model and $T=100$, the rates of correct model selection range between 62% and 66% for AIC, 89% and 93% for SIC, and 78% and 82% for HQ. When $T=200$, a 5%–10% improvement is observed for AIC and HQ, while SIC selects the correct model at a rate of at least 96% for all values of d .

(v) Very interesting conclusions are obtained by aggregating the results in two main categories, the first involving the models that have a polynomial trend (General and Polynomial) and the second consisting by the models that do not have a polynomial trend. Then we observe that when the true model is the General or the Polynomial, all three criteria identify the existence of a polynomial trend

at rates of more than 88% even when $T=100$. In particular, AIC exhibits the best performance, identifying the existence of a polynomial trend in at least 96% of the cases when $d=7.5$ or 10, and with a 100% success when $d=15$ or $d=30$. When $T=200$, all three criteria exhibit practically a 100% success in identifying the polynomial trend.

(vi) When the true data generating process does not involve a polynomial trend (Oscillatory-I and Oscillatory-II models), things are not so clear cut. Although in all cases the three criteria do not select models that have a polynomial trend at rates of at least 57%, SIC and HQ exhibit some bias towards the Polynomial model when $T=100$, while when $T=200$, AIC performs worst among the three criteria, selecting models that do not have a polynomial trend approximately 80% of the trials.

Combining these remarks we may conclude to the following three “rules of thumb” concerning model selection using AIC, SIC and HQ, between the competing models under consideration for the empirically relevant case of $T=100$:

(a) When all criteria select Oscillatory-I or -II models then it is highly probable that the true generating process of the series has an oscillatory trend and does not have a quadratic trend.

(b) The same holds (probably with a slightly lower confidence) when SIC and HQ select Oscillatory-I or -II models and AIC selects the General model.

(c) When all criteria select the General model, then it is highly probable that the true generating process of the series has both quadratic and oscillatory trends.

These rules of thumb do not depend on the value of d and, although they do not cover all possible combinations, they concern the cases for which conclusions may be drawn with a relatively high conviction.

3 Empirical Results

We focus on the prices of the main fuel and mineral resources (see also Slade, 1982, Berck and Roberts, 1996, and Ahrens and Sharma, 1997, among others). Historical real prices (at constant 1998 U.S. dollars) for aluminum, copper, iron, lead, nickel, silver, tin and zinc were obtained from U.S. Geological Survey for the period 1900–2010, while historical real prices for bituminous coal, petroleum and natural gas (at constant 2005 U.S. dollars) were collected from Energy Information Administration for the periods 1949–2010, 1900–2010 and 1922–2010, respectively. As far as measurement units are concerned, we have used \$/ton for aluminum, copper, iron, lead, nickel, tin, zinc and bituminous coal, \$/kgr for silver, \$/barrel for petroleum and \$/(1000 cubic feet) for natural gas.

First, we present the results that are based on the three information criteria under consideration, namely AIC, SIC and HQ. The competing models are the ones defined above, namely the General, Polynomial, Oscillatory-I and Oscillatory-II models. All of the four models are estimated by GLS, for each of the eleven commodities. The error term, u_t , is assumed to follow either an AR(1) or an AR(2) or an ARMA(1,1) process. The results from these three alternative specifications are largely the same, and therefore we discuss the results only for the AR(1) case.

Before we present our results, a few remarks on the selected method for accounting for the serial correlation of the error term are in order: In particular, instead of the parametric GLS corrections, we could alternatively employ non-parametric corrections, such as the ones suggested by Newey and West (1987). The employment of these methods requires the applied researcher to make several choices concerning the kernel (e.g. Bartlet, Parzen or Quadratic Spectral) and the

truncation lag or bandwidth parameter, S_T . To this end, Andrews (1991) demonstrated that the Quadratic Spectral kernel is the best with respect to an asymptotic truncated mean square error criterion in the class of kernels that necessarily generate positive semi-definite estimators of the covariance matrix in finite samples. The bandwidth parameter, S_T , may be selected by data dependent methods, such as the parametric methods suggested by Andrews (1991), or the non-parametric ones suggested by Newey and West (1994). Furthermore, Andrews and Monahan (1992) found that prewhitening of u_t is likely to improve the performance of the nonparametric estimators in finite samples.

However, in the empirical problem under study we have observed that the error term u_t exhibits a very high degree of persistence. In such a case, the nonparametric corrections are likely to produce misleading inferences on the trend coefficients, even when the sample size is quite large. In his Monte Carlo study, Vogelsang (1998) found that when u_t is a near-to-unit root process, the nonparametric corrections produce Wald tests that suffer from severe size distortions. As the largest root, ρ , approaches unity, the empirical sizes become very large and deteriorate with the sample size, since the unit-root asymptotics become dominant. On the other hand, the parametric GLS corrections exhibit much better properties, producing test statistics with empirical sizes very close to their corresponding nominal ones. Moreover, when ρ is close to one, GLS was found to exhibit very good power properties (see also Canjels and Watson, 1997).

The GLS-based results, tabulated in Table 4, may be summarized as follows:

(i) For seven commodities, namely, for copper, iron, lead, silver, tin, zinc and natural gas, all criteria select either Oscillatory-I or Oscillatory-II models. Therefore, the conditions required by rule of thumb (a) are satisfied. This implies that

evidence suggests existence of oscillatory trends and absence of quadratic trends in the real prices of these commodities.

(ii) For nickel, AIC selects the General model, while both HQ and SIC select the Oscillatory-II model. These are the conditions required by rule of thumb (b), which implies existence of an oscillatory trend and absence of a quadratic trend in nickel's real price as well.

(iii) For coal, all criteria select the General model, that is, the conditions required by rule of thumb (c) are satisfied. This, in turn, implies that the real price of coal exhibits both quadratic and oscillatory trends. Moreover, the choice of $d=7.5$ in the selected model corresponds to a pattern with multiple U-shapes.

(iv) As far as aluminum is concerned, AIC selects the General model, while SIC and HQ select the Polynomial model. The results of the Monte Carlo simulations (Table 3) do not support a specific model for this case, although they offer some evidence of a quadratic trend in the real price of aluminum.

(v) Finally, AIC and HQ select the polynomial model for the real price of petroleum, while SIC selects the Oscillatory-II model with $d=30$. Examining carefully the results of the Monte Carlo simulations in Table 3, we observe that SIC selects the specific model only when the true model is indeed the Oscillatory-II. On the other hand, there is a small but not negligible probability that AIC and HQ select the Polynomial model, while the series is generated by an Oscillatory-II model with $d=30$. Therefore, this combination of selected models can be interpreted as an indication of oscillatory trend (and not a quadratic one) in the real price of petroleum.

According to the previous remarks and the results of Table 4, the evidence supports that the real prices of copper, lead, silver, tin, nickel, petroleum and

natural gas are better described by the Oscillatory-II model. In other words, the amplitudes of their oscillatory trends remain constant throughout our sample. On the other hand the real price of zinc seems to be better described throughout our sample by either the Oscillatory-II model, or the Oscillatory-I model, where the value of c_4 is negative and statistically significant. Therefore, the empirical evidence does not support an increasing trend (either linear or quadratic) in their real price of eight out of the eleven natural resources under study, namely, in the price of copper, lead, silver, tin, zinc, nickel, petroleum and natural gas. On the other hand, evidence supports the existence of both oscillatory and polynomial (linear or quadratic) trends in the real prices of iron and coal. Finally, the analysis provides some evidence that the real price of aluminum follows a quadratic trend without rejecting, however, the simultaneous existence of an oscillatory trend as well.

(TABLE 4 AROUND HERE)

4 Conclusions

This paper revisits the literature on the long-run trend of natural resource real prices. Simple price models that support oscillatory trend behavior are introduced and tested against the standard quadratic (polynomial) trend price model supported by the relevant literature, via the model selection criteria of Akaike (AIC), Schwarz (SIC) and Hannan and Quinn (HQ).

In order to assess the performance of the model selection criteria, a Monte Carlo study is conducted involving models with either a polynomial or an oscillatory

trend, as well as a more general model that nests both oscillatory and polynomial trend models. The results of the Monte Carlo simulations reveal cases where the combination of the selections made by AIC, SIC and HQ, supports with relatively high conviction the existence of oscillatory trends only. When, however, the three criteria select the general model, the results of the Monte Carlo study support the existence of both polynomial and oscillatory trends.

The aforementioned models are then estimated using the series of real prices of eleven major natural resource commodities. For each commodity, the models selected by AIC, SIC and HQ are obtained. For nine commodity price series, namely, for copper, iron, lead, silver, tin, zinc, natural gas, petroleum and nickel, the combinations of the selected models fall into the category where the results of the Monte Carlo study do not support the existence of quadratic trends. For the seven of these nine commodities, namely for copper, lead, silver, tin, nickel, petroleum and natural gas, the simplest oscillatory model is selected, which excludes linear trends and increasing oscillation amplitudes, while the oscillatory models selected for zinc do not involve an increasing linear trend as well. On the other hand, both a quadratic and an oscillatory trend are identified for the real price of coal. As far as aluminum is concerned, the selected models do not fall into a high conviction category, although there is some evidence that its real price follows a trend that has at least a polynomial component.

Given the very simple structure of the oscillatory models introduced in this study, even the more general model that nests the polynomial and the oscillatory models is quite simple. However, it is worth noting that for the majority of the commodity prices under consideration, an oscillatory model was selected against the more general one. The evidence against the existence of a polynomial compo-

ment in the trend of the natural resource real prices has strong implications for the debate on increasing, or not, natural resources scarcity, and as a consequence, on developing policy interventions for optimal long-run natural resource pricing and conservation.

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Tables

Table 1. Estimated Parameters of the General model for Aluminum ($d = 7.5, 10, 15$ and 30).

Estimated Parameters	d			
	7.5	10	15	30
c_0	11.353	15.675	16.733	18.676
c_1	-0.246	-0.362	-0.366	-0.577
c_2	0.002	0.002	0.002	0.003
c_3	1.479	-0.912	-0.989	-2.077
c_4	-0.026	0.025	-0.009	0.090
ρ	0.627	0.626	0.590	0.590
σ_u^2	1.347	1.348	1.331	1.330

Table 2. Misspecification Effects from Omitting Oscillatory Components (5% Empirical Sizes)

d	$H_0:$	$T = 100$			$T = 200$		
		$c'_1 = c_1$	$c'_2 = c_2$	$\rho = 0$	$c'_1 = c_1$	$c'_2 = c_2$	$\rho = 0$
7.5		86.52	36.88	9.48	45.86	0	100
10		48.3	4.58	8.38	30.44	0	100
15		29.42	11.38	47.78	0.02	0.02	100
30		100	77.34	50.72	0	0	100

Table 3. Performance of Information Criteria: Percent Selections of the Competitive Models (% , rounded to the nearest integer)
A. True models have polynomial terms (General, Polynomial)

T	<i>d</i>	AIC				SIC				HQ			
		7.5	10	15	30	7.5	10	15	30	7.5	10	15	30
100	True=Gen.	43	41	74	72	8	10	32	31	22	23	56	54
	Pol.	53	57	26	28	80	85	67	69	70	73	44	46
	OsI	4	2	0	0	12	5	1	0	8	4	0	0
	OsII	0	0	0	0	0	0	0	0	0	0	0	0
200	True=Gen.	100	100	100	100	98	99	98	100	100	100	100	100
	Pol.	.0	0	0	0	2	1	2	0	0	0	0	0
	OsI	0	0	0	0	0	0	0	0	0	0	0	0
	OsII	0	0	0	0	0	0	0	0	0	0	0	0
100	Gen.	24	23	25	24	3	3	3	3	10	10	11	11
	True=Pol.	73	74	75	76	92	92	97	97	86	86	89	89
	OsI	3	3	0	0	4	5	0	0	4	4	0	0
	OsII	0	0	0	0	1	0	0	0	0	0	0	0
200	Gen.	20	20	19	20	1	1	1	1	6	7	6	6
	True=Pol.	80	80	81	80	99	99	99	99	94	93	94	94
	OsI	0	0	0	0	0	0	0	0	0	0	0	0
	OsII	0	0	0	0	0	0	0	0	0	0	0	0
B. True models do not have any polynomial terms (Oscillatory-I and -II)													
T	<i>d</i>	AIC				SIC				HQ			
		7.5	10	15	30	7.5	10	15	30	7.5	10	15	30
100	Gen.	14	16	16	18	1	2	1	2	6	6	6	8
	Pol.	27	27	5	12	25	26	2	14	27	29	4	13
	True=OsI	53	54	23	70	50	56	11	84	55	58	18	79
	OsII	6	3	56	0	24	16	86	0	12	7	72	0
200	Gen.	19	20	19	19	1	1	1	1	7	7	6	6
	Pol.	0	0	0	0	0	0	0	0	0	0	0	0
	True=OsI	81	80	72	81	99	99	63	99	93	93	75	94
	OsII	0	0	9	0	0	0	36	0	0	0	19	0
100	Gen.	17	14	12	15	1	1	1	1	6	4	4	5
	Pol.	2	10	12	11	1	5	5	4	2	8	9	7
	Os.I	15	14	14	9	5	5	4	3	10	10	8	6
	True=OsII	66	62	62	65	93	89	90	92	82	78	79	82
200	Gen.	13	12	12	12	0	0	0	0	3	3	2	2
	Pol.	0	3	2	0	0	1	1	0	0	2	2	0
	Os.I	15	14	14	15	3	3	3	3	9	7	8	9
	True=OsII	72	71	72	73	97	96	96	97	88	88	88	89

Table 4. Natural Resource Prices: Model Selection and Estimation (p -values below)

Metal	Criteria	Model	c_0	c_1	c_2	c_3	c_4	ρ	d
ALUMINUM	AIC	Gen.	18.676	-0.577	0.003	-2.077	0.090	0.590	30
			0.004	0.197	0.389	0.779	0.065	0.000	
	HQ, SIC	Pol.	13.451	-0.280	0.002			0.661	
			0.000	0.000	0.000			0.000	
COAL	AIC, HQ, SIC	Gen.	0.584	-0.012	0.000	-0.054	0.000	0.839	7.5
			0.000	0.000	0.000	0.024	0.279	0.000	
COPPER	AIC, HQ, SIC	OsII	2.761			0.847		0.801	7.5
			0.000			0.063		0.000	
NATURAL GAS	AIC, HQ, SIC	OsII	0.065			-0.030		0.721	30
			0.000			0.000		0.000	
IRON	AIC, HQ, SIC	OsI	0.827			-0.418	0.009	0.808	10
			0.000			0.000	0.000	0.000	
LEAD	AIC, HQ, SIC	OsII	1.245			0.194		0.777	7.5
			0.000			0.164		0.000	
NICKEL	AIC	Gen.	4.884	-2.240	0.019	21.440	0.260	0.637	30
			0.697	0.013	0.019	0.140	0.009	0.000	
	HQ, SIC	OsII	15.912			-3.784		0.760	
			0.000			0.067		0.000	
PETROLEUM	AIC, HQ, SIC	Pol.	0.192	-0.005	0.000			0.804	
			0.079	0.216	0.031			0.000	
	SIC	OsII	0.621			-0.266		0.860	
			0.000			0.002		0.000	30
SILVER	AIC, HQ, SIC	OsII	162.510			90.690		0.719	10
			0.006			0.043		0.000	
TIN	AIC, HQ, SIC	OsII	18.977			-4.404		0.851	15
			0.000			0.056		0.000	
ZINC	AIC, SIC	OsI	1.459			0.482	-0.005	0.598	7.5
			0.000			0.012	0.017	0.000	
	HQ	OsII	1.440			0.214		0.663	
			0.000			0.220		0.000	10