Internal versus External Liquidity: Investment Efficiency under Market Frictions

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Abstract

This paper develops a framework to analyze how firms respond to liquidity shocks, focusing on the trade-off between internal liquidity hoarding and external asset sales. Building on Holmström and Tirole (1998), and extending insights from Stein (2012), the model incorporates asset market frictions, investor behavior, and asymmetric information. Asset sales generate externalities that tighten financial constraints and distort investment. These effects become technological when interacting with pledgeable asset limits. Policy interventions—liquidity guarantees and government asset purchases—can mitigate inefficiencies and restore optimal investment. The framework offers insights into financial regulation and crisis response in incomplete markets with heterogeneous firms.

JEL Classification: E22; E44; G01: D58

1 Introduction

Liquidity shocks pose significant challenges to firms, particularly when financial markets are incomplete and asset prices are sensitive to aggregate conditions. This paper examines how firms choose between internal liquidity hoarding and external asset sales to manage interim funding gaps. We build on the foundational work of Holmström and Tirole (1998), who analyze firm liquidity demand in a dynamic moral hazard setting. Their model highlights the role of pledgeable assets and credit constraints in shaping optimal financing contracts. We extend the analysis to incorporate externalities in asset markets and the role of investor behavior, which influence asset prices and firm-level decisions. In our model, firms face a stochastic liquidity shock after initial investment. To accommodate these shocks, they can either reserve cash ex ante or rely on asset sales ex post. While external liquidity allows for greater initial investment, it exposes firms to asset price risk, especially in fire-sale conditions, and to underpricing due to asymmetric information. These risks generate externalities that affect other firms', tightening financing constraints and distorting strategic choices. We introduce investors who allocate wealth between productive capital and liquidity provision. Their decisions influence asset prices and, indirectly, firm behavior. The equilibrium exhibits inefficiencies due to pecuniary externalities and adverse selection. We show that policy interventions, such as government asset purchases or liquidity guarantees can improve outcomes by mitigating these distortions.

The paper contributes to the literature on financial frictions, liquidity management, and crisis policy. It offers a tractable framework for understanding how market structure and information asymmetries affect firm-level decisions and aggregate investment efficiency. The problem of liquidity shocks and funding gaps has been examined by several authors, notably Holmstrom and Tirole (1998) and Stein (2012).[1] Moreover, recent empirical work (e.g., Acharya et al., 2011; Ivashina and Scharfstein, 2010) highlights how liquidity shocks and asset sales during crises affect firm-level financing and aggregate investment. This paper contributes to this literature by formalising the trade-off between internal and external liquidity in a setting with endogenous asset prices and investor behavior.

The main contribution of this paper is to examine externalities in funding markets arising from the impact of asset prices on firms financial constraints and also the impact of associated information problems. We show that policy interventions,

[1] Stein (2012) focuses on financial intermediaries and macroprudential regulation, our model shifts the focus to non-financial firms and their strategic responses to liquidity shocks.

such as government asset purchases or liquidity guarantees, can improve outcomes by mitigating these distortions. The closest paper to this paper is Holmstrom and Tirole (1998), who study the determinants of the firm's liquidity demand in a dynamic moral hazard model. In their model, there are three periods. At date 0, the firm raises funds to invest in a variable-sized project that pays off at date 2. At date 1, the firm experiences a liquidity shock. The shock is a random fraction of the date 0 investment and represents the amount of additional investment that must be made to continue the project. If the necessary funds can be raised, the project proceeds, delivering a stochastic date 2 return that depends on the entrepreneur's effort. They show that the optimal date 0 contract between the firm and the outside investors limits both the initial investment level and the amount that the firm is allowed to spend on the liquidity shock, both constraints being proportional to the firm's initial assets. Because the firm is credit-constrained, the second-best solution trades-off the benefits of a higher initial investment against the increased likelihood of having to terminate the project early and see it all go to waste. This solution can be implemented in several ways. One is to give the firm all the necessary funds in advance but adds a liquidity covenant, in which the firm promises to set aside a certain amount of funds to cover future liquidity needs. Alternatively, intermediaries could fund future liquidity needs via a credit line. The approach in this paper although similar in its basics differs in several crucial aspects leading to quite different results.

First, the basic model focusses on firms choice of internal versus external funding of liquidity shocks to cash flows. External funding of such shocks allows greater investment but exposes the firm to asset price risk and in particular fire-sale risk that imposes externalities on funding strategies. Also, in the presence of asymmetric information, it exposes the firm to potentially greater underpricing risk. The argument that these externalities creates the rationale for policy intervention draws on Shleifer and Vishny (1992, 1997).[2] In our model firms have limited amount of pledgeable initial assets that limits the scale of their activity. The firm is faced with a divisible constant returns project choice. For an initial investment, the project yields its stochastic return in two periods time. At an interim date the firm is subject to a liquidity shock, which must be funded, for-else the firm will be liquidated but continuation is positive net-present value. At the initial date the firm raises debt finance from competitive financiers to either be invested directly in the project or held in a

[2] On fire sales, see also Kiyotaki and Moore (1997), Gromb and Vayanos (2002), Morris and Shin (2004), Allen and Gale (2005), Fostel and Geanakoplos (2008), Brunnermeier and Pedersen (2009), Stein (2009), Caballero and Simsek (2010), and Geanakoplos (2010).

cash reserve to cover the liquidity shock. At this date (or during the first period), the entrepreneur chooses costly effort, which determines the success probability of the project. Firms differ in the probability of the liquidity shock and it is this that determines whether the firm will, other things being equal, choose to keep some borrowed funds as cash and operate the project at a reduced scale, or invest fully in the project and sell some part of the project in the event of the liquidity shock. Selling assets is better than additional borrowing as that involves reserving limited pledgeable asset capacity at the initial date and cutting back on the scale of a positive net-present value project. The choices that entrepreneurs make are dependent upon the market conditions for asset sales. The first possibility we consider is that entrepreneurs trade assets. Hoarders, when not being hit by a shock will be able to use, unused liquidity to by assets from non-hoarders hit by a shock. In the event that both types are hit by a shock assets sell at fire-sale prices. We broaden the model and introduce investors, who have limited wealth but can choose to commit resources to buy assets from illiquid non-hoarders both in states when they compete with hoarders and in fire-sale states, when they act competitively with each other but absent demand from hoarders.

The model's equilibrium properties exhibit some inefficiencies. These are first shown for the basic model with investors who playy a crucial role in the model. The focus is on the interim market for assets and the impact on entrepreneurs' strategic choices. In choosing their holdings of liquid assets relative to productive capital, investors maximise their returns but do not consider the impact that their decisions have on asset prices in liquidity sales by entrepreneurs. This in turn, in affecting the value of such sales, impacts financing constraint in liquidity event states and thereby the decisions of entrepreneurs beyond pure redistribution. We show that competitive equilibria can lead to excessive reliance on external liquidity, suboptimal investment, and distorted effort incentives. In generalising the model to consider some simple degree of asymmetric information relating to project returns, we see how the relative price of internal versus external liquidity is impacted and that this is exhibited in both incentive and advantageous selection effects for all entrepreneurs.

The final sections of the paper concern policy. We demonstrate that liquidity guarantees and government asset purchases can mitigate fire-sale externalities and improve selection, especially under asymmetric information. At a fundamental level the model has incomplete markets. If entrepreneurs could engage in state contingent futures contracts and maintain incentives, the first-best would obtain. This would be achieved if at the initial date the firm could undertake the first-best level of investment and cover the interim liquidity shock through borrowing against the final returns on the project but without violating the pledgeable asset constraint. If such contracts are not feasible, there may be a role for the government to step in and provide liquidity funding using its ability to guarantee and enforce loans. By the same token, if this is feasible, in eliminating the need for interim asset sales there will be no externalities arising from the impact of investor behaviour on these markets. If this intervention is not possible, then if the government enters the market to buy assets in the event of liquidity shocks at prices consistent with social efficiency, it can restore efficiency. In the case of asymmetric information and the equilibrium distortions in incentives and the composition of investment, the above inefficiencies are compounded by market distortions impacting the relative value of firms investment and effort choices and the size of the market. To correct these inefficiencies the government will need to impact the relative returns to the entrepreneurs strategic choices and the marginal cost of funds to all entrepreneurs.

To sum up, this paper contributes to the literature on liquidity management and financial frictions in five key ways. First, it formalizes the trade-off between internal liquidity hoarding and external asset sales in a dynamic setting with endogenous asset prices and matching frictions. Second, it introduces investors who allocate wealth between productive capital and liquidity provision, showing how their behavior influences asset prices and firm strategies. Third, it identifies pecuniary externalities arising from asset sales and demonstrates how these become technological when they interact with constraints on pledgeable assets, distorting real decisions such as effort and investment scale. Fourth, the model incorporates asymmetric information about firm productivity and analyzes pooled debt contracts, revealing how external liquidity provision leads to both adverse selection—by attracting lower-productivity firms, and incentive distortions, by weakening effort due to asset sales and fire-sale pricing. Finally, the paper evaluates policy interventions, including liquidity guarantees and government asset purchases, and characterizes the conditions under which these tools restore efficiency by mitigating fire-sale dynamics and improving selection.

The remainder of the paper is organized as follows. Section 2 presents the baseline model of liquidity management, contrasting internal and external strategies and deriving firm-level outcomes. Section 3 introduces asset market dynamics and matching frictions, highlighting how liquidity shocks propagate through asset sales. Section 4 analyzes investor behavior and equilibrium asset pricing, emphasising the feedback between investor expectations and firm decisions. Section 5 formalises the externalities in asset sales and compares the competitive equilibrium to the social planner's solution. Section 6 extends the model to incorporate asymmetric information and pooled financing, identifying distortions in effort and selection. Section 7 discusses the typology of liquidity shocks and strategic implications for firms. Section 8 evaluates policy interventions and outlines the conditions under which guarantees and asset purchases improve efficiency. The paper concludes with implications for financial regulation and crisis response design.

2 Baseline Model of Liquidity Management[3]

Entrepreneurs initially choose investment in a project or technology with which they are endowed. They must also choose a financial policy and effort to maximise expected wealth. Firms are subject to liquidity shocks and must take this into account when choosing their strategies. Firms invest in capital at date $t = 0$ that yields a return at date $t = 2$. At an intermediate date $t = 1$, firms are faced with a liquidity shock, which is intrinsic to the project. Investment is financed with borrowing. Firm's investment policy is constrained by capital market constraints, in particular the total level of financing is constrained by credit rationing because of limited pledgeable assets, P. Let the face value of debt be D, the promised payment to financiers. Funds raised can be used to finance investment, K, or held as liquid cash, C. The investment yields a return at date $t = 2$ of aXK, with probability $\theta(E)$, return rate $X > 1$ and productivity parameter $a > 1$. The productivity parameter at this stage is assumed to fixed and common to all entrepreneurs. The success probability $\theta(E)$, is a function of effort, with $\theta' > 0$ and $\theta'' < 0$ and cost of effort E. Liquid assets, C, can be used to insure against a negative liquidity shock, L, which occurs with probability $(1 - v)$ at date $t = 1$. The liquidity shock and its probability of occurring are a characteristic of the entrepreneurs project and are independent of any financing choices. The marginal cost of funds and the return on cash holdings is zero.

The alternative to the above investment and funding strategy is not to hold cash and to invest more in which case, in the event of a liquidity shock of size L, to cover the cost of the shock the firm must sell assets. The amount of assets that must be sold will depend upon the market for assets and hence the price of assets in the event of the shock. The price achieved will be worse if many firms are faced with the same shock and firms in a similar industry would be the best buyers. If the shock is idiosyncratic, the price will be denoted by q^I, which will be higher than if the shock is systemic, in which case the price is $q^F \leq q^I$. This difference will be positive and

[3] As the paper uses alot of notation. An appendix includes a glossary of terms.

bigger the greater the degree of industry specificity. Note that the price q^F matters to all firms. Individual firms will act to maximise profits and only be concerned with the marginal impact of q^F. However, the price of the assets for sale will reflect the average behaviour of affected firms.

2.1 Hoarding

Let K^h be capital and C^h liquid assets. The firms liabilities are debt, B raised against pledgeable assets, P. The NPV of projects is entrepreneurs equity,

$$K^h + C^h = B^h = P \tag{1}$$

Liquid assets raised, C^h, is liquidity hoarding that is used to cover a liquidity shock of L that occurs with probability $(1 - \nu)$ at date $t = 1$. Because the expected return on the project exceeds unity, the constraint $C^h \geq L$ will be binding, so henceforth, $C^h = L$. The firm invests K^h at date $t = 0$ in a project which generates an income at date $t = 2$ of aXK^h with probability $\theta(E)$.

Assume that cash raised at date $t = 0$ to cover the liquidity shock is used at date $t = 1$ if the shock occurs. If the shock does not occur, the cash is retained in the firm and paid to equity or debt in the good state at date $t = 2$. In the bad state at date $t = 2$, there are two possibilities; either the retained cash is paid to equity or to debt, thereby reducing the face value of debt. Let us first consider that retained cash is paid to equity, then the return on equity if effort is applied is

$$\begin{gathered} U^h = \theta(E^h)v(aXK^h + L - D^h) + \theta(E^h)(1 - v)(aXK^h - D^h) \\ +(1 - \theta(E^h))vL - E^h = \theta(E^h)(aXK^h - D^h) + vL - E^h \end{gathered} \tag{2}$$

and the debt payment satisfies

$$\theta(E^h)D^h \geq B = K^h + L \tag{3}$$

The firm's date $t = 0$ choice of effort E^h is chosen to maximise

$$U^h = \theta(E^h)(aXK^h - D^h) + vL - E^h \tag{4}$$

with first order condition

$$\theta'(E^h)(aXK^h - D^h) - 1 = 0 \tag{5}$$

In the alternative case where the retained cash is paid to debt holders, the entrepreneur's return is $\theta(E^h)(aXK^h - D^h)$ and the debt payment satisfies $\theta(E^h)D^h + vL \geq B = K^h + L$, and the incentive constraint is $\theta'(E^h)(aXK^h - D^h) = 0$. We will only consider the first case, although the second returns the same expected return to the entrepreneur.

2.2 Non-Hoarding

Now suppose that there is no hoarding. Then assuming that capital raised up to the pledgeable assets threshold, P,

$$K^{nh} = B^{nh} = P \tag{6}$$

If borrowing at date $t = 0$ takes place up to this threshold, there is no scope for further borrowing at date $t = 1$ to meet the liquidity shock. The firm could choose a smaller scale of investment and so retain some borrowing capacity as an alternative to meeting the shock through asset sales but this is more expensive than asset sales.[4] Hence, debt at date $t = 0$ is issued to fund investment and the shock L is met with asset sales in the bad state at date $t = 1$, with the firm selling a fraction α^{nh} of the project. Then

$$\begin{aligned} U^{nh} &= \theta(E^{nh})v(aXK^{nh} - D^{nh}) \\ &+\theta(E^{nh})(1-v)(aXK - \alpha^{nh}aXK^{nh} - D^{nh}) - E^{nh} \\ &= \theta(E^{nh})(aXK^{nh} - D^{nh}) - (1-v)\alpha^{nh}\theta(E^{nh})aXK^{nh} - E^{nh} \end{aligned} \tag{7}$$

$$\theta^H D^{nh} \geq K^{nh} \tag{8}$$

The firm's date $t = 0$ choice of effort E^h is chosen to maximise

$$U^{nh} = \theta(E^{nh})(XK^{nh} - D^{nh}) - (1-v)\alpha^{nh}\theta(E^{nh})aXK^{nh} - E^{nh} \tag{9}$$

[4] Note that the value of the asset sael at a price of $q < 1$, is $\alpha^{nh}\theta(E^{nh})aXK^{nh} = L/q$ which in turn will exceed the expected value of any additional debt finance to cover the liquidity shortfall, $\theta(E^{nh})\Delta D^{nh} = L$. However, to do this the firm needs to retain borrowing capacity of $\Delta P = L = \Delta K^{nh}$, sacrificing $\theta(E^{nh})X\Delta K^{nh}$ for sure against a potential saving of financing costs at date $t = 1$ with probability $(1 - v)$

with first order condition

$$\theta'(E^{nh})(aXK^{nh} - D^{nh}) - (1-v)\alpha^{nh}\theta'(E^{nh})aXK^{nh} - 1 = 0 \tag{10}$$

In this case, the share of the project that needs to be sold to cover the liquidity shock depends upon the price, q, of the share of the project sold, so that the necessary sale satisfies

$$q\alpha^{nh}\theta(E^{nh})aXK^{nh} = L, \text{ or } \alpha^{nh}\theta(E^{nh})aXK^{nh} = L/q \tag{11}$$

where the price q depends upon the market conditions for assets at date $t = 1$.

In order to compare entrepreneur returns under internal and external liquidity we first need to consider the effort choice, by comparing (5) and (10). Consider condition (9) and note that given $K^{nh} = K + L$, then if $q = 1$, this condition is equivalent to (5), in which case $E^{nh} = E^h$ and $\theta(E^{nh}) = \theta(E^h)$. Then the return to the entrepreneur in (7) is equivalent to that in (2). But if $q < 1$, if the liquidity event occurs, the entrepreneur has to sell more of the project so that $\theta(E^h) > \theta(E^{nh})$

We are now able to compare the return to the entrepreneur under hoarding and non-hoarding. The former can be written as $\theta(E^h)(aXK - D^h) + vL - E^h = \theta(E^h)aXK - (K + L) + vL - E^h$. For the latter, $\theta(E^{nh})(aXK^{nh} - D^{nh}) - (1 - v)\theta(E^{nh})\alpha^{nh}aXK^{nh} - E^{nh} = \theta(E^{nh})aXK^{nh} - K^{nh} - (1 - v)L/q - E^{nh}$. Thus the comparison is

$$\theta(E^h)aXK^h - (K^h + L) + vL - E^h \gtrless \theta(E^{nh})aXK^{nh} - K^{nh} - (1-v)L/q - E^{nh} \tag{12}$$

but $K^{nh} = K + L$, so hoarding will be preferred if

$$\theta(E^h)aXK^h - \theta(E^{nh})aXK^{nh} + vL - E^h > -(1-v)L/q - E^{nh} \tag{13}$$

Assuming that the liquidity shock L is sufficiently large and $\theta(E^h)aXK^h - \theta(E^{nh})aXK^{nh} - (E^h - E^{nh}) < 0$, this condition is more likely to be satisfied the lower q and the lower aX and noting that $L < L/q$, the lower v. This says that firms that are subject to a higher probability of liquidity shocks, low v, in considering the trade-off between a smaller scale of project and having reserves of cash to insure against having to liquidate assets at discounted prices, see the latter as dominating the former.

The comparison hinges on several factors: Effort distortion: Asset sales reduce the marginal return to effort, lowering $\theta(E^h)$ relative to $\theta(E^{nh})$. Fire-sale discount: If $q < 1$, external liquidity becomes more costly. Shock probability: Lower v increases the expected cost of asset sales, favoring hoarding. This inequality is more likely to hold when: q is low (deep fire-sale discounts), v is low (high shock probability), aX is low (lower project productivity), and effort distortions are significant.

3 Asset Market Dynamics and Matching Frictions

Now assume that entrepreneurs are distributed over an interval according to the value of v. At the value v^* an individual entrepreneur is indifferent between internal and external liquidity. Then, from (13) at any given v^*, given monotonicity and continuity, given for $q < 1$, $L < L/q$, lower v entrepreneurs prefer internal liquidity. We now introduce the possibility of a broader market for assets. Firms that prefer internal liquidity, "hoard liquidity", have the strategic advantage that if they are not hit by a liquidity shock, they will have unused liquidity and will be able to buy assets from firms that are hit by shocks and have to sell assets.

Suppose that at date $t = 1$, hoarders can buy assets from non-hoarders, if they have unused liquidity at a price of $q_I \leq 1$. The conditional (on having funds to purchase assets) probability that they can find a firm willing to sell is given by π^b so there is a probability $(1 - \pi^b)$ that they will not be able to buy assets. We also allow for possible rationing of asset sales through the rationing fraction $0 \leq f \leq 1$, to be examined later. Then the net return on investment with hoarding is

$$\theta(E^h)aXK - E^h + v[\pi^b(f\frac{L}{q_I} + (1-f)L) + (1-\pi^b)L] \tag{14}$$

At this stage we set $f = 1$. The net return to investment and reliance on outside liquidity is

$$\theta(E^{nh})aXK^{nh} - E^{nh} - (1-v)[\pi^s\frac{L}{q_I} + (1-\pi^s)\frac{L}{q_F}] \tag{15}$$

where, π^s is the conditional probability of finding a buyer, so being a seller.

At date $t = 0$, firm's must decide upon a strategy, to hoard liquidity or to plan to sell assets in adverse circumstances. However, as can be seen, this will depend upon parameters and crucially upon forecast market conditions, the conditional matching probabilities. The problem with meeting liquidity needs with asset sales is matching sellers to buyers. Note that the possibility of hoarders not being able to buy assets arises if the non-hoarders do not have to sell assets because they are not hit with a liquidity shock. Similarly, the possibility that non-hoarders cannot sell at the price q_I arises if the hoarders are also hit with a liquidity shock and are not in a position to buy. In this case assets have to be sold at a fire-sale price, which we set at q_F.

This is made clearer through the table below.

The states in which trades between firms can take place are by definition non-fire-sale states. In fire-sale states all entrepreneurs need liquidity, so there are no buyers, only sellers. In the non-fire-sale states, in which there are sellers and buyers, if there is a match in the market between a firm with liquidity and one that is selling assets to cover a shortfall, the price of the transaction is q_I, which will be better than selling to an alternative use buyer.[5] In order to generate fire-sale and non-fire-sale states, we add some correlation of shocks, otherwise, with independent shocks sellers will always be able to find buyers. Let v be the probability that firm i does not suffer a liquidity shock. The table is for two firms but is the basis for a general case for a population divided into two groups, hoarders and non-hoarders.

Firm i (Hoarder)	Firm j (Non-Hoarder)	Joint Probability	Market State
No shock	No shock	v·μ	No trade
No shock	Shock	v·(1-μ)	Non--fire-sale state (trade possible)
Shock	No shock	(1-v)·(1-ε)	Non--fire-sale state (trade possible)
Shock	Shock	(1-v)·ε	Fire-sale state (no buyers)

There are four states for the liquidity event: one where neither firm suffers a liquidity event $v\mu$; two where one does and the other does not $v(1-\mu)+(1-v)(1-\varepsilon)$ and one where both do, $(1-v)\varepsilon$. In the state where only the hoarder suffers the liquidity event, there is no asset sale. When the non-hoarder suffers the liquidity event, he needs to sell assets to the hoarder. When both entrepreneurs suffer the shock, fire-sale states, hoarders need their liquidity to cover the shock that they share in common with the

[5] We assume that prices are set competitively and not by bilateral bargaining with specialist buyers who may have bargaining power. The model could be modified to introduce a wedge between reservation prices for specialist and non-specialist buyers.

non-hoarders and non-horders' assets will sell at fire-sale prices. Then the probability that a hoarder finds a seller is $\pi^b = (1-\mu)$, and the probability that a non-hoarder finds a buyer is $\pi^s = (1-\varepsilon)$. These conditional probabilities are the equilibrium matching probabilities in our model.

At this stage, assume that the only degree of heterogeneity in the population of entrepreneurs is in the probability v. Let the total number of entrepreneurs be N. The distribution of entrepreneurs according to the variable v over the interval $[0,1]$ is given by the distribution function $F(v)$. At date $t=1$, hoarders total buying capacity is $F(v^*)N(1-\mu)$ and total sales from non-hoarders is $(1-F(v^*))N(1-\varepsilon)$. Absent other buyers, in equilibrium the number of asset sellers in non-fire-sale states must equal the number of buyers, so the price in these states must be consistent with market clearing

$$F(v^*)N(1-\mu) = (1-F(v^*))N(1-\varepsilon) \tag{16}$$

or $F(v^*)/1-F(v^*) = (1-\varepsilon)/(1-\mu)$. Then to ensure that hoarders are induced in sufficient numbers to hoard liquidity, asset sellers, who must sell, have to take a discount so $q_I < 1$ to clear the market. In the fire-sale states, that occur when hoarders are also hit by a shock, the sales from non-hoarders is $(1-F(v^*))N\varepsilon$. The fire-sale effect favours hoarding as asset sellers take a discount. Given q_F the value of q_I determines the strategy, to hoard or not to hoard of an individual entrepreneur with a given value of v. The equilibrium value of q_I is one which, when forecasted rationally, induces a marginal $v=v^*$ that satisfies the equilibrium condition (16).

Note that if $(1-\nu)\varepsilon$ is high and q_F is low, then, low v firms will avoid being exposed to fire sale outcomes by hoarding liquidity, so v^* will be higher, which will benefit those high v firms that do not hoard, who will achieve better prices for assets in non-fire-sale states.

4 Investor Behavior and Asset Price Formation

The role of investors in providing liquidity and influencing asset prices has been explored in Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002). Our model extends this by incorporating investor competition with hoarders and analyzing the implications for equilibrium asset pricing and firm strategy.

In equilibrium, the willingness of investors to allocate liquidity toward purchasing assets in fire-sale states depends critically on the expected return from such purchases. This return must be competitive with alternative uses of their capital, such as investment in productive assets. Consequently, asset prices, particularly in fire-sale conditions, must adjust to ensure that investors are sufficiently incentivised to participate.

An endogenous value of fire-sale assets requires a market for fire-sale assets, which depends upon there being buyers. We assume that their exist specialist investors who are in the market for assets. They hold capital to buy assets for speculative returns, the expected (risk-adjusted) return they get from assets held for fire-sale purchases must equal the alternative return on these funds. Then in fire-sale states, they must buy all assets from the total mass of selling firms, S, $M = S$. In non-fire-sale states, they compete with hoarders to buy assets, $M = S - H$, where H is the total mass of hoarders, who are buyers.

Investors have wealth W that they allocate between liquidity, M, with return R, and investment in productive capital, which generates a return $\phi(W - M)$, with $\phi(0) = 0$, $\phi' > 0$ and $\phi'' < 0$. Their optimisation problem is

$$\max\{RM + \phi(W - M)\} \tag{17}$$
$$\text{where } R = F(v^*) + (1 - F(\nu^*))[(1-\varepsilon)(1-f)\frac{L}{q_I} + \varepsilon\frac{L}{q_F} + (1-\varepsilon)fL]$$

We can assume that if both hoarders and investors submit orders to buy from non-hoarders in non-fire sale states, they are rationed proportionately, so that $f = H/(M+H)$. The first-order-condition for the choice of M is

$$R = \phi' \tag{18}$$

This equation solves for M,

$$M = W - \phi'^{-1}(R) \tag{19}$$

This implies that the return on asset purchases must equal the marginal return on productive investment. The return R itself depends on the equilibrium asset prices in both normal and fire-sale states, denoted q_I and q_F respectively. In fire-sale states, investors are the sole buyers, and must absorb the entire supply of assets from distressed firms. To clear the market, the price q_F must be sufficiently low to make the expected return attractive. In normal states, investors compete with hoarders, and the price reflects both demand sources. The extent of rationing between hoarders and investors affects the equilibrium price and thus the return R.

The key insight is that the equilibrium asset prices must adjust downward to induce the desired level of investor liquidity provision. If prices are too high, investors will allocate less liquidity, leading to insufficient market clearing and exacerbating fire-sale discounts. Conversely, if prices are too low, investors may over-allocate liquidity, potentially crowding out hoarders and distorting firm incentives.

This mechanism introduces a feedback loop: investor expectations about asset prices influence their liquidity allocation, which in turn affects asset prices and firm behavior. The model captures this interdependence and shows how price formation in asset markets is endogenous to investor participation, with implications for efficiency and policy.

At date $t = 1$, the market conditions are either normal or fire-sale. In the former, the normal market equilibrium

$$(1 - f)M + fH = S \tag{20}$$

where, $S = (1 - F(v^*))N$ and $H = F(v^*)N$, so

$$M = \frac{1}{1 - f}(1 - (1 + f)F(v^*))N \tag{21}$$

Given the values of f, S, H, q_F and v^*, this equation determines q_I.

In the fire-sale case, $f = 0$, and

$$M = S = (1 - F(v^*))N \tag{22}$$

In this case the expected return will need to be the right value, where the demand from arbitrageurs on the left-hand-side depends on expectations of returns. This equation determines q_F. Note that to ensure that arbitrageurs commit sufficient funds, M, to buy assets, the prices q^F and q^I will have to be sufficiently low. In particular, the extent of rationing in the normal market will impact the extent of firesale discounts to induce the equilibrium value of M.

5 Pecuniary and Technological Externalities in Asset Sales

In decentralized asset markets, firms and investors act competitively, taking prices as given and ignoring the broader impact of their decisions on market outcomes. However, when asset sales depress prices, they impose pecuniary externalities on other firms by tightening financing constraints and reducing continuation values. These externalities become technological when they interact with constraint, such as limited pledgeable assets. that affect real decisions like effort, investment scale, and liquidity strategy. This section formalizes the nature of these externalities and shows how competitive equilibrium leads to excessive reliance on external liquidity, inefficient asset pricing, and suboptimal investment. We contrast this with the social planner's problem, where liquidity provision is chosen to internalize these effects and restore efficiency.

In the competitive equilibrium, investors allocate liquidity, M, to asset purchases based solely on private returns, without accounting for how their actions affect asset prices and, through them, the financing constraints of other firms. When firms sell assets to meet liquidity needs, this leads to a pecuniary externality, when the resulting price impact tightens constraints for all firms exposed to asset sales, reducing continuation values and distorting effort incentives. These effects are not internalized by individual investors, leading to excessive liquidity provision and inefficient asset pricing. We formalize this by comparing the competitive equilibrium to the social planner's solution, where liquidity is allocated to maximize aggregate surplus, taking into account the endogenous response of asset prices, matching frictions, and firm behavior. In the competitive equilibrium, firms and investors act independently, optimising based on private returns without internalizing the broader effects of their decisions on asset prices and financial constraints. Investors allocate liquidity to asset purchases purely based on expected returns, which can lead to excessive fire-sale discounts and distorted firm behavior. Asset prices are determined by decentralized trading and may fall sharply in systemic liquidity events, tightening constraints for all firms and reducing continuation values. In contrast, the social planner's solution coordinates liquidity provision and asset pricing to internalize these externalities. The planner recognizes how investor liquidity affects asset prices and firm incentives, and sets prices to reflect social value rather than marginal private valuations. This induces more firms to hoard liquidity, reducing exposure to fire-sales and improving aggregate investment efficiency. The planner would prefer higher asset prices (i.e., less aggressive investor buying), which would induce more firms to hoard liquidity, reducing exposure to fire-sales and improving aggregate investment efficiency.

We will address this issue more formally. Entrepreneurs make their choice of strategy taking the parameters as given; and with fair competitive financial terms, they choose the strategy that yields them the maximum expected wealth. That is for given v, $U^h(v) \gtrless U^{nh}(v)$. Investors take the decisions of entrepreneurs as given and choose how many funds to allocate to buying assets from firms at date $t = 1$, M, according to condition (18). The marginal entrepreneur, with $v = v^*$, is indifferent to the two strategies $max\{U^h(v^*), U^{nh}(v^*)\}$. The social optimum must maximise total social surplus. Assuming financiers break even, this is the sum of all active entrepreneurs expected wealth and the expected returns of investors:

$$S = \int_{\underline{v}}^{v^*} \{\theta(E^h)aXK - (K+L) - E^h \tag{23}$$

$$+v[\pi^b(f\frac{L}{q_I} + (1-f)L) + (1-\pi^b)L]\}dF(v)$$

$$+\int_{v^*}^{\overline{v}} \{\theta(E^{nh})aXK^{nh} - K^{nh} - E^{nh}$$

$$-(1-v)[\pi^s\frac{L}{q_I} + (1-\pi^s)\frac{L}{q_F}]\}dF(v) + \{RM + \phi(W-M)\}$$

where $\pi^b = (1-\mu)$, and $\pi^s = (1-\varepsilon)$. The social planner will be constrained by the hoarders and non-hoarders pledgeable income constraints, (1) and (6); investor optimisation, (18); market clearing in assets (16); and efforts incentive compatibility, $\theta'(E) = aXK$. Then the social planner chooses M to satisfy:

$$\frac{dS}{dM} = \int_{\underline{v}}^{v^*} v\pi^b[Lf(\frac{-dq_I/dM}{q_I^2}) + \frac{L}{q_I}\frac{\partial f}{\partial M} - L\frac{\partial f}{\partial M}]dF(v) \tag{24}$$

$$+\int_{v^*}^{\overline{v}} (1-v)[\pi^s L(\frac{dq_I/dM}{q_I^2}) + (1-\pi^s)L(\frac{dq_F/dM}{q_F^2})]dF(v) + R - \phi' = 0$$

The upper part of the expression concerns non–fire-sale states (normal market), the lower part concerns fire-sale states and the last term is the return on liquid assets minus the marginal return on productive assets. The terms in this expression are price sensitivities dq_I/dM, dq_F/dM, how asset prices respond to investor liquidity; these affect firm financing constraints and continuation values. Matching frictions $\frac{\partial f}{\partial M}$, reveal how liquidity affects the ability of firms to sell assets in normal states; this impacts the expected value of asset sales and firm strategy. The final term, $R - \phi'$ is the investor opportunity cost., as the planner internalises the cost of diverting funds from productive capital. The state probabilities: v, $1-v$, probabilities of firms being in good or bad liquidity states weight the planner's concern across different types of firms.

The point here is that in choosing M, the social planner recognises the impact of the investors cash position on the values of assets in asset sales and consequentially the impact on the magnitude of asset sales to fund continuation of all impacted firms. In general from the investors first-order condition $dR/dM = -\phi''(W - M)$. Note that for illustration, in the special case when $f = 1$ and $q_I = 1$, by (18) $\varepsilon d\frac{1}{q_F}/dM = -\varepsilon\frac{dq_F/dM}{q_F^2} = -\phi'' > 0$, or $\frac{dq_F/dM}{q_F^2} = \phi''/\varepsilon$. Note that $\frac{L}{q_I}\frac{\partial f}{\partial M} - L\frac{\partial f}{\partial M} < 0$. In an equilibrium with investors, the second-term in the social planner's optimisation problem dominates the first-term, so making the sum of the two terms negative, making the social return to investors supplying liquidity less than R so that the socially optimal level of investor liquidity is less than the competitive equilibrium level, $M^s < M^c$. This means that the planner would not set prices in asset sales as low as the competitive economy, which means that on balance there would be less investment as more firms use internal liquidity.

6 Asymmetric Information and Pooled Financing Distortions

In the preceding sections, firms were treated as homogeneous in their productivity. We now introduce heterogeneity in entrepreneurial ability, captured by a productivity parameter $a > 1$, which is privately known to each firm. This extension allows us to examine how asymmetric information interacts with financing constraints and liquidity strategies. In particular, we explore how pooling contracts—necessitated by the inability of lenders to observe a affect effort incentives and selection into internal versus external liquidity regimes.

The analysis reveals that external liquidity provision exacerbates both incentive distortions and adverse selection, leading to inefficient investment and a misallocation of financing across firms, We now assume that firms differ in terms of the productivity parameter, $a > 1$. This productivity variable is distributed over the population according to $G(a)$, with support $[\underline{a}, \overline{a}]$. If the parameter a is private information, there is no incentive for entrepreneurs to reveal this to lenders. Borrowing and any other financing will therefore take place on pooled terms. Consider the impact on the two financing choices faced by entrepreneurs and illustrated in Section 1, keeping all other features of the problems the same.

First, consider internal financing of the liquidity shock. In this case the entrepreneurs return is modified by replacing the probability terms by the terms dependent on productivity term. Similarly, the entrepreneurs incentive constraint is modified. The principal modification is that the face value of debt now satisfies a pooled condition involving the conditional average probability of success. So $\theta(E^h(a))D^h \geq B = K + L$

is replaced by

$$\rho = \int_{a^*}^{\overline{a}} \theta(E^h(a))\overline{D}^h \frac{dG(a)}{1-G(a^*)} = B = K + L \tag{25}$$

where $\overline{D}^h$ is the face value for all borrowers, borrowing in this way. For a given fixed payment $\overline{D}^h$, the marginal entrepreneur has an ability a^*such that $U^h = \theta(E^h(a^*))(a^*XK - \overline{D}^h) + vL - E^h = 0$. From condition (5), $\theta'(E^h)(aXK^h - D^h) - 1 = 0$, $\frac{\partial E^h}{\partial a} = \theta'(E^h)X/\Delta > 0$, where $\Delta = \theta''(E^h)(aXK - \overline{D}^h) < 0$. Then using the implicit function theorem, $\frac{\partial \theta}{\partial a} = \theta'(E^h)\frac{\partial E^h}{\partial a} > 0$. Similarly, $\frac{\partial \theta}{\partial \overline{D}^h} = \theta'(E^h)\frac{\partial E^h}{\partial \overline{D}^h} < 0$. Only entrepreneurs with ability $a > a^*$ apply for finance. In a pooling equilibrium $\overline{\theta}^h = \int_{a^*}^{\overline{a}} \theta(E^h(a))\frac{dG}{1-G(a^*)} > a^*$. Given that the financiers break even, the social surplus from the marginal entrepreneur, for whom $U^h = 0$, is $(\theta(E^h(a^*)) - \overline{\theta}^h)\overline{D}^h < 0$. Moreover, in equilibrium

$$\frac{d\rho}{d\overline{D}^h} = (\overline{\theta}^h - \theta(E^h(a^*))\frac{dG(a^*)}{1-G(a^*)}) + \overline{D}^h \int_{a^*}^{\overline{a}} \frac{\partial\theta(E^h(a))}{\partial \overline{D}^h}\frac{dG(a)}{1-G(a^*)} \tag{26}$$

where her in the first term (the advantageous-selection effect), $(\overline{\theta}^h - \theta(E^h(a^*))$ is the difference between average and marginal success probability, which .is positive by (16). That is if marginal entrepreneurs are lower quality than the average, more debt attracts worse types. Moreover, $\frac{\partial\theta(E^h(a))}{\partial \overline{D}^h} < 0$ increasing debt weakens effort incentives. This term being negative reflects how higher debt reduces pledgeable income via lower effort.

Now consider the alternative of external liquidity provision. The same modifications as above are required for the entrepreneur's return function and incentive constraint. For a given fixed payment $\overline{D}^{nh}$, the marginal entrepreneur has an ability a^{**}such that $U^{nh} = \theta(E^{nh}(a^{**}))(XK^{nh} - \overline{D}^{nh}) - (1-v)\overline{\theta}^{nh}\alpha^{nh}XK^{nh} - E^{nh} = 0$. From condition (10), $\theta'(E^{nh})(aXK^{nh} - \overline{D}^{nh}) - (1-v)\theta'(E^{nh})\alpha^{nh}aXK^{nh}$, $\frac{\partial E}{\partial a} = [\theta'(E^h)X - (1-v)\theta'(E^{nh})\alpha^{nh}XK^{nh}]/\Delta > 0$, where $\Delta = \theta''(E^{nh})(aXK^{nh} - \overline{D}^{nh}) - (1-v)\theta''(E^{nh})\alpha^{nh}aXK^{nh} < 0$. Again, using the implicit function theorem, $\frac{\partial \theta}{\partial a} = \theta'(E^{nh})\frac{\partial E^{nh}}{\partial a} > 0$ and $\frac{\partial \theta}{\partial \overline{D}^{nh}} = \theta'(E^{nh})\frac{\partial E^{nh}}{\partial \overline{D}^{nh}} < 0$.

Noting that $K^{nh} > K^h$ and that the magnitude of the sale of assets to cover the liquidity shortfall, α^{nh}, will depend upon the price of assets in the state at date $t = 1$, but will be lower than it would be on fair terms for marginal entrepreneurs due to $\overline{\theta}^{nh} > \theta(E^{nh}(a^{**}))$. Hence, $a^{**} < a^*$. The principal modification is again that the face value of debt now satisfies a pooled condition involving the conditional average probability of success. So $\theta(E^{nh})D^{nh} \geq B = K^{nh}$ is replaced by

$$\rho = \int_{a^{**}}^{\overline{a}} \theta(E^{nh})(a)\overline{D}^{nh} \frac{dG}{1 - G(a^{**})} = B = K^{nh} \tag{27}$$

with $\overline{D}^{nh} > \overline{D}^{h}$. Again, given that the financiers break even, the social surplus from the marginal entrepreneur is $(\theta(E^{nh}(a^{**})) - \overline{\theta}^{nh})\overline{D}^{nh} < 0$. Moreover, in equilibrium

$$\frac{d\rho}{d\overline{D}^{nh}} = (\overline{\theta}^{nh} - \theta(E^{nh}(a^{**}))\frac{dG}{1 - G(a^{**})}) + \overline{D}^{nh} \int_{a^{**}}^{\overline{a}} \frac{\partial\theta(E^{nh}(a^{**}))}{\partial\overline{D}^{nh}} \frac{dG}{1 - G(a^{*})} \tag{28}$$

The first term (the advantageous-selection effect) is positive by (16) and the second term (the incentive effect) is negative but both will be bigger than in the case of internal finance.

The first term captures the (advantageous) selection effect: how the marginal entrepreneur compares to the average. If the marginal type is worse than the average, this term is positive, increasing $\overline{D}^{nh}$ worsens the pool. The second term captures the incentive effect: how increasing debt affects effort and thus success probability. This term is negative, higher debt weakens incentives. But both will be bigger than in the case of internal finance. That is, the crucial point is that $a^{**} < a^{*}$ means that the subsidy in pooled financing is greater for non-hoarding (external liquidity), which will move firms towards hoarding (internal liquidity). This means less investment but also lower exposure to asset sales and in particular fire-sales.

We can summarise the above result in a proposition:

Proposition: Comparative Distortions in Pooled Debt Contracts

Let firms differ in productivity $a \in [\underline{a}, \overline{a}]$ and let debt contracts be pooled across types. Then, under external liquidity provision:

1.The advantageous selection effect $(\overline{\theta}^{nh} - \theta(E^{nh}(a))$ is larger than under internal liquidity, i.e., $(\overline{\theta}^{nh} - \theta(E^{nh}(a)) > (\overline{\theta}^{h} - \theta(E^{h}(a))$ because external liquidity attracts lower-productivity firms (i.e., $a_{nh} < a_{h}$).

2.The incentive effect $\frac{\partial\theta(E^{nh}(a))}{\partial\overline{D}^{nh}}$ is more negative than under internal liquidity: $\frac{\partial\theta(E^{nh}(a))}{\partial\overline{D}^{nh}} < \frac{\partial\theta(E^{h}(a))}{\partial\overline{D}^{h}}$ due to greater dilution of effort incentives from asset sales and fire-sale pricing.

3. Therefore, the total distortion in pledgeable income from increasing debt is greater under external liquidity.

This result implies that pooled debt contracts are more distortionary under external liquidity. The planner should prefer internal liquidity for high-productivity firms to preserve effort incentives and reduce adverse selection.

7 Liquidity Shock Typology and Strategic Implications

The incompleteness of markets in our economy means that firms are over exposed to liquidity risk. Let us suppose that at date $t = 1$, the liquidity shock is either permanent or the result of a delay in a payment that will come in a period's time. If the shock is permanent then matters are the same as above, with the shock having to be met from current cash. Choosing to ensure that the firm has enough pledgeable income to cover a bank loan equal to the shock will give the same outcome as in the case of liquidity hoarding. However, if the shock is a timing shock, then the optimal strategy is for the firm to invest to the maximum and ignore the liquidity shock which has an expected value of zero. So that if the shock occurs with probability $(1 - \nu)$ at date $t = 1$ but will be revered with the same probability at date $t = 2$, the firm borrows $C = L$, pledging the unused borrowing capacity at date $t = 1$ and repays the loan at date $t = 2$. That is, the firm enters into a repurchase agreement with a bank. It borrows L and pledges residual borrowing capacity that it has against accounts receivable. This strategy clearly dominates liquidity hoarding or asset sales but only works because the liquidity shock is only a timing shock.

A problem arises if the liquidity shock may be permanent or transitory, in which case there is a signal extraction problem. Moreover there may be a problem in reopening accounts receivable if the floating charge is not well defined. However,there is also the problem that the ability to repo can be affected by the perceived value of the firm's investments. For example, if the rate of default on assets increases, then the firm cannot repo enough assets to withstand the shock. Then the absence of liquidity means that the firm must sell assets and increase the rate of default. If the firm cannot increase the default rate the liquidity problem becomes a bankruptcy problem.

8 Policy Interventions: Guarantees, Asset Purchases, and Market Design

As noted, at a fundamental level the model has incomplete markets. If entrepreneurs could engage in state contingent futures contracts and maintain incentives, the first-best would obtain. This would be achieved if at the initial date the firm could undertake the first-best level of investment and cover the interim liquidity shock through borrowing against the final returns on the project but without violating the pledgeable asset constraint. If, as we have argued, such contracts are not feasible without violating the pledgeable asset constraint, there may be a role for the government to step in and provide liquidity funding using its ability to guarantee and enforce loans. By the same token, if this is feasible, in eliminating the need for interim asset sales, there will be no externalities arising from the impact of investor behaviour on these markets. If this intervention is not possible, then if the government enters the market to buy assets in the event of liquidity shocks at prices consistent with social efficiency, it can restore efficiency.

In the case of asymmetric information and the equilibrium distortions in incentives and the composition of investment, the above inefficiencies are compounded by market distortions impacting the relative value of firms investment and effort choices and the size of the market. To correct these inefficiencies the government will need to impact the relative returns to the entrepreneurs strategic choices and the marginal cost of funds to all entrepreneurs. In the presence of incomplete markets and asymmetric information, decentralized asset sales generate externalities that distort firm behavior and aggregate investment. Two policy tools can mitigate these inefficiencies: liquidity guarantees and government asset purchases. These reponses have been discussed in Hanson et al. (2011) and Caballero and Krishnamurthy (2008). However, our framework formalises the conditions under which such interventions restore efficiency by mitigating fire-sale externalities and improving selection.

Liquidity guarantees allow firms to borrow against future returns without violating pledgeable asset constraints. By reducing the need for interim asset sales, guarantees eliminate fire-sale externalities, as firms no longer depress asset prices. They also preserve effort incentives, since financing is less sensitive to market conditions and Improve selection, by enabling high-productivity firms to invest at scale. However, guarantees require credible enforcement and may introduce moral hazard if firms anticipate bailouts. Their effectiveness hinges on the government's ability to screen firms and enforce repayment.

Government Asset Purchases in which the government acts as a buyer in asset markets during liquidity shocks can be beneficial. By purchasing assets at socially efficient prices, the government supports asset prices, reducing pecuniary externalities. This crowds in investment, especially for firms relying on external liquidity,stabilises investor expectations and improves liquidity provision. This intervention is particularly valuable when asymmetric information leads to underpricing and adverse selection. However, it requires accurate valuation and may distort private incentives if mispriced. In equilibrium, both tools can restore efficiency, but their design must account for the nature of shocks, firm heterogeneity, and investor behavior. The planner's optimal policy may involve a hybrid approach, guarantees for high-productivity firms and asset purchases to stabilize markets during systemic shocks.

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Appendix A: Symbol Reference Guide

Symbol	Definition
K	Capital invested at date t = 0
C	Liquid cash reserves held at t = 0
L	Size of the liquidity shock at t = 1
D	Face value of debt
B	Total borrowing raised at t = 0 (equals K + C)
P	Pledgeable asset value (credit constraint)
a	Productivity parameter of the project
X	Return rate on capital investment
$\theta(E)$	Success probability of the project, increasing in effort E
E	Entrepreneurial effort
q_I	Asset price in idiosyncratic (non-fire-sale) states
q_F	Asset price in systemic (fire-sale) states
α^{nh}	Fraction of project sold to meet liquidity needs
v	Probability that firm i does not suffer a liquidity shock
μ	Probability that firm j does not suffer a liquidity shock
ε	Probability that firm j is matched with a buyer
$\pi^b = 1 - \mu$	Probability that a hoarder finds a seller
$\pi^s = 1 - \varepsilon$	Probability that a non-hoarder finds a buyer
f	Rationing fraction in asset markets between hoarders and investors
M	Liquidity allocated by investors to asset purchases
W	Total wealth of investors
R	Return on liquidity allocated to asset purchases
$\phi(W - M)$	Return on productive investment by investors
$\theta(E(a))$	Success probability as a function of effort and productivity