Budgets, Redistribution and the Real Impacts of Monetary and Fiscal Policy.

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Abstract

The focus of this paper is on the impact of monetary and fiscal policy, with a focus on the level and maturity structure of government debt. We will explore these through an inter-temporal Metzleric model with capital accumulation and a government funding channel. The inter-tempoporal model is Metzleric in spirit and developed using a well-known framework due to Blanchard (1986) with a government budget constraint. Individuals have infinite horizons but finite life expectancy. Individuals hold assets, including fiat money directly but insure against leaving unplanned bequests by trading annuity contracts. An extended version of the model allows for borrowing and lending, with the default risk on loans being insured. This creates a wedge between borrowing and lending rates,which generated policy impacts. A final extension of the model is the introduction of simple income shocks to consumers budgets. Again, we assume that individuals insure against these to smooth out their impact. The insurance mechanism proposed to address this issue places demands upon the design of public debt. The policies considered are debt financed tax cuts, variations in the composition of government debt and changes in the inflation target. The exercises are purely qualitative and are undertake for comparative steady states that abstract from transitional dynamics.

JEL Classification:E21;E22;E44;E52;E62;H62;H63.

1 Introduction

The model we propose is a dynamic equilibrium model of consumption and investment decisons with wealth accumulation and a consolidated government budget constraint. The model is in the spirit of Metzler (1951), which was designed to understand how open-market operations have real effects because central bank issuance of money to buy claims on capital reduces private sector receipts from capital, returned to the private sector as tax cuts, but the tax cuts are not fully capitalised, so non-money wealth declines. Households then have to be induced to hold a higher ratio of real money balances to capital, putting downward pressure on interest rates. Savings then increases to rebuild wealth. The price level does not therefore increase to offset the increase in the nominal stock of money as would be the case if the tax cut was fully capitalised. The non-neutrality of this policy can be interpreted as a redistribution of resources from future to current tax payers (see for example Barro (1974)). The maturity structure of government debt and hence debt management policy is not something that can be examined in the essentially static Metzler model, although as Mundell (1965) and others have shown, because of its focus on wealth, it could be adopted to consider some inter-temporal problems in a taxonomic format. This includes highlighting, for example, capital market imperfections and more detail on the burden of the debt.

The Metzler model comprised a savings equals investment relationship (goods market equilibrium), a money market equilibrium and an explicit wealth constraint. It did not include an explicit government budget constraint and did not develop forward looking optimising consumption, asset holding and investment decisions. Let us consider the role of the government budget constraint. This immediately draws to our attention the notion that fiscal policy can impact real interest rates and the price level. If the government budget constraint is treated as a "uses and sources of funds" condition, then given some predetermined paths for taxes and government expenditure and level of debt finance, the endogenous variable is the path of seigniorage and thereby the path of the future money supply. In this sense, the government budget constraint is satisfied for all policy paths, not as an equilibrium condition. An alternative view, known as the "fiscal theory of the price level" (see Woodford (1995) and (2003)), hypotheses that the government's budget constraint is not satisfied for arbitrary price levels but only at equilibrium price levels. In this case the level of nominal government liabilities plays a critical role in determining the price level. Note that, under this theory, the value of government liabilities is equal to the present value of fiscal surpluses and seigniorage payments. Of course, this theory focusses attention on a tension between the ability of the government to borrow more on the basis of an increased value of fiscal surpluses as opposed to the price level satisfying an equilibrium condition.

Barro (1979) argued that government debt should be long-term to allow the government to smooth taxes, as implied by dynamic Ramsey taxation, insured from shocks including short-term debt roll-over risk. Gale (1990) provides a general analysis of the design of government debt in an OLG framework with incomplete markets, in which the debt instuments perform a role in achieving risk sharing between generations. In the spirit of Gale, Angeletos (2002) constructs an inter-temporal model with incomplete markets, in particular the absence of Arrow Debreu securities, or indeed as many linearly independent traded securities and options as states (see Ross 1976). In his model, the market value of long-term debt varies with equilibrium interest rates. This endogenous variation in the debt burden insures the government against the need to raise either the tax rate or the level of debt when fiscal conditions turn bad, for example during wars. This means that the government can smooth out impacts without testing its sequence of sustainable surpluses. Other authors, notably Gibaud, Nosbusch and Vayanos (2013), have considered the role of short and long-maturities of debt in an OLG framework, when both one-period and two-period bonds are used to insure against interest rate risk and generations act as clienteles for bonds. When arbitrageurs have limited risk capacity and individuals are sufficiently risk averse, longer-term bond returns include a term premium. Greenwood et al (2015) argue that long-term debt does avoid roll-over risk, but this has to be traded-off against the monetary services and hence lower interest costs of short-term debt.

Our focus in this paper is on the impact of debt financed fiscal policy and the maturity structure of government debt. We will explore these through an inter-temporal Metzleric model with capital accumulation and a government funding channel. The inter-temporal Metzler model is developed using a well-known framework due to Blanchard (1986) with a government budget constraint. Individuals have infinite horizons but finite life expectancy. Although individuals hold assets, including fiat money directly, they insure against leaving unplanned bequests by trading annuity contracts. An extended version of the model allows for borrowing and lending, with the default risk on loans being insured, with premia on loans being equivalent to default risk in loan rates. This creates a wedge between borrowing and lending rates but allows for the capitalisation of wage income at the same rate for borrowers and lenders. A final extension of the model is the introduction of simple income shocks to consumers' budgets. Again, we assume that individuals want to insure against these to smooth out their impact. The insurance mechanism proposed to address this issue places demands upon the design of public debt. The policies considered are debt financed tax cuts, variations in the composition of government debt and changes in the inflation target. The exercises are purely qualitative and are undertake for comparative steady states that abstract from transitional dynamics. The real impacts of the policy changes all follow from individual household budgets not consolidating the government's budget in an invariant way.

2 Agents

2.1 Households

The basic inter-generational model of Blanchard (1986) is extended to address government finance and in particular the structure of public debt. At date t, the population is N_t. At each date a proportion $(1-\gamma)$ of the population die and a proportion $\psi = (1-\gamma)$ are born, so the population is constant in both size and demographic structure. New-born individuals start to work immediately. Individuals are assumed to supply labour inelastically every period for a wage of $W_{t,s}$ and pay taxes, $T_{t,s}$. At each date a surviving individual suffers a negative temporary shock to wage income (full or partial unemployment) of $\delta W_{t,s}$ with probability v.

In this model, a new-born cohort and all surviving cohorts, regardless of when they were born have the same life expectancy and so face the same optimisation problem. Agents face two risks, at each date they may die with probability $\psi = (1-\gamma)$. There is also a risk of income loss at each of $\delta W_{t,s}$ with probability v. Individuals invest $A_{t,s}$ in return bearing assets. A a proportion ω is invested in real assets, equities, yielding a return of $(1+r_t)$ and the remaining proportion, $(1-\omega)$, in nominal government bonds paying a composite nominal return of return of $(1+i_t^B)$ or in real terms $(1+i_t^B)\frac{P_t}{P_{t+1}}$, where P_t is the price level. The total return on on a unit investment in the portfolio is $Z_{t,s} = (1+z_t) = [\omega(1+r_t)+(1-\omega)(1+i_t^B)\frac{P_t}{P_{t+1}}]$ per period. In developing the model, we abstarct from return risk and so do not solve for a portfolio rule, ω_t.[1] Individuals can also hold real money balances. To hedge mortality risk individuals also enter into annuity contracts arranged by competitive intermediaries that break even. The annuities avoid any unplanned bequests, they pay $(1-\gamma)Z_{t,s}$ to the survivors from the dyers estates. Hence,

[1] The most general probability distribution is admissible: a probability density over continuous $r's$, or finite positive probabilities at discrete values of r. In its simplest form, we assume independence between yields at different times so that $F(r_0, r_1, ..., r_t..., r_\infty) = F(r_0)F(r_1)...F(r_t)$. If at each date r_t can take two values with equal probability of $\frac{1}{2}$ of either r_t^h or r_t^l with $r_t^h > (1+i_t^B)\frac{P_t}{P_{t+1}} - 1 > r_t^l$ and $E_t r_t = \frac{1}{2}r_t^h + \frac{1}{2}r_t^l$.

the total return in the event of survival at each date is $Z_{t,s}/\gamma$ and zero otherwise. The additional exposure to income losses at every date after the first of $\delta W_{t,s}$ can be hedged by paying premiums to an insurance company. The premiums are paid across periods to smooth consumption. The aggregate present value of these premiums equals the present value of expected payouts. This element of social insurance means that there is sharing of income shocks across cohorts. We will develop the model initially by abstracting from the wage income shocks and their insurance and return to them later in an extended version of the basic model.

Preferences for a cohort born at date $s \leq t$ are iso-elastic, defined over consumption and real money balances:

$$U(t,s) = E_s \sum_{t=s}^{\infty} (\beta\gamma)^t [\frac{C_{t,s}^{1-\alpha}}{1-\alpha} + \frac{\phi}{1-\sigma}(\frac{M_{t,s}}{P_t})^{1-\sigma}] \tag{1}$$

where $\alpha \geq 0$, and with $\alpha = 1$ is logarithmic utility. The elasticity of marginal utility or coefficient of relative risk aversion parameter, α, balances income and substitution effects, with the income effect reinforcing the substitution effect for $\alpha > 1$. The per period cohort budget constraint is:

$$C_{t,s} + \gamma E_t(\Lambda_{t,t+1} \frac{Z_{t+1}A_{t+1,s}}{\gamma}) + \frac{M_{t+1,s}}{P_t} = \frac{Z_t A_{t,s}}{\gamma} + (1 + i_t^M)\frac{M_{t,s}}{P_t} + (W_{t,s} - T_{t,s}) \tag{2}$$

Any interest on money holding i_t^M is assumed to be zero.

Maximising (1) subject to (2), defining $J(A_{t,s}, \frac{M_{t,s}}{P_t})$ as the value function of the dynamic-programming problem so

$$J(A_{t,s}, \frac{M_{t,s}}{P_t}) = \max[\frac{C_{t,s}^{1-\alpha}}{1-\alpha} + \frac{\phi}{1-\sigma}(\frac{M_{t,s}}{P_t})^{1-\sigma}] + \beta\gamma E_t J(A_{t+1,s}, \frac{M_{t+1,s}}{P_{t+1}}) \tag{3}$$

Initially we will assume that return bearing assets are perfect substitutes so that we do not solve for ω. Hence, choosing $\{C_{t,s}\}_{t=s}^{t=\infty}$ and $\{\frac{M_{t,s}}{P_t}\}_{t=s}^{t=\infty}$, optimality requires

$$\Lambda_{t,t+1} = \beta(\frac{C_{t+1,s}}{C_{t,s}})^{-\alpha} \tag{4}$$

and

$$\phi(\frac{M_{t,s}}{P_t})^{-\sigma}C_{t,s} = 1 - E_t(\frac{P_t}{P_{t+1}}\Lambda_{t,t+1}) = \frac{i_t^S}{1+i_t^S} \tag{5}$$

which inverting, yields

$$\frac{M_{t,s}}{P_t} = [\phi C_{t,s}\frac{1+i_t^S}{i_t}]^{\frac{1}{\sigma}} \tag{6}$$

Note, solving from the cohort budget constraint by forward substitution

$$\sum_{\tau=0}^{\infty}(\gamma)^{\tau}E_t(\Lambda_{t,t+\tau}C_{t+\tau,s}) = \sum_{\tau=0}^{\infty}\gamma^{\tau}E_t(\Lambda_{t,t+\tau}W_{t+\tau,s}) - \sum_{\tau=0}^{\infty}\gamma^{\tau}E_t(\Lambda_{t,t+\tau}T_{t+\tau,s}) + A_{t,s} + \frac{M_{t,s}}{P_t} \tag{7}$$

where

$$\Lambda_{t,t+\tau} = \prod_{j=0}^{j=\tau}\frac{1}{(1+r_{t+j})}$$

$$\lim_{T\to\infty}\Lambda_{t,t+T}A_{t+T,s} = 0 \tag{8}$$

or

$$\sum_{\tau=0}^{\infty}(\beta\gamma)^{\tau}E_t(\Lambda_{t+\tau}C_{t+\tau,s}) = H_{t,s} - \overline{T}_{t,s} + A_{t,s} + \frac{M_{t,s}}{P_t} \tag{9}$$

Also,

$$H_{t,s} = \sum_{\tau=0}^{\infty}\gamma^{\tau}E_t(\Lambda_{t,t+\tau}W_{t+\tau,s}) \tag{10}$$

$$\overline{T}_{t,s} = \sum_{\tau=0}^{\infty}\gamma^{\tau}E_t(\Lambda_{t,t+\tau}T_{t+\tau,s}) \tag{11}$$

The consumption function for the cohort follows from (4) and (9):[2]

$$C_{t,s} = \frac{H_{t,s} - \overline{T}_{t,s} + A_{t,s} + \frac{M_{t,s}}{P_t}}{\sum_{t=0}^{\infty}\gamma^{\frac{t}{\alpha}}[\beta(E_{z_t}Z_t)^{1-\alpha}]^{\frac{t}{\alpha}}}$$

To guarantee convergence of the forward iteration in the denominator, $\gamma(\beta(E_{z_t}Z_t)^{1-\alpha})^{\frac{1}{\alpha}} < 1$,

$$C_{t,s} = \{1 - [\gamma\beta(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\}(H_{t,s} - \overline{T}_{t,s} + A_{t,s} + \frac{M_{t,s}}{P_t}) \tag{12}$$

In this consumption function, the portfolio-return term takes expectations with

[2]For more details on this derivation see Levhari and Mirman (1977).

respect to the probability distribution of z_t. The impact of the return term depends critically upon the value of α, turning on the critical value of $\alpha = 1$ or log utility. For $\alpha < 1$, the return derivative is negative and for $\alpha > 1$, the return derivative is positive.

We can now aggregate over cohorts. With a constant birth rate $\psi = (1-\gamma) \geq 0$ and a survival rate $\gamma \geq 0$, total consumption is

$$C_t = (1-\gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} C_{t,s} \tag{13}$$

Analogously

$$A_t = (1-\gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} A_{t,s} \tag{14}$$

$$H_t = (1-\gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} H_{t,s} \tag{15}$$

$$\overline{T}_t = (1-\gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} T_{t,s} \tag{16}$$

and finally, aggregate money demand is

$$\frac{M_t}{P_t} = (1-\gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} \frac{M_{t,s}}{P_t} \tag{17}$$

2.2 Firms

When individuals are born, they begin work and so long as they survive they accumulate wealth by investing in the economy's competitively run technology. The dividends from the technology are paid to surviving individuals and to dying individuals as part of annuity payments. Firms are competitive and employ capital and labour to produce goods. The production function is Cobb-Douglas, $Y = F(K, L) = K^{\varepsilon} L^{1-\varepsilon}$. The profits of firms are paid as dividends to the owners of equities, who in this economy are mutual insurance companies. Hence, $\widetilde{D}_t = \alpha \widetilde{Y}_t - I_t - \Phi(I_t)$, where $I_t = K_t - (1-\delta) K_{t-1}$ is investment and $\Phi(I_t)$ with $\Phi' > 0$ and $\Phi'' > 0$ are convex adjustment costs. The value of equities, the claim on the income from capital is

$$Q_t = D_t + \Lambda_{t+1} Q_{t+1} \tag{18}$$

Given

$$\lim_{T \to \infty} \Lambda_{t,t+T} Q_{t+T} = 0 \tag{19}$$

$$Q_t = \sum_{\tau=0}^{\infty} E_t(\Lambda_{t+\tau} D_{t+\tau}) \tag{20}$$

Firms invest to maximise the value of equity, so $Q_t' - 1 = \Phi'(I_t)$, which, if $\Phi(I_t) = \frac{1}{2\kappa} I_t^2$, yields:

$$I_t = \kappa(q_t - 1) \tag{21}$$

where $q_t = Q_t' = \sum_{\tau=0}^{\infty} E_t(\Lambda_{t+\tau} \varepsilon (\frac{K_{t+\tau}}{L_{t+\tau}})^{\varepsilon - 1})$.

2.3 Government

We first write down the uses and sources of funds identity in nominal terms for the central government (the treasury) in nominal terms,

$$P_t G_t + i_t \widehat{B}_{t-1} \equiv P_t T_t + (\widehat{B}_t - \widehat{B}_{t-1}) + CBR_t$$

where G_t is government expenditure, T_t is taxes, $\widehat{B}_t$ is the total value of outstanding government debt, CBR_t are receipts from the central bank, and i_t is the weighted average nominal interest rate. The monetary authority (central bank) uses and sources of funds condition is

$$(B_t^{CB} - B_{t-1}^{CB}) + CBR_t \equiv i_t B_{t-1}^{CB} + (\overline{M}_t - \overline{M}_{t-1})$$

where B_t^{CB} are government bonds held by the central bank and $\overline{M_t}$ is the outstanding stock of high-powered (base) money. Consolidating the two conditions, letting $B_t \equiv \widehat{B}_t - B_t^{CB}$ be the nominal stock of government bond held by the public and expressing in real terms we get

$$G_t + i_t \frac{B_{t-1}}{P_t} \equiv T_t + (\frac{B_t}{P_t} - \frac{B_{t-1}}{P_t}) + (\frac{\overline{M}_t}{P_t} - \frac{\overline{M}_{t-1}}{P_t}) \tag{22}$$

With different maturities of debt, the total real value of government debt, $\frac{B_t}{P_t}$, is comprised of short-term (one period) government bonds and long-term government bonds:

$$\frac{B_t}{P_t} \equiv \frac{B_t^S}{P_t} + \frac{B_t^L}{P_t} \tag{23}$$

Each period, some constant fraction, θ, of long-term bonds, B_t^L, mature, so conversely $(1-\theta)$ do not mature, and B_t^{LNew} new bonds are issued, $B_t^L = (1-\theta)B_{t-1}^{LOld} + B_t^{LNew}$. All short-term bonds are by definition, new. In turn

$$i_t^L B_t^L = (1-\theta) i_t^L B_{t-1}^{LOld} + i_t^L B_t^{LNew} \tag{24}$$

If $\eta = B_t^S / B_t$ is the fraction of debt that is short-term and the average maturity of long-term debt is $1/\theta$, then the average debt maturity is given by $\eta + (1-\eta)(1/\theta)$.

The interest rates on the bonds satisfy the inter-temporal pricing conditions, respectively for short-term bonds

$$1 = E_t(\Lambda_{t+1} \frac{P_t}{P_{t+1}} (1 + i_t^S)) \tag{25}$$

and for long-term bonds

$$1 = E_t(\Lambda_{t+1} \frac{P_t}{P_{t+1}} (1 + i_t^{LNew} + g_t)) \tag{26}$$

where $g_t = (B_t^{LOld} - B_{t-1}^{LOld})/B_{t-1}^{LOld}$ is the capital gain (loss) per period due to interest rate movement. Also, we have the term-structure relationship:

$$(1 + i_t^{LNew}) = E_t \prod_{j=0}^{j=L} \Lambda_{t+j} (1 + i_{t+j}^S)^j \tag{27}$$

The government budget constraint is:

$$\frac{B_t^S + B_t^L}{P_t} + \frac{M_t}{P_t} = \frac{(1+i_t^S)B_{t-1}^S + (1+i_t^L)B_{t-1}^L}{P_t} + (1+i_t^M)\frac{M_{t-1}}{P_t} + (\psi+\gamma)\left(T_{t+\tau} - G_{t+\tau}\right) \tag{28}$$

with $B_t^L = (1-\theta)B_{t-1}^{LOld} + B_t^{LNew}$. Clearly given $\psi = (1-\gamma)$, $(\psi+\gamma) = 1$, reflecting the stable population with non-survivors being replaced by births. Using $1 = E_t(\Lambda_{t+1}\frac{P_t}{P_{t+1}}(1+i_t))$ and integrating the budget constraint gives

$$\frac{B_t^S + B_t^L}{P_t} + \frac{M_t}{P_t} = \frac{1}{(1+i_t)}[\sum_{\tau=0}^{\infty} E_t(\Lambda_{t,t+\tau}(s_{t+\tau}^f + s_{t+\tau}^s)) + \lim_{T\to\infty} E_t(\Lambda_{t,t+T}(s_{t+T}^f + s_{t+T}^s))] \tag{29}$$

Here, $s_{t+\tau}^f = (T_{t+\tau} - G_{t+\tau})$; and $s_{t+\tau}^s = (i_{t+\tau}^S - i_{t+\tau}^M)(\frac{\overline{M}_\tau}{P_\tau})$ is seigniorage. The seiniorage term is revenue and hence a source of funds the government gets to use to purchase goods or save taxes, by being able to fund purchases through the monetary base, on which it pays a lower rate of interest, $0 \leq i_t^M < i_t^S$. Here we assume that the interest rate on the monetary base i_t^M is zero. This means that if the government borrows from the central bank, its effective borrowing rate is zero so that monetary creation is a pure inflation tax at the rate of growth of the money supply denoted by μ.

Also, we have

$$\lim_{T\to\infty} E_t(\Lambda_{t,t+T}(s_{t+T}^f + s_{t+T}^s)) = 0 \tag{30}$$

which is the transversality condition. Satisfaction of this condition is necessary for the forecast sequence of fiscal and seigniorage surpluses to be sustainable. This means that the debt-to-income ratio can be kept under control. If the transversality condition is satisfied for all policy sequences for positive prices, these policies are called Ricardian. If policy paths exist for which the transversality condition is not satisfied for all price paths, but only at equilibrium prices, these policies are called non-Ricardian. In the latter case, the government's budget condition is an equilibrium condition. At equilibrium prices, the transversality condition will be satisfied for both Ricardian and non-Ricardian government policies.

3 Market Equilibrium

If return bearing assets, equities and bonds, are perfect substitutes, there is only one market equilibrium condition for these assets. The aggregate market equilibrium condition for financial assets is

$$(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}A_{t,s} = Q_t + \frac{B_t}{P_t} \tag{31}$$

The left-hand-side of this equation is equal to the asset holding held by all of the surviving cohorts up until date t. Money is held by households and must equal the value of the money supply in equilibrium:

$$(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}[\phi C_{t,s}\frac{1+i_t^S}{i_t^S}]^{\frac{1}{\sigma}} = \frac{\overline{M}}{P_t} \tag{32}$$

We also have the consolidated government budget condition:

$$\frac{B_t^S + B_t^L}{P_t} + \frac{\overline{M}_t}{P_t} = \frac{1}{(1+i_t)}[\sum_{\tau=0}^{\infty}E_t(\Lambda_{t,t+\tau}(s_{t+\tau}^f + s_{t+\tau}^s)) + \lim_{T\to\infty} E_t(\Lambda_{t,t+T}(s_{t+T}^f + s_{t+T}^s))] \tag{33}$$

Goods market equilibrium is given by:

$$C_t + I_t + \Phi(I_t) + G_t = Y_t \tag{34}$$

From (13), using (12), (14) and (16) we obtain:

$$C_t = \{1 - [\gamma\beta(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\}[H_t - \overline{T}_t + A_t + \frac{M_t}{P_t}]$$

By substitution of (31) to eliminate A_t and consolidating income from labour (wages) and capital (dividends) into a single income term, Y_t,

$$C_t = \{1 - [\gamma\beta(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\}[\sum_{\tau=0}^{\infty}\gamma^{\tau}E_t(\Lambda_{t+\tau}(Y_{t+\tau})) - \overline{T}_t + Q_t + \frac{B_t}{P_t} + \frac{\overline{M}}{P_t}] \tag{35}$$

Then letting $S_t = Y_t - C_t$, we can replace (34) with

$$S_t = I_t + \Phi(I_t) + G_t \tag{36}$$

which is the economy's IS curve.

The nominal rate of interest is a policy variable set by a target rule (a Taylor rule). According to Taylor's original version of the rule, the nominal interest rate should respond to divergences of actual inflation rates from target inflation rates and of actual Gross Domestic Product (GDP) from potential GDP:

$$i_t^S = \pi_t^* + r_t^* + a_\pi(\pi_t - \pi^*) + a_y(Y_t - \overline{Y}) \tag{37}$$

where a_π and a_y are policy weights. Here, however, we are assuming fully flexible prices and no output gap, $Y_t = \overline{Y}$. Moreover, we assume that $a_\pi > 1$.

In this economy, the real rate of interest is set to satisfy (36). The short-term nominal rate of interest is a policy variable set to achieve the inflation objective set by (37). This means that the money market equilibrium condition (32) takes this rate as given so that it it solves for the quantity of money. Assuming that the all of the right-hand side variables in the condition (33) are exogenously set policy variables, if the transversality condition (30) is satisfied, the consolidated government budget condition (33) is an equilibrium condition that determines the price level.

4 Policy

4.1 Ricardian Equivalence

From the government budget condition (33), with the birth rate equal to the death rate but keeping terms for comparison purposes:

$$\Delta T_t + \sum_{\tau=0}^{\infty} E_t \Lambda_{t,t+\tau} (\psi + \gamma)^\tau \Delta T_{\tau+\tau} = 0 \tag{38}$$

with $\psi = 1-\gamma$. For households alive at date t, using the lifetime budget constraint, the impact is

$$\Delta T_t + \sum_{\tau=0}^{\infty} E_t \Lambda_{t,t+\tau} (\gamma)^{\tau} \Delta T'_{t+\tau} > 0 \tag{39}$$

The reason the two effects are different is that the future tax liabilities are shared with yet unborn tax payers, $\psi > 0$, so for all τ, $\Delta T'_{t+\tau} < \Delta T_{\tau+\tau}$. If there is no birth, $\psi = 0$, $\Delta T'_{t+\tau} = \Delta T_{\tau+\tau}$, then the net wealth effect on consumers is zero. In the model, however, the assumption of a zero-death rate and no birth to keep the population constant delivers neutrality, however, this is an artifact of the model assumption, $\psi = 1 - \gamma$. The crucial element for non-neutrality is a positive birth rate.[3] The unborn are not part of the surviving population's decision problem but will pay some future taxes. So as there is birth in the model, $(1 - \gamma) > 0$, we have non-neutrality and more so if future tax increases are further into the future. That is, the debt payment impact falls disproportionately on yet unborn generations. The positive wealth effect in turn implies that $-\Delta T_t + \frac{\Delta B_t}{P_t} < 0$. This means that consumers at date t increase consumption, so that saving must increase by less than the tax cut. Hence savings must increase by less than the value of government bonds issued to finance the tax cut. But if savings increase by less than the value of the tax cut, the real rate of interest will increase and investment and the capital stock will be lower. At the same time, the present value of government surpluses will be lower, and given the sequence of fiscal surpluses and seiniorage revenues, this necessitates an increase in the price level. Note also that this implies an decrease in the value of real money balances, thereby necessitating an increase in the nominal rate of interest. At a constant inflation rate this is consistent with a higher real interest rate and in turn a lower level of financial wealth.

4.2 Debt Maturity

Barro (1979) started a literature examining the role of the maturity structure of governemnt debt as one of intetemporal tax smoothing. This sees the fundamen-

[3]This point is emphasised in Buiter (1988).

tal problem as one of meeting the government's commitments but at the same time setting taxes to minimise distortions. However, in the current context we assume lump sum taxes. The approach taken here is to consider debt policy as one of smoothing shocks through a dynamic insurance mechanism. The model provides a simple way to see the implications of the maturity structure of the public debt. In the model we present, any non-neutrality of government debt policy will emerge from the composition of government debt impacting the budget sets of households and thereby their consumption-savings decisions. The problem here is seen as a Modigliani-Miller problem, along the lines of Wallace (1981) and Chamley and Polemarchakis (1984). Wallace looked at the impact of changes in the composition of the government's (inclusive of the central bank) balance sheet on household budget sets and market equilibrium conditions. The analysis he conducts is similar to the general equilibrium analysis of corporate financial policy undertaken by Stiglitz (1969). Consider an initial equilibrium with a particular composition of government debt held by households to provide a particular profile of payment to them. At this equilibrium, household budget sets, asset holdings and consumption demands are given. Equilibrium determines equilibrium market returns on stock and bonds and the value of money holdings. Implied annuity transfers are also determined. Suppose that at this equilibrium, debt maturity is changed by substituting one maturity for another, so say average maturity is increased by substituting longer-term debt for short-term debt. This will be irrelevant if households view the new maturity structure as a perfect substitute for the initial structure, with no impact of the change on the distribution of tax liabilities needed to finance the total payments on the debt.

Let us suppose that initially all of the debt is short term. Then the government debt is a sequence of short-term government bonds that must be rolled over to equal the present value of primary surpluses and seigniorage payments. This implies a sequence of tax payments and hence $\{s_\tau^f\}_{\tau=t}^{\tau=\infty}$ satisfying equation (33). We need to understand the impact of debt policy. The initial equilibrium with the government rolling over short-term bonds translates into the composition of the sequence of household budget sets and consumption decisions. In equilibrium there is a path of discount factors $\{\Lambda_{t+\tau}\}_{\tau=0}^{\tau=\infty}$, price levels $\{P_{t+\tau}\}_{\tau=0}^{\tau=\infty}$ and hence implied interest rates satisfying (25) and (26).

Now consider a policy of substituting long-term debt for short-term debt, with $\Delta\frac{B_t^S}{P_t}+\Delta\frac{B_t^L}{P_t}=0$. This impacts the left-hand-side of (33) through a change in debt composition only. This change in the composition of the debt will not impact the tax financed payments on the total amount of debt if a sequence of rolled over short-term debt is a perfect substitute for long-term debt, so that any long-term bond can be costlessly decomposed into a sequence of short term bonds. Then issuing long-term bonds will be equivalent in expected value to the sequence of rolled over short-term bonds they replace in household portfolios, provided the real (inflation adjusted) sequence of payments to the bond-holders is the same. If long-term bonds can be used to satisfy tax liabilities without any additional cost, the maturity substitution will be neutral in its effects.

On the other hand, if the substitution of longer-term for short-term debt changes the profile of tax liabilities by pushing these payments further into the future, then on the right-hand-side of (33), the pattern of primary surpluses must change. The payments to bond-holders in the immediate term must be reduced and longer-term payments must be increased. This is neutral from the stand-point of the government's budget constraint. This in turn means that the price level does not change. However, it reduces tax payments in the short-term and increases them in the longer-term. These later payments are a transfer of tax burden in part to future, yet unborn, tax-payers. Hence, the net wealth of current consumers is increased and so is their consumption. The non-neutrality and real impact is therefore of a similar nature to the case of non-Ricardian equivalence discussed above.

4.3 Tobin Mundell Effect

Mundell (1965) and Tobin (1965) considered the impact on investment and capital holding through a portfolio balance effect. Now consider the impact of a change in the inflation target. In this model consider an increase in π^* in the Taylor rule (37). This must be accommodated by an increase in monetary growth. This in turn raises the nominal interest rate, which in (32) reduces the demand for money and so to maintain the inflation target the stock of money must be reduced. The reduction in money holding causes a substitution towards interest bearing assets. This will be enhanced by a negative impact on the value of outstanding government bonds. Given the stock of government bonds in the model, this necessitates an increase in holdings of equity in real capital. This must reduce real interest rates, which increases Q and so investment and capital accumulation. At the same time if $\alpha > 1$, so that the impact of lower returns on consumption in (35) is negative, so saving increases. In equilibrium, the difference between the real and nominal interest rates satisfies the Fisher equation and so at lower real rates the change in the nominal rate of interest will be less than the change in the inflation target.

Of course, the above reasoning has to be modified in our model. Our model has money in the utility function as in Sidrauski (1967). In the pure infinite horizon case with an infinite horizon, which obtains with a zero-birth rate (and no death), super-neutrality obtains, so the steady state real interest rate and capital stock is independent of the inflation rate. Consider the special case of a zero birth and death rate, $\gamma = 1$. In that case the real interest rate must equal the rate of time preference, $r = \rho$, and therefore the capital stock is independent of the monetary growth rate. Moreover, consumption is unaffected. An increase in monetary growth, μ, increases the nominal interest rate one-for-one and therefore reduces holdings of real money balances. It can be shown that real seignorage revenues, $\mu\frac{M_t}{P_t}$, increase, so that lump-sum taxes fall and human wealth increases. Moreover, the fall in non-human wealth, caused by the reduction in real money balances, is exactly off set by the increase in human wealth, so that total wealth and the consumption of physical goods are unaffected. Marini and Van der Ploeg (1988) show that when $\gamma = 1$, so lives are infinite, and the sub-utility function is weakly separable in consumption and real balances, as in our model, the real part of the dynamic system separates out from the monetary part and therefore monetary growth does not affect the transitional dynamics of the real variables. However, when the sub-utility function is not weakly separable, monetary growth affects the marginal propensity to consume goods out of total wealth and thus affects the dynamics, but it does not affect the steady-state value of capital (see Fischer, 1979).

Now consider the general case of finite lives, $\gamma < 1$ and a positive birth rate. Now changes in monetary growth affect the steady-state value of the capital stock. Even a weakly separable sub-utility function generates this non-neutrality result. With finite lives and positive birth and death rates an increase in monetary growth leads in the long run to an equal increase in inflation, a fall in the real interest rate, an increase in capital, output, and consumption. In this model there is also a fall in the level of real money balances. Of course, as Marini and van der Ploeg (1988) point out, this effect is very similar to the conventional Mundell-Tobin effect. This non-neutrality arises, because with finite lives a wedge is driven between the discount rate used to calculate the value of government surpluses, which in the absence of risk and $\psi + \gamma = 1$ is r, and the one used to calculate human wealth, $r + (1 - \gamma)$. This can be seen in the algebra of the model along similar lines as in the case of Ricardian equivalence examined above. Monetary growth raises seigniorage revenues (despite a fall in real money balances) and therefore reduces lump-sum taxes. Households human wealth increases because lump-sum taxes fall. This impact is enhanced by an increase in wages and a fall in the real rate of interest.

5 Extending the Model

We now extend the model to accommodate some agent heterogeneity along the lines examined extensively by Farmer (see for example Farmer (2016)). The simple extension is to have two types of individual born at each date who differ purely in terms of their time preference. There is a high time preference group 1, with time preference factor β_1, who want to consume more earlier and indeed may want to borrow in the early phase of life. There is a second group with lower time preference, $\beta_2 > \beta_1$, who never borrow and can act as lenders to borrowers.

Borrowing could take the form of short sales of return bearing assets, combined with life insurance. Here the borrower, borrows a portfolio of assets which are then sold. The borrower then uses the proceeds to finance consumption but will have to ensure that they have saved enough later to repurchase the assets and settle the short position. However, if they die before the account is settled, at least in part, the lender is exposed to either a full or partial default. This could be handled by changing the nature of annuities contracts in a complex way. An alternative is pure consumption loans. The lender will offer consumption loans to the impatient borrower. The loans are held in lenders portfolios along-side government bonds and equities and are used by borrowers to fund consumption. To ensure that in the event of death the loan can be repaid and offer a return equivalent to long positions in assets combined with equities, the loans must be insured by short-positions in annuities backed by long-positions in return bearing assets or command higher rates with a significant default risk premium paid in solvent states.

Let us suppose that borrowing takes place, borrowers plan to repay loans later but in the event that they die their debts are paid by life insurance. The lender will be paid for sure either from the surviving borrower or from a insurance company. The insurance company collects insurance risk premia from borrowers, pooling loans across borrowers and setting premia on competitive fair terms. We assume that insurance companies are endowed with capital from history so that they can meet debts of borrowers who die in debt.

We assume that the proportion of type β_1 born at each date is $0 < \lambda < 1$. The consumption-savings decisions of the two groups are technically the same as above, the principal difference is that there are two groups differing by discount rate and the recognition that net borrowers have no assets to transfer in the event of their death, rather debts that must be covered by life-insurance. We do not examine the dynamic programming problem in any detail.[4] For each group, the consumption functions are the same linear functions as in (13). For the high-time preference group, let the flow budget constraint be written as

$$\begin{aligned} C^1_{t,s} + \gamma E_t(\Lambda_{t,t+1}\frac{Z_{t+1}A^1_{t+1,s}}{\gamma}) + \gamma\frac{M^1_{t+1,s}}{P_t} - \gamma E_t(\Lambda_{t,t+1}\frac{(1+i^S_t)}{\gamma}L^+_{t+1,s}) \\ -\gamma V^*_{t,s} = \frac{Z_t A^1_{t,s}}{\gamma} + L^+_{t,s} + (1+i^M_t)\frac{M^1_{t,s}}{P_t} + (W_{t,s} - T_{t,s}) \end{aligned} \tag{40}$$

In the uses of funds on the left hand side of the expression along-side the loan repayment term is the premium, $V^*_{t,s}$, paid to the insurance company to cover the solvency risk of loans. In the borrowing phase, $A^1_{t,s} = 0$ and and borrowing $L^{1+}_{t,s} > 0$ in this phase. From the lenders perspective they are not exposed to solvency risk on these loans, so they attract the same interest rate as the short-term borrowing rate. For the β_1 group the consumption function is

$$C^1_{t,s} = \{1 - [\gamma\beta(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\}(H^1_{t,s} - \overline{T}_{t,s} - \overline{V}^*_{t,s} + A^1_{t,s} + \frac{M^1_{t,s}}{P_t}) \tag{41}$$

where $\overline{V}^*_{t,s}$ is the capital value of the insurance premiums.

For the β_2 group, who do borrow but lend and are on the opposite side of the

[4] The household's dynamic programme is treated as smooth in the sense that there are no non-convexities between phases of the household's life cycle, such as switching between borrowing and lending. Hence in the case of a hard no borrowing constraint binding, we will treat it as fixed. In a more general framework we would need to consider left and right derivatives of the value function with respect to the state variable $A_{t,s}$ at points of phase transition and discontinuity in the consumption control variable, $C_{t,s}$.

competitively priced consumption loans, $L^{+}_{t+1,s}$, the flow budget constraint is

$$C^2_{t,s} + \gamma E_t(\Lambda_{t,t+1}\frac{Z_{t+1}A^2_{t+1,s}}{\gamma}) + \gamma\frac{M^2_{t+1,s}}{P_t} + \gamma E_t(\Lambda_{t,t+1}\frac{(1+i^S_t)}{\gamma}L^{+}_{t+1,s}) \tag{42}$$
$$= \frac{Z_t A^1_{t,s}}{\gamma} - L^{+}_{t,s} + (1+i^M_t)\frac{M^1_{t,s}}{P_t} + (W_{t,s} - T_{t,s})$$

The cohort's consumption function is

$$C^2_{t,s} = \{1 - [\gamma\beta(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\}(H^2_{t,s} - \overline{T}_{t,s} + A^2_{t,s} + \frac{M^2_{t,s}}{P_t}) \tag{43}$$

The aggregate consumption function is

$$C_t = \lambda[(1-\gamma)\sum_{s=-\infty}^{t}\gamma^{t-s}C^1_{t,s}] + (1-\lambda)[(1-\gamma)\sum_{s=-\infty}^{t}\gamma^{t-s}C^2_{t,s}] \tag{44}$$

This aggregate function replaces (35) in the equilibrium condition (34).

The net aggregate return bearing asset holding is

$$\lambda[(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}A^1_{t,s} \tag{45}$$
$$+(1-\lambda)(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}A^2_{t,s}]$$

If return bearing assets are perfect substitutes, this quantity replaces the term on the left-hand-side of (31).

Now consider the case when it is not possible to borrow to access human capital to enhance current consumption as is assumed above. That is, individuals cannot short-sell interest bearing assets or buy life-insurance with borrowed funds. However, if there is a binding borrowing constraint for the β_1 group, consumption in the first phase of life involves:

$$C^1_{t,s} = \frac{Z_t A_{t,s}}{\gamma} + (1+i^M_t)\frac{M^1_{t,s}}{P_t} + (W_{t,s} - T_{t,s}) \tag{46}$$

This means that any increase in $W_{t,s}$ or reduction in $T_{t,s}$ for this group will result in a one for one increase in consumption, $C^{1i}_{t,s}$.

Suppose that at time t, of the high-discount group β_1, a particular cohort born at date s is constrained. This is indicated by the zero-one indicator variable $\sigma_{t,s}$ being equal to one. Then, the net demand for return bearing assets in (31) becomes

$$\lambda(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}A^1_{t,s} + (1-\lambda)(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}(1-\sigma_{t,s})A^2_{t,s} \tag{47}$$

5.1 Policy

Given the simple modification of the model to include heterogeneity within cohorts, we reconsider the two simple exercises undertaken above.

5.1.1 Ricardian Equivalence

The changes to the model do not fundamentally change the governments intertemporal budget condition. From the government budget condition, with the birth rate equal to the death rate

$$\Delta T_t + \sum_{\tau=0}^{\infty} E_t\Lambda_{t,t+\tau}(\psi+\gamma)^{\tau}\Delta T_{\tau+\tau} = 0 \tag{48}$$

For households alive at date t, the combined impact on net wealth of the tax change is

$$\lambda[\Delta T_t+\sum_{\tau=0}^{\infty}E_t\Lambda_{t,t+\tau}(\gamma)^{\tau}(1-\sigma_{t,s})\Delta T'_{t+\tau}]+(1-\lambda)[\Delta T_t+\sum_{\tau=0}^{\infty}E_t\Lambda_{t,t+\tau}(\gamma)^{\tau}\Delta T'_{t+\tau}] \tag{49}$$

where the terms $\sigma_{t,s}=1$ for some s indicates that some β_1 group cohorts may be credit constrained and not able to access future wages and so do not discount the tax liability. This becomes material through the total impact on consumption:

$$\Delta C_t = \lambda[(1-\gamma)\sum_{s=-\infty}^{t}\gamma^{t-s}\{1-[\gamma\beta_1(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\} \tag{50}$$
$$[\Delta T_t + \sum_{\tau=0}^{\infty} E_t\Lambda_{t,t+\tau}(\gamma)^{\tau}(1-\sigma_{t,s})(\Delta T'_{t+\tau} + \Delta\overline{V}^*_{t,s})]$$
$$+\lambda[\sum_{s=-\infty}^{t}\sigma_{t,s}(\Delta T'_t + \Delta\overline{V}^*_t)] + (1-\lambda)[(1-\gamma)\sum_{s=-\infty}^{t}\gamma^{t-s}\{1-[\gamma\beta_2(E_{z_t}Z_t)^{1-\alpha}]^{\frac{1}{\alpha}}\}$$
$$(\Delta T_t + \sum_{\tau=0}^{\infty} E_t\Lambda_{t,t+\tau}(\gamma)^{\tau}\Delta T'_{t+\tau})]$$

This ΔC_t term is made up of three big terms. The first term is the impact on the unconstrained cohorts of the β_1 group; and the second the impact on the constrained cohorts of this group; whilst the last term is the impact on the unconstrained β_2 group. All three of the consumption changes are positive. The first and third, because some of the future tax liabilities are borne by unborn households. The presence of the term $\psi > 0$ in (48) means that $\Delta T'_{t+\tau} < \Delta T_{\tau+\tau}$ for all cohorts and for all τ. Note, however, that all members of the β_1 group who borrow will benefit additionally from the tax cut by effectively borrowing at the government borrowing rate and save paying insurance premia, $\Delta\overline{V}^*_t < 0$, on these loans as the loans are financed through future lump-sum taxes. There is also a more significant effect from the presence of the credit constrained members of the β_1 group, who will pay future taxes but as they are constrained, spend the tax cut. Again, the impacts on the β_2 will be zero if $\psi = 0$, as $\Delta T'_{t+\tau} = \Delta T_{\tau+\tau}$. However, borrowing cohorts and in particular constrained cohorts will still see a positive impact on net wealth and consumption from effectively improved borrowing terms even when $\psi = 0$.

5.1.2 Debt Maturity

In the extended model the issue of debt maturity is more complex. The debt market can be divided into clienteles who have different preferences for longer and shorter-term funding. This raises the potential for the government to cater to the demands of the market. Additional non-neutralities arise if different groups consumption opportunities are affected asymmetrically by changes in the government's debt policy. Here we consider segmenting the market for bonds, so that the bond portfolio required to finance the consumption for the β_1 group may differ from that for the β_2 group but for the moment assume that no households are credit constrained, $\sigma_{t,s} = 0$ for all s.

We assume short-term bonds are held between the β_1 group and the β_2 group with $\eta^{S1} + \eta^{S2} = 1$; and similarly for long-term bonds $\eta^{L1} + \eta^{L2} = 1$. The return bearing asset holdings of the β_1, impatient group, is equal to the supply

$$\lambda(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}A_{t,s}^{1} = Q_t^1 + \eta^{S1}\frac{B_t^S}{P_t} + \eta^{L1}\frac{B_t^L}{P_t} \tag{51}$$

They wish to consume relatively more now and less later in life but will need to build up assets later to ensure that they are solvent in the limit. The demand for return bearing assets and annuity contracts by cohorts on the left-hand-side of this expression imply a certain stream of payments to survivors. Older types must have relatively lower payments as they have consumed more earlier, this is the so-called humped savings hypothesis. This means that the portfolio of assets will need to deliver more income in the immediate future to match liabilities

Next consider the β_2, more patient group:

$$(1-\lambda)(1-\gamma)\sum_{s=-\infty}^{t}(\gamma)^{t-s}A_{t,s}^{2} = Q_t^2 + \eta^{S2}\frac{B_t^S}{P_t} + \eta^{L2}\frac{B_t^L}{P_t} \tag{52}$$

At date t, this group of survivors from previous cohorts and the cohort born at date t will consume less than the β_1 group with a preference for later consumption, so that they hold more assets and more annuities. This group will save more and hold more assets and annuity contracts and thereby allocate more of their human capital to more distant consumption.

The question arises as to whether the government can cater to this (and so reduce its own funding costs) by issuing longer-term bonds that substitute for rolling over short-term bonds, which will raise welfare only if it overcomes a constraint. Rolling over short-term bonds or indeed liquidating long-term bonds are equivalent if there is no roll over risk or early-liquidation risk.

6 Further Extension of the Model

In the analysis conducted so far, all that matters is distribution of tax liabilities. It is this that determines payments to debt, which could be perpetuities The retirement of bonds at some point will limit the burden of any taxes implied by debt falling on future generations. The same outcome can be achieved by rolling over short-term debt. Of course, this result is due to perfect substituted between maturities and in particular, the absence of liquidation risk for long-term bonds or roll-over risk for short-term bonds.

6.1 Shocks to Government Expenditure

Let us first consider the illustrative example of a shock to the government's budget and ask how the government may meet shocks in a way that smooths or insures the impact. The optimal way of meeting the contingent needs is to be able to cover the income losses through a holding of (Arrow-Debreu) state-contingent claims. Angeletos (2002) provides a general framework in which the maturity structure of public debt provides the opportunity for the government to construct dynamic insurance against shocks.[5] He shows that to span the state-space, the economy needs as many linearly independent income streams as states of the world at each date and the rebalancing of the portfolio at each date. To achieve this with different maturities is only be possible if the different maturities are not co-linear. In his framework the government uses debt policy as part of an optimal policy of funding (random) government expenditures with Ramsey taxation, when also

[5] Bohn (1990) analyses the optimal structure of government debt in a stochastic environment. He shows in a model with distortionary taxes, the government should smooth tax rates over states of nature as well as over time. Government liabilities should be structured to hedge against macroeconomic shocks that affect the government budget. The optimal structure of government liabilities generally includes some" risky" securities which are state-contingent in real terms.

faced with income and interest rate shocks. The optimal maturity structure of government debt provides perfect insurance and allows the government to sustain an invariant rate of taxation. Holding long-term debt and investing in a short-term asset can hedge the budget against both random variation in government expenditure and aggregate income, as well as against the risk of refinancing the outstanding debt at variable interest rates. However, it is still optimal to transform the initial debt to a perpetuity, so as to insulate the budget from the risk of refinancing public debt at variable interest rates.

Consider the government's budget condition (33) where the left-hand side is the value of the government's liabilities and the right-hand side is the value of its assets, the present value of fiscal surpluses. The right-hand-side may be subject to shocks. Assume that there is the possibility of a negative shock at each date to fiscal surpluses because of a jump in government expenditure, that is with probability v^f the current fiscal surplus falls by $\Delta^f s_t^f$ because of an increase in government expenditure and in the event of such a change assume by condition (37) that the short-term rate of interest jumps up by $\Delta^i i_t^S$, the possibility of these jumps is reflected in the term-structure and bond prices. Following Angeletos (2002), suppose that the government structures its liabilities by making all long-term bonds perpetuities but also issuing some bonds to finance a reserve fund, R_t. The value of government bonds pre the fiscal shock and contributions to the reserve fund is $\frac{B_{t-1}}{P_t}$. Post the fiscal shock and contributions to the reserve fund, the value of the debt is $\frac{B_t}{P_t}$. The change in the value of bonds $\frac{B_t}{P_t}$ due to the interest rate change is $\Delta^i \frac{B_t}{P_t}$. The right-hand-side of (33) is the present value of fiscal surpluses, written as $PV(s_t^f)$. The difference in the value of $PV(s_t^f)$ due to the interest rate change is $\Delta^f PV(s_t^f)$. The profile of payments on the perpetuities is such that $\frac{B_t}{P_t} = PV(s_t^f)$ and $\Delta^i(\frac{B_t}{P_t}) = \Delta^f PV(s_t^f)$, moreover, $\Delta^f PV(s_t^f) = \Delta^f s_t^f + \Delta^i[\frac{B_{t-1}}{P_t} - s_t^f]$, which combine with the government's uses and sources of funds condition $(1+i_t^S)R_t = \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} + s_t^f$ to give $\Delta^i(\frac{B_t}{P_t}) = \Delta^f s_t^f + \Delta^i[\frac{B_t}{P_t} - (1+i_t^S)R_t]$.

The last condition yields

$$R_t = \frac{\Delta^f s_t^f}{\Delta^i(1+i_t^S)}$$

and substituting into the uses and sources of funds condition

$$\frac{B_t}{P_t} = [\frac{B_{t-1}}{P_t} - s_t^f] + \frac{(1+i_t^S)\Delta^f s_t^f}{\Delta^i(1+i_t^S)}$$

The optimal investment in the reserve funds R_t makes sure that the increase in returns after a fiscal shock is just enough to compensate for the shortfall in the primary surplus. The optimal perpetuity, on the other hand, is equal to this investment plus the historical level of debt. We will comment on this policy at the end of the next sub-section.

6.2 Shocks to Household Income

Now introduce a simple change in the model. The problem we consider here is different from the above, in that households are subject to income shocks that they wish to insure. For each cohort there is an additional risk of income loss at every date after the first of $\delta W_{t,s}$ with probability v^I. This shock affects every member of the cohort in the same way and are distributed iid over time. Hence, it cannot be mitigated by within cohort insurance or contingent transfers. Insurance necessitates sharing the risk with other cohorts. The additional exposure to income losses can, however, be hedged by paying premia to an insurance company; or a government run scheme with the premia being a tax. Let us suppose that the premia are paid across periods to smooth consumption. The income shock insurance is arranged by zero profit insurance companies, which take in and invest premia and make payments in the event of loss. Competition and actuarial fairness mean that the aggregate present value of these premia equals the present value of expected payouts. This element of social insurance means that there is sharing of income shocks across cohorts. This is modelled as an adjustment to the per-period wage income stream. The per-period budget constraint of a cohort is

$$C_{t,s} + \gamma E_t(\Lambda_{t,t+1}\frac{Z_{t+1}A_{t+1,s}}{\gamma}) + \gamma\frac{M_{t+1,s}}{P_t} = \tag{53}$$
$$\frac{Z_t A_{t,s}}{\gamma} + (1+i_t^M)\frac{M_{t,s}}{P_t} + [W_{t,s}(1-\delta_t) + X_{t,s} - V_{t,s}^{**} - T_{t,s}]$$

The income shocks are offset by payments of $X_{t,s}$. The expected value of the income shocks is $\sum_{\tau=0}^{\infty}\gamma^\tau v^I E_t(\Lambda_{t,t+\tau}\delta W_{t+\tau,s})$. The present value of premiums $V_{t,s}^{**}$, is $\overline{V}_{t,s}^{**} = \sum_{\tau=0}^{\infty}\gamma^\tau E_t(\Lambda_{t,t+\tau}V_{t+\tau,s}^{**})$, where $V_{t+\tau,s}^{**}$ is constant.

The optimal way of meeting the contingent needs of the insurance company to be able to cover the income losses is through a holding of (Arrow-Debreu) state-contingent claims. We assume that this is not possible and that insurance companies take in premia and pool risks and invest in bonds of a limited range of maturities. If an insurance company has sufficient reserves to meet current income losses that would allow the appropriate cover. Such an insurance company would then be in a position to take in premia and offer to cover losses on a fair basis. The reserves of the fund could be held in long-term bonds with some being liquidated in the event of a negative shock. If the initial endowment of the insurance fund is an issue, then a government run scheme, where the initial balance of the fund is obtained by issuing long-term bonds and servicing the debt with additional premium type taxation may be appropriate.

Suppose that the insurance premiums and any reserves of the insurance fund are held in a portfolio of perpetuities, which insulates the insurance fund from any period by period variations in the short-term rate of interest. Then, if an income shock occurs, the insurance company must liquidate bonds to cover the loss. However, in the absence of liquidity risk, low prices for bonds at this date, there is no cost to this strategy. Suppose on the other hand, that the insurance company invests in one-period bonds. Then, in the event of a loss it uses the realised value of some one-period bonds to cover the loss. In the event that the loss does not occur it rolls-over the bonds by purchasing new ones. If there is no exposure to roll-over risk from rising short-term interest rates, there is no cost to this strategy. Hence it is a matter of indifference which strategy is used.

There is the possibility that at each date the short-term interest rate jumps. At each date there is a potential shock to interest rates, they may jump up by $\Delta^i i_t^S$, then holding short-term bonds may be expensive. This arises as the one-period cost of the strategy includes a refinancing cost if the bonds are not needed with probability $(1 - v^I)$ to fund payouts, this cost is $\Delta^i i_t^S B_t^{S*}$. On the other hand, if liquidating long-term bonds with probability v^I may be expensive because of of the impact of a negative liquidity effect on liquidated bonds of $\Delta^I B_t^{L*}$. Given that the probability of the liquidity event is exogenous, the choice of strategy involves a simple comparison of $(1-v)\Delta^i i_t^S B_t^{S*}$ to $v\Delta^I B_t^{L*}$. Given that this is loss is possible at every date, this comparison arises at every date. So if the cost of roll-over risk is greater than liquidation cost risk, it means that it is optimal to hold enough short-term bonds in reserve to meet current needs and hold longer-term bonds, perpetuities, as reserve to meet later needs as they arise.

The examples above concern insulating the government budget from shocks to the fiscal surplus on the one hand and insuring households from income shocks on the other. In both cases the crucial issue is to smooth the impact of the shocks and avoid short-term exposure to interest rate risk. However, funding the government budget or the insurance schemes portfolio with long-term bonds, perpetuities, does impose a burden on yet unborn cohorts and will other-things-being equal increase consumption relative to what it otherwise would be. In the case of income shocks, the trade-off may involve considering the convex costs of rollover-risk against the liquidation risk associated with long-term bonds.

7 Efficiency Properties of the Model

In this section we examine some basic efficiency properties of the model. Blanchard (1986) undertakes an evaluation of the efficiency of his model in which he examines its dynamics and steady state properties. Here, we undertake similar exercises but only examine the steady state. As in Diamond (1965) and Blanchard we consider Phelps (1961)-Koopmans (1965) efficiency. It is easier to make the main point using the special case of log utility in which $\alpha = 1$. Note that savings is given by

$$S = (Y - T) - C \tag{54}$$

Substituting for consumption and income as $Y = W + rQ$. Letting $\beta = 1/1 + \rho$, and approximating $[1 - \beta\gamma] \approxeq \rho + (1 - \gamma)$, which is exact in the limit of continuous time:

$$S = W + rQ - (\rho + (1 - \gamma))[\frac{W}{r + (1 - \gamma)} - \overline{T}_t + Q + \frac{B_t}{P_t} + \frac{\overline{M}}{P_t}] \tag{55}$$

Collecting terms

$$S = W\frac{[r - \rho]}{(r + (1 - \gamma))} + Q[r - \rho - (1 - \gamma)] - (\rho + (1 - \gamma))[-\overline{T}_t + \frac{B_t}{P_t} + \frac{\overline{M}}{P_t}] \tag{56}$$

Let the fiscal and monetary terms $\overline{T}_t = \frac{B_t}{P_t} = \frac{\overline{M}}{P_t} = 0$. On a balanced growth path on which $S_t = I_t$ with $K_t = K_{t-1}$, so $I_t = \delta K_{t-1}$. The solution must lie between two values of K: $r(K^*)$, which satisfies $r(K^*) \approxeq \rho$; and $r(K^{**}) \approxeq \rho + (1 - \gamma)$.

The modified golden rule solution obtains when $\gamma = 1$ and $T_t = Q = \frac{B_t}{P_t} = \frac{\overline{M}}{P_t} = 0$ and r satisfies $r(\widehat{K}) = \rho$. In the absence of trade with unborn generations, or current generations caring in the sense of Barro (1974) about future generations through a bequest motive, the equilibrium interest rate in the model in the paper may well be below $r(\widehat{K})$.[6] Then alternative assets can improve efficiency by reducing capital holding. Some level of money holding will substitute

[6] Barro (2020) notes that the general Phelps-Koopmans efficiency condition that the riskless rate exceeds the growth rate, $r > g$, does not hold in data (see Abel et. al. (1989) and Blanchard (2019)). However, he argues that the condition holds when r is based on risky returns (on equity) but not when r is based on safe returns (approximated by treasury bills). He argues that in a stochastic growth model r is replaced by a risky expected return that incorporates a significant equity premium. To resolve the equity-premium puzzle of Mehra and Prescott (1985), he uses a simple model with disaster risk based on Rietz (1988) and Barro (2006).

for over-accumulation of capital as will holding government bonds of any maturity structure, so long as some of the implied tax liabilities fall on unborn cohorts. That is some degree of financial crowding out will increase welfare in well understood ways. Let us address these issues below.

The extent to which money holding can increase and displace capital in portfolios is limited by the demand for money and requires setting interest rates lower. A lower nominal interest rate target cannot be separated from the inflation target, so this necessarily involves reducing the inflation target. Turning to debt financing, this can only displace capital if the debt constitutes net wealth as in the original Diamond formulation. However, there is an additional avenue which concerns a bubble-like feature of government debt as examined by Tirole (1986) and Gali (2014 and 2020) as well as others.[7]

From a household's perspective, the transversality conditions on their lifetime budget constraint (infinite horizon but finite life expectancy) is satisfied inclusive of the taxes they expect to pay. However, from the government's perspective, if the economy is dynamically inefficient there is value at infinity. Households are not able to access this value by issuing securities, but the government can. Hammond (1975) examined the problem as a poverty, or isolated generation game, that can be understood along the lines of Samuelson (1958) and Shell (1971).[8] Transfers of

[7]The concern of this paper is closest to that of Tirole. Gali is concerned with showing the existence of bubbles and then showing how the random fluctuation in the value of bubles impacts output in a model with some price stickiness and hence a stochastic output gap.

[8]Shell (1971) give a simple illustration of Samuelson's (1958) problem using a chocolate bar economy analogy. The economy consists of a infinite sequence of two-period overlapping generations. Each generation is born with one bar of a perishable good. An autarkic solution is overcome by a sequence of IOU's, which facilitates trade between generations. If this works, the IOU each period is exchanged for the perishable good and so every generation to infinity has consumption when young and old except for the first, which gets the bonus of being able to consume all of its endowment when young. This additional unit of consumption has essentially come from infinity and is the bubble value of the IOU. Every generation in the sequence is better of but there is a problem that every generation has a temptation to deviate and not accept the previous generation' s IOU and issue their own IOU.

This led Hammond (1975) to consider the problem as a super game with punishments for those who do not honour IOUs. In his case the transfers between generations were pension payments but the problem is the same. However, his solution involves individuals yet unborn inheriting an understanding that they must punish coexisting generations if they failed to pay a fair pension by not paying them a fair pension or honour IOUs and thereby see the need to act the same way themselves.

Shell and others have suggested that the efficient solution and the value of the bubble is envalue from infinity through sequences of IOUs may be sustainable in a super-game setting but if not then a social security or inter-temporal tax-based redistribution scheme may be effective. In its most stark form government debt is essentially interest-bearing money (pieces of paper). The payments of interest in an inefficient economy can be made by essentially taxing resources trapped at infinity. In this case households place a finite capital value on their expected tax liabilities, but the government can finance repayment of debt by rolling over debt and hence tax payments to infinity.[9] Then, it is this feature of debt as opposed to taxing unborn generations, that increases efficiency. Note that in inefficient equilibria with bubble values, the transversality condition on the government's inter-temporal budget constraint is not satisfied, so that the equation cannot be used to define a unique price level as an equilibrium outcome.

8 Conclusion

The paper has developed a dynamic Metzleric model that incorporates Blanchard type intergenerational cohorts and an intertemporal government budget constraint. The model incorporates heterogeneity, with two types of individual born at each date, patient and impatient. Insurance markets are introduced to allow risks to be traded and to provide a simplification of the model and to allow us to focus on some simple channels for the real effects of policy. The model is set up to provide a simple setting in which to investigate substituting tax for debt finance, changes in the maturity structure of government debt and inflationary finance. Real effects occur because of redistribution owing to finite horizons, incompleteness of markets and differential discount rates. The final section of the

forced by the government accepting the IOU as the only medium in which to settle tax liabilities.

[9] The Ponzi scheme nature of government finance described (see Barro 2020) assumes that the government can always meet required payments through taxes. This requires that the government remains solvent, so that the value of payments on debt must be bounded by the value of the governments tax base. Uncertainty in the evolution of GDP implies that the rolling-over forever of the government's debt generates a positive probability that the debt would eventually exceed the government's collateral, thereby triggering sovereign default. In this case, Ponzi borrowing in risk-free form by the government can be ruled out

paper examines some simple efficiency properties of the market equilibrium and hence the basis for government intervention.

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