Agency Costs, Investment and Debt Overhang

D C WEBB

London School of Economics

June 22, 2022

Abstract

The paper develops a dynamic model of firm investment with agency problems. The approach taken follows that of Cao et.al. (2019) but integrates some features of Clementi and Hopenhayn (2006) and Bias et.al. (2011). A firm is faced with multi-period projects and needs to raise outside finance but cannot commit to honest reporting of income, so incentives must ensure honest reporting. This agency induced wedge between the cost of internal and external funds impacts investment policy. High cash flow reports increase the entrepreneur's equity stake and this tends to relieve the agency problem, thereby leading to more investment and earlier exercise of investment opportunities.. Faced with sequential investments, the wedge between the cost of internal and external finance can affect the way the firm ranks projects. In particular, projects that generate net cash flow quickly but are of relatively lower net present value may be prioritised so as to keep leverage and financial servicing costs low before higher net present value projects that deliver net cash flows more slowly are initiated. Even though the firm's capital structure is designed to mitigate this problem, leverage in particular has real effects upon investment policy. We also show how the moral-hazard problem interacts with a Myers (1977) debt-overhang problem and generates an interaction between leverage and the timing of exercise of growth options.

JEL Classification: D86;G30;G31;G32.

1 Introduction

This paper is concerned with the impact of a firm's financial policies on its level of investment when there is moral hazard arising from outside finance. This means that the volume of internal funds is important for investment, so that depending upon the firm's current funding, investment may be lower than optimal so that internally generated funds can be used to finance subsequent investment. Firms that have growth opportunities must manage leverage and debt service costs to limit agency costs and thus time growth.

A number of papers have developed simple discrete time, finite horizon models of corporate finance of given investments in the presence of moral hazard and have highlighted interesting properties of optimal financial policy. Key contributions are Bolton and Scharfstein (1990), Innes (1990) and Holmstrom and Tirole (1997). A number of more recent papers have added to our understanding of more general inter-temporal investment problems with repeated moral hazard. Gromb (1999) extends Bolton and Sharfstein's analysis to an infinite horizon. Other papers apply recursive techniques developed to handle multi-period moral hazard problems (see Green (1987), Spear and Srivasta (1987) and Thomas and Worral (1990)) to consider dynamic investment -financing decisions. Quadrini (2003), analyses the investment problem in a stationary environment with a simple moral hazard problem, where non-convexities arise because of lumpy liquidation. Clementi and Hopenhayn (2006) consider a similar non-stationary problem in which the firm requires injections of working capital, again with an emphasis on the interface between the principal-agent problem and investment with lumpy liquidation. These papers also place emphasis on the conditions under which financial contracts are renegotiation proof. DeMarzo and Fishman (2007a) consider a firm with given investments and examine the determination of the optimal financial policy in the presence of repeated moral hazard. In a companion paper DeMarzo and Fishman (2007b) provide a general discrete-time analysis of a class of agency problems and the implications for corporate investment and growth. Both DeMarzo and Sannikov (2006) and Biais et al (2007) develop continuous time versions of the financing problem for a firm with given investments but repeated moral hazard in financing. These papers essentially generalise the analysis in DeMarzo and Fishman (2007a) to continuous time and provide a variety of elegant results characterizing the solution to the agency problem and its implementation through financial contracts. Biais et al (2007) actually derive the continuous time problem as the limit of an infinite-horizon discrete time problem and in doing so illustrate the optimal financial policy in both discrete and continuous time. However, these papers do not examine the interaction of the firm's financial policy with its real investment decisions.[1] Later papers, for example Bolton et. al (2011) and DeMarzo et. al. (2012) consider the interaction of multi-period agency, security design, capital structure and real investment policy. Similar issues are considered in Biais et. al. (2011), who pay particular attention to the design of incentives to induce desired investment performance. Cao et. al. (2019) develop a discrete time model of financial frictions and the "q" theory of investment.

The present paper considers a model in which a firm is faced with multi-period projects offering different cash flow profiles. The approach taken follows that of Cao et. al. but integrates some features of Bias et. al. (2011). We investigate how the investment policy is affected by the agency problem arising with external finance. In particular, the firm needs to raise outside finance but cannot commit to honest reporting of income. We note and refer to Biais et. al. (2011) that the mis-reporting model is isomorphic to other moral hazard models with private benefits (Tirole 2006) or indeed costly effort (de Meza Webb (2000) and numerous others). Hence, incentives must be put in place to ensure honest reporting.[2] This agency induced wedge between the cost of internal and external finance impacts investment policy. The main result of the present paper is to show how a broad set of agency problems interact with investment policy. The behaviour of the firm is sensitive to the timing of cash flows, the variance of the company's earnings and the market interest rate. In this framework,

[1] DeMarzo and Sannikov (2006) and Biais et al (2007) provide a rigorous derivation of the link between the derivation of the corporate capital structure and the valuation of corporate liabilities.

[2] Alberquerque and Hopenhayn (2004) study lending and firm investment dynamics with limited contract enforcement in a symmetric information environment. As Hopenhayn and Clementi (2006) note this model has quite different implication for financing and investment than the moral hazard model.

capital structure is chosen to maximize financing capacity. The optimal capital structure gives the agent a sufficient equity stake in the firm and access to credit so as to finance a constrained level of investment, whilst maintaining incentives. Long-term debt, or bonds, is used to ensure that the appropriate incentives are in place, so that the unused financial capacity is used for profitable investments. This policy may involve the firm over-borrowing in the first instance. Only when some threshold value of the entrepreneur's value function is achieved and after currently profitable investments have been made will dividends be paid. It is shown that the firm may trade-off NPV against cash flow. The model presented in the paper yields a simple monotonically decreasing link between investment and the extent of the agency problem. High cash flow reports increase the entrepreneur's equity stake and this tends to relieve the agency problem, thereby leading to more investment and earlier exercise of investment opportunities. Moreover, the paper identifies a virtuous circle in which there is positive serial correlation between cash flow and investment and of investment with investment over time. That is "success breeds success". This naturally means that in this model as in for example the models of Clementi and Hopenhayn (2006) and DeMarzo and Fishman (2007b), the agency problem and the importance of current cash flow is greatest for early stage firms.

Even though firms capital structure is designed to mitigate this problem, leverage in particular has real effects upon investment policy. Faced with multi-period investment projects and sequential investment opportunities, we show how the moral-hazard problem interacts with a Myers (1977) debt-overhang problem and generates an interaction between leverage and the timing of exercise of investment options. We also show that faced with sequential investments, the moral hazard problem in driving a wedge between the cost of internal and external finance can affect the way firms rank projects. In particular, projects that generate net cash flow quickly but are of relatively lower net present value may be prioritised so as to keep leverage and financial servicing costs low before higher net present value projects that deliver net cash flows later are initiated.

The principal empirical observations that this paper and related literature address, concern the link between variations in firm level investment and financial factors. In particular, the observed relationship between investment and current and anticipated agency problems and thus the importance of internal net worth (or equity). Hubbard (1998) provides a survey of the principal findings in the empirical literature relating to the link between investment and measures of internal versus external finance. The link of this investment behaviour to cash flow is found by Devereux and Schianterelli (1990) and Himmelberg and Gilchrist (1998). The latter also find that these effects are less significant for larger, more mature firms. Various authors have observed the apparent smoothing of adjustment costs in investment and that this may be complemented by the adaptation of firms' investment behaviour to agency problems in raising outside finance, see for example Fazzari and Petersen (1993), Almeida, Campello and Weisbach (2004) and Almeida and Campello (2005). Authors, including Whited (1992) have also found that the impact of financial constraints will be less for firms with lower levels of leverage. Finally, Fazzari, Hubbard and Petersen (1998, 2000) find that only firms with low levels of agency cost that place a low premium on cash pay dividends. The principal empirical prediction of the present paper relates the Himmelberg and Gilchrist (1998) finding of a link between financial constraints and thereby how the firm is financed to firm size and growth and cash flow risk. The model in the paper predicts that serial correlation in investment will be highest for growth firms that have high levels of investment options. This effect is more pronounced for this type of firm if the agency problem is significant, as measured by a low level of external equity, a high level of leverage, a significant reliance on internal cash flow to finance growth and low dividend payout.

2 General Problem

We consider a firm whose projects can only be managed by an entrepreneur. The entrepreneur has initial wealth of A_0. If A_0 is less than the initial capital requirement, the entrepreneur must raise finance from a financier. Both the entrepreneur and the financier are risk neutral. The entrepreneur wishes to maximize

$$W_t = E_t \sum_{s=t}^{s=\infty} \beta^s C_s,$$

where $0 < \beta = 1/(1+\rho) < 1$ is a discount factor. The financier wishes to maximize,

$$F_t = E_t \sum_{s=t}^{s=\infty} \widehat{\beta}^s Y_s,$$

$\widehat{\beta} = 1/(1+\widehat{\rho})$. Here C_s is the cash payment from the firm to the entrepreneur and Y_s is the payment from the firm to the financier at date s. The entrepreneur is assumed to be more impatient than the financier, but will still does not want to finance consumption through the firm if agency cost constraints are binding. The firm's technology returns $R(K_t, \omega_t)$ each period in state ω_t, where K_t is capital, with $R_K > 0$ and $R_{KK} < 0$. The state follows a Markov process, with transition probabilities $\pi(\omega_{t+1}|\omega_t)$. Each period the firm's project can be continued or liquidated. In order to continue the project at each date, capital must be retained in the project.

When the entrepreneur raises external finance there is the following agency problem. At each date the entrepreneur-agent observes income directly and can mis-report and retain some fraction θ of the difference between actual and reported income. So if the true state is ω_t, by reporting state $\widehat{\omega}_t < \omega_t$. the agent gains $\theta[R(K_t, \omega_t) - R(K_t, \widehat{\omega}_t)]$. Therefore, to ensure that the entrepreneur behaves honestly he must face the right incentives.

The entrepreneurs dynamic programming problem is

$$W(K_t, F_t, \omega_t) = \max\{C_t + \beta E_t W(K_{t+1}, F_{t+1}, \omega_{t+1})\} \tag{1}$$

subject to the financier's participation constraint:

$$E_t\widehat{\beta}F_{t+1}(\omega_{t+1}) + R(K_t, \omega_t) \geq C_t + Y_t + F_t + I_t + J(I_t) \tag{2}$$

and the incentive compatibility condition

$$\theta[R(K_t, \omega_t) - R(K_t, \widehat{\omega}_t)] \leq \beta[W(K_{t+1}, F_{t+1}, \omega_{t+1}) - W(\widehat{K}_{t+1}, \widehat{F}_{t+1}, \widehat{\omega}_{t+1})] \tag{3}$$

and the definition:

$$K_{t+1} \equiv K_t + I_t \tag{4}$$

Note that $V(K_t, W_t, \omega_t) = F(K_t, W_t, \omega_t) + W_t$ and $V_W = F_W + 1 > 0$ so that $F_W > -1$ and $W_F < -1$. Hence the incentive constraint can be written as

$$F(\omega_{t+1}) \leq \overline{F}(\omega_{t+1}) \tag{5}$$

where $\overline{F}(\omega_{t+1})$ is the maximum amount of external fiance in sate ω_{t+1} consistent with maintaining incentives.

2.1 Full Information Benchmark

Consider the full-information problem in which the incentive constraint is not relevant. Substituting (2) into (1)

$$\begin{aligned} W(K_t, F_t, \omega_t) + F(\omega_t) \;&=\; \max\{R(K_t, \omega_t) - C_t - Y_t - F_t - I_t - J(I_t) \\ &\quad +\beta E_t W(K_{t+1}, F_{t+1}, \omega_{t+1}) + \widehat{\beta} E_t F_{t+1}(\omega_{t+1})\} \end{aligned} \tag{6}$$

Let $\beta = \widehat{\beta}$, using $V_t = F_t + W_t$ so in the absence of an agency problem the value function $V(K_t, \omega_t)$ satisfies

$$V(K_t, \omega_t) = \max\{R(K_t, \omega_t) - C_t - Y_t - F_t - I_t - J(I_t) + \beta E_t V(K_{t+1}, \omega_{t+1})\}$$

so we have the standard investment problem. Choosing I_t:

$$-1 - J'(I_t) + \beta E_t V_K(K_{t+1}, \omega_{t+1}) \tag{7}$$

Defining marginal q, q^m

$$q_t^m = \beta E_t V_K(K_{t+1}, \omega_{t+1}) \tag{8}$$

so we have

$$-1 - J'(I_t) + q_t^m = 0 \tag{9}$$

Then by the envelope condition

$$V_K(K_t, \omega_t) = R_K(K_t, \omega_t) + \beta E_t V_K(K_{t+1}, \omega_{t+1})$$

Moving one period ahead and taking expectations and using (8)

$$q_t^m = \beta E_t R_K(K_{t+1}, \omega_{t+1}) + \beta E_t q_{t+1}^m \tag{10}$$

or

$$E_t R_K(K_{t+1}, \omega_{t+1}) = \frac{1}{\beta} q_t^m - E_t q_{t+1}^m = \rho_t - (E_t q_{t+1}^m - q_t^m) \tag{11}$$

By forward substitution, (10) solves for

$$q_t^m = \beta \sum_{s=t}^{s=\infty} E_t \beta^s R_K(K_t, \omega_t)$$

Finally, note that in equation (11), marginal q_m can be replaced by average q if the technology return and adjustment cost functions are linear homogeneous, so that $V_K(K_t, \omega_t) = V(K_t, \omega_t)/K_t$.

2.2 The Problem with Agency Constraints

We now introduce the incentive constraint as a binding constraint, which defines borrowing capacity. The value function incorporating the constraints is

$$W(K_t, F_t, \omega_t) = \max\{C_t + \beta E_t W(K_{t+1}, F_{t+1}, \omega_{t+1})\} \tag{12}$$

$$+\lambda_t[\widehat{\beta} E_t F(\omega_{t+1}) + R(K_t, \omega_t) - C_t - Y_t - F_t - I_t - J(I_t)] + E_t \mu_{t+1}[\overline{F}(\omega_{t+1}) - F(\omega_{t+1})]$$

Choosing I_t,

$$\beta E_t W_K(K_{t+1}, F_{t+1}, \omega_{t+1}) + \lambda_t \widehat{\beta} E_t \frac{dF(\omega_{t+1})}{dK_{t+1}} - \lambda_t[1 + J'(I_t)] = 0 \tag{13}$$

and choosing $F(\omega_{t+1})$ for each state realisation ω_{t+1},

$$\beta W_F(K_{t+1}, F_{t+1}, \omega_{t+1}) + \lambda(\omega_t)\widehat{\beta} - \mu(\omega_{t+1}) = 0 \tag{14}$$

and using the envelope condition $W_F(K_{t+1}, F_{t+1}, \omega_{t+1}) = -\lambda(\omega_{t+1})$

$$\lambda_t = \frac{\beta}{\widehat{\beta}}\lambda_{t+1} + \frac{1}{\widehat{\beta}}\mu_{t+1} \tag{15}$$

The multiplier λ_{t+1} measures the marginal value of additional funds in terms of the entrepreneur's wealth at date $t + 1$, in a particular state. Hence, λ_{t+1}/λ_t, is the marginal rate of substitution between dates t and $t+1$. The multiplier $\pi(\omega_{t+1}|\omega_t)\mu_{t+1} = dW_t/d\overline{F}(\omega_{t+1})$, measures the marginal cost of a binding borrowing constraint in a particular state. If $\mu_{t+1} = 0$ and $\lambda_t = \lambda_{t+1} = 1$ and $\beta_t = \widehat{\beta}_t$, then the unconstrained solution decribed above obtains.

Moreover, we have the implied pricing condition

$$\lambda_t = E_t(\lambda_{t+1} \frac{R_K(K_{t+1}, \omega_{t+1}) - 1 - J'(I_{t+1}) - Y_{t+1} - F_{t+1}}{1 + J'(I_t) - \widehat{\beta} E_t F(\omega_{t+1})}) \tag{16}$$

Now we can determine average q, denoted by q^a,

$$q^a = \frac{\beta E_t W(K_{t+1}, F_{t+1}, \omega_{t+1}) + E_t \widehat{\beta} F(\omega_{t+1})}{K_{t+1}}$$

From (13)

$$\lambda_t = \frac{\beta E_t W_K(K_{t+1}, F_{t+1}, \omega_{t+1})}{[1 + J'(I_s)] - \widehat{\beta} E_t \frac{dF(\omega_{t+1})}{dK_{t+1}}]}$$

and noting that $q_t^m = 1 + J'(I_t)$, so

$$\lambda_t [q_t^m - \widehat{\beta} E_t \frac{dF(\omega_{t+1})}{dK_{t+1}}] = \beta E_t W_K(K_{t+1}, F_{t+1}, \omega_{t+1})$$

Now if $W/K = W_K$ and $F/K = dF/dK$, we can write

$$\lambda_t [q_t^m - q_t^a + \beta E_t W_K(K_{t+1}, F_{t+1}, \omega_{t+1})] = \beta E_t W_K(K_{t+1}, F_{t+1}, \omega_{t+1})$$

so

$$q_t^m - q_t^a = \frac{\lambda_t - 1}{\lambda_t} \beta E_t W_K(K_{t+1}, F_{t+1}, \omega_{t+1}) \tag{17}$$

which is positive if $\lambda_t > 1$ when $\mu_{t+1} > 0$.[3]

2.3 Incentives and Financing

Let us now explicitly consider incentives. The entrepreneur's continuation value under truth-telling must be greater than the return under lying. That is $W(K_{t+1}, F_{t+1}, \omega_{t+1}) \geq W(\widehat{K}_{t+1}, \widehat{F}_{t+1}, \widehat{\omega}_{t+1})$. This constraint defines the borrowing limit, with the maximum amount

[3] In an elegant continuous time setting Bolton et. al. (2011) show that this wedge is related to the marginal value of internal funds or liquidity.

that can be pledged out of income being $(1-\theta)R(K_t,\omega_t)$. If outside finance takes the form of a sequence of one-period pure discount claims, $Y_t = 0$ and $F_t > 0$, then the value of outside finance per period is limited by this value so that $\overline{F}(\omega_t) = (1-\theta)R(K_t,\omega_t)$. If the financier's claim is longer-term with maturity date t^*, then the value of pledgeable claims is defined recurcisvely to equal $\overline{F}(\omega_t) = F_t = E_t \sum_{s=t}^{s=t^*} \widehat{\beta}^s (1-\theta)R(K_s,\omega_s)$. Given this determination of the value of the long-term financial capacity, the claim is equivalent to an income bond and is therefore equivalent to a financial capacity defined as a sequence of one-period claims, with period by period refinancing.

Note that when W_t is low and if $K_0 - A_0 = F_0$, is relatively high $W_t < W_t^{**} = \theta R(K_t,\omega_t)$, then for $W_t < W_t^{**}$, to maintain incentives there has to be a positive probability of liquidation given by $x(W_t) = (W_t^{**} - W_t)/W_t^{**}$. The value of the firm in this region is given by a linear combination of the liquidation value $L(K_t)$ and $V(W_t^{**})$. If $W_t > W_t^{**}$, then $x(W_t) = 0$ and all incentives are linked to income. So long as the entrepreneur is raising outside finance and the agency problem persists, the firm pays no dividends and under truth-telling, the entrepreneur's position evolves according to $W_{t+1} = (1+\rho)(W_t + \theta R(K_t,\omega_t))$. If at some point the firm achieves an optimum level of the capital stock and the need for external finance $F_t = 0$, then W_t is held at W_t^*, with income paid to the entrepreneur as dividends, so it is only once this threshold is reached that dividends are paid and the firm is self-financed.

2.4 Properties of the Solution to the Investment Problem

Now we consider the impact of the agency problem on the firm's investment policy. This is felt first through the depressing effect that the difference in the entrepreneur's stake under honest reporting over dishonest reporting has on incentives. In the early phase of the firm's life, it is heavily reliant on outside finance but pledgeable income may be limited. Over time, if the firm does well and W_t is built up, the difference $W_t - \widehat{W}_t$ gets bigger allowing more external finance to be supplied because the entrepreneur has more skin in the firm's continuation.

The above yields a simple monotonically decreasing link between investment and the extent of the agency problem. High cash flow reports increase the entrepreneur's equity stake and this tends to relieve the agency problem, thereby leading to more funds being advanced by the financier. Thus there is positive serial correlation between cash flow and investment and of investment with investment over time.

In, for example, the models of Clementi and Hopenhayn (2006) and DeMarzo and Fishman (2007a), the agency problem and the importance of current cash flow is greatest for firms with capital held significantly below the level that would obtain in the absence of the constraint. These firms are small relative to their optimal size. Increased cash-flow risk increases the cost to the financier of providing incentives. In particular, the more variable cash-flow, the more expensive it is to provide incentives. Intuitively, we might expect this to depress the firm's current capital stock and investment rate and create caution in expanding it.

Finally, in this framework, if the entrepreneur has low initial funds and is reliant on external finance he will have a preference for projects that generate more cash quickly. Hence, if faced with a choice of two mutually exclusive investment plans, with one generating cash earlier than the other, even if the latter is intrinsically higher net present value, as we will demonstrate later, the entrepreneur may prefer the former.

3 Implementation by the Optimal Financial Policy.

In the above, we see that the entrepreneur must have an equity stake. In particular we need to make sure that the entrepreneur always has a sufficient stake in the company going forward. The financier allows the entrepreneur access to contingent lump-sum transfers. Given the scale of the firm, the entrepreneur always has a high enough equity stake to prefer efficient continuation, to diverting income. If the incentive constraint is binding, the share in future income matches the gain from lying. To insure that incentives are maintained in the light of shocks, the financier provides a credit facility to the firm, which can be drawn upon as a function of reported income. As noted by Hart and Moore in a number of papers (see for example Hart and Moore (1994)) and as rigorously demonstrated by DeMarzo and Sannikov (2006) and Biais et al (2007), the role of long-term debt is to adjust the profit rate so that the entrepreneur's return is consistent with truthful reporting. This is a feature of the present model. There exists an optimal level of debt, such that if debt is too high the entrepreneur will simply run down the credit balance. On the other hand, if it is too low the entrepreneur will build up cash to reduce risk.

The financier manages his exposure to the project. Starting with an initial advance of $K_0 - A_0$, the financier receives an income of Y_t from which the cost of capital inclusive of agency costs is deducted and a further advance to the entrepreneur of $I_t + J_t$ is made and so on period by period. In general, at each date the financier must have a claim, $F(W_t, \omega_t)$, worth at least as much as the opportunity cost of capital advanced to the project. On the other hand, in the event that no capital is advanced to the project, the project is liquidated for $L(K_t)$ and this is recovered by the financier. In the event of the firm continuing: For $0 \leq W_t < W_t^{**}$, $F(W_t, \omega_t) \geq F(W_t^{**}, \omega_t)$ and in this region after income is realised, the firm is liquidated with probability x_t and all income is paid to the financier. For $W_t^{**} \leq W_t \leq W_t^{*}$, $x_t = 0$ and $F(W_t^{**}, \omega_t) \geq F(W_t, \omega_t) \geq F(W_t^{*}, \omega_t)$, and again all income is paid to the financier. Finally, for $W_t \geq W_t^{*}$, the agency constraint is no longer binding, so that capital can be supplied at the first-best level and the value of the financiers position is held at the reflecting barrier $F(W_t^{*}) = 0$.

There is a maximum level of sustainable outside finance $\overline{F}$, which corresponds to the lowest level of W in the the region where $F'(W) < 0$. There are two possible cases: the corresponding value of equity is either zero or positive. In the first case the contract is renegotiation proof, and $\overline{F} = L(K_t)$, the liquidation value. The entrepreneur's outside finance is fully collateralized. In the second case, $\overline{F} > L(K_t)$ and the claims of financiers can exceed the value of collateral. In this case, the extent of outside funding is governed by the incentive constraint and hence the credible amount of value that can be guaranteed to the financier, which is the present value of income not needed to maintain the entrepreneur's commitment to the firm without cheating.

The above solution to the dynamic investment-financing problem can be implemented in a simple way. The firm is financed with debt, short-period debt B_t that must be repaid each period so that new debt must be issued each period, and equity, S_t. The entrepreneur has to have a sufficient equity stake, θS_t. The financier agrees to supply $F_0 = K_0 - A_0$ and also agrees to fund subsequent investment needs by extending debt finance as a function of reported income. The gross income stream paid to the financier must at least meet repayment of the capital advanced and interest. The financier's claim to cash flows is $F_t = V_t - W_t$. This investment is held as stocks and bonds with value $(1-\theta)S_t + B_t$. Then at date $t = 0$, $F_0 = (1-\theta)S_0 + B_0$ and subsequently $F_t = (1-\theta)S_t + B_t$. The entrepreneur's equity position grows with good income realisations and contracts with poor ones. Only when the entrepreneur's equity stake is high enough and debt has been paid-off can the entrepreneur be trusted not to cheat. The crucial point is that the entrepreneur has to have a sufficiently large equity stake to maintain incentives but at the same time must also have to make contractual payments, debt service payments to the financier, thereby reducing leverage as quickly as possible so as to keep the constraining effect of agency costs on investment to a minimum.

At each date we impose the incentive condition that the entrepreneur prefers, or is indifferent between continuing and taking his share of the capital advanced to the project as a special dividend and then defaulting so long as $W_t \leq W^*$. If $W_t < W^*$, then all income is used to pay bondholders until $B_t = 0$, capital is supplied up to the limit implied by the incentive constraint. If $W_t > W^*$ and $B_t = 0$, then the first-best level of investment is incentive compatible and dividends can be paid. This feature of the optimal contract is a feature of bilateral financial arrangements that only trigger dividends when certain performance targets have been reached, see Biais et al (2007), DeAngelo, De Angelo and Stultz

(2006) and Kaplan and Stromberg (2003 and 2004).

Because the value function W_t is concave in F_t; at low levels of W_t, the debt-equity ratio is high and the cost of providing incentives $dW_t/d\overline{F}(\omega_{t+1})$ is high. Hence, the incentive to reduce debt and build up capital is high. In this region, investment is heavily cash-flow constrained and a premium is placed on building up the equity value of the firm.

4 Growth Options

In the problem we have examined, maximising the entrepreneur's wealth is consistent with maximising the value of the firm as the objective function. A concern emerges if the outside financier holds risky debt, which can be motivated by reference to the classic Myers (1977) problem. Myers starts from the perspective of a firm that has an existing set of operations financed with equity, S_t^o, and risky debt, B_t^o so $V_t^o = S_t^o + B_t^o$. Suppose that at any date, there is a probability δ that the project fails. In this event, the cash return from the project is zero and the liquidation value of the project, $L(K_t)$ is realised. In the event of default, the financier receives the residual value of the project and the entrepreneur nothing. The firm is then faced with an initially unforeseen growth opportunity, not priced into initial security returns with stand alone value V_t^g that costs I_t. Suppose that this option was to be financed with new debt, B_t^g, that is not a project specific claim and that the original debt, B_t^o, has a senior claim on all income including that from the option. The the new debt must be fairly priced but the exercise of the option reduces the default risk of the original debt, so that $\Delta B_t^o > 0$, and even though $V_t^g - I_t > 0$, it is possible that $S_t^o < 0$, so that it is not in initial shareholders' interests to exercise the option. Myers calls this a "debt overhang problem". Of course, this solution is not renegotiation proof. If the option is not exercised the initial debt holders will be worse-off and will be willing to cut the face value of their claim to ensure that $S_t^o \geq 0$ and the growth option is exercised.

We now wish to understand the implications of the above in the context of the contracting model we have developed. Of course, it will matter whether the growth opportunity is anticipated or unanticipated. Denote the capital invested in the initial project as K_t^o. First, introduce the possibility that at some date τ, after the initial project has been commenced but before the optimal value, K^{o*}, is reached, the firm has an initially unanticipated growth option. This has a return function $R^g(K_s^g, \omega_t)$, with $R_K^g > 0$ and $R_{KK}^g < 0$. It requires an initial capital outlay of K_τ^g, where τ is the exercise date of the growth option, and subsequent capital investments of I_t^g. The investment cannot be implemented with K_τ^g from internal funds, to ensure that the project is undertaken, the entrepreneur needs financier participation, so in order to gain the commitment of the financier, the financier must ensure that the entrepreneur has the incentive to make agreed payments. If the project is positive net present value, it can generate additional equity value. However, the ability of the entrepreneur to implement the project will be affected by the spill-over effect of the growth option on the value of the initial financial contract and thereby reducing the value of the growth option to the entrepreneur.

We first consider the problem of investing in the growth option when there are no spillover effects. This applies when there are no agency problems, so there is no problem of limiting the entrepreneur's access to outside funds, so that outside and internal finance are perfect substitutes. In this case, the growth option will be undertaken on an unconstrained basis and undertaken when it adds to the entrepreneur's equity, which will be consistent with maximising the unconstrained value of the firm. In the context of the problem we have examined above with the basic moral hazard problem, the additivity principle is still maintained if there are no spillover effects, in the sense that the initial project does not create costs for the new project, in the form of an agency cost of debt. However, if the initial project is financed with risky debt, this may not be the case. We approach this problem by way of an example and then extend our formal model to address it in more detail.

Let W_t^o be the value of the entrepreneur's position from the initial project with the financier's position been given by F_t^o. The growth option yields a gross value to the entrepreneur, inclusive of the growth option of W_t^A with the financier's position given by $F_t^A = F_t^o + F_t^g$. The growth option will be exercised if it can be financed, so that $F_t^g \geq 0$ and the equity value of $W_t^A \geq W_t^o$, is increased. If the growth option is entirely separable from the initial investment, the growth option is analogous to the initial investment and fully additive. In this case, the option should be exercised, if it is positive net present value, as son as it nmaterialises. This is not the case if the initial project is financed at least in part with risky debt and this debt is a senior claim on the firm's total income stream and assets, so that in an insolvency event it has first claim. Suppose, therefore, that the initial investment involved the issue of an initial amount of outside finance, $F_0^o = K_0 - A_0$ and subsequent finance from the financier until self-sufficiency is obtained. Moreover, suppose that $W_t^o < W_t^{o*}$ so there is a positive probability of default. Then, if the investment option is undertaken, some of the value generated will increase the value of the initial financial claim F_t^o, by ΔF_t^o. In raising the value of this claim, the agency costs constraining the initial investment are reduced, by allowing the entrepreneur to achieve self-finance of this project earlier, thereby raising W_t^o by ΔW_t^o. The entrepreneur will sanction the growth option investment with initial funding of F_0^g if $W_t^A \geq W_t^o$ and $W_t^A > \widehat{W_t^A}$, but the increase in W_t^A is constrained by the agency cost of debt overhang ΔF_i^o. The optimal exercise of the investment option will trade-off the agency cost of debt overhang against the benefits of alleviating the agency costs of moral hazard constraining the initial project.[4]

We begin with a firm that has initiated an investment programme with outside finance. The initial project is positive net present value on a stand-alone basis and is implemented as adding to share-holder equity when financed with outside funds through a dynamic incentive compatible financial arrangement. This project forms the basis for a sustainable financial plan, consisting of income and expenditure projections and hence EBITDA projections. The

[4] Chen and Manso (2017) examine how debt overhang, macroeconomic risks and agency problems interact. Distortions caused by agency problems will affect investors more in recessions than in booms. As the size of agency conflict due to debt overhang (as measured by the potential transfer from equity holders to debt holders) depends on the riskiness of debt, for a given investment opportunity, the transfers from equity holders to debt holders in a typical procyclical firm will tend to concentrate in bad times, when entrepreneur net worth is lower and debt is riskier.

firm's EBITDA is used to pay financiers and pay down outside liabilities. The firm is then faced with a growth opportunity, which in order to be exercised, has to add to the value of the entrepreneur's equity stake and be part of a sustainable financial plan comprising of the initial investment's and the growth opportunity's income and expenditure streams.

As soon as the growth opportunity materialises, the problem can be written recursively starting with the growth opportunity and working back to the initial investment. In the absence of any agency problems, the outcome of the investment problem will be the unconstrained first best. If the growth option is known and is of positive net present value it will be implemented as the solution to the full-information benchmark case examined above, which applies under self-finance or with external finance but no agency problem, so that $\mu_{t+1} = 0$. Execution of this project involves the outlay of K^g_τ, followed by subsequent investments of I^g_t. Given this solution, we step backwards to the initial decision in which the investment outlay of K^o_0 is made, followed by K^o_t and K^g_0 in turn followed by I^o_t, which must satisfy an additive problem as outlined below.

In the presence of agency problems, matters are more complex. The first point to address is the impact of any initial discrete start up investment cost for the growth option, K^g_0. To undertake the growth option, the entrepreneur must secure funding. If this was unanticipated when the initial investment was initiated and the cash flows are not separate, then the value of the initial financial claim will be impacted, $\Delta F^o_t > 0$, and hence a reduction in the constraint on the funding of the initial investment so that $\Delta W^o_t > 0$. This only applies to the extent that F^o_t is risky and hence W^o_t is exposed to this risk. This is greatest when W^o_t is low relative to W^{o*}_t, so that the leverage of the initial project remains high. However, when the project is initiated, the jump increase in W^A_t net of ΔF^o_t and inclusive of any increase in W^o_t, must be positive. In other words, the debt-overhang effect is a transfer from the entrepreneur to the initial-financier. This transfer cannot be too large, so that the entrepreneur participates, $W^A_t \geq W^o_t$ and incentives are maintained, $W^A(K^o_{t+1}+K^g_{t+1}, F^o_{t+1}+F^g_{t+1}, \omega_{t+1}) \geq \widehat{W^A_t}(\omega_{t+1})$. Thus, even though the growth opportunity is positive net present value, its exercise will depend upon the debt overhang effect, which is lower at higher values of W_t^o, so the entrepreneur waits for this value to be high enough relative to W_t^{o*} before exercising the growth opportunity.

The entrepreneur's value function is additive in the initial investment and the growth opportunity. In this specification, the two investments are treated as having separate adjustment cost functions. The problem has to be specified before and after a point of discontinuity when the growth option is exercised. After the discontinuity the problem is written as:

$$W^A(K_t^o + K_t^g, F_t^o + F_t^g, \omega_t) = \max\{C_t^g + C_t^o + \beta E_t W^A(K_{t+1}^o + K_{t+1}^g, F_{t+1}^o + F_{t+1}^g, \omega_{t+1}\} \quad (18)$$

This maximum is achieved subject to the financier's participation condition:

$$E_t\widehat{\beta}F_{t+1}^g(\omega_{t+1}) + E_t\widehat{\beta}F_{t+1}^o(\omega_{t+1}) + \widehat{m}(K_t^g, K_t^o, \omega_t) \geq C_t^g + Y_t^g + F_t^g + I_t^g + J(I_t^g) \qquad (19)$$

The term $m(K_t^g, K_t^o, \omega_t) = R(K_t^o, \omega_t) + R(K_t^g, \omega_t)$ is total EBITDA from both investments. EBITDA net of the payment to the initial project, is given by $\widehat{m}(K_t^g, K_t^o, \omega_t) = \max\{m(K_t^g, K_t^o, \omega_t) - (C_t^o + Y_t^o + F_t^o + I_t^o + J(I_t^o)), 0\}$, reflecting the notion that the initial project has first claim on EBITDA.

The incentive compatibility condition is

$$W^A(K_{t+1}^o + K_{t+1}^g, F_{t+1}^o + F_{t+1}^g, \omega_{t+1}) \geq \widehat{W_t^A}(\omega_{t+1}) \qquad (20)$$

which, following the earlier argument, can be written as

$$F^g(\omega_{t+1}) + F^o(\omega_{t+1}) \leq \overline{F}(\omega_{t+1}) \qquad (21)$$

Before the point of discontinuity, the growth option terms are absent. This can be accommodated in the above specification through an a zero-one indicator function multiplying all of the terms relating to the growth option, g, which is zero before the discontinuity. However, to initiate the growth option a discrete outlay of K^g_τ is required. Moreover, even though the growth option has become known at some date after the initial investment is initiated, the firm is still faced with a choice of when to exercise it.

At the point of discontinuity, K^g_τ is invested in the growth opportunity, so at this point total investment is $K^o_\tau + K^g_\tau$, and total external finance is $F^o_\tau + F^g_\tau$, with $F^g_\tau = K^g_\tau$. The optimisation programme needs to determine the conditions that must hold at the time of dicontinuity, τ. To the right of this point the value function is $W(K^o_\tau + K^g_\tau, F^o_\tau + F^g_\tau, \omega_\tau) \geq W(K^o_\tau, F^o_\tau, \omega_\tau)$ and to the left, the reverse. In other words, the point of discontinuity is a point of overtaking. In essence the agency costs of external finance decline as F^o_t as is reduced and are overtaken by the forgone value in not exercising the growth option. Given the point of discontinuity, we now have two problems, one before this point and one afterwards.

The Lagrangian after the growth option is exercised is

$$
\begin{aligned}
W(K^o_t + K^g_t, F^o_t + F^g_t, \omega_t) &= \max\{C^g_t + C^o_t + \beta E_t[W(K^o_{t+1} + K^g_{t+1}, F^o_{t+1} + F^g_{t+1}, \omega_{t+1})]\} \\
&+\lambda_t[\widehat{\beta} E_t(F^g_{t+1}(\omega_{t+1}) + F^o_{t+1}(\omega_{t+1})) + \widehat{m}(K^g_t, K^o_t, \omega_t) - C^g_t - Y^g_t - F^g_t - I^g_t - J(I^g_t)] \\
&+\mu_{t+1} E_t[\overline{F}(\omega_{t+1}) - F^g(\omega_{t+1}) - F^o(\omega_{t+1})] \qquad (22)
\end{aligned}
$$

Choosing I^g_t

$$
\beta E_t W_K(K^o_t + K^g_t, F^o_t + F^g_t, \omega_t) + \lambda_t \widehat{\beta} E_t \frac{dF^g(\omega_{t+1})}{dK^g_{t+1}} + \lambda_t \widehat{\beta} E_t \frac{dF^o(\omega_{t+1})}{dK^g_{t+1}} - \lambda_t[1 + J'(I^g_s)] = 0 \quad (23)
$$

The spill-over term $\widehat{\beta} E_t \frac{dF^o(\omega_{t+1})}{dK_{t+1}}$, will be zero if the effect is already anticipated in the pricing of the bonds. Choosing F^g_{t+1} for each state realisation ω_{t+1},

$$
\beta W_F(K^o_t + K^g_t, F^o_t + F^g_t, \omega_t) + \lambda_t \widehat{\beta} - \mu_{t+1} = 0
$$

and using the envelope condition $W_F(K_t^o + K_t^g, F_t^o + F_t^g, \omega_t) = -\lambda(\omega_{t+1})$

$$\lambda_t = \frac{\beta}{\widehat{\beta}}\lambda_{t+1} + \frac{1}{\widehat{\beta}}\mu_{t+1} \tag{24}$$

Note again that if the incentive constraint is binding, $\mu_{t+1} > 0$ and $\lambda_t > 1$.

The above problem is for a single entrepreneur or firm financed by a single financier. The growth option involves an initial investment followed by a sequence of further investments, implemented subject to adjustment costs and agency costs. The option may or may not be in the firm's plans at the date the original investment is initiated. Once the option is initiated, the optimisation problem is a consolidated problem, with a single financier participation condition and a single incentive constraint for each state. This means that if the value function for the entrepreneur is given by $W_t^A = W(K_t^o + K_t^g, F_t^o + F_t^g, \omega_t)$, then the two thresholds for the value function (eqivalent to W_t^{**} and W_t^*) are given by W_t^{A**} and W_t^{A*}, with random liquidation up to the first value and internal finance above the second. The second phase involves the balanced management of optimal investment in both the growth option and the initial investment, inclusive of adjustment and agency costs.

The outlined solution assumes that the entrepreneur contracts with a single financier for both projects and does not have the option to contract with another financier for the second project. If the financial claims issued were all one-period claims, then at each date the contract would be renegotiation proof and indeed there would be no over-hang problem. However, with longer-dated claims this is not the case. Then to ensure that new projects are not funded through dilution a la Hart and Moore (1995), the initial financier will require that his claim has priority, through a protective covenant.

5 Timing of Investments

In the previous section we have considered how the agency or moral hazard problem impacts the timing of exercise of growth options that have to be realised after the initial investment.

Here leverage and cash flow coverage (EBITDA) of debt service payments plays an important role in the development of the firm's growth policy. However, we have not considered the impact that the agency problem will have on the timing of investments. Let us therefore suppose that the entrepreneur has access to two investment projects denoted by 1 and 2, which in the above replace o and g. The question we focus on is the timing of projects when projects have different net-present values but also differ in cash-flow profiles (timing). In the absence of agency problems, the self-finance solution obtains and both projects should be initiated immediately if they are positive net present value. In the presence of the basic moral hazard problem, we have seen that the entrepreneur's initial level of wealth A_0 and subsequent net-worth W_t determines the extent of external financial resources he can command, F_t. This means that the timing of investments becomes important. Suppose that on a stand-alone basis $V_0^1 > V_0^2$ but project 2 generates net-cash more quickly, earlier, than project 1. The wedge between the cost of external and internal funds for this project when the incentive constraint binds, $\mu_{t+1} > 0$, and reliance on expensive external funds (and so $q_t^m - q_t^a > 0$) will, in expectation, be of shorter duration for project 2 than for project 1. As ths project gnerates cash more quickly, the time taken to pay of external finance will be expected to be less. Unless the initial capital outlay K_0^2 is significantly greater than K_0^1, project 2 allows the entrepreneur to generate cash internally and build up net worth, through W_t, relatively quickly, by repaying the financier more quickly and hence keeping leverage lower before project 1 is initiated.

Following the same reasoning as in the last section, we break the optimisation problem into two parts. The discontinuity at the at the exercise point for the second investment initiated, manifests itself with the left and right derivatives of the value function W_t again being unequal at the point of discontinuity. At the point of discontinuity at time τ, the given value of K_τ^1 is invested in the slower project, so at this point total investment is $K_\tau^2 + K_\tau^1$. In turn, total, external finance is $F_\tau^o + F_\tau^g$, with $F_\tau^g = K_\tau^g$. The optimisation programme needs to determine the value of K_τ^2 at the discontinuity and the endogenous date of the discontinuity,

τ. To the right of this point, the value function is $W(K_\tau^2+K_\tau^1, F_\tau^2+F_\tau^1, \omega_\tau) \geq W(K_\tau^2, F_\tau^2, \omega_\tau)$, this is reversed to the left of the discontinuity.

Given the point of discontinuity, we have two problems, one before this point and one afterwards. In the first phase, the problem involves only the quick project 2, that generates cash and pays off external finance quickly, thereby relieving the impact of agency costs, with the problem being as in the first section of the paper. The second project is exercised when at the margin the saving in agency cost is off-set by the uplift in net-present value. The second phase again involves the balanced, optimal investment in both project 2 and 1, inclusive of adjustment and agency costs. The analytics of the problem are as before.

6 Other Incentive Problems

It has been recognised for a long time that serious agency problems arise when there is a separation of ownership and control. Firms are run by mangers who may have divergent interests from investors (financiers). Jensen (1986) advanced a free-cash-flow theory. This theory is based on the notion that when positive net present value investments have been exhausted, mangers will spend cash generated on privately beneficial projects. Jensen argued that to mitigate this problem, a firm should issue long-term debt that forces or commits it to pay cash out, thereby preventing "misuse" of cash.[5] In a similar vein, Hart and Moore (1990) argued that managers may attempt to raise more finance to fund negative net present value activities. They argue that firms may be able to issue claims that dilute existing claims, in particular outside-equity, to finance such actions. They argue that long-term senior debt limits the scope for this dilution by forcing firms to pay cash out and therefore necessitating raising new finance to fund new investments on fair terms. Thus, in both cases, long-term debt is used to constrain managers incentive to waste resources. In this sense, a Myers' debt overhang is created to constrain the waste of either free-cash-flow or undiluted equity. There is of course the problem that the debt can constrain both good (positive NPV) and bad

[5] This theory is developed in Stulz (1990).

(negative NPV) investments. In the former case there will be an incentive to renegotiate debt, to reduce the debt-overhang and allow the investment to be undertaken.

The key point of the above discussion is that the agency problems of free-cash-flow and equity dilution are in theory most acute for firms that have exhausted positive net-present-value investments. The agency model we have examined above in its basic form examines the evolution of the firm's investment and financing problem, with the agency problem declining if the firm has a series of positive cash flow outcomes that enables the entrepreneur to achieve self-finance and no longer be constrained from obtaining first-best investment because of agency problems. The crucial point here is that the financier limits the entrepreneur's access to funds but incentivises him to pay down the financiers position, whilst being committed to truthful reporting. Here the agency problem is at its greatest when reliance on external finance, leverage, is high. Moreover, we have argued that investment in growth options may be delayed until cumulative firm performance brings overall leverage down to a point that incentives on the combined projects can be maintained and any debt over-hang problem mitigated.

7 Risk Shifting

In the above, higher cash-flow risk increases the cost of maintaining incentives and so makes it more expensive for the firm to finance its investments. This can be seen in condition (3), where the magnitude of the term $[W(K_{t+1}, F_{t+1}, \omega_{t+1}) - W(\widehat{K}_{t+1}, \widehat{F}_{t+1}, \widehat{\omega}_{t+1})]$ reflects the variance of cash flows. When debt is risky, increases in cash-flow risk increase the variability of cash flow in the region below W_t^{***}, which increases the probability of termination and also reduces the value of equity. DeMarzo and Sannikov(2006) relate this to the asset substitution problem in corporate finance (see Jensen and Meckling (1976)) and argue that in this type of contracting environment the above mechanism precludes the problem. The asset substitution problem is an incentive problem that is eliminated if both the entrepreneur and financier hold only equity stakes in the company but in this model, the entrepreneur must have a big enough claim to ensure no cheating. However, we have also seen that the optimal financial policy must ensure that the financier is paid of through a series of contractual payments, namely debt service payments. But it is precisely this type of capital structure, in which the entrepreneur holds a leveraged convex claim that the asset substitution problem exists. That is, after debt is issued, there is an incentive to switch to higher risk investments but the entrepreneur would like to commit to a low risk strategy.

With debt financing, if the firm has increased cash-flow risk, then agency costs are incurred and this will, as we have seen, lead to a lower level of capital accumulation so long as the now more severe incentive constraint binds. But consider the firm at the early stage of its development, when after it obtains initial finance the debt-equity ratio is high and the agency problem is significant. At this stage, the entrepreneur-equity holder, who has a deeply out of the money convex claim, may be tempted to incur the burden of increased agency costs for a gain at the expense of the outside financier, who holds a lot of debt. Of course, given sequential rationality, in the sub-game perfect equilibrium of the financing game, the financier would anticipate any shift in the cash-flow risk, and price the debt accordingly. To mitigate this problem, the financier needs a contingent claim that in the event of an increase in risk allows him to increase his equity stake. The terms of this contract would have to be modified to satisfy a risk shifting incentive constraint along the equilibrium path. This convertible contract, in this case a convertible bond, was proposed as the incentive-compatible contract in the original Jensen-Meckling framework by Green (1984). A complex variant of this contract could play a role at some stage in the early history of the financing of the firm in our model and indeed this is often a feature of venture capital contracts (see for example Schmidt (2003)).

8 Conclusion

This paper has studied investment under uncertainty when there are adjustment costs in changing the capital stock and agency problems in financing investment. The agency problem arises from only the entrepreneur observing returns and needing to be incentivised by financiers to act truthfully. The paper demonstrates the interaction of the adjustment costs of changing durable investment and the agency problem arising from external financing. The former means that the timing of investment depends upon these costs. The latter means that the entrepreneur's net worth and the firm's cash flows are important. Low current income realisations reduce borrower net worth and add to agency problems that constrain investment plans. Thus the firm will be biased in its investment choices towards projects that generate cash rather than long-run value. Moreover, cash flow risk will increase the value of investment options but will also increase agency costs. The paper demonstrated the nature of these interactions. In addition it characterised the financial policy of the firm and how this interacts with its investment policy, linking this to the severity of the agency problem and the maturity of the firm, more mature firms have exhausted their investment options.

The model presented in the paper yields a number of predictions. Firms' investments, particularly growth firms will be constrained by agency problems arising from low entrepreneur net worth (equity), relative to the optimal firm size. The agency problem will have a greater effect when the constrained level of the capital stock is significantly below the first-best. Growth firms with significant agency problems, arising when investment is significantly constrained, will tend to have relatively high ratios of entrepreneur to financier equity and high levels of debt. Moreover, only mature firms, which have low agency problems will pay dividends and then only after current profitable investments have been made. Moreover, if the agency problem is severe, it can seriously impact the exercise of growth options. Also, if the firm has to use finacial capacity to invest in projects requiring lumpy initial investments, there will not only be an impact upon timing but also on the prioritisation of projects, with an initial preference for cah flow over net-present -value.

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