Kalman Filter Learning Versus Bounded Rationality in a Heterogeneous Agent NK Model

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KF Learning versus Bounded Rationality

Motivation:

- FIRE models often struggle to reproduce the persistence observed in actual macroeconomic data
- Agents in FIRE models are assumed to know the *model* (RE) and *state variables* (FI)
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This Paper: Propose a NK model with RE and bounded rationality agents
$\Rightarrow$ Comparison among different models
$\Rightarrow$ Information assumption is important

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- BR agents: AU learning + adaptive expectations about *external* variables
  - Observe local variables and nominal interest rate with no lag, aggregate with a lag

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Bayesian Estimation: estimate the models with macroeconomic data
$\Rightarrow$ BR and RE with II improve the model fit and the persistence of the model
$\Rightarrow$ RE(II)-BR > Pure RE(II) > RE(PI)-BR > Pure BR > Pure RE

Comment # 1: Alternative misspecified forecasting rules?

AU learning: agents only have knowledge of their own objectives and constraints, do not have economic model of determination of aggregate variables

A simple example:

$$c_t = \sum_{s=t}^{\infty} \left[(1-\beta)\, E_t^* y_s - \sigma \beta E_t^* r_s \right]$$

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– Convenient approach. Are the findings dependent on this particular forecasting rules?

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- Estimated shock persistence are quite different across models
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  - But..., price mark-up ρ_{MS} is estimated to be 0.39 in Pure RE (PI), 0.97 in all other models

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- Survey information serves as an *additional restriction* and helps differentiate between models
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*– Estimate the models with macroeconomic and **survey** data simultaneously, if substantial differences in the likelihood (e.g. Hommes et al. (2023))*

Final remarks

- Growing literature on deviations from FIRE
- This paper: **information assumption** is important in the empirical comparison of RE and BR NK models
- Potential for policy implications