

Comment on Marcet and Singleton's "Learning about Bond Prices"  
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**Question:** This paper studies how learning about bond prices may play an important role in understanding the empirical behavior of bond yields.

**Approach:** The paper employs a model of "**Internal Rationality (IR)**".

- Agents are fully rational but their understanding of how markets work is not perfect, i.e., they don't know the pricing function well and learn about unobserved underlying shocks to prices,
- In other literature, agents know the pricing function and learn about fundamentals.

## Results:

- Such a simple model of IR generates relevant equations with variances and correlations in closed form, making comparative statics clear.
- Interesting mechanism: There is a self-referential transmission of slope movements. Just because the slope is high in some period, this causes the perceived underlying slope to go up, driving future slope up, driving future perceived slope up, and so on.
- This model of IR could improve the match of several important empirical observations, including high volatility puzzles.

## Comment 1

A key assumption is investors' belief on bond yield.

$$(1) \quad -\log(Q_t^n)/n = \mathbf{a}_t + \mathbf{s}_t n + \phi^n \pi_t^e + u_t^n,$$

where  $u^n$  is a time-maturity specific error. The level and slope of  $\mathbf{a}_t$ ,  $\mathbf{s}_t$  are unobserved. Agents will learn about their underlying values by filtering the observed yield.

With simplified assumptions,

- investors know the true intercept:  $\mathbf{a}_t = -\log(\delta) + \gamma g^c$
- but they are uncertain about  $\mathbf{s}_t$  and believe  $\mathbf{s}_t = \mathbf{s}_{t-1} + \omega_t$
- under RE,  $\mathbf{s}_t = 0$

This is a fine specification but what is a microfoundation for this specification, i.e., based on some decision theory?

## Comment 2

The mechanism of self-referential transmission appears to be related to self-fulfilling expectations (RE) in macro theory in which multiplicity of momentary equilibria plays a key role (see Hirano and Stiglitz' paper).

In this model, the equilibrium dynamics appear to be uniquely determined. What is it so, or is it always the case?

## Reference

Tomohiro Hirano, and Joseph E. Stiglitz 2022. "Land Speculation, Booms, and Busts with Endogenous Phase Transitions: A Model of Economic Fluctuations with Rational Exuberance" NBER Working Paper No. 29745

### Comment 3

It is assumed that all bonds with  $n > 1$  are resold one period after being purchased.

Is this assumption realistic?

Consider another extreme case. Suppose investors hold bonds until maturity. Then, would the result still hold?

## Comment 4

The government issues bonds with different maturity,  $n = 1, \dots, N$ .

This implies that in general equilibrium, the government receives some revenues. At the same time, the government uses lump sum tax.

But I don't see it in the budget constraint of investors.

$$(2) \quad P_t c_t + \sum_{n=1}^N Q_t^n B_t^n = w_t P_t + \sum_{n=1}^N Q_t^{n-1} B_{t-1}^n$$

Does this mean the model is a partial equilibrium model? It appears to me that we need to think about the government budget constraint in a consistent manner with investors' budget constraint.

## Comment 5

This model of Internal Rationality could explain several important empirical observations that are difficult to be justified with models of RE. This is obviously a great advantage.

What are the advantages of this model over other models with learning pioneered by Evans and Honkapohja?

## Comment 6 (minor)

As deviations from RE, I've heard of the following literature.

- Near-Rational model by Akerlof and Yellen in 80s.
- Heterogeneous information (Angeletos and his collaborators etc.)
- Heterogenous belief (Scheinkman, Xiong, Hong etc.)

Conceptually, what are differences from these literatures?

Is there any survey paper in behavioral finance so that non-experts like me can study comprehensively?

## Comment 7 (minor) on rational expectations

Obviously, rational expectations may be extreme. Nonetheless, rational expectations can be considered as the benchmark case.

Economists have been wedded to models of rational expectations, not so much because they necessarily believed that individuals were so rational and foresighted, but because without that assumption, economics seemed untethered.

An interesting direction? may be that even a small deviation from RE can have a markedly different result and implication, including some discontinuity.

By the way, in the paper, there are following statements.

- The profession often takes the view that if investors are rational, asset prices should equal the discounted sum of future flows.
- Much of the modern finance theory is predicted on the premise that asset prices are determined by the true discounted value of future asset payments, and that this follows whenever investors are rational"

In contrast, Hirano and Toda (2023) provide Bubble Necessity Theorem in workhorse macroeconomic models (with rational agents and rational expectations).

- Asset prices cannot equal fundamental values. In other words, there are benchmark cases in which the notion that asset prices should basically reflect fundamentals is wrong.

Reference: Tomohiro Hirano, and Alexis Akira Toda 2023. "Bubble Necessity Theorem," arXiv:2305.08268