Competition and Incentives with Motivated Agents*

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Abstract

A unifying theme in the literature on organizations such as public bureaucracies and private non-profits is the importance of missions, as opposed to profit, as an organizational goal. Such mission-oriented organizations are frequently staffed by motivated agents who subscribe to the mission. This paper studies incentives in such contexts and emphasizes the role of matching principals' and agents' mission preferences in increasing organizational efficiency and reducing the need for high-powered incentives. The framework developed in this paper is applied to non-profits, school competition, and incentives in the public sector.

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1 Introduction

The late twentieth century witnessed an historic high in the march of market capitalism with unbridled optimism in the role of the profit motive in promoting welfare in the production of private goods. Moreover, this generated a broad consensus on the optimal organization of private good production through privately-owned competitive firms. When it comes to the provision of collective goods, no such consensus has emerged.[1] Debates about the relative merits of public and private provision still dominate.

This paper suggests a contracting approach to the provision of collective goods which cuts across the traditional public-private divide. It focuses on two key issues: (i) how to structure incentives and (ii) the role of competition between providers. At its heart is the idea that organizations for the provision of collective goods cohere around a *mission.*[2] Thus production of collective goods can be viewed as *mission-oriented*.

Not all activities within the public-sector are mission-oriented. For example, in some countries, governments own car plants. While this is part of the public-sector, the optimal organization design issues here are no different than those faced by GM or Ford. Not all private sector activity is profit-oriented. Universities, whether public or private, have many goals at variance with profit maximization.

The missions pursued in the provision of collective goods come from the underlying motivations of the individuals (principals and agents) who work in the mission-oriented sector. Workers are typically *motivated agents,* i.e. agents who pursue goals because they perceive intrinsic benefits from doing so. There are many examples – doctors who are committed to saving lives, researchers to advancing knowledge, judges to promoting justice and soldiers to defending their country in battle. Viewing workers as mission-oriented makes sense when the output of the mission-oriented sector is thought of as producing collective goods. The benefits and costs generated by mission-oriented production organizations are typically not priced. In addition, donating one's income earned in the market is likely to be an imperfect substitute to joining and working in such an organization in the presence of agency costs or because individuals care not just about the levels of these collective goods, but their personal involvement in their production (i.e., a "warm glow").

It is well known from the labor literature on compensating differentials that employment choices and wages depend on taste-differences (Rosen, 1986). This paper explores how this economizes on the need for explicit monetary incentives while accentuating the importance of non-pecuniary aspects of organization design in increasing effort. Thus mission choice can affect the productivity of the organization. For example, a school

[1]We use the term collective good as opposed to the stricter notion of a public good. Collective goods in this sense also include merit goods. This label also includes a good like education to which there is a commitment to collective provision even though the returns are mainly private.

[2]See, for example, Wilson (1989) on public bureaucracies and Sheehan (1998) on non-profits. Tirole (1994) is the first paper to explore the implications of these ideas for incentive theory.

curriculum or method of discipline that is agreed to by the whole teaching faculty can raise school productivity.

However, mission preferences typically differ between motivated agents. Doctors may have different views about the right way to treat ill patients and teachers may prefer to teach to different curriculums. This suggests a role for organizational diversity in promoting alternative missions and competition between organizations in attracting those whose motivational preferences best fit with one another. We show that there is direct link between such sorting and an organization's productivity.

The insights from the approach have applications to a wide variety of organizations including schools, hospitals, universities and armies. The primitives are not whether the organization is publicly or privately owned but the production technology, the motivations of the actors and the competitive environment. We also abstract (for the most part) from issues of financing.

We benchmark the behavior of the mission-oriented part of the economy against a profit-oriented sector where standard economic assumptions are made – profit seeking and no non-pecuniary agent motivation. This is important for two reasons. First, we get a precise contrast between the incentive structures of profit-oriented and mission-oriented production. Second, the analysis casts light on how changes in private sector productivity affect optimal incentive schemes operating in the mission-oriented sector. This has implications for debates about how pay-setting in public sector bureaucracies responds to the private sector.

Our approach yields useful insights into on-going debates about the organization of the mission-oriented sector of the economy. For example, it offers new insights into the role of competition in enhancing productivity in schools. More generally, it suggests that one of the potential virtues of private non-profit activity is that it can generate a variety of different missions which improve productivity by matching managers and workers who have similar mission preferences. An analogous argument can be made in support of decentralization of public services. However, on the flip side, public bureaucracies, whose policies can be imposed by politicians, may easily become de-motivated. While matching on mission preferences is potentially productivity enhancing, it also leads to conservatism and can raise the cost of organizational change.

This paper contributes to an emerging literature which studies incentive issues outside of the standard private goods model.[3] One strand of this puts weight on the multitasking aspects of non-profit and government production along the lines of Holmstrom and Milgrom (1991). Another emphasizes the career concerns aspects of bureaucracies (Dewatripont, Jewitt and Tirole (1999), Alesina and Tabellini (2003)). These two are brought together in Acemoglu, Kremer and Mian (2003). However, these all work with standard motivational assumptions. This paper shares in common with Akerlof and Kranton (2003), Benabou and Tirole (2003), Dixit (2001), Francois (2000), Murdock

[3] See Dixit (2002) for a survey of this literature.

(2002), and Seabright (2003) the notion that non-pecuniary aspects of motivation matter.[4] In common with Crawford and Sobel (1982) and Aghion and Tirole (1997) our approach places emphasizes how non-congruence in organizational objectives can play a role in incentive design. However, we explore the role of matching principals and agents – selection rather than incentives – as a way to overcome this.[5]

The remainder of the paper is organized as follows. In the next section, we lay out the basic model, study optimal contracts and matching of principals and agents. Section three explores applications of the model and section four concludes.

2 The Model

2.1 The Environment

A "firm" consists of a risk neutral principal and an agent who is needed to carry out a project. The project's outcome (which can be interpreted as quality) can be high or low: $Y_H = 1$ ('high' or 'success') and $Y_L = 0$ ('low' or 'failure'). The probability of the high outcome is the effort supplied by the agent, e, at a cost $c(e) = e^2/2$. Effort is unobservable and hence non-contractible. We assume that the agent has no wealth and so cannot put in a performance bond. Thus, a limited liability constraint operates which implies that the agent has to be given a minimum consumption level of $\underline{w} \geq 0$ every period. Because of the limited-liability constraint, the moral hazard problem has bite. This is the only departure from the first-best in our model.

We assume that each principal has sufficient wealth so as not to face any binding wealth constraints, and that the principal and agent each can obtain an autarchy payoff of zero.

The mapping from effort to the outcome is the same for all projects. We also assume that agents are identical in their ability to work on any type of project. Projects differ exclusively in terms of their missions. A "mission" consists of attributes of a project that make some principals and agents value its success over and above any monetary income they receive in the process. This could be based on what the organization does (charitable versus commercial), how they do it (environment friendly or not), who is the principal (kind and caring versus strict profit-maximizer) and so on. Allowing agents to have preferences over their work environment follows a long tradition in labor economics (see, for example, Rosen, 1986).

In our basic model missions are exogenously given attributes of a project associated

[4]Some of these ideas consider the possibility that intrinsic motivation can be affected by the use of explicit incentives (see also Titmuss (1970), Frey (1997)). We treat the level of intrinsic motivation as given.

[5]See Ackerberg and Botticini (2002), Dam and Perez-Castrillo (2001), and Lazear (2000) for approaches to principal agent problems where sorting is important.

with a particular principal. In section 2.3, we examine consequences of endogenizing mission choice.

There are three types of principals and agents labelled $i \in \{0, 1, 2\}$. The types of the principals and the agents are perfectly observable. If the project is successful, a principal of type i receives a payoff of $\pi_i > 0$. All principals receive 0 if a project is unsuccessful. Principals of type 0 have the same preferences as in the standard principal-agent model, i.e., π_0 is entirely monetary. However π_1 and π_2 may have a non-pecuniary component. To focus exclusively on horizontal aspects of matching between principals and agents we assume that $\pi_1 = \pi_2 \equiv \hat{\pi}$.[6]

Some agents care about the mission of the organization for which they work. Formally this implies that the payoff of such agents depends on their type, and the type of the principal for whom they work. Like principals, all agents are assumed to receive 0 if the project fails irrespective of who they are matched with. Agents of type 0 have standard pecuniary incentives – their utility depends positively on money and negatively on effort. Since they are motivated solely by money, they do not care intrinsically about which organization they work for. In contrast an agent of type1 (type 2) receives a non-pecuniary benefit of $\overline{\theta}$ from project success if he works for a principal of type 1 (type 2) and $\underline{\theta}$ if matched with a principal of type 2 (type 1).where $\overline{\theta} > \underline{\theta} \geq 0$.[7]

The payoff of an agent of type j who is matched with a principal of type i when the project succeeds can therefore be summarized as:

$$\theta_{ij} = \begin{cases} 0 & i = 0 \text{ and/or } j = 0 \\ \underline{\theta} & i \in \{1, 2\}, j \in \{1, 2\}, i \neq j \\ \overline{\theta} & i \in \{1, 2\}, j \in \{1, 2\}, i = j. \end{cases}$$

We will refer to the parameter θ_{ij} as *agent motivation* and agents of type 1 and 2 as *motivated* agents. We will refer to the economy as being divided into a *mission-oriented sector* (i.e., $j = 1, 2$) and a *profit-oriented sector* (i.e., $j = 0$).

We make

Assumption 1:

$$\max\{\pi_0, \hat{\pi} + \overline{\theta}\} < 1.$$

This ensures that there is an interior solution for effort in all possible principal agent matches.

The analysis of the model is in three steps. We first solve for optimal contracts for an exogenously given match of a principal of type i and an agent of type j. Contracts

[6] We use the term *horizontal* matching to describe a situation where there is no difference in the quality of organizations when principals and agents are efficiently matched. The standard *vertical* matching model looks at situations where some principals and agents are more productive regardless of who they match with. We will briefly return to the implications of vertical sorting in section 3.2.

[7] These payoffs are contractible, unlike in Hart and Holmstrom (2002) where non-contractibility of private benefits plays an important role. Also, these are independent of monetary incentives, which is contrary to the assumption in the behavioral economics literature (see Frey, 1997).

between principals and agents have two components: a *fixed* wage w_{ij} which is paid regardless of the project outcome and a bonus b_{ij} which the agent receives if the outcome is Y_H. We initially take the agent's reservation payoff $\overline{u}_j \geq 0$ to be exogenously given. Second, we consider the extension to endogenous missions which makes θ_{ij} endogenous. Third, we study matching of principals and agents where the reservation payoffs are endogenously determined.

2.2 Optimal Contracts

As a benchmark, consider the first best case where effort is contractible. This will result in effort being chosen to maximize the joint payoff of the principal and the agent. This effort level will depend on agent motivation and hence the principal-agent match. However, the contract offered to the agent plays no allocative role in this case.[8] Thus, while matching may raise efficiency, it has no implications for incentives in the first best. It is straightforward to calculate that the joint surplus in any principal agent pair is equal to $\frac{1}{2}(\pi_i + \theta_{ij})^2$.[9]

In the second best, effort is not contractible. The principal's optimal contracting problem under moral hazard solves:

$$\max_{\{b_{ij}, w_{ij}\}} u_{ij}^p = (\pi_i - b_{ij})\, e_{ij} - w_{ij} \tag{1}$$

subject to:

(i) the *limited liability constraint* requiring that the agent be left with at least $\underline{w}$:

$$b_{ij} + w_{ij} \geq \underline{w}, w_{ij} \geq \underline{w}. \tag{2}$$

(ii) the *participation constraint* of the agent that:

$$u_{ij}^a = e_{ij}\,(b_{ij} + \theta_{ij}) + w_{ij} - \frac{1}{2}e_{ij}^2 \geq \overline{u}_j. \tag{3}$$

(iii) the *incentive-compatibility constraint* which stipulates that the effort level maximizes the agent's *private* payoff given (b_{ij}, w_{ij}):

$$e_{ij} = \arg\max_{e_{ij}\in[0,1]} \left(e_{ij}\,(b_{ij} + \theta_{ij}) + w_{ij} - \frac{1}{2}e_{ij}^2 \right).$$

This constraint can be simplified to:

$$e_{ij} = b_{ij} + \theta_{ij} \tag{4}$$

[8] Any values of b_{ij} and w_{ij} such that the agent gets at least $\underline{w}$ in all states of the world, and his expected payoff is $\overline{u}_j$ will work.

[9] The Pareto-frontier is a straight-line with slope equal to minus one and intercepts on both axes equal to the joint surplus.

so long as $e_{ij} \in [0,1]$.[10]

We will restrict attention to the range of reservation payoffs for the agent in which the principal earns a non-negative payoff. Let $\overline{v}_{ij}$ be the value of the reservation payoff of an agent of type j such that a principal of type i makes zero expected profits under an optimal contract. Also, the participation constraint of the agent may not bind if the reservation payoff is very low due to the presence of limited liability. Let $\underline{v}_{ij}$ denote the value of the reservation payoff such that for $\overline{u}_j \geq \underline{v}_{ij}$ the agent's participation constraint binds. In the appendix we show that $\overline{v}_{ij}$ and $\underline{v}_{ij}$ are positive real numbers under our assumptions, and $\underline{v}_{ij} < \overline{v}_{ij}$.[11]

A further assumption is needed to guarantee the existence of optimal contracts under moral hazard. In particular, the payoff from project success to the principal and/or the agent must be high enough to offset the agency costs due to moral hazard, and ensure both parties get non-negative payoffs. The following assumption provides a sufficient condition for this to be true for any principal-agent match:

Assumption 2:

$$\frac{1}{4}\left[\min\{\pi_0, \hat{\pi}\}\right]^2 - \underline{w} > 0.$$

The following Proposition characterizes the optimal contract. All proofs are presented in the Appendix.

Proposition 1: *Suppose Assumptions 1-2 hold. An optimal contract* $(b^*_{ij},\, w^*_{ij})$ *between a principal of type* i and an *agent of type* j *given a reservation payoff* $\overline{u}_j \in [0, \overline{v}_{ij}]$ *exists, and has the following features:*

(i) The fixed payment is set at the subsistence level, i.e., $w^*_{ij} = \underline{w}$

(ii) The bonus payment is characterized by

$$b^*_{ij} = \begin{cases} \max\{0, \frac{\pi_i - \theta_{ij}}{2}\} & \text{if } \overline{u}_j \in [0, \underline{v}_{ij}] \\ \sqrt{2\left(\overline{u}_j - \underline{w}\right)} - \theta_{ij} & \text{if } \overline{u}_j \in [\underline{v}_{ij}, \overline{v}_{ij}]. \end{cases}$$

(iii) The optimal effort level solves: $e^*_{ij} = b^*_{ij} + \theta_{ij}$.

The first part of the proposition shows that the fixed wage payment is set as low as possible. Other than the agent's minimum consumption constraint, the agent is risk-

[10] This will be the case under the optimal contract. The bonus payment b_{ij} will never optimally set to be greater than or equal to the principal's payoff from success π_i because then the principal will be receiving a negative expected payoff. Therefore by Assumption 1, $e_{ij} < 1$. Also, it is never optimal to offer a negative bonus to the agent. Because the limited liability constraint requires that $b_{ij} + w_{ij} \geq \underline{w}$ and so this is feasible only if $w_{ij} > \underline{w}$. But by increasing b_{ij} and decreasing w_{ij} while keeping the agent's utility constant, effort would go up and the principal would be better off. Therefore, $e_{ij} > 0$.

[11] We also show that $\overline{v}_{ij}$ is less than the joint surplus under the first-best, which indicates the presence of agency costs.

neutral and does not care about the spread between his income in the two states. Hence, the principal will want to make this as small as possible.

The second part characterizes optimal bonus payments and the third part characterizes optimal effort, which follows directly from the incentive-compatibility constraint. Limited liability implies that the principal cannot induce the first-best level of effort in the presence of moral hazard.[12] In choosing b the principal faces a trade-off between providing incentives to the agent (setting b higher) and transferring surplus from the agent to himself (setting b lower). Accordingly, the reservation payoff of the agent plays an important role in determining b, and the higher is the reservation payoff, the higher is b.

Another important parameter is the motivation of the agent. For the same level of b, an agent with greater motivation will supply higher effort. From the principal's point of view, b is a costly instrument of eliciting effort. Since agent motivation is a perfect substitute for b, motivated agents receive lower incentive pay in the optimum. The various possibilities can be classified in three cases which depend on the value of the reservation payoff, and whether the agent values project success more than the principal:

Case 1: If the agent is more motivated than the principal and the outside option is low, then $b^*_{ij} = 0$, i.e., there should optimally be no incentive pay.

Case 2: If the principal is more motivated than the agent, then

$$b^*_{ij} = \frac{1}{2}\left(\pi_i - \theta_{ij}\right).$$

In this case, the principal sets incentive pay equal to half the difference in the principal and agent's valuation of the project.

Case 3: If the outside option is high then

$$b^*_{ij} = \sqrt{2\left(\overline{u}_j - \underline{w}\right)} - \theta_{ij}.$$

The optimal incentive pay, in this case, is set by the outside market with a discount depending in size on the agent's motivation.

The third part of the Proposition characterizes optimal effort which depends on the sum of the agent's motivation and bonus payment. In the first case, the principal relies solely on agent motivation while in the second and third cases, an additional incentive is provided. In case 3, the effort level is entirely determined by the outside option.

We now offer three corollaries of this proposition which are useful to understanding its implications for incentive design. The first describes what happens in the profit-oriented sector

[12] Making the agent a full residual claimant (i.e., $b_{ij} = \pi_i$) will result in first best effort, but expected profits will be negative making this unattractive to the principal.

Corollary 1: *In the profit-oriented sector* $(i = 0)$ *the optimal contract is characterized by:*

(i) *The fixed payment is set at the subsistence level, i.e.,* $w_{0j}^* = \underline{w}$ $(j = 0, 1, 2)$.

(ii)The bonus payment is characterized by

$$b_{0j}^* = \begin{cases} \frac{\pi_0}{2} & \text{if } \overline{u}_j \in [0, \underline{v}_{0j}] \\ \sqrt{2(\overline{u}_j - \underline{w})} & \text{if } \overline{u}_j \in [\underline{v}_{0j}, \bar{v}_{0j}] \end{cases}$$

for $j = 0, 1, 2.$

(iii) The optimal effort level solves: $e_{0j}^* = b_{0j}^*$ $(j = 0, 1, 2.)$

This follows directly from the fact that $\theta_{0j} = 0$ *for* $j = 0, 1, 2$. Notice that case 1 above is no longer a possibility – the agent in the profit oriented sector must always be offered incentive pay to put in effort.

The next two corollaries are regarding the mission-oriented sector and illustrate the importance of matching principals and agents.

Corollary 2: *Suppose that* $\bar{u}_0 = \bar{u}_1 = \bar{u}_2$. *Then, in the mission-oriented sector* $(i = 1, 2)$, *effort is higher and the bonus payment lower if the agent's type is the same as that of the principal.*

To see this, observe from part (ii) of Proposition 1, that the bonus paid to the agent is decreasing in his motivation and is zero if the agent is more motivated than the principal. Moreover, the bonus is higher if i differs from j. The observation that effort is lower combines parts (ii) and (iii) of Proposition 1. Hence, organizations with "well-matched" principals and agents will have higher levels of productivity if the reservation payoffs of agents $(\bar{u}_j)$ to the mission-oriented sector is the same for all types.

Corollary 3: *Suppose that* $\bar{u}_0 = \bar{u}_1 = \bar{u}_2$. *Then, in the mission-oriented sector* $(i = 1, 2)$ *bonus payments and effort are negatively correlated in a cross-section of organizations.*

Thus, holding constant the reservation payoffs of agents $(\bar{u}_j)$, productivity (i.e., optimal effort) and incentive pay will be (weakly) negatively correlated across organizations. The heterogeneity among such organizations is driven only by preferences of the agent which affects both effort and incentive payments. This is a pure *selection* effect. Holding the characteristics of the principals and the agent constant, greater incentive pay would lead to higher effort and higher productivity, as in the standard principal-agent model.

These last results are useful in demonstrating the costs to having poor matching of principals and agents in a world where there are motivated agents. In section 2.4, we show how endogenous matching of principal-agent pairs and endogenous determination of the agents' reservation payoffs can increase efficiency.

2.3 Endogenous Motivation

In this section, we discuss how our framework can be extended to make the motivation of agents endogenous by allowing the principal to pick the mission of the organization.[13] Suppose that both the principal and the agent care about the mission. Let missions, x, be real numbers over the unit interval $X = [0, 1]$. For concreteness sake, x could be a school curriculum with 0 denoting secular education and 1 denoting a high degree of religious orientation. Let the non-pecuniary benefits of the principal and the agent conditional on project success be affected by the mission choice. Formally, $\pi_{ij} = g^i(x_{ij})$ and $\theta_{ij} = h^j(x_{ij})$ where $x_{ij} \in X$ ($i = 1, 2$ and $j = 1, 2$).[14] The basic model can be thought as a case in which the mission is not contractible and is picked by the principal after he hires an agent. In this case

$$x^*_{ij} = \arg\max_{x_{ij} \in X} \left\{ g^i(x) \right\}$$

which is independent of the agent's type.[15]

However, if the mission choice is contractible, then it might be optimal for the principal to use the mission choice to incentivize the agent – either by picking a "compromise" mission somewhere between the principal and agent's preferred outcome or even picking the agent's preferred mission. A full treatment of this is beyond the scope of this paper. However, to illustrate the issues involved, consider case 2 of Proposition 1. Suppose that $g^i(x) = P - \frac{1}{2}(x - \alpha_i)^2$ and $h^j(x) = A - \frac{1}{2}(x - \alpha_j)^2$ where $\alpha_i \in X$ and $\alpha_j \in X$ are the "ideal" missions of principals of type i and agents of type j, and $P > A$. In this case, the agent is given a bonus payment of $\frac{1}{2}(\pi_i - \theta_{ij})$ and the optimal effort level is $e^*_{ij} = \frac{1}{2}(\pi_i + \theta_{ij})$. The principal's expected payoff in this case is $e^*_{ij}(\pi_i - b^*_{ij}) - \underline{w} = \frac{1}{4}(\theta_{ij} + \pi_i)^2 - \underline{w}$. The optimal mission if contractible will, therefore solve:

$$x^*_{ij} = \arg\max_{x_{ij} \in X} \frac{1}{4} \left\{ g^i(x) + h^j(x) \right\}^2.$$

It is straightforward to show that the optimal mission choice is given by $x^*_{ij} = \frac{\alpha_j + \alpha_i}{2}$.[16]

This compromise mission increases θ_{ij} relative to the case where the principal picks his ideal mission of α_i. Thus compromising on the mission will reduce the need for

[13] Endogenous motivation or mission preference could be the result of "socialization" of agents by principals (see Akerlof and Kranton, 2003).

[14] Note that in the case of endogenous missions, we use π_{ij} to denote the payoff of the principal since her mission choice may depend on the type of agent with whom she is matched.

[15] Thus, $\bar{\theta} = h^j(x^*_{jj})$, $\underline{\theta} = h^j(x^*_{ij})$ for $i = 1, 2$, $j = 1, 2$ and $i \neq j$.

[16] Without loss of generality, suppose $\alpha_i > \alpha_j$. Observe that a value of x that exceeds α_i or is less than α_j will never be chosen since it is dominated by choosing $x = \alpha_i$ or $x = \alpha_j$. The problem in this case is to choose x to maximize $\frac{1}{4}\left(g^i(x) + h^j(x)\right)^2$ subject to the constraint $g^i(x) \geq h^j(x)$ (which is one of the conditions that characterizes case 2 of Proposition 1). Notice that $g^i(x) + h^j(x)$ is a concave function which attains its global maximum at $\frac{\alpha_i + \alpha_j}{2}$. The first derivative of $\left(g^i(x) + h^j(x)\right)^2$ is

incentive pay, i.e. b^*_{ij} will be lower. However, overall effort (and hence the productivity of the organization) will be greater. This illustrates how, absent perfect matching, mission choice can be manipulated to raise agent motivation and is a substitute for financial motivation.

Clearly, the assumption that mission choice is either non-contractible or perfectly contractible is an extreme one. In reality, mission choice is likely to be subject to incentive problems and one of the key elements of organization design would have the aim of influencing mission choice. For example, choosing non-profit status or giving agents discretion in mission-setting could be viewed as mechanisms through which a principal pre-commits not to choose missions that may be viewed negatively by agents. In Besley and Ghatak (2004) we explore this issue in greater detail.

Although the mission can "bridge the gap" between the principal and agent, it is no substitute for having them agree on the mission in the first place. In the next section, we explore how this comes about.

2.4 Competition

A key feature of our model is that the types of the principals and agents affect organizational efficiency. In this section, we consider what happens when the different sectors compete for agents reverting to the case where the mission is exogenous.[17] We study this without modeling the competitive process explicitly, focusing instead on the implications of stable matching. We look for allocations of principals and agents which are immune to a deviation in which *any* principal and agent can negotiate a contract which makes both of them strictly better off. Were this not the case then we would expect rematching to occur.

First, we need to introduce some additional notation. Let $\mathcal{A}_p = \{p_0, p_1, p_2\}$ denote the set of types of principals and let $\mathcal{A}_a = \{a_0, a_1, a_2\}$ denote the set of types of agents. Following Roth and Sotomayor (1989), the matching process can summarized by a one-to-one matching function $\mu : \mathcal{A}_p \cup \mathcal{A}_a \to \mathcal{A}_p \cup \mathcal{A}_a$ such that (i) $\mu(p_i) \in \mathcal{A}_a \cup \{p_i\}$ for all $p_i \in \mathcal{A}_p$ (ii) $\mu(a_j) \in \mathcal{A}_p \cup \{a_j\}$ for all $a_j \in \mathcal{A}_a$ and (iii) $\mu(p_i) = a_j$ if and only if $\mu(a_j) = p_i$ for all $(p_i, a_j) \in \mathcal{A}_p \times \mathcal{A}_a$. A principal (agent) is unmatched if $\mu(p_i) = p_i$ ($\mu(a_j) = a_j$). What this function does is to assign each principal (agent) to at most one agent (principal) and allows for the possibility that a principal (agent) remains

$2\left(g^i(x) + h^j(x)\right)\left(\frac{dg^i(x)}{dx} + \frac{dh^j(x)}{dx}\right)$. The unique critical point of $\frac{1}{4}\left(\theta_a(x) + \theta_p(x)\right)^2$ is therefore $\frac{\alpha_i+\alpha_j}{2}$. Notice that the derivative is strictly positive for all $x \in [\alpha_j, \frac{\alpha_i+\alpha_j}{2})$ and strictly negative for all $x \in (\frac{\alpha_i+\alpha_j}{2}, \alpha_i]$. Therefore, the function $\left(g^i(x) + h^j(x)\right)^2$ and affine transformations of it are pseudo-concave, and so the function attains a global maximum at $x = \frac{\alpha_i+\alpha_j}{2}$ (see Simon and Blume, 1994, p. 527-28). As $P > A$, the constraint $g^i(x) \geq h^j(x)$ is satisfied at $x = \frac{\alpha_i+\alpha_j}{2}$.

[17] It would be straightforward to extend the model to incorporate matching with endogenous missions. An earlier version of the paper studied this case.

unmatched, in which case he is described as "matched to himself".

Let n_i^p and n_j^a denote the number of principals of type i and the number of agents of type j in the population. We assume that $n_1^a = n_1^p$ and $n_2^a = n_2^p$ to simplify the analysis. However, the population of principals and agents of type 0 need not be balanced – we consider both *unemployment,* i.e., $n_0^a > n_0^p$, and *full employment*, i.e., $n_0^a < n_0^p$. We assume that a person on the "long-side" of the market gets none of the surplus which pins down the equilibrium reservation payoff of all types of agents.[18]

From the analysis in the previous section, for a given value of $\overline{u}_j$, we can uniquely characterize the optimal contract between a principal of type i and an agent of type j. We begin by showing that any stable matching must have agents matched with principals of the same type. This is stated as:

Proposition 2: *Consider a matching μ and associated optimal contracts (w_{ij}^*, b_{ij}^*) for $i = 0, 1, 2$ and $j = 0, 1, 2$. Then this matching is stable only if $\mu(p_i) = a_i$ for $i = 0, 1, 2$.*

This result says that all stable matches must have principals and agents matched assortatively. This argument is a consequence of the fact that, for any fixed set of reservation payoffs, an assortatively matched principal agent pair can always generate more surplus than one where the principal and agent are of different types.[19]

This result allows us to focus on assortative matching. The next two results characterize the contracts and the optimal effort levels in two cases – full-employment and unemployment in the profit-oriented sector.

In the full employment case, principals compete for scarce agents with latter capturing all of the surplus. This sets a floor on the payoff that a motivated agent can be paid. Whether the participation constraint is binding now depends on how π_0 compares with θ_{ij} and $\hat{\pi}$. Let

$$\xi \equiv \max\{\overline{\theta}, \hat{\pi}\} + \overline{\theta}.$$

We assume that when the mission-oriented and profit-oriented sectors compete for agents, then mission-oriented production is viable:

Assumption 3:

$$\overline{\theta} + \hat{\pi} \geq \pi_0.$$

The following proposition characterizes the optimal contracts and optimal effort levels under the stable matching in the full-employment case:

Proposition 3: *Suppose that $n_0^a < n_0^p$ (full employment in the profit-oriented sector). Then the following matching μ is stable: $\mu(a_j) = p_j$ for $j = 0, 1, 2$ and the associated optimal contracts have the following features:*

[18] For the case $n_0^a = n_0^p$ there is a range of possible values of the reservation payoff.

[19] This requires a non-standard matching argument because of our focus on horizontal sorting. Recent results on assortative matching in non-transferable utility environments by Legros and Newman (2003) cannot be applied in our setting.

(i) The fixed payment is set at the subsistence level, i.e., $w_{jj}^* = \underline{w}$ *for* $j = 0, 1, 2$.
(iii) The bonus payment in the mission-oriented sector is:

$$b_{11}^* = b_{22}^* = \frac{1}{2} \max\left\{\xi, \pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right\} - \overline{\theta}$$

and the bonus payment in the profit-oriented sector is:

$$b_{00}^* = \frac{\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}}{2}.$$

(iii) The optimal effort level solves: $e_{jj}^* = b_{jj}^* + \overline{\theta}$ *for* $j = 1, 2$ *and* $e_{00}^* = b_{00}^*$.

The proposition illustrates how competition and incentives interact. There are two effects.

First, there is a *matching effect*. This restricts the range of contracts observed in the mission-oriented sector. This also raises organizational productivity – which follows using the logic of Corollary 2 given assortative matching. If the outside option is not binding, then this is achieved with concomitant *reductions* in incentive pay.[20] In our set up is all agents in the mission-oriented sector receive the same incentive payment in equilibrium, and are equally productive.

Second, there is an *outside option effect.* Competition among principals pins down the equilibrium value of the outside option. With full employment in the profit-oriented sector, the expected payoff of profit-oriented principals is driven to zero with agents capturing all the surplus from profit-oriented production. The reservation utility of a motivated agent is set by what he could obtain by switching to the profit-oriented sector. A sufficiently productive profit-oriented sector ($\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}} > \xi + \overline{\theta}$) leads to a binding outside option and a mission oriented sector that uses more incentive pay. Thus the outside option effect can also raise productivity, but by increasing incentive pay for agents.

Proposition 3 also gives a sense of when incentives will be less high-powered in mission-oriented production with motivated agents. Even if the participation constraint binds, the mission-oriented sector incentive pay is at the private sector level less $\overline{\theta}$. Without the participation constraint binding, incentive pay in the mission-oriented sector is zero if $\overline{\theta} > \hat{\pi}$ which also implies that incentives are more high powered in the profit-oriented sector.[21]

We now consider what happens if there is unemployment in the profit-oriented sector and profit-oriented principals are able to extract all the surplus from this agent (at least

[20] Matching can improve productivity even under the first-best. The analysis of the second-best offers insights on the effect of matching on the pattern of incentive pay.

[21] It is possible to have more high-powered incentives in the mission-oriented sector, but only if the participation constraint is not binding, and $\hat{\pi}$ is high relative to π_0 and $\overline{\theta}$.

in so far as the limited liability constraint permits). The supply price of motivated agents is now determined by their unemployment payoff. The following proposition characterizes this case:

Proposition 4: *Suppose that $n_0^a > n_0^p$ (unemployment in the profit-oriented sector). Then the following matching μ is stable: $\mu(a_j) = p_j$ for $j = 0, 1, 2$ and the associated optimal contracts have the following features:*

(i) The fixed payment is set at the subsistence level, i.e., $w_{jj}^* = \underline{w}$ for $j = 0, 1, 2$.

(iii) The bonus payment in the mission-oriented sector is:

$$b_{11}^* = b_{22}^* = \frac{1}{2}\xi - \overline{\theta}$$

and the bonus payment in the profit-oriented sector is:

$$b_{00}^* = \frac{\pi_0}{2}.$$

(iii) The optimal effort level solves: $e_{jj}^* = b_{jj}^* + \overline{\theta}$ for $j = 1, 2$ and $e_{00}^* = b_{00}^*$.

The effect of competition on incentives now acts purely through the matching effect. The presence of unemployment unhinges incentives in the mission-oriented and profit-oriented sectors of the economy since the only outside option is being unemployed. Principals earn positive profits and employed agents in both sectors earn a rent relative to the unemployed in this case.

Contrasting the results in Proposition 1 with those in Propositions 2, 3 and 4 yields some insight into the role of competition in the mission-oriented sector and its role in changing the pattern of incentive pay and improving productivity. The results in Propositions 2, 3 and 4 correspond to an idealized situation of frictionless matching. They provide a benchmark for what can be achieved in a decentralized economy and how matching can raise productivity and affect the structure of incentive pay.

3 Applications

The benchmark for our analysis is the case where principals and agents are matched and allocated by endogenously determined reservation payoffs as illustrated in Propositions 3 and 4. However, even in a world of motivated agents, there may be frictions which prevent this idealized outcome from being attained. These comprise natural frictions such as search costs and asymmetric information. However, there may also be "artificial" frictions due to government policies. A number of government policies in recent years have been in the direction of reducing these artificial barriers to a competitive, decentralized system of collective service provision.[22] This may involve reforms within the public sector or else initiatives to foster greater involvement of the non-profit sector in service provision. The model developed here is well-placed to think through the implications of such developments.

In this section, we discuss three main contexts in which the ideas apply. We begin with a discussion of non-profit organizations. We then discuss how school production might fit with the model. Finally, we discuss public bureaucracies.

3.1 Non-profit Organizations

The notion of a mission-oriented organization staffed by motivated agents corresponds well to many accounts of non-profit organizations. The model emphasizes why those who care about a particular cause are likely to end up as employees in mission-oriented non-profits. This finds support in Weisbrod (1988), who observes that "Non-profit organizations may act differently from private firms not only because of the constraint on distributing profit but also, perhaps, because the motivations and goals of managers and directors ... differ. If some non-profits attract managers whose goals are different from those managers in the proprietary sector, the two types of organizations will behave differently." (page 31). He also observes that "Managers will ... sort themselves, each gravitating to the types of organizations that he or she finds least restrictive – most compatible with his or her personal preferences" (page 32).[23]

Weisbrod also cites persuasive evidence to support the idea that such sorting is important in practice in the non-profit sector. The notion of a mission-oriented organization is however somewhat more far-reaching than that of a non-profit. For example, such sorting can be very important in "socially responsible" for-profit firms such as the Body Shop.[24]

How exactly non-profit status facilitates greater sorting on missions raises interesting issues. If the organization can contract over the mission up front as in section 2.3, then it should make no difference whether there is a formal non-profit constitution. Thus, as argued by Glaeser and Shleifer (2001), adopting non-profit status must have its roots in contracting imperfections. This would be relevant if the principal has some incentive to act opportunistically ex post in a way that diverts the mission from what the agent would like. This would make it difficult to recruit motivated agents or "demotivate" those already in the organization to the extent that opportunism was not anticipated. The

[22] These are sometimes known as *quasi-market* reforms (see, for example, Legrand and Bartlett (1993))

[23] See Glaeser (2002) for a model of non-profits where workers and managers of non-profits have something like our mission-preferences, i.e., caring directly about the output of the firm.

[24] On the website of the Body Shop, their "values" are described as follows: "We consider testing products or ingredients on animals to be morally and scientifically indefensible" and "We believe that a business has the responsibility to protect the environment in which it operates, locally and globally" (see http://www.thebodyshop.com/).

possibility of such "mission drift" would also speak in favor of having a board of trustees that will safe-guard the mission. It also shows the importance of having a motivated principal, i.e. someone who is dedicated to the mission, running an organization.[25]

Empirical studies suggest that in industries where both for-profits and non-profits are in operation, such as hospitals, the former sector make significantly higher use of performance-based bonus compensation relative to base salary for managers (Ballou and Weisbrod, 2002 and Bertrand, Hallock, and Arnould 2003). It is recognized in the literature that managers may care about the outputs produced by the hospital or the patient. However, researchers are unable to explain this empirical finding. In the words of Ballou and Weisbrod (2002): "While the compensating differentials may explain why levels of compensation differ across organizational forms, it does not explain the differentials in the use of strong relative to weak incentives." Our framework provides a simple explanation for this finding. In addition, Bertrand, Hallock, and Arnould (2003) find that the spread of managed care in the US, which increases market competition, induced significant changes in the behavior of non-profit hospitals. In particular, although the relationship between economic performance and top managerial pay in nonprofit hospitals is on average weak, they found that it strengthens with increases in HMO penetration. In terms of our model, this can be explained as the effect of an increase in the profitability of the for-profit sector (π_0) which tightens the participation constraints of the managers.

Our framework also underlines the value of diversity in the non-profit sector provided that there are variety of views in the way in which collective goods should be produced (as represented by the mission preferences). Weisbrod (1988) emphasizes just this role of non-profit organizations in achieving diversity. For example, he observes that non-profits will likely play a more important role in situations where there is greater underlying diversity in preferences for collective goods. For example, he contrasts the U.S. and Japan suggesting that its greater cultural heterogeneity is partly responsible for the greater importance of non-profit activity in the U.S.. Our analysis of the role of competition in sorting principals and agents on mission preferences underpins the role of diversity in achieving efficiency. Better matched organizations can result in higher effort and output. Hence, diversity is not only good for the standard reason, namely, consumers get more choice, but also in enhancing productive efficiency.

Non-profit organizations rely on heterogeneous sources of finance – a mixture of private donors, government grants and endowments. The analysis so far as abstracted from such issues by supposing that the principal has a source of wealth. Hence, the analysis best fits the case of a well-endowed organization. But given the importance of external finance in practice, it is interesting to think through the implications of introducing a third group of actors – donors – who contribute towards the organization.[26]

[25] This suggests that promotion of insiders may be important in such organizations as a way of preserving the mission.

[26] See Glaeser (2002) for a related attempt to consider the role of donors in the governance of non-

Like agents, we would expect donors to pick organizations on the basis of the missions that they pursue. When such matching is perfect, the existence of outside financiers raises no new issues.

The more interesting case arises when donors have mission preferences that differ from those of any matched principal-agent pair in the economy. They can then seek to influence organizations by offering a donation that is conditional on changing the mission of the organization. But our analysis suggests that externally enforced mission changes come at a cost since the agent (and possibly the principal) will become demotivated and the organization will become less productive.[27] This leads us to conjecture that endowment finance will generally be associated with higher levels of productivity in the non-profit sector.

The role of the donor can also give some insight into the difference between public and private finance. In publicly funded organizations, the government plays the role of donor. It can use this role to influence mission choice. We would expect its mission preferences to be determined either by electoral concerns or constitutional restrictions (e.g., maintaining a neutral stance with respect to religious issues). The government may be able to provide financial support to some private organizations but if it does so, it might tend to distort their missions towards its preferred style of provision. But in doing so, it can reduce productivity since agents will be less motivated as a consequence. Indeed, when the US President George W. Bush announced the policy of federal support for faith-based programs in 2001, some conservatives expressed concerns that involvement with the government will cost churches intensity and integrity.[28] Thus, we would expect government funded organizations on average to be less efficient than those privately financed through *endowments*. However, whether they are more or less productive than those funded by private *donations* is less clear given the earlier discussion.

3.2 Educational Providers

Educational providers are a key example of motivated agents regardless of whether they are publicly or privately owned. The approach developed here provides some useful insights into the role of competition and incentives in improving school performance.[29] Moreover, the model works equally well for thinking about publicly and

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profits.

[27]Formally, both π_i and θ_{ii} will be lower.

[28]See “Leap of Faith” by Jacob Weisberg, February 1, 2001, Slate (http://www.slate.msn.com).

[29]As Hoxby (2003) points out while the empirical literature suggests that there are productivity differences across schools and that competition may affect these, there has been relatively little theoretical work on determinants of school productivity. Hoxby (1999) is a key exception. She models the impact of competition in a model where there are rents in the market for schools, and argues that a Tiebout like mechanism may increase school productivity. Other approaches to the issue, such as Epple and Romano (2002), have emphasized peer-group effects (i.e., school quality depends on the quality of the mean student) but as far as “supply side” factors are concerned they assume that some schools are

privately owned schools without invoking the implausible assumption that the latter are profit-maximizing.

Some schooling policies, specifically those restricting diversity in mission choice, have served to prevent the kind of decentralized outcome studied in Propositions 3 and 4. However, recent policies to encourage entry and competition between schools may allow schools to emerge with more distinctive missions. For example, in the U.K., prime minister Tony Blair has been emphasizing the importance of diversity in his education policy. In the U.S., initiatives to encourage charter schools is based on the idea of creating schools that cater to community needs. The competitive outcome that we characterize can be thought of as the outcome from an idealized system of decentralized schooling in which schools compete by picking different missions and attracting teachers who are most motivated to teach according to them.

To think through these issues formally, consider the model of mission choice introduced in section 2.3. For simplicity, we will focus on the allocation of a balanced population of teachers (agents) to schools taking the outside option in the profit-oriented sector as given. In this context, a mission could be a curriculum or a method of teaching.

At one extreme is a centralized world where schools are forced to adopt homogeneous missions as a matter of government policy. Suppose that this mission is $x = \frac{\alpha_1+\alpha_2}{2}$ which is set between the preferred missions of the two types of principals and agents. Even if principals and agents match together on the basis of mission preferences, there is no improvement in school productivity as principals and agents payoffs depend on x which is fixed exogenously.

Now suppose that the government offers the freedom to schools to set their own missions. This could be by allowing new schools to enter or by allowing existing schools to change their missions and to compete for teachers on the basis of their mission preferences. Applying the logic of Proposition 2, we will now have schools with missions α_1 and α_2 with principals and agents matched on the basis of their mission preferences.

The model predicts that this form of competition will yield increases in school productivity in *all* schools – all agents and principals will have higher levels of motivation than when missions are homogenous.[30] Thus, theoretically at least, school competition of this form is "a rising tide that lifts all boats" to use Hoxby's (2002) phrase. The model also provides an alternative explanation for why some schools (such as Catholic schools) can be more productive by attracting teachers whose mission-preferences are closely aligned with those of the school management.

The general point is that a decentralized schooling system where missions are developed at the school level will tend to be more productive (as measured in our model by equilibrium effort) than a centralized one in which a uniform mission is imposed on

more productive than others for exogenous reasons. Akerlof and Kranton (2002) provides important insights in the economics of education using ideas from sociology.

[30] This result holds true whether or not the outside option binds as long as it remains fixed exogenously by the profit-oriented sector and is the same for all motivated agents.

schools by government.[31] Moreover, this is true regardless of whether we are talking about public or private schools.[32]

Allowing more competition through mission choice reallocates teachers across schools and improves efficiency while reducing the need for incentive pay. Thus, our approach shows that competition between schools can have effects on productivity in schools without creating a need for incentive pay.[33]

A general concern with school competition is that sorting leads to inequality. This would happen in our model if there were vertical rather that horizontal differentiation between the principals and agents – specifically some agents have high θ whichever principal they are matched with and some principals have high π regardless of the agent they match with. In this case, it is possible to show that, in a stable matching, high θ agents will be matched with high π principals.[34] Applied to schools, this predicts high and low productivity schools with some children segregated in poor quality schools.[35] However, centralizing mission choice is not a solution to this problem unless certain kinds of mission preferences and levels of motivation happen to be correlated. The optimal policy would require incentives for highly motivated teachers to work with less motivated principals.

3.3 Incentives in Public-Sector Bureaucracies

Our model can also cast light on the some more general issues in the design of incentives in public bureaucracies. Disquiet about traditional modes of bureaucratic organiza-

[31] The approach offered here is distinct from existing theoretical links competition and productivity in the context of schools. For example, yardstick competition has been used extensively in the U.K. where "league tables" are used to compare school performance. Whether such competition is welfare improving in the context of schools is moot since the theoretical case for yardstick comparisons is suspect when the incentives in organizations are vague or implicit as in the case of schools (see, for example, Dewatripont, Jewitt and Tirole (1999)). Another possible paradigm for welfare-improving school competition rests on the possibility that it can increase the threat of liquidation with a positive effect on teacher effort (Schmidt, 1997). This possibility could easily be incorporated into our model as a force that increases the cost to the agent (in this case a teacher) of the outcome where the output is Y_L.

[32] But arguably our model offers an excessively rosy view of competition. Missions may be driven by ideology, religious or political concerns some of which may not contribute to the social good. With horizontally differentiated schools, society could also end up being fragmented, making it more difficult to solve collective action problems. In a more realistic world of multi-dimensional missions, the issue for school policy will be which aspect of the mission to decentralize.

[33] This holds if the outside option is fixed. However, if school competition raises the outside options of teachers, then it could lead to more use of incentive of pay. For a review of recent debates about incentive pay for teachers (see Hanushek, 2002).

[34] This is the more standard result from the matching literature. Since we have non-transferable utility (due to limited liability), we can use the insights from Legros and Newman (2003).

[35] This differs from the standard model of school segregation based on peer group effects in production – see, for example, Epple and Romano (2002).

tion has lead to a variety of policy initiatives to improve public sector productivity. The so-called *New Public Administration* emphasizes the need to incentivize public bureaucracies and to empower consumers of public services.[36] Relatedly, Osborne and Gaebler (1993) describe a new model of public administration emphasizing the scope for dynamism and entrepreneurship in the public sector. Our framework suggests an intellectual underpinning for these approaches. However, by focussing on mission-orientation, which is also a central theme of Wilson (1989), we emphasize the fundamental differences between incentive issues in the public sector and those that arise in standard private organizations.

The results developed here give some insight into how to offer incentives for bureaucrats when there is a competitive labor market. Our framework implies that public sector incentives are likely to be more low-powered because it specializes in mission-oriented production. It therefore complements existing explanations based on multi-tasking and multiple principals for why we would expect public sector incentives to be lower powered than private sector incentives (Dixit, 2002). It provides a particularly clean demonstration of this as the production technology is assumed to be identical in all sectors.

In a public bureaucracy, we might think of the principal's type being chosen by an electoral process. The productivity of the bureaucracy will change endogenously if there is a change in the mission if the principal is replaced, unless there is immediate "re-matching". This provides a possible underpinning for the difficulty in re-organizing public-sector bureaucracies and a decline in morale during the process of transition. Over time, as the matching process adjusts to the new mission, this effect can be undone and so we might expect the short and long-run responses to change to be rather different. As Wilson (1989, p. 64) remarks, in the context of resistance to change in bureaucracies by incumbent employees, "..one strategy for changing an organization is to induce it to recruit a professional cadre whose values are congenial to those desiring the change." This suggests a potentially efficiency-enhancing role for politicized bureaucracies where the agents change with changes in political preferences.

The approach also gives some insight into how changes in private sector productivity necessitate changes in public sector incentives. Consider as a benchmark, the competitive outcome in Proposition 3. Changes in productivity that affect both sectors in the same way will have a neutral effect. However, unbalanced productivity changes that affect one sector only may have implications for optimal contracts. To see this, consider an exogenous change in π_0. In a situation of full employment as described in Proposition 3, then even if public employees initially receive a rent above their outside option, the voluntary participation constraint will eventually bind. The model predicts that this will lead to greater use of high-powered incentives in the public sector.[37]

[36] See Barzelay (2001) for background discussion.

[37] In the unemployment case described in Proposition 4, private-sector productivity does not affect public-sector productivity. Hence, we would expect issues concerning the interaction between public and private pay to arise predominantly in tight labor markets.

Putting together insights from Propositions 3 and 4, we can throw some light on why the arguments of the New Public Management to promoting incentives in the public sector became popular, as it did in countries like New Zealand and the U.K. in the 1980s. There were two components. First, the U.K. experienced a fall in motivation among principals and agents in the public sector under the Prime Ministership of Margaret Thatcher due to her efforts to change the mission of public sector bureaucracies. However, since this was done in the time of high unemployment, Proposition 4 predicts that there was little consequence for public sector incentives. However, in the 1990s, there was a return to full employment and a rise in private sector wages – raising π_0 in terms of our model. This, as Proposition 4 predicts, caused the public sector to consider schemes that mimic private sector incentives.

The model can also cast light on another component of the New Public Management – attempts to empower beneficiaries of public programs. Examples include attempts to involve parents in the decision-making process of schools and patients in that of the public health system. This is based on the view that public organizations work better when members of their client group get representation and can help to shape the mission of the organization. The model developed here suggests that this works well provided that teachers and parents share similar educational goals. Otherwise, attempts by parents to intervene will simply increase mission conflict which can reduce the efficiency of organizations.

One key issue that frequently arises in discussions of incentives in bureaucracies is corruption. By attenuating the property rights of the principal, corruption can motivate the agent and may have superficial similarities with our model here. But there are two key differences: corruption is purely pecuniary and it is not "value creating".[38] The insights developed here are quite distinct from incentive problems due to corruption.

A common complaint about public bureaucracies is that they are conservative and resist innovation.[39] Our model can make sense of this idea. In a profit-oriented organi-

[38] Our framework can capture the differences formally if we suppose that $\pi_i = B_i - R_i$ and $\theta_{ij} = \mu R_j$ where R_j is the amount that an agent of type j "steals" from a principal of type i. (The cost of stealing is parametrized by $\mu \leq 1$.) Assuming for simplicity that the agent's outside option is zero, the optimal contract (applying Proposition 1) is now:

$$\left(b^*_{ij}, w^*_{ij}\right) = \left(\frac{B_i - (1+\mu) R_i}{2}, \underline{w}\right).$$

The corresponding effort level is

$$e^*_{ij} = b^*_{ij} + \mu R_j = \frac{B_i - (1-\mu) R_i}{2}.$$

So long as $\mu < 1$, both the principal and agent are worse off because of corruption. Also, the productivity of the organization is decreasing in R_j in this case.

[39] The ensuing argument could equally be applied to religious organizations, advocacy groups, and NGOs which are often accused of being rigid in their views and approaches.

zation, any change that increases the principals payoff, π_0, will be adopted. However, in a mission oriented organization, the preferences of the agent need also to be considered. Consider, for the sake of illustration, case 2 of Proposition 2. The principal's expected payoff in this instance is $e^*_{ij}(\pi_i - b^*_{ij}) - \underline{w} = \frac{1}{4}(\theta_{ij} + \pi_i)^2 - \underline{w}$. Then a mission oriented organization will innovate only if $\pi_i + \theta_{ij}$ is larger – it is optimal for the principal to factor in the effect that it has on the motivation of the agent. If innovations reduce θ_{ij}, they might be resisted even if π_i is higher. If we think of π_i as predominantly a financial payoff, then innovations that pass standard financial criteria for being worthwhile (raising π_i) may be resisted in mission-oriented sectors of the economy. Since much of the drive for efficiency in the public sector uses financial accounting measures, this could explain why public bureaucracies are often seen as conservative and resistant to change.

4 Concluding Comments

The aim of this paper has been to explore competition and incentives in mission-oriented production. These ideas are relevant to discussing organizations where agents have some non-pecuniary interest in the organization's success. Key examples are non-profits, public bureaucracies and educational providers. With motivated agents, there is less need for incentive pay. There is also a premium on matching on mission preferences.

Much remains to be done to understand the issues better. It is important in particular to understand how the existence of motivated agents affects the choice of organizational form. The analysis also cries out for a more complete treatment of the sources of motivation and the possibility that motivation is crowded in or out by actions that the principal can take.[40]

In this paper, we have maintained a sharp distinction between mission-oriented and profit-oriented sectors. However, private firms frequently adopt missions. In future work, it would be interesting to develop the content of mission choice in more detail and to understand how mission choice interacts with governance of organizations and market pressures.

[40] Frey (1997), Bénabou and Tirole (2003) and Akerlof and Kranton (2003) make important progress in this direction.

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5 Appendix : Proofs

First observe that the first best effort is $\pi_i + \theta_{ij}$ and first best surplus is: $\frac{1}{2}(\pi_i + \theta_{ij})^2 - \underline{w} = S_{ij}$. To prove Proposition 1, we proceed by proving several useful Lemmas. Substituting for e_{ij} using the incentive compatibility constraint, we can rewrite the optimal contracting problem in section 3.1 as:

$$\max_{\{b_{ij}, w_{ij}\}} u_{ij}^p = (\pi_i - b_{ij})(b_{ij} + \theta_{ij}) - w_{ij}$$

subject to:

$$\begin{aligned} w_{ij} &\geq \underline{w} \\ u_{ij}^a &= \frac{1}{2}(b_{ij} + \theta_{ij})^2 + w_{ij} \geq \overline{u}_j. \end{aligned}$$

This modified optimization problem involves two choice variables, b_{ij} and w_{ij}, and two constraints, the limited-liability constraint and the participation constraint. The objective function u_{ij}^p is concave and the constraints are convex. Now we are ready to prove:

Lemma 1: *Under an optimal incentive contract at least one of the participation and the limited liability constraints will bind.*

Proof: Suppose both constraints do not bind. As the participation constraint does not bind, the principal can simply maximize his payoff with respect to b_{ij} which yields

$$b_{ij} = \max\left\{\frac{\pi_i - \theta_{ij}}{2}, 0\right\}$$

and the corresponding effort level would be

$$e_{ij} = b_{ij} + \theta_{ij} = \max\left\{\frac{\pi_i + \theta_{ij}}{2}, \theta_{ij}\right\}.$$

Since the PC is not binding, and by assumption $w_{ij} > \underline{w}$, the principal can reduce w_{ij} by a small amount without violating any of these two constraints. This will not affect e_{ij}, and yet increase his profits. This is a contradiction and so the principal will reduce w_{ij} until either the limited-liability constraint or the participation constraint binds. **QED**

Lemma 2: *Under an optimal incentive contract, if the limited liability constraint does not bind, then (i) e_{ij} is at the first-best level; and (ii) the principal's expected payoff is strictly negative.*

Proof: We prove the equivalent statement, namely, if e_{ij} is not at the first-best level then the limited liability constraint must bind. As $b \leq \pi_i$, effort cannot exceed the first-best level. The remaining possibility is that e_{ij} is less than the first-best level. Suppose this is the case, i.e., $e_{ij} = b_{ij} + \theta_{ij} < \pi_i + \theta_{ij}$.We claim that in this case the limited-liability constraint must bind. Suppose not. That is, we have an optimal contract (b_{ij}^0, w_{ij}^0) such that $b_{ij}^0 < \pi_i$ and $w_{ij}^0 > \underline{w}$. Suppose we reduce w_{ij}^0 by ε and increase b_{ij}^0 by an amount such that the agent's expected payoff is unchanged. Since the agent chooses effort to maximize his own payoff we can use the envelope theorem to ignore the effects of changes in w_{ij} and b_{ij} on his payoff via e_{ij}. Then $du_{ij}^a = e_{ij}db_{ij} + dw_{ij} = 0$. The effect of these changes on principal's payoff is $du_{ij}^p = de_{ij}(\pi_i - b_{ij}) - (e_{ij}db_{ij} + dw_{ij})$. The second term is zero by construction and the first term is positive and so the principal is better of. This is a contradiction. This proves the first part of the lemma. Next we show that if the limited-liability constraint does not bind then the principal's expected payoff is strictly negative. From the first part of this lemma, if the limited-liability constraint does not bind, then $e_{ij} = \pi_i + \theta_{ij}$.From the incentive compatibility constraint, this implies $b_{ij} = \pi_i$.Since $w_{ij} > \underline{w}$ (the limited-liability constraint does not bind) and $\underline{w} \geq 0$, this immediately implies the principal's expected payoff $u_{ij}^p = -w_{ij} < 0$. **QED**

Lemma 3: *Suppose Assumption 2 holds. Then $\overline{v}_{ij}$ is a strictly positive real number that does not exceed S_{ij}.*

Proof: By Lemma 2, if the principal's payoff is non-negative then the limited-liability constraint must bind. Therefore, $w_{ij} = \underline{w}$. Given the modified version of the optimal contracting problem stated at the beginning of this section, the only remaining variable to solve for is b_{ij}.The agents payoff is increasing in b_{ij}. Therefore we can solve for b_{ij} from the equation $(\pi_i - b_{ij})(b_{ij} + \theta_{ij}) - \underline{w} = 0$, namely, the principal's expected payoff is equal to 0.Being a quadratic equation it has two roots, but the higher one is the relevant one since the agent's payoff is increasing in b_{ij}. This is:

$$b_{ij} = \frac{\pi_i - \theta_{ij} + \sqrt{(\pi_i + \theta_{ij})^2 - 4\underline{w}}}{2}.$$

Substituting this into the agent's payoff function, we get

$$\overline{v}_{ij} = \frac{1}{2}\left(\frac{\theta_{ij} + \pi_i + \sqrt{(\theta_{ij} + \pi_i)^2 - 4\underline{w}}}{2}\right)^2 + \underline{w}.$$

By Assumption 2, $(\theta_{ij} + \pi_i)^2 - 4\underline{w} > 0$ and so $\overline{v}_{ij}$ is a real number. It is strictly positive as $\pi_i > 0$ and $\theta_{ij} > 0$. Also, as $\underline{w} \geq 0$, $\overline{v}_{ij} \leq S_{ij}$ with the equality holding if $\underline{w} = 0$. **QED**

Lemma 4: Suppose Assumption 2 holds. Then $\underline{v}_{ij}$ lies in the real interval $(0, \overline{v}_{ij})$.

Proof: Suppose the participation constraint does not bind. By Lemma 1, the limited-liability constraint binds and

$$b_{ij} = \max\left\{\frac{\pi_i - \theta_{ij}}{2}, 0\right\}.$$

The agent's payoff is $\frac{1}{2}(b_{ij}^* + \theta_{ij})^2 + \underline{w} = \frac{1}{8}\left(\theta_{ij} + \max\{\pi_i, \theta_{ij}\}\right)^2 + \underline{w}$. This is a positive real number as $\pi_i > 0$ and $\theta_{ij} > 0$. There are two cases, depending on whether π_i is greater than or less than θ_{ij}. In the former case it is clear upon inspection that $\underline{v}_{ij} < \overline{v}_{ij}$. In the latter case, we need to show that $\frac{\theta_{ij}+\pi_i+\sqrt{(\theta_{ij}+\pi_i)^2-4\underline{w}}}{2} > \theta_{ij}$. Upon simplification this condition is equivalent to $\pi_i\theta_{ij} - \underline{w} > 0$. By Assumption 2, $\frac{1}{4}\pi_i^2 - \underline{w} > 0$. In the present case, by assumption $\theta_{ij} > \pi_i$. Therefore $\pi_i\theta_{ij} > \pi_i^2 > \frac{1}{4}\pi_i^2$ and so this condition holds given Assumption 2. **QED**

Proof of Proposition 1: Now we are ready to characterize the optimal contract and prove existence. By Lemma 1 and Lemma 2, the only relevant cases are when the limited-liability constraint binds but the participation constraint does not bind, and when both the participation constraint and the limited-liability constraint binds. From the proof of Lemma 4, the former case can be usefully split into two separate cases depending on whether π_i is greater than or less than θ_{ij}.This means there are three cases to study:

Case 1: The participation constraint does not bind and $\theta_{ij} > \pi_i$. We have already established in the proof of Lemma 1 that in this case the limited-liability constraint will bind and that:

$$\begin{aligned} b_{ij}^* &= \max\left\{\frac{\pi_i - \theta_{ij}}{2}, 0\right\} = 0 \\ w_{ij}^* &= \underline{w} \\ e_{ij}^* &= b_{ij}^* + \theta_{ij} = \theta_{ij}. \end{aligned}$$

From Lemma 4, the agent's payoff is $\frac{1}{2}\theta_{ij}^2 + \underline{w}$. Since the participation constraint does not bind by assumption in this case, the following must be true:

$$\frac{1}{2}\theta_{ij}^2 > \overline{u}_j - \underline{w}.$$

The principal's payoff is

$$\left(b_{ij}^* + \theta_{ij}\right)\left(\pi_i - b_{ij}^*\right) - \underline{w} = \theta_{ij}\pi_i - \underline{w}.$$

Case 2: The participation constraint does not bind and $\theta_{ij} \leq \pi_i$. In this case:

$$\begin{aligned} b_{ij}^* &= \max\left\{\frac{\pi_i - \theta_{ij}}{2}, 0\right\} = \frac{\pi_i - \theta_{ij}}{2} \\ w_{ij}^* &= \underline{w} \\ e_{ij}^* &= b_{ij}^* + \theta_{ij} = \frac{\pi_i + \theta_{ij}}{2} \end{aligned}$$

From Lemma 4 the agent's payoff is $\frac{1}{8}(\pi_i+\theta_{ij})^2+\underline{w}$ in this case. Since the participation constraint does not bind by assumption in this case, the following must be true:

$$\frac{1}{8}(\pi_i+\theta_{ij})^2 > \overline{u}_j - \underline{w}.$$

The principal's payoff is

$$\left(b^*_{ij}+\theta_{ij}\right)\left(\pi_i - b^*_{ij}\right) - \underline{w} = \frac{1}{4}(\pi_i+\theta_{ij})^2 - \underline{w}.$$

Case 3: The participation constraint and the limited-liability constraint binds. These constraints then uniquely pin down the two choice variables for the principal. In particular, we get

$$\begin{aligned} w^*_{ij} &= \underline{w} \\ b^*_{ij} &= \sqrt{2\left(\overline{u}_j - \underline{w}\right)} - \theta_{ij} \end{aligned}$$

using which and the incentive compatibility constraint we get

$$e^*_{ij} = b^*_{ij} + \theta_{ij} = \sqrt{2\left(\overline{u}_j - \underline{w}\right)}.$$

As $b^*_{ij} \leq \pi_i$, $e^*_{ij} = \sqrt{2\left(\overline{u}_j - \underline{w}\right)} \leq \pi_i + \theta_{ij}$. Therefore, $\overline{u}_j - \underline{w} \leq \frac{1}{2}\left(\pi_i+\theta_{ij}\right)^2$. Notice that in this case $b^*_{ij} > 0$ as that is equivalent to $\overline{u}_j - \underline{w} > \frac{1}{2}\theta^2_{ij}$ and this must be true because otherwise the participation constraint would not bind and we would be in the previous case. The payoff of the agent in this case is by assumption,

$$u^a_{ij} = \overline{u}_j.$$

The principal's payoff is:

$$u^p_{ij} = \sqrt{2\left(\overline{u}_j - \underline{w}\right)}\left(\pi_i + \theta_{ij} - \sqrt{2\left(\overline{u}_j - \underline{w}\right)}\right) - \underline{w}.$$

From the proof of Lemma 3 this is equal to zero if $\overline{u}_j = \overline{v}_{ij}$. Therefore, so long as $\overline{u}_j \leq \overline{v}_{ij}, u^p_{ij} \geq 0$.

Finally, we must check that the optimal contract exists. The principal's expected payoff when $\overline{u}_j = 0$ and $\theta_{ij} \geq \pi_i$ is $\theta_{ij}\pi_i - \underline{w}$. By Assumption 2 this is positive for $i = 1, 2$ and $j = 1, 2$. The principal's expected payoff when $\overline{u}_j = 0$ and $\theta_{ij} < \pi_i$ is $\frac{1}{4}(\pi_i+\theta_{ij})^2 - \underline{w}$. By Assumption 2 this is positive. In both the cases above the agent receives a strictly positive payoff even though $\overline{u}_j = 0$. In the first case, the agent's payoff is $\frac{1}{2}\theta^2_{ij} + \underline{w}$ and in the second case it is $\frac{1}{8}(\pi_i+\theta_{ij})^2 + \underline{w}$ which are strictly positive real numbers by Lemma

4. On the other extreme, if the principal's expected payoff is set to zero, the agent's expected payoff under the optimal contract is $\overline{v}_{ij}$ which is a strictly positive real number by Lemma 3. For all $\overline{u}_j \geq \underline{v}_{ij}$, the participation constraint binds and the principal's payoff is a continuous and decreasing function of $\overline{u}_j$, and so an optimal contract exists for all $\overline{u}_j \in [0, \overline{v}_{ij}]$. **QED**.

Proof of Proposition 2: Let $\beta_{ij} \equiv \pi_i + \theta_{ij}$ for $i = 0, 1, 2$ and $j = 0, 1, 2$ and let $\gamma_{ij} = \max\left\{\frac{\beta_{ij}}{2}, \theta_{ij}\right\}$. Let z_j be the reservation payoff of an agent of type j ($j = 0, 1, 2$). Then from the proof of Proposition 1 the payoff of to a principal in the mission-oriented sector ($i = 0, 1, 2$) when matched with a motivated agent ($j = 0, 1, 2$) is given by:

$$\Pi^*_{ij}(z_j) = \begin{cases} \pi_i\theta_{ij} - \underline{w} & \textit{for} \quad \pi_i < \theta_{ij} \quad \textit{and } z_j - \underline{w} < \frac{1}{2}\gamma^2_{ij} \\ \frac{\beta^2_{ij}}{4} - \underline{w}, & \textit{for} \quad \pi_i \geq \theta_{ij} \quad \textit{and } \; z_j - \underline{w} < \frac{1}{2}\gamma^2_{ij} \\ \sqrt{2(z_j - \underline{w})}(\beta_{ij} - \sqrt{2(z_j - \underline{w})}) - \underline{w}, & \textit{for} \qquad \frac{1}{2}\gamma^2_{ij} \leq z_j - \underline{w} \leq \overline{v}^a_{ij} \end{cases}$$

From the proof of Proposition 1, $\Pi^*_{ij}(z_j)$ is (weakly) decreasing in z_j for all $i = 0, 1, 2$ and all $j = 0, 1, 2$. First consider principals in the mission-oriented sector. As $\pi_1 = \pi_2 = \hat{\pi}$, for any given value of $z_0 = z_1 = z_2 = z$, $\Pi^*_{ii}(z) > \Pi^*_{ij}(z)$ for $i = 1, 2$ and for all $j = 0, 1, 2$. Next consider principals in the profit-oriented sector. For any given value of $z_0 = z_1 = z_2 = z$, $\Pi^*_{00}(z) = \Pi^*_{01}(z) = \Pi^*_{02}(z)$. We now demonstrate that all stable matches must be assortative.

Suppose that there is a stable non-assortative match with reservation payoffs (z_0, z_1, z_2). Since $n^a_1 = n^p_1$ and $n^a_2 = n^p_2$, there must be at least one match involving a principal of type i ($i = 1, 2$) and an agent of type $j \neq i$ ($j = 0, 1, 2$). We show that this leads to a contradiction.

Of the various possibilities, we can eliminate right away the one where a principal of type i ($i = 1, 2$) is matched with an agent of type $j \neq i$ ($j = 0, 1, 2$) and correspondingly, an agent of type i is unmatched. Such an agent receives the autarchy payoff of 0 and so a principal of type i ($i = 1, 2$) cannot possibly prefer to hire an agent of type $j \neq i$ as $\Pi^*_{ii}(0) > \Pi^*_{ij}(z_j)$ for all $i = 1, 2,$ for all $j \neq i$, and $z_j \geq 0$. Given this there are three types of non-assortative matches that we need to consider.

First, a principal of type i ($i = 1, 2$) is matched with an agent of type 0 and correspondingly an agent of type i is matched with a principal of type 0. Stability implies a principal of type i would not wish to bid away an agent of type i from a principal of type 0. This implies $\Pi^*_{i0}(z_0) \geq \Pi^*_{ii}(z_i)$ which in turn implies that $z_i > z_0$ as $\Pi^*_{ii}(z_0) > \Pi^*_{i0}(z_0)$. Similarly, the fact that a principal of type 0 prefers to hire an agent of type i ($i = 1, 2$) over an agent of type 0 implies that $\Pi^*_{0i}(z_i) \geq \Pi^*_{00}(z_0)$ which in turn implies $z_0 \geq z_i$. But that is a contradiction.

Second, a principal of type 1 is matched with an agent of type 2 and correspondingly, an agent of type 1 is matched with a principal of type 2. By stability a principal of type 1 would not wish to bid away an agent of type 1 from a principal of type 2. This implies $\Pi^*_{12}(z_2) \geq \Pi^*_{11}(z_1)$, which in turn implies that $z_1 > z_2$ since $\Pi^*_{11}(z_2) > \Pi^*_{12}(z_2)$. Similarly, the fact that principal two does not want to bid away agent two implies that $\Pi^*_{21}(z_1) \geq \Pi^*_{22}(z_2)$. But by a similar argument this implies that $z_2 > z_1$. This is a contradiction.

Third, a principal of type i ($i = 1, 2$) is matched with an agent of type 0, an agent of type i is matched with a principal of type $j \neq i$ ($j = 1, 2$), and an agent of type j is matched with a principal of type 0. Repeating the arguments used above, the fact that a principal of type i ($i = 1, 2$) prefers an agent of type 0 to an agent of type i implies $z_i > z_0$. Similarly, as a principal of type $j \neq i$ ($j = 1, 2$) prefers an agent of type i to an agent of type j implies $z_j > z_i$. Together, these two inequalities imply $z_j > z_0$. However, the fact that a principal of type 0 (weakly) prefers to hire an agent of type j to an agent of type 0 implies $z_0 \geq z_j$ which is a contradiction.

Therefore there is no stable non-assortative match. **QED.**

Proof of Proposition 3: By Proposition 2, we can restrict attention on assortative matches. Since $n^p_0 > n^a_0$, there are unemployed profit-oriented principals. Therefore, all employed principals in the profit-oriented sector must be earning zero profits. The stated contracts are optimal according to Proposition 1 relative to a common reservation payoff for all types of agents of:

$$\hat{u} = \frac{1}{8}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right)^2 + \underline{w}.$$

From the proof of Lemma 3, this is the payoff that an agent of any type who is matched with a principal of type 0 receives when the principal's expected payoff is zero. Accordingly, this is the relevant reservation payoff of all agents under full employment. We proceed to prove that the proposed assortative matching is stable.

All employed principals in the profit-oriented sector are earning zero profits. They cannot therefore attract away an unmotivated agent from another profit-oriented principal without earning a negative profit. Hence the matching *within* the unmotivated sector is stable.

An agent of type j ($j = 1, 2$) receives a payoff of $v^a_j = \max\left\{\frac{1}{8}\left(\xi + \overline{\theta}\right)^2 + \underline{w}, \hat{u}\right\} \equiv \hat{v}^a$. Since this is the same for both types of motivated agents, and $\Pi^*_{ii}(z) > \Pi^*_{ij}(z)$ for $i = 1, 2$ and for all $j = 0, 1, 2$, the proposed matching is stable *within* the mission-oriented sector.

Finally, we show that matching between the profit-oriented and mission-oriented sectors is stable.

Let us define the following function to simplify notation:

$$g(x_1, x_2) \equiv \sqrt{2\left(x_1 - \underline{w}\right)}\left(x_2 - \sqrt{2\left(x_1 - \underline{w}\right)}\right) - \underline{w}.$$

This gives the payoff of a principal under an optimal contract when the participation constraint is binding, the reservation payoff of the agent is x_1 and the joint payoff of the principal and the agent from success is x_2 (e.g., if the principal is type i and the agent is type j then $x_2 = \pi_i + \theta_{ij}$). We check if a principal of type 0 can offer an agent of type 1 or 2 a payoff of $\hat{v}^a$ and still be strictly better off than he is in the proposed matched with an unmotivated agent. Currently such a principal earns an expected payoff of 0. If he hires an agent of type j ($j = 1, 2$) the participation constraint will bind since $\frac{1}{8}(\xi + \overline{\theta})^2 + \underline{w} \geq \frac{1}{8}\pi_0^2 + \underline{w}$ for $i = 1, 2$ (by Assumption 3). There are two cases to consider. First, $\hat{v}^a > \hat{u}$. Then the maximum payoff that a principal of type 0 can earn from an agent of type j ($j = 1, 2$) is $g(\hat{v}^a, \pi_0) < g(\hat{u}, \pi_0)$ as $\hat{v}^a > \hat{u}$. But by construction $g(\hat{u}, \pi)$ = 0 in the full-employment case and so such a move is not attractive. Similarly, if $\hat{v}^a = \hat{u}$, the maximum payoff that a principal of type 0 can earn from an agent of type j ($j = 1, 2$) is $g(\hat{u}, \pi_0)$ which is the same that he earns in his current match.

Next we show that a principal of type i ($i = 1, 2$) will not find it profitable to attract an unmotivated agent who earns $\hat{u}$. A principal of type i ($i = 1, 2$) can earn at most $g(\hat{u}, \hat{\pi})$ from such a move which is strictly less than $g(\hat{u}, \hat{\pi} + \overline{\theta})$ which is what he was earning before, in case the participation constraint was binding. Now let us consider the possibility that the participation constraint was not binding. Notice that $g(\hat{u}, \hat{\pi}) = \frac{1}{2}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right) \left\{\hat{\pi} - \frac{1}{2}\left(\pi_0 + \sqrt{\pi_0^2 - 4\underline{w}}\right)\right\} - \underline{w} \leq \frac{1}{4}\hat{\pi}^2 - \underline{w}$ (since the expression $y(a - y)$ is maximized at $y = \frac{a}{2}$). As the participation constraint was not binding by assumption in this case, the principal was earning either $\hat{\pi}\overline{\theta} - \underline{w}$ (if $\overline{\theta} > \hat{\pi}$) or $\frac{\left(\hat{\pi}+\overline{\theta}\right)^2}{4} - \underline{w}$ (if $\overline{\theta} \leq \hat{\pi}$). In the former case, as $\overline{\theta} > \hat{\pi}$, $\hat{\pi}\overline{\theta} - \underline{w} > \frac{1}{4}\hat{\pi}^2 - \underline{w}$. In the latter case, $\frac{1}{4}\hat{\pi}^2 - \underline{w} \leq \frac{\left(\hat{\pi}+\overline{\theta}\right)^2}{4} - \underline{w}$ for all $\overline{\theta} \geq 0, \hat{\pi} > 0$. Thus, the proposed matching is stable as claimed. **QED.**

Proof of Proposition 4: The stated contracts are optimal contract according to Proposition 1 and Corollary 1 relative to a common reservation payoff of zero. This is what we would expect as $n_0^p < n_0^a$ and so there are unemployed agents. The rest of the proof is similar to that of Proposition 3, and is hence omitted. **QED**.