GV 507:
Lectures on Political Competition and Welfare

Professor Timothy Besley*

Michaelmas Term 1999

Contents

*Disclaimer: These notes are not guaranteed to be error free. Please bring any problems that you notice to the attention of the lecturer.

1. The Economic Environment

There are N citizens who have to make a social decision about a set policies denoted by $x \in \mathcal{A}$, where $\mathcal{A}$ denotes the set of feasible policies. Citizen's preferences over policy are denoted $V^i(x, j)$ (where $i = 1, ..., N$) ,and j denotes the identity of the policy maker. This specification allows for the many other possibilities beyond purely self-interested policy making. For example, citizens may be altruistic or paternalistic. Individuals may also care about who holds office. One extreme possibility is that individuals care only about holding office. However, they could also care intrinsically about selecting a particular citizen to make policy on their behalf.

Feasibility is also defined broadly. Naturally, it includes technological feasibility as a constraint. However, it may also embody constraints on information available to the policy maker. It may also embody constitutional restrictions on policy instruments if these are relevant. The following gives a few examples that illustrate policy making environments that fit the model.

Example: (One dimensional political science environment) The standard in the formal political science literature considers a set of policy alternatives $\mathcal{A}$ which is lie in m-dimensional Euclidean space with each citizen i having distance preferences over these alternatives with ideal point α_i; i.e., $V^i(x, j) = -\|\alpha_i - x\|$ for all $j = 1, .., N$.

Example: (Negative Income Tax Model) A standard public economics example is where $x = (t, T)$ are an income tax rate, t, and an income guarantee level, T. Individuals have (identical) preferences over consumption, c, and labor supply, ℓ, denoted by $u(c, \ell)$ and differ in their wage rates (which are representative of earning abilities) denoted by a_i. In this case

$$V^i(t, T) = v(t, T, a_i) = \max_{c,\ell} \{u(c, \ell) : c = a_i\ell(1 - t) + T\}.$$

Let $\ell(a(1 - t), T)$ denote the optimal labor supply and $c(a(1 - t), T)$ the optimal consumption level of an individual of ability a. The feasible set of policies, $\mathcal{A}$, are the values of (t, T) which satisfy the government budget constraint

$$(1 - t)\sum_{i=1}^{N} a_i\ell(a_i(1 - t), T) = NT.$$

The notion of a Condorcet winner is central to political economy modeling. Its genesis is the eighteenth century work of the Marquis de Condorcet who showed that majority rule (modeled as sequential pairwise comparisons) need not select a particular outcome. Indeed it could lead to cycles between alternatives. A Condorcet winner exists if there is some policy alternative that beats all others in pairwise comparisons. We define for the case where voters' preferences do not depend on j since this is the case studied in most applications in public economics. A particular policy outcome x_c is a (strict) Condorcet winner in the set $\mathcal{A}$ if there is no other policy $x \in \mathcal{A}/\left\{x_c\right\}$, which is (strictly) preferred to it by a majority of the population.

The most common incarnation of a Condorcet winner is in cases where the median citizen's policy alternative is decisive over all others. This is possible in both of the examples laid out above. In the first example preferences are single-peaked. If $m = 1$, i.e., there is one-dimension to the policy space, then the Condorcet winner is at the median ideal point. When $m > 1$, there is no guarantee that a median exists without further strong assumptions on the distribution of types (see, for example, Mueller (1992)). For our second example, Roberts (1977) has shown that there is a Condorcet winner if $y\left(t, T, a\right) \equiv a\ell\left(a\left(1-t\right), T\right)$ is increasing in a for all $(t, T) \in [0, 1] \times \Re$. The Condorcet winner in that instance is the level of redistribution preferred by the median ability group.[1] Perturbing the model to give the government a second policy instrument, such as a public good or an income tax with more than one tax bracket, requires much more stringent assumptions.

More generally, the existence of a Condorcet winner requires that the type space and/or the policy space to be severly restricted. A vast literature has grown up around the fact that Condorcet winners are not to be expected in most interesting economic environments. Perhaps the most straightforward and well-known environment in which a Condorcet winner does not exist is a game of pure distribution. Suppose that the government has to divide a cake of size one. In that case x is an element of the N-dimensional unit simplex, the latter being the set $\mathcal{A}$. It is easy to see that for any randomly selected alternative in $\mathcal{A}$, another can be found that beats it in a pairwise comparison under majority rule.

[1] Gans and Smart (1995) shows that this is equivalent to a single crossing property in the preferences $v\left(t, T, a\right)$. To see this, observe that, using Roy's identity, $y\left(t, T, a\right) = -\frac{\partial v(t,T,a_i)/\partial t}{\partial v(t,T,a_i)/\partial T}$. Hence Robert's condition is equivalent to the indifference curves (drawn in policy space) being ordered appropriately.

2. Representative Democracy

Our discussion of resource allocation and political competition will focus on representative democracy . In practice a large number of decisions about taxes and expenditures are determined in multi-issue elections where voting is over candidates rather than policies. We describe models of representative democracy as four stage games. Stage one is an entry stage in which the number of candidates for office is determined. Stage two is a campaign stage where each candidate announces which policy he will implement if elected. At stage three, citizens vote over the candidates who have entered and at stage four policies are implemented.

2.1. Policy Choice

The aim is to build a model that nests as special cases, the Downsian model of political competition and the citizen-candidate approach. They differ crucially in terms of the assumed motivation of candidates for office and the extent of commitment to policy announcements.

After the election has been won by some candidate, he/she must choose which policy to implement. This depends upon what is assumed about candidate preferences and the possibility of commitment. Associated with each candidate will be a preferred policy stance We contrast two very different models. In a Downsian model of policy choice, each candidate is associated with some policy stance given by:

$$\widehat{x}_i =^{\arg\max}_{x} \left\{ V^i(x) | \, x \in \mathcal{A} \right\}. \tag{2.1}$$

We will suppose that this unique for each citizen.

The policy outcome, however, may not be $\widehat{x}_i$ if the campaign announcements are binding. Let X_i denote the campaign announcement of candidate $i \in \mathcal{C}$. Then we will suppose that the actually policy outcome will be

$$x_i^* = h\left(\widehat{x}_i, X_i\right).$$

In the case of full commitment, $x_i^* = X_i$, while in the case of no commitment, $x_i^* = \widehat{x}_i$. We assume that $x = h\left(x, X\right)$ if $x = X, i.e.$, if the announcement and the optimal policy agree, then this is the policy implemented after the election. The relationship in $h\left(\cdot\right)$ should ideally be endogenous. However, this would require a dynamic model for a satisfactory treatment. Alesina (1988) models political competition in a repeated game where broken promises lead voters to punish politicians. Only for vanishingly small rates of discount would politicians be able to commit to policies that were not functions of their preferences.[2]

Given the policy selection convention, we can associate a utility imputation $(v_{1i}, ..., v_{Ni})$ associated with each candidate's election, where $v_{ji} = V^j(x_i^*, i)$ is individual j's utility if i is elected. If there are no candidates, we assume that a default policy x_0 is selected. We denote the utility imputation in this case as $(v_{10}, ..., v_{N0})$, where $v_{j0} = V^j(x_0, 0)$.

2.2. Voting

Given a candidate set $\mathcal{C} \subset \mathcal{N}$, and a policy announcements for each candidate $X = \{X_i\}_{i \in \mathcal{C}}$, for $X_i \in \mathcal{A}$, each citizen j makes a voting decision. He may vote for any candidate in $\mathcal{C}$ or he may abstain. Let $\alpha_j \in \mathcal{C} \cup \{0\}$ denote his decision. If $\alpha_j = i$ then citizen j casts his vote for candidate i, while if $\alpha_j = 0$ he abstains. A vector of voting decisions is denoted by $\alpha = (\alpha_1, ..., \alpha_N)$. Given $\mathcal{C}$ and α, let $P^i(\mathcal{C}, \alpha)$ be the probability that candidate i wins. Under plurality rule, this is the candidate with the most votes. We assume that ties are broken by randomly selecting from among the tying candidates.

We assume that citizens correctly anticipate the policies that would be chosen by each candidate and act so as to maximize their expected utilities. In the language of the voting literature, therefore, we are "assuming" that voters vote *strategically* as opposed to sincerely.[3] We define a *voting equilibrium* to be a vector of voting decisions α^* such that for each citizen $j \in \mathcal{N}$ (i) α_j^* is a best response to α_{-j}^*; i.e.,

$$\alpha_j^*(\mathcal{C}, X) \in \arg\max \left\{ \sum_{i \in \mathcal{C}} P^i\left(\mathcal{C}, \ \left(\alpha_j, \alpha_{-j}^*\right)\right) V^j(h(\widehat{x}_i, X_i)) \mid \alpha_j \in \mathcal{C} \cup \{0\} \right\}, \quad (2.2)$$

and (ii) α_j^* is not a weakly-dominated voting strategy.[4] The requirement that voters do not use weakly-dominated voting strategies is standard in the voting

[2] See all Dixit, Grossman and Gul (1998) for further progress in this direction.

[3] Voting sincerely means simply voting for your most preferred candidate, without considering the consequences of your vote for the outcome of the election.

[4] A voting decision α_j is weakly dominated for citizen j if there exists $\widehat{\alpha}_j \in \mathcal{C} \cup \{0\}$ such that

$$\sum_{i \in \mathcal{C}} P^i(\mathcal{C}, (\widehat{\alpha}_j, \alpha_{-j})) v_{ji} \geq \sum_{i \in \mathcal{C}} P^i(\mathcal{C}, (\alpha_j, \alpha_{-j})) v_{ji}$$

for all α_{-j} with the inequality holding strictly for some α_{-j}.

literature. It implies that citizens never vote for their least preferred candidate. Thus, in two candidate elections, it implies that citizens vote sincerely. It is straightforward to show that a voting equilibrium exists for any non-empty candidate set $\mathcal{C}$, although it need not be unique. Even when citizens have strict preferences over the available candidates, there can be multiple voting equilibria in elections with three or more candidates.[5]

2.3. Campaigning

In the campaign stage, each candidate can announce a proposed policy X_i to maximize their expected utility. There are two possible roles for these announcements. They may affect voting behavior or they can affect the policy outcome if the candidate wins. We look for a Nash equilibrium in such announcements. Formally for $i \in \mathcal{C}$, let $u^i(\mathcal{C}, \alpha(\mathcal{C}, X) = \sum_{j\in\mathcal{C}} P^j(\mathcal{C}, \alpha(\mathcal{C}, X))V^i\left(h\left(\widehat{x}_j, X_j\right)\right)$, be the expected utility of a particular candidate, then for all non-empty candidate sets $\mathcal{C}$, let

$$\widehat{X_i} \in \arg\max\left\{u^i(\mathcal{C}, \alpha(\mathcal{C}, X_i, X_{-i}) : X_i \in \mathcal{A}\right\}. \tag{2.3}$$

An equilibrium collection of announcements $\widehat{X}\left(\mathcal{C}\right) = \left\{\widehat{X_i}\left(\mathcal{C}\right)\right\}_{i\in\mathcal{C}}$ is such that (2.3) holds for all $i \in \mathcal{C}$.

2.4. Entry

We now consider the entry process. For reasons that will become clear below, we consider the possibility that only some sub-set of citizens (the *eligible* citizens) can stand for office. We will denote this set by $\mathcal{D} \subset \mathcal{N}$. There are D citizens in that set labeled $i = 1, ..., D$. We model entry as a game played between this sub-set of citizens. Such citizen's pure strategies are denoted $s^i \in \{0, 1\}$, where $s^i = 1$ denotes entry. A pure strategy profile is denoted by $s = (s^1, ..., s^D)$. Given s, the set of candidates in is $\mathcal{C}(s) = \{i \mid s^i = 1\} \subset \mathcal{D}$. There is a common cost (possibly small) of entering to become a candidate, denoted by δ.

We will assume that entry decisions must form a Nash equilibrium. Let $\alpha(\mathcal{C}, \widehat{X}\left(\mathcal{C}\right))$ be the vector of voting decisions that citizens anticipate when the

[5]It is possible to introduce "refinements" which reduce the set of voting equilibria in multi-candidate elections (see our discussion paper (Besley and Coate (1995a)) for an example). However, any such refinement appears likely to remove perfectly reasonable voting behavior and therefore generate false precision to the theory.

candidate set is $\mathcal{C}$. Given this, the expected payoff to any citizen i from a particular pure strategy profile s is given by:

$$U^i(s;\alpha(\cdot)) = \sum_{j\in\mathcal{C}} P^j(\mathcal{C}(s),\alpha(\mathcal{C}(s),\widehat{X}(\mathcal{C}(s))))v_{ij} + P^0(\mathcal{C}(s))v_{i0} - \delta s^i, \qquad (2.4)$$

where $P^0(\mathcal{C})$ denotes the probability that the default outcome is selected. Thus, $P^0(\mathcal{C})$ equals one if $\mathcal{C} = \emptyset$ and zero otherwise. Citizen i's payoff is therefore the probability that each candidate j wins multiplied by i's payoff from j's preferred policy, less the entry cost if he chooses to enter.

To ensure the existence of an equilibrium, it is sometimes necessary to allow the use of mixed strategies. Let γ^i be a mixed strategy for citizen i, with the interpretation that γ^i is the probability that i runs for office. The set of mixed strategies for each citizen is then the unit interval $[0,1]$. A mixed strategy profile is denoted by $\gamma = (\gamma^1, ..., \gamma^D)$ and citizen i's expected payoff from the mixed strategy profile γ is denoted by $u^i(\gamma;\alpha(\cdot))$.[6] An *equilibrium of the entry game* is then a mixed strategy profile $\hat{\gamma}$ with the property that there is a voting equilibrium $\alpha(\mathcal{C})$ such that for all $i \in \mathcal{D}$, $\hat{\gamma}^i$ is a best response to $\hat{\gamma}_{-i}$. The entry game is a *finite* game so that we can apply the standard existence result due to Nash (1950) to conclude that an equilibrium exists. Equilibria can be of two types, pure strategy equilibria in which $\gamma = s$ for some $s \in \{0,1\}^N$ and mixed strategy equilibria in which $\gamma^i \in (0,1)$ for some citizen i. As will be shown in the next section, pure strategy equilibria exist quite broadly and thus will be the main focus of attention.

2.5. Equilibrium

A political equilibrium, is a collection of entry decisions γ, a function describing campaign announcements as a function of candidate sets $\widehat{X}(\mathcal{C})$, and a function describing voting behavior $\alpha\left(\mathcal{C},\widehat{X}(\mathcal{C})\right)$ such that (i) γ is an equilibrium of the entry game given $\widehat{X}(\mathcal{C})$ and $\alpha\left(\mathcal{C},\widehat{X}(\mathcal{C})\right)$, (ii) for all non-empty candidate sets $\mathcal{C}$, $\widehat{X}(\mathcal{C})$ is a campaign equilibrium and (iii) for all non-empty candidate sets $\mathcal{C}$, $\alpha\left(\mathcal{C},\widehat{X}(\mathcal{C})\right)$ is a voting equilibrium. Existence of equilibrium can be a problem in certain specifications of the model, an issue to which we return below.

[6] This is given by $u^i(\gamma;\alpha(\cdot)) = \prod_{j=1}^N \gamma^j U^i(1,...,1;\alpha(\cdot)) + \prod_{j=2}^N \gamma^j(1-\gamma^1)U^i(0,1,...,1;\alpha(\cdot))$ $+ ...... + \prod_{j=1}^N (1-\gamma^j)U^i(0,....,0;\alpha(\cdot))$.

2.6. The Downsian Model

The Downsian approach to political equilibrium (and some of its extensions to allow entry) are embedded in the above model. We now show what will happen when the model is specialized to its Downsian version. Downs assumed that candidates cared exclusively about winning. Taken literally in the present model, this means that we assume that preferences of the candidates are

$$V^i(x_i, j) = \begin{cases} \Delta & if\ i = j \\ 0 & \text{otherwise.} \end{cases} \tag{2.5}$$

We can interpret Δ as the rent from holding office. In this set-up policy does not matter at all, making it credible for a candidate to offer any policy at the campaign stage. Downs (1957) did not present a full specification of his model in game-theoretic terms. The interpretation of his model given here follows Feddersen, Sened and Wright (1990).[7]

Downs focused on the case where there where only two candidates ($\#\mathcal{C} = 2$). We begin with that case. The equilibrium choice of announcements (which are also policies in this model are):

$$\widehat{X_i} = \arg\max \{\Delta P_i(\mathcal{C}; \alpha(\mathcal{C}, X_i, X_{-i})) : x \in \mathcal{A}\}. \tag{2.6}$$

is candidate i's preferred policy choice. We will work with the case where voter preferences do not depend upon j. Then, let x_c denote a Condorcet winner in $\mathcal{A}$ which for the remainder of this section will be assumed to exist. We then have the following.

Proposition 1. *Suppose that a Condorcet winner exists in $\mathcal{A}$. Then, the unique Nash equilibrium in campaign announcements $\left\{\widehat{X_i}\right\}_{i\in\mathcal{C}}$ has $\widehat{X_i} = x_c$ for all $i \in \mathcal{C}$.*

Thus the Downsian model predicts convergence to a Condorcet winner. This provides a powerful underpinning for the common practice of assuming that the outcome preferred by the median voter is selected in political equilibrium. It is equally well-known that this approach (in its simple guise developed here) requires the existence of a Condorcet winner for it to work. If there is no policy that beats every other in pairwise comparisons then there is no Nash equilibrium in pure strategies.

[7]Downs' model might also be approach by supposing that candidates care about policy and winning, but are able to commit to future policy.

The convergence result can be generalized to the case of more than two candidates if entry is costly; this is shown in Feddersen, Sened and Wright (1990). Their analysis restricts entry to Downsian candidates, i.e. those who have preferences of the form (2.5). Thus suppose that the set $\mathcal{D}$ contains only such candidates with $D = \#\mathcal{D}$. Since entry is costly, all candidates in the race must receive the same number of votes which implies that each candidate will win with equal probability. Thus, the payoff to a citizen who chooses to enter when the candidate set is $\mathcal{C}$ is $\frac{\Delta}{\#\mathcal{C}} - \delta$. We now have the following:

Proposition 2. *(Feddersen, Sened and Wright (1990)) In a Downsian model with entry, there is an equilibrium where each candidate chooses x_c and where the the number of candidates is the maximum of D or the largest integer d such that $\frac{\Delta}{d} - \delta > 0$.*

In this equilibrium each candidate chooses the same policy outcome.[8] Entry then serves purely as a rent dissipation device. The standard Downsian assumption that there are only two candidates can, in this model, only be justified by the appeal to large entry barriers that make unprofitable for others to enter and share the rent from holding office. As with the basic two-candidate Downsian model, the result relies in this Proposition relies on the existence of a Condorcet winner.

2.7. The Citizen-Candidate Model

There are two principal feautures of the citizen-candidate model. First, it allows for entry by any kind of candidate ($\mathcal{D} = \mathcal{N}$). It is also assumed that announcements made about policy prior to the election have no force since candidates will simply implement their preferred policy if they win, i.e., $x_i^* = \widehat{x}_i$. In this instance, the campaign stage of the model is like cheap talk where candidates make anoucements with no binding force. There can be no real role for campaigning in policy formation, although it can potentially act as a device to select particular

[8]In Feddersen, Sened and Wright (1990) this is the unique equilibrium because they assume a one-dimensional model with a continuous policy space and voters who are strictly globally risk averse. This rules out equilibria where candidates adopt distinct positions and tie. For such equilibria to survive, no voter must prefer the gamble that he faces to to switching to another candidate whom he leads to win with certainty. In any model with large numbers of voters with preferences that vary continuously over the policy space, it is very unlikely that such equilibria will be possible. It would be straightforward, although tedious to formulate a condition that makes the Feddersen, Sened and Wright equilibrium the unique outcome in our somewhat more general set-up.

voting equilibria. Campaigning might also serve a role in extensions to situations of imperfect information.

Equilibria of the citizen-candidate model can be in pure or mixed strategies. The former exist quite boradly and will be the primary focus on the analysis. However, there are reasonable cases in which pure strategy equilibria fail to exist. We begin with an example where the only equilibria are in mixed strategies. It uses a well-known framework from public economics: public provision of private goods when individuals can opt out and consume in the private sector (see, for example, Stiglitz (1974)).

Example: (Mixed Strategies with Non-Single Peaked Preferences) The polity is divided into three groups; rich, middle class and poor. Their sizes are N_R, N_M, and N_P. We assume that $\frac{N}{2} > N_M > Max\{N_R, N_P\} + 1$ and also that $N_i \neq N_j$ for $i, j \in \{P, M, R\}$. Society must choose the level of public provision of a private good, such as public health care or education. Each citizen also has the option of buying the good in the market, making no public provision a policy option. We assume that there is a unit demand for the publicly provided good. However, quality may differ. We allow quality provided in the public sector to be at one of two levels, q_L and q_H, with L standing for low and H for high. Thus the set of social alternatives is $\{0, q_L, q_H\}$. We assume that the status quo point is zero provision.

Citizens in each group have identical tastes and order policy choices as follows:

$$\begin{gathered} v_R(0) > v_R(q_L) > v_R(q_H) \\ v_M(q_H) > v_M(0) > v_M(q_L) \\ v_P(q_L) > v_P(q_H) > v_P(0) \end{gathered}$$

These preferences can be justified by the fact that the rich always prefer to use the private sector and are forced to pay taxes for the poor and middle classes to consume in the public sector. The middle class use the public sector only if quality is high and would rather have no public sector than one that they did not use. Finally, the poor prefer low quality provision to high because they have to finance some of the tax burden associated with the public sector and quality is a normal good. That preferences can have this property is shown by Stiglitz (1974) for the case of public education. There is no Condorcet winner in this environment. Low quality would lose to zero provision; zero provision would lose to high quality; and high quality would lose to low quality. Thus the Downsian approach again produces no pure strategy equilibrium. It is also true that our approach yields no pure strategy equilibria (for sufficiently small δ).[9] However, there are interesting mixed strategy equilibria.[10]

We focus on equilibria where one citizen from each of the three groups enters with positive probability. We label the representatives from each of the groups as M, P and R. The normal form of the game between these three citizens is as follows

Middle Class

		Enter	*Not Enter*
Poor:	*Enter*	$(v_R(q_H)-\delta, v_M(q_H)-\delta, v_P(q_H)-\delta)$	$(v_R(0)-\delta, v_M(0), v_P(0))$
	Not Enter	$(v_R(q_H)-\delta, v_M(q_H)-\delta, v_P(q_H))$	$(v_R(0)-\delta, v_M(0), v_P(0))$

Rich Enter:

Middle Class

		Enter	*Not Enter*
Poor:	*Enter*	$(v_R(q_L), v_M(q_L)-\delta, v_P(q_L)-\delta)$	$(v_R(q_L), v_M(q_L), v_P(q_L)-\delta)$
	Not Enter	$(v_R(q_H), v_M(q_H)-\delta, v_P(q_H))$	$(v_R(0), v_M(0), v_P(0))$

Rich do not Enter:

There are two payoff matrices, where M choose the column, P chooses the row and R chooses the payoff matrix. The Appendix proves

Proposition 3. *For sufficiently small δ, there is a unique mixed strategy equilibrium of this three person game given by:*
$\gamma_P = 1, \gamma_M = \frac{v_R(0)-v_R(q_L)-\delta}{v_R(0)-v_R(q_H)}$ *and* $\gamma_R = \frac{\delta}{v_M(q_H)-v_M(0)}$.

It can also be verified that, given the three representatives of each group are entering with these probabilities, no other citizen has an incentive to enter. Thus, the three representatives M, P and R entering with probabilities γ_M, γ_P and γ_R and every other citizen entering with probability zero is a mixed strategy equilibrium of the entry game. In this equilibrium, as δ gets small, the probability of the poor individual being selected to choose policy goes to one. Thus the policy outcome is low quality public provision with the rich and the middle class consuming

[9] This is proven in the Appendix.

[10] In this example, because of the discrete set of policy alternatives, it is very easy to calculate mixed strategy equilibria for the Downsian model. There is a unique equilibrium of this form which involves each party choosing each alternative with probability 1/3.

in the private sector. This is interesting since the biggest group (the middle class) almost always get their least preferred policy. In effect, the equilibrium involves the poor and rich ganging up on the middle class to keep them out of power.

Turning now to pure strategy equilibria, we focus on two cases: those with one and two candidates standing. We begin with one-candidate equilibria which are rare in an important sense. Nonetheless, they provide an excellent link between the citizen-candidate and Downsian models. The key result will use the notion of a Coondorcet winner in a somewhat less restrictive sense than that needed in a Downsian model. Suppose then that preferences do not depend upon the identity of the winning candidate. Then, let $\mathcal{A}^* \subset \mathcal{A}$ denote the set of (feasible) policies that are optimal for some citizen and let x_c^* to denote the Condorcet winner in this set (if it exists). It is possible for x_c^* exists when x_c does not. For example Snyder and Kramer (1988) consider an extension of the income tax model discussed above to have two income tax bands. They show that the median citizen's preferred income tax schedule is a Condorcet winner in the set of alternatives that are optimal for some citizen.[11] They then argue that this a reasonable for prediction for what will be the political equilibrium. This is not reasonable in a Downsian world where candidates can propose tax schedules that are not in $\mathcal{A}^*$. However, the citizen-candidate model offers a framework in which political competition takes place across policies $\mathcal{A}^*$.

The following result uses this weaker notion of a Condorcet winner to tie clarify the link between the citizen-candidate approach and the Downsian model.

Proposition 4. *(Besley and Coate (1997a)) Suppose that $V^i(x, j)$ is independent of j for all $i \in \mathcal{N}$, and that a Condorcet winner, x_c^*, exists in $\mathcal{A}^*$, then*

(i) if citizen i running unopposed is an equilibrium of the entry game for sufficiently small δ, $x_i^* = x_c^*$ and

(ii) if $x_i^* = x_c^* \neq x_0$ then citizen i running unopposed is an equilibrium of the entry game for sufficiently small δ.

This establishes an essential equivalence between the policy outcome being a Condorcet winner in $\mathcal{A}^*$ and a one candidate equilibrium for the case where citizens are motivated purely by purely policy concerns.

We now consider what happens if there are also rents from holding office as was posited by Feddersen, Sened and Wright's characterization of the Downsian

[11]See Persson and Tabellini (1994) for an approach that also uses the fact that candidates optimize once elected.

model with entry. Suppose then that

$$V^i(x_i, j) = \begin{cases} v^i(x) + \Delta & if\ i = j \\ v^i(x) & \text{otherwise.} \end{cases} \tag{2.7}$$

This generalizes the preferences that we posited for the Downsian model by allowing for some policy preference. Suppose now that a Condorcet winner exists in $\mathcal{A}^*$ and let $\mathcal{M}$ denote the set of citizens whose preferred policy outcome is x_c^*, with $M = \#\mathcal{M}$. We now obtain a result exactly parallel to that obtained by Feddersen, Sened and Wright for these more general preferences.

Proposition 5. *Suppose that preferences are as in (2.7), then there is a political equilibrium of the citizen-candidate model where each candidate chooses x_c^* and the equilibrium number of candidates is the maximum of M and the largest integer m such that $\frac{\Delta}{m} - \delta > 0$.*

This result allows for the possibility of a significant number of candidates competing for office all with identical policy preferences. As in the Downsian model, entry dissipates the rents of holding office. Each candidate in the proposed equilibrium will provide the Condorcet winner. The outcome described here looks very similar to that predicted by the Downsian model with entry even though we have assumed that candidates have policy preferences. However, we have done so without assuming that entry is only possible by candidates who are motivated by holding office. These results are relevant only in cases where a Condorcet winner exists and, while these are weaker than in the standard model, they are still fairly stringent. We therefore needed an approach that works more generally.

Two candidate equilibria of the citizen-candidate model exist under reasonable weak condition as demonstrated in Besley and Coate (1997a). The key proposition from that paper is Proposition 3. Intuitively, two candidate equilibria must have two properties. First, none of the two candidates in the race should wish to exit. A necessary condition for this is that both should have a positive probability of winning. This requires that each wins with probability 1/2. Since voting is sincere in two candidate contests, this requires that each candidate is equally popular in terms of sincere preferences. Second, no candidate should be able to enter and beat either of the candidates in the race. This depends on the voting equilibrium played in the three candidate that would ensue were any third candidate to enter. Our notion of voting equilibrium is very permissive in three candidate contests. Indeed, there will always be a voting equilibrium in which all individuals continue to vote as they would in the initial two candidate equilibrium, so that the entrant gets no votes. This is the case regardless of how popular the entrant might be in terms of sincere preferences over candidates. Hence, constructing voting equilibria that support two candidate equilibria is fairly easy.

This argument for the construction of two candidate equilibria does suggest that Duverger's hypothesis that there is something special about two candidate competition under plurality rule has merit. Here it comes from the fact that voting equilibrium has bite only in two candidate competitions, making it easy to construct entry deterring voting strategies consistent with rational behavior. This falls short of the even stronger claims of Palfrey (1989) and Cox (1997) that Duverger's hypothesis in candidate sets with any numbers of candidates because (generically) no two candidates can be equally popular.

In terms of policy outcomes, these two candidate equilibria differ from the Downsian model by allowing for the possibility of policy divergence between one or more actors in political competition. Thus the model gets away from the notion that median voters will dominate policy outcomes. This squares with the extensive empirical literature that has found little support for the predictions of the median voter model. (see Romer and Rosenthal (1979)). The model of Besley and Coate (1997a), unlike its sister incarnation (Osborne and Slivinski (1996)), puts no a priori limits on the kind of policy divergence that can be entertained. There is nothing to say that political competition needs have any kind of centrist tendency. This conclusion is avoided by Osborne and Slivinski (1986) since they assume sincere voting, guaranteeing that moderate third candidates will beat extreme candidates.

Two candidate equilibria can remain in the presence of rents to holding office. Suppose, for example, that preferences are of the form given in (2.7). It is still possible to have two candidate equilibria which do not exhibit full rent dissipation. This is because disagreement over other policies is sufficient to deter entry. Citizens might be motivated by earning the rent from holding office. However, if they enter and attract votes from one of the existing candidates, they may just end up inducing their least favored candidate to win. Hence, there are voting equilibria where entry is deterred. The result in Feddersen, Sened and Wright and in Proposition 2 above does, therefore, seem special to assuming that all candidates have the same policy preference. In that case entrants will reduce ex ante rents to zero. This logic does not carry over to equilibria where candidates have diverse policy preferences.

2.8. Assessment

We have provided general framework that nests two different models of representative democracy. The strengths and weaknesses of these two models should now be apparent. Downs model gave a reason to believe that the median voter would reign supreme in plurality rule electoral systems. As such it apparently made the task of putting together economics and politics rather straightforward. However, its Achilles heel lies in the fact that it can apply only in environments where we have a Condorcet winner in the policy space. This rules out all but a handful of interesting policy problems.

The citizen-candidate approach works for arbitrary policy spaces and has pure strategy equilibria in many of them. By forcing the candidates to choose their optimal policy once elected it rules out the kind of futile campaign politics that result from the non-existence of Condorcet winners. However, the model is not entirely satisfactory as the charibdis of multiplicity displaces the scylla of non-existence.

Even when there is a Condorcet winner, the citizen-candidate model does not necessarily pick it out. We are skeptical that refinements will help to reduce the set of equilibria very much. Nor is it clear that this would always be desirable. Consider, for example, the two candidate equilibria where coordination failure among voters ruling our entry. It is not clear that one would want to rule these out without properly specifiying institutional solutions to the coordination failure problem. It seems unlikely, therefore, that we could find a convincing selection criterion. It is possible that history and institutions are the key to gaining a better grasp. However, this escape seems banal. Extensions of the model to incorporate more features of political equilibrium may however show which kinds of equilibrium phenomenon are more reasonable in different settings. Before leaving this issue, it is only fair to draw the analogy with models of market behavior. There too, it is well appreciated that models that have any kind of generality rarely yield unique predictions.

3. Normative Analysis of Political Equilibria

This section discusses what normatively desirable properties (if any) equilibria of our representative democracy model possess. Prior to this, it is important to consider what normative criteria might be appropriate in this context. There are three broad criteria that can be used and each has some role in the existing literature. The outcome from policy chosen in a representative democracy consists of a policy vector x and a citizen to implement it j. We refer to this as a selection $(x, j) \in \mathcal{A} \cup \mathcal{N}$. Viewed ex ante, the outcome is a probability distribution of selections with randomness due to either mixed strategies or ties among candidates.

Our first criterion of interest is *Pareto efficiency.* A selection $(x, j) \in \mathcal{A} \cup \mathcal{N}$ is Pareto efficient if and only if there is no other policy alternative $(x', j') \in \mathcal{A} \cup \mathcal{N}$ such that $V^i (x'.j') > V^i (x, j)$ for all $i = 1, ..., N$. There is an immense amount of work in public economics that has looked at the implications of Pareto efficiency. The Samuelson rule for the provision of a public good and its variants are of this form. Much of the optimal tax literature can be interpreted in this way. Stiglitz (1982) and Harris (1980) explicitly take this approach.

If x includes a set of lump-sum taxes and transfers then we will obtain first best frontier. However, more commonly, the set of policies is restricted to "distortionary" policy instruments. This is the case of the second best Pareto frontier. Thus in the case of taxes, the benchmark for efficiency is a tax system which satisfies the dictates of the Ramsey rule.

In the income the set of Pareto efficient taxes can be computed as follows. Let (t_j, T_j) be individual j's individually preferred tax rate, i.e. that solves:

$$\begin{gathered} Max\ V^j (t, T) \\ \text{subject to} \\ (1-t) \sum_{i=1}^{N} a_i \ell (a_i (1-t), T) = NT \end{gathered}$$

Then, consider the highest and the lowest tax vectors that are generated this way (the lowest will typically be zero). An income tax system is Pareto efficient provided that it lies between these maximal and minimal tax systems.

It is well known that, particularly when redistributive instruments are limited, then Pareto efficiency may be a very weak criterion with many different policy outcomes qualifying. There is an extensive literature showing that public provision of private goods such as health and education, and minimum wages all become Pareto efficient policies under some conditions in a second-best world.

To get a more determinate criterion for policy, it is therefore often suggested that some distributional criterion is needed. This can be embodied in a social welfare function.. Thus consider a function $w((x, j)) = W\left(V^1 (x, j), ..., V^N (x, j)\right)$ which maps from policy into social welfare as a function of individuals underlying utilities from policy. This compares gains and losses from certain policies and weighs them up. A *welfare maximizing* policy for social welfare function $W(\cdot)$ then solves

$$\begin{gathered}\max_{(x,j)} w((x,j)) \\ \text{subject to} \\ (x,j) \in \mathcal{A} \cup \mathcal{N}.\end{gathered}$$

A vast literature in public economics, pulled together for the first time in Atkinson and Stiglitz (1980) characterizes policy that solves problems of this form.

Many economists use a third criterion which at first sight does not appear to correspond to either of these two approaches. Suppose that, for all $j \in \mathcal{N}$, $V^i(x,j) = y_i + u^i(x)$ where y_i is a private consumption good so that utility is transferable in this good and $u^i(x)$ is surplus (measured in units of y) from policy vector x. Typically, we can think that $u^i(x) = b^i(x) - c^i(x)$ where $b^i(x)$ is a benefit function and $c^i(x)$ a cost function from policy vector x.[12] Then a policy is said to be *surplus maximizing* if and only if it solves

$$\begin{gathered}\max_x \sum_{i=1}^N u^i(x) \\ \text{subject to} \\ x \in \mathcal{A}.\end{gathered}$$

Although widely used, surplus maximization rarely has a satisfactory interpretation as either an efficiency or distributional criterion. Surplus maximization makes most sense when there are lump-sum transfers in the set of policy instruments. In that case, social surplus policies that make social surplus larger can be separated from those that make it smaller. It then makes sense to maximize social surplus, which is essentially just an efficiency criterion. The problem with this interpretation is assuming that there are lump-sum transfers. There is a large number of well understood arguments for not doing so. In full recognition of this, most analyses that use the social surplus criterion do not assume lump sum transfers are available. Without such transfers, policies that increase social surplus will typically do so by generating gainers and lowers. In that case, to justify wishing to maximize social surplus one would have to fall back on assuming that one is socially indifferent to the distribution of income. However, assuming such a welfare function is rather arbitrary and would not appear to have any particularly attractive ethical basis.

[12] For example, in the standard case of a pure public good g financed by a uniform head tax, the benefit is tyically $B(\theta_i, g)$ with θ_i being a preference parameter and the cost is g/N.

Writings in political economy have frequently used the term political failure to describe situations in which governments misallocate resources. The main purpose is then to suggest that this kind of failure should be weighed against market failure in order to establish whether there is a case for government intervention. Here, we will be interested in situations in which the outcome in political equilibrium does not satisfy one or more the normative criteria laid out above.

Besley and Coate (1997b) have argued that adopting a policy choice that is not Pareto efficient in political equilibrium provides a sensible definition of *political failure*. This notion parallels the widely used idea of market failure — a market is said to have failed when there is a feasible reallocation of resources that makes everyone better off. Below, we will discuss what is understood about political failure in the existing literature. This is a potentially important departure between political economy analysis and traditional normative analysis since a benevolent planner would always choose policy from the economy's Pareto frontier.

We would argue that second best Pareto efficiency of policy choices is the most natural and appropriate efficiency benchmark for assessing the performance of particular institutions for making policy choices. However, much of the literature has used other criteria. Perhaps the most widely used criterion is maximization of social surplus. We will discuss the implications of this below in some specific models. More generally, however, one can consider evaluating policy outcomes according to objectives that permit trade-offs between gainers and losers. One possibility is to consider whether a particular policy selected by a political process maximizes some social welfare function. One could then critique political processes for failing to deliver maximal social welfare function. Note, however, that provided that the policy choice is efficient in our sense, then this amounts purely to a criticism of the way in which the political process favors certain interests against any others.

Buchanan has set up a very different normative standard in his work. He compares the outcome attained from a political process to a policy vector x_0 which is the outcome that would prevail with not government intervention at all. Then he defines political failure as a situation in which the political process selects a policy outcome which does not Pareto dominate x_0. This criterion was first suggested by Wicksell and corresponds to a situation where there would be unanimous consent to the proposed policy over x_0 — so-called unanimity rule. One policy that would never be acceptable on this criterion would be a purely redistributive change (at least any change of that sort which could not be sustained by purely altruistic sentiments or the party who has been made worse off).

Buchanan's and Besley and Coate's definitions are different — neither one implies the other. The difference is illustrated in Figure 1. Let A denote the no government situation. Since we allow the government can engage in certain wealth creating activities we suppose that the Pareto frontier available with government intervention lies outside that available with no government. Consider point B which is on a higher Pareto frontier and hence is efficient according to the definition given above. However, this point is not a Pareto improvement over point A. Hence, Buchanan would describe a political process that resulted in a point like B being selected as a political failure. However, according to Besley and Coate's definition it is not. Now consider point C. According to Buchanan this is not a political failure as it a Pareto improvement relative to A. However, Besley and Coate would describe this as a political failure since it is not on the available Pareto frontier.

3.1. Efficiency

We begin by considering efficiency in the Downsian model. So suppose that $V^i(x, j)$ does not depend upon j for all voters, and that a Condorcet winner exists. We then have the following (obvious) result:

Proposition 6. *Let x_c be a strict Condorcet winner in $\mathcal{A}$, then it is Pareto efficient in $\mathcal{A}$.*

The argument is straightforward. Suppose that there is a policy choice which is better for everyone, then clearly it must be better for a majority. Hence, the set of Condorcet winner's must be a sub-set of the set of Pareto efficient policy choices. Any model, Downsian or otherwise, that selects a Condorcet winner must result in an efficient policy choice. Hence, the equilibrium cannot exhibit political failure. This point appears to be poorly understood. Consider, for example, the argument that political equilibrium is inefficient because the median voter's choice of public goods does not satisfy the Samuelson rule. This misunderstands the distinction between first and second best efficiency. For a Condorcet winner to exist, the median voter must be restricted to a simple tax policy (such as a head tax).[13] Thus first best efficiency is unattainable *by assumption*. In contrast, the Samuelson rule is relevant in cases where lump-sum taxation is available. The divergence between these two rules for public good provision is unrelated to the fact that one is generated by a positive model and the other by a normative model.

[13] This is necessary to ensure that a Condorcet winner exists.

The median voter's preferred public goods level is efficient in the set of outcomes achievable when there is a restriction on tax instruments.

This result on policy efficiency carries over to all policy outcomes generated by the citizen-candidate model provided that the policy maker is happy to be in office, i.e., $V^i(x,i) \geq V^i(x,k)$ for all $k \in \mathcal{N}$. Since the policy choice must be optimal for some citizen, it is certainly not possible to make the winning citizen better off and hence to generate a Pareto improvement. As stressed in Besley and Coate (1997a), this is a rather trivial observation. Its main interest is two-fold. First, that there are a number of extensions of the simple model where the fact that an individual who is elected maximizes does not imply that the selection is Pareto efficient. These will be discussed in detail in the sequel. Second, the literature on government efficiency does not appear to have recognized that this routine application of a standard economic concept yields a clear-cut conclusion in simple environments. As observed in our discussion of efficiency in the Downsian model, the literature is replete with discussions of whether policy making is efficient where an application of this result would reveal the result to be trivial.

While we believe that the result is best thought of as being trivial, it is surprising that it also appears to be controversial. There is sometimes a concern that the utility of the policy maker is allowed to count in the assessment of efficiency. If it did not, then it is clear that there is no presumption that this simple form of efficiency would hold. However, this seems to us to be a peculiar judgment and, essentially, a distributional judgment rather than an assessment of efficiency.

We now consider two extensions of our model in which representative democracy can fail to be Pareto efficient. Both work by making the identity of the policy maker important. The first is a case where there are differing levels of policy making competence and the second is a model with multiple equilibria where different policy makers can induce different investment equilibria when in office.

Example: (Differing Competence) The analysis of policy competence follows Besley and Coate (1997a). They generalize the model presented so far, to allow for citizens to differ in their choice sets. Thus they suppose that the feasible policies when citizen i is chosen to make policy is $\mathcal{A}^i$. They give an example in terms of differing competence to produce public goods. Here, we will consider an example to illustrate the kind of inefficiency. Suppose that there is a single policy at stake so that x is a scalar (and $\mathcal{A} = [0, X]$), and that preferences over policy can be written as

$$V^i(x,j) = v(x,\theta_i) - \beta_j$$

where θ_i is a policy parameter and β_j is a parameter which represents whether j is competent or not. We suppose that $\beta_j = 0$ is the policy maker is competent and $\beta_j = \beta > 0$ if the policy maker is incompetent. Thus, incompetent policy makers induce losses in utility for all citizens and every citizen would prefer a policy selection that involved a competent policy maker. We suppose that $\theta_i \in \{\lambda_1, ..., \lambda_H\}$ with $H < N$, and that for every preference type there is at least one competent citizen. This implies that any political equilibrium where an incompetent citizen makes policy can be Pareto dominated.

We now construct a two-candidate political equilibrium where incompetent candidates stand and are elected. To this end, consider a case where two incompetent candidates are standing against each other and differ sufficiently in their policy preferences so that each attracts half the electorate. Clearly the whole polity would prefer to displace such candidates with competent ones. However, our entry mechanism does not guarantee that this will happen, even as the cost of entry goes to zero. The line of reasoning should now be familiar and is the problem of coordination in voting equilibria. There is nothing to guarantee that the three candidate race between a competent and incompetent candidate of the same type against a third candidate of a different type will result in all voters coordinating on the competent candidate. If only a small number fail to do so, then the third candidate will win.

Example: (Coordination Failure in Private Sector Decisions) We now turn to another possible sort of inefficiency in a model. This is the possibility of coordination failure in private sector decisions. Models of this kind have been much analyzed following the influential work of Murphy, Shleifer and Vishny (1988). Here, consider what happens when individuals who are elected can make policy choices that solve the problem. For this, we need to model an underlying game played between private sector individuals as a function of the policy decisions. Consider a world in which there is a single government decision and each private sector agent makes a discrete decision $y_i \in \{0, 1\}$. The payoff of citizen i is $f^i\left(\sum_{i=1}^{N} y_i, x\right)$. Thus there is an externality in private sector decisions. We assume that $f^i(\cdot, x)$ is increasing and convex. This makes private sector actions strategic complements and hence it is possible for there to be multiple private sector equilibria. It also implies that equilibria in which more citizens choose $y = 1$ are better for every citizen.

We assume the following move order for the policy selection stage of the model. First, the elected citizen makes a policy choice $x \in \mathcal{A}$, and then all citizens make their private sector decision. Then associated with a particular policy choice and citizen j who is elected will be a number of equilibrium investors denoted $N^*(x,j)$. With the possibility of multiple investment equilibria, there is no guarantee that $N^*(x,j)$ is unique. Viewed another way, this allows us to index the policy choice on the identity of the policy maker independently of their policy choice. Thus, it is theoretically possible for a citizen to be preferred to another simply because they are associated with a better investment equilibrium. We assume that each citizen correctly anticipates the investment equilibrium associated with citizen j. Now we have that for this model

$$V^i(x,j) = f^i(N^*(x,j),x)$$

and $x_j^* = \arg\max\{f^i(N^*(x,j),x) : x \in \mathcal{A}\}$ as usual. Pareto dominance of particular policy makers is now possible because of the differences in the investment equilibria. In particular, it is quite possible to have two citizens j and k such that $f^i\left(N^*\left(x_j^*,j\right),x_j^*\right) > f^i(N^*(x_k^*,k),x_k^*)$. The most obvious example would be two citizens with the same policy preferences one of whom is associated with a superior investment equilibrium. We now ask whether in political equilibrium, we would expect only efficient citizens to be victorious. The results and reasoning closely follow the competence example that we studied above. In a one candidate equilibrium, there is a guarantee that only efficient citizens can win. However, in cases where there are two candidates with polarized preferences, it is quite possible to have candidates who are polarized who do not solve the coordination failure problem.

The analysis of this section has showed that as far as pure policy choices go, Pareto efficiency is guaranteed. However, in models where voters care about the identity of the policy maker due to differences in competence or in their ability to coordinate the economy can result in political equilibria that are not Pareto efficient. Thus, while the concept of efficiency that we employed may appear trivial on the surface, there are cases where is does not hold.

3.2. Distributional Issues

The outcome under representative democracy as modeled here reflects the winning candidate's policy preferences. As we have emphasized throughout our discussions, there is no particular reason to define these narrowly — they could very well embody a good deal of concern for policy beyond self-interested concerns. Nonetheless, there is no particular reason to think that representative democracy as modeled by our approach will lead to particularly equitable policy outcomes.

We begin with the case where a Condorcet winner exists. In typical applications, this is the policy preference of the median citizen in a one-dimensional space. Thus only if the outcome that maximizes their preference is deemed normatively desirable will any kind of broader social objective be furthered. This is likely to require restrictive assumptions about the underlying environment. Consider, for example, whether the Condorcet winner is also a surplus maximizing policy. This would mean that choosing a policy that maximized $u^j(x)$ subject to $x \in \mathcal{A}$, where j is the median individual in some suitable dimension will yield the same policy that maximizes $\sum_{j=1}^{N} w^j(x)$ subject to $x \in \mathcal{A}$. Bergstrom (1979) explored this question for the case where x is a single public good and demonstrated that the conditions on preferences and the distribution in that case are extremely restrictive. There is no reason to believe that the findings in that example are unrepresentative.

There is one case where representative democracy yields a surplus maximizing outcome. This is where the policy maker has a full set of lump-sum taxes available to him as part of x, and has an objective function that is a increasing in citizens' utilities. (This includes the trivial case where the policy maker is purely selfish.) If we suppose, in addition, that there is some lower bound on utilities which the lump-sum taxes must repect, then the policy maker acts like a discriminating monopolist for those individuals whose utility does not count in his preferences. The outcome is then surplus maximizing.[14] In this world, the policy outcome will be invariant to who is chosen as policy maker and elections serve purely to determine which set of individuals get more than their reservation utility. This result is not surprising. In normative models, one can only justify the surplus maximizing case as being interesting when the policy maker has a full set of lump-sum transfers. This is also the case where it is interesting in positive models. Whenever lump-sum taxes are limited, then it is unlikely that political equilibrium will be surplus maximizing. However, it is precisely in such cases that it is uninteresting as a normative criterion too!

Results with a median voter flavor do suggest that some policies will be "centrist". However, this does depend on the kinds of policies that one has in mind. For policies of "global" interest such as national income tax rates or defence, this seems reasonable. However, issues of more specialized interest "local public goods", there is a general concern that minorities will not be favored. For example in a one-dimensional environment where x is a discrete public good which is

[14] This result breaks down if the policy maker does not respect other citizen's preferences over x and/or there is a lower bound on individual's incomes rather than utilities.

valued by some subset of the population, we would predict that the public good would go unprovided in the simple framework discussed here. To treat these issues properly would, however, require us to go beyond the simple model discussed so far. Many policy decisions are made in centralized legislatures consisting of individuals from a number of geographical jurisdictions. They then choose to make policy according to legislative decision making rules. One strand of this literature has suggested that social norms of universalism actually lead to excessive provision of local public goods (see Johnson, Shepsle and Weingast (1988)).[15]

The centrist tendency on global public goods does, however, require some revision in light of the citizen-candidate approach with its divergent two-candidate equilibria. This is best seen in a one-dimensional model where there is little guarantee (without involving sincere voting) that policy making will converge to the centre (however defined). Thus, it seems possible that political competition will result in policy divergence and positions that result in some minority of the population getting their preferred view. This can appear in a more extreme guise in multi-dimensional settings as the following example illustrates.

Example: (The tyranny of the minority): Far from predicting that minorities are always disfavored, multi-dimensional models along the lines developed here can explain why it is possible to have situations in which majorities are not represented. The following example, based on Besley and Coate (1997b), illustrates this possibility in a specific economic environment where the government uses an income tax to finance public goods and an income guarantee. Here, we present a slightly more reduced form version of that. There are two policy instruments (x_A, x_B) and preferences are separable so that $V^i(x_A, x_B) = v\left(x_A, \theta_A^i\right) + v\left(x_B, \theta_B^i\right)$ where $\left(\theta_A^i, \theta_B^i\right)$ denotes two parameters that represent policy preferences in each policy dimension. Preferences are single peaked in each policy dimension. We assume that there are two possible values of $\theta_A^i \left(\in \left\{\underline{\gamma}, \overline{\gamma}\right\}\right)$, and that a majority of the population have policy preference $\underline{\gamma}$. There are $H < N$, types of preferences in the other dimension so that $\theta_B^i \in \{\lambda_1, ..., \lambda_H\}$. Let μ denote the median preference in that dimension. Each citizen can therefore be viewed as being located along one of the two lines illustrated in Figure 2.

The only candidate for a Condorcet winner in this environment is the policy outcome preferred by a citizen with preferences $\left(\underline{\gamma}, \mu\right)$. However, this may not beat all other policies in pairwise comparisons and hence would be vulnerable

[15] Coate (1997) integrates a number of approaches to legislative decision making into the citizen-candidate framework.

to entry by other citizen-candidates. There are a number of possible two and three candidate equilibria.[16] Perhaps the most striking possibility is of a two candidate equilibrium with opposing preferences in policy dimension B, but with candidates who agree about the policy outcome in dimension A and both have policy preference $\overline{\gamma}$. This can happen provided that disagreement in the A policy dimension is not too fierce. In this equilibrium, a majority of the population prefer the policy outcome in dimension A preferred by a citizen with policy preference $\underline{\gamma}$. Again the logic of coordination failure in voting is critical to this kind of result. It could not happen with sincere voting as it is clear that all citizens would switch to an entrant with policy preference $\underline{\gamma}$.

The discussion so far offers a somewhat more negative view of the likelihood of obtaining egalitarian outcomes than some of the literature. The probabilistic voting approach to representative democracy, reviewed in Couglin (1988), paints a more optimistic view of the policy outcome. These models take a Downsian view of candidate's motivations. However, they suppose that voting is sincere, with random shocks to voter's intentions. With appropriate assumptions about the structure of these shocks, a Condorcet winner can exist that beats all other policies probabilistically and will be selected in a Downsian equilibrium. The policy chosen is equivalent to maximizing a weighted sum of individual's utilities and hence corresponds to *some* social welfare function. This result is best thought of as a direct generalization of the standard finding that the median voter's utility is maximized in the non-random Downsian framework. Instead of trying to appeal only to a median voter, uncertainty about voter intentions make it optimal for vote maximizing parties to spread around their largesse to appeal to voters away from the median.[17] This does suggest the prospect of a more egalitarian policy outcome in general.[18] Another Downsian approach which suggests a more egalitarian view is Myerson (1994) who studies mixed strategy equilibria of a Downsian model where the government's policy is pure distributive (deciding how to divide a cake among the population). He shows that candidates who care solely about winning will offer an ex ante equal division of resources to all citizens.

[16] There may also be equilibria with more than three candidates.

[17] In an interesting application of these ideas Dixit and Londregan (1995) discusses incentives for parties who care only about winning to cultivate minorities with transfers when there is also a fixed dimension of difference across the candidates.

[18] The achievements of this view have, however, been overplayed. First, the determinants of the distributional outcome become essentially the random shocks to voter preferences. This is rather unsatusfactory foundation for a theory of distribution. The existence issue is also open in these models.

Candidate motivation is also an important aspect of distribution in a representative democracy. Downsian candidates can be thought of as cynical policy makers. However, they are not self-interested when it comes to policy making. Yet, there is a significant literature that has worried about whether candidates who will maximize their own well-being once in office will be selected in political equilibrium. The political agency literature beginning with Barro (1970) and Ferejohn (1973) rests on this premise. We now discuss how the model of representative democracy developed so far can cast light on this. Perhaps the best known tradition in political economy that has focused on self-interested behavior, is Brennan and Buchanan (1980)'s Leviathan model. They postulate that incumbents will maximize the amount that they can extract from citizens when in office. We now consider whether political competition can sort in less self-interested individuals. In a world of purely selfish individuals every citizen has an incentive to run for office. If in the limit, everyone would stand and the lucky candidate would maximize revenues.

It may be possible to avoid this outcome with only a few altruists. Altruistic candidates would find it easy to attract support from self-interested ones and hence would fair better in electoral competition. The former's incentives to enter the race may therefore be enhanced and we would expect them to monopolize political office. We now explore this logic further using a simple distribution game.

Consider a model where the task of the incumbent is to choose how to distribute a stock of wealth, W, which can be divided in any way desired. Thus $\mathcal{A} = \left\{ x \in \Re_{+}^{N} \mid \sum_{i \in \mathcal{N}} x_i = W \right\}$. There are two types of citizens. *Selfish* citizens have preferences $V^i(x, 1) = V^i(x, 0) = x_i$, while *altruistic* citizens have preferences: $V^i(x, 1) = V^i(x, 0) = \frac{1}{N} \sum_{j \in \mathcal{N}} u(x_j)$ where $u(\cdot)$ is increasing and strictly concave. The latter care about something akin to social welfare in conventional models and, if elected, divide the wealth equally. By contrast, selfish individuals consume everything themselves. We assume that if nobody runs, then the wealth is lost.[19] These assumptions yield the following result.

Proposition 7. *Suppose that there are at least two altruists in the polity and that* $u(\frac{W}{N}) - \left[\frac{N-1}{N} u(0) + \frac{1}{N} u(W)\right] > \delta$. *Then, the only pure strategy equilibria involve a single altruist running uncontested.*

[19] Thus the wealth is best interpreted as something that is created and then distributed by government.

Thus for small enough costs, the only pure strategy equilibria involve government by an altruist. Only a few altruists are needed for representative democracy to avoid Leviathan. While our example allowed representative democracy to produce a completely equitable outcome, this is not a general conclusion. It casts doubt on the reasonableness of the pure Leviathan model, rather than suggesting a rosy picture where equity always prevails. Factionalism where leaders favor certain sub-groups in society seems perfectly possible in our model, and the electoral success of fascism in the twentieth century, makes it hard to be sanguine that democracy can avoid the tyranny of ideologies that advocate extreme forms of repression against certain populations. Understanding when such extremism can arise in our model is an important issue for investigation.

4. Dynamic Analysis

In this section, we discuss the issues involved in extending the above discussion to include dynamic models of the policy process. We study only two periods since this suffices to make the main points of interest. We consider two ways of making the analysis dynamic — either by incorporating government or private sector capital accumulation. We suggest a notion of equilibrium for representative democracy that is consistent with forward looking behavior by voters and policy makers and which works in these environments.

4.1. An approach to politico-economic equilibrium

We extend the model developed to far to allow for capital accumulation by government and citizens. Private sector accumulation decisions are represented in a vector of state variables, one for each citizen: $s = \{s_1, ..., s_N\}$, where $s_i \in \mathcal{S}_i$, a compact set with $\mathcal{S} = \cup_{i=1}^{N} \mathcal{S}_i$. The government may also take a "dynamic" decisions which we denote by $g \in \mathcal{G}$, also a compact set. The s_i state variables can be thought of as human or physical capital and the g variable is either a public investment or a decision to finance current spending by issuing debt. In our two period set-up, the values of the state variables are determined once-and-for-all in period one. Let $x_t \in \mathcal{A}_t$ to denote the "static" policies, i.e., those that only affect only contemporaneous payoffs. Preferences of citizen i are assumed intertemporally additive:

$$V_1^i (x_1, s_i, g) + V_2^i (x_2, s_i, g) .$$

We also need a notation to describe feasible policies. For period one policies, this is denoted $(x_1, g) \in A^1(s)$ and for period two $x_2 \in A^2(s, g)$. Note that period one private investment choices enter these constraints. This is important as we shall see below.

Example: (Income tax model extended): Let $x_j = (t_j, T_j)$ with $j \in \{1, 2\}$ and suppose that second period abilities depend upon the period one investments by the government and citizens. Thus $a_2^i = h(a_1^i, s_i, g)$. Suppose also that private investment costs c units of the consumption good. Then $V_1^i(x_1, s_i, g) = v(a_1^i(1-t_1), T_1 - cs^i)$ and $V_2^i(x_2, s_i, g) = v(h(a_1^i, s_i, g)(1-t_2), T_2)$. In this case

$$\mathcal{A}^1(s) = \left\{(t_1, T_1, g) : \sum_{i=1}^{N} a_1^i \ell^i \left(a_1^i(1-t_1), T_1 - cs_i\right) t_1 \geq NT_1 - cg_1\right\}$$

and

$$\mathcal{A}^2(s, g) = \left\{(t_2, T_2) : \sum_{i=1}^{N} g\left(a_1^i, s_i, g\right) \ell^i \left(g\left(a_1^i, s_i, g\right)(1-t_2), T_2\right) t_2 \geq NT_2\right\}.$$

We will use versions of this model in all of the examples that we develop in the next sub-section.

To describe policy in a dynamic model, we use the notion of a *policy-cum-investment sequence* (PCIS) denoted $\{x_1, g, s, \pi_2\}$ where $\pi_2(s, g) : \mathcal{A}_2(s, g) \to [0, 1]$ and $\pi_2 = \{\pi_2(s, g)\}_{(s,g) \in \mathcal{S} \cup \mathcal{G}}$ are probability distributions over period two policy. (Thus the second period policy choice will be $x_2 \in \mathcal{A}^2(s, g)$ with probability $\pi_2(x_2; s, g)$.) This comprises a vector of period one policy and investment decisions and a probability distribution over period two policy. The latter is conditioned on the particular period one investment decisions by the government and the private sector. We will show that this notion provides a useful vehicle for studying two-period political equilibria since models of political competition can be viewed as generating sequences of this kind.

We are interested in studying situations in which investment choices are decentralized, i.e. chosen privately by the citizens. In making their choices, it is crucial to consider how future policy depends on investment. Let $\mathcal{H}$ denote the set of probability distributions over x_2. Then, a *policy function* $\Pi_2 : \mathcal{S} \cup \mathcal{G} \to \mathcal{H}$ maps from investment decisions into probability distributions over period two policies and summarizes how investment decisions affect future policy. We shall see below how political equilibrium generates a particular policy function.

For a given policy function, investment decisions must form a Nash equilibrium. Given a pure strategy profile $s \in \mathcal{S}$, individual i's payoff is

$$W^i(s, x_1, g, \Pi_2(s,g)) = V_1^i(x_1, s_i, g) + \sum_{x_2 \in \Delta(\Pi_2(s,g))} \Pi_2(s,g) V_2^i(x_2, s_i, g)\ )$$

where $\Delta(\cdot)$ denotes the support of the probability distribution. A pure strategy *equilibrium of the investment game* given $((t_1, T_1), \Pi_2)$ is a strategy profile s such that for each citizen i, s_i is a best response to s_{-i} given (x_1, g, Π_2). Let $\sigma(x_1, g, \Pi_2)$ denote a Nash equilibrium strategy profile and let $\Omega(x_1, g, \Pi_2)$ denote the set of investment profiles that can be generated as Nash equilibria given policies (x_1, g, Π_2). We assume that the latter is non-empty for all (x_1, g, Π_2) that we consider.

We now sketch how a PCIS can be generated in our model of representative democracy. Basically, we require that our underlying four stage game is played in each period conditional on the state variables in that period.[20] We begin at the policy making stage At the policy making stage in period two, the candidate's preferred policy is $\widehat{x}_{2i}(s,g) \in \arg\max\{V_2^i(x_2, s_i, g) : x_2 \in \mathcal{A}^2(s,g)\}$. Voters then vote over candidates in period two given that the policy outcome will be $x_{2i}^* = h(\widehat{x}_{2i}(s,g), X_{2i})$ where X_{2i} is the campaign promise of citizen i if he is a candidate. This gives rise to a utility imputation associated with each citizen being elected denoted $v_{ik}^2 = V_2^i(x_{2k}^*, s_i, g)$. We then solve for a voting equilibrium associated with any period two candidate set, $\mathcal{C}_2$, and a set of campaign "promises" $\{X_{2i}\}_{i \in \mathcal{C}_2}$. We then require that such promises form Nash equilibrium. Voting and campaign equilibria are both functions of the state variables (s,g) in addition to being functions of $\mathcal{C}_2$. Finally, we look for an equilibrium of the entry game among the eligible population. This is denoted $\gamma_2(s,g)\ \ (= (\gamma_{21}(s,g), \gamma_{22}(s,g), ..., \gamma_{2N}(s,g)))$.

An *equilibrium of the period two election* given the first period investment decisions (s,g) is a vector of entry decisions γ_2, a function describing campaign behavior $X_2(\cdot)$ and a function describing citizens' anticipated voting behavior $\alpha_2(\cdot)$ such that (i) for each citizen j, γ_{2j} is a best response to γ_{2-j} given (s,g) and $\alpha_2(\cdot)$, and (ii) for all non-empty candidate sets $\mathcal{C} \subset \mathcal{N}$, the anticipated campaigning behavior forms a Nash equilibrium given (s,g) and (iii) for all non-empty candidate sets $\mathcal{C} \subset \mathcal{N}$, the anticipated voting behavior is a voting equilibrium given (s,g).

[20] This will necessarily be incomplete. The interested reader can find details sketched in Besley and Coate (1998) and Besley and Coate (1997c).

Associated with any equilibrium of the period two election $(\gamma_2, X_2(\cdot), \alpha_2(\cdot))$, are two probability distributions. The first is the distribution over the citizen who chooses period two policy. This is denoted by $r_2(\gamma_2, X_2(\cdot), \alpha_2(\cdot))$, with $r_2(\gamma_2, X_2(\cdot), \alpha_2(\cdot))(i)$ being the probability that citizen i will be the policy maker and $r_2(\gamma_2, X_2(\cdot), \alpha_2(\cdot))(0)$ the probability that nobody runs. The second is the probability distribution over second period policies, denoted by $\pi_2(\gamma_2, \alpha_2(\cdot), X_2(\cdot); s, g)$,with $\pi_2(\gamma_2, X_2(\cdot), \alpha_2(\cdot), e)(x_2)$ denoting the probability that the second period policy outcome is (x_2). These two probability distributions are related by:

$$\pi_2(\gamma_2, X_2(\cdot), \alpha_2(\cdot); s, g)(x_2) = \sum_{j \in \{i \in \mathcal{N} \cup \{0\} : x_{2i}(s,g) = x_2\}} r_2(\gamma_2, X_2(\cdot), \alpha_2(\cdot))(j). \tag{4.1}$$

This completes the analysis of the period two political equilibrium.

We now turn to period one policy choices. In making first period decisions, citizens must anticipate what will happen in period 2. This will depend upon the period one investment profile. Suppose that they anticipate that the equilibrium of the second period entry, voting and campaigning games. Then the period two political equilibrium generates the policy function $\pi_2(e, g) \equiv \Pi_2(s, g)$. The period one incumbent citizen must anticipate the period one investment equilibrium, $\sigma(x_1, g, \Pi_2)$, and maximizes utility given this and the period two policy function $\Pi_2 \left(= \{\Pi_2(s, g)\}_{(s,g) \in \mathcal{S} \cup \mathcal{G}}\right)$. Citizen k's optimal choice $(t_{1i}(\Pi_2), T_{1i}(\Pi_2))$ belongs to the set

$$\begin{gathered} \underset{(x_1, g)}{\arg\max}\ W^k(\sigma(x_1, g, \Pi_2), x_1, g, \Pi_2(\sigma(x_1, g, \Pi_2), g)) \\ \text{subject to} \\ (x_1, g) \in A_1(\sigma(x_1, g, \Pi_2)) \\ \text{and} \\ \pi_2(s, g) \equiv \Pi_2(s, g) \ \text{ for all } \ (s, g) \in \mathcal{S} \cup \mathcal{G}. \end{gathered} \tag{4.2}$$

We assume that the solution to (4.2) is well-defined and unique. Let $(\widehat{x}_{k1}, \widehat{g}_k)$ denote the policy that solves this.

In many respects, equation (4.2) is the most important in seeing the difference between static and dynamic models of political competition and has been the focus of most attention in existing analyses to which we refer below. There are two key observations. First, period one policies can affect investment decisions and hence future policy choices. Thus there is a potential strategic role for policies that affect only current payoffs directly but which change private investment incentives.

Second, government policies that are explicitly dynamic can affect future political outcomes directly as well as through their effects on current investments. Below, we show that these have profound implications for thinking about efficiency of government policy choices in democratic settings.

To close the model, it is now necessary to specify then voting, campaigning and entry games that select a particular citizen in period one. There will be no need to do so explicitly here. A politico-economic equilibrium of the model is then a political equilibrium in each period and a set of private investment decisions in period one.

We now consider how the Downsian model can be extended to this environment. The main restriction of applying this approach is its reliance on the existence of a Condorcet winner. The issues for period two are exactly the same as those that arise in the static model. However, further issues arise in the extension to a dynamic environment.

To illustrate this, we consider the case where the only source of capital accumulation is by the government. Hence, the only kind of intertemporal linkage in the model is that created by the fact that the government investment decision has dynamic consequences. The period two payoff in this world will be $V_2^i(x_2, g)$ for citizen i and the feasible policies are described by $x \in \mathcal{A}^2(g)$. Treating g as parametric (which is legitimate in period two), then the condition for a Condorcet winner in this problem is standard and presents no particular complications over a one period model. Let us suppose that a period two Condorcet winner exists and is denoted $x_2^c(g)$. If this outcome is predicted by the model of political competition, then the period two probability distribution over policy is degenerate and attaches all its probability mass to the Condorcet winner. We can therefore define the two period payoff over period one policy as

$$W^i(x_1, g) = V_1^i(x_1, g) + V_2^i(x_2^c(g), g).$$

If we now seek a Condorcet winner to solve the period one political equilibrium, it is clear that deriving conditions will need assumptions about how x_2^c depends upon g and how period two preferences depend upon g. This is much more demanding than in the standard model and requiring that $V_1^i(x_1, \cdot)$ is single peaked will not typically suffice. Hence, it is rather unlikely that a Condorcet winner will exist in this kind of dynamic model even if within period preferences are single peaked.

A common response to the problem of non-existence of a Condorcet winner has been to work with models where the strategic component of the problem (represented by the in this example by the dependence of x_c^2 on g) is not internalized by voters. In effect, this says that voters vote myopically. This is the approach taken, for example, in Alesina and Rodrik (1991).

By contrast, the citizen candidate model faces not particular equilibrium existence problems in the dynamic extension. Provided that the period policy choice decision in (4.2) is well-defined, a political equilibrium exists. That does not mean, however, that it is easy to solve for equilibria in particular applications. In the following section, we use a dynamic citizen candidate model to illustrate some policy effects that arise in dynamic political economy models.

4.2. Policy Making in the Dynamic Model

Extensions to a dynamic environment add three main features to the study of policy making. The first of these is the failure of the political system to commit to future compensation for losers distorts intertemporal capital accumulation incentives. The second of these is strategic policy making whereby an incumbent uses a policy instrument to influence future political outcomes. The third arises in models with private capital accumulation where accumulation decisions alter future political equilibria. In the next three sub-sections, we develop examples to illustrate these possibilities and argue that each can lead to a deviation away from efficiency.

4.2.1. Failure to Commit to Future Compensation

It is in the very nature of political equilibrium that policy makers cannot commit future policies. Thus there is a link between the political economy literature and the literature on planners who are assumed unable to commit to future tax policy and are tempted to exploit the fact that capital is supplied inelastically after investment decisions have been made (see, for example, Fischer [1980]). In that problem, however, the tastes of the planner are constant — it is just that his incentives change over time. Political economy models add the feature that the policy makers preferences can change depending on which citizens he represents.

Our first problem of commitment failure arises precisely because there is some possibility that the identity of the future policy maker be different. If the economic reform is being considered that creates gainers and losers, then commitment to compensation may be required as a condition of successful reform. Our first example, from Besley and Coate (1998) is based on this possibility.[21]

[21] In a related analysis Dixit and Londregan [1995] shows how this can lead to misallocation

Example: (Non-commitment to future compensation) We are in the world of the income tax example laid out at the beginning of this section, where the government can redistribute via an income tax and can invest so as to raise second period abilities. The population is equally divided into two ability types — low and high with N_L denoting the set of low ability individuals and N_H the set of high ability individuals. Individuals supply labor inelastically and care only about their consumption. Let a_L denote the first period ability of low ability individuals and a_H ($> a_L$) that of high individuals. The government can undertake a discrete investment at a cost of C and can raise the ability of high ability individuals. Thus $g \in \{0, 1\}$ with $a_{2i}(1) = a_H + \delta$ for $i \in N_H$, while $a_{2i}(1) = a_L$ for $i \in N_L$. We assume that $\delta/2 > C/N$. This implies that any equilibrium in which the public investment is not undertaken can be Pareto dominated by using the period two tax policy to compensate the low ability individuals. We now construct an equilibrium of the citizen candidate model in which the investment is not undertaken. The key observation is that if a type H is elected in period 2, then he will choose a zero rate of redistributive taxation, any commitment to doing otherwise is not credible. The political equilibrium in both periods has a low and a high ability individual competing for office. Each candidate is elected with probability 1/2. When $g = 0$, the probability distribution over second period policies generated by this equilibrium selects $(1, \overline{a})$ with probability $\frac{1}{2}$ and $(0, 0)$ with probability $\frac{1}{2}$. When $g = 1$, the probability distribution selects $(1, \overline{a} + \delta/2)$ with probability $\frac{1}{2}$ and $(0, 0)$ with probability $\frac{1}{2}$. In the period one election a low ability citizen is elected with probability $\frac{1}{2}$ and will set a tax rate of 100%. He will not introduce the investment if $\delta/4 < C/N$. A high ability citizen sets a zero tax rate and undertakes the investment, thereby selecting $(0, -C/N, 1)$ as the first period policy. Assuming that $\delta/4 < C/N$, therefore, a Pareto inefficient policy sequence is selected with probability $\frac{1}{2}$.

If the identity of the policy maker were known to be the same in both periods then this problem would not arise. The lack of commitment is generated by the possibility of having a policy maker with different tastes in period two. The kind of context envisaged here is quite general. The reform process in Eastern Europe where the potential losers were initially in control of policy might be a case in point. The key observation in terms of our general theme is that the policy choice in political equilibrium can be Pareto dominated. Hence, this can legitimately be viewed as a case of political failure.

in private sector capital accumulation.

4.2.2. Strategic Policy Making

The problem discussed in the last section arises even if the political process is unaffected by the reform decision. Issues of strategic policy making arise when the current incumbent can undertake some action that influences the future policy outcome. There is a now a sizeable literature that has looked at this possibility in a variety of contexts. Persson and Svensson [1990] and Tabellini and Alesina [1991] investigated whether a government might have an incentive to accumulate debt to influence future policy making. Thus a conservative incumbent might realize that debt accumulation could reduce the spending proclivities of future liberal incumbents. Aghion and Bolton [1990} and Milesi-Ferretti [1994] illustrate how policy today can be used to affect the identity of the future policy maker who is chosen to make policy. The importance of durable policies at changing future political outcomes is also studied in a more abstract context in Glazer [1990]. Besley and Coate [1998] place these examples in a unified framework along the lines developed here and relates the findings to the notion of political failure.

To illustrate this, we now develop an example. This looks at private capital accumulation when borrowing opportunities are limited along the lines of the literature reviewed Benabou (1996). This creates a link between period one redistribution and private investment decisions. This can initiate a strategic role for redistribution as current redistribution influences future policy outcomes. Depending upon the exact nature of the gainers and losers this can generate incentives for greater or less redistribution than would otherwise occur. In our example, strategic concerns lead to less redistribution than would otherwise occur. The outcome achieved in politico-economic equilibrium can be Pareto dominated by policy with greater period one redistribution.

Example: (Strategic use of redistributive policy) Suppose that the population is divided into three groups; poor, middle class and rich. Let N_R, N_M, N_P denote the number of rich, mobile, and poor individuals respectively. Suppose that the number of poor citizens equals the number of rich and mobile citizens; i.e., $N_P = N_M + N_R$. The Assume that individuals supply labor inelastically and care only about their consumption in each period. There are two ability levels — low (a_L) and high (a_H). In period one, the poor and mobile citizens have ability a_L which for convenience we normalize to zero. The middle class have an investment opportunity which enables them to earn a_H. However, this has a positive cost c and, we assume, borrowing opportunities are absent. The latter implies that the mobile can invest only if there is sufficient period one redistribution. Redistribution is via a negative income tax scheme with tax rate $t \in [0,1]$ and guarantee $T \in \Re^{+}$. Let $A = \frac{N_R a_R}{N}$ be average period one income. The problem is interesting only in the case where $A > c$, so that societal mean income exceeds the cost of investment. Then there is a tax rate in $[0,1]$ that will enable the mobile citizens to invest. We now consider whether the equilibrium will redistribute sufficiently to allow citizens with investment opportunities to invest in equilibrium.

Proposition 8. *There is a politico-economic equilibrium in which a poor and mobile citizen compete in period one. The mobile citizen sets $t = 1$ if he is elected and all mobile citizens make their investment. If $c \in \left[\frac{a_R}{4}, A\right]$ a poor citizen does not redistribute sufficiently for the mobile citizens to invest.*

This outcome is Pareto dominated by a policy where a poor individual who is elected in period one redistributes epsilon more in period one holding the period two tax policy of full redistribution fixed. The problem is that this is not consistent with political equilibrium. Once the middle class have invested, they no longer support the redistributive policy that the poor desire and will side with rich against redistribution.[22]

The example shows that myopic political economy with links between capital accumulation and redistribution can miss out on some effects. The argument that incomplete borrowing will result in greater redistribution by government is not warranted. This depends upon the exact pattern of gainers and losers. Only in a model with forward looking political behavior can the issue be properly addressed.

4.2.3. Aggregate Capital Accumulation and Political Equilibrium

The essence of strategic policy making is that current policies are used to manipulate future equilibrium outcomes. This arises because a particular citizen exerts a non-negligible influence on policy outcomes by his period one actions. We now turn to affects on policy outcomes due to aggregate capital accumulation decisions. Even though each citizen exerts a negligible influence on the policy outcome, collectively they have an effect. This intertemporal link is another important aspect of dynamic political economy models.

The effects studied in this section are germane to the growing literature that looks at policy making in growth models. Alesina and Rodrik [1994] and St. Paul

[22] In fact, as shown in the proof of Proposition 8, the rich will actually choose to redistribute in period one if they are elected as this helps to promote mobile investment provided that $\frac{3}{4}a_R > c$.

and Verdier [1997] develop models that link taxes and accumulation via voting without allowing voters to be forward looking. Krusell, Quadrini and Rios-Rull [1994] pursue a similar approach while allowing for forward looking voting behavior. We are interested in whether political economy effects lead to inefficiencies. Our normative benchmark is the *Ramsey equilibrium* where a government commits to a sequence of tax rates at the beginning of time while citizens make optimal investment decisions.[23]

We explore two aspects of these issues here. We begin with a model where current investment affects the type of the citizen selected to choose policy in period two. We study a model where capital accumulation increases redistributive taxation by increasing inequality. This leads individuals with investment opportunities not to exploit them to the full. In our second model citizens are ex ante identical. There are two political equilibria — one where every citizen invests and individuals are identical ex post and another where investment is incomplete so that there is inequality and redistributive taxation ex post. In both examples a Ramsey equilibrium can be found that dominates politico-economic equilibrium.

Example: (Shifting policy makers) Suppose that there are three groups of citizens: there are N_P poor, N_M middle class and N_R rich, each comprising less than half the population. Their period one earnings abilities are respectively $a_1^P < a_1^M < a_1^R$. The investment opportunity is discrete and open only to the rich so that $a_1^P = a_2^P$, $a_1^M = a_2^M$ and $g\left(a_1^R, 1\right) > a_1^R$. We assume that $g\left(a_1^R, 1\right) - a_1^R > c$, so that investing is worthwhile for any rich person if there is no redistributive taxation. Each citizen supplies one unit of labor inelastically and cares only about their consumption in each period. We assume that

$$\frac{N_P a_1^P + N_M a_1^M + N_R a_1^R}{N} < a_1^M < \frac{N_P a_1^P + N_M a_1^M + N_R g\left(a_1^R, 1\right)}{N}. \tag{4.3}$$

This says that the ability of the middle class is higher than the mean if the rich do not invest and lower if they do. We also assume that the middle class vote is pivotal in the sense that $N_P + N_M > N_R$ and $N_R + N_M > N_P$.

For this environment, we have[24]

[23] Formally, this is a case where investment decisions are $\sigma\left(x_1, g, \Pi_2\right)$ for some constant policy function Π_2.

[24] The example uses an investment equilibrium is in (symmetric) mixed strategies. This is not for the usual reason where mixed strategies solve a problem of non-existence of an investment equilibrium. The pure strategy equilibria in this example work only by having each investor believe that they can influence future policies unliaterally. This does not seem very unnatural.

Proposition 9. *For large enough N, there is a politico-economic equilibrium where a fraction (less than one) of the rich invest. Both periods have no redistributive taxation implemented by a single rich or middle class citizen running unopposed.*

In this example the rich fear that if they invest then this will change the policy outcome to induce greater redistributive taxation. Thus investment incentives are blunted. The outcome can be Pareto dominated by a Ramsey equilibrium that commits to zero taxation in both periods as all of the rich will invest under such conditions.

Example: (Multiple Pareto Ranked Political Equilibria) We consider the income tax environment discussed above. Everyone starts out identical with a common wage rate, $a_{1i} = a_L$. Preferences within each period are quasi-linear, and quadratic, i.e. of the form $x_i - (\ell_i)^2/2$. The investment decision is discrete, i.e., $s_i \in \{0, 1\}$ and $a_{i2} = g(a_{i1}, 1) = a_H > a_L$ for all i. We assume $(a_L)^2 > c$ and $\frac{1}{2}\left[(a_H)^2 - (a_L)^2\right] > c$. The first condition says that an individual has sufficient resources to invest under normal period labor supply conditions (so that there is no need to borrow to invest) and the latter that the investment pays off if there is no taxation in period two.

The Appendix proves:

Proposition 10. *If $\frac{1}{8}\left[\frac{(a_H)^2+(a_L)^2}{(a_H)^2}\right]^2 < \frac{c}{\left((a_H)^2-(a_L)^2\right)}$, there are two politico-economic equilibria for this environment. In the first of these, everyone invests in period one and the tax rate on income is zero in both periods. In the second of these, less than half the population invests, there is no redistributive taxation in period one, but there is redistribution in period two.*

One of these equilibria implements the Ramsey optimum with zero taxation. (This is a trivial election in which no-one stands for office in both periods.) The other one has a positive level of redistributive taxation in period two with a zero level in period one. The period redistribution is driven from the fact that non-investors are in the majority. The condition in the Proposition guarantees that there is an investment equilibrium with less than half the population choosing to invest.

The Ramsey equilibrium in which everyone invests Pareto dominates that where only some individuals invest. The issue once again is commitment. Everyone would be better of in a world in which society committed not to redistribute in period two. However, anticipating period two redistribution becomes self-fulfilling.[25]

4.3. Assessment

Dynamic political economy models have now been applied in a variety of contexts. In contrast to static models, there is no good reason to expect that the outcome will be efficient even if the identity of the policy maker does not matter. Thus political failure as defined by Besley and Coate (1998) abounds in dynamic environments. Having understood this, the next step is to reconsider the design of decision making institutions. In the context of public investment this may mean trying to design institutions that separate the evaluation of public sector projects from the general process of redistribution decisions. Our analysis of the strategic use of redistribution decisions may suggest that it is better to deal with such problems by tackling the market failure directly rather using the rather less direct policy of redistribution. It is clear that we are only beginning to understand these issues and it is essential to take our understanding forward to the level of institution design if we are to capitalise further on our understanding of these issues.

[25] The logic here is similar to that developed in Grossman and Helpman (1995). They show how having policies generated by lobbying activity can lead to multiple steady state equilibria in a model of private capital accumulation. A similar argument is also developed in St Paul and Verdier (1997).

References

[1] Aghion, Philippe and Patrick Bolton, [1990], "Government Debt and the Risk of Default: A Politico-Economic Model of the Strategic Role of Debt," in Rudiger Dornbusch and Mario Draghi (eds), *Public Debt Management: Theory and History*, Cambridge University Press: Cambridge, England.

[2] Alesina, Alberto, [1988], "Credibility and Policy Convergence in a Two-Party System with Rational Voters," *American Economic Review*, 78(4), 796-806.

[3] Alesina, Alberto and Dani Rodrik, [1994], "Distributive Politics and Economic Growth", *Quarterly Journal of Economics*, 109(2), 465-90.

[4] Atkinson, Anthony B. and Joseph E. Stiglitz, [1980], *Lectures on Public Economics*, New York: McGraw Hill.

[5] Becker, Gary, [1985], "Public Policies, Pressure Groups and Dead Weight Costs," *Journal of Public Economics*, 28, 329-347.

[6] Benabou, Roland, [1996], "Growth and Distribution," *NBER Macroeconomics Annual 1996*,

[7] Bergstrom, Theodore, [1979], "When Does Majority Rule Supply Public Goods Efficiently?" *Scandinavian Journal of Economics*, 81, 216-226.

[8] Besley, Timothy and Stephen Coate, [1997a], "An Economic Model of Representative Democracy," *Quarterly Journal of Economics*, 112, 85-114.

[9] Besley, Timothy and Stephen Coate, [1997b], "Analyzing the Case for Government Intervention in a Representative Democracy," *STICERD Discussion Paper No.*

[10] Besley, Timothy and Stephen Coate, [1997c], "Private Capital Accumulation in a Representative Democracy," typescript.

[11] Besley, Timothy and Stephen Coate, [1998], "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis", *American Economic Review*, forthcoming.

[12] Brennan, Geoffrey, and James M. Buchanan, [1980], *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, Cambridge: Cambridge University Press.

[13] Buchanan, James M., [1967], *Public Finance in Democratic Process*, Chapel Hill: University of North Carolina Press.

[14] Coate, Stephen,

[15] Coughlin, Peter, [1992], *Probabilistic Voting Theory*, Cambridge: Cambridge University Press.

[16] Cox, Gary W., [1997], *Making Votes Count*, Cambridge: Cambridge University Press.

[17] Dixit, Avinash, [1996], *The Making of Economic Policy*, Cambridge: MIT Press.

[18] Dixit, Avinash, Furuk Gul and Gene Grossman, [1998], "A Theory of Policy Compromise," typescript.

[19] Dixit, Avinash, and John Londregan, [1995], "Redistributive Politics and Economic Efficiency," *American Political Science Review*, 89(4), 856-66.

[20] Downs, Anthony, [1957], *An Economic Theory of Democracy*, New York: Harper Collins.

[21] Feddersen, Timothy J., Itai Sened and Stephen G. Wright, [1990], "Rational Voting and Candidate Entry under Plurality Rule," *American Journal of Political Science*, 34, 1005-16.

[22] Fischer, Stanley, [1980], "Dynamic Inconsistency, Cooperation, and the Benevolent Dissembling Government" *Journal of Economic Dynamics and Control*, 2(1), 93-107.

[23] Gans, Joshua and Michael Smart, [1996], "Majority Voting with Single-Crossing Preferences," *Journal of Public Economics,* 59(2), 219-37.

[24] Glazer, Amihai, [1989], "Politics and the Choice of Durability," *American Economic Review*, 79, 1207-13.

[25] Grossman, Gene, and Elhanan Helpman, [1997], forthcoming *Economic Journal.*

[26] Grossman, Gene, and Elhanan Helpman, [1994], "Protection for Sale," *American Economic Review.*

[27] Hammond, Peter, [1979], "Straightforward Incentive Compatibility in Large Economies", *Review of Economic Studies*, 46, 263-282.

[28] Harris, Richard, [1980], "Pareto Efficient Commodity Taxation," *Journal of Public Economics*,

[29] Harsanyi, John, [1973], "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points," *International Journal of Game Theory*, 2, 1-23.

[30] Kramer, Gerald H., [1978], "Existence of Electoral Equilibrium," in Peter C. Ordeshook, ed, *Game Theory and Political Science*, New York: New York University Press.TI:

[31] Krusell, Per, Quadrini, Vincenzo and Jose-Victor Rios-Rull, [1994], "Politico-Economic Equilibrium and Economic Growth," University of Pennsylvania, CARESS Working Paper #94-11, forthcoming in *Journal of Economic Dynamics and Control.*

[32] Milesi-Ferretti, Gian-Maria and Enrico Spolaore, [1994], "How Cynical Can an Incumbent Be? Strategic Policy in a Model of Government Spending, " *Journal of Public Economics*, 55, 121-40.

[33] Mueller, Dennis, [1990], *Public Choice II*, Cambridge: Cambridge University Press.

[34] Myerson, Roger B., [1993], "Incentives to Cultivate Favored Minorities under Alternative Electoral Systems", *American-Political-Science-Review*, 87(4), 856-69.

[35] Myerson, Roger B. and Robert Weber, [1993], "A Theory of Voting Equilibria", *American Political Science Review,* 87(1), 102-14.

[36] Osborne, Martin J. and Al Slivinski, [1996], "A Model of Political Competition with Citizen-Candidates," *Quarterly Journal of Economics*, 111, 65-96.

[37] Palfrey, Thomas R., [1984], “Spatial Equilibrium with Entry”, *Review of Economic Studies*, 51, 139-56.

[38] Persson, Torsten and Guido Tabellini, [1994], “Representative Democracy and Capital Taxation”, *Journal of Public Economics*, 55(1), 53-70.

[39] Persson, T. and L. Svensson, [1989], “Why a Stubborn Conservative Would Run A Deficit: Policy with Time-Inconsistent Preferences,” *Quarterly Journal of Economics*, 104, 325-346.

[40] Roberts, Kevin, [1977], “Voting over the Income Tax Schedules,” *Journal of Public Economics*, 8(3), 329-40.

[41] Romer, Thomas and Howard Rosenthal, [1979], “The Elusive Median Voter,” *Journal of Public Economics*, 12, 143-70

[42] Snyder, James M., and Kramer, Gerald H., [1988], “Fairness, Self-interest, and the Politics of the Progressive Income Tax”, *Journal of Public Economics*; 36(2), 197-230.

[43] Stiglitz, Joseph E., [1974], “The Demand for Education in Public and Private Schools,” *Journal of Public Economics,* 3, 349-385.

[44] Stiglitz, Joseph E., [1982], *Journal of Public Economics*,

[45] Tabellini, Guido and Alberto Alesina, [1990], “Voting on the Budget Deficit,” *American Economic Review*, 80, 37-49.

[46] Usher, Dan, [1994], “The Significance of the Probabilistic Voting Theorem,” *Canadian Journal of Economics*, 27, 433-45.

[47] Weingast, Barry, Kenneth Shepsle, and C. Johnsen, [1981], “The Political Economy of Benefits and Costs: A Neo-classical Approach to Distributive Politics,” *Journal of Political Economy*, 89, 642-64.

[48] Wittman, Donald, [1989], “Why Democracies Produce Efficient Results”, *Journal of Political Economy*, 97, 1395-426.

5. Appendix A: Proofs

Proof of Proposition 4: The proof proceeds by establishing a series of three claims.

Claim 1. *If δ is sufficiently small, there are no pure strategy equilibria.*

Proof. We show first that there are no equilibria with just one candidate. Suppose that citizen i were running unopposed. If $i \in N_R$, then condition (ii) of Proposition 3 is not satisfied for $k \in N_M$. With a rich and middle class candidate, every sincere partition would have the middle class candidate winning and if δ is sufficiently small, $v_{kk} - v_{ki} = v_M(q_H) - v_M(0) > \delta$. If $i \in N_M$, then condition (ii) is violated for any $k \in N_P$ and if $i \in N_P$ then (ii) is violated for $k \in N_R$.

Next, we show that there are no two candidate equilibria. Proposition 4 of Besley and Coate (1997a) tells us that there must exist a sincere partition with $\#N_i = \#N_j$. This is not possible unless i and j belong to the same group. But in this case, $v_{ii} = v_{ji} = v_{ij} = v_{jj}$, and condition (i) of Proposition 4 in Besley and Coate (1997) is violated.

Finally, we demonstrate that there are no equilibria with three or more candidates. Suppose not, then there are three possibilities: (a) the set of winning candidates, $W(\mathcal{C}, \alpha(\mathcal{C}))$, contains citizens from only one group; (b) $W(\mathcal{C}, \alpha(\mathcal{C}))$ contains citizens from two groups; (c) $W(\mathcal{C}, \alpha(\mathcal{C}))$ contains citizens from all three groups. Begin with case (a). If all the winning candidates are rich, then there is no poor person in the race given that zero public provision is their least preferred outcome. But then there can be no middle class candidates either, which implies that there is a single rich candidate - a contradiction. The other cases are dealt with by similar reasoning. Now consider case (b) and suppose that the winning set is drawn from the rich and middle class. With no citizens being indifferent between rich and middle class candidates, there can be only one winning candidate of each type. However, since all the poor and the middle class must be voting for the middle class candidate, there can be no rich individual in the winning set - a contradiction. Other variants of case (b) can be dealt with in this way. Finally, there is case (c). Again, there can be only one winning candidate of each type. But then the winning candidate from each group must be receiving all the votes from individuals of his type. Since $N_M > Max\{N_R, N_P\}$ this contradicts the supposition that there is a winning candidate from each group. ∎

For δ sufficiently small, there is a unique mixed strategy equilibrium of the three person game between M, P, and R given by:

$$\gamma_P = 1, \gamma_M = \frac{v_R(0) - v_R(q_L) - \delta}{v_R(0) - v_R(q_H)} \text{ and } \gamma_R = \frac{\delta}{v_M(q_H) - v_M(0)}$$

Proof. Observe first that, when δ is small, there can be no mixed strategy equilibria in which one or more of the three players does not enter with positive probability. Now note that $(\gamma_P, \gamma_M, \gamma_R) > (0, 0, 0)$ is a mixed strategy equilibrium if and only if the following three conditions are satisfied:

$$(1 - \gamma_R) [\Delta_P(q_L, 0) - \gamma_M \Delta_P(q_H, 0)] \geq \delta, \tag{5.1}$$

with $\gamma_P = 1$ if the inequality is strict;

$$[\gamma_R + (1 - \gamma_P)(1 - \gamma_R)] \Delta_M(q_H, 0) \geq \delta \tag{5.2}$$

with $\gamma_M = 1$ if the inequality is strict;

$$\gamma_P [\Delta_R(0, q_L) - \gamma_M \Delta_R(0, q_H)] \geq \delta \tag{5.3}$$

with $\gamma_R = 1$ if the inequality is strict, where $\Delta_P(q_L, 0) = v(q_L) - v(0)$, etc. Observe that condition (5.1) implies that $\gamma_R < 1$ and, since $\Delta_R(0, q_L) < \Delta_R(0, q_H)$, then condition (5.3) implies that $\gamma_M < 1$. It follows that (5.2) must bind so that

$$(1 - \gamma_R) = \left(\frac{\Delta_M(q_H, 0) - \delta}{\gamma_P \Delta_M(q_H, 0)} \right) \tag{5.4}$$

Note that

$$\frac{\Delta_M(q_H, 0) - \delta}{\gamma_P \Delta_M(q_H, 0)} [\Delta_P(q_L, 0) - \gamma_M \Delta_P(q_H, 0)] > \frac{\Delta_M(q_H, 0) - \delta}{\Delta_M(q_H, 0)} [\Delta_P(q_L, 0) - \Delta_P(q_H, 0)] .$$

Thus (5.1) implies that if δ is sufficiently small, then $\gamma_P = 1$. From (5.3) we then find that $\gamma_M = \frac{\Delta_R(0, q_L) - \delta}{\Delta_R(0, q_H)}$. Together with (5.4) this proves the result. ∎

Claim 2. *Let $(\gamma_P, \gamma_M, \gamma_R)$ be the mixed strategy equilibrium for the three player game described in the previous Claim. Then, for δ sufficiently small, the following strategy profile constitutes an equilibrium of the entry game:*

$$\gamma^i = \gamma_M, \ \gamma^j = \gamma_P, \ \gamma^k = \gamma_R, \text{ and } \gamma^g = 0 \text{ for all } g \notin \{i, j, k\},$$

where $i \in N_M$, $j \in N_P$ and $k \in N_R$.

Proof. It suffices to check that no other citizen wishes to enter the race given that citizens' i, j and k are entering with the stated probabilities. Note first that no poor citizen, other than j, wishes to enter because a poor individual is already in the race with probability one. Entry could only reduce the likelihood that a poor individual is selected to govern by splitting the vote for the poor candidate. Consider then a rich citizen. Making use of the fact that $\gamma_P = 1$, the expected payoff from a rich citizen (other than k) entering is bounded above by:

$$(1 - \gamma_R) \left[\Delta_R(0, q_L) - \gamma_M \Delta_R(0, q_H)\right].$$

This is an upper bound, since an additional rich individual entering may cause a rich individual not to be elected by splitting the vote for the rich candidate. As shown in the proof of the previous Claim, the fact that $\gamma_R < 1$ implies that

$$[\Delta_R(0, q_L) - \gamma_M \Delta_R(0, q_H)] = \delta.$$

Thus no rich citizen, other than citizen k, wishes to enter. A similar argument rules out entry by middle class citizens other than citizen i. ∎

Proof of Proposition 6: We begin by establishing the following lemma:

Lemma 1. *Suppose that $\mathcal{C}$ contains both altruistic and selfish citizens. Then if $\alpha \in E(\mathcal{C})$, $W(\alpha, \mathcal{C})$ contains only altruistic citizens.*

Proof. If there is an altruistic candidate in the race then for any altruistic citizen or any selfish citizen not in the race voting for an altruistic candidate is the only weakly undominated strategy. For a selfish citizen in the race, voting for himself and voting for an altruist are the only weakly undominated strategies. Thus if $\alpha \in E(\mathcal{C})$, the maximum number of votes received by any selfish candidate is one. The result follows if we can show that at least one altruist must receive two or more votes. This is clear, if at least one citizen is not in the race. If all citizens are in the race, then weak dominance does not rule out the possibility that all citizens receive one vote. However, with two or more altruists this would not be an equilibrium. An altruist could achieve his preferred outcome by simply switching his vote to another altruist. ∎

With this Lemma, we can now prove the proposition. Proposition 3 and the assumption on entry costs imply that a single altruist running uncontested is a pure strategy equilibrium. Thus it suffices to eliminate the possibility of any other equilibria. Note first that there exist no pure strategy equilibria in which both altruists and selfish citizens enter. The Lemma implies that all selfish candidates would be better off not entering. There can also be no pure strategy equilibria in which only selfish candidates enter. The Lemma implies that an altruist would win if he entered. The condition on entry costs implies that an altruist would enter if he could win. By not entering, the altruist receives a payoff of $\frac{N-1}{N}u(0)+\frac{1}{N}u(W)$, while if he entered he would receive a payoff of $u(\frac{W}{N})-\delta$. Finally, it is clear that there can exist no pure strategy equilibria in which more than one altruist enters. One of the altruists would be better off not entering and saving the entry costs. ∎

Proof of Proposition 8: We now consider the politico economic equilibria of this model. Begin in period two. If the mobile citizens have invested, then there is a two candidate equilibrium where a poor citizen stands against a rich or mobile citizen. Each wins with probability 1/2, the poor citizen sets $t=1$ if they win and the other candidate sets $t=0$. If the mobile citizens have not invested, then there is an equilibrium with either a poor or mobile citizen standing, with winning candidate choosing $t=0$. Thus if $A>c$, then redistribution in period one will result in a change in the period two political equilibrium. Specifically, it will lead to less period two redistribution.

We now consider the policy choice of each type of citizen in period one. If a rich citizen is elected, it is easy to see that he prefers to redistribute just enough in period one so that mobile citizen invests if $\frac{3}{4}a_R>c$. A mobile citizen will wish to set $t=1$ in period one. This enables him to invest. A poor citizen's period one redistribution decision depends upon c. If $c<\frac{a_R}{4}$ he chooses to redistribute in period one. However, if $c\in\left[\frac{a_R}{4},A\right]$ then he will choose to redistribute to a level just below c/A where it is impossible for a mobile citizen to invest.

Now consider the period one political equilibrium. There is an equilibrium where a mobile citizen stands against a poor citizen each winning with probability one half. ■

Proof of Proposition 9: To calculate the politico-economic equilibrium for this environment, Let t_2^i be the desired tax rate of an individual of type i in period 2. It is clear that $t_2^R=0$ and $t_2^P=1$, i.e. the rich do no redistribution and while the poor do a maximal amount. The middle class's preference depend upon the amount of period one investment. Let n_R be the number of rich investors. Define n_R^* as the critical number of rich investors which would lead a middle class policy maker to select to redistribute. Formally, this is defined as the smallest value of n_R such that $a_1^M<\frac{N_Pa_1^P+N_Ma_1^M+n_Rg\left(a_1^R,1\right)}{N}$. Using this,

$$t_2^M = \begin{cases} 1 & if\ n_R \geq n_R^* \\ 0 & otherwise. \end{cases}$$

The period two political equilibrium is now as follows. If $n_R < n_R^*$, then a single rich person runs for office and is elected unopposed. If $n_R \geq n_R^*$, then a single poor person runs and is elected unopposed.

Let p denote the probability that a given rich person invests. Then distribution of the number of investors, n in a population of N, will have a binomial distribution $b(n; N, p) = \binom{N}{n} (p^n (1-p)^n)$. Let $B(n; N, p) = \sum_{x=0}^{n} b(x; N, p)$ be the probability that less than n_R^* rich invest.

A mixed strategy equilibrium must make all investors indifferent between their two pure strategies. Hence, we the equilibrium value of p must solve

$$\begin{aligned} &B(n_R^* - 1; N_R - 1, p)\, g\left(a_1^R, 1\right) + [1 - B(n_R^* - 1; N_R - 1, p)]\, \overline{a}(1) - c \\ &\quad = B(n_R^*; N_R - 1, p)\, a_1^R + [1 - B(n_R^*; N_R - 1, p)]\, \overline{a}(0) \end{aligned}$$

where

$$\overline{a}(i) = \left[\frac{N_M a_1^M + N_P a_1^P}{N} + \frac{1}{N}\left(\sum_{x=0}^{N} x b(x; N_R - 1, p) + i\right) g\left(a_1^R, 1\right)\right]$$

If an individual chooses not to invest then the probability that there will be redistributive taxation is $B(n_R^*; N_R - 1, p)$ while it is $B(n_R^* - 1; N_R - 1, p)$is if he chooses to invest. The right hand side refers to the case where the number of investors is less than n_R^* The Nash equilibrium must have a strictly positive probability of investment. However, investment is also less than complete, i.e., $0 < p < 1$. Note that the equilibrium survives in large economy as N_P, N_M and N_R all tend to infinity, holding their respective fractions in the population fixed. ■

Proof of Proposition 10: The policy in period two depends upon how many individuals invested in period one and what type of policy maker is selected, i.e. whether or not that individual invested. Let $\theta \in \Theta \equiv \left\{0, \frac{1}{N}, \frac{2}{N}, ..., 1\right\}$ denote the fraction of individuals who invest in period one. Let $\left(t_2^H(\theta), T_2^H(\theta)\right)$ denote the equilibrium period two tax vector when a fraction θ of the population has invested. No matter what fraction of the population has invested, an individual who has invested in period one will not wish to redistribute if elected. Hence,

$t_2^H(\theta) = T_2^H(\theta) = 0$, where H stands for policy being selected by an individual with ability a_H.

If an individual with ability a_L is in office in period two, then the tax rate does depend upon θ. Let $\overline{y}(t_2, \theta)$ be the mean period two earnings when a fraction θ have invested. Then, the choice of period two tax rate of a low ability type solves

$$t_2^L(\theta) = \arg\max \{v(t_2, t_2\overline{y}(t_2, \theta), a_L)\} \quad i \in \{L, H\}.$$

Using the first order condition and the functional form assumption on preferences, the equilibrium tax rate is

$$t_2^L(\theta) = \frac{A(\theta) - (a_L)^2}{\left(2A(\theta) - (a_L)^2\right)} \geq 0 \tag{5.5}$$

where $A(\theta) = \theta(a_H)^2 + (1-\theta)(a_L)^2$. A citizen who has not invested favors some redistributive taxation as long as $\theta > 0$. The tax rate chosen is increasing in θ. If citizens anticipate that a low ability type will be in power in period two, and a fraction θ of the population will invest, the expected utility gain to investing is given by

$$v\left(t_2^L(\theta), t_2^L(\theta)\overline{y}\left(t_2^L(\theta), \theta\right), a_H\right) - v\left(t_2^L(\theta), t_2^L(\theta)\overline{y}\left(t_2^L(\theta), \theta\right), a_L\right) - c. \tag{5.6}$$

Differentiating (5.6) with respect to θ in this case reveals that as the fraction of investors increases, the gain from investing diminishes.[26] This is because more investors implies that the low ability type will raise the level of redistributive taxation.

The second equilibrium is asymmetric with a fraction less than one of individuals investing. Let n denote the number of investors. If n/N is less than 1/2, then the non-investors find themselves in a majority in period two. In that case, some redistribution will take place according to (5.5) and the utility gain from investment in such states will be measured by (5.6). A particular value of n is an equilibrium if conditions analogous to (**??**) and (**??**) above hold. The following result, giving conditions for an equilibrium of this form, is proven in the Appendix.[27]

[26]It is easy to check that this is a general result, i.e. does not depend upon the assumption of the quasi-linear, quadratic preferences.

[27]For large enough N there is not pure strategy equilibrium where nobody invests. The logic is similar to that used in Lemma 1 above.

Lemma 2. *As the polity becomes large, there is an equilibrium where less than half the population invests if*

$$\frac{1}{8}\left[\frac{(a_H)^2+(a_L)^2}{(a_H)^2}\right]^2 < \frac{c}{\left((a_H)^2-(a_L)^2\right)}.$$

Proof: As N becomes large, the fraction of investors will be such that each citizen is indifferent between investing and not investing. Hence, the equilibrium fraction of investors will satisfy

$$v\left(t_2^L(\delta), T_2^L(\delta)\cdot\overline{y}\left(t_2^L(\delta),\delta\right), a_H\right) - v\left(t_2^L(\delta), T_2^L(\delta)\cdot\overline{y}\left(t_2^L(\delta),\delta\right), a_L\right) = c. \tag{5.7}$$

There is no equilibrium of this form with $\delta > 1/2$. To see this, remember that $t_2^L(\delta), T_2^L(\delta) = (0,0)$ in this case and we know that the left hand side of the above equation exceeds the right hand side in that case. We will demonstrate conditions for an equilibrium with $\delta < 1/2$. Equilibria with $\delta = 1/2$ are possible but not generic.

Condition (5.7) is equivalent to

$$\frac{1}{2}\left(\frac{\delta\Delta+(a_L)^2}{2\delta\Delta+(a_L)^2}\right)^2\Delta = c.$$

where $\Delta \equiv (a_H)^2-(a_L)^2$. From this, it is clear that the left hand side exceeds the right hand side at $\delta = 0$, and as we have already shown, the left hand side is decreasing in δ. Hence, a sufficient condition for a mixed strategy equilibrium of the desired form is that

$$\frac{1}{2}\left(\frac{\frac{\Delta}{2}+(a_L)^2}{\Delta+(a_L)^2}\right)^2\Delta < c$$

which is the condition given in the text. ■

This lemma completes the proof. ■