Job Turnover and the slope of the Phillips Curve

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Abstract

This paper relates the observed flatter Phillips Curve to the rise in labour turnover, as well as the weakening of collective bargaining. In a New Keynesian model of sticky wages, workers or unions discount future wage income with a low discount factor if there is a strong flow of job turnover. In the New Keynesian wage Phillips Curve, this implies that future inflation is discounted more heavily than without job turnover. A low coefficient for future inflation is consistent with empirical estimates such as Gali and Gertler (1999). A Phillips Curve that is less forward looking due to job turnover will appear to be flatter in the short run, because turnover creates a downward bias for the slope if it is not accounted for. In the long run, the Phillips Curve is much flatter, and is no longer vertical or near-vertical. The paper then derives the optimal monetary policy in such a setup. In particular, the price targeting result of the Ramsey policy is violated when there is turnover. In the presence of steady state distortions, the long run inflation target is not zero, and there is no full mean reversion of the price level after a cost-push disturbance.

1 Introduction

The Phillips Curve, which dates back to A.W. Phillips seminal paper (1958), is central to macroeconomics. While the main point of contention has long been the degree of inflation persistence that it implies, its shape has also been questioned recently. The aftermath of the 2008 financial crisis has shown unemployment sharply increasing and then sharply falling, while inflation

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has remained low and positive. The relationship seems to have broken down.
This would suggest that the short-run Phillips curve has become flatter, as
evidenced by Blanchard et al. (2015) or Ball and Mazumder (2014).

The idea of a vertical, or near-vertical long-run Phillips Curve, has also
been questioned. In a recent Peterson policy brief (2016), Blanchard argues
that the long-run Phillips curve has become flatter, largely due to inflation
expectations anchoring at zero or low levels. As such, there would be a real
tradeoff between output and inflation in the long run.

Some explanations such as menu costs and anchored expectations have
been put forward, but they either lack microfoundations or tractability, which
would be useful for welfare analysis. Others, relate it to globalisation: while
there is a global Phillips curve, global value chains and increased competition
make domestic Phillips curves less relevant (see Carney, 2017).

This paper, instead, relates these evolutions to job turnover. The advantage
of this microfoundation is that it is more observable and more tractable.
As we shall see in the next subsection, there has been a secular trend in job
turnover and other features of the labour market over the past decades. This
paper shows how it can explain the evolution of the Phillips curve: a flatter
long run curve which is no longer vertical or near vertical. And in the short
run, the curve will look flatter than if turnover is properly accounted for.
The optimal monetary policy, in terms of inflation target and stabilisation,
are then derived.

**Job turnover**

In the New Keynesian wage Phillips curve models, such as the one pioneered
by Erceg, Henderson and Levin (2000), workers (or unions) set staggered
wages optimally. Current (wage) inflation depends on future (wage) inflation
expectations as well as the output gap. In the log linear approximation, the
coefficient of future inflation is $\beta$, the riskless discount factor.

However, when there is a significant probability that a worker quits, or
that he will be fired and replaced by someone else, the net present value
of his job will be discounted with a lower factor than the risk-less discount
factor. The probability of being laid off should not matter for the worker,
as the union acts as an insurance mechanism. Because the union is assumed
to split the wage income between employed and unemployed members, the
employee does not lose his income when he is laid off. But turnover relates
to quits and personal dismissals, not layoffs. And it is not the purpose of
the union to insure against these, so the turnover probability is a relevant
discount factor. It makes the wage setting decision, and hence the wage
Phillips curve, less forward looking.
Figure 1 comes from the job tenure survey from the OECD, for people aged 25 – 54. The proportion of people less than a year into their job is a good indicator of yearly job turnover, though temporary contracts probably overstate the figure. In most countries, there has been an increase in the less than one year proportion of workers, which indicates a rising turnover. This can also be seen with the increase of the less than three years proportion, which is less sensible to temporary employment. Last, the proportion of people more than ten years into their job has fallen across most countries. This is highly suggestive of an increased turnover.

The increasing share of temporary contracts, and the recent rise in the “gig economy” (part time contracts, self-employed contractors, zero-hours in Britain) are also likely to weaken collective bargaining in favour of more individual bargaining, as suggested by Haldane (2017). This would suggest lower wages, but also less forward looking decisions, which is the point of this paper. Table 2 shows how the share of temporary contracts has evolved over time (again using OECD data). While the upward trend is not always monotonic and varies in magnitude across countries, it is relatively strong, especially in countries like France, Italy or the Netherlands).
Calvo meets perpetual youth

A crucial assumption is that when a worker quits (or is dismissed) and is replaced, the wage stickiness will be (at least partially) transmitted to the entrant worker. The entrant does not renegotiate its wage immediately, and has to abide by the wage of the previous incumbent it has replaced. Or equivalently, there is no difference between incumbents and entrants in their distribution of wages. This assumption is consistent with Hall (2005), Blanchard and Gali (2007) or Gertler and Trigari (2009) who combine nominal rigidities and labour search frictions. Gertler, Huckfeldt and Trigari (2016) find no evidence that the wage of new hires is more cyclical than for existing workers, so that it is equally rigid.

This model of entry has some perpetual youth flavour as in Blanchard (1985). As hinted by Weil (1989), the crucial feature in these models is not so much the probability of death of the agent, but the fact that new agents are born at any instant, and they don’t have a say (or a possibility to contract) over decisions made before their birth. Here, when a new worker starts a job, he is bound, by the decisions of his predecessor – or this is the case if wages are set by a union, which only cares about the welfare of its existing
members. The externality between existing and new agents creates the extra discounting.

Related literature

Snower and Tesfaselassie (2017) derive a positive optimal long run inflation target in the presence of job turnover, but they do not investigate the short run properties much. Bilbiie, Ghironi and Melitz (2012; 2016) as well as Bilbiie, Fujiwara and Ghironi (2014) look at the optimal long run monetary policy in similar setup: sticky prices with firm entry and exit. In their model, the exit probability affects the Phillips curve and the optimal long run Ramsey policy. While these papers use a Rotemberg instead of a Calvo framework, and inflation offsets different long run distortions, the intuition, as well as the assumption that new workers cannot reset their wage, is largely the same \(^1\). But this paper shows how turnover leads to a flatter long run Phillips curve, and a perceived flatter curve in the short run. It also explains how the optimality of price targeting is broken, compared to the classical result in Woodford (2003), Benigno and Woodford (2004), or Gali (2008). Last, it shows how optimal short run policies are affected.

Different explanations have been put forward for the recently flatter Phillips Curve. Ball and Mazumder (2011) suggest that with menu costs, price changes will be less frequent when inflation is low, and the resulting Phillips Curve will be flatter. Blanchard (2016) relies on anchored inflation expectations. My approach has the advantage of tractability and observable microfoundations, which allow for a welfare analysis. While the labour market has been highlighted as a possible driver of the flatter Phillips Curve (see Haldane, 2017 or chapter 2 of the October 2017 World Economic Outlook), no proper model has been suggested yet. The idea of a global Phillips Curve – inflation reacting to global not domestic conditions – has also been floated (eg. Carney, 2017), but again without a proper undrlying model.

This paper also belongs in the stream of literature that tries to reassess the New Keynesian model in light of the Great Depression and the Zero Lower Bound. While this paper introduces an extra discounting in the Phillips curve(s), other papers have introduced a discount factor in the Euler equation instead, to explain the forward guidance puzzle. McKay, Nakamura and Steinsson (2016) build a model of incomplete financial markets which leads to a discounted Euler equation, while in Del Negro, Giannoni and Patterson

\(^1\)In my Calvo framework, workers adopt the wage distribution of existing workers. In a Rotemberg setup, it is assumed that new workers (or firms) take the existing symmetric wage (or price), and are not free to choose their starting wage (price) optimally.
(2013), it comes from a Blanchard-Yaari model of perpetual youth for households which is similar to this paper, where it applies to firms and workers instead. The interaction between a discounted Phillips curve and a discounted Euler equation has been partially studied by Gabaix (2016).

Last, this paper is related to the literature on the optimal level of inflation, which does not solely rely on the Phillips curve. In their handbook chapter (2011), Schmitt-Grohe and Uribe document such other motives for positive inflation. If the price stickiness exhibits a quality bias (Schmitt-Grohe and Uribe, 2009), then a positive inflation will simply ensure that the hedonic price level remains constant. If wages are more rigid downwards than upwards, positive inflation will make relative wage adjustments easier (Olivera, 1964; Akerlof, Dickens and Perry, 1996; Kim and Ruge-Murcia, 2009). A positive amount of inflation might also be useful to increase the nominal interest rate safely above zero, in case the zero lower bound needs to be avoided (Adam and Billi, 2006; Reifschnieder and Williams, 2000).

The paper is organized as follows: Section 2 builds a New Keynesian model with sticky prices and wages, as well as firm and product turnover. The non linear Phillips curves are derived and linearly approximated. Section 3 investigates the prediction of a flatter Phillips curve in the short, middle and long run. Last, Section 4 solves the welfare maximization problem, both in the non linear (steady state inflation) and quadratic setups (optimal stabilisation).

2 The model

2.1 A microfounded model

The model of wage rigidities closely follows Gali’s (2008) notations, with monopolistic competition in the labour market. There is a continuum of wage-setting worker types, indexed by $j \in [0, 1]$.

Households and firms

Let me first look at the household. A worker of type $j$ maximizes a utility

$$E_0 \sum_{t \geq 0} \beta^t U(C_t(j), N_t(j))$$

The period utility function $U$ is separable in consumption and labour. The utility of consumption $C$, $u(C)$, is a concave function with inverse elasticity of intertemporal substitution $\sigma$, while the disutility of labour $N$, $v(N)$ is
convex with an inverse Frish elasticity $\phi$. The utility from consumption and disutility from labour are scaled by a parameter $\lambda$:

$$U(C_t(j), N_t(j)) = u(C_t) - v(N_t(j)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \lambda \frac{N_t(j)^{1+\phi}}{1+\phi}$$ (2)

Perfect competition is assumed in the goods market. The production function has diminishing returns to labour $N_t$, with a labour elasticity $(1-\alpha)$:

$$Y_t = N_t^{1-\alpha}$$

Labour is a CES aggregate of the labour of each type $j$, with a wage elasticity of substitution $\epsilon$:

$$N_t = \left[ \int_0^1 N_t(j)^{1-1/\epsilon} \, dj \right]^{1/\epsilon}$$

The aggregate wage index $W_t$ is

$$W_t = \left[ \int_0^1 W_t(j)^{1-1/\epsilon} \, dj \right]^{1/\epsilon}$$

The amount of labour of type $j$ employed by firm $i$ is

$$N_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon} N_t$$

Worker $j$ maximizes the expected discounted utility (1) subject to the budget constraint

$$P_tC_t(j) + Q_tB_t(j) = B_{t-1}(j) + (1 - \tau_t)W_t(j)N_t(j) + D_t + T_t$$

where $\tau_t$ is a proportional labour tax (or subsidy) on his labour compensation $W_t(j)N_t(j)$, $D_t$ is the dividend from owning a diversified portfolio of firms, and $T_t$ is a lump sum transfer (or tax) from the government. New bonds $B_t(j)$ can be bought or sold at price $Q_t$, the stochastic discount factor of the household. Let us assume a balanced government budget in each period, so that the lump-sum transfer (or tax) is financed by the tax on labour:

$$T_t = \tau_t W_t N_t$$

This ensures that consumption and output are equal in each each period. Because of perfect competition in the goods market, prices are equal to marginal costs, or

$$P_t = MC_t = W_t \frac{N_t^\alpha}{1 - \alpha}$$
Hence the real wage is linked to output as

\[ \Omega_t = (1 - \alpha)Y_t^{-\alpha/(1 - \alpha)} \]

With decreasing returns to scale, firms make a profit \( D_t = \alpha P_t Y_t \).

As in Erceg et al. (2000) or Gali (2008), let us assume markets with complete contingent for consumption but not leisure. This ensures full consumption smoothing across agents.

**Lemma 1** With complete markets, there is full consumption smoothing:

\[ \forall (t, j), \quad C_t(j) = C_t = Y_t \]

*The Euler equation of consumption pins down the riskless discount factor*

\[ Q_t = E_t^{\beta} \frac{P_t}{P_{t+1}} \frac{u'(C_{t+1})}{u'(C_t)} = \beta \frac{P_t}{P_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \]  

(3)

*The labour supply decision for a worker \( j \) in problem (1) is equivalent to maximizing the following quantity in each period*

\[ u'(Y_t) (1 - \tau) W_t(j) N_t(j) - \lambda \frac{N_t(j)^{1+\phi}}{1 + \phi} \]  

(4)

**Distortions and dispersions**

Let us define the first-best and flexible outcomes. Using the utility and production function, the natural level of output is

\[ Y^* = \left( \frac{1 - \alpha}{\lambda} \right)^{\frac{1}{\alpha + \phi + \sigma}} \]

**Lemma 2** In the flexible outcome, the real wage \( \Omega = \frac{W}{P} \) is a markup \( \mu \) above the marginal rate of substitution of the worker:

\[ \mu = \left( \frac{\epsilon}{(\epsilon - 1)(1 - \tau)} \right) \]

*The flexible price output is*

\[ \bar{Y} = \left( \frac{1 - \alpha}{\lambda \mu} \right)^{\frac{1}{\sigma + \phi + \sigma}} = Y^* \left( \frac{1}{\mu} \right)^{\frac{1}{\sigma + \phi + \sigma}} \]
The markups depend on the wage elasticity – with a high elasticity, the markup is close to 1. But it also depends on the wage tax $\tau$. A positive tax creates an additional wedge, but a subsidy can offset – partially or fully – the inefficiency caused by the finite wage elasticity. Unless the subsidies fully offset the wedges ($\mu = 1$), the flexible output will be inefficiently low as $\bar{Y} < Y^n$.

With staggered wages, the price and wage dispersions will be costly in terms of welfare. Heterogeneity in prices requires longer hours for each individual to enjoy the same utility of consumption. And the aggregate number of hours must increase when wages are heterogeneous, to produce the same amount of goods.

**Lemma 3** The aggregate utility function can be written

$$
\int_0^1 U(C_t, N_t(j)) dj = \bar{Y}^{1-\sigma} \left[ \left( \frac{\bar{Y}}{Y_t} \right)^{1-\sigma} - \frac{1-\alpha}{1+\varphi} \Delta_t \left( \frac{\bar{Y}}{Y_t} \right)^{1+\varphi} \right] 
$$

with the price and wage dispersions

$$
\Delta_t = \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon(1+\phi)} dj \geq 1
$$

### 2.2 Sticky wages and the Phillips Curve

**Worker discounting**

A fraction $\theta$ of workers have sticky wages, and a fraction $\delta$ keeps their job from one period to another; the two are independent. The discount factor accounts for the price and the firms survival probabilities $\theta$ and $\delta$. Instead of maximizing the discounted sum of expression (4) with a discount factor $\beta$, the applicable rate of time preference will be $\beta \theta \delta$: disutility of labour – attached to a wage and a worker – is discounted by $\beta \theta \delta$, while the labour compensation is discounted by $\theta \delta Q_t$.

It is assumed that when a worker is replaced, the new worker cannot automatically renegotiate his wage. Instead, he faces the same probability of sticky wages than existing workers. If they were completely free to choose new wages, the effect would die out; but as long as the new wage partly takes into account the wage of existing workers, the effect would be lessened but not die out. This gives a discrepancy between the joint survival probability $\theta \delta$ of the optimal wage setting decision, and the true wage stickiness $\theta$ that is featured in the dynamics of the aggregate wage and dispersion. This is the
cause of the flatter wage Phillips curve. In their Rotemberg setup, Snower and Tesfaiellassie (2017) (or Bilibie Ghironi and Melitz, 2012;2016) assume that new workers (or firms) start with the symmetric wage (price) of existing workers (firms). It is similar to here: entrants are bound by incumbents. As mentioned before, evidence from Gertler, Huckfeldt and Trigari (2016) tends to support this assumption.

It is also possible to think about the case where it is the union which sets the wage of workers of type \( j \), and the union insures workers against layoffs but not quits or dismissals. When a worker quits, or is dismissed, we can assume that he leaves his labour type and finds a different occupation, where wages are set by a different union. As such, if the union maximizes the utility of its existing members, employed or not, it will have a short discounting horizon. And it will not take into account the utility of future members, because they do not belong to this union yet.

**The non linear Phillips curve**

When a worker is free to set a wage \( w_t(j) \), he seeks to maximize the discounted sum of the wage compensation minus the utility, defined in expression (4).

\[
E_t \sum (\theta \beta \delta)^{T-t} \left[ u'(Y_T) \frac{(1 - \tau_T) w_t(j) N_T(j)}{P_T} - \lambda N_T(j)^{1+\phi} \right]
\]

**Lemma 4** The reoptimizing price \( w^*_t \) is :

\[
(w^*_t)^{1+\phi} = \frac{E_t \sum (\theta \beta \delta)^{T-t} \mu_t \left( \frac{W_t}{W_T} \right)^{t-\epsilon} \lambda N_T^{1+\phi}}{E_t \sum (\theta \beta \delta)^{T-t} \left( \frac{W_t}{W_T} \right)^{t-\epsilon} \Omega_T u'(Y_T) N_T} = \left( \frac{K_t}{F_t} \right) \tag{7}
\]

with recursive terms \( F_t \) and \( K_t \)

\[
F_t = (1 - \alpha)Y_t^{1-\sigma} + \theta \beta \delta \mu_t F_{t+1} \Pi_{t+1}^{-1}
\]

\[
K_t = \mu_t \lambda Y_t^{1+\phi} + \theta \beta \delta \mu_t K_{t+1} \Pi_{t+1}^{1+\phi} \tag{8}
\]

This is where the job survival probability, \( \delta \) plays a role, compared to the standard model. \( \delta \) is an extra factor, appearing here in the worker’s discounting, through the recursive \( F_t \) and \( K_t \). In the recursive equation, \( F_t \) depends on the expected future value \( E_t F_{t+1} \), multiplied by the inflation and a discount factor \( \theta \beta \delta \). The exact same phenomenon occurs for the recursive term \( K_t \). As we just saw for the price Phillips curve with \( \delta \), the \( \delta \) makes these two terms less forward looking than in the standard model, and it makes the wage Phillips curve flatter, as we will see with the linear approximation.
In each period, only a fraction \((1 - \theta)\) of wages are reoptimized at the value \(w_t^*\), while a fraction \(\theta\) still follows the previous distribution of wages, with an aggregate \(W_{t-1}\). Using the definition of the aggregate wage, the wage level \(W_t\) is a weighted aggregate of the previous wage level \(W_{t-1}\) and the current optimal wage \(w_t^*\):

\[
W_t^{1-\epsilon} = \theta W_{t-1}^{1-\epsilon} + (1 - \theta)(w_t^*)^{1-\epsilon}
\]

This provides the dynamics for the wage inflation and dispersion

\[
\frac{1 - \theta \Pi_t^{\epsilon-1}}{1 - \theta} = w(\Pi_t) = \left(\frac{w_t^*}{W_t}\right)^{1-\epsilon} = \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon-1}{1+\phi}} \quad (10)
\]

\[
\Delta_t = \theta \Delta_{t-1} \Pi_t^{\epsilon(1+\phi)} + (1 - \theta) w(\Pi_t)^{\frac{(1+\phi)}{1-\epsilon}} \quad (11)
\]

**Linear quadratic setup**

Although we will look at the optimal steady state level of inflation that the non linear model yields, it is useful to derive a linear quadratic approximation around a zero inflation steady state. In the flexible price steady state, there is no inflation (\(\Pi = 1\)), and no dispersion (\(\Delta = 1\)). The steady state values \(\bar{Y}, \bar{\Omega}, \bar{F}\) and \(\bar{K}\) are easy to pin down. Let us define the percentage deviation of each variable: \(\pi_t = \log \Pi_t\), and \(d_t = \log \Delta_t\). Similarly \(y_t, \omega_t, f_t\) and \(k_t\) denote log deviations of the capital-letter variables from the steady state.

**Proposition 1** The linear wage Phillips curve is

\[
\pi_t = \kappa y_t + \beta \delta E_t[\pi_{t+1}] \quad (12)
\]

with

\[
\kappa = \left(\frac{\phi+\alpha}{1-\alpha} + \sigma\right) \frac{(1-\theta)(1-\theta \beta \delta) \theta}{1+\phi \epsilon}
\]

This linear wage Phillips curve is broadly similar with the standard wage Phillips curve in a model of price and wage stickiness. Current wage inflation positively depends on the output gap and future expected wage inflation, and negatively on the real wage. However, two differences stand out. The coefficient \(\kappa\) is slightly different as it features the parameter \(\delta\). But most importantly, future inflation is discounted by \(\beta \delta\) instead of simply \(\beta\). In terms of intuition, this is because \(\beta \delta\) is now the discount factor that is applicable to the job tenure of the worker.
3 A flatter Phillips curve

3.1 Predictions of the model

Non vertical long run Phillips curve

The long run version of (12) implies a flatter long-run Phillips curves, and it is no longer vertical or nearly vertical as without turnover:

$$\bar{\pi} = \frac{\kappa}{1 - \beta\delta}\bar{y}$$

When $\delta$ is smaller than 1, $\kappa$ increases slightly. However the increasing effect on the denominator $(1 - \beta\delta)$ largely dominates. This means that long run inflation will depend less strongly on the long run output gap, and the curve is not as vertical.

Property 1 In the long run Phillips curve between inflation and output of the form $\bar{\pi} = \chi\bar{y}$, the coefficient $\chi$ decreases with turnover ($\delta$ falls):

$$\chi = \left(\frac{\phi + \alpha}{1 - \alpha} + \sigma\right) \frac{(1 - \theta)}{\theta(1 + \phi)} \frac{1 - \theta\beta\delta}{1 - \beta\delta}$$

Because the linear equation is only an approximation of a highly non-linear model, it is useful to see the impact of turnover on the non linear long run Phillips Curve. In steady state the price and wage inflation must be equalized: $\Pi = \Pi$. Taking the steady state in equations (8), (9) and (10), output can be written in terms of inflation

Lemma 5 The non linear long-run Phillips curve is

$$\left(\frac{Y}{\bar{Y}}\right)^{\frac{\phi + \alpha}{1 - \alpha} + \sigma} = \left[\frac{1 - \theta\beta\delta\Pi^{(1 + \phi)}}{1 - \theta\beta\delta\Pi^{-\gamma}}\right]^{\frac{1 + \alpha}{\gamma - 1}}$$

(13)

Figure (3) displays the output level $Y$ associated to a long run (annualized, price and wage) inflation $\Pi$. When $\Pi = 1, Y = 1$ (the flex price case). As $\Pi$ increases, there is a limited output gain, at least to the first order. With turnover ($\delta < 1$), the long run tradeoff is flatter than in the normal case without. This was true for the linear approximation of the curves around zero inflation, and it is also true for the non linear case.
Short and middle run

In equation (12), the coefficient of the output gap does not fall with more turnover (a fall in $\delta$). The coefficient $\kappa$ is (slightly) decreasing in $\delta$, so it increases when the survival probability falls. The intuition is that with a lower discount factor, more weight is put on current economic conditions, so inflation reacts more strongly to current output. However, let us look at two cases where the Phillips curve would be perceivedly flatter. Equation (12) can be iterated forward:

$$\pi_t = \kappa y_t + \beta \delta E_t [\pi_{t+1}] = \kappa \sum_{k \geq 0} (\beta \delta)^k E_t y_{t+k}$$

Let us assume that the output gap is serially correlated:

$$y_t = \rho_y y_{t-1} + u_t$$

with $u_t$ a mean-zero disturbance. Then we can write inflation as

$$\pi_t = \frac{\kappa}{(1 - \rho_y \beta \delta)} y_t$$
Property 2  The slope of a traditional Phillips Curve displaying only current inflation and output will depend on the ratio

\[
\frac{(1 - \theta \beta \delta)}{(1 - \rho_y \beta \delta)}
\]

As long as \( \rho_y > \theta \) (the output gap being more persistent than wages), the slope will decrease when \( \delta \) falls (turnover increases).

Let us also look at an estimated New Keynesian Phillips curve with a restricted \( \beta \), if the turnover is not accounted for. Using the assumptions above,

\[
\pi_t - \beta E_t [\pi_{t+1}] = \kappa y_t - \beta (1 - \delta) E_t [\pi_{t+1}]
\]

\[
\pi_t - \beta E_t [\pi_{t+1}] = \kappa \left[ y_t - \beta (1 - \delta) \sum_{k \geq 1} (\beta \delta)^k E_t y_{t+k} \right]
\]

Property 3  The estimated slope in this case will be

\[
\kappa^* = \frac{\text{cov}(\pi_t - \beta E_t [\pi_{t+1}], y_t)}{\text{var}(y_t)} = \frac{(1 - \beta \rho_y)}{(1 - \rho_y \beta \delta)} \kappa
\]

As long as \( \rho_y > \theta \) (the output gap being more persistent than wages), the slope will decrease when \( \delta \) falls (turnover increases).

This is the case in the empirical estimates of Gali and Gertler (1999), where they use marginal costs instead of the output gap. They estimate \( \pi_t = \lambda mc_t + \beta E_t \pi_{t+1} \). The estimated coefficient of marginal costs, \( \lambda \), depends on the assumption about the coefficient of future inflation, \( \beta \). When this coefficient is restricted to \( \beta = 1 \), the estimated value of \( \lambda \) is smaller than when there is no restriction and \( \beta \) takes a lower value.

Remark  We have to assume here that the output gap is more persistent than sticky wages (\( \rho_y > \theta \)) in order to generate a downward bias in the traditional PC, and the restricted New Keynesian PC. This is not difficult as \( \rho_y \approx 0.95 \) in the US for example. However, such an assumption would not be necessary in a Rotemberg setup. In such a setup, the coefficient \( \kappa \) does not depend on turnover. Assuming \( y_t = \rho_y y_{t-1} + u_t \) as before, \( (1 - \rho_y \beta \delta) \) increases when \( \delta \) falls, so the traditional and restricted New Keynesian slopes are always smaller with turnover.
3.2 Empirical results

I rely on data from the OECD to test a wage Phillips curve between inflation and cyclical unemployment\(^2\). I have 21 countries, between 1996 and 2014 (or fewer years for some countries). Cyclical unemployment \(u_t\) is defined as unemployment minus the NAIRU, or structural unemployment. Wage growth is the yearly percentage increase in nominal compensation per worker. For turnover, I rely on the job tenure survey. While the proportion of worker who have been in their job for less than a year is not a perfect metrics for the rate of yearly job turnover, it is nevertheless a relatively good indicator. Therefore my turnover variable \(\tau_t\) is the proportion of worker between 25 and 54 who have been in their job for a year or less.

I run two regressions. The first is a short run expectation-based curve:

\[
\pi_t = \gamma(\tau_t)u_t + \beta(\tau_t)\pi_{t+1} + v_t
\]

where \(v_t\) is an error term. \(\gamma(\tau_t)\) is expected to be negative, and decrease slightly with turnover \(\tau_t\) (a slightly steeper curve). \(\beta(\tau_t)\) is positive and smaller than 1, and it should decrease with \(\tau_t\). In order to test the effect of turnover on these coefficients, I add the cross terms \((\tau_t \times u_t)\) and \((\tau_t \times \pi_{t+1})\) in the regression. The two estimates are expected to be negative. I also add time and country fixed effects in the regression. Last, to rule out common trends in turnover and the coefficients, I also allow a trend in the coefficients. As such, the equation can be written

\[
\pi_{n,t} = \alpha_n + \alpha_t + \gamma_1 u_{n,t} + \gamma_2 (\tau_{n,t} \times u_{n,t}) + \gamma_3 (t \times u_{n,t}) + \beta_1 (\tau_{n,t} \times \pi_{n,t+1}) + \beta_2 (t \times \pi_{n,t+1}) + v_{n,t}
\]

The results are coherent with the predictions of the model. The effect of turnover on the coefficient of future inflation is negative and significant, as predicted. And allowing for a trend in the coefficient does not make turnover insignificant. Contrary to the prediction, the unemployment coefficient increases with turnover (which makes the curve flatter). But this effect was predicted to be small, and in the data the change is positive but insignificant. The flatter unemployment coefficient might be caused by less frequent wage changes as in the menu costs model of Ball and Mazumder (2011): changes in wage will be less frequent as inflation and volatility declined over the past decades.

It is also insightful to look at the case of a restricted \(\beta\). If the on future inflation is restricted to the riskless discount factor (about 0.96 yearly), we

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\(^2\)It as long been argued, for example in Gali and Gertler (1999), or in Gali (2011) that inflation output Phillips curve are difficult to estimate, while they work better with real marginal costs or unemployment.
can see that the coefficient on unemployment is reduced to less than a third, from $-0.3$ to $-0.09$. And the effect of turnover on the unemployment coefficient is magnified under this restriction, and this is consistent with my earlier predictions.

Now let us look at the medium run Phillips curve. If unemployment is serially correlated of order 1, we saw that the Phillips curve could also be written

$$\pi_t = \tilde{\gamma}(\tau_t)u_t + \tilde{v}_t$$

$\tilde{v}_t$ is the new error term, and $\tilde{\gamma}(\tau_t)$ is predicted to be negative and increasing (a flatter curve) with turnover $\tau_t$. As before, I allow for time and country fixed effects, I include the term ($\tau_t \times u_t$) in the regression, which is expected to be positive. To rule out common trends in turnover and the coefficient, I again allow a trend in the coefficient. As such, the equation can be written

$$\pi_{n,t} = \tilde{\alpha}_n + \tilde{\alpha}_t + \tilde{\gamma}_1 u_{n,t} + \tilde{\gamma}_2 (\tau_{n,t} \times u_{n,t}) + \tilde{\gamma}_3 (t \times u_{n,t}) + \tilde{v}_{n,t}$$

The effect of turnover on the coefficient is positive, which is consistent with the predictions. It is not significant at the 10% level, but it is not less significant when a trend effect is allowed. As such, these results are consistent with the idea that turnover creates a flatter middle run Phillips curve.
4 Price or inflation targeting?

4.1 Turnover and price targeting

As we will see, introducing turnover into a standard New Keynesian model has strong implications for the optimal Ramsey policy. Let us first define the aggregate welfare function.

Welfare function

While workers discount future wages with the probability of job turnover, individuals do not die in my model. Therefore, the aggregate utility function of the social planner is simply the aggregation of each household’s utility given in equation (1). Using equation (5), this is

\[
E_0 \sum_{t \geq 0} \beta^t U(C_t, N_t(j)) = E_0 \sum_{t \geq 0} \beta^t \tilde{\gamma}^{1-\sigma} \left[ \left( \frac{\bar{Y}}{Y_t} \right)^{1-\sigma} \frac{1-\alpha}{1+\sigma} \Delta_t \left( \frac{\bar{Y}}{Y_t} \right)^{\frac{1+\phi}{1-\sigma}} \right] \tag{14}
\]

In terms of intuition, it is easier to look at the optimality of price targeting in a quadratic setup. When steady state distortions are small, the approximation of (14) and (11) bring a quadratic approximation that is not different from the case without turnover. This is because turnover plays no direct role in the utility function, or the dynamics of the dispersions.

Lemma 6 The second order approximation of the aggregate utility is

\[
U = -\sum_{t \geq 0} \beta^t \left[ \kappa (y_t - y^n)^2 \frac{1}{2} + (1 - \alpha) \epsilon_{w} \pi_t^2 \right] + \frac{1-\alpha}{1+\sigma} \Delta_t \left( \frac{\bar{Y}}{Y_t} \right)^{\frac{1+\phi}{1-\sigma}} \tag{17}
\]

with \(y^n = \log \frac{\bar{Y}}{Y}\) and \(\kappa = (\sigma + \frac{\phi+\alpha}{1-\alpha} \frac{(1-\sigma^w)(1-\sigma^\bar{Y})}{\theta^w} \frac{1}{1+\phi_{w} \neq \kappa}\)
Contrary to $\kappa$, $\delta$ does not appear in $\tilde{\kappa}$, which is exactly the same coefficient as in the case with no turnover. This is because the distortion is discounted with the discount factor of the household, where the death shocks play no role.

Let us also assume cost push shocks in the Phillips curve:

$$\pi_t = \kappa y_t + \beta \delta E_t \pi_{t+1} + u_t$$

with $u_t$ the cost push shock, an error term. We allow it to be an $AR(1)$ process with autocorrelation $\rho_u$ ($\rho_u = 0$ denoting the white noise case).

The optimality of price targeting

**Proposition 2** When $\delta = 1$, price targeting is optimal for the Ramsey policy: even with steady state distortions, the long run optimal level of inflation is zero; while inflation reacts to cost push shocks in the short run, this is accompanied by deflation in the future, so that there is full mean reversion of the price level. In other words, there is long-run price targeting in response both to long term distortions and short term cost push shocks.

When $\delta < 1$, price targeting is no longer optimal: long run inflation is non zero if there are steady state distortions; in response to cost push shocks, some deflation in the future offsets the initial response of inflation, but there is no longer full mean reversion of the price level. In other words, price targeting does not hold anymore.

The intuition is as follows: in the benchmark, by committing to give up some discretion in the future, the planner has some extra discretion in the present correct cost push shocks, or an inefficient steady state. So that price stability is optimal from today’s perspective, but there is an incentive to renege tomorrow. With the death shock, firms are less responsive to commitments, so that the current gain in terms of commitment no longer offsets the inefficiency in the future. Thus, even with a credible commitment, inflation will always be used to offset cost push shocks or steady state inefficiencies.

To better grasp the logic, it is useful to compare the Ramsey policy, which is history dependent, to an optimal state dependent policy. While such a solution is not Ramsey optimal, it features no dynamic inconsistency. We can call this solution Markovian, or Recursively Pareto Optimal as in Brendon and Ellison (2015). Let us assume that the optimal inflation is a function of the natural rate of output and the current cost push shock: $\pi_t = \bar{\pi} + \gamma \pi u_t$.

In such a Markovian setup, the optimal inflation is not zero even without turnover. This is because the long run Phillips Curve is not vertical without turnover, and a very little amount of inflation is welfare improving. On
the other hand, the Ramsey policy in this case is to allocate the current and future inflation differently. A high inflation is used in the short run, in exchange for no inflation in the long run. While this is not time-consistent, it is optimal from today’s point of view. With turnover, the Markov optimal inflation is higher due to the flatter long run curve. And the Ramsey policy still uses more inflation in the short run, but not zero in the long run.

In response to cost push shocks, the difference between the Markov and Ramsey policy is more important. The Ramsey policy commits to offset current inflation with future disinflation in response to cost push shocks, and this commitments improves the tradeoff in the short run. Because the Markov policy is not history dependent, it cannot promise future disinflation, and hence mean reversion of the price level. When turnover is introduced, there is no longer full mean reversion of the price level, but it does not impact the Markov policy much.

![Figure 4: Ramsey and Markov policy in response to wage cost-push shocks](image)

4.2 Long run optimal inflation

In this subsection, we derive the optimal steady state inflation implied by the non-linear model. While a closed form expression was available for the long run Phillips curve, the optimal level of steady-state inflation (for a given amount of steady state distortions) can only be defined implicitly. As
such, it is useful to calibrate most of the parameters, to provide a graphical illustration. As in Gali, let us calibrate $\alpha = 0.25$, $\beta = 0.99$, $\epsilon = 8$, $\theta = 0.66$, $\phi = 0.11$ and $\sigma = 0.16$. Now we need to find values for $\delta$. Let us consider a low turnover scenario ($\delta = 0.95$, or an average duration of 5 years) and an intermediate scenario with $\delta = 0.90$.

It is a well known feature that in the presence of steady state distortions, the optimal Ramsey policy of the New Keynesian model does not bring a constant level of inflation. While there is a small output-inflation tradeoff, the Ramsey policy dictates to frontload some of the inflation at the beginning, with a reduced inflation in the future. This brings the classical time inconsistency problem: it is optimal to promise zero or low inflation in the future, while having a higher rate of inflation temporarily. But in the future, there is an incentive to renege on past promises of low inflation.

Thus we have two ways to define the optimal long run inflation. One is to look at the long run solution of the Ramsey policy: we solve the dynamic Ramsey problem, with the discounted utility function and the dynamic Phillips curve constraints, and look at the long run solution. But this runs into the issue of inconsistency, and the long run rate of inflation is not optimal for the current period.

If the aim is to have a constant rate of inflation that is applicable both to the short and long run, we can instead look at the long run constraints, and maximize utility subject to them. As such, we are restricting ourselves to the set of constant inflation rates. Instead of solving the dynamic problem and restrict to the time-invariance solution, we impose time-invariance before solving the maximization.

In the case of the time invariant solution, one simply maximizes the per period objective function of the social planner (5), subject to the long run output inflation Phillips curve (13) and the expression of the long run dispersion

$$\Delta = \frac{(1 - \theta)w(\Pi)^{\phi(1+\phi)}}{1 - \theta \Pi^{(1+\phi)}}$$

Intuitively, inflation helps to bring output closer to its natural level – but too much inflation reduces output as the curve is non linear – but it increases the price and wage dispersions, which reduce utility.

$$L = \left\{ + \Phi_1 \left[ \ln \left( \frac{1}{1-\theta \beta(1+\phi)} w(\Pi)^{\phi(1+\phi)} \right) - \left( \frac{\phi + \alpha}{1-\alpha} \right) \ln Y \right] \right\}$$

$$+ \Phi_2 \left[ (1 - \theta \Pi^{(1+\phi)}) \Delta - (1 - \theta) w(\Pi)^{\phi(1+\phi)} \right]$$
As illustrated in figure (5), this brings a positive amount of inflation, even when $\delta = 1$. The optimal inflation increases as $\delta$ decreases.

For the timeless Ramsey policy, we write the full dynamic Lagrangian (with $Y_t$, $\Omega_t$, $K_t$ and $F_t$ renormalized to flex price values).

The social planner maximizes the discounted sum of the per period utilities (5), subject, in each period, to the recursive expressions of $F_t$ and $K_t$ (equations 8 and 9), the ratio $\frac{K_t}{F_t}$ (equation 10), as well as the dynamics of $\Delta_t$ (equations 11).

Intuitively the tradeoffs are similar to the time invariant problem: inflation increases output at the first order, but increases the costly price and wage distortions. However, the fully dynamic setting is different from the previously static one. The Lagrangian of the problem writes

$$L = \sum \beta^t \left( \begin{array}{c} \left[ \frac{1}{1-\sigma} Y_t^{1-\sigma} - \frac{1-\sigma}{1+\phi} Y_t^{1+\phi} \Delta_t \right] \\ + \phi_{1,t} \left[ K_t w(\Pi_t)^{1+\phi} - F_t \right] \\ + \phi_{2,t} \left[ F_t - Y_t^{1-\sigma} - \theta \beta \delta \Pi_t F_{t+1} \Pi_{t+1}^{1-\phi} \right] \\ + \phi_{3,t} \left[ K_t - Y_t^{1+\phi} - \theta \beta \delta \Pi_t K_{t+1} \Pi_{t+1}^{1+\phi} \right] \\ + \phi_{4,t} \left[ \Delta_t - \theta \Delta_{t-1} \Pi_t^{1+\phi} - (1 - \theta) w(\Pi_t)^{\phi(1+\phi)} \right] \end{array} \right)$$

After taking the first order conditions, we look at the steady state value of each constraint and multiplier. Figure (5) displays the optimal rate of inflation depending on the amount of steady state distortions, for different values of $\delta$. When $\delta = 1$, we have the classic result of zero inflation in the long run, but it increases as this parameter decreases.

Figure 5 displays the constant and timeless Ramsey steady state inflation depending on the natural output $Y^n > 1$, for different values of $\delta^n$. The constant policy is in blue while the Ramsey policy is in dashed red. With more frequent death shocks, the optimal level of constant inflation is higher. When $\delta = 1$, there is a small level of inflation for the constant policy, but no inflation for the timeless Ramsey policy: this is the optimality of price stability. However, when death shocks are introduced, the optimal level of inflation increases with the output gap, for both the constant and timeless cases. For a large output gap ($Y^n >> 1$) and large death shocks, the optimal annual level of inflation is in the order of 1 – 3% annually.

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5 Conclusion

This paper constructed a New Keynesian model with Calvo wage stickiness, as well as job turnover. I show how this leads to a Phillips Curve that is far less forward looking. When looking at a medium run Phillips Curve, with persistent output or unemployment disturbances, this can account for a flatter curve. If the coefficient of future inflation is restricted in a standard NK Phillips Curve, this creates a bias on the estimate of the slope of the Phillips Curve, and this bias increases with more turnover. This prediction is tested on OECD data and is not rejected empirically. In the long run, the Phillips Curve is also flatter, and no longer vertical or near-vertical.

I show how turnover breaks the optimality of price stability. Price stability is no longer optimal, and inflation expectations are more anchored than in traditionnal Phillips curves. As such the optimal Ramsey policy no longer targets the price level in response to cost push shocks. If this turnover is large, and if the steady state distortions are high enough, the optimal level of inflation can reach $1 - 2\%$ annually. In fact, if there was partial price and wage indexation, the optimal inflation would be higher, or a same amount of inflation would be rationalized by a lower turnover or steady state distortion.

One fruitful avenue of future research would be to investigate the empirics in greater details. Phillips curves can be more informative if we don’t impose...
the restriction that they are vertical or quasi vertical in the long run. Also, a cross section of different sectors, and different types of workers - eg, temporary vs. permanent employees - could provide additional evidence. Another fruitful avenue could be to endogenize this turnover. If turnover becomes endogenous, they might be influenced by inflation and the output gap, in a more complex DSGE model. Stabilising this turnover could become part of the monetary authority’s goal.

References


