# On the Efficiency of Competitive Equilibria with Pandemics*

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**Abstract**

The epidemiological literature suggests that virus transmission occurs only when individuals are in relatively close contact. We show that if society can control the extent to which economic agents are exposed to the virus and agents can commit to contracts, virus externalities are local, and competitive equilibria are efficient. The Second Welfare Theorem also holds. These results still apply when infection status is imperfectly observed and when agents are privately informed about their infection status. If society cannot control virus exposure, then virus externalities are global and competitive equilibria are inefficient, but the policy implications are very different from those in the literature. Economic activity in this version of our model can be inefficiently low, in contrast to the conventional wisdom that viruses create global externalities and result in inefficiently high economic activity. If agents cannot commit, competitive equilibria are inefficient because of a novel pecuniary externality.

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# 1 Introduction

In this paper, we argue that pandemics induce local externalities. We begin by noting that the epidemiological evidence suggests that viruses travel only relatively short distances. This property implies that infected individuals can transmit the virus to others only if they are engaged in relatively close contact. We will refer to such interactions as *meetings.* If society can control the extent to which people are exposed to the virus by choosing the types of people who meet each other without loss of output, then the external effects induced by viruses are appropriately thought of as local externalities. We show that with *controllability* of exposure and *commitment* to contracts, competitive equilibria are efficient. To illustrate the key role of controllability, we show that if people cannot control their exposure to the virus, the externalities turn out to be global, and the equilibrium is inefficient. To illustrate the key role of commitment, we show that without commitment, the local external effects induce a novel pecuniary externality, and equilibria are inefficient. We show that our efficiency results continue to hold even if agents are privately informed about their infection status. Our methods can be adapted straighforwardly to a wide variety of environments with local externalities or local public goods, and our results suggest that competitive equilibria remain efficient in these settings.

Much of the existing literature on the external effects of viruses treats these external effects as inducing global externalities. A typical formulation is to assume that the rate at which new infections form depends on the existing masses of infected and susceptible individuals as well as on measures of aggregate economic activity. Not surprisingly, competitive equilibria in such environments are inefficient, and corrective policies, including Pigouvian taxes, are desirable. Put simply, there is no technical difference in the existing literature between the externalities induced by the emission of greenhouse gases and the externalities induced by the transmission of viruses. Given that the epidemiological literature decisively shows that virus transmission is relatively local, and given that exposure is controllable, we argue that the global-externality view is inappropriate for the analysis of pandemics. Instead, analyzing pandemics requires a theory in which pandemics are treated as local externalities.

We develop such a theory by combining elements from the literatures on competitive search, epidemiology, and club goods. Models in the competitive search literature use the metaphor of islands to describe competition among firms and households, which take prices as given. This metaphor seems particularly appropriate for analyzing environments in which people can control their exposure to the virus. In equilibrium, it turns out that islands differ in the extent to which people are exposed to the virus. This metaphor is also closely related to the original idea in the local-public-goods literature of locations competing with each other to provide local public goods (see, for example, Tiebout, 1956). From the epidemiology literature, we adapt the SIR model to study how viruses transmit infections. From the club goods literature, we adapt the idea that the value of belonging to a club (in this case, an island) depends on the composition of its members.

For expositional reasons, we begin with the simplest version of our model, in which each individual's infection status is public information. Agents can be of one of three types: susceptible, infected, and recovered. Production takes place on "work" islands. In each work island, a single consumption good is produced each period, using a constant-returns-to-scale technology for which labor is the only input. This technology is the same across all work islands. In addition, the economy has a "home" island, in which no production takes place. Each work island is characterized by a triple of wages, one for each type. The interpretation is that any firm that produces in a work island must pay a worker of a particular type the wage associated with that type on that island. Each agent supplies one unit of indivisible labor and chooses a lottery over the work and home islands. Competitive insurance firms provide consumption insurance over the outcomes of the lotteries. Each firm employs at most one worker. A constant-returns-to-scale matching technology describes how firms and workers on a particular island are matched with each other. Unmatched workers on work islands are described as unemployed. This formulation is essentially the same as that in the competitive search literature.

Our main point of departure from the competitive search literature is that we allow for local externalities in the form of infections. We assume that individuals on a particular work island randomly "meet" each other in the process of production.[1] A susceptible worker who meets an infected worker is infected with a probability

[1]While we explicitly model infections as occurring in "production," we think of individuals as getting infected in the process of conducting economic transactions such as consumption, recreation, and other leisure activities. Our model can be easily generalized to allow for infection in such activities.

that is given by the infection technology. Given the assumption that meetings in an island are random, the probability that a susceptible worker is infected on a particular island depends on the fraction of infected agents relative to the total number of workers on that island; that is, the infection technology has constant returns to scale. Since no production takes place on the home island, no meetings do either, and so the probability of infection is zero. Workers derive utility from consumption and suffer a utility cost if they are infected. Infected workers recover with a probability given by the infection technology. We assume for simplicity that recovered workers cannot get re-infected. The proportion of susceptible, infected, and recovered workers in period zero is exogenously given.

Our definition of competitive equilibrium is standard. We show that any competitive equilibrium is efficient, and furthermore, any efficient allocation can be implemented as a competitive equilibrium with suitably chosen lump-sum taxes andtransfers. In equilibrium, islands vary in their degree of exposure to the virus. In this sense, our model captures the idea that controllability is essential to ensure that competitive equilibria are efficient. Our finding is in sharp contrast to those in much of the literature. The reason for the contrast is that the literature models virus transmission as a global externality. Since the epidemiological literature makes a convincing case for virus transmission as a local phenomenon, modeling this transmission as a local externality seems to be the appropriate way of analyzing policy interventions. Our result shows that once we allow for controllability, competitive forces will produce efficient outcomes. It turns out that with public information, the equilibrium can be implemented as a sequence of static equilibria so that commitment plays no role.

We show that our efficiency result does not depend on the assumption that types are perfectly observable. To do so, we analyze an environment in which some workers are infected but asymptomatic; therefore, neither they nor anyone else knows that they are infected. We allow workers to sign contracts with insurance firms that can offer intertemporal insurance contracts as well as help solve the coordination problem typical in models with public goods. We show that the welfare theorems continue to hold in this environment. Our efficiency result does not depend on the assumption of public information. We show that the equilibrium outcomes with private information coincide with those with public information, and are, therefore, efficient, as long as recovered agents' types are public information. The intertemporal contracts offered by insurance firms play a central role in implementing the equilibrium, and equilibrium outcomes cannot be implemented by sequences of static equilibria.

It turns out that in the competitive equilibrium, susceptible agents have strong incentives to mix with recovered agents, since there is a positive congestion externality associated with the presence of a type of worker who is known not to be infected. These incentives imply that, in the competitive equilibrium, initially recovered agents consume more than their marginal product and susceptible agents consume less than their marginal product. The result that the equilibrium has mixing illustrates that our efficiency results do not depend on complete sorting by infection status. We go on to show that our efficiency results continue to hold with multiple occupations with complementarities as long as infection status and occupational status are independent. In equilibria with multiple occupations, we have mixing in islands of different types of people employed in different types of occupations. These findings are of interest because Stiglitz (1982) and others have criticized models that obtain efficiency with local public goods because they seem to depend on equilibria being completely sorting.

Our efficiency results do not hold if virus exposure is not controllable. To understand the role of controllability, we consider a version of our model in which the economy has only one work island. In this case, the externalities induced by the pandemic are (almost) global rather than local. Not suprisingly, competitive equilibria are no longer efficient. We show, however, that the nature of the inefficiency and the associated optimal policy interventions are very different from those commonly supposed in the literature that models pandemics as global externalities. For example, in a static version of our model, it turns out that susceptible agents work too little in the competitive equilibrium, compared with the efficient outcome. The reason is that susceptible agents who allocate more labor to the work island confer a positive congestion externality to other susceptible agents on that island. They do so because by working, they increase the probability that other susceptible agents will meet them rather than meeting infected agents. No susceptible agent internalizes this benefit that is conferred on other susceptible individuals. While in the dynamic model it is in principle possible for susceptible agents to work too much, we show that they work too little for a wide range of parameter values. We go on to show that policy interventions that use type-independent Pigouvian taxes are worse than targeted lockdown policies. Within the class of such Pigouvian policies, we show that susceptible agents work more in the efficient outcome than in the competitive equilibrium. Taken together, these findings suggest that the conventional wisdom that economic activity in a competitive equilibrium is inefficiently too high in a pandemic is incorrect.

We find that with imperfect observability of infection status, commitment, in addition to controllability, is crucial for our efficiency result. To understand the role of commitment, we consider an environment in which workers can quit at any time and join other insurance firms. We show that this form of limited commitment induces a pecuniary externality so that the equilibrium outcomes are inefficient. This pecuniary externality arises because the value of outside options for the worker depends on the population mix of workers by infection status. This population mix is determined by the decisions of all firms in previous periods. No individual firm internalizes that its decisions affect the population mix and therefore the value of the outside option. The planner, however, does internalize the effect on the value of the outside option. Thus, the solution to the planning problem differs from the competitive equilibrium.

Finally, we show that in the controllable version of our model, competitive equilibria are efficient even if the infection technology has increasing returns to scale as in Farboodi et al. (2020) and Acemoglu et al. (2020).

**Literature**

An extensive epidemiological literature has addressed how viruses are transmitted (see Kermack and McKendrick, 1927; Morawska et al., 2020; Bourouiba et al., 2014; Bourouiba, 2020; Somsen et al., 2020). The consistent finding in this literature is that viruses require individuals to come into relatively close contact with each other in order for infections to spread. This literature compels economic analysts to regard pandemics as situations with local rather than global externalities.

An extensive literature in economics has analyzed various aspects of pandemics. See, for example, Eichenbaum et al. (2020), Farboodi et al. (2020), Toxvaerd and Rowthorn (2020), Toxvaerd (2019), Bethune and Korinek (2020), Goodkin-Gold et al. (2020), Bisin and Gottardi (2020), Moser and Yared (2020), Atkeson (2020), Alvarez et al. (2020), Glover et al. (2020), Acemoglu et al. (2020), Baqaee et al. (2020), Berger et al. (2020). Some of this literature explicitly analyzes the inefficiencies associated with pandemics (see Eichenbaum et al., 2020; Toxvaerd and Rowthorn, 2020; Toxvaerd, 2019; Goodkin-Gold et al., 2020; Bisin and Gottardi, 2020; and Bethune and Korinek, 2020.) All of the papers in this area regard pandemics as creating global externalities.

Our formulation has obvious and immediate predecessors in the literature on local public goods and that on clubs. See, for example, Tiebout (1956), Buchanan (1965), Stiglitz (1982), Cole and Prescott (1997), and Ellickson et al. (1999). This literature has discussed how local public goods and clubs can be efficiently provided as long as firms and households compete effectively with each other. We have adopted many ideas from this literature.

We find it convenient to formulate the local-public-goods problem using ideas from the competitive/directed search literature. See, for example, Peters (1984), Moen (1997), Guerrieri et al. (2010), and Wright et al. (2021). Following that literature, we think of firms as choosing locations that are indexed by a variety of characteristics, including prices. This formulation is particularly convenient for studying the provision of local public goods.

## 2 A Pandemic Model with Controllability

In this section, we develop a model in which infection status is perfectly observable and virus exposure is controllable. We show that the welfare theorems hold and that the competitive equilibrium can be implemented as a sequence of static equilibria so that commitment plays no role.

Consider a discrete-time, finite-horizon model that combines elements of models from the literatures on competitive search, epidemiology, and club goods.[2] The model has a continuum of workers of mass one and a continuum of locations, denoted by $j \in \mathcal{J}$ and referred to as "islands." The model has one consumption good per period. In one of the islands, denoted by $j = 0$ and described as the "home" island, no production takes place. In each of the other islands, described as the "work" islands, the consumption good is produced according to a constant, returns-to-scale production function in which one unit of labor produces $A$ units

[2]Our results immediately apply to the infinite-horizon model if we consider equilibria that are limits of the finite-horizon equilibria.

of the consumption good. Workers are endowed with one unit of time per period. Each worker is in one of three mutually exclusive health states, $\eta \in \{S, I, R\}$, where $S$, $I$, and $R$ denote susceptible, infected, and recovered types, respectively. In this section, we assume that health states are publicly observable. Let $\boldsymbol{\mu}_t = (\mu_{St}, \mu_{It}, \mu_{Rt})$ denote the masses of susceptible, infected, and recovered agents at the beginning of period $t$ so that

$$\mu_{St} + \mu_{It} + \mu_{Rt} = 1. \tag{1}$$

Infected agents can transmit the disease to susceptible agents only if they meet susceptible agents in one of the islands. Susceptible agents at the beginning of period $t$ are workers who have not been infected in any previous period. Infected agents are workers who are infected and currently infectious. Recovered agents are workers who were previously infected but are not currently infectious. We assume that recovered agents cannot be re-infected and that the allocations for each individual agent can depend only on the current type and the current period.[3] This assumption is without loss of generality, since health states are publicly observable. In later sections, when types are not perfectly observable, we will allow allocations to depend on private and public histories.

The allocation of labor time is indivisible in the sense that a worker can allocate labor time to at most one island in any period. We allow for lotteries so that workers choose a probability distribution over which islands to allocate their labor time to.[4] Let $\mathbf{l}_{\eta t} = (l_{j\eta t})_{j \in \mathcal{J}}$ denote a probability measure over islands for agents of type $\eta$ in period $t$ so that $\int l_{j\eta t} dj = 1$. Here, $l_{j\eta t}$ is also the fraction of agents of type $\eta$ who are allocated to island $j$. We can interpret these lotteries in two equivalent ways. One way is that $l_{j\eta t}$ is the probability that a worker of a given type is allocated to a given island. Another is to think of workers as belonging to families whose members are all of the same type, so that $l_{j\eta t}$ is the fraction of family members allocated to island $j$.

Next, we describe how infections propagate. Let $\lambda_{j\eta t} \equiv \mu_{\eta t} l_{j\eta t} / L_{jt}$ be the fraction of type $\eta$ on island $j$ in period $t$, where $L_{jt} \equiv \sum_\eta \mu_{\eta t} l_{j\eta t}$ is the total labor supply on the island. We assume that the probability that a susceptible agent is infected depends only on $\lambda_{jIt}$. This infection technology has constant returns

[3]Our results continue to hold if we allow agents to die.

[4]The lotteries are inessential for public information but play an important role in later sections. We introduce lotteries here so as to use common notation throughout our analysis.

to scale because the probability that a susceptible agent is infected on an island depends on the ratio of the labor supply of infected agents relative to total labor supply on that island. In island 0, we assume that infections do not occur, and so the infection probability is zero.[5]

For expositional purposes, in most of our analyses, we assume that the infection technology is linear in that

$$\psi\left(\lambda_{jIt}\right) = \chi\lambda_{jIt} \text{ for } j \neq 0 \text{ and } L_{jt} > 0, \tag{2}$$

where $\chi > 0$ is a constant. An alternative technology that maintains constant returns to scale is as follows. Suppose that in the process of production, a susceptible person randomly meets $M$ other agents per unit of time. The probability of infection in any single meeting is proportional to the fraction of infected agents on the island and is given by $\hat{\chi}\lambda_{jIt}$ for some constant $\hat{\chi} > 0$. Thus, the probability of not being infected in $M$ meetings is $\left(1-\hat{\chi}\lambda_{jIt}\right)^M$, and so the probability that a susceptible agent is infected on island $j$ is $1 - \left(1-\hat{\chi}\lambda_{jIt}\right)^M$. Our results hold with this specification of the infection technology. In Section 6, we consider a class of non-constant, returns-to-scale infection technologies and show that our main results are unchanged.

An infected person in period $t$ exits the infection state with probability $\alpha > 0$ and enters the recovered state. The mass of agents of each type in the economy as a whole then evolves according to the Markov transition matrix

$$\mu_{t+1} = \begin{bmatrix} \mu_{St+1} \\ \mu_{It+1} \\ \mu_{Rt+1} \end{bmatrix} = \begin{bmatrix} 1 - \int_{j\neq 0} l_{jSt}\psi\left(\lambda_{jIt}\right) dj & 0 & 0 \\ \int_{j\neq 0} l_{jSt}\psi\left(\lambda_{jIt}\right) dj & 1-\alpha & 0 \\ 0 & \alpha & 1 \end{bmatrix} \begin{bmatrix} \mu_{St} \\ \mu_{It} \\ \mu_{Rt} \end{bmatrix}. \tag{3}$$

Here, the integrals in the Markov matrix are evaluated only over those islands in which $L_{jt} > 0$.

We think of each island as indexed by wage rates for each type, denoted by $\mathbf{w}_{jt} = \{w_{j\eta t}\}$. Competitive production firms choose the islands on which they locate. Let $\gamma_{jt}$ be the mass of firms that locate in island $j$ in period $t$. Workers and firms on

[5]The assumption that the probability of infection is zero on the home island is purely for convenience. Our results would be unchanged if the probability of infection on the home island were strictly positive but lower than that on the work islands.

island j are matched according to a matching technology $M(L_{jt}, \gamma_{jt})$. We assume that this technology is constant returns to scale. It is convenient to define the market tightness on island j by $\theta_{jt} \equiv \gamma_{jt}/L_{jt}$. As is standard, let $m_w(\theta_{jt}) = M(1, \gamma_{jt}/L_{jt})$ be the probability that a worker is matched with a firm and $m_f(\theta_{jt}) = M(L_{jt}/\gamma_{jt}, 1)$ be the probability that a firm is matched with a worker on island j in period t. Upon being matched, a firm-worker pair produces A units of the final good per unit of time if a positive measure of workers are present on that island and zero otherwise. The assumption that a positive measure of workers must be present in order for production to take place captures the idea that the production process necessarily involves meetings among workers. If we allowed individual workers to produce on their own, it is trivially feasible to produce the final good without any meetings and without any associated infections. Our assumption rules out such trivial possibilities. Finally, we also assume that unmatched workers do not produce but can be infected.

Agents' preferences over the final consumption good are given by

$$U(c) = \sum_{t \geqslant 0} \beta^t u(c_t),$$

where $u(\cdot)$ satisfies the usual assumptions of continuity, monotonicity, differentiability, and concavity. Infected workers suffer an additive utility cost given by $\kappa$.

An allocation is a tuple $Z = (\boldsymbol{\mu}, \boldsymbol{\Theta}, \mathbf{l}, \boldsymbol{\lambda}, \mathbf{c})$, where $\boldsymbol{\mu} \equiv \{\boldsymbol{\mu}_t\}$, $\boldsymbol{\Theta} \equiv \{\theta_{jt}\}$, $\mathbf{l} = \{l_{j\eta t}\}$, $\boldsymbol{\lambda} = \{\lambda_{j\eta t}\}$, $\mathbf{c} \equiv \{c_{\eta t}\}$, and $\boldsymbol{\mu}_t$ is the mass of each type at the beginning of each period t, $\theta_{jt}$ is the market tightness on island j in period t, $l_{j\eta t}$ is the fraction of agents of type $\eta$ allocated to island j in period t, $\lambda_{j\eta t}$ is the mass of type $\eta$ agents on island j, and $c_{\eta t}$ denotes the consumption for an agent of type $\eta$. Note that the mass of firms on any island can be recovered from the labor input of workers and the market tightness on that island.

Clearly, $\lambda_{j\eta t}$ is implied from the state $\mu_{\eta t}$ and the labor allocation $l_{j\eta t}$ as long as $L_{jt} > 0$. We include $\boldsymbol{\lambda}$ as part of the allocation, because in our definition of competitive equilibrium, we will need to endow agents with beliefs about the probability of meeting different types of agents if they happen to choose an island that has no agents in it.

Note that in defining an allocation, we do not consider lotteries over consumption or allow for consumption to depend on the island to which a given worker is allocated. We do so because given our assumption that the utility cost of infection is separable from the utility of consumption, the planner optimally chooses to give all workers of a given type the same consumption level regardless of the realization of the lottery. In our analysis of competitive equilibrium, we assume that perfect insurance markets are available to insure against these realizations. An allocation is feasible if, for all $t$, it satisfies

$$\sum_{\eta} \mu_{\eta t} c_{\eta t} \leqslant \int_{j \neq 0} \sum_{\eta} \left( \mu_{\eta t} m_w \left( \theta_{jt} \right) A l_{j\eta t} \right) dj, \tag{4}$$

$$\int l_{j\eta t} dj = 1, \tag{5}$$

$$\lambda_{j\eta t} = \frac{\mu_{\eta t} l_{j\eta t}}{L_{jt}}, \text{ for all } L_{jt} > 0, \tag{6}$$

and (3). Equation (4) is the resource constraint: it requires that aggregate consumption be less than or equal to aggregate output.

We have described an environment in which the extent of virus exposure depends on the mix of agents of various types in an island. In our environment, it is feasible to allow for any mix of agents without loss of output, and so the mix of agents can be controlled without sacrificing output. In this sense, virus exposure is controllable.

An allocation is associated with a set of *active* islands to which at least one type of agent is assigned with positive probability. Formally, let the set of active islands at time $t$ be denoted by

$$\Gamma_t \equiv \{ j \in \mathcal{J} : l_{j\eta t} > 0 \text{ for some } \eta \in \{S, I, R\} \}.$$

**Definition.** An allocation is Pareto optimal if it is feasible and there is no feasible allocation that makes some type $\eta$ strictly better off without making some other type strictly worse off.

Next, we turn to defining a competitive equilibrium. For simplicity, in doing so we assume that no intertemporal insurance markets are available.[6] Since we allow

[6]We show later that competitive equilibria with restricted markets yield Pareto optimal allocations, so that allowing for intertemporal and insurance markets cannot improve outcomes.

only for static trade, commitment plays no role. We use a recursive representation of each agent's problem. Let $\boldsymbol{\lambda}_t(\boldsymbol{\mu}_t) = \{\lambda_{j\eta t}(\boldsymbol{\mu}_t)\}_j$ denote the fraction of type $\eta$ agents on island j in period t, and let $\boldsymbol{\Theta}_t(\boldsymbol{\mu}_t) = \{\theta_{jt}(\boldsymbol{\mu}_t)\}$ denote the market tightness in each island. Each individual susceptible agent takes as given the fraction of infected agents, market tightness, and the evolution of the state $\boldsymbol{\mu}_{t+1} = G(\boldsymbol{\mu}_t)$. In terms of the labor allocation, we think of each agent as choosing a probability distribution over islands. Let $l_j$ denote the probability that the agent chooses island j. Each susceptible agent chooses consumption and the labor probability distribution to solve

$$V_t(S, \boldsymbol{\mu}_t) = \max_{c, l_j} u(c) + \int_j l_j \left(\psi\left(\lambda_{jIt}\right)\left[-\kappa + \beta V_{t+1}(I, \boldsymbol{\mu}_{t+1})\right] + \beta\left(1 - \psi\left(\lambda_{jIt}\right)\right) V_{t+1}(S, \boldsymbol{\mu}_{t+1})\right) dj,$$

subject to

$$c = \int_{j \neq 0} l_j m_w\left(\theta_{jt}(\boldsymbol{\mu}_t)\right) w_{jSt} dj,$$

$$\int l_j dj = 1.$$

Note that the consumption of the agent is simply the expected value of labor earnings over all work islands. This formulation of the budget constraint captures the idea that agents can insure themselves perfectly in terms of their consumption regardless of the realization of the lottery. That is, insurers can diversify away the idiosyncratic risk associated with the realization of the lottery. We suppress the insurance firms for notational convenience. In defining a competitive equilibrium, it is useful to define the value associated with choosing a particular island j with probability one in period t and returning to the optimal strategy in all future periods for arbitrary beliefs $\hat{\boldsymbol{\lambda}}_t$. This value is given by

$$\hat{V}_t\left(j, S, \boldsymbol{\mu}_t; \hat{\boldsymbol{\lambda}}_t\right) = u(c) + \psi\left(\hat{\lambda}_{jIt}\right)\left[-\kappa + \beta V_{t+1}(I, \boldsymbol{\mu}_{t+1})\right] + \beta\left(1 - \psi\left(\hat{\lambda}_{jIt}\right)\right) V_{t+1}(S, \boldsymbol{\mu}_{t+1}),$$

where

$$c = \begin{cases} m_w\left(\theta_{jt}(\boldsymbol{\mu}_t)\right) w_{jSt}, & \text{if } j > 0 \\ 0, & \text{if } j = 0 \end{cases}.$$

The values for the other types are defined similarly.

**Definition.** An equilibrium is an allocation $(\boldsymbol{\mu}, \boldsymbol{\Theta}, \mathbf{l}, \boldsymbol{\lambda}, \mathbf{c})$, values $\{V_t(\eta, \boldsymbol{\mu}_t)\}_{\eta, t}$, and a measure of active islands $\Gamma_t(\boldsymbol{\mu}_t) = \{j \in \mathcal{J} : l_{j\eta t}(\boldsymbol{\mu}_t) > 0 \text{ for some } \eta \in \{S, I, R\}\}$ such that

1. $l_{\eta t}(\boldsymbol{\mu}_t)$ solves each agent's recursive problem;

2. $m_f(\theta_{jt}) \sum_\eta \lambda_{j\eta t} [A - w_{j\eta t}] \leqslant 0$ for all j, with equality if $j \in \Gamma_t$;

3. for any $j \in \Gamma_t$, $\lambda_{j\eta t}$ satisfies (6);

4. the law of motion $\boldsymbol{\mu}_{t+1} = G(\boldsymbol{\mu}_t)$ for the state is given by (3);

5. for any $j \in \Gamma_t^c$, if $A - w_{j\eta t} > 0$ for all $\eta$ then $m_f(\theta_{jt}) = 0$ and $m_w(\theta_{jt}) = 1$;

6. for $j \in \Gamma_t^c$ such that $\hat{V}_t\left(j, \eta, \boldsymbol{\mu}_t; \hat{\boldsymbol{\lambda}}_t\right) < V_t(\eta, \boldsymbol{\mu}_t)$ for all $\hat{\boldsymbol{\lambda}}_t$, $\lambda_{j\eta t} = 0$.

To understand these conditions, note that conditions 1, 3, and 4 are entirely standard. Condition 2 is a free-entry condition which guarantees that firm profits are non-negative. Conditions 5 and 6 impose our refinements. To understand 5, consider an inactive island $j \in \Gamma_t^c$ such that $A - w_{j\eta t} > 0$ for all $\eta$. Firms would make strictly positive profits if they believed that they would be able to hire workers on that island. The free-entry condition 2 requires that on such an island, either no workers join—that is, $\lambda_{j\eta t} = 0$—or firms believe that the probability that they are matched with workers is zero—that is, $m_f(\theta_{jt}) = 0$. The spirit of this refinement is that on an island where wages are less than marginal product for every type of worker, the mass of potential firms is large relative to the mass of potential workers seeking to locate on that island. This refinement is satisfied if workers are forced to mix across all islands with strictly positive probability through, say, a tremble. In this case, the mass of firms that would seek to locate on an island with strictly positive profits would have to be very large to prevent profits from being arbitrarily large.

To understand refinement 6, consider an inactive island j that makes some type $\eta$ strictly worse off under all possible beliefs. The refinement requires that the fraction of type $\eta$ on that island is zero in equilibrium. This refinement can be thought of as arising from a reasonable restriction on higher-order beliefs. If there is no set of beliefs for which a set of agents of type $\eta$ would switch to island j, then in equilibrium, agents of type $\hat{\eta}$ should reasonably believe that the probability of meeting agents of type $\eta$ in island j is zero.

Next, we characterize the equilibria. A *mixing* equilibrium is one in which there exists some t and some island $j \in \Gamma_t$ such that $l_{jIt} > 0$ and $l_{jSt} > 0$. A *sorting* equilibrium is one in which for all t and all islands $j \in \Gamma_t$, if $l_{jIt} > 0$, then $l_{jSt} = 0$.

An equilibrium has *cross-subsidization* if there exist some $t$ and some $j \in \Gamma_t$ such that for some $\eta$,$\eta'$ with $l_{j\eta t}, l_{j\eta' t} > 0$, we have that $w_{j\eta} < A$ and $w_{j\eta'} > A$.

**Proposition 1** (Characterization)**.** *Any competitive equilibrium features sorting and has no cross-subsidization and no unemployment in the sense that* $m_w(\theta_{jt}) = 1$ *for all* $t$ *and* $j \in \Gamma_t$.

All proofs, except those mentioned below, are in the Online Appendix. In the competitive equilibrium, agents consume $A$ units of the final good in each period, and susceptible agents never get infected. Thus, in equilibrium, all susceptible people are assigned to a separate island in which their wage is given by $A$, all infected people are assigned to a separate island in which their wages are also given by $A$, and recovered people are arbitrarily assigned to any island where their wage is given by $A$. The proof that in equilibrium, susceptible agents are assigned to a separate island relies critically on the idea that the environment allows wages to depend on infection status and allows for islands in which wages are different from productivity $A$. That is, it is feasible to allocate susceptible workers to islands in which wages are lower and virus exposure is also lower than on other islands. This feasibility captures the idea that virus exposure is controllable.

Note that in our baseline formulation, there are no vacancy-posting costs and no involuntary unemployment in equilibrium. In Appendix B, we allow for vacancy posting costs and show that the main results are unchanged.

Given the equilibrium characterization, it follows immediately that the equilibrium is Pareto optimal.

**Proposition 2.** *The competitive equilibrium is Pareto optimal.*

Next, we establish a version of the Second Welfare Theorem. To do so, it is first useful to define histories for all agents. The initial history for any agent simply consists of that agent's type, so $h_0 = \eta_0$. The aggregate initial history is simply $H_0 = \boldsymbol{\mu}_0$. The allocation in period 0 for an individual agent is given by $z_0 = (c_0, \mathbf{l}_0)$, and the firm allocation is given by $\boldsymbol{\gamma}_0$. The individual history $h_t$ is recursively defined as

$$h_t = (h_{t-1}, \eta).$$

The individual-allocation rule specifies the consumption and labor allocation as a function of individual histories: $z_t(h_t) = (c_t(h_t), l_{jt}(h_t))$. The aggregate history is given by

$$H_t = (\boldsymbol{\mu}_t, \boldsymbol{\gamma}_{t-1}, H_{t-1}).$$

The firm-allocation rule specifies firm allocations as a function of aggregate histories, denoted by $\boldsymbol{\gamma}_t(H_t)$. A collection of individual-allocation rules and firm-allocation rules induces a probability distribution over histories denoted by $\pi_t(h_t)$ in the following manner:

$$\pi_{t+1}(h_t, S) = \pi_t(h_{t-1}, S)\left(1 - \int_{j\neq 0} l_{jt}(h_{t-1}, S)\,\chi\lambda_{jIt}dj\right), \tag{7}$$

where

$$\lambda_{jIt} = \frac{\sum_{h_{t-1}} \pi_t(h_{t-1}, I)\, l_{jt}(h_{t-1}, I)}{\sum_{h_{t-1}} \sum_{\eta} \pi_t(h_{t-1}, \eta)\, l_{jt}(h_{t-1}, \eta)},$$

$$\pi_{t+1}(h_{t-1}, S, I) = \pi_t(h_{t-1}, S)\int_j l_{jt}(h_{t-1}, S)\,\chi\lambda_{jIt}dj, \tag{8}$$

$$\pi_{t+1}(h_{t-1}, I, I) = (1-\alpha)\,\pi_t(h_{t-1}, I), \tag{9}$$

$$\pi_{t+1}(h_{t-1}, I, R) = \alpha\pi_t(h_{t-1}, I), \tag{10}$$

$$\pi_{t+1}(h_{t-1}, R, R) = \pi_t(h_{t-1}, R). \tag{11}$$

Thus, given some utility levels $\{\underline{V}(I), \underline{V}(R)\}$, any Pareto optimal allocation solves the following programming problem:

$$\max \sum_{t\geqslant 0} \beta^t \sum_{h_t} \pi(h_t \mid S)\left[u(c_t(h_t \mid S)) - \mathbf{1}_{\{\eta_t = S\}}\int_j l_{jt}(h_t \mid S)\,\psi(\lambda_{jIt})\,dj\kappa - \mathbf{1}_{\{\eta_t = I\}}\kappa\right], \tag{12}$$

where $h_t \mid S$ refers to histories in which $h_0 = S$, subject to

$$\sum_{t\geqslant 0} \beta^t \sum_{h_t} \pi(h_t \mid h_0)\left[\int_j l_{jt}(h_t \mid h_0)\left[u(c_t(h_t \mid h_0)) - \mathbf{1}_{\{\eta_t = I\}}\kappa\right]\right] \geqslant \underline{V}(h_0), \quad h_0 \in I, R,$$

$$\sum_{h_t} \pi(h_t \mid h_0)\, c_t(h_t) \leqslant \sum_{h_t} \pi(h_t \mid h_0)\left[\int_{j\neq 0} m_w(\theta_{jt})\, A l_{jt}(h_t)\,dj\right], \tag{13}$$

$$\int l_{jt}(h_t)\,dj = 1, \tag{14}$$

and (7)-(11). As we vary $\underline{V}(I)$, $\underline{V}(R)$, we can trace out the Pareto frontier.

**Proposition 3.** *Consider any allocation that is Pareto optimal. There exists a lump-sum tax system which supports that outcome as an equilibrium.*

## 2.1 Multiple Occupations and/or Multiple Commodities

For expositional reasons, we have considered a model with a single type of output good and a single input. In this section, we show that our results continue to hold if we allow individuals to differ in the type of the labor input and/or in the types of goods that they consume. To develop this extension, suppose first that the economy has $M$ different types of labor inputs and a single final good. The technology for producing the final good is

$$Y_t = Af(L_1, ..., L_M),$$

where $L_i$ denotes the amount of labor of input type $i$. These input types can be interpreted as occupations. Each household in this economy specializes in the type of labor input that it can supply. The fraction of households that can supply labor of input type $i$ is $\nu_i$.

We assume that the probability of being infected on a particular island depends only upon the aggregate labor supply by infected, susceptible, and recovered people and continues to be given by (2). The probability of infection does not depend on the composition of occupation types on the island. We also assume that the initial fraction of agents who are infected is the same across all occupation types.

The definition of an allocation is unchanged except that we now have to index $\mu, l, \lambda, c$ by the occupation type in addition to the infection type. Wages in each island are now indexed by both the occupation type and the infection type. With this definition of wages and allocations, the definition of competitive equilibrium is essentially the same as in the single-occupation-type economy.

It is immediate that if both the infection probabilities and the initial distribution of infected agents are independent of occupation type, Proposition 1 continues to hold in the sense that the the competitive equilibrium has sorting, no cross-subsidization and no unemployment. It is also immediate that the analogs of the First and Second Welfare Theorems hold.

We can also straightforwardly extend our economy to an economy with multiple consumption goods. To see how the economy can be extended, suppose that the economy has $N$ different types of goods. The technology for producing each one of these goods is given by

$$Y_i = A_i L,$$

where the subscript $i$ denotes the type of consumption good. Households' utility over these consumption goods is given by $u(c_1, ..., c_N)$. An allocation is defined in a similar way to that above. A competitive equilibrium now consists of a vector of prices for each consumption good as well as wage rates. Again, it is straightforward to prove that Proposition 1 and the welfare theorems continue to hold in this environment.

## 3 A Pandemic Model with Imperfect Observability

In the previous section with observable types, the equilibrium allocation had perfect separation between susceptible and infected agents, which immediately implied the welfare theorems. We show that the efficiency results continue to hold even if we allow for asymptomatic infected agents who are indistinguishable from susceptible agents. The possible agent types are now described by $\eta \in \{U_S, U_I, I, R\}$ with initial masses denoted by $\mu_{\eta 0}$. Let $U_S$ denote susceptible agents and $U_I$ denote infected asymptomatic agents. Agents of type $U_S$ and $U_I$ cannot be distinguished from each other and therefore must receive the same allocations. Let $U$ denote the type of such an agent, which we refer to as the *unknown* type, and $\mu_{Ut} = \mu_{U_S t} + \mu_{U_I t}$ for all $t$. Let $I$ denote the symptomatic infected agents and $R$ denote the recovered agents. We assume that recovered agents can be identified even if they were previously asymptomatic. This assumption is purely for convenience. Agents of type $U_I$ become symptomatic with probability $\phi$ and recover with probability $\alpha$. Similarly, newly infected agents are also symptomatic with probability $\phi$.

We assume that the economy has a large number of *insurance* firms. We introduce these firms for two reasons. First, they provide agents the opportunity to purchase inter-temporal insurance contracts. With imperfectly observable types, agents can get infected in equilibrium and thus would like to insure themselves. Second, as we discuss below, insurance firms help solve the coordination problem that arises in environments with local public goods. Neither the insurance motive nor the coordination issues arise with perfectly observable types, and thus we abstracted from insurance firms in the previous section to simplify notation.

We assume that competitive insurance firms offer *contracts* in period 0 contingent on the entire history of an individual. Both firms and workers are commited to these contracts. The relevant individual history in period $t$ is given by $h_t = (\eta_0, ..., \eta_t)$, with $\eta_t \in \{U, I, R\}$. Insurance firms offer contracts that specify consumption and labor for each individual history taking as given the public history. Let $z = (\mathbf{c}, \mathbf{l}) = \{c_t(h_t), l_{jt}(h_t)\}$ denote an arbitrary contract, and let $z^* = (\mathbf{c}^*, \mathbf{l}^*)$ denote the equilibrium contract.

Let

$$\lambda_{Ijt} = \frac{\sum_{h_{t-1}} [\pi_t(h_{t-1}, U_I) l_{jt}(h_{t-1}, U) + \pi_t(h_{t-1}, I) l_{jt}(h_{t-1}, I)]}{\sum_{h_{t-1}} \sum_{\eta} [\pi_t(h_{t-1}, \eta) l_{jt}(h_{t-1}, \eta)]}$$

denote the fraction of infected agents on island $j$, where infected agents include those who are symptomatic as well as those who are asymptomatic. An allocation induces an evolution of the masses of various types of agents according to

$$\pi_{t+1}(h_{t-1}, U_S, U_I) = (1-\phi)\pi_t(h_{t-1}, U_S) \int_{j\neq 0} l_{jt}(h_{t-1}, U) \chi \lambda_{Ijt} dj, \tag{15}$$

$$\pi_{t+1}(h_{t-1}, U_S, I) = \phi \pi_t(h_{t-1}, U_S) \int_{j\neq 0} l_{jt}(h_{t-1}, U) \chi \lambda_{Ijt} dj, \tag{16}$$

$$\pi_{t+1}(h_{t-1}, U_S, U_S) = \pi_t(h_{t-1}, U_S) \left[1 - \int_{j\neq 0} l_{jt}(h_{t-1}, U) \chi \lambda_{Ijt} dj\right], \tag{17}$$

$$\pi_{t+1}(h_{t-1}, U_I, U_I) = (1-\phi)(1-\alpha)\pi_t(h_{t-1}, U_I), \tag{18}$$

$$\pi_{t+1}(h_{t-1}, U_I, I) = \phi(1-\alpha)\pi_t(h_{t-1}, U_I), \tag{19}$$

$$\pi_{t+1}(h_{t-1}, U_I, R) = \alpha \pi_t(h_{t-1}, U_I), \tag{20}$$

$$\pi_{t+1}(h_{t-1}, R, R) = \pi_t(h_{t-1}, R). \tag{21}$$

An allocation is resource feasible if

$$\sum_{h_t} \pi_t(h_t) c_t(h_t) \leqslant \sum_{h_t} \pi_t(h_t) \int_{j\neq 0} m_w(\theta_{jt}) A l_{jt}(h_t) dj.$$

Production firms are identical to those described earlier. Let $L^*_{jt}$ be the mass of workers allocated to island j in period t by all other insurance firms. An individual insurance firm takes $L^*_{jt}$ as given, as well as the masses of agents of various types employed by other firms. If $L^*_{jt} > 0$, then firms take as given the relative mass of infected agents in that island, $\lambda^*_{jIt}$. This relative mass is given by

$$\lambda^*_{jIt} = \frac{\sum_{h_{t-1}} \left[\pi^*_t\left(h_{t-1}, U_I\right) l^*_{jt}\left(h_{t-1}, U\right) + \pi^*_t\left(h_{t-1}, I\right) l^*_{jt}\left(h_{t-1}, I\right)\right]}{L^*_{jt}}, \tag{22}$$

where

$$L^*_{jt} = \sum_{h_{t-1}} \sum_{\eta} \left[\pi^*_t\left(h_{t-1}, \eta\right) l^*_{jt}\left(h_{t-1}, \eta\right)\right].$$

In other words, an individual insurance firm does not internalize the effect of its choices on the infection probability on an island in which there is a positive mass of agents signed by other insurance firms.

Let $\tilde{\pi}_0\left(h_0\right)$ denote the mass of agents of type $h_0$ that an individual insurance firm attracts in period 0. Given these initial masses, let $\tilde{\pi}_t\left(h_{t-1}, h_t\right)$ denote the mass of agents associated with history $\left(h_{t-1}, h_t\right)$ who are signed by this firm. These masses evolve according to (15)-(21) if $L^*_{jt} > 0$. If $L^*_{jt} = 0$, we assume that the relative mass of infected agents on that island is given by

$$\lambda_{jIt} = \frac{\sum_{h_{t-1}} \left[\tilde{\pi}_t\left(h_{t-1}, U_I\right) l_{jt}\left(h_{t-1}, U\right) + \tilde{\pi}_t\left(h_{t-1}, I\right) l_{jt}\left(h_{t-1}, I\right)\right]}{\sum_{h_{t-1}} \sum_{h_t} \left[\tilde{\pi}_t\left(h_{t-1}, h_t\right) l_{jt}\left(h_{t-1}, h_t\right)\right]}. \tag{23}$$

Thus, in this case, the insurance firm internalizes the effects of its choices on the infection probability on that island. This arises from our assumption that pandemics generate local externalities and that an insurance firm can no longer be considered "small" on an island if it is the only one allocating agents to that particular island.

A contract is an allocation for a particular firm i, denoted by $z_i$. It is convenient here to define explicitly the problem these firms solve, which is

$$\max_{z, \tilde{\pi}_0(\eta_0)} \left( \sum_{t \geqslant 0} Q_t \sum_{h_0} \sum_{h_t} \tilde{\pi}\left(h_t \mid h_0\right) \left[ \int_{j \neq 0} m_w\left(\theta_{jt}\right) A l_{jt}\left(h_t \mid h_0\right) - c_t\left(h_t \mid h_0\right) \right] \right), \tag{24}$$

subject to

$$\tilde{\pi}_0(h_0)\sum_{t\geqslant 0}\beta^t\sum_{h_t}\frac{\tilde{\pi}_t(h_t\mid h_0)}{\tilde{\pi}_0(h_0)}\nu_t(h_t\mid h_0)\geqslant\tilde{\pi}_0(h_0)\underline{V}(h_0),\qquad\forall h_0,$$

where

$$\nu_t(h_t\mid h_0)\equiv u(c_t(h_t\mid h_0))-\int_{j\neq 0}l_{jt}(h_t\mid h_0)\mathbf{1}_{\{\eta_t=U_S\}}\psi(\lambda_{jIt})\kappa-\mathbf{1}_{\{\eta_t=U_I,I\}}\kappa,\quad(25)$$

and (15)-(21), where $\lambda_{jIt}$ is given by (23) if $L^*_{jt}=0$ and (22) if $L^*_{jt}>0$.

As in the case in which types are perfectly observable, virus exposure is controllable in that it is feasible to allow for any mix of agents of various publicly observable types without loss of output. Thus, virus exposure can be controlled without sacrificing output.

We now define a competitive equilibrium.

**Definition.** A competitive equilibrium is an allocation $z^*$, prices $Q_t$, market tightness $\boldsymbol{\Theta}$, and market utilities $\{\underline{V}(h_0)\}_{h_0}$ such that

1. given prices $Q_t$ and market utilities $\underline{V}(h_0)$, $\{z^*,\pi_0(h_0)\}$ solves (24);

2. $m_f(\theta_{jt})\sum_\eta\lambda_{j\eta t}[A-w_{j\eta t}]\leqslant 0$ for all j, with equality if $j\in\Gamma_t$;

3. $\{\underline{V}(h_0)\}_{h_0}$ is such that the firms make zero profits; that is, the value of (24) is zero;

4. the resource constraint is satisfied:
$$\sum_{h_t}\pi_t(h_t)c_t(h_t)=\sum_{h_t}\pi_t(h_t)\int_{j\neq 0}m_w(\theta_{jt})Al_{jt}(h_t)\,dj,\ \forall t;$$

5. for any $j\in\Gamma^c_t$, if $A-w_{j\eta t}>0$ for all $\eta$ then $m_f(\theta_{jt})=0$ and $m_w(\theta_{jt})=1$.

Without loss of generality, we can consider a representative insurance firm that allocates agents to islands in which $L^*_{jt}=0$. In the following proposition, we show that any competitive equilibrium features no involuntary unemployment and no mixing between U- and I-type agents.

**Proposition 4** (Characterization)**.** *In any competitive equilibrium, there is no mixing between* $U$*- and* $I$*-types, and no unemployment in the sense that* $m_w(\theta_{jt}) = 1$ *for all* $t$ *and* $j \in \Gamma_t$.

Since mixing the $U$ and $I$ types on the same island increases the infection probability for the $U$ types without any additional benefit, it is always optimal to separate these two types. By contrast, pooling $U$ and $R$ agents is valuable, since such pooling lowers the infection probability for the $U$ types and leaves the $R$ types unaffected. This observation suggests that firms will pool these types in equilibrium. We now prove the existence of such a pooling equilibrium; moreover, in this equilibrium the initial $U$ agents pay a premium in order to pool with the initially recovered agents. We also show that this equilibrium is efficient. To do so, it is useful to define the maximum value that the initially unknown types would receive if they never mixed with initially infected or initially recovered agents. This value is

$$\underline{V}(U) = \max \sum_{t \geqslant 0} \beta^t \sum_{h_t} \pi(h_t \mid U) \left[ u(c_t(h_t \mid U)) - l_t(h_t \mid U) \mathbf{1}_{\{\eta_t = U_S\}} \psi(\lambda_{It}) \kappa - \mathbf{1}_{\{\eta_t = U_I, I\}} \kappa \right],$$

subject to

$$\sum_{h_t} \pi(h_t \mid U)(c_t(h_t \mid U) - l_t(h_t \mid U) A) \leqslant 0, \quad \forall t,$$

and

$$\lambda_{It} = \frac{\sum_{h_{t-1}} [\pi_t(h_{t-1}, U_I) l_t(h_{t-1}, U)]}{\sum_{h_{t-1}} \sum_{\eta = \{U, R\}} [\pi_t(h_{t-1}, \eta) l_t(h_{t-1}, \eta)]}.$$

Similarly, let $\underline{V}(R)$ and $\underline{V}(I)$ define the maximum values that the initially recovered and symptomatic infected agents would receive if they did not mix with any other agents. These are:

$$\underline{V}(R) = \sum_{t=0}^{T} \beta^t u(A),$$

$$\underline{V}(I) = \sum_{t \geqslant 0} \beta^t \sum_{h_t} \pi_t(h_t \mid I) \left[ u(A) - \mathbf{1}_{\{\eta_t = I\}} \kappa \right].$$

Finally, let $V_\eta$ denote the equilibrium value for the initial $\eta$ types.

**Proposition 5.** *A competitive equilibrium with the following properties exists:*

- *There is mixing between* $U$ *and* $R$ *type agents.*

- $V_U > \underline{V}(U)$, $V_R > \underline{V}(R)$, *and* $V_I = \underline{V}(I)$.
- *The equilibrium is efficient.*

*Proof.* See Appendix. □

In such a competitive equilibrium, both initially unknown and initially recovered types are made strictly better off than if they were on their own. In particular, the initially recovered types consume an amount greater than their marginal product, while the initially unknown types consume an amount less than their marginal product. This is because the unknown-type agents are willing to give up some consumption in order to pool with recovered-type agents, since they reduce the infection probability. Note that if there were an initial mass of vaccinated agents, then they would be identical to recovered agents and so would also get higher consumption in equilibrium.

We emphasize that commitment by firms and workers is critical in ensuring efficiency. Obviously, if firms cannot commit, they will not honor contracts that generated negative present value of profits for any history. Later, we show that the equilibrium is inefficient if workers cannot commit. We also emphasize that insurance firms play a crucial role in solving the coordination problem that arises because of these local externalities. If agents were to individually decide how to allocate their time across islands, equilibria in which unknown-type and recovered agents are on separate islands could arise. In such cases, forcing recovered and unknown-type agents to inhabit the same island would make both types strictly better off. Insurance firms solve this coordination problem, since they understand how infections transmit between agents who sign contracts with them. Similar coordination problems arise in the local public goods literature and are solved by the presence of "clubs."

## 3.1 Efficiency with Private Information

Suppose now that the types of $U$ and $I$ agents are privately observed and that the type of $R$ agents is public information. We show that the equilibrium with public information described above is also an equilibrium with private information. To see this result, note that in the equilibrium with public information, type-$I$ agents receive consumption equal to $A$, and type-$U$ agents receive consumption less than or equal to $A$. Thus, type-I agents prefer to reveal their type rather than pretend to be of type $U$, and $U$-type agents prefer to reveal their type rather than pretend to be I. The reason is that in the equilibrium with public information, they receive a utility at least as large as that if they pooled with I types. The assumption that $R$ types are public information is important because otherwise an I type would have an incentive to pretend to be an $R$ type. Clearly, the equilibrium is efficient. We summarize these results in the following proposition.

**Proposition 6.** *Suppose that* $U$ *and* $I$ *types are private information, but the* $R$ *type is public information. Then, the equilibrium characterized in Proposition 5 is still an equilibrium and is efficient.*

# 4 A Pandemic Model without Controllability

A crucial assumption underlying the efficiency results in the previous section is that virus exposure is controllable, so pandemics generate local externalities. In this section, we consider a special case of our model in which virus exposure is not controllable, so pandemics generate (almost) global externalities. This is the assumption made by much of the economics literature studying pandemics. While it is unsurprising that equilibria will typically be inefficient, we show that the implications for optimal policy are more subtle than those in that literature.

Consider a version of our benchmark model with only one work island, denoted $j = 1$, and a home island, denoted $j = 0$.[7] We assume that the work island is indexed by $w_{1\eta} = A$ for all $\eta \in \{U, I, R\}$. Note that in this environment, it is not feasible to allow for any mix of agents of various types without loss of output. That is, the mix of agents cannot be controlled without sacrificing output. In this sense, virus exposure is not controllable.

As in the previous section, insurance firms offer contracts that specify consumption and labor allocations for each individual history, taking as given the public history. Let $z = (\mathbf{c}, \mathbf{l}) = \{c_t(h_t), l_t(h_t)\}$ denote an arbitrary contract, and let $z^* = (\mathbf{c}^*, \mathbf{l}^*)$ denote the equilibrium contract. Since we have only one work island, we let $l_t$ denote the labor allocation to the work island and $1 - l_t$ denote the labor allocation to the home island. We think of competitive insurance firms

[7]The externalities here are almost global, because agents are exposed to the virus only if they are present on the work island.

as simultaneously offering contracts to all agents in period 0 as functions of their initial histories $h_0 = \eta_0$. Each agent chooses the contract that offers the highest utility. Let $V_0(h_0)$ denote the highest utility offered by all other firms. We think of this highest utility as the market price. Let $Q_t$ denote the Arrow-Debreu price for consumption in period $t$. Insurance firms trade with each other at these prices. An individual firm takes as given these market prices and the labor allocations chosen by other firms when offering a contract that maximizes expected profits. These labor allocations induce infection probabilities according to

$$\psi\left(\lambda_{It}^{*}\right) = \chi \frac{\sum_{h_{t-1}} \left[\pi_t\left(h_{t-1}, I\right) l_t^{*}\left(h_{t-1}, I\right) + \pi_t\left(h_{t-1}, U_I\right) l_t^{*}\left(h_{t-1}, U\right)\right]}{\sum_{h_{t-1}} \sum_{\eta} \pi_t\left(h_{t-1}, \eta\right) l_t^{*}\left(h_{t-1}, \eta\right)}.$$

Let $\tilde{\pi}(h_0)$ denote the mass of agents of type $\eta_0$ attracted to this particular contract, and let $\tilde{\pi}(h_t \mid h_0)$ be the corresponding mass of these agents in period $t$ with history $h_t$. The individual firm takes market prices and the equilibrium allocations induced by the choices of other firms as given and solves

$$\max \left( \sum_{t \geqslant 0} Q_t \sum_{h_0} \sum_{h_t} \tilde{\pi}\left(h_t \mid h_0\right) \left[l_t\left(h_t\right) A - c_t\left(h_t\right)\right] \right), \tag{26}$$

subject to

$$\tilde{\pi}_0\left(h_0\right) \sum_{t \geqslant 0} \beta^t \sum_{h_t} \frac{\tilde{\pi}_t\left(h_t \mid h_0\right)}{\tilde{\pi}_0\left(h_0\right)} \nu_t\left(h_t \mid h_0\right) \geqslant \tilde{\pi}_0\left(h_0\right) \underline{V}\left(h_0\right), \qquad \forall h_0,$$

and the laws of motion for types given by (15)-(21) for the single-work-island case in which the fraction of infected agents is given by $\lambda_{It}^{*}$. Obviously, in equilibrium, $\tilde{\pi}_0(h_0) = \mu_{\eta 0}$.

## 4.1 Competitive Equilibrium

**Definition 1.** A competitive equilibrium is then an allocation $z^{*}$, prices $Q_t$, and market utilities $\{\underline{V}(h_0)\}_{h_0}$ such that

1. given $Q_t$ and market utilities $\underline{V}(h_0)$, $\{z^{*}, \pi_0(h_0)\}$ solves (26);

2. the resource constraint is satisfied:

$$\sum_{h_t} \pi(h_t) [l_t(h_t) A - c_t(h_t)] = 0, \quad \forall t, h_t.$$

Note that if the resource constraint is satisfied, the insurance firms make zero profits.

**Proposition 7.** *In any competitive equilibrium, there is no cross-subsidization, and symptomatic infected and recovered agents supply one unit of labor in the work island in all periods. If* $u'(0) A > \kappa/(1-\beta)$*, there is mixing in the sense that unknown-type agents work a positive amount in at least one period.*

The proposition says that in contrast to the controllable case, unknown-type and symptomatic infected agents will mix in equilibrium.

## 4.2 Efficiency of Competitive Equilibrium

We now show that when competition across islands is restricted, competitive equilibria are in general inefficient. A Pareto optimal allocation for the model without controllability is defined identically to that in the model with controllability with the restriction that the economy has only one work island.

**Proposition 8.** *The competitive equilibrium is inefficient.*

The reason for the failure of the First Welfare Theorem is that private insurance firms do not internalize the effect of the choice of labor supply on the probability of infection. For example, increasing the labor supply of symptomatic infected agents increases the probability of infection for susceptible agents and thus lowers their welfare. This effect is not internalized by private firms offering contracts to individual agents.

While the presence of an externality in the model without controllability is clear, the relationship between the optimal and the equilibrium labor supply allocation is more subtle. We illustrate this result by comparing the optimal and equilibrium allocations when the planner has access to a full set of instruments with those when the planner has access only to untargeted Pigouvian taxes.

First, we characterize the set of Pareto optimal allocations that maximize the welfare of unknown-type agents while leaving symptomatic infected and recovered agents at least as well off as in the competitive equilibrium. To do so, we develop an assumption that ensures that the pandemic is sufficiently costly that lockdowns are desirable. We start from an allocation in which all agents are working and the proportion of symptomatic infected agents is $\varepsilon \in (0,1)$. In this case, reducing the mass of symptomatic infected agents who work to zero confers a utility gain of $\chi\kappa\varepsilon$ to each unknown susceptible agent. For symptomatic infected agents to be at least as well off as in the competitive equilibrium, each of them must be compensated with $A\varepsilon$ units of consumption. This amount can be raised by reducing the consumption of each unknown-type agent by $A\varepsilon/\mu_{U0}$. The utility cost of this reduction is $u'(A)\,A\varepsilon/\mu_{U0}$ to a first order. We say that pandemics are socially costly if the marginal benefits from this policy exceed its marginal costs. In our model, the condition for pandemics to be socially costly is

$$u'(A)\,\frac{A}{\mu_{U0}} - \mu_{U_S 0}\chi\kappa < 0. \tag{27}$$

Under the assumption that pandemics are socially costly, we show that if the measure of initially infected agents is sufficiently small and the probability of becoming symptomatic sufficiently large, optimal policy requires all symptomatic-infected agents to stay at home.

**Lemma 1.** *Suppose that (27) holds and $\phi$ is sufficiently large. Then, there exists some $\mu_I^*, \mu_{U_I}^* > 0$ such that if $\mu_{I0} < \mu_I^*$ and $\mu_{U_I0} < \mu_{U_I}^*$, the solution to the Pareto problem has no symptomatic infected agents working, and unknown-type agents working more than in the competitive equilibrium. Recovered agents supply one unit of labor in both the equilibrium and the Pareto optimal allocation.*

This lemma shows that if pandemics are socially costly and the fraction of initially infected agents is sufficiently small, the policy that maximizes the welfare of the unknown-type agents while leaving symptomatic infected and recovered agents at least as well of as in the competitive equilibrium involves subsidizing symptomatic infected agents to stay home and allowing all unknown and recovered types to work. When the share of initial asymptomatic infected agents $\mu_{U_I0}$ is small and the probability of becoming symptomatic is large, unknown-type agents work more than they would in the competitive equilibrium. The payments to the symptomatic infected agents are obtained by taxing unknown-type agents at work.

Note the sharp contrast with the competitive equilibrium, where all infected agents always work.

## 4.3 Simple Pigouvian Taxes

Suppose that the government does not have access to targeted policies and can only levy untargeted Pigouvian taxes on labor income. We first consider the optimal Pigouvian taxes in a static model and then consider the dynamic model.

### 4.3.1 Pigouvian taxes in a static model

Assume that the government can impose a linear tax on labor income $\tau$ on all agents, with proceeds rebated lump-sum to the agents. Since we are interested in the role of these taxes to correct externalities, we abstract from redistributive concerns and assume that the tax revenue collected from a particular type is rebated lump-sum to that type.

Standard duality arguments imply that one can write the insurance firm's problem as maximizing the welfare of the initially unknown types subject to a budget constraint and the participation constraints for the other agents. From Proposition 7, we know that all symptomatic infected and recovered agents supply one unit of labor and consume $A$ units of the consumption good. Thus, the problem of the firm is

$$\max_{l_0(U)\in[0,1]} u\left(\left(1-\tau\right)Al_0\left(U\right)+T\right)-\xi_0 l_0\left(U\right)\chi\lambda_I^*\kappa-\left(1-\xi_0\right)\kappa, \tag{28}$$

where $\xi_0 = \mu_{U_S 0}/\mu_{U0}$ and

$$\lambda_I^* = \frac{\mu_{I0}+\mu_{U_I 0}l_0^*\left(U\right)}{\mu_{U0}l_0^*\left(U\right)+\mu_{I0}+\mu_{R0}}.$$

Note that all symptomatic infected and recovered agents continue to supply one unit of labor, since working does not yield disutility to these agents. The first-order condition for this problem is

$$u'\left(\left(1-\tau\right)Al\left(U\right)+T\right)A\left(1-\tau\right)-\xi_0\chi\lambda_I^*\kappa=0. \tag{29}$$

Let $l\left(U;\tau,T,l^*\left(U\right)\right)$ denote the best-response function that solves this problem. It must be that

$$l^*(U;\tau) = l(U;\tau,\tau l^*(U), l^*(U)). \tag{30}$$

Note that $l^*(U;0)$ refers to the equilibrium labor supply with no taxes. It is straightforward to show that the optimal untargeted Pigouvian tax implements the solution to the following problem:

$$\max_{l}\ u(Al) - \xi_0 l\chi \frac{\mu_{I0} + \mu_{U_I 0} l}{\mu_{U0} l + \mu_{I0} + \mu_{R0}} \kappa - (1-\xi_0)\kappa. \tag{31}$$

Let $l^{opt}(U)$ be the solution to this problem. We first consider a local perturbation of $l$ around the equilibrium labor supply with no taxes, $l^*(U;0)$. Consider the total derivative of (31):

$$u'(lA)\,A dl - \xi_0 \chi \lambda_I \kappa dl - \xi_0 \chi \frac{\partial \lambda_I}{\partial l} \kappa dl. \tag{32}$$

Evaluating (32) at $l = l^*(U;0)$ and using the first order condition (29) yields $-\xi_0\chi\partial\lambda_I/\partial l \kappa dl$. We have

$$\frac{\partial \lambda_I}{\partial l} = \mu_{U0} \frac{(\mu_{I0} + \mu_{R0})\,\mu_{U_I 0} - \mu_{I0}}{(\mu_{U0} l + \mu_{I0} + \mu_{R0})^2},$$

which is negative iff $\mu_{U_I 0} \leqslant \mu_{I0}(\mu_{I0} + \mu_{R0})^{-1}$. Thus, if $\mu_{U_I 0}$ is small, at least locally, the welfare of the $U$ agents can be increased by forcing them to work more than in the competitive equilibrium. This result establishes that the conventional wisdom that economic activity in the competitive equilibrium is always too high relative to the efficient level is incorrect, at least in the static model. Our result shows that economic activity can be *lower* than the efficient level, at least locally. The reason for our result is straightforward. When the mass of asymptomatic infected agents is relatively small, increasing the mass of $U$ agents who work reduces the probability that any susceptible agent will be infected. In effect, in our economy, the labor supply of unknown-type agents induces a positive congestion externality by increasing the probability that a susceptible agent will meet a susceptible agent and reducing the probability that a susceptible agent will meet a symptomatic infected agent. In a competitive equilibrium, no individual unknown-type agent internalizes this effect.

Under restrictions on policies, it turns out that for a wide variety of utility functions, the economic activity is globally too low in the competitive equilibrium.

**Lemma 2.** *Suppose that* $u(c) = \log(c)$. *Then, for sufficiently small* $\mu_{U_I0}$, $l^{opt}(U) > l^*(U;0)$. *Thus, the optimal Pigouvian policy is a subsidy on labor.*

For utility functions of the form $u(c) = c^{1-\sigma}/(1-\sigma)$ we have computed a variety of numerical examples and shown that in every case, for sufficiently small $\mu_{U_I0}$, $l^{opt}(U) > l^*(U;0)$.

### 4.3.2 Pigouvian taxes in a dynamic model

Consider next the inefficiency in the labor supply of susceptible agents in the dynamic model. For ease of exposition, for this subsection we assume that $\mu_{U_I} = 0$ and $\phi = 1$ so that there are no asymptomatic agents. However all our results go through if $\mu_{U_I}$ is sufficiently small and $\phi$ is sufficiently close to one. Let $\lambda^*_{It}$ be the equilibrium fraction of infected agents relative to the total mass of agents on the work island, which is given by

$$\lambda^*_t = \frac{\mu^*_{It}}{\mu^*_{Ut} l^*_t(U) + \mu^*_{It} + \mu^*_{Rt}}$$

since symptomatic infected and recovered agents always work. Let the mass of agents associated with an individual firm who were initially of type $i \in \{U, I, R\}$ be denoted by $\boldsymbol{\pi}_t = (\pi_t(U), \pi_t(I), \pi_t(R))$. As in the static case, initial I and R agents work one unit on all future periods and consume A. We can then write the dual problem of the firm recursively as choosing $(c_\eta, l_\eta)$ to maximize

$$V_t(\boldsymbol{\pi}_t, \lambda^*_{It}) = \max_{c_t(\eta), l_t(\eta)} \sum \pi_t(\eta) \left[ u(c_t(\eta)) - l_t(U)\psi(\lambda^*_{It})\kappa \mathbf{1}_{\{\eta=U\}} - \kappa \mathbf{1}_{\{\eta=I\}} \right] \quad (33)$$
$$+ \beta V_{t+1}\left(\boldsymbol{\pi}_{t+1}, \lambda^*_{It+1}\right),$$

subject to

$$\sum_\eta \pi_t(\eta) c_t(\eta) \leqslant \sum_\eta \pi_t(\eta) l_t(U)(1-\tau)A + T$$

and

$$\boldsymbol{\pi}_{t+1} = \begin{bmatrix} \pi_{t+1}(U) \\ \pi_{t+1}(I) \\ \pi_{t+1}(R) \end{bmatrix} = \begin{bmatrix} 1 - l_t(U)\psi(\lambda^*_{It}) & 0 & 0 \\ l_t(U)\psi(\lambda^*_{It}) & 1-\alpha & 0 \\ 0 & \alpha & 1 \end{bmatrix} \begin{bmatrix} \pi_t(U) \\ \pi_t(I) \\ \pi_t(R) \end{bmatrix}. \quad (34)$$

It is straightforward to show that for any $t > 0$, $c_\eta = c_{\eta'}$. The first order condition for $l_t(U)$ (noting that $l_t(I) = l_t(R) = 1$) is:

$$\pi_t(U)\, u'(c_t(U))\, A(1-\tau) - \psi(\lambda^*_{It})\,\kappa - \beta \frac{\partial V_{t+1}}{\partial \pi_{t+1}(U)} \psi(\lambda^*_{It})$$
$$+\beta\psi(\lambda^*_{It}) \left( \pi_t(U) \frac{\partial V_{t+1}}{\partial \pi_{t+1}(U)} - \pi_t(I) \frac{\partial V_{t+1}}{\partial \pi_{t+1}(I)} \right) = 0.$$

As in the static problem, the optimal Pigovian tax chosen at time $t$ implements the solution to a planning problem in which the planner chooses $l_t(U)$ while internalizing its effect on the infection probabilities.

We consider a local perturbation of $l_t(U)$ around the equilibrium choice $l^*_t(U; 0)$. This welfare change is given by the total derivative of (33). Note that changing $l^*_t(U; 0)$ affects welfare in three ways. The first is the effect on the probability of infection for susceptible agents in the current period. The second is that $l^*_t(U; 0)$ affects the transition law for the mass of agents of various types, (34), and hence affects the probability of infection in future periods. Evaluating the total derivative at the competitive-equilibrium allocation, we obtain

$$\underbrace{-\mu^*_{Ut} l^*_t(U;0)\, \psi'(\lambda^*_{It}) \frac{\partial \lambda^*_{It}}{\partial l^*_t(U;0)} \kappa + \beta \left[ \frac{\partial V_{t+1}}{\partial \pi_{It+1}} - \frac{\partial V_{t+1}}{\partial \pi_{Ut+1}} \right] \mu^*_{Ut} l^*_t(U;0)\, \psi'(\lambda^*_{It}) \frac{\partial \lambda^*_{It}}{\partial l^*_t(U;0)}}_{\text{externality from current infection}}$$
$$+ \underbrace{\beta \frac{\partial V_{t+1}}{\partial \lambda^*_{It+1}} \frac{\partial \lambda^*_{It+1}}{\partial l^*_t(U;0)}}_{\text{externality from future infection}}.$$

See the proof of Lemma 3 for details. Whether such a local perturbation increases welfare depends on the combination of the two externalities that arise in the competitive equilibrium. The first arises because increasing the labor supply of unknown-type agents in the current period changes the infection probability implied by $\lambda^*_{It}$ for all susceptible agents in the current period. We label this the *externality from current infection*. The second externality arises because increasing the labor supply by unknown-type agents in the current period changes the infection probability in future periods implied by $\lambda^*_{t+1}, \lambda^*_{t+2}, \ldots$. We label this effect the *externality from future infection*.

The current-infection externality in turn has a static component, which is identical to that in the static model, and a dynamic component. Given that $\mu_{U,0} = 0$ and $\phi = 1$, the sign of the static component is positive, since an increase in the mass of unknown-type agents has a positive congestion externality, as discussed earlier. The dynamic component arises because a change in aggregate labor supply alters the infection rates for each individual firm and affects the masses of various types of agents associated with that firm. While the sign of the dynamic component is ambiguous in general, in the appendix, we show that if $\alpha$ is sufficiently small or $T = 2$, this term is positive.

The future-infection externality arises because the change in $l_t^* (U; 0)$ affects the probability of infection in future periods. It is straightforward to show that the continuation value is decreasing in the probability of infection in future periods implied by $\lambda_{t+1}^*$. Since an increase in current labor supply reduces the relative mass of susceptible to infected agents, it is easy to show that the infection probability is increasing in $l_t^* (U; 0)$. Thus, in contrast to the static model, in the dynamic model reducing the labor supply of unknown-type agents may raise welfare. We summarize these results in the following lemma.

**Lemma 3.** *The effect on welfare of a small increase in labor supply of unknown-type agents from the competitive equilibrium can be decomposed into an externality from current infection and an externality from future infection. If $\alpha$ is sufficiently small or $T = 2$, welfare rises because of the externality from current infection and falls because of the externality from future infection. The overall effect on welfare is ambiguous.*

In Figure 1, we plot the period-0 aggregate labor supply in the competitive equilibrium, the Pareto optimal allocation, and under the optimal Pigouvian policies as we vary parameters in our model. We see that, quite generally, aggregate labor supply under these policies is *larger* than in the competitive equilibrium.[8] Thus, economic activity is inefficiently too low in the competitive equilibrium. The conventional presumption in the pandemic literature is that just like those caused by greenhouse gases, the externalities created by pandemics cause economic activity to be too high in a competitive equilibrium. The policy implication in this literature is that curbing economic activity through lockdowns and other such measures is beneficial. We have shown that economic activity is too low and that stimulating it through various policies may well be beneficial.

[8]In Appendix C, we plot the aggregate labor supply in the second period as well as the labor supply for each type.

Figure 1: Employment in the first period as a function of parameters.

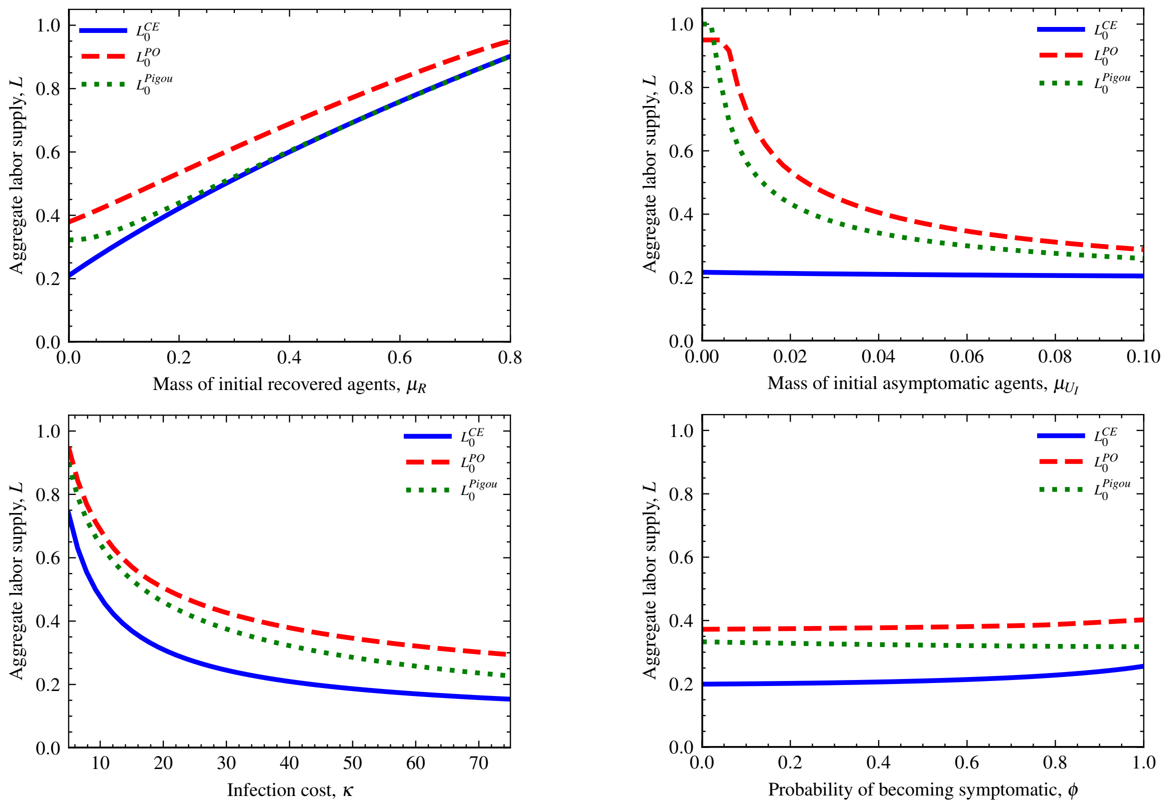


**Figure Notes**. Aggregate employment $L^{CE}$ in the competitive equilibrium, $L^{PO}$ in the Pareto optimum, and $L^{Pigou}$ with optimal Pigouvian taxes. Default parameter values: $\sigma = 2$ (CRRA), $\beta = 0.9$ (discount factor), $A = 2$ (productivity of labor), $\chi = 1$ (infectivity rate), $\kappa = 40$ (cost of infection), $\phi = 0.5$ (symptomatic rate), $\alpha = 0.1$ (recovery rate), $\mu_{U_S} = 0.9025$ (mass of unknown susceptible), $\mu_{U_I} = 0.0475$ (mass of asymptomatic infected), $\mu_I = 0.05$ (mass of symptomatic infected). The upper-left panel varies the initial mass of recovered agents, $\mu_R$, so that $\mu_{U_S} + \mu_{U_I} + \mu_R = 0.95$. The upper-right panel varies the initial mass of asymptomatic infected so that $\mu_{U_S} + \mu_{U_I} = 0.95$.

# 5 A Pandemic Model without Commitment

Thus far, we have assumed that firms and workers can commit to contracts. We now ask if our efficiency result continues to hold when workers can unilaterally walk away from contracts. We show that in this case, the competitive equilibrium can be inefficient because of the presence of a novel pecuniary externality.

Assume for ease of exposition that the economy lasts for two periods, $t = 0, 1$. We assume that insurance firms whose contracts are accepted in period 0 are committed to those contracts, but workers are not. Specifically, workers at the beginning of period 1 can leave their current insurance firms and accept contracts offered by other firms.

Associated with any competitive equilibrium is a set of market utilities for each type in periods 0 and 1. Let $\bar{V}_0(h_0)$ denote the market utilities for type $h_0$ and $\bar{V}_1(h_1)$ denote the market utilities for type $h_1$ in period 1. The contracts offered by insurance firms in period 0 must have a continuation utility in period 1 at least as large as $\bar{V}_1(h_1)$. Similarly, the contracts offered by poaching firms in period 1 must also offer a utility level at least as large as $\bar{V}_1(h_1)$. We begin with the problem of poaching firms in period 1. This problem is

$$\max_{c_1(h_1),\tilde{\pi}_1(h_1)} \sum_{h_1} \tilde{\pi}_1(h_1)\left[\int_{j\neq 0} m_w(\theta_{j1})\, A l_{j1}(h_1) - c_1(h_1)\right], \tag{35}$$

subject to

$$\tilde{\pi}_1(h_1)\left[v_1(h_1 \mid h_0)\right] \geqslant \tilde{\pi}_1(h_1)\,\bar{V}_1(h_1), \quad \forall h_1,$$

where $v_1(h_1 \mid h_0)$ was defined in (25). Since we look for equilibria in which period-0 firms retain all their workers, market clearing requires that poaching firms do not employ any workers. That is, $\tilde{\pi}_1(h_1) = 0$ solves (35).

The period-0 insurance firm's problem is

$$\max_{z,\tilde{\pi}_0(\eta_0)} \left(\sum_{t\geqslant 0} Q_t \sum_{h_0}\sum_{h_t} \tilde{\pi}(h_t \mid h_0)\left[\int_{j\neq 0} m_w(\theta_{jt})\, A l_{jt}(h_t \mid h_0) - c_t(h_t \mid h_0)\right]\right), \tag{36}$$

subject to

$$\tilde{\pi}_0(h_0)\sum_{t\geqslant 0}\beta^t \sum_{h_t} \frac{\tilde{\pi}_t(h_t \mid h_0)}{\tilde{\pi}_0(h_0)} v_t(h_t \mid h_0) \geqslant \tilde{\pi}_0(h_0)\,\bar{V}(h_0), \qquad \forall h_0$$

and the limited commitment constraint

$$\tilde{\pi}_1(h_1 \mid h_0)\, v_1(h_1 \mid h_0) \geqslant \tilde{\pi}_1(h_1 \mid h_0)\,\bar{V}_1(h_1), \quad \forall h_1, \tag{37}$$

and (15)-(21), where $\lambda_{jIt}$ is given by (23) if $L^*_{jt} = 0$ and (22) if $L^*_{jt} > 0$. Market clearing requires that the masses of worker types that solve (36) equal the population masses of these types.

**Definition 2.** A competitive equilibrium consists of an allocation $\{z_0^*, z_1^*\}$, prices $\{Q_0, Q_1\}$, market tightness $\{\boldsymbol{\Theta}_0, \boldsymbol{\Theta}_1\}$, and market utilities $\left\{\bar{V}_0(h_0), \bar{V}_1(h_1)\right\}_{h_0,h_1}$ such that

1. given $\bar{V}_1(h_1)$, poaching firms solve (35) and cannot attract any workers in the sense that if $\tilde{\pi}_1$ solves this problem, $\tilde{\pi}_1 = 0$;
2. given the market utilities, the allocations solve (36); furthermore, the masses of agents chosen by insurance firms equal population masses;
3. $\boldsymbol{\pi}_1$ is induced from $\boldsymbol{\pi}_0$ by the allocations $z_0^*$ in that they satisfy (15)-(21);
4. $m_f(\theta_{jt}) \sum_\eta \lambda_{j\eta t}[A - w_{j\eta t}] \leqslant 0$ for all j, with equality if $j \in \Gamma_t$, for all $t \geqslant T-1$;
5. the market utilities are such that insurance firms make zero profits;
6. the resource constraint is satisfied:
$$\sum_{h_t} \pi_t(h_t) c_t(h_t) = \sum_{h_t} \pi_t(h_t) \int_{j\neq 0} m_w(\theta_{jt}) A l_{jt}(h_t)\, dj, \quad \forall t \geqslant 0;$$
7. for any $j \in \Gamma_t^c$, if $A - w_{j\eta t} > 0$ for all $\eta$, then $m_f(\theta_{jt}) = 0$ and $m_w(\theta_{jt}) = 1$ for all $t \geqslant 0$.

It will be useful to define a static competitive equilibrium in which the economy lasts for one period. This competitive equilibrium is simply a collection of allocations, market tightness, and market utilities such that all firms solve (35), insurance and production firms make zero profits, the resource constraint is satisfied, and our refinement holds. For future use, note that the allocations in a static competitive equilibrium depend on masses of various types in the population.

Next, we show that any competitive equilibrium is inefficient.

**Proposition 9.** *With limited commitment, the competitive equilibrium is inefficient.*

In the proof, we show that the competitive equilibrium is inefficient because of an externality. To understand this externality, note first that in a limited-commitment economy, the outcomes in the last period coincide with those in a static, competitive equilibrium. The utility levels in this equilibrium depend on the masses of types in the population. Next, note that an individual insurance firm in period 0 takes as given the continuation utility levels of its workers and understands that the choice of work effort determines the probabilities that its workers in the last period will be of various types. It does not, however, internalize that the continuation utilities will be determined by the population distribution of the various types. Finally, note that by contrast, the social planner internalizes that the choice of work effort determines both the probability that an individual worker's type will change and the resulting change in the population distribution in the last period. Thus, the competitive equilibrium is inefficient.

# 6 A Model with an Alternate Infection Technology

So far, we have assumed a simple linear infection technology. We now consider a generalization of the infection technology in the spirit of Acemoglu et al. (2020). As we will show, the efficiency results in the model with controllability do not depend on the form of the infection technology. However, the implications for optimal policy in the model without controllability can depend on the technology.

We start by discussing the model with controllability. Assume that if the masses of agents are given by $\boldsymbol{\mu}$, the probability of infection in a given work island is (with some abuse of notation)

$$\psi\left(\boldsymbol{\mu}, \mathbf{l}\right) = \chi \frac{\mu_{U_I} l_j\left(U\right) + \mu_I l_j\left(I\right)}{\left[\mu_U l_j\left(U\right) + \mu_I l_j\left(I\right) + \mu_R l_j\left(R\right)\right]^{2-\omega}}, \tag{38}$$

where $\omega \in [1, 2]$ is the parameter that governs the returns to scale in the infection technology. With $\omega = 1$, the technology displays constant returns, and with $\omega \in (1, 2]$, it displays increasing returns. It is important to note that with $\omega = 1$, technology (38) nests our baseline case, while with $\omega = 2$, it nests the quadratic technology used in the literature.

For simplicity, we restrict attention to the static model. With increasing returns, we need to add a minimum size constraint on the problem confronting insurance firms. This constraint is

$$\sum_{\eta} \pi\left(\eta\right) l_j\left(\eta\right) \geqslant \underline{L} \mathbf{1}_{l_j > 0}, \quad j \neq 0. \tag{39}$$

Without this constraint, if the technology displays increasing returns to scale, firms will have incentives to continuously split the population of U types across an increasing number of islands, and thus there will be no equilibrium.

The problem for the insurance firm is

$$\max_{z,\tilde{\pi}(\eta)} \left( \sum_{\eta} \tilde{\pi}(\eta) \left[ \int_{j\neq 0} m_w(\theta_j) A l_j(\eta) - c(\eta) \right] \right), \tag{40}$$

subject to

$$\tilde{\pi}(\eta) \left[ u(c(\eta)) - \int_{j\neq 0} l_j(\eta) \mathbf{1}_{\{\eta=U_S\}} \psi(\boldsymbol{\mu}, \mathbf{l}) \kappa - \mathbf{1}_{\{\eta=U_I, I\}} \kappa \right] \geqslant \tilde{\pi}(\eta) \underline{V}(\eta)$$

and (39).

Assume that

$$\pi(\eta) \geqslant \underline{L}, \quad \forall \eta,$$

so population masses are large enough to meet this constraint.

We now prove that if an equilibrium exists, it is efficient. Any equilibrium must be of the following form: the representative firms will choose $J+1$ work islands so that all the symptomatic infected agents will be on a single island. On each of the other J islands, the firm will allocate a fraction $\pi(U) l(U)/J$ of the U types and a fraction $\pi(R)/J$ of the recovered agents. The number of islands J and the labor allocation in each island, $l(U)$, satisfy

$$\pi(U) l(U) + \pi(R) \geqslant J\underline{L}.$$

Therefore, if an equilibrium exists, it must solve

$$\max_{\{c(\eta)\}_{\eta\in\{U,R\}} J, l(U)} u(c(R))$$

$$\pi(U) c(U) + \pi(R) c(R) = AJ[\pi(U) l(U) + \pi(R)],$$

$$u(c(U)) - \xi J l(U) \chi \psi(\boldsymbol{\mu}, \mathbf{l}/J) \kappa - (1-\xi) \kappa \geqslant V_U,$$

$$\pi(U) l(U) + \pi(R) \geqslant J\underline{L},$$

where $\xi = \mu_{U_S}/\mu_U$, for some $V_U$. It is immediate that this is just a Pareto problem, and thus the equilibrium must be efficient. We summarize this result in the following proposition.

**Proposition 10.** *With an increasing returns to scale technology and an appropriate minimum size constraint, any competitive equilibrium is efficient.*

Proving the existence of an equilibrium with this technology is challenging because the problem is highly non-convex.

## 6.1 Model without Controllability

We now study how optimal policies change in the model without controllability and with this more general infection technology. Since there is no role for insurance, we drop the insurance firms and just consider incentives of individual agents to supply labor. As was the case before, all symptomatic infected and recovered agents choose to spend all of their time on the work island. The problem for an unknown-type agent is

$$\max_{l} u\left(Al\right) - \xi l \chi \frac{\mu_{U_I} l^* + \mu_I}{\left(\mu_U l^* + \mu_I + \mu_R\right)^{2-\omega}} \kappa - \left(1 - \xi\right) \kappa,$$

where $l^*$ is the labor supply of all other $U$-type agents, and $\xi$ is the mass of unknown susceptible agents relative to that of unknown-type agents. The equilibrium level of labor supply of the $U$ type agent is $\min\{l^*, 1\}$, where $l^*$ solves

$$u'\left(Al\right) A = \xi \chi \frac{\mu_{U_I} l^* + \mu_I}{\left(\mu_U l^* + \mu_I + \mu_R\right)^{2-\omega}} \kappa.$$

As in the baseline model, it is straightforward to see that the competitive equilibrium is inefficient. Specifically, agents do not internalize the effect of their labor supply on the infection probability. Moreover, using a similar argument, we note that if the mass of asymptomatic infected agents is small, in the efficient allocation the planner has them spend all their time on the home island and finances their consumption by taxing $U$ and $R$ types.

We also consider the optimal Pigouvian tax policy that maximizes the welfare of the U-type agents. As in the earlier section, this policy solves

$$\max_{l} u(Al) - \xi l \chi \frac{\mu_{U_I} l^* + \mu_I}{(\mu_U l^* + \mu_I + \mu_R)^{2-\omega}} \kappa - (1-\xi)\kappa.$$

In Figure 2, we plot the aggregate labor supply in the competitive equilibrium, in the efficient allocation, and under the optimal Pigouvian policy.

Figure 2: Employment as a function of the returns-to-scale parameter.

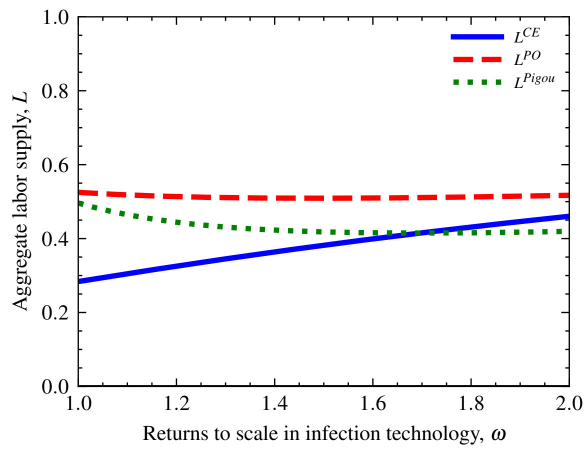


**Figure Notes**. Aggregate employment $L^{CE}$ in the competitive equilibrium, $L^{PO}$ in the Pareto optimum, and $L^{Pigou}$ with optimal Pigouvian taxes. Default parameter values: $\sigma = 2$ (CRRA), $A = 2$ (productivity of labor), $\chi = 1$ (infectivity rate), $\kappa = 40$ (cost of infection), $\mu_{U_S} = 0.9025$ (mass of unknown susceptible), $\mu_{U_I} = 0.0475$ (mass of asymptomatic infected), $\mu_I = 0.05$ (mass of symptomatic infected).

# 7 Conclusion

We show that if virus exposure is controllable and agents can commit to contracts, pandemics generate local externalities, and competitive equilibria are efficient. This result is in sharp contrast to the literature that models pandemics as yielding global externalities. If we assume that virus exposure is not controllable, pandemics generate global externalities, and competitive equilibria are inefficient. In this case, however, aggregate economic activity is inefficiently low in the competitive equilibrium. We show that this result arises because of a positive congestion externality associated with the labor supply of susceptible agents. By considering an environment with one-sided commitment and showing that equilibria are inefficient because of the presence of a novel pecuniary externality, we also show that the effiiciency result depends crucially on commitment.

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# Appendix

**Proposition** 5. A competitive equilibrium with the following properties exists:

- There is mixing between $U$ and $R$ type agents.
- $V_U > \underline{V}(U), V_R > \underline{V}(R)$, and $V_I = \underline{V}(I)$.
- The equilibrium is efficient.

*Proof.* To characterize the competitive equilibrium, consider the Pareto frontier. Clearly, in any Pareto optimal allocation there is no mixing of infected agents with other agents. Let island $1$ be the island to which all symptomatic infected agents are assigned. We consider a Pareto problem in which the initially infected simply receive $A$ units of consumption in all periods. We let $V_U$ denote the utility of agents who are initially of unknown type. We consider the problem of maximizing the utility of the initially recovered agents, whose value is denoted $V_R\left(V_U\right)$. As we vary $V_U$, we trace out the Pareto frontier. This Pareto problem is then given by

$$V_R\left(V_U\right) = \max \sum_{t \geqslant 0} \beta^t \sum_{h_t} \pi_t\left(h_t \mid R\right) u\left(c_t\left(h_t \mid R\right)\right), \tag{41}$$

subject to

$$\sum_{\eta_0} \sum_{h_t} \pi_t\left(h_t \mid \eta_0\right)\left[A \int_{j \neq 0} l_{jt}\left(h_t \mid \eta_0\right) dj - c_t\left(h_t \mid \eta_0\right)\right] \geqslant 0, \qquad \forall t, h_t,$$

$$c_t\left(h_t \mid I\right) = A, \quad l_{1t}\left(h_{t-1}, I \mid I\right) = 1, \qquad \forall t,$$

and

$$\sum_{t \geqslant 0} \beta^t \sum_{h_t} \frac{\pi_t\left(h_t \mid U\right)}{\pi_0\left(U\right)} v_t\left(h_t \mid U\right) \geqslant V_U,$$

where $\lambda_{jIt}$ is given by (23).

Next, we prove the following lemma.

**Lemma 4.** *Any Pareto optimal allocation must have mixing between agents of unknown types and recovered agents.*

*Proof.* Consider the infection probability of agents of unknown type in some period:

$$\psi\left(\lambda_{jIt}\right)=\chi\frac{\sum_{h_{t-1}}\left[\pi_t\left(h_{t-1},U_I\right)l_{jt}\left(h_{t-1},U\right)+\pi_t\left(h_{t-1},I\right)l_{jt}\left(h_{t-1},I\right)\right]}{\sum_{h_{t-1}}\sum_{\eta}\left[\pi_t\left(h_{t-1},\eta\right)l_{jt}\left(h_{t-1},\eta\right)\right]}.$$

As we increase the mass of recovered agents assigned to island j, the probability of infection decreases. Thus, unknown-type agents are willing to give up some consumption to be mixed with recovered agents. Assigning recovered agents to an island with unknown-type agents and increasing the consumption of recovered agents by a suitable amount—that is, by reducing the consumption of unknown-type agents as to satisfy the resource constraint—is Pareto improving. Q.E.D.

**Lemma 5.** $V_R\left(V_U\right)$ *is a decreasing function. Moreover,* $V_R\left(\underline{V}\left(U\right)\right)>\underline{V}\left(R\right)$ *and* $\lim_{V_U\to\infty}V_R\left(V_U\right)=\sum_{t=0}^{T}\beta^t u\left(0\right)$.

*Proof.* The proof that $V_R$ is a decreasing function follows from the inspection of the programming problem. Suppose $V_U=\underline{V}\left(U\right)$. Reallocating some of the unknown-type agents to mix with the recovered agents reduces the infection probability of the unknown-type agents. Thus, these agents will be willing to give up some of their consumption to mix with the recovered agents. This consumption can be redistributed to the initial recovered agents, and thus $V_R\left(\underline{V}\left(U\right)\right)>\underline{V}\left(R\right)$. The last statement of the lemma follows by inspection of the programming problem. Q.E.D.

Next, we prove a lemma in which we provide an alternative characterization of the firm's problem (24).

**Lemma 6.** *The firm's problem (24) can be written as*

$$\max_{z,\tilde{\pi}_0(\eta_0)}\tilde{\pi}_0\left(R\right)\left(\sum_{t\geqslant 0}\beta^t u\left(c_t\left(h_t\mid R\right)\right)-\underline{V}\left(R\right)\right), \tag{42}$$

*subject to*

$$\left(\sum_{\eta_0}\sum_{h_t}\tilde{\pi}\left(h_t\mid\eta_0\right)\left[\int_{j\neq 0}m_w\left(\theta_{jt}\right)Al_{jt}\left(h_t\mid\eta_0\right)-c_t\left(h_t\mid\eta_0\right)\right]\right)=0\quad\forall t\geqslant 0,\ \forall h_t, \tag{43}$$

$$\tilde{\pi}_0(\eta_0)\sum_{t\geqslant 0}\beta^t\sum_{h_t}\frac{\tilde{\pi}_t(h_t\mid\eta_0)}{\tilde{\pi}_0(\eta_0)}v_t(h_t\mid h_0)\geqslant\tilde{\pi}_0(h_0)\underline{V}(h_0),\quad\forall h_0\neq R.$$

*Proof.* Consider the dual of the firm's problem in (24) given by

$$\max_{z,\tilde{\pi}_0(\eta_0)}\tilde{\pi}_0(R)\left(\sum_{t\geqslant 0}\beta^t u(c_t(h_t\mid R))-\underline{V}(R)\right)\tag{44}$$

$$\sum_{t\geqslant 0}Q_t\sum_{\eta_0}\sum_{h_t}\tilde{\pi}(h_t\mid\eta_0)\left[\int_{j\neq 0}m_w(\theta_{jt})Al_{jt}(h_t\mid\eta_0)-c_t(h_t\mid\eta_0)\right]=0\tag{45}$$

$$\tilde{\pi}_0(\eta_0)\sum_{t\geqslant 0}\beta^t\sum_{h_t}\frac{\tilde{\pi}_t(h_t\mid\eta_0)}{\tilde{\pi}_0(\eta_0)}v_t(h_t\mid h_0)\geqslant\tilde{\pi}_0(h_0)\underline{V}(h_0),\quad\forall h_0\neq R.$$

In any competitive equilibrium, the resource constraint implies that profits must be zero period-by-period. Thus, if a contract is part of a competitive equilibrium, we can replace (45) by the requirement that profits be zero in every period. This requirement is simply (43). Note that once we have solved this problem, we can simply set prices so that firms have no incentive to engage in inter-temporal trade.

*Proof of Proposition 5.* Consider the firm's problem (24). As shown above, this problem is equivalent to (42). By an earlier argument, we know that there is no cross-subsidization in favor of the initial asymptomatic infected types, so $c_t(h_t\mid I)=A,\ l_t(h_t\mid I)=1$. Thus, we can separate out the allocations of the initial symptomatic infected types from the above problem. From Proposition 4, it is also optimal to pool all the U and R types on the same island and have the symptomatic infected on a separate island. Thus, as long as the solution has $\tilde{\pi}_0(R)>0$, we can write the problem as (assuming no equilibrium unemployment, which is true by an earlier argument)

$$V_R^E(V_U)=\max_{z,\tilde{\pi}_0(\eta_0)}\sum_{t\geqslant 0}\beta^t u(c_t(h_t\mid R)),\tag{46}$$

subject to

$$\left(\sum_{\eta_0}\sum_{h_t}\frac{\tilde{\pi}(h_t\mid\eta_0)}{\tilde{\pi}_0(R)}\left[\int_{j\neq 0}Al_{jt}(h_t\mid\eta_0)-c_t(h_t\mid\eta_0)\right]\right)=0\quad\forall t\geqslant 0,\ \forall h_t,$$

$$\sum_{t \geqslant 0} \beta^t \sum_{h_t} \frac{\tilde{\pi}_t (h_t \mid U)}{\tilde{\pi}_0 (U)} \nu (h_t \mid U) \geqslant V_U.$$

Notice that in this problem the masses $\tilde{\pi}_0 (U)$ and $\tilde{\pi}_0 (R)$ do not show up separately but only as a ratio $\tilde{\pi}_0 (U) / \tilde{\pi}_0 (R)$. For example,

$$\frac{\tilde{\pi}_1 (U, U)}{\tilde{\pi}_0 (R)} = \frac{\tilde{\pi}_0(U) \left[1 - \phi \xi_0 l_0(U) \chi \lambda_{I0} - \alpha (1 - \xi_0) - (1 - \alpha) \phi (1 - \xi_0)\right]}{\tilde{\pi}_0(R)},$$

where $\xi_0 = \mu_{U_S 0} / (\mu_{U_S 0} + \mu_{U_I 0})$ is a constant. A similar argument holds for all future periods/histories. Thus, the problem can be written as one in which the firm chooses $z$ and the relative mass $\rho = \tilde{\pi}_0 (U) / \tilde{\pi}_0 (R)$. Suppose $V_U < \underline{V} (U)$. Then, it must be that the consumption of the U types is less than $A \int_{j \neq 0} l_{jt} dj$. Then, the value of the firm's problem can be increased by increasing the mass of unknown-type agents attracted to the firm, allocating them to an island of their own, and providing them with the appropriate level of consumption needed to deliver $V_U$. Thus, $\rho (V_U) = \infty$. Suppose next that $V_U$ is arbitrarily large. Then, $V_R^E (V_U)$ is arbitrarily small and initially recovered agents in particular receive consumption less than $A$. Then, the value of the firm's problem can be increased by reducing the mass of unknown-type agents relative to recovered agents and providing them the consumption. Thus, for $V_U$ large enough, $\rho (V_U) = 0$. By continuity, it follows that there exists some $V_U^* > \underline{V} (U)$ such that $\rho (V_U^*) = \pi_0 (U) / \pi_0 (R)$.

Next, we show that $V_R^E (V_U^*) > \underline{V}_R$. Suppose that this is not the case and that $V_R^E (V_U^*) \leqslant \underline{V}_R$. If $V_R^E (V_U^*) < \underline{V}_R$; then, the solution to the firm's problem would involve choosing $\rho = 0$ and thus giving utility $\underline{V}_R$ to the recovered agents. Thus, suppose that $V_R^E (V_U^*) = \underline{V}_R$. Consider a deviating contract that chooses $\tilde{\rho} = \rho - \varepsilon$ for $\varepsilon > 0$ but small. In this case, since there are relatively more recovered agents, by pooling R and U-type agents, the probability of infection for the U-type agents decreases, which implies that their participation constraint is slack. Clearly, in this case, the recovered agents can be made strictly better off, which is a contradiction.

To see why this allocation constitutes a competitive equilibrium, notice that if all other firms offer the contracts associated with the allocation above, no individual firm can profitably deviate. Moreover, the firm's problem at $V_U = V_U^*$ corresponds to a Pareto problem. Thus, the equilibrium is efficient, and it has pooling between U and R types.

□

# Online Appendix

*"On the Efficiency of Competitive Equilibria with Pandemics"*

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## A Omitted Proofs

**Proposition 1.** Any competitive equilibrium features sorting and has no cross-subsidization and no unemployment in the sense that $m_w(\theta_{jt}) = 1 \; \forall t$ and $j \in \Gamma_t$.

*Proof.* Consider the last period T. We will begin by showing that for any $j \in \Gamma_T$ with $l_{jIT} > 0$, $w_{jIT} \geqslant A$. To see this result, suppose, by way of contradiction that $w_{jIT} < A$. Then, consider some $j' \in \Gamma_T^c$ such that $w_{jIT} < w_{j'IT}$ and $w_{j'\eta T} < A$ for all $\eta$. From equilibrium condition 5, we have that $m_w(\theta_{j'T}) = 1$. Thus, an infected agent is strictly better off by choosing island $j'$; this result is a contradiction. A similar argument establishes that $w_{jRT} \geqslant A$ for $j \in \Gamma_T$.

We use this result to show that there is no cross-subsidization in period T. To see this result, suppose there exists some island $j \in \Gamma_T$ such that $w_{jST} < A$. Then, consider some island $j' \in \Gamma_T^c$ with $w_{jST} < w_{j'ST}$ and $w_{j'\eta T} < A$ for all $\eta$. From equilibrium condition 5, we have that $m_w(\theta_{j'T}) = 1$. From equilibrium condition 6, we have that neither infected nor recovered agents will choose island $j'$. Thus, the probability of infection in island $j'$ is zero. Therefore, the susceptible agent is strictly better off by choosing island $j'$; this result is a contradiction.

Next, we show that there is no unemployment in period T. Suppose there exists some $j \in \Gamma_T$ such that $m_w(\theta_{jT}) < 1$. Suppose that $l_{jIT} > 0$. Consider an island $j' \in \Gamma_T^c$ such that $m_w(\theta_{jT}) w_{jIT} < w_{j'IT} < A$ and $w_{j'\eta' T} < m_w(\theta_{jT}) w_{j\eta' T}$ for all other $\eta'$. Then, condition 5 says that $m_w(\theta_{j'T}) = 1$ and so the infected agent is made strictly better off by switching to this island. Next, suppose that $l_{jST} > 0$. Consider an island $j' \in \Gamma_T^c$ such that $m_w(\theta_{jT}) w_{jST} < w_{j'ST} < A$ and $w_{j'\eta' T} < m_w(\theta_{jT}) w_{j\eta' T}$

for all other $\eta'$. Then, condition 6 of the equilibrium implies that the infection probability on island $j'$ is zero. Moreover, condition 5 says that $m_w(\theta_{j'T}) = 1$, and so the susceptible agent is made strictly better off by switching to this island.

We complete the argument for period $T$ by showing that the equilibrium is separating. To do so, suppose that the equilibrium has mixing so that there is some $j \in \Gamma_T$ such that $l_{jST}, l_{jIT} > 0$. Consider some island $j' \in \Gamma_T^c$ such that $w_{j'\eta T} < w_{j\eta T}$ for all $\eta$ and $w_{jST} - \kappa\psi(\lambda_{jIT}) < w_{j'ST}$. Then, condition 5 guarantees that $m_w(\theta_{j'T}) = 1$, and condition 6 guarantees that $\lambda_{j'IT} = 0$. Thus, the susceptible agent is made strictly better off by switching islands, and we have a contradiction.

These arguments imply that $V_T(S, \mu_T) \geqslant V_T(I, \mu_T)$ for all $\mu_T$. Next, consider period $T-1$. Using the monotonicity result, we can repeat all the arguments above to show that there is no cross-subsidization, unemployment, or mixing in period $T-1$. The argument for the other periods follows by induction. □

**Proposition 3.** Consider any allocation that is Pareto optimal. There exists a lump-sum tax system which supports that outcome as an equilibrium.

*Proof.* Using arguments similar to those in Proposition 1, we have that any allocation in which susceptible agents get infected is dominated by an allocation in which these agents are assigned to an otherwise identical island with no infected agents. It follows that susceptible agents never get infected in a Pareto optimal allocation. Since productivity in island 0 is strictly below that in any other island, and since the social planner can always assign enough firms to any island so that the probability of unemployment is zero, we have that aggregate per capita output is simply $A$. Now consider any Pareto optimal allocation that assigns some level of consumption to each agent as a function of that agent's type. By appropriately choosing lump-sum taxes, it is immediate that this Pareto optimal allocation can be implemented in a competitive equilibrium. □

**Proposition 4.** In any competitive equilibrium, there is no mixing between $U$- and $I$-types, and no unemployment in the sense that $m_w(\theta_{jt}) = 1$ for all $t$ and $j \in \Gamma_t$.

*Proof.* As a first step, notice that in any equilibrium, zero profits on the part of production firms implies that for any t and island j,

$$\sum_{h_{t-1}} \sum_{\{\eta : l_{jt}(h_{t-1},\eta)>0\}} \left(\pi_t \left(h_{t-1}, \eta\right) w_{j\eta t}\right) = A.$$

Thus, since we are considering insurance firms, it is without loss of generality to focus on equilibria such that $w_{j\eta t} = A$ for all $j \in \Gamma_t$ and $\eta$ such that $l_{jt}\left(h_{t-1}, \eta\right) > 0$ for some $h_{t-1}$.

Next, we show that mixing between U and I types can never be part of an equilibrium. To see this, suppose that there is mixing in some period t and on some island j. The period utility for type $(h_{t-1}, U)$ on this island is given by

$$u\left(c_t^*\left(h_{t-1}, U\right)\right) - \frac{\pi_t^*(h_{t-1}, U_S)}{\pi_t^*(h_{t-1}, U)} l_{jt}^*\left(h_{t-1}, U\right) \psi\left(\lambda_{jIt}^*\right) \kappa - \frac{\pi_t^*(h_{t-1}, U_I)}{\pi_t^*(h_{t-1}, U)} \kappa,$$

where

$$\lambda_{jIt}^* = \frac{\sum_{h_{t-1}} \left[\pi_t^*\left(h_{t-1}, U_I\right) l_{jt}^*\left(h_{t-1}, U\right) + \pi_t^*\left(h_{t-1}, I\right) l_{jt}^*\left(h_{t-1}, I\right)\right]}{\sum_{h_{t-1}} \sum_{\eta} \left[\pi_t^*\left(h_{t-1}, \eta\right) l_{jt}^*\left(h_{t-1}, \eta\right)\right]}.$$

Notice that

$$\lambda_{jIt}^* > \tilde{\lambda}_{jIt} = \frac{\sum_{h_{t-1}} \left[\pi_t^*\left(h_{t-1}, U_I\right) l_{jt}^*\left(h_{t-1}, U\right)\right]}{\sum_{h_{t-1}} \sum_{\eta \neq I} \left[\pi_t^*\left(h_{t-1}, \eta\right) l_{jt}^*\left(h_{t-1}, \eta\right)\right]}.$$

Consider an island $j' \in \Gamma_t^c$ such that $w_{j'It} = A - \varepsilon$, where $\varepsilon$ is such that

$$\begin{aligned} & u\left(c_t^*\left(h_{t-1}, U\right) - \varepsilon\right) - \frac{\pi_t^*(h_{t-1}, U_S)}{\pi_t^*(h_{t-1}, U)} l_{jt}^*\left(h_{t-1}, U\right) \psi\left(\tilde{\lambda}_{jIt}\right) \kappa - \frac{\pi_t^*(h_{t-1}, U_I)}{\pi_t^*(h_{t-1}, U)} \kappa \\ > & u\left(c_t^*\left(h_{t-1}, U\right)\right) - \frac{\pi_t^*(h_{t-1}, U_S)}{\pi_t^*(h_{t-1}, U)} l_{jt}^*\left(h_{t-1}, U\right) \psi\left(\lambda_{jIt}^*\right) \kappa - \frac{\pi_t^*(h_{t-1}, U_I)}{\pi_t^*(h_{t-1}, U)} \kappa, \end{aligned}$$

and $w_{j\eta t} < A$ for all $\eta \neq I$. Consider an alternate contract that is identical to the equilibrium except that $\tilde{l}_{j't}\left(h_{t-1}, I\right) = l_{jt}^*\left(h_{t-1}, I\right)$, $\tilde{l}_{jt}\left(h_{t-1}, I\right) = 0$ and $\tilde{c}_t\left(h_{t-1}, U\right) = c_t^*\left(h_{t-1}, U\right) - \varepsilon$. By condition 4, $m_w\left(\theta_{j't}\right) = 1$, so this contract is feasible, makes type $(h_{t-1}, U)$ strictly better off, and leaves all other types at least as well off as before. Thus, there exists a feasible contract that yields strictly positive profits for the firm; and we again have a contradiction.

Finally, we argue that there is no unemployment. Suppose that for some $t$ and $j \in \Gamma_t$, $m_w(\theta_{jt}) < 1$. Then, consider an alternate contract where the insurance firm moves all agent types from $j$ to some $j' \in \Gamma_t^c$ such that

$$m_w(\theta_{jt})\, w_{j\eta t} < w_{j'\eta t} < A$$

for all $\eta$. From condition 4, $m_w(\theta_{j't}) = 1$, and so this contract leaves the firm with strictly larger resources and leaves all agents at least as well off as before. Thus, the firm can make strictly positive profits, which is a contradiction. □

**Propisition 7.** In any competitive equilibrium, there is no cross-subsidization, and symptomatic infected and recovered agents supply one unit of labor in the work island in all periods. If $u'(0)A > \kappa/(1-\beta)$, there is mixing in the sense that unknown-type agents work a positive amount in at least one period.

*Proof.* Define

$$\Pi(\eta_0) \equiv \sum_{t \geqslant 0} Q_t \sum_{h_t} \pi(h_t \mid \eta_0) \left[ l_t(h_t \mid \eta_0) A - c_t(h_t \mid \eta_0) \right]$$

to be the profits for the insurance firm associated with each type $\eta_0$. Since there is perfect competition, it must be that firms make zero profits; i.e.,

$$\sum_{\eta_0} \tilde{\pi}(\eta_0)\, \Pi(\eta_0) = 0.$$

We now argue that $\Pi(\eta_0) = 0$ for all $\eta_0$. Suppose not. Then $\Pi(\eta_0) > 0$ for some $\eta_0$. This implies that there exists some $t$ and $\hat{h}_t$ such that $l_t\left(\hat{h}_t \mid h_0 = \eta_0\right) A - c_t\left(\hat{h}_t \mid h_0 = \eta_0\right) > 0$. Consider a deviating firm offering the following contract:

$$\tilde{c}_t(h_t \mid h_0 = \eta_0) = c_t(h_t \mid h_0 = \eta_0), \forall h_t \neq \hat{h}_t,$$

$$\tilde{c}_t\left(\hat{h}_t \mid h_0 = \eta_0\right) = c_t\left(\hat{h}_t \mid h_0 = \eta_0\right) + \varepsilon,$$

where $0 < \varepsilon < l_t \left(\hat{h}_t \mid h_0 = \eta_0\right) A - c_t \left(\hat{h}_t \mid h_0 = \eta_0\right)$, and

$$\tilde{c}_t \left(h_t \mid h_0 \neq \eta_0\right) = 0, \forall h_t.$$

Clearly, for this deviating firm, $\tilde{\pi}\left(h_0 = \eta_0\right) = \pi\left(h_0 = \eta_0\right)$ and $\tilde{\pi}\left(h_0 \neq \eta_0\right) = 0$. Therefore, this firm makes strictly positive profits, yielding a contradiction. Consequently, there is no cross-subsidization.

Next, consider the contract offered to the types $\eta_0 = I$. Suppose that $l_t \left(h_t \mid I\right) < 1$ for any $t$ and $h_t$. Notice that increasing $l$ strictly increases the profits for the firm and leaves the participation constraint for this type unchanged. Thus, by increasing $l_t \left(h_t \mid I\right)$, the firm makes strictly positive profits, yielding a contradiction. This, along with the result that there is no cross-subsidization, implies that $c_t \left(h_t \mid I\right) = A$. An identical argument holds for $h_0 = R$.

Finally, consider the contract offered to types $\eta_0 = U$. Suppose this contract specifies $l_t \left(h_{t-1}, U\right) = 0$ for all $t$. Then, the no-cross subsidization result implies that $c_0 \left(U\right) = 0$. Consider a firm offering the following deviating contract, which is identical to the equilibrium contract, except

$$\tilde{l}_0 \left(h_0 = U\right) = \varepsilon > 0,$$

$$\tilde{c}_0 \left(h_0 = U\right) = \varepsilon A.$$

Clearly, the firm continues to make zero profits under this contract. The change in welfare for the initial unknown type agents is given by

$$\begin{aligned}
\Delta W(U) = {} & u(\varepsilon A) - \left(1-\xi_0\right)\kappa - \xi_0 \varepsilon \psi\left(\lambda_{I0}^*\right)\kappa + \left[\beta\left(1-\xi_0\right)(1-\alpha)\phi + \beta\xi_0\varepsilon\psi\left(\lambda_{I0}^*\right)\phi\right] V_1(U, I) \\
& + \left[\beta\left(1-\xi_0\right)(1-\alpha)(1-\phi) + \beta\xi_0\left[1-\varepsilon\psi\left(\lambda_{I0}^*\right)\right] + \beta\xi_0\varepsilon\psi\left(\lambda_{I0}^*\right)(1-\phi)\right] V_1(U, U) \\
& + \beta\left(1-\xi_0\right)\alpha V_1(U, R) - \left[u(0) - \left(1-\xi_0\right)\kappa + \beta\left(1-\xi_0\right)(1-\alpha)\phi V_1(U, I)\right] \\
& - \left[\left[\beta\left(1-\xi_0\right)(1-\alpha)(1-\phi) + \beta\xi_0\right] V_1(U, U) + \beta\left(1-\xi_0\right)\alpha V_1(U, R)\right],
\end{aligned}$$

where $\xi_0 = \mu_{U_s0} / \left(\mu_{U_s0} + \mu_{U_I0}\right)$. Differentiating the above expression with respect to $\varepsilon$ and evaluating at $\varepsilon = 0$ yields

$$u'(0)A - \xi_0\psi\left(\lambda_{I0}^*\right)\kappa + \beta\xi_0\phi\psi\left(\lambda_{I0}^*\right)\left[V_1(U, I) - V_1(U, U)\right].$$

Note that in any equilibrium contract, the insurance firm will completely insure future consumption of all initial U types. Let $c_t$ denote the consumption associated with the original contract. Therefore,

$$V_1(U,U) \leqslant \sum_{t=1}^{T} \beta^{t-1} u(c_t),$$

and

$$V_1(U,I) \geqslant \sum_{t=1}^{T} \beta^{t-1} u(c_t) - \frac{1-\beta^T}{1-\beta}\kappa.$$

Hence, the above derivative is bounded from below by

$$u'(0)A - \kappa - \beta\frac{1-\beta^T}{1-\beta}\kappa = u'(0)A - \frac{1-\beta^{T+1}}{1-\beta}\kappa,$$

which is strictly positive given our assumption. Thus, a deviating firm can construct a contract that makes both it and the unknown-type agent strictly better off, yielding a contradiction. Thus, there is mixing in at least one period. □

**Proposition 8.** The competitive equilibrium is inefficient.

*Proof.* As a first step, notice that using standard duality arguments and the result that in any equilibrium, profits must be zero in every period (i.e., the resource constraint must hold period by period), we can write the problem of the firm as

$$\sum_{t\geqslant 0}\beta^t \sum_{h_t} \pi(h_t \mid U)\left[u(c_t(h_t\mid U)) - l_t(h_t \mid U)\,\mathbf{1}_{\{\eta_t=U_S\}}\psi(\lambda^*_{It})\,\kappa - \mathbf{1}_{\{\eta_t=I,U_I\}}\kappa\right],$$

subject to

$$\sum_{h_t} \pi(h_t)\left[l_t(h_t)A - c_t(h_t)\right] \geqslant 0, \qquad \forall t \geqslant 0, h_t,$$

$$\sum_{t\geqslant 0}\beta^t \sum_{h_t} \pi(h_t \mid \eta_0)\left[u(c_t(h_t \mid \eta_0)) - \mathbf{1}_{\{\eta_t=I\}}\kappa\right] \geqslant \underline{V}(\eta_0), \text{for } \eta_0 \in \{I,R\},$$

and (15)-(21) for the single-work-island case. Compare this problem with that of the planner described in (12), but restricted to the one-work-island case. The proof follows immediately from the observation that the planner internalizes the effects of the labor allocation on the relative mass of infected agents on the work island,

$\lambda_{It}$, and thus on the probability of infection $\psi(\lambda_{It})$. Since this is not internalized by the firm, competitive equilibria in general will be inefficient. □

**Lemma 1.** Suppose that (27) holds and $\phi$ is sufficiently large. Then, there exists some $\mu_I^*, \mu_{U_I}^* > 0$ such that if $\mu_{I0} < \mu_I^*$ and $\mu_{U_I0} < \mu_{U_I}^*$, the solution to the Pareto problem has no symptomatic infected afents working, and unknown-type agents working more than in the competitive equilibrium. Recovered agents supply one unit of labor in both the equilibrium and the Pareto optimal allocation.

*Proof.* Assume first that $\mu_{U_I0} = 0$ and $\phi = 1$. In the model with a single work island, the relevant individual history in period t is given by $h_t = (\eta_0, ..., \eta_t) = \eta^t$. Thus, we can write the period-0 planner's problem as follows

$$\max_{\{c_t(\eta^t), l_t(\eta^t), \pi_t(\eta^t)\}} u(c_0(U)) + l_0(U)\chi\lambda_{I0}(\pi_0, l_0)[-\kappa + \beta V_1(U, I)]$$
$$+ [1 - l_0(U)\chi\lambda_{I0}(\pi_0, l_0)]\beta V_1(U, U),$$

where (with some abuse of notation)

$$\lambda_{It}(\pi_t, l_t) = \left(\frac{\sum_{h_{t-1}} \pi_t(h_{t-1}, I) l_t(h_{t-1}, I)}{\sum_{h_{t-1}} \sum_{\eta} \pi_t(h_{t-1}, \eta) l_t(h_{t-1}, \eta)}\right),$$

$$V_1(U, I) = u(c_1(U, I)) - \kappa + \beta\alpha V_2(U, I, R) + \beta(1 - \alpha) V_2(U, I, I),$$

$$V_1(U, U) = u(c_1(U, U)) + l_1(U, U)\chi\lambda_{I1}(\pi_1, l_1)[-\kappa + \beta V_2(U, U, I)]$$
$$+ (1 - l_1(U, U)\chi\lambda_{I1}(\pi_1, l_1))\beta V_2(U, U, U),$$

subject to

$$u(c_0(I)) - \kappa + \sum_t \beta^t \sum_{h_t} \frac{\pi_t(h_t \mid I)}{\pi_0(I)}[u(c_t(h_t \mid I)) - \mathbf{1}_{\eta_t = I}\kappa] \geqslant V^{ce}(I),$$

$$u(c_0(R)) + \sum_t \beta^t \sum_{h_t} \frac{\pi_t(h_t \mid R)}{\pi_0(R)} u(c_t(h_t \mid R)) \geqslant V^{ce}(R),$$

$$\sum_{h_t} \pi_t(h_t) c_t(h_t) \leqslant \sum_{h_t} \pi(h_t) l_t(h_t) A, \quad \forall t,$$

$$\pi_t\left(h^{t-2},U,U\right)\leqslant\pi_{t-1}\left(h^{t-2},U\right)\left(1-l_{t-1}\left(h^{t-2},U\right)\chi\lambda_{It-1}\left(\pi_{t-1},l_{t-1}\right)\right), \tag{47}$$

$$\pi_t\left(h^{t-2},U,I\right)\geqslant\pi_{t-1}\left(h^{t-2},U\right)l_{t-1}\left(h^{t-2},U\right)\chi\lambda_{It-1}\left(\pi_{t-1},l_{t-1}\right), \tag{48}$$

$$\pi_t\left(h^{t-2},I,I\right)\geqslant(1-\alpha)\,\pi_{t-1}\left(h^{t-2},I\right),$$

$$\pi_t\left(h^{t-2},I,R\right)\leqslant\alpha\pi_{t-1}\left(h^{t-2},I\right),$$

$$\pi_t\left(h^{t-2},R,R\right)\leqslant\pi_{t-2}\left(h^{t-2},R\right).$$

Note that in this formulation of the problem, we have included $\pi_t\left(h_t\right)$ as a choice variable and the definition as a constraint. The inequalities in these constraints are written so that the associated multiplier is positive. For example if the inequality in (47) is reversed, then clearly the planner will always choose $\pi_t\left(h^{t-2},U,U\right)=1$, which violates feasibility. Similarly, if the inequality in (48) is reversed, the planner would always choose $\pi_t\left(h^{t-2},U,I\right)=0$, again violating feasibility. These constraints will always bind.

Let $\iota_t$ be the multiplier on the feasibility constraints. Taking the first order conditions with respect to $c_0\left(U\right), c_1\left(U,I\right),$ and $c_1\left(U,U\right)$ yields

$$u'\left(c_0\left(U\right)\right)=\pi_0\left(U\right)\iota_0,$$

$$l_0\left(U\right)\chi\left(\frac{\pi_0\left(I\right)l_0\left(I\right)}{\sum_\eta\pi_0\left(\eta\right)l_0\left(\eta\right)}\right)\beta u'\left(c_1\left(U,I\right)\right)=\iota_1\pi_1\left(U,I\right),$$

and

$$\left(1-l_0\left(U\right)\chi\left(\frac{\pi_0\left(I\right)l_0\left(I\right)}{\sum_\eta\pi_0\left(\eta\right)l_0\left(\eta\right)}\right)\right)\beta u'\left(c_1\left(U,U\right)\right)=\iota_1\pi_1\left(U,U\right).$$

Using the definitions of $\pi_1\left(U,\eta_1\right)$ implies that $c_1\left(U,U\right)=c_1\left(U,I\right)$, and combining these equations yields

$$\frac{\beta u'\left(c_1\left(U,\eta_1\right)\right)}{u'\left(c_0\left(U\right)\right)}=\frac{\iota_1}{\iota_0},\quad \eta_1\in\{U,I\}.$$

A similar argument implies that

$$\frac{\beta u'(c_1(R,R))}{u'(c_0(R))} = \frac{\iota_1}{\iota_0}.$$

Therefore, if $\pi_0(I) = 0$, it must be that $c_0(U) = c_0(R) = A$.

Let $\nu_1(\eta_0, \eta_1)$ be the multiplier on the constraint for $\pi_1(\eta_0, \eta_1)$. The derivative of the planning problem with respect to $l_0(I)$ is

$$\begin{aligned}
&- l_0(U)\chi \frac{\pi_0(I)(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}\kappa \\
&+ l_0(U)\chi \frac{\pi_0(I)(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}\beta\left[V_1(U,I) - V_1(U,U)\right] \\
&+ \iota_0\pi_0(I)A - \left[\nu_1(U,U) + \nu_1(U,I)\right]\pi_0(U)l_0(U)\chi \frac{\pi_0(I)(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}.
\end{aligned} \tag{49}$$

We can factor out $\pi_0(I)$ from this expression and write the residual after substituting the earlier first order conditions as

$$\begin{aligned}
&\frac{u'(c_0(U))}{\pi_0(U)}A - l_0(U)\chi \frac{(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}\kappa \\
&l_0(U)\chi \frac{(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}\beta\left[V_1(U,I) - V_1(U,U)\right] \\
&- \left[\nu(U,U) + \nu(U,I)\right]\pi_0(U)l_0(U)\chi \frac{(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}.
\end{aligned}$$

Notice that since $c_1(U,I) = c_1(U,U)$, $V_1(U,I) \leqslant V_1(U,U)$. Evaluating this expression at $\pi_0(I) = 0$, we obtain an expression that is bounded from above by

$$\frac{u'(c_0(U))}{\pi_0(U)}A - l_0(U)\chi \frac{(\pi_0(U)l_0(U) + \pi_0(R)l_0(R))}{\left(\sum_\eta \pi_0(\eta)l_0(\eta)\right)^2}\kappa. \tag{50}$$

Clearly, in this case $l_0(U) = l_0(R) = 1$, and so the above expression is (since $\pi_0(U) = \mu_{U0}$)

$$u'(A)\frac{A}{\mu_{U0}} - \chi\kappa < 0$$

owing to the assumption. Because of the Maximum theorem the policy functions are continuous in $\mu_{I0} = \pi_0(I)$. Therefore, there exists some $\mu^*_{I0} > 0$ such that for $\mu_{I0} < \mu^*_{I0}$, the derivative (49) is strictly negative for any $l_0(I) > 0$. Thus, it must be that for $\mu_{I0} < \mu^*_{I0}$, $l_0(I) = 0$. Since the infection probability is zero, $l_0(U) = 1$. Given that $l_0(I) = 0$, we have that the total mass of infected in period 1 is $\pi_1(I, I) < \pi_0(I)$. Moreover, $\pi_1(U, I) = 0$, and $\pi_1(U, U) = \pi_0(U)$.

Next, we take the derivative with respect to $l_1(I, I)$. This derivative is

$$\pi_1(I, I)(1 - l_0(U)\chi\lambda_{I0})\beta$$

times

$$\begin{aligned}
&\frac{u'(c_1(U, U))}{\pi_0(U)}A - l_1(U, U)\chi\frac{\left[\sum_{(\eta_0,\eta_1)\neq(I,I)}\pi(\eta_0, \eta_1)\, l_1(\eta_0, \eta_1)\right]}{\left(\sum_{\eta_0,\eta_1}\pi(\eta_0, \eta_1)\, l_1(\eta_0, \eta_1)\right)^2}\kappa \\
&l_1(U, U)\chi\frac{\left[\sum_{(\eta_0,\eta_1)\neq(I,I)}\pi(\eta_0, \eta_1)\, l_1(\eta_0, \eta_1)\right]}{\left(\sum_{\eta_0,\eta_1}\pi(\eta_0, \eta_1)\, l_1(\eta_0, \eta_1)\right)^2}\beta\left[V_2(U, U, I) - V_1(U, U, U)\right] \\
&- \left[\nu_2(U, U, U) + \nu_2(U, U, I)\right]\pi_1(U, U)\, l_1(U, U)\chi\frac{\left[\sum_{(\eta_0,\eta_1)\neq(I,I)}\pi(\eta_0, \eta_1)\, l_1(\eta_0, \eta_1)\right]}{\left(\sum_{\eta_0,\eta_1}\pi(\eta_0, \eta_1)\, l_1(\eta_0, \eta_1)\right)^2}.
\end{aligned} \tag{51}$$

An identical argument for period 1 implies that $V_2(U, U, I) - V_1(U, U, U) \leqslant 0$. Therefore, if $\pi_1(I, I) = 0$, the expression above is bounded above by

$$\frac{u'(A)}{\pi_0(U)}A - \chi\kappa < 0.$$

Hence, there exists some $\mu^*_{I1}$ such that if $\pi_1(I, I) < \mu^*_{I1}$, then $l_1(I, I) = 0$. We can repeat this argument for all periods $t = \{0, 1, ..., T\}$ and generate a sequence $\{\mu^*_{It}\}_{t\geqslant 0}$. Define $\mu^*_I = \min\{\mu^*_{It}\}$. So, for $\mu_{I0} < \mu^*_I$, we have that $l_t(\eta^{t-1}, I) = 0$ and $l_t(\eta^{t-1}, U) = 1$ for all t, and there are no new infections in any period.

Now suppose that $\mu_{U_I 0} > 0$ and $\phi < 1$ so that there are asymptomatic infected agents. Then, one can show that the equivalent expression to (50) is

$$\frac{u'(c_0(U))}{\pi_0(U)}A - \frac{\mu_{U_s 0}}{\mu_{U0}}l(U)\chi\frac{\{\pi(U)l(U)[1 - \mu_{U_I 0}] + \pi(R)l(R)\}}{\left(\sum_{\eta_0}\pi(\eta_0)l(\eta_0)\right)^2}\kappa.$$

Since $\pi_0(U) = \mu_{U0}$, taking the limit as $\mu_{U_I 0} \to 0$ yields

$$\frac{u'(A)}{\mu_{U0}} A - \chi\kappa < \frac{u'(A)}{\mu_{U0}} A - \frac{\mu_{U_S 0}}{\mu_{U0}} \chi\kappa < 0,$$

since $l_0(U) \to 1$. The second inequality follows directly from the assumption. Therefore, there exists some $\mu^*_{I0}, \mu^*_{U_I 0} > 0$ such that for $\mu_{I0} < \mu^*_{I0}$ and $\mu_{U0} < \mu^*_{U_I 0}$, the derivative with respect to $l_0(I)$ is strictly negative for any $l_0(I) > 0$. Thus in this case, it must be that $l_0(I) = 0$. A similar argument to that above implies that there is a sequence $\left\{\mu^*_{It}, \mu^*_{U_I t}\right\}$ such that if $\mu_{It} < \mu^*_{It}, \mu_{U_I t} < \mu^*_{U_I t}$, then $l_t(h^{t-1}, I) = 0$. Next, recall that (if $l_t(h^{t-1}, I) = 0$),

$$\begin{aligned}
\mu_{U_I t} = \pi_t\left(h^{t-1}, U_I\right) &= (1-\alpha)(1-\phi)\pi_{t-1}\left(h^{t-2}, U_I\right) \\
&\quad + (1-\phi)\pi_{t-1}\left(h^{t-2}, U_S\right) l_{t-1}\left(h^{t-2}, U\right) \times \\
&\quad \times \chi \frac{\sum_{h^{t-2}}\left[\pi_{t-1}(h^{t-2}, U_I)\, l_{t-1}(h^{t-2}, U)\right]}{\sum_{h^{t-2}}\left[\pi_{t-1}(h^{t-2}, U)\, l_{t-1}(h^{t-2}, U) + \pi_{t-1}(h^{t-2}, R)\right]} \\
&< \pi_{t-1}\left(h^{t-2}, U_I\right) = \mu_{U_I t-1}
\end{aligned}$$

for $\phi$ sufficiently close to one. Similarly, $\mu_{It} < \mu_{It-1}$ if $\phi$ sufficiently close to one. Define $\mu^*_I = \min\{\mu^*_{It}\}$ and $\mu^*_{U_I} = \min\left\{\mu^*_{U_I t}\right\}$. Then for $\mu_{I0} < \mu^*_I$, $\mu_{U_I 0} < \mu^*_{U_I}$, and $\phi$ sufficiently close to one, we have that $l_t(\eta^{t-1}, I) = 0$ for all $t$. □

**Lemma 2.** Suppose that $u(c) = \log(c)$. Then, for sufficiently small $\mu_{U_I 0}$, $l^{opt}(U) > l^*(U; 0)$. Thus, the optimal Pigouvian policy is a subsidy on labor.

*Proof.* Suppose that $u(c) = \log(c)$. Then, using (29) in equilibrium, we have that

$$\frac{1}{l^*_0(U;0)} = \xi_0 \chi \left(\frac{\mu_{I0} + \mu_{U_I 0} l^*_0(U;0)}{\mu_{U0} l^*_0(U;0) + \mu_{I0} + \mu_{R0}}\right)\kappa.$$

As $\mu_{U_I 0} \to 0$, we have

$$l^*_0(U;0) \to \min\left\{\max\left\{\frac{\mu_{I0} + \mu_{R0}}{\chi\mu_{I0}\kappa - \mu_{U0}}, 0\right\}, 1\right\}.$$

The optimal untargeted Pigouvian taxes implement the solution to the following problem:

$$\max_{l} \; \log(lA) - \xi_0 l \chi \left( \frac{\mu_{I0} + \mu_{U_I 0} l}{\mu_{U0} l + \mu_{I0} + \mu_{R0}} \right) \kappa - (1 - \xi_0) \kappa.$$

The first order condition with respect to $l$ is

$$\frac{1}{l} - \xi_0 \chi \left( \frac{\mu_{I0} + \mu_{U_I 0} l}{\mu_{U0} l + \mu_{I0} + \mu_{R0}} \right) \kappa + l \xi_0 \chi \frac{(\mu_{I0} + \mu_{R0}) \mu_{U_I 0} - \mu_{U0} \mu_{I0}}{(\mu_{U0} l + \mu_{I0} + \mu_{R0})^2} \kappa = 0.$$

In the limit as $\mu_{U_I} \to 0$, $l$ solves

$$\mu_{U0}^2 l^2 + [2\mu_{U0} - \chi \mu_{I0} \kappa] (\mu_{I0} + \mu_{R0}) l + (\mu_{I0} + \mu_{R0})^2 = 0.$$

This is a quadratic equation whose roots are given by

$$l = (\mu_{I0} + \mu_{R0}) \frac{[\chi \mu_{I0} \kappa - 2\mu_{U0}] \pm \sqrt{[2\mu_{U0} - \chi \mu_{I0} \kappa]^2 - 4\mu_{U0}^2}}{2\mu_{U0}^2}.$$

Given these results, we know that the planner's welfare is strictly increasing at $l = 0$ and has two critical points given by the above equation. Thus, the solution, $l^{opt}(U)$, must be either the smallest root or one. If $l^{opt}(U) = 1$, then clearly $l^{opt}(U) \geqslant l^*(U; 0)$ when $\mu_{U_I 0} = 0$. Suppose that $l^{opt}(U)$ is given by the smallest root. Then, the difference $l^{opt}(U) - l^*(U; 0)$ has the same sign as

$$\frac{[\chi \mu_{I0} \kappa - 2\mu_{U0}] - \sqrt{[2\mu_{U0} - \chi \mu_{I0} \kappa]^2 - 4\mu_{U0}^2}}{2\mu_{U0}^2} - \frac{1}{\chi \mu_{I0} \kappa - \mu_{U0}}$$
$$= \left[ \frac{\chi \mu_{I0} \kappa [\chi \mu_{I0} \kappa - 3\mu_{U0}] - (\chi \mu_{I0} \kappa - \mu_{U0}) \sqrt{\chi \mu_{I0} \kappa (\chi \mu_{I0} \kappa - 4\mu_{U0})}}{2\mu_{U0}^2 (\chi \mu_{I0} \kappa - \mu_{U0})} \right].$$

Let us consider the numerator of the above expression. We want to prove that it is positive; i.e.,

$$\chi \mu_{I0} \kappa [\chi \mu_{I0} \kappa - 3\mu_{U0}] - (\chi \mu_{I0} \kappa - \mu_{U0}) \sqrt{\chi \mu_{I0} \kappa (\chi \mu_{I0} \kappa - 4\mu_{U0})} \geqslant 0,$$

which equivalent to showing that

$$(\chi \mu_{I0} \kappa) [\chi \mu_{I0} \kappa - 3\mu_{U0}]^2 - (\chi \mu_{I0} \kappa - \mu_{U0})^2 (\chi \mu_{I0} \kappa - 4\mu_{U0}) \geqslant 0$$

or

$$\mu_{U0}^2 \geqslant 0,$$

which proves the result. Thus, at $\mu_{U_I 0} = 0$ we have that $l^{opt}(U) > l^*(U;0)$. By continuity, the result continues to hold for sufficiently small $\mu_{U_I 0}$. □

**Lemma 3.** The effect on welfare of a small increase in labor supply of unknown-type agents from the competitive equilibrium can be decomposed into an externality from current infection and an externality from future infection. If $\alpha$ is sufficiently small or $T = 2$, welfare rises because of the externality from current infection and falls because of the externality from future infection. The overall effect on welfare is ambiguous.

*Proof.* Differentating the planner's problem with respect to $l_U$ and evaluating at $l_t^*(U;0)$ yields

$$-\mu_{Ut} l_t^*(U;0)\, \psi'(\lambda_t^*) \frac{\partial \lambda_t^*}{\partial l_U} \kappa + \beta \left( \frac{\partial V_{t+1}}{\partial \mu_{It+1}} \frac{\partial \mu_{It+1}}{\partial \lambda^*} + \frac{\partial V_{t+1}}{\partial \mu_{Ut+1}} \frac{\partial \mu_{Ut+1}}{\partial \lambda_t^*} \right) \frac{\partial \lambda_t^*}{\partial l_U} + \beta \frac{\partial V_{t+1}}{\partial \lambda_{t+1}^*} \frac{\partial \lambda_{t+1}^*}{\partial l_U}. \tag{52}$$

We know from the analysis of the static model that for sufficiently small $\mu_I$, $\partial \lambda_t^* / \partial l_U < 0$. We now analyze the second term. Consider the continuation value $V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda_{t+1}^*\right)$, which equals

$$\max_{c_{\eta t+1}, l_{\eta t+1}} \sum \mu_{\eta t+1} \left[ u\left(c_{\eta t+1}\right) - l_{Ut+1} \psi\left(\lambda_{t+1}^*\right) \kappa \mathbf{1}_{\eta=U} - \kappa \mathbf{1}_{\eta=I} \right] + \beta V_{t+2}\left(\boldsymbol{\mu}_{t+2}, \lambda_{t+2}^*\right),$$

subject to

$$\sum \mu_{\eta t+1} c_{\eta t+1} = \sum \mu_{\eta t+1} l_{\eta t+1} A,$$

$$\mu_{Ut+2} = \mu_{Ut+1}\left(1 - l_{Ut+1}\psi\left(\lambda_{t+1}^*\right)\right),$$

$$\mu_{It+2} = (1-\alpha)\,\mu_{It+1} + \mu_{Ut+1} l_{Ut+1} \psi\left(\lambda_{t+1}^*\right),$$

$$\mu_{Rt+2} = \mu_{Rt+1} + \alpha \mu_{It+1}.$$

Let $\phi_{t+1}, \beta\nu_{Ut+1}, \beta\nu_{It+1}, \beta\nu_{Rt+1}$ be the respective multipliers on the above constraints. Differentiating the value with respect to $\mu_{Ut+1}$ and $\mu_{It+1}$ yields

$$\begin{aligned}\frac{\partial V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda^*_{t+1}\right)}{\partial \mu_{Ut+1}} &= \left[u\left(c_{Ut+1}\right) - l_{Ut+1}\psi\left(\lambda^*_{t+1}\right)\kappa\right] - \phi_{t+1}\left[c_{Ut+1} - l_{Ut+1}A\right] \\ &+ \beta\nu_{Ut+1}\left(1 - l_{Ut+1}\psi\left(\lambda^*_{t+1}\right)\right) + \beta\nu_{It+1} l_{Ut+1}\psi\left(\lambda^*_{t+1}\right)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda^*_{t+1}\right)}{\partial \mu_{It+1}} &= \left[u\left(c_{It+1}\right) - \kappa\right] - \phi_{t+1}\left[c_{It+1} - l_{It+1}A\right] \\ &+ \beta\nu_{It+1}\left(1-\alpha\right) + \beta\nu_{Rt+1}\alpha.\end{aligned}$$

It is also useful to compute

$$\frac{\partial V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda^*_{t+1}\right)}{\partial \mu_{Rt+1}} = u\left(c_{It+1}\right) - \phi_{t+1}\left[c_{Rt+1} - l_{Rt+1}A\right] + \beta\nu_{Rt+1}.$$

The first order conditions of the problem are

$$u'\left(c_{\eta t+1}\right) = \phi_{t+1},$$

$$-\mu_{Ut+1}\psi\left(\lambda^*_{t+1}\right)\kappa + \phi_{t+1}\mu_{Ut+1}A - \beta\nu_{Ut+1}\mu_{Ut+1}\psi\left(\lambda^*_{t+1}\right) + \beta\nu_{It+1}\mu_{Ut+1}\psi\left(\lambda^*_{t+1}\right) = 0. \tag{53}$$

and

$$\nu_{\eta t+1} = \frac{\partial V_{t+2}\left(\boldsymbol{\mu}_{t+2}, \lambda^*_{t+2}\right)}{\partial \mu_{\eta t+2}}, \forall \eta.$$

Equation (53) can be written as

$$\psi\left(\lambda^*_{t+1}\right)\kappa - \phi_{t+1}A + \beta\nu_{Ut+1}\psi\left(\lambda^*_{t+1}\right) - \beta\nu_{It+1}\psi\left(\lambda^*_{t+1}\right) = 0.$$

Next, we have

$$\begin{aligned}\nu_{It} - \nu_{St} =& \frac{\partial V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda^*_{t+1}\right)}{\partial \mu_{It+1}} - \frac{\partial V_{t+1}\left(\boldsymbol{\mu}_{t+1}, \lambda^*_{t+1}\right)}{\partial \mu_{Ut+1}} \\ =& -\kappa + \phi_{t+1}A + \beta\nu_{It+1}\left(1-\alpha\right) + \beta\nu_{Rt+1}\alpha \\ &+ l_{Ut+1}\left[\psi\left(\lambda^*_t\right)\kappa - \phi_{t+1}A + \beta\nu_{Ut+1}\psi\left(\lambda^*_t\right) - \beta\nu_{It+1}\psi\left(\lambda^*_t\right)\right] - \beta\nu_{St+1}.\end{aligned}$$

Substituting (53) into the above equation yields

$$-\left[1-\psi\left(\lambda_{t+1}^{*}\right)\right]\kappa-\beta\nu_{Ut+1}\left[1-\psi\left(\lambda_{t+1}^{*}\right)\right]+\beta\nu_{It+1}\left[1-\alpha-\psi\left(\lambda_{t+1}^{*}\right)\right]+\beta\nu_{Rt+1}\alpha$$

or

$$\nu_{It}-\nu_{St}=-\left[1-\psi\left(\lambda_{t+1}^{*}\right)\right]\kappa+\beta\alpha\left[\nu_{Rt+1}-\nu_{It+1}\right]+\beta\left[1-\psi\left(\lambda_{t+1}^{*}\right)\right]\left[\nu_{It+1}-\nu_{Ut+1}\right]. \tag{54}$$

We will show that $\nu_{It}-\nu_{Ut}\leqslant 0$ using an induction argument. Clearly, for $t=T$ this holds, since $\nu_{\eta T+1}=0$. Suppose that

$$\nu_{It+1}-\nu_{Ut+1}\leqslant 0$$

and

$$-\left[1-\psi\left(\lambda_{t+1}^{*}\right)\right]\kappa+\beta\alpha\left[\nu_{Rt+1}-\nu_{It+1}\right]\leqslant 0.$$

Then,

$$\nu_{It}-\nu_{Ut}\leqslant 0,$$

and

$$\begin{aligned}&-\left[1-\psi\left(\lambda_{t}^{*}\right)\right]\kappa+\beta\alpha\left[\nu_{Rt}-\nu_{It}\right]\\=&-\left[1-\psi\left(\lambda_{t}^{*}\right)\right]\kappa+\beta\alpha\left[\kappa+(1-\alpha)\beta\left[\nu_{Rt+1}-\nu_{It+1}\right]\right],\end{aligned}$$

which is less than zero for sufficiently small $\alpha$.

Finally, recall that

$$\mu_{Ut+1}=\mu_{Ut}\left(1-l_{Ut}\psi\left(\lambda_{t}^{*}\right)\right)$$

and

$$\mu_{It+1}=(1-\alpha)\mu_{It}+\mu_{Ut}l_{Ut}\psi\left(\lambda_{t}^{*}\right).$$

Therefore,

$$\begin{aligned}\frac{\partial\mu_{Ut+1}}{\partial\lambda_{t}^{*}}&=-\mu_{Ut}l_{Ut}\psi'\left(\lambda_{t}^{*}\right)\\\frac{\partial\mu_{It+1}}{\partial\lambda_{t}^{*}}&=\mu_{Ut}l_{Ut}\psi'\left(\lambda_{t}^{*}\right).\end{aligned}$$

Thus,

$$\beta \frac{\partial V_{t+1}\left(\mu_{t+1},\lambda^*_{t+1}\right)}{\partial \mu_{It+1}} \frac{\partial \mu_{It+1}}{\partial \lambda^*} \frac{\partial \lambda^*_t}{\partial l^*_U} + \frac{\partial V_{t+1}\left(\mu_{t+1},\lambda^*_{t+1}\right)}{\partial \mu_{Ut+1}} \frac{\partial \mu_{Ut+1}}{\partial \lambda^*} \frac{\partial \lambda^*_t}{\partial l^*_U}$$
$$= -\beta\left[\nu_{It} - \nu_{Ut}\right] \mu_{Ut} l_{Ut} \psi'\left(\lambda^*_t\right) \frac{\partial \lambda^*_t}{\partial l^*_U}$$

which is strictly positive if $\alpha$ is sufficiently small. Note also that in a two period model, this expression is unambiguously negative, since $\nu_{Ut+1}, \nu_{It+1}, \nu_{Rt+1} = 0$.

Finally, consider the last term in (52). We have

$$\begin{aligned}\frac{\partial V_{t+1}\left(\mu_{t+1},\lambda^*_{t+1}\right)}{\partial \lambda^*_{t+1}} &= -\mu_{Ut+1} l_{Ut+1} \psi'\left(\lambda^*_{t+1}\right)\kappa - \nu_{Ut+1}\mu_{Ut+1} l_{Ut+1}\psi'\left(\lambda^*_{t+1}\right) \\ &\quad + \nu_{It+1}\mu_{Ut+1} l_{Ut+1}\psi'\left(\lambda^*_{t+1}\right) \\ &= -\mu_{Ut+1} l_{Ut+1}\psi'\left(\lambda^*_{t+1}\right)\kappa + \mu_{Ut+1} l_{Ut+1}\psi'\left(\lambda^*_{t+1}\right)\left[\nu_{It+1} - \nu_{Ut+1}\right] \leqslant 0\end{aligned}$$

and

$$\lambda^*_{t+1} = \frac{\mu_{It+1}}{\mu_{Ut+1} l^*_{Ut+1} + \mu_{It+1} + \mu_{Rt+1}} = \frac{1}{\frac{\mu_{Ut+1}}{\mu_{It+1}} l^*_{Ut+1} + 1 + \frac{\mu_{Rt+1}}{\mu_{It+1}}}.$$

Since $\mu_{Ut+1}$ is decreasing in $l^*_U$ and $\mu_{It+1}$ is increasing in $l^*_U$, $\partial \lambda^*_{t+1}/\partial l^*_U > 0$. Thus, the last term in (52) is negative. □

**Proposition 9.** With limited commitment, the competitive equilibrium is inefficient.

*Proof.* Our first result is that period-0 firms make zero profits in each period. This results follows immediately from the firm's zero-profit condition, along with the resource constraints. We use this result to argue that the limited commitment constraints with respect to period 1 for the period zero firm given in (37) must be binding. Suppose that this is not the case and that the limited commitment constraint was slack for some type—say, type $h_1$. Now consider the poaching firm's problem. If this firm were to offer the same period-1 allocation as the period firm, it would make zero profits by our first result. But the participation constraint for this poaching firm for type $h_1$ is slack. Thus, the poaching firm can reduce the consumption in period 1 of type $h_1$ and strictly increase its profits while continuing to respect the participation constraints. Thus, the limited commitment constraints must be binding. These results imply that the allocations in period 1 coincide with the static competitive equilibrium in that period. In particular, these allocations depend only on the distribution of types at the beginning of the period and not on what an individual's type may have been in the preceeding period. The two results also imply that we can write the period-0 firm's problem as one of maximizing period-0 profits, subject to a period-0 participation constraint in which the firm takes as given that the continuation utilities of workers will coincide with the static competitive equilibrium in period 1 given by

$$\max_{z,\tilde{\pi}_0(h_0)} \left( \sum_{h_0} \tilde{\pi}(h_0) \left[ \int_{j \neq 0} m_w(\theta_{jt}) A l_{j0}(h_0) - c_0(h_0) \right] \right), \tag{55}$$

subject to

$$\tilde{\pi}_0(h_0) \left[ v_0(h_0) + \beta \sum \tilde{\pi}_1(h_1 \mid h_0) \bar{V}_1(h_1) \right] \geqslant \tilde{\pi}_0(h_0) \bar{V}_0(h_0), \qquad \forall h_0. \tag{56}$$

Note that an individual insurance firm does not internalize that the continuation utilities for any type depend on the distribution of types in the population. Next, we can use the zero profit condition to write the period 0 firm's problem 55 in its dual form:

$$\max_{z,\tilde{\pi}(h_0)} u(c_0(R)) + \beta \bar{V}_1(R),$$

subject to

$$\sum_{h_t} \tilde{\pi}_t(h_t) \left[ \int_{j \neq 0} A l_{jt}(h_t) \, dj - c_t(h_t) \right] = 0, \forall t \in \{0, 1\}$$

and (56). Now consider a social planner who seeks to maximize utility of the initial recovered agents subject to the resource constraints and the constraints that all other agents be made at least as well off as in the competitive equilibrium and the requirement that continuation utilities coincide with those in a static competitive equilibrium. This planner, unlike individual insurance firms, recognizes that continuation utilities depend on the population masses in period 1. Clearly, the solutions to the planning problem and the problem of an individual insurance firm are different, so any competitive equilibrium is inefficient. □

# B A Model with Vacancy Costs

We now consider an extension of our framework in which firms have to pay a vacancy cost $\kappa_\nu$ in order to attract workers. The definition of an allocation is unchanged. The resource constraint is given by

$$\sum_{\eta} \mu_{\eta t} c_{\eta t} + \kappa_\nu \int \gamma_{jt} dj \leqslant \int_{j \neq 0} \sum_{\eta} \left(\mu_{\eta t} m_w\left(\theta_{jt}\right) A l_{j\eta t}\right) dj.$$

Define $(w^*, \theta^*)$ as the solution to the following problem:

$$\max_{\theta, w} m_w\left(\theta\right) w, \tag{57}$$

subject to

$$m_f\left(\theta\right)\left(A - w\right) = \kappa_\nu.$$

We will show that the equilibrium wage and tightness will be given by $(w^*, \theta^*)$. We now define an equilibrium for this environment.

**Definition.** An equilibrium is an allocation $(\boldsymbol{\mu}, \boldsymbol{\Theta}, \mathbf{l}, \boldsymbol{\lambda}, \mathbf{c})$, values $\{V_t\left(\eta, \boldsymbol{\mu}_t\right)\}_{\eta,t}$, and a measure of active islands $\Gamma_t\left(\boldsymbol{\mu}_t\right) = \{j \in \mathcal{J} : l_{j\eta t}\left(\boldsymbol{\mu}_t\right) > 0 \text{ for some } \eta \in \{S, I, R\}\}$ such that

1. $l_{\eta t}\left(\boldsymbol{\mu}_t\right)$ solves each agent's recursive problem;
2. $m_f\left(\theta_{jt}\right) \sum_{\eta} \lambda_{j\eta t}\left[A - w_{j\eta t}\right] \leqslant \kappa_\nu$ for all j, with equality if $j \in \Gamma_t$;
3. for any $j \in \Gamma_t$, $\lambda_{j\eta t}$ satisfies (6);
4. the law of motion $\boldsymbol{\mu}_{t+1} = G\left(\boldsymbol{\mu}_t\right)$ for the state is given by (3);
5. for any $j \in \Gamma_t^c$, if $w_{j\eta t} = w \neq w^*$ for all $\eta$, then $\theta_{jt}$ solves $m_f\left(\theta\right)\left(A - w\right) = \kappa_\nu$;
6. for $j \in \Gamma_t^c$ such that $\hat{V}_t\left(j, \eta, \boldsymbol{\mu}_t; \hat{\boldsymbol{\lambda}}_t\right) < V_t\left(\eta, \boldsymbol{\mu}_t\right)$ for all $\hat{\boldsymbol{\lambda}}_t$ then $\lambda_{j\eta t} = 0$.

The only condition that changes from the baseline is the refinement in condition 5. The condition says that for an inactive island that has constant wages for each type not equal to $w^*$, the agent's beliefs about the market tightness on that island are such that at that wage firms make zero profits. As we mentioned above, we will show that $w^*$ is indeed the equilibrium wage, and so this condition says that agents' beliefs about market tightness in off-equilibrium-path islands is such that firms always make zero profits.

We now characterize the competitive equilibrium.

**Proposition 11** (Characterization)**.** *Any competitive equilibrium is separating, has no cross-subsidization, and the equilibrium wage rate and market tightness are given by* $(w^*, \theta^*)$.

*Proof.* Consider the last period T. Notice that for any island $j \in \Gamma_T$, it must be that $l_{jIT} > 0$ and $w_{jIT} \geqslant w^*$. Suppose that this is not the case and that $w_{jIT} < w^*$. In equilibrium, firms must make zero profits on this island. Consider some $j' \in \Gamma_T^c$ such that $w_{j'\eta T} = w^*$ for all $\eta$. From equilibrium condition 5, we have that $\theta_{j'T} = \theta^*$. Given the definition of $(w^*, \theta^*)$, the infected agent is strictly better off by choosing island $j'$; this is a contradiction. A similar argument establishes that $w_{jRT} \geqslant w^*$ for $j \in \Gamma_T$ such that $l_{jRT} > 0$.

We use this result to show that there is no cross-subsidization in period T. To see this result, suppose there exists some island $j \in \Gamma_T$ such that $w_{jST} < w^*$. Then, consider some island $j' \in \Gamma_T^c$ with $w_{jST} < w_{j'\eta T} = w^* - \varepsilon$ for all $\eta$ and small $\varepsilon$. From equilibrium condition 5, $\theta$ is such that firm's make zero profits. From equilibrium condition 6, we have that neither infected nor recovered agents will choose island $j'$. Thus, the probability of infection in island $j'$ is zero. Therefore, for small enough $\varepsilon$, the susceptible agent is strictly better off by choosing island $j'$; this result is a contradiction.

We complete the argument for period T by showing that the equilibrium is separating. To do so, suppose that the equilibrium has mixing so that there is some $j \in \Gamma_t$ such that $l_{jST}, l_{jIT} > 0$. Consider some island $j' \in \Gamma_t^c$ such that $w_{j'\eta T} = w^* - \varepsilon$ for all $\eta$. As was the case before, the market tightness on this island is given by condition 5. Condition 6 guarantees that $\lambda_{j'IT} = 0$. Thus, for sufficiently small $\varepsilon$, the susceptible agent is made strictly better off by switching, and we have a contradiction.

These arguments imply that $V_T(S, \boldsymbol{\mu}_T) \geqslant V_T(I, \boldsymbol{\mu}_T)$ for all $\boldsymbol{\mu}_T$. Next, consider period $T-1$. Using the monotonicity result, we can repeat all the arguments above to show that there is no cross-subsidization, or mixing in period $T-1$. The argument for the other periods follows by induction. □

We now show that the equilibrium allocation is efficient.

**Proposition 12.** *The competitive equilibrium is Pareto optimal.*

*Proof.* In equilibrium there are no new infections. In the absence of infections, the Pareto optimal allocation maximizes the agent's utility subject to the resource constraint

$$\max_{\gamma, c_w, c_u} m_w\left(\frac{\gamma}{L}\right) u\left(c_w\right) + \left(1 - m_w\left(\frac{\gamma}{L}\right)\right) u\left(c_u\right),$$

subject to

$$L m_w\left(\frac{\gamma}{L}\right) c_w + L\left(1 - m_w\left(\frac{\gamma}{L}\right)\right) c_u + \gamma\kappa_v \leqslant M\left(\gamma, L\right) A,$$

where $L$ is the mass of agents. It is easy to see that this problem is equivalent to

$$\max u\left(c\right),$$

subject to

$$c + \theta\kappa_v \leqslant m_w\left(\theta\right) A,$$

which can be written as

$$\max_{\theta}\left[m_w\left(\theta\right) A - \theta\kappa_v\right].$$

Since $m_f\left(\theta\right) = m_w\left(\theta\right)/\theta$, this maximization problem is identical to (57). Thus, the equilibrium is optimal. □

# C Additional Figures

Figure 3: Employment in the last period in the UIR model.

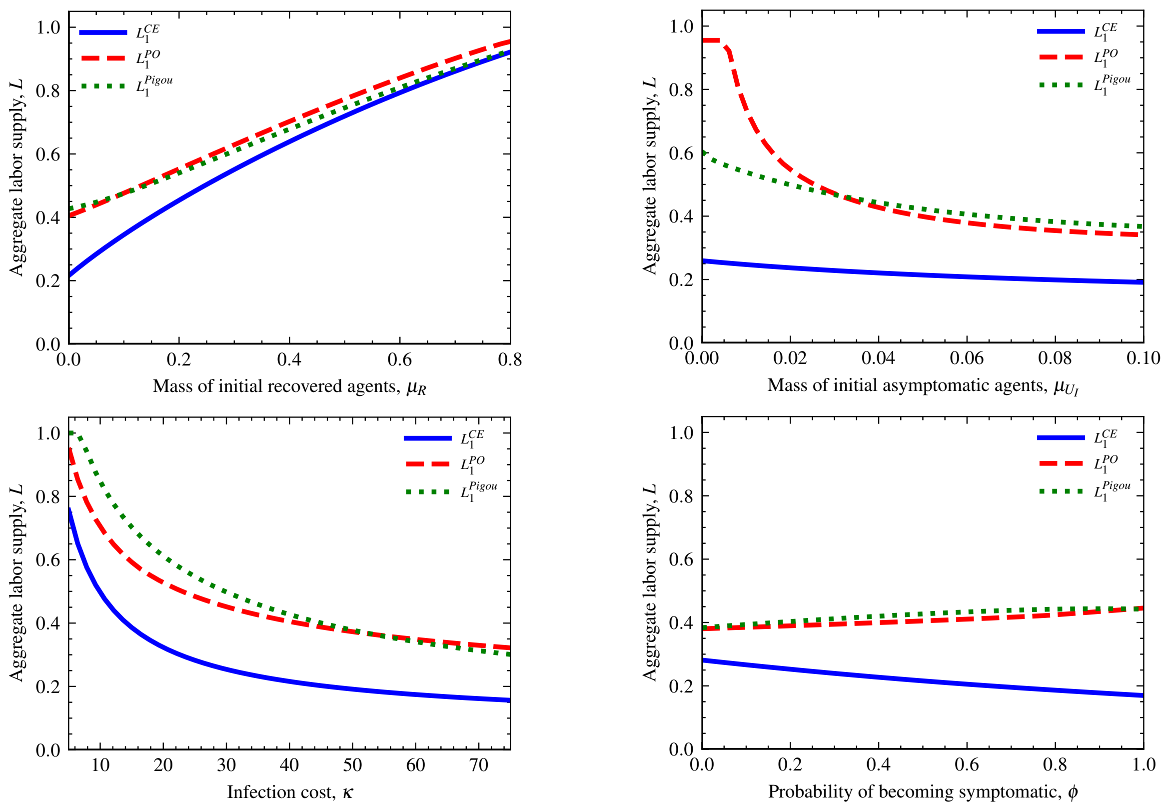

Figure 4: Individual employment in the first period in the UIR model.

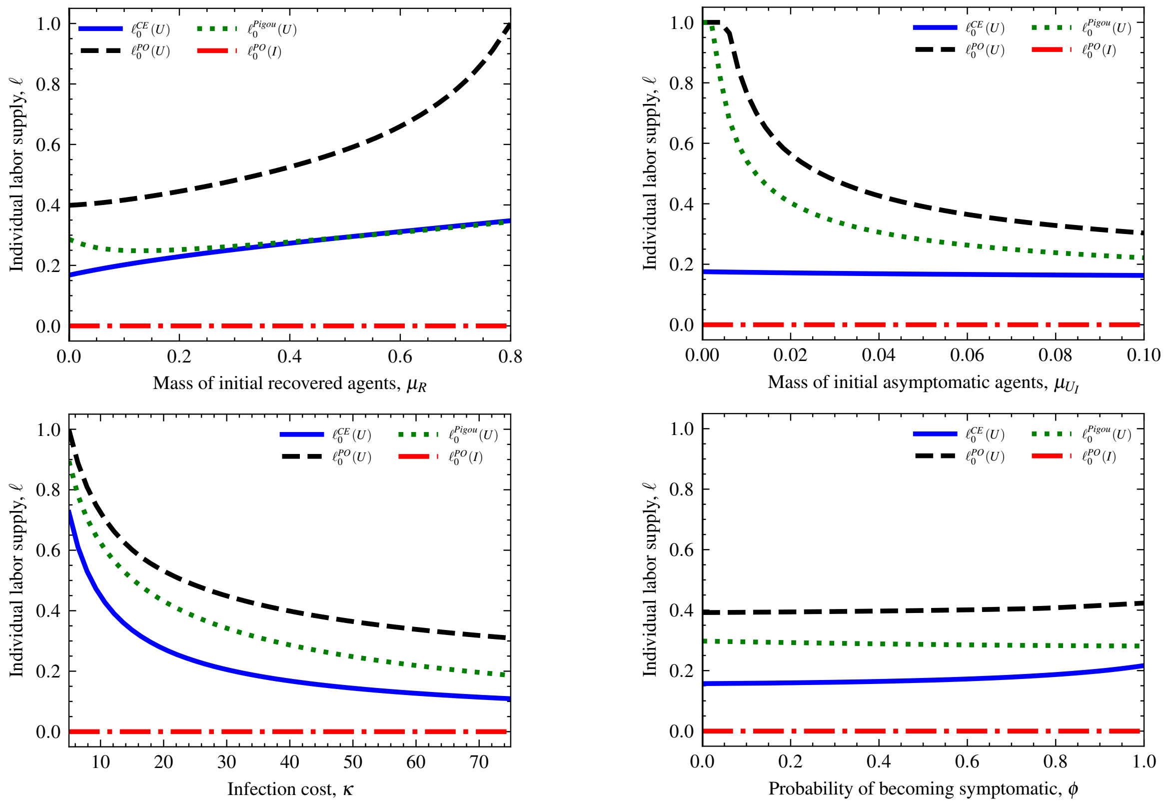

Figure 5: Individual employment in the last period in the UIR model.