ELECTION INVERSIONS AND THE U.S. ELECTORAL COLLEGE

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Overview

- An *election inversion* is an event
  - which can occur under U.S. Electoral College or any other districted electoral system.
- An election inversion occurs when the candidate (or party) who wins the most votes from the nationwide electorate fails to win the most electoral votes (or seats) and therefore loses the election
  - Such terms as ‘election reversal,’ ‘reversal of winners,’ ‘wrong winner,’ ‘representative inconsistency,’ ‘compound majority paradox,’ and ‘referendum paradox’ are also used.
  - Such an event actually occurred in the 2000 Presidential election
- In so far as this phenomenon may be ‘paradoxical,’ it is of a somewhat different character from most other ‘paradoxes of voting,’
  - in that it may arise even if there are only two candidates (or other alternatives).
  - Moreover, it is straightforward in nature and an occurrence is readily apparent.
  - However, the underlying factors that produce election inversions are less apparent and there is consider-able confusion about the circumstances under which election inversions occur.

Overview (cont.)

- With specific respect to the U.S. Electoral College, this presentation will
  - review some manifestations of election inversions,
  - analyze their sources,
  - establish logical extreme bounds on the phenomenon, and
  - estimate the frequency of Presidential election inversions in various data sets:
    - actual elections,
    - hypothetical uniform swings from actual elections,
    - random (Bernoulli) elections (in the manner of Feix, Lepelley, Merlin, and Rouet), and
    - simulated elections in which votes fluctuate randomly about the comtemporary historical pattern.
  - In addition, the presentation examines how these frequencies may change under other proposed variants of the Electoral College such as the ‘district system,’ the ‘proportional system,’ etc.

The Problem of Election Inversions

- Any *indirect* or *districted* electoral system can produce an *election inversion.*
  - That is, the candidate or party that wins the most popular votes nationwide may fail to win
    - the most (uniform) districts or
    - (non-uniform) districts with the greatest total weight, and
  - thereby loses the election.

- **Footnote: The first scientific work on this subject was done by May (1948). Chambers (2005) demonstrates (in effect) that no neutral (between candidates) districted electoral rule can satisfy "representative consistency," i.e., avoid election inversions . Also see Laffond and Laine (2000)**

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PROBABILITIES OF CERTAIN ELECTION RESULTS

KENNETH MAY, Carleton College*

1. Introduction. In any indirect electoral system or in the election of any representative body, the results will not in general correspond exactly to the votes of the electorate. In particular, it is possible for a candidate or a party to win a majority of the electors or representatives with a minority of the popular vote. What is the probability of such a result?† The answer depends, of course, upon many factors, including the method of election, the number of districts, voters and parties, and the frequency distributions of election outcomes in the districts.

* Presented in part to the Minnesota Section of the Mathematical Association of America at St. Cloud, Minnesota, May 10, 1947.

† The question was proposed to the author by Professor G. L. Field of the Department of Government, Wayne University. The writer is indebted to Professors J. Neyman and H. Lewy of the University of California for helpful suggestions.

The U.S. Electoral College

- The President of the United States is elected,
  - not by a direct national popular vote, but
  - by an indirect Electoral College system in which (in almost universal practice since the 1830s)
    - separate state popular votes are aggregated by adding up electoral votes awarded on a winner-take-all basis to the plurality winner in each state.
  - Each state has electoral votes equal in number to its total representation in Congress.
- Therefore the U.S. Electoral College is a *districted* electoral system.
- Most such systems have *uniform districts*, which have
  - equal weight (e.g., a single parliamentary seat),
  - reflecting (approximately) equal populations and/or numbers of voters.
- In contrast, Electoral College "districts" (i.e., states) are (highly) unequally weighted
  - State electoral votes vary with population and at present range from 3 to 55.

Examples of Election Inversions

- Election inversions are actually more common in some (“Westminster”) parliamentary systems (with uniform districts) than in U.S. Presidential elections.
- Most such elections are very close with respects to both seat and votes,
  - but not all (e.g., Canada, 1979)
- However, many such inversions result from votes for third, etc., parties,
  - a factor not considered in the following US-focused analysis.

Election		Leading Parties	Pop. Vote %	Seats
UK	1929	Conservatives	38.1	260
		Labour	37.1	287
UK	1951	Labour	48.78	297
		Conservatives	47.97	302
UK	1974 (Feb.)	Conservatives	37.2	297
		Labour	37.9	301
NZ	1978	Labour	40.4	40
		National	39.8	51
NZ	1981	Labour	39.0	43
		National	38.8	47
CA	1979	Liberals	40.1	114
		Conservatives	35.1	136

Electoral College Election Inversions

- The U.S. Electoral College has produced three manifest election inversions,
    - all of which were very close with respect to popular votes, and
    - two of which were very close with respect to electoral votes.
  - plus one massive but "latent" election inversion that is not usually recognized.

Manifest EC Election Inversions

Election	EC Winner	EC Loser	EC Loser's 2-P PV%
2000	271 [Bush (R)]	267 [Gore (D)]	50.27%
1888	233 [Harrison (R)]	168 [Cleveland (D)]	50.41%
1876	185 [Hayes (R)]	184 [Tilden (D)]	51.53%

- The 1876 election was decided (just before the inauguration) by an Electoral Commission that, by a bare majority and straight party-line vote, awarded all of 20 disputed electoral votes to Hayes.
  - Unlike Gore and Cleveland, Tilden won an absolute majority (51%) of the *total* popular vote (for all parties/candidates).

The 1860 Election: A Latent But Massive Inversion

Candidate	Party	Pop. Vote %	EV
Lincoln	Republican	39.82	180
Douglas	Northern Democrat	29.46	12
Breckinridge	Southern Democrat	18.09	72
Bell	Constitutional Union	12.61	39

Total Democratic Popular Vote	47.55%
Total anti-Lincoln Popular Vote	60.16%

- Two inconsequential inversions (between Douglas and Breckinridge and between Douglas and Bell) are manifest.
- It may appear that Douglas and Breckinridge were *spoilers* against each other.
  - Under a direct popular vote system, this would have been true (if we suppose that, had one Democratic candidate withdrawn, the other would have inherited all or most of the his support).
  - But in fact, under the Electoral College system, Douglas and Breckinridge were *not* spoilers against each other.

A Counterfactual 1860 Election

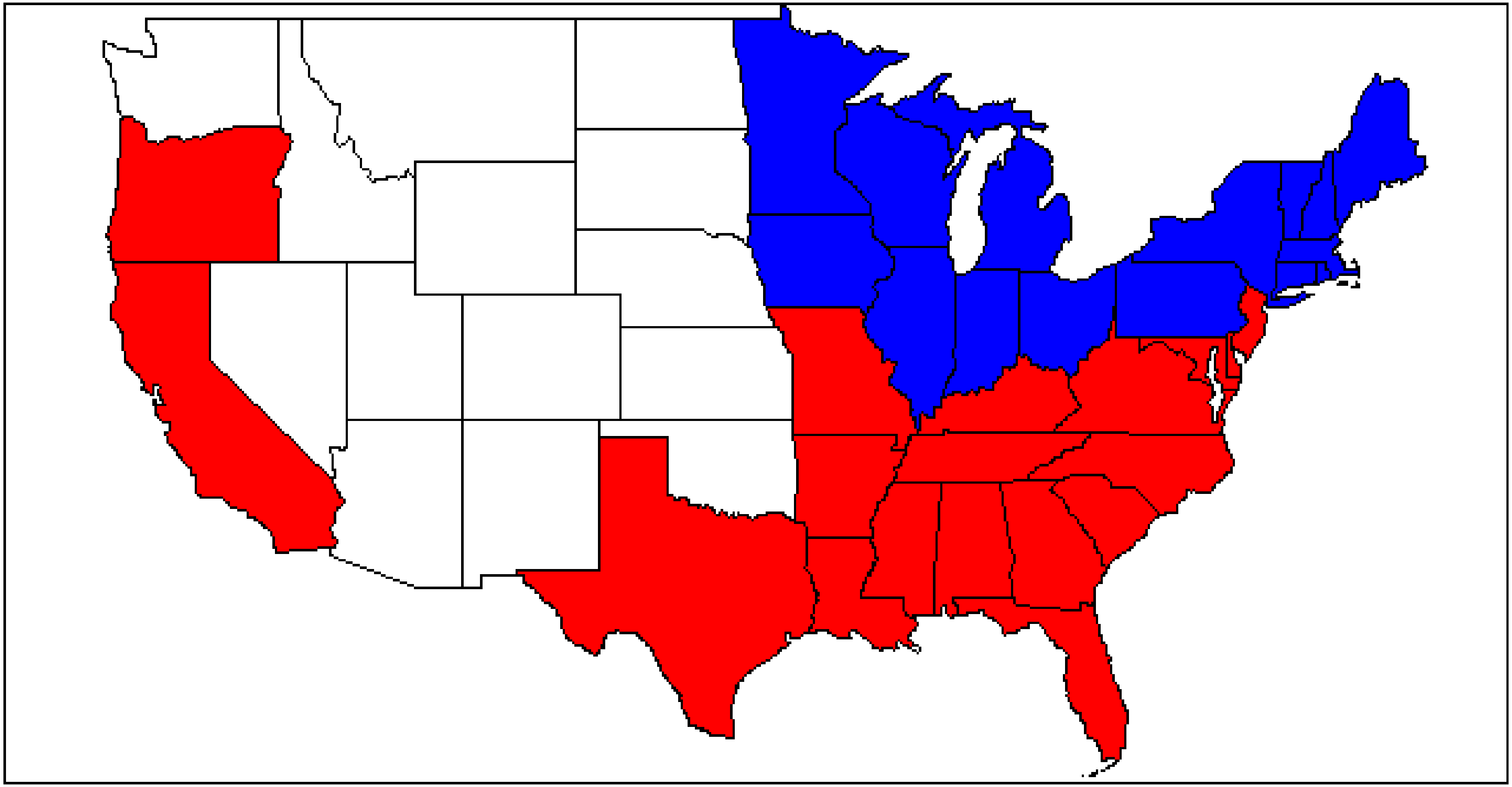
- Suppose the Democrats could have held their Northern and Southern wings together and won *all* the votes captured by each wing separately.
  - Suppose further that it had been a Democratic vs. Republican "straight fight" and that the Democrats *also* won *all* the votes that went to Constitutional Union party.
  - And, for good measure, suppose that the Democrats had won *all* NJ electoral votes (which for peculiar reasons were actually split between Lincoln and Douglas).

- Here is the outcome of this counterfactual 1860 election:

Party	Pop. Vote %	EV
Republican	39.82	169
Anti-Republican	60.16	134

Of all the states Lincoln won, he won by less than an absolute majority only in CA and OR.

A Counterfactual 1860 Election



LINCOLN (169) vs. ALL ANTI-REPUBLICANS (134)

The Probability of Election Inversions: First Cut – Historical Estimate

- Number of Inversions/Number of elections (since 1828)

  3/46 = .06522

  4/46 = .08696

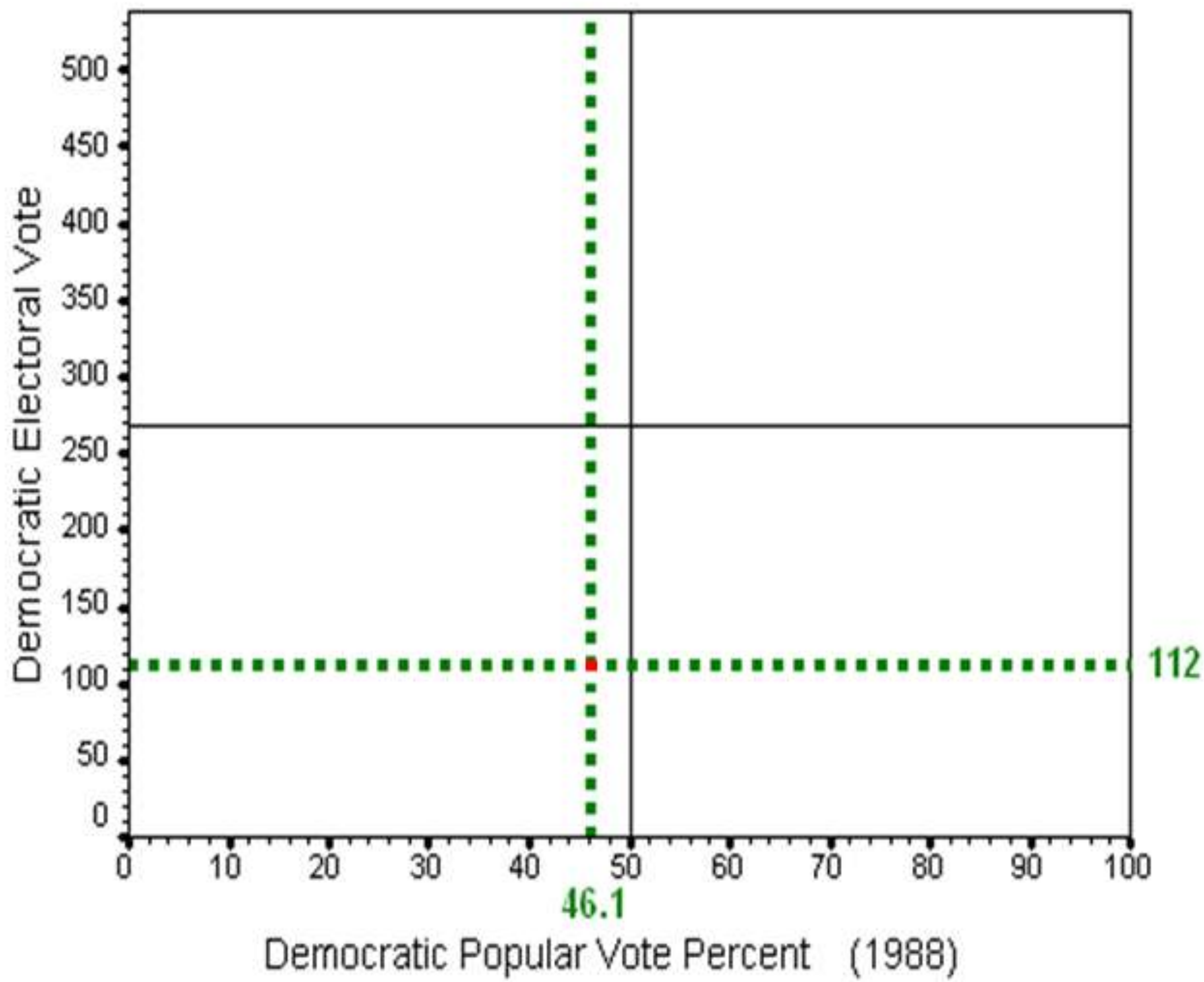
- Clearly an important determinant of the probability of an election inversion is the probability of a close division of the popular vote.

The PVEV Step Function

- We turn to a more informative historical and empirical analysis of election inversions, using *state-by-state popular vote percentages* to produce what I call the Popular Vote-Electoral Vote or *PVEV step function* for each election.
  - The PVEV is based on the kind of *uniform swing analysis* pioneered by Butler (1946), which has also be called
    - *hypothetical (single-year) swing analysis* (Niemi and Fett, 1986),
    - the *Bischoff method* (Peirce and Longley, 1981),
    - and it has also been employed by Nelson (1974), Garand and Parent (1991), and others.
- The PVEV function
  - is a cumulative distribution function and is therefore (weakly) monotonic, and
  - is a step function because, while the independent variable (PV) is essentially continuous, the dependent variable (EV) is discrete (taking on only whole number values and jumps up in discrete steps).

The PVEV Step-Function: 1988 as an Example

In 1988, the Democratic ticket of Dukakis and Bentsen received 46.10% of the two-party national popular vote and won 112 electoral votes (though one was lost to a "faithless elector").



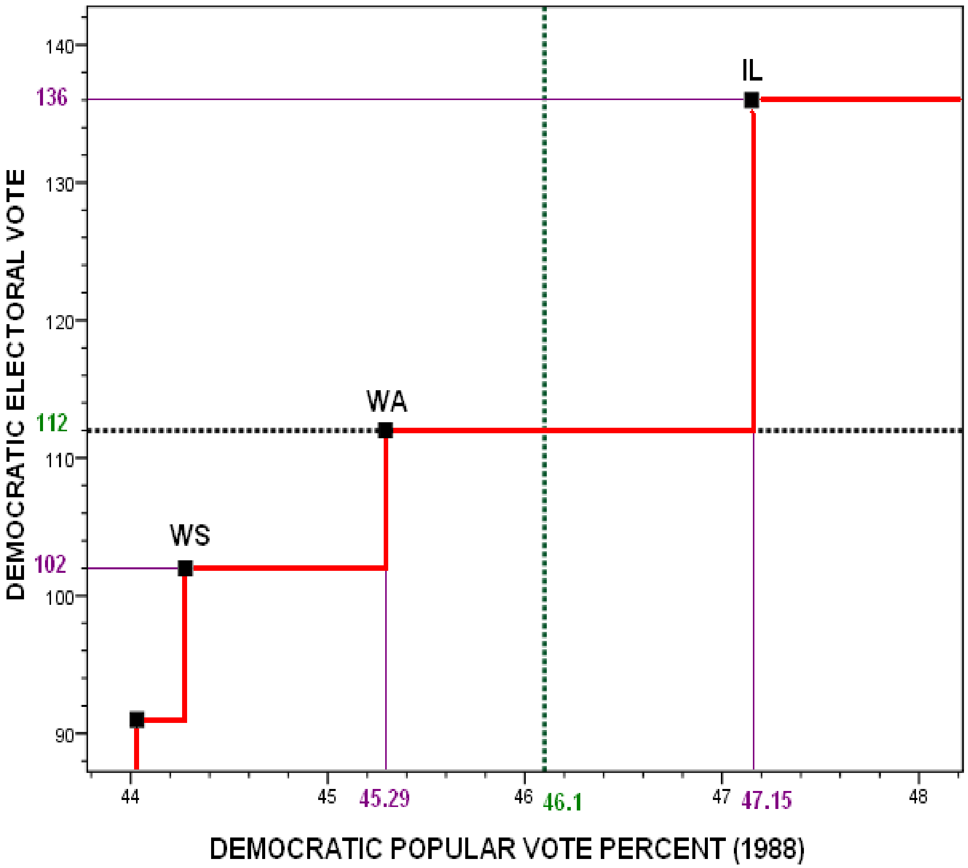
1988 Example (cont.)

Of all the states that Dukakis carried, he carried Washington (10 EV) by the smallest margin of 50.81%.

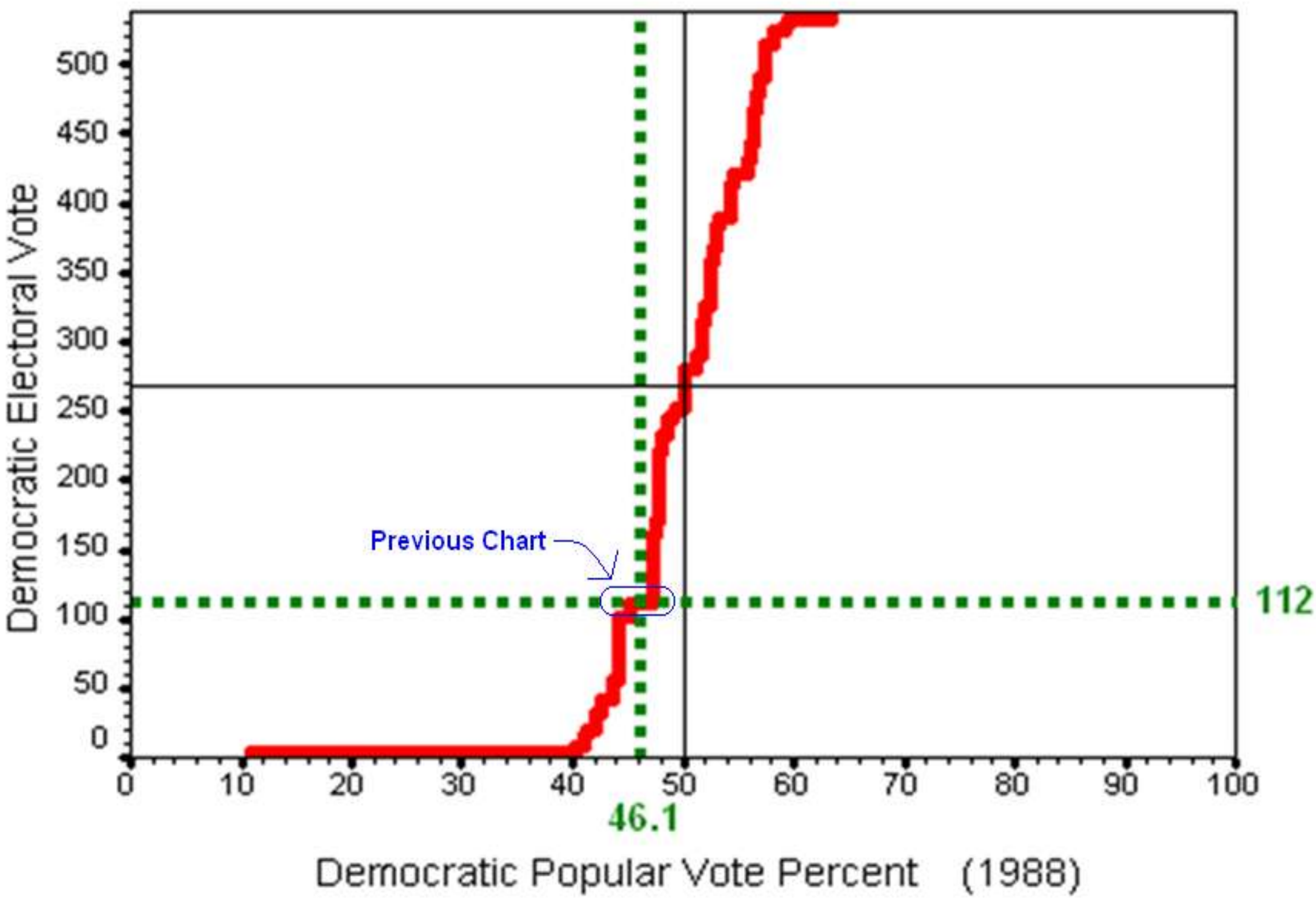
- If the Dukakis national popular vote of 46.10% were (hypothetically) to *decline* by 0.81 percentage points *uniformly* across all states (to 45.29%), WA would tip out of his column (reducing his EV to 102).

Of all the states that Dukakis failed carry, he came closest to carrying Illinois (24 EV) with 48.95%.

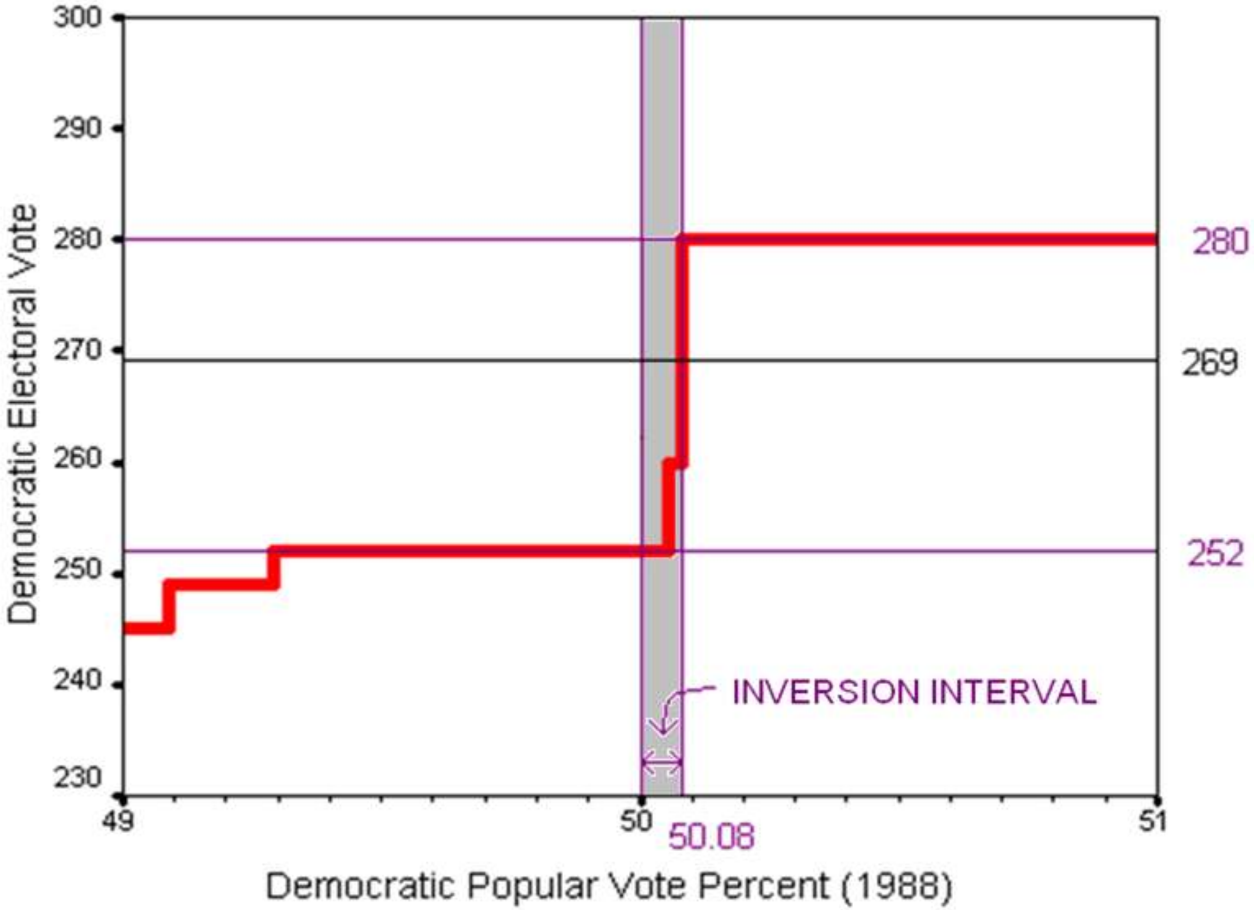
- If the Dukakis popular vote of 46.10% were (hypothetically) to *increase* by 1.05 percentage points *uniformly* across all states (to 47.15%), IL would tip into his column (increasing his EV to 136).



The PVEV Step Function for 1988



Zoom in on PV ≈ 50%

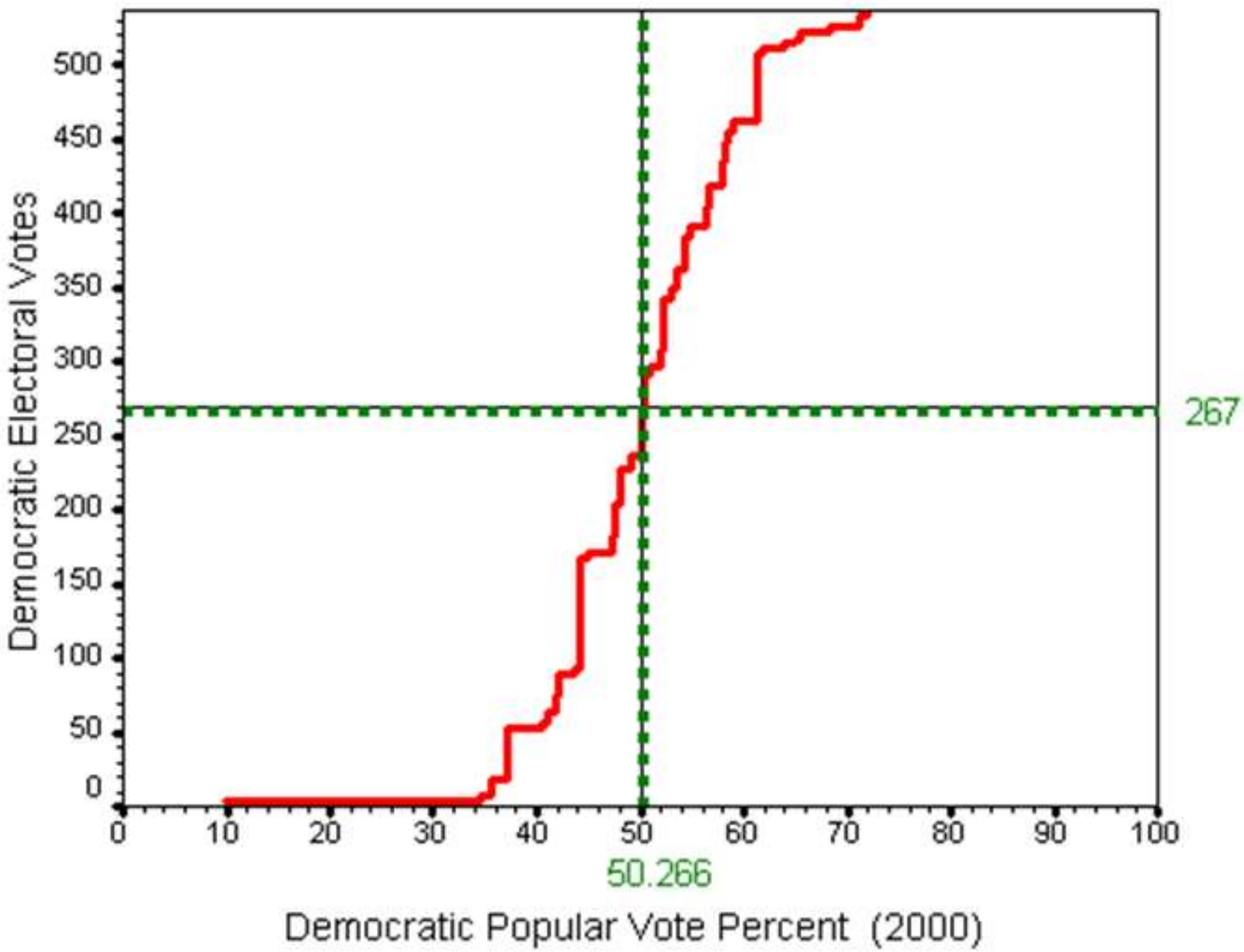


2000 vs. 1988

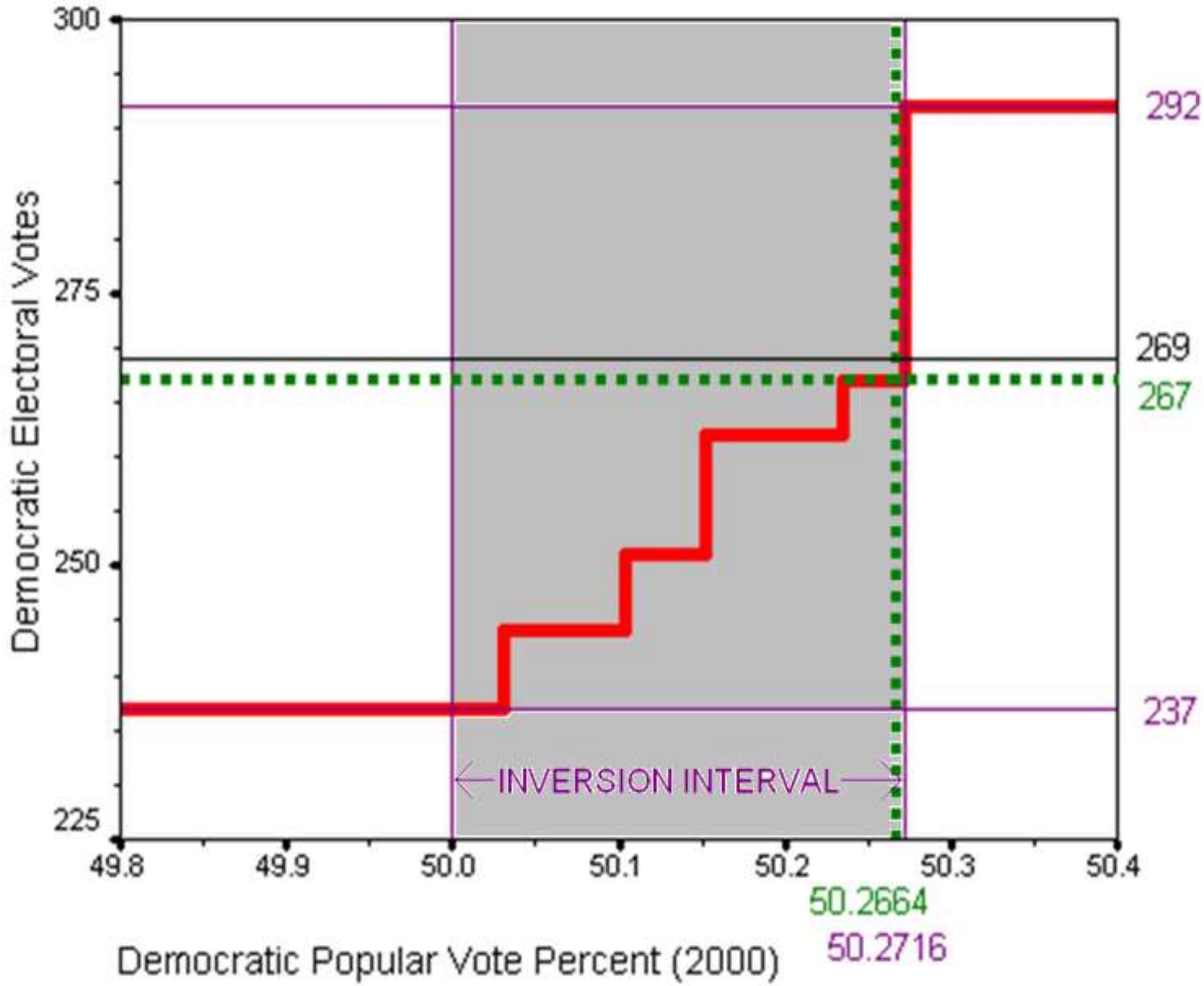
- The 2000 election produced an actual election inversion.
  - However, the election inversion interval was (in absolute terms) only slightly larger in 2000 than in 1988:
    - DPV 50.00% to 50.08% in 1988
    - DPV 50.00% to 50.27% in 2000

- The key difference between the 2000 and 1988 elections was not the magnitude of the inversion interval but that 2000 was *much* closer.
  - The actual D2PV was 50.267%, putting it (just) within the inversion interval,
    - though if the 2000 inversion interval had been as small as the 1988 interval (but still pro-Republican) Gore would have won.

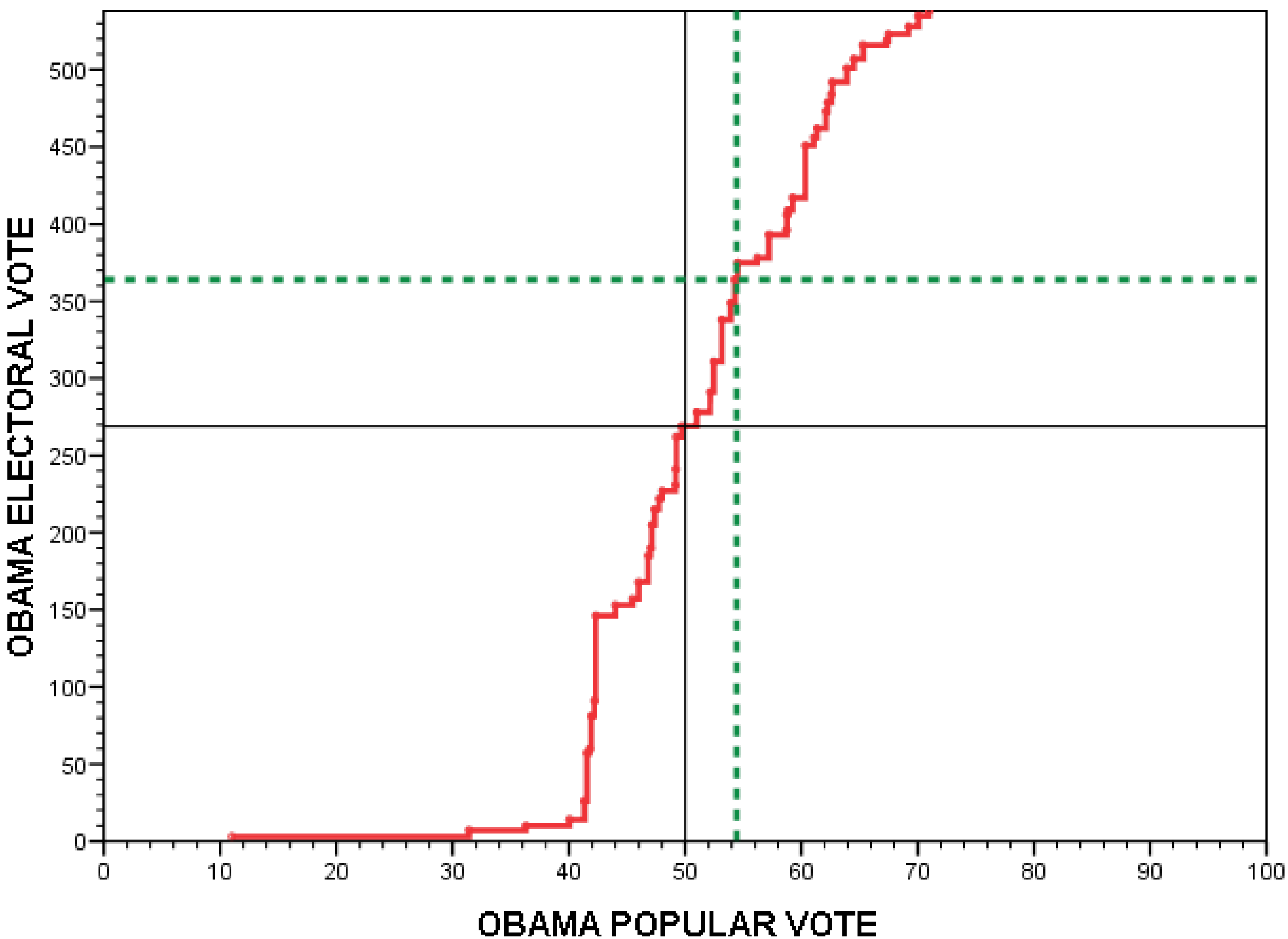
The PVEV Step Function for 2000



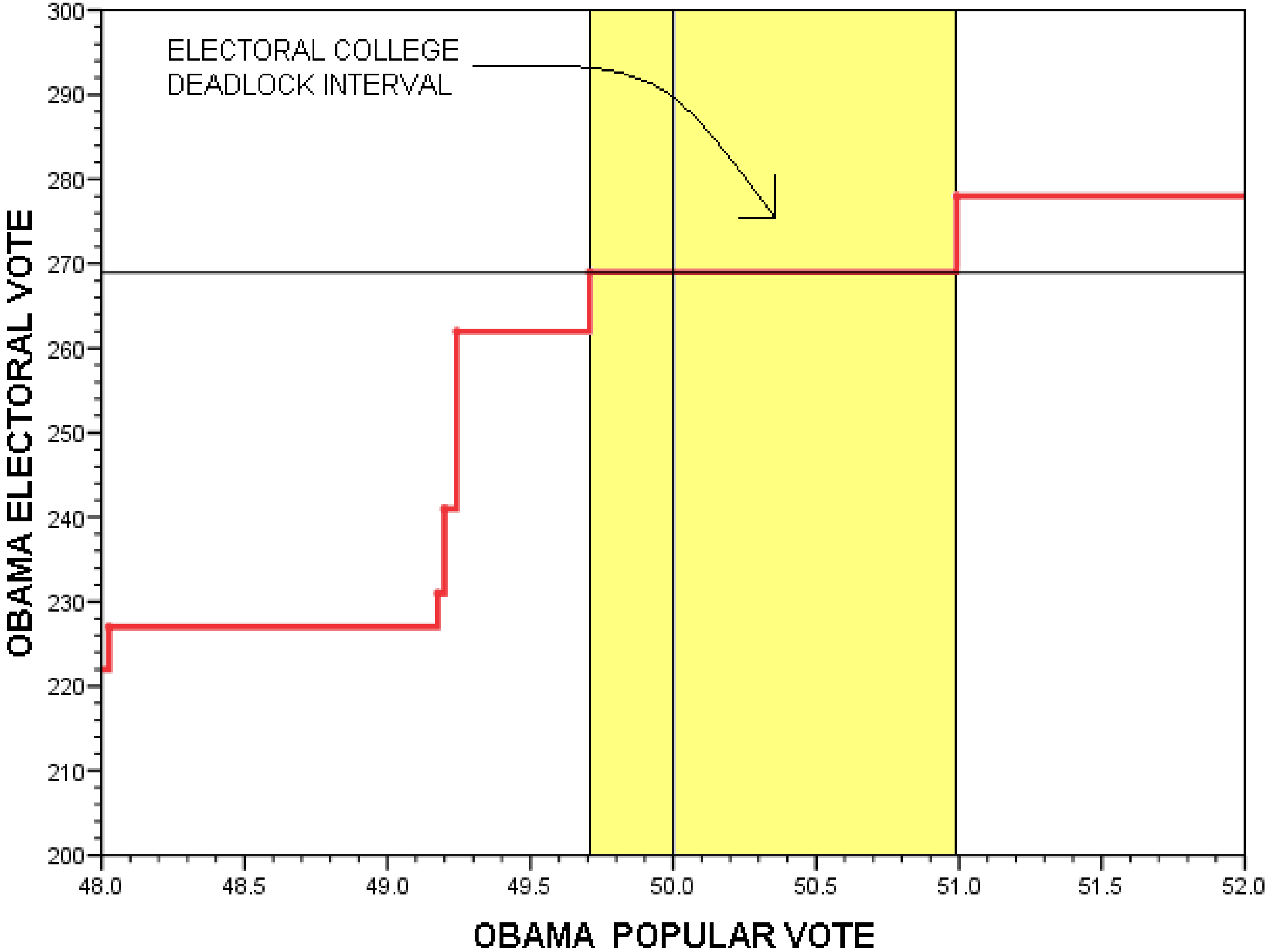
Zoom in on PV ≈ 50%



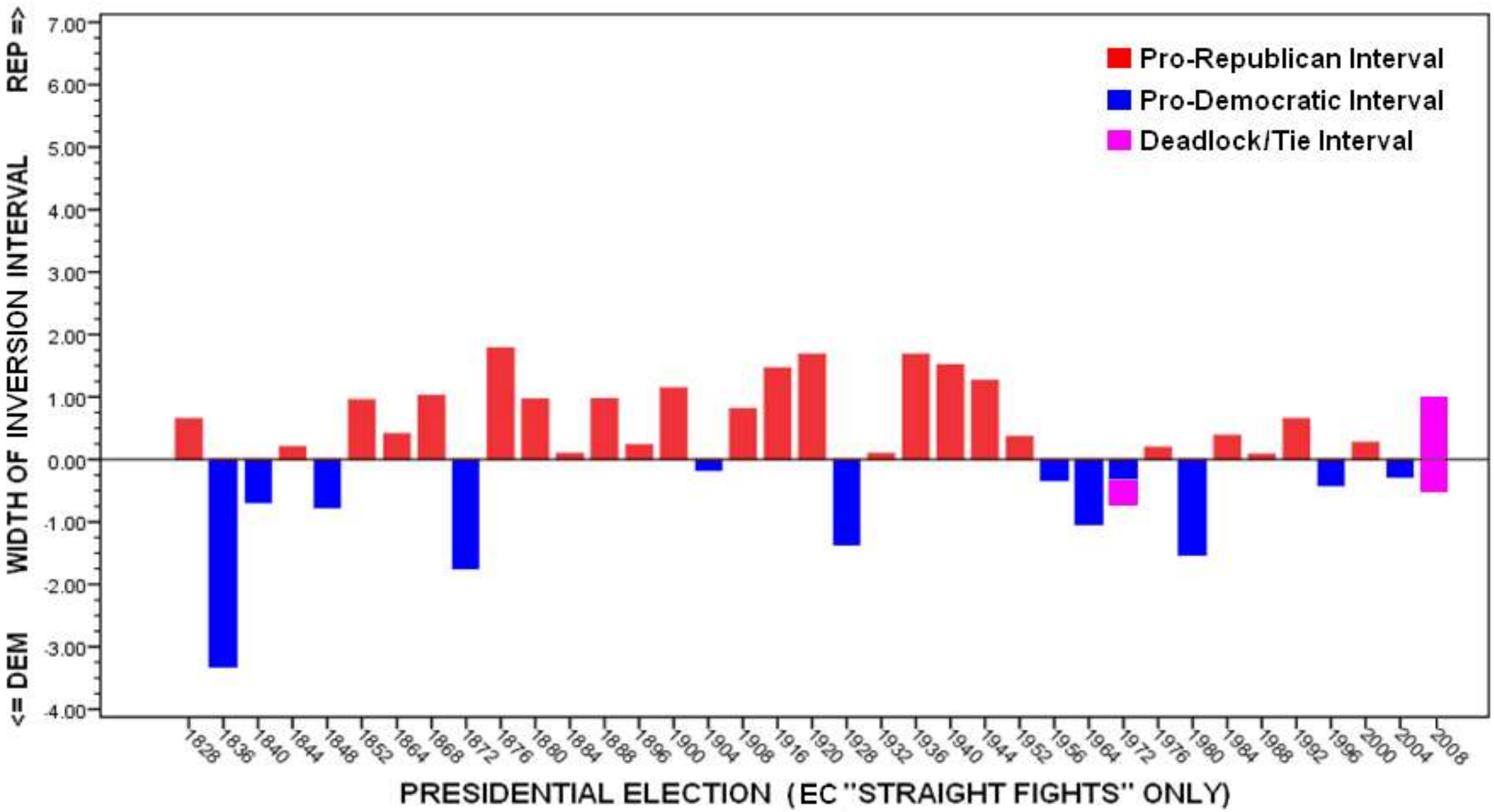
The PVEV Step Function for 2008



Zoom in on PV ≈ 50%



Historical Magnitude and Direction of Election Inversion Intervals

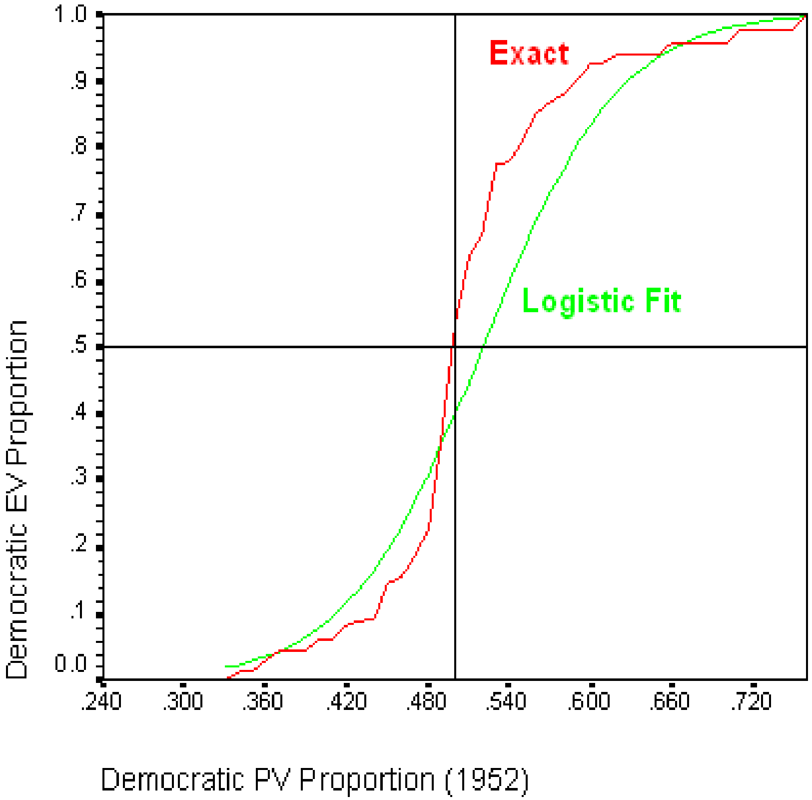


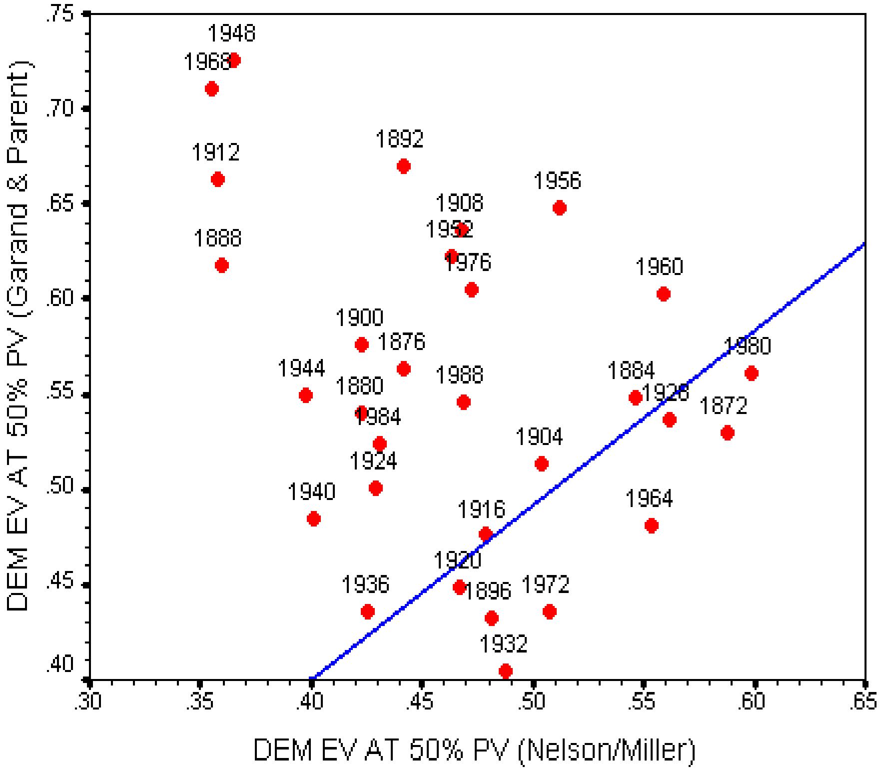
The Probability of Election Inversions: Second Cut – Historical Estimate

- Summary statistics for all EC straight fight elections since 1828:
  - Democratic (or Whig/Republican) two-party popular vote percent is approximately normally distributed,
    - with SD ≈ 6%.
    - Let's set the mean at 50%
  - Mean Magnitude of Inversion Interval ≈ 1%
  - This implies the probability of election inversion ≈ 0.066
- Summary statistics for all EC straight fight elections since 1988:
  - Democratic (or Republican) two-party popular vote percent:
    - SD ≈ 3.4%
    - Again let's set mean at 50%
  - Mean Magnitude of Inversion Interval ≈ 0.34%
  - This implies the probability of election inversion ≈ 0.04

Footnote: Garand and Parent (1991)

- G&P (in effect) use the PVEV function for each election to estimate the best fitting two-parameter logistic S-curve from 1872 through 1984 and they examine this best fitting curve in the vicinity of PV = 50%.
  – This smooth curve suggest quite different, and usually much larger and (preponderantly pro-Democratic), inversion intervals estimates of than those produced by the PVEV function itself.





Two Distinct Sources of Possible Election Inversions

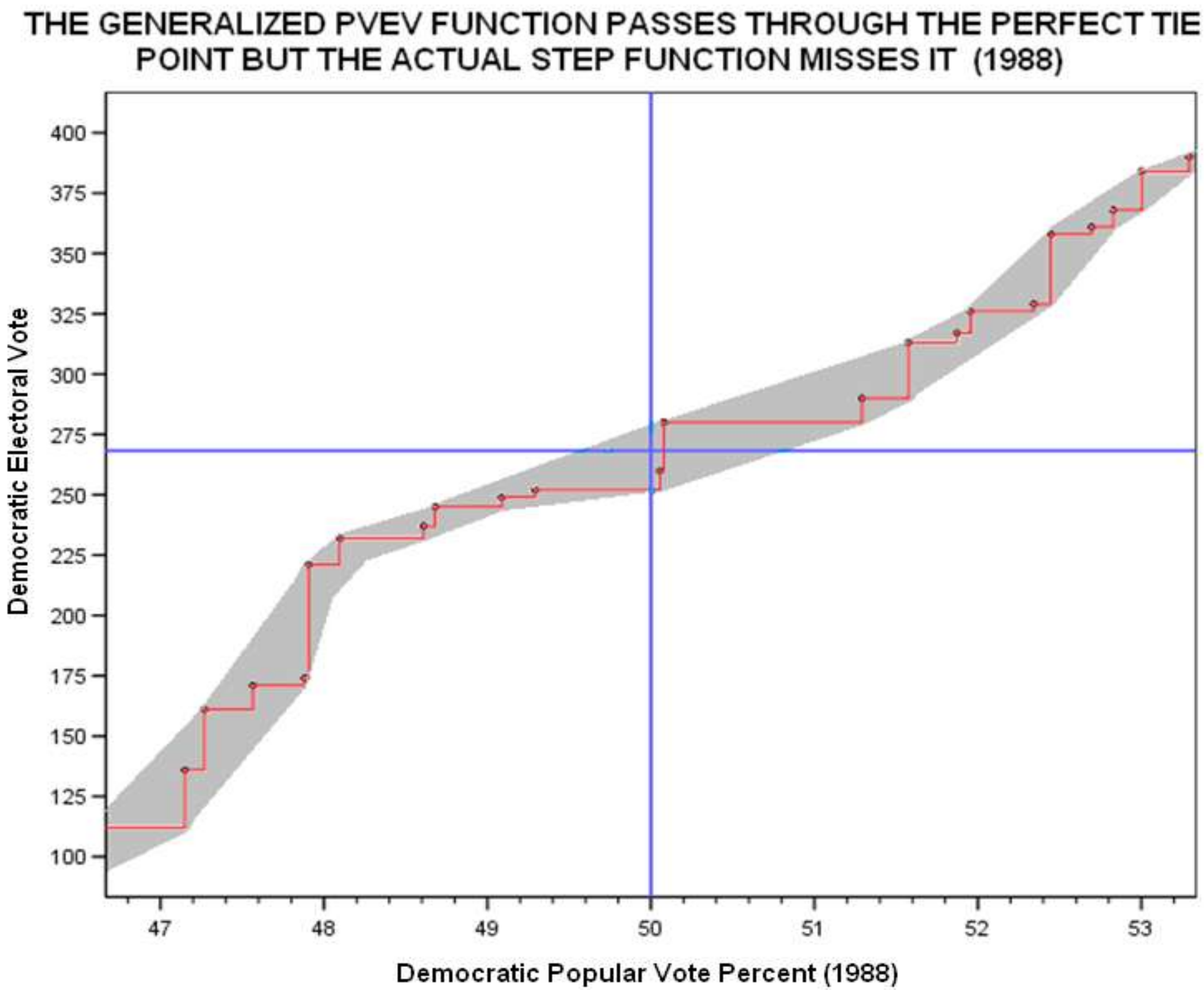
- The PVEV step-function defines a particular *electoral landscape*,
  - i.e., an interval scale on which all states are located with respect to the *relative partisan* (with respect to Presidential preference) *composition of their electorates ;*
  - for example, in the 1988 electoral landscape WA is 1.86% more Democratic than Illinois.
- Put otherwise, if two elections with two different (PV and EV) outcomes had the same landscape, their state-by-state popular votes would be perfectly correlated,
  - but their regression equations would have different constant terms, reflecting different pro- or anti-Democratic swings up and down over the same landscape.
- The PVEV visualization makes it evident that there are two distinct ways in which election inversions may occur.

The First Source of Possible Inversions

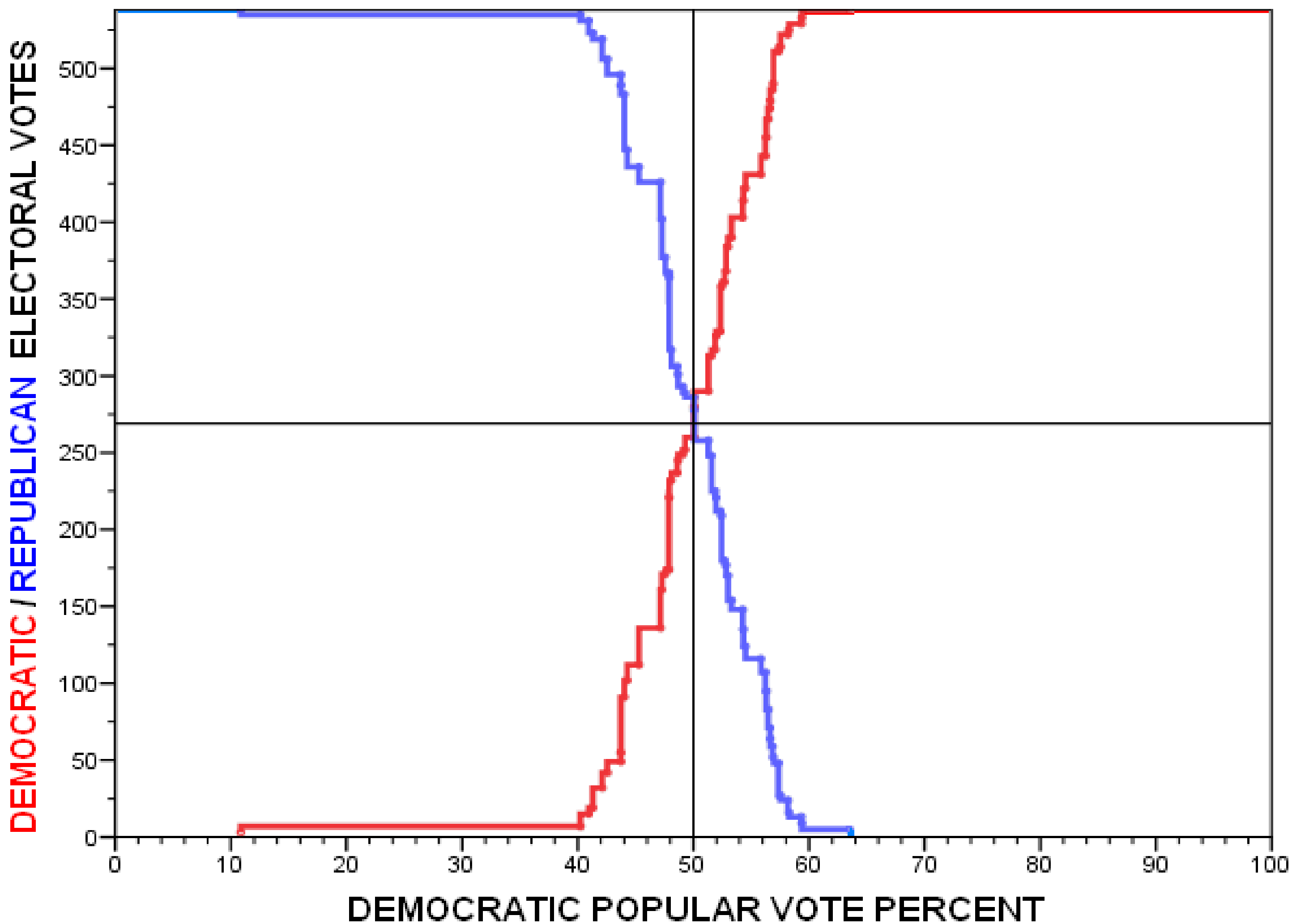
- The first source of possible election inversions is invariably present.

- An election reversal may occur as a result of the (non-systematic) “rounding error” (so to speak) necessarily entailed by the fact that the PVEV function moves up in *discrete steps*.
  - That is to say, as the Democratic vote swings upwards, almost certainly the *pivotal state* (that gives the Democratic candidate 270 or more electoral votes) will essentially never tip into the Democratic column precisely as the Democratic popular vote crosses the 50% mark.
    - In any event, a specific electoral landscape allows (in a sufficiently close election) a “wrong winner” of one party only.
    - But small perturbations in some landscapes would allow a “wrong winner” of either party.

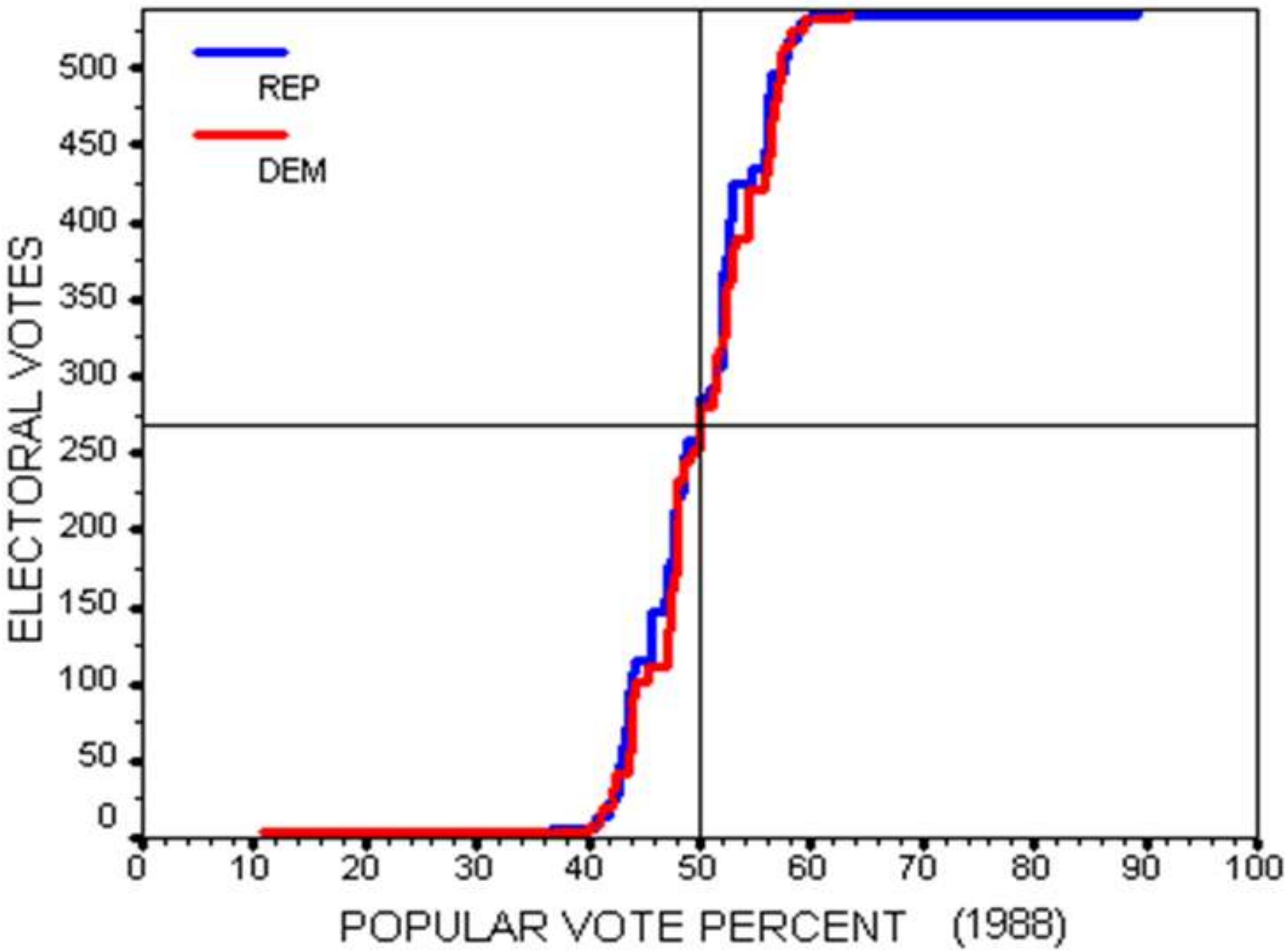
First Source (cont.)



PVEV for 1988 Showing Both DEV and REV by D2PV (Almost Symmetric)



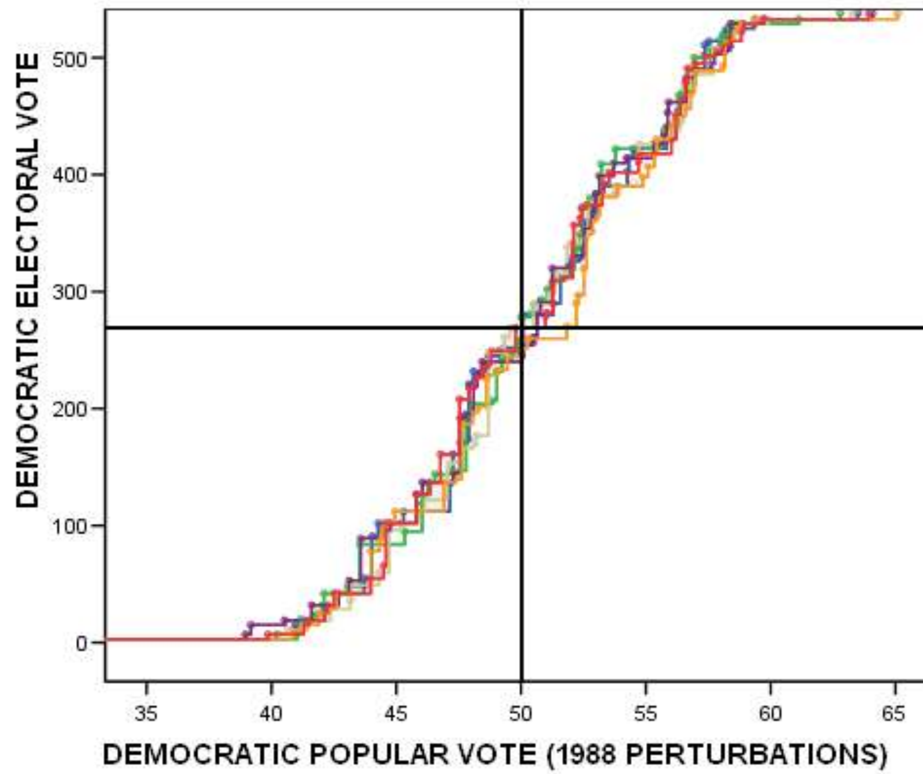
Democratic vs. Republican PVEV in 1988 (Almost Identical)



First Source (cont.)

- The previous 1988 chart (and similar charts for all recent elections [including 2000 and 2008 included here]) provide a clear illustration of possible election inversions due to such “rounding error” only.

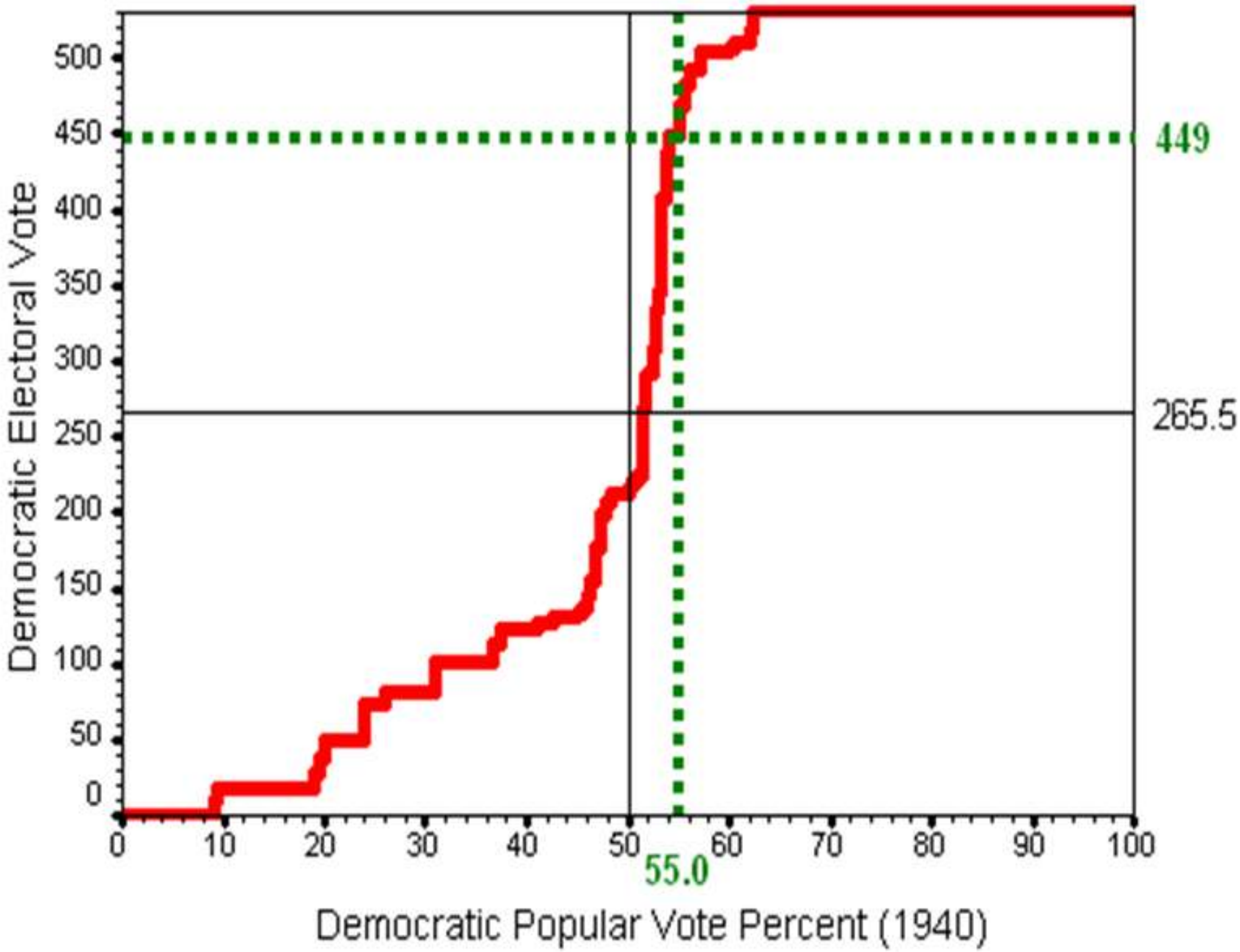
- Moreover, if the 1988 election
  - had been much closer (in popular votes) and
  - the electoral landscape slightly perturbed,
  - Dukakis might have been a wrong winner instead of Bush.



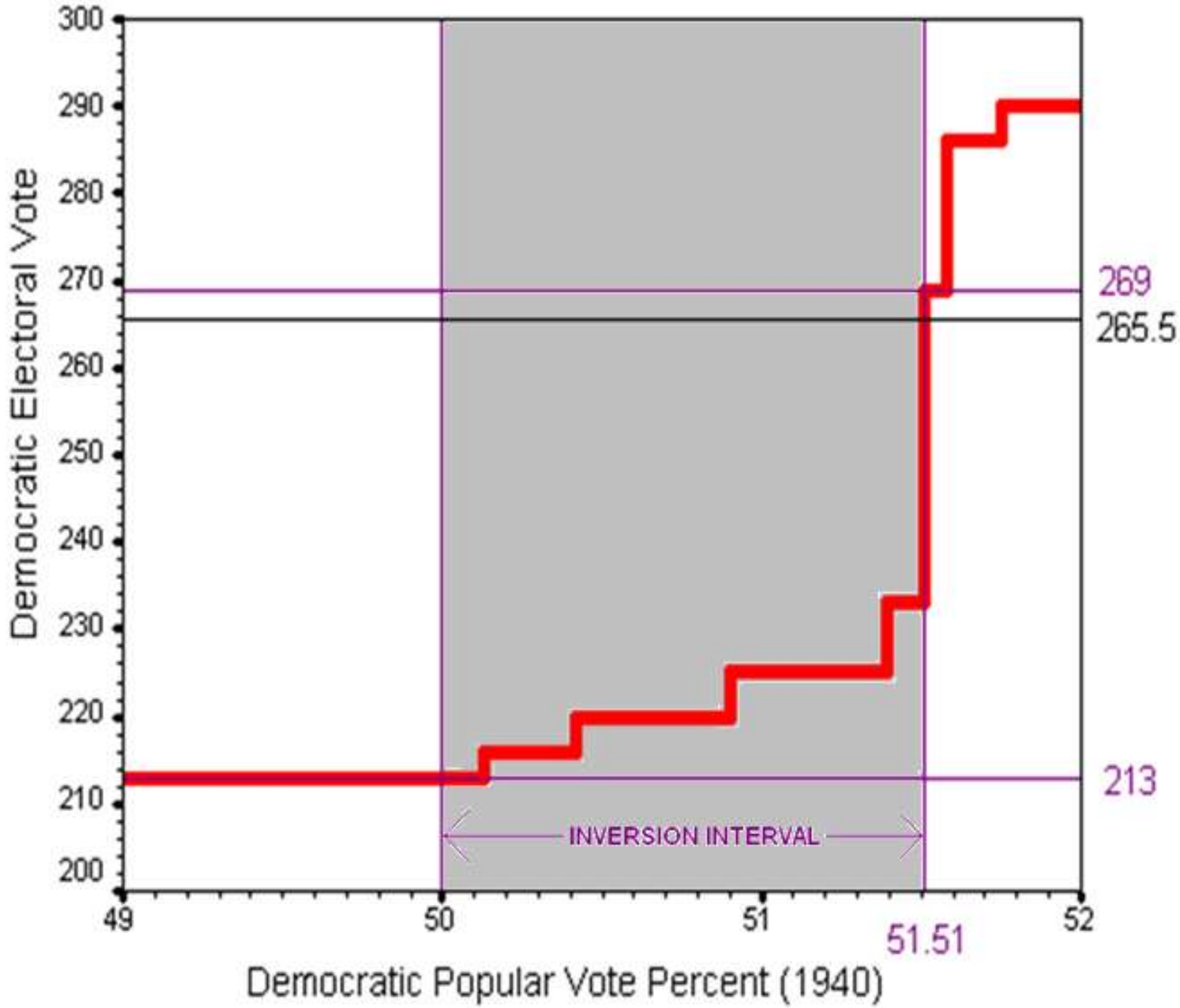
The Second Source of Possible Election Inversions

- Second, an election reversal may occur as result of a (systematic) *asymmetry* or (*partisan*) *bias* in the general character of the PVEV function,
  - particularly in the vicinity of PV = 50%.
  - In this event, small perturbations of the electoral landscape will not change the partisan identity of potential wrong winners.
- In times past (specifically, in the New Deal era and earlier), there was a clear (pro-Republican) asymmetry in the PVEV function that resulted largely from the electoral peculiarities of the old "Solid South," in particular,
  - its *overwhelmingly Democratic popular vote percentages*, combined with
  - its *strikingly low voting turnout*.

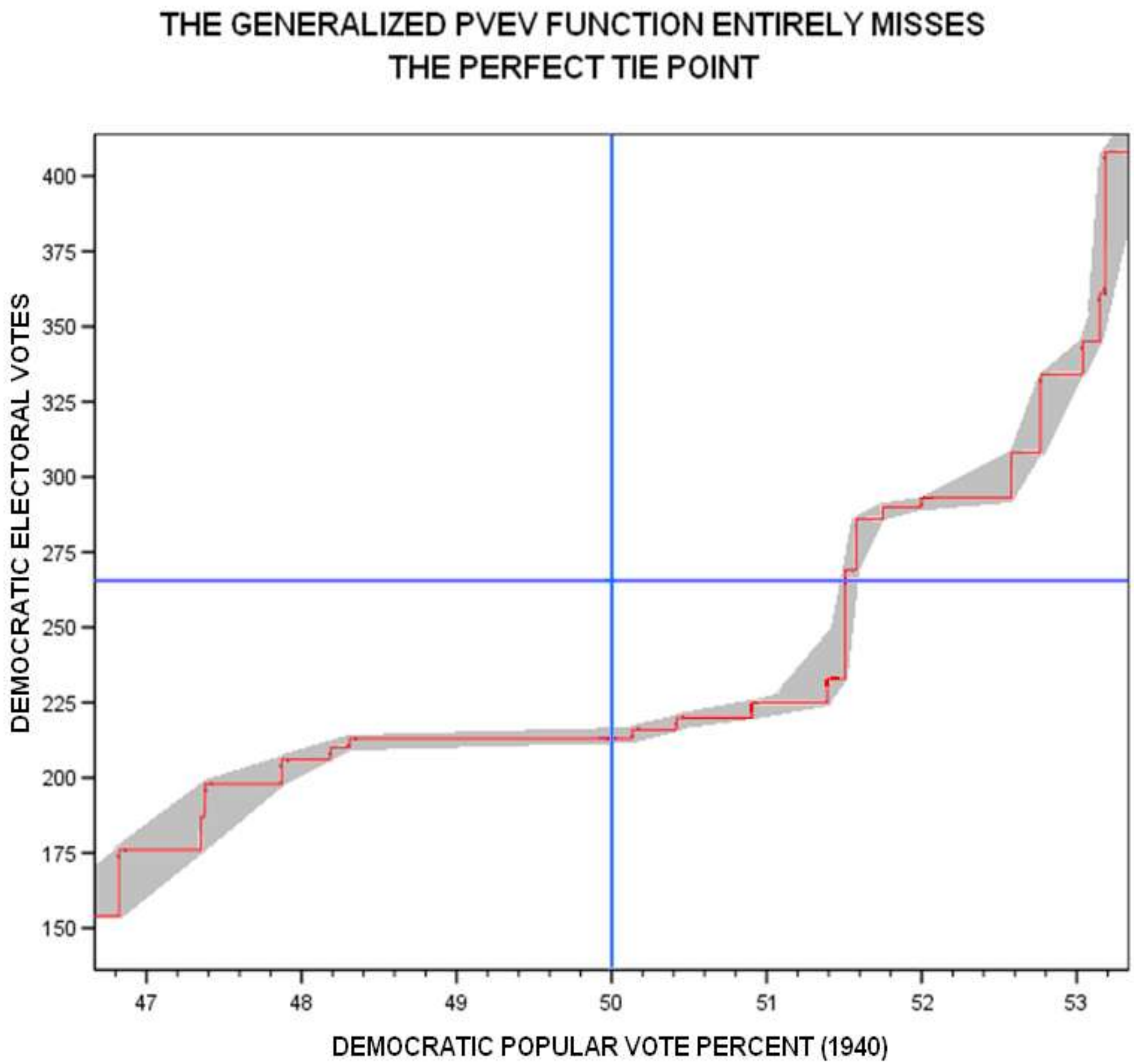
An Asymmetric PVEV: 1940



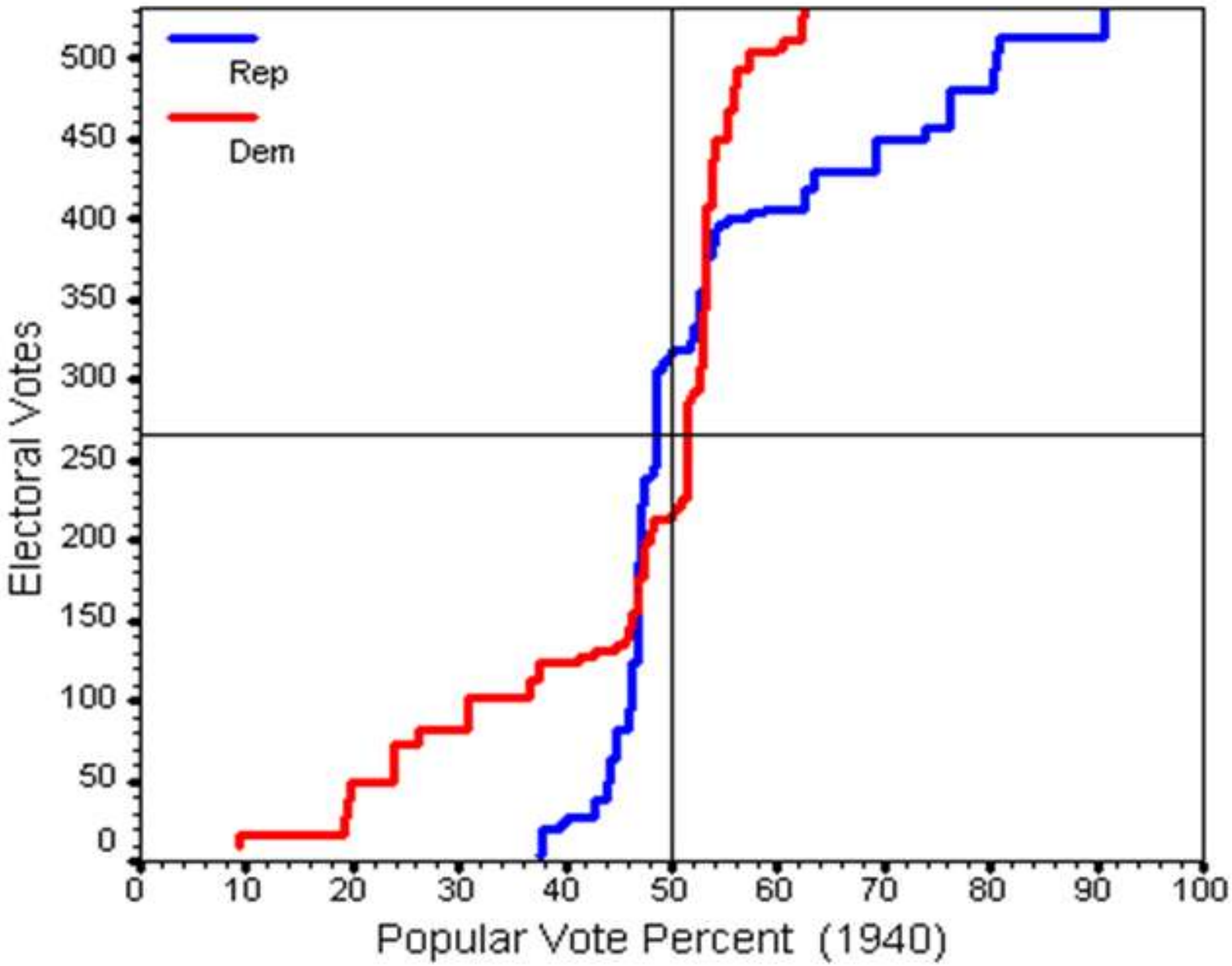
Zoom in on the 1940 Inversion Interval



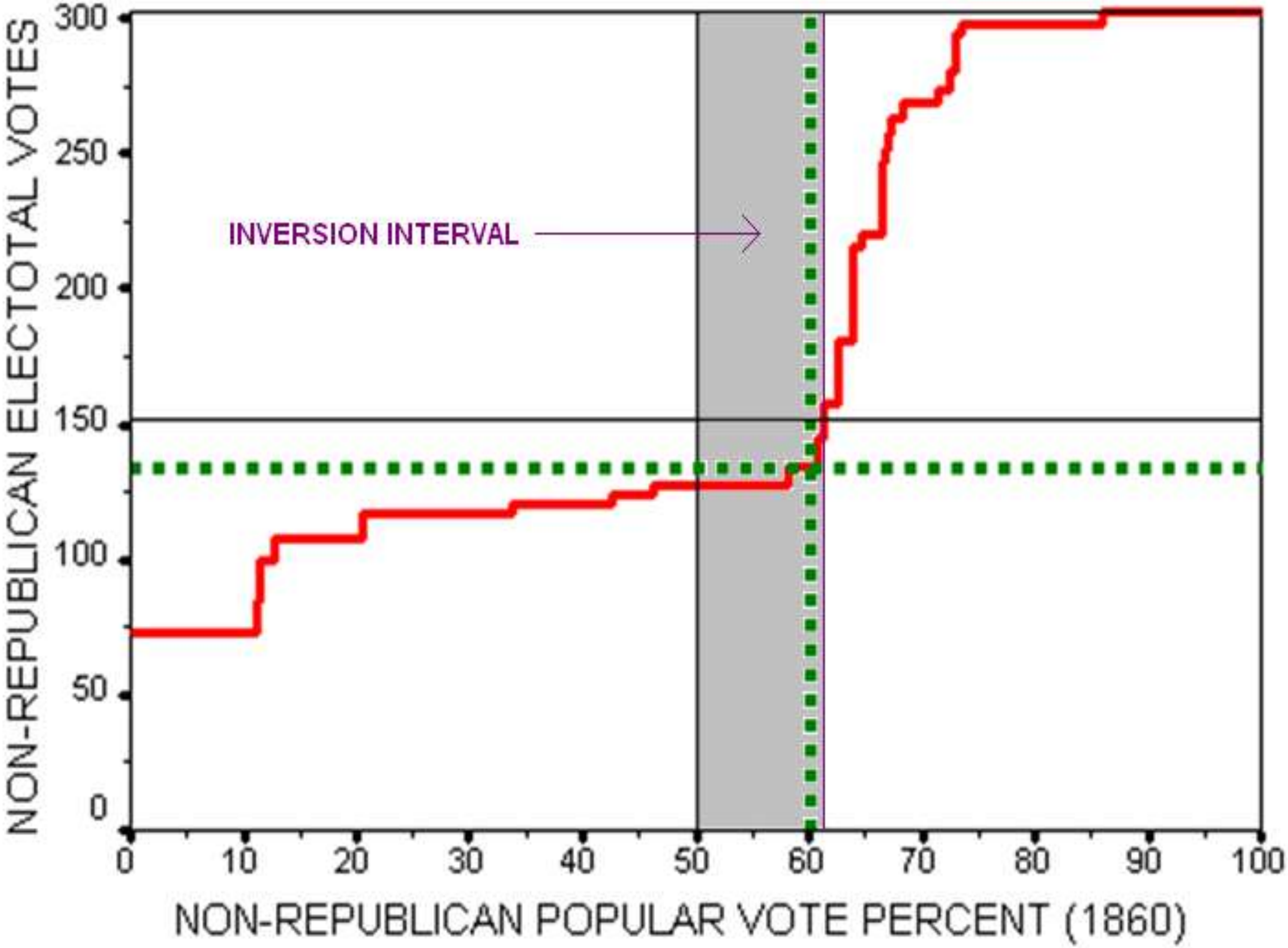
Generalized PVEV Function in 1940



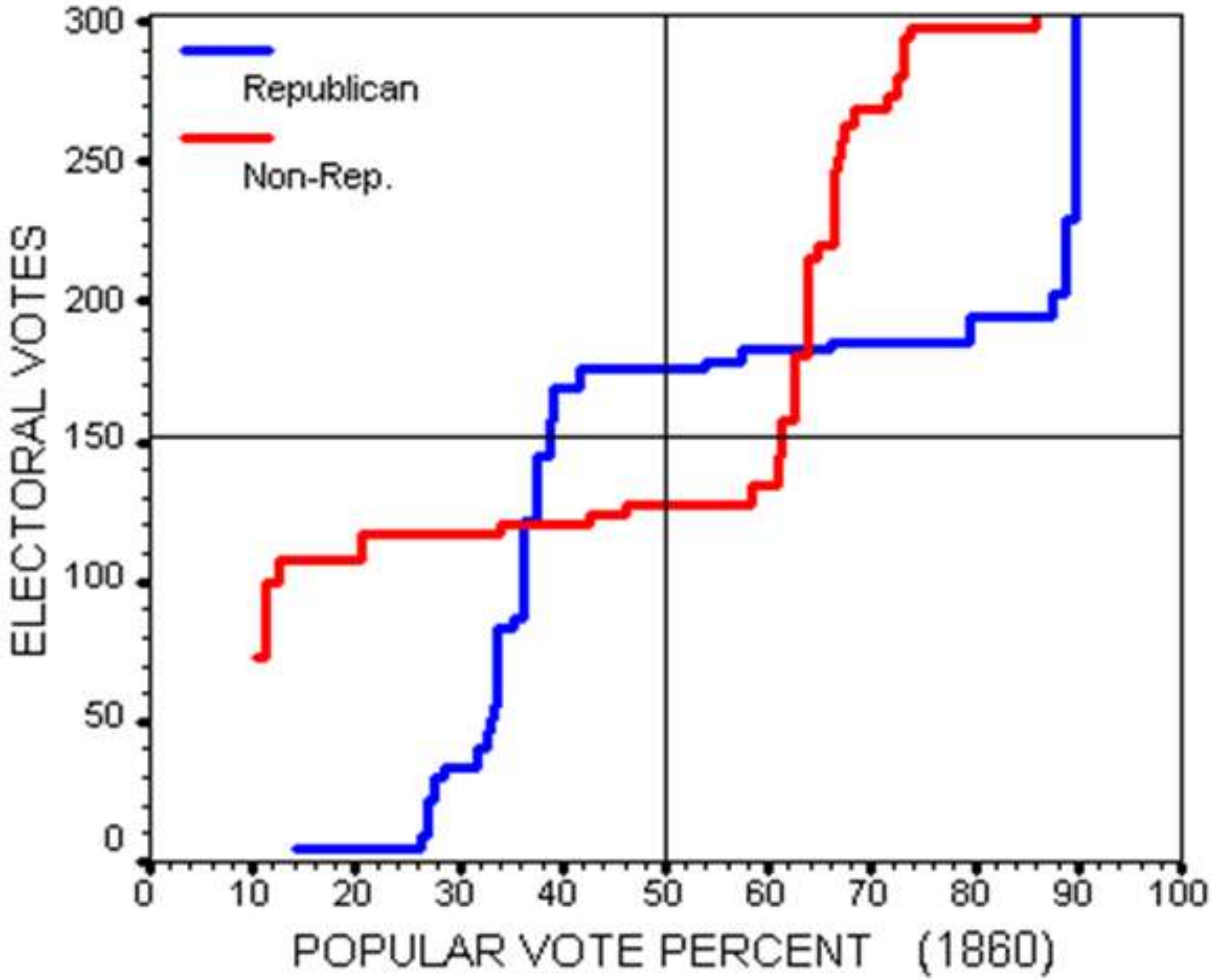
Democratic vs. Republican PVEV in 1940



The Non-Republican PVEV in 1860



Non-Republican vs. Republican PVEV in 1860



Two Sources of Asymmetry or Bias in the PVEV

- Asymmetry or bias in the PVEV function can result from either or both of two distinct phenomena:
  - *apportionment effects* (which subsumes what are often singled out as a third effect, i.e., *turnout effects*), and
  - *distribution effects*.

- Either effect alone can produce election inversions.

- In combination, they can either reinforce or counterbalance each other.

Apportionment Effects

- A *perfectly apportioned* districted electoral system is one in which each state's [district's] electoral vote [voting weight] is precisely proportional to the total popular vote cast in that state/district.
  – Apportionment effects are thereby eliminated.
- It follows that, in a perfectly apportioned system, if a party (or candidate) wins X% of the electoral vote, it *carries states [districts] that collectively cast X% of the total popular vote*.
  – Note that this says nothing about the popular vote margin by which the party/candidate wins (or loses) individual states.
  – In particular, this does *not* say that the party wins X% of the popular vote.

Apportionment Effects (cont.)

- The concept of a perfectly apportioned electoral system is an analytical tool.
  - As a practical matter, an electoral system can be perfectly apportioned only retroactively,
    - i.e., only after the popular vote in each state/district is known.

- *Apportionment effects* refer to whatever causes deviations from perfect apportionment.

Imperfect Apportionment of Electoral Votes

- The U.S. Electoral College system is (very) imperfectly apportioned, for numerous of distinct reasons.
  - House (and electoral vote) apportionments are anywhere from two (e.g., in 1992) to ten years (e.g., in 2000) *out of date*.
  - House seats (and electoral votes) are apportioned on the basis of *total population* [see footnote below], not on the basis of
    - the *voting age population*, or
    - the *voting eligible population*, or
    - the number of *registered voters*, or
    - the number of *actual voters* in a given election.
    - All these factors vary considerably from state to state.
  - House seats (and electoral votes) must be apportioned in whole numbers and therefore can't be precisely proportional to *anything*.
  - Small states are guaranteed a *minimum of three* electoral votes, due to their guaranteed one House seat and two Senate seats.
- Similar imperfections apply (in lesser or greater degree) in all districted systems.

- **Footnote: In addition, until 1864 House seats were apportioned on the basis of the total free population plus three fifths of "all other persons."**

Footnote: Apportionment Effects (cont.)

- In highly abstract (and rather confusing) analysis of its workings, Alan Natapoff (an MIT physicist) largely endorsed the workings Electoral College (particularly its within-state winner-take-all feature) as a vote counting mechanism, but he proposed that each state's electoral vote be made precisely proportional to its share of the national popular vote.
  - This implies that
    - electoral votes would not be apportioned until after the election, and
    - would not be apportioned in whole numbers.
    - Actually Natapoff proposes perfect apportionment of "House" electoral votes while retaining "Senatorial" electoral effects
      - in order to counteract the "Lion [Banzhaf] Effect."
  - Such a system would bring about perfect apportionment and eliminate apportionment effects from the Electoral College system (while fully retaining its distribution effects).
  - Elections inversions can still occur under Natapoff's perfectly apportioned system (due to distribution effects).
  - Natapoff's perfectly apportioned EC system creates seemingly perverse turnout incentives in "non-battleground" states,
    - though he views this as a further advantage of his proposed.

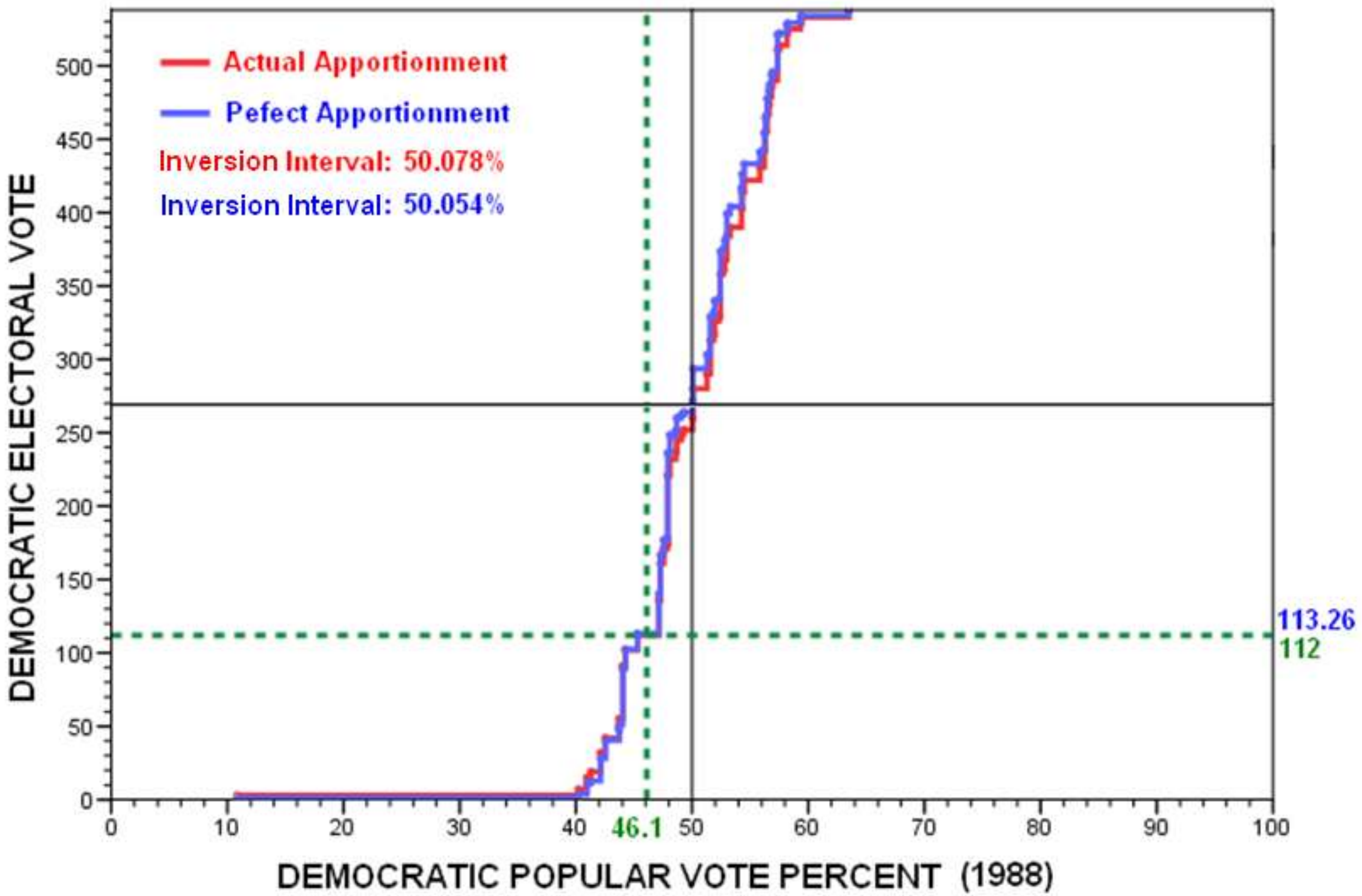
Alan Natapoff, "A Mathematical One-Man One-Vote Rationale for Madisonian Presidential Voting Based on Maximum Individual Voting Power," ***Public Choice*****, 88/3-4 (1996)**

Imperfect Apportionment (cont.)

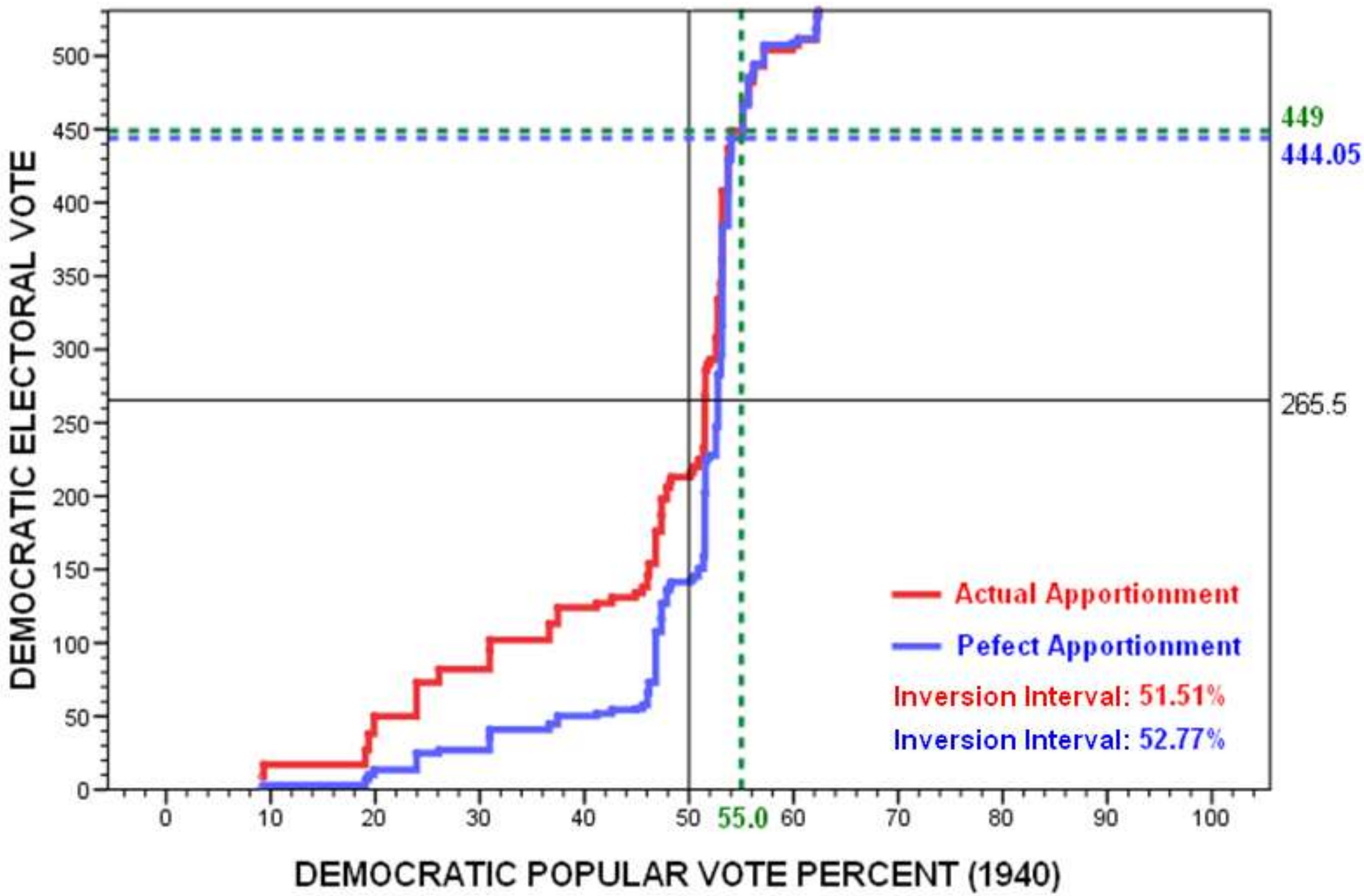
- Imperfect apportionment may or may not create bias in the PVEV function.
  - This depends on the extent to which state (or district) advantages with respect to apportionment effects is correlated with their support for one or other candidate or party.

- We can separate apportionment effects from distribution effects by recalculating the PVEV function
  - with electoral votes retroactively and precisely (and therefore fractionally) reapportioned on the basis of the total (two-party) popular vote in each state.
- Any residual bias in the PVEV function must be due to distribution effects.

1988 PVEV Based on Perfect vs. Imperfect Apportionment



1940 PVEV Based on Perfect vs. Imperfect Apportionment



Apportionment Effects in 1860

- As noted before, the grand daddy of all election inversions occurred in the (counterfactual variant of) the 1860 election.
  - The 1860 electoral landscape exhibited the same kind of bias as 1940 (reflecting low turnout and Republican weakness in the South) but to an even more extreme degree, especially in the second respect.

- The 1860 election was based on highly imperfect apportionment.
  - The southern states (for the last time) benefited from the 3/5 compromise pertaining to apportionment.
  - Southern states had on average smaller populations than northern states and therefore benefited disproportionately from the small state guarantee.
  - Even within the free population, suffrage was more restricted in the south than in the north.
  - Turnout among eligible voters was lower in the south than the north.

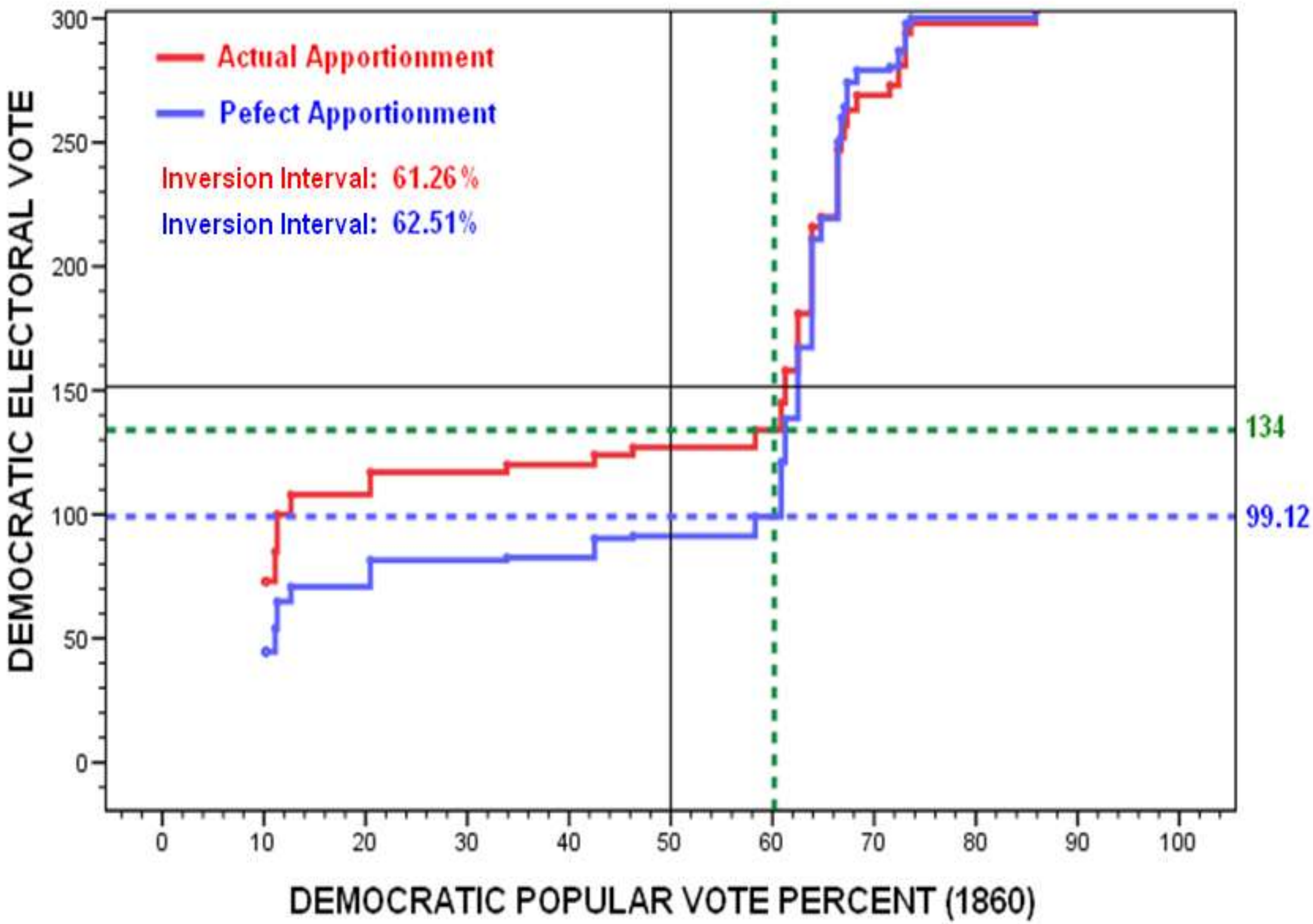
Apportionment Effects in 1860 (cont.)

- But *all* of these apportionment effects favored the south and therefore the Democrats.

- Thus the pro-Republican election inversion was (more than) *entirely due to distribution effects*.

- The magnitude of the inversion interval in 1860 would have been even (slightly) greater in the absence of the counter-balancing apportionment effects.

1860 PVEV Based on Perfect vs. Imperfect Apportionment



Distribution Effects

- Distribution effects in districted electoral system result from the *winner-take-all at the district/state level* character of these systems.
  - Such effects can be powerful even under
    - uniformly districted (one district-one seat/electoral vote) systems, and
    - perfectly apportioned systems.
  - One candidate's or party's vote may be more "efficiently" distributed than the other's, causing an election inversion independent of apportionment effects.

- *Footnote:* Consistent with the strongly two-party nature of U.S. Presidential (and other) elections, everything in this presentation – and the discussion of distribution effects in particular – supposes that minor parties play no significant role in elections.
  - Presidential elections for which this supposition is inappropriate have been excluded from analysis.
  - The present analysis therefore is not adequate for a discussion of potential election inversions in the UK and Canada in the present era, where support for regional minor parties plays an important role in the translation of votes into seats for the two major parties.

Distribution Effects (cont.)

- Here is the simplest possible example of a distribution effect producing an election inversion in a small, uniform, and perfectly apportioned district system.

- Nine voters are perfectly apportioned into three districts.
- The individual votes for candidates D and R in each district are as follows: (R,R,D) (R,R,D) (D,D,D).

	Popular Votes	Electoral Votes
D	5	1
R	4	2

R's votes are more "efficiently" distributed, so R wins a majority of electoral votes with a minority of popular votes.

The 25% vs. 75% Rule

- If the number of districts is fairly large and the number of voters is very large, the most extreme logically possible example of an election inversion in a perfectly apportioned system results when
  - one candidate or party wins just over 50% of the popular votes in just over 50% of the uniform districts or in non-uniform districts that collectively have just over 50% of the voting weight.
  - These districts also have just over 50% of the popular vote (because apportionment is perfect).
  - The winning candidate or party therefore wins just over 50% of the electoral votes with just over 25% (50 + ε % times 50 + ε %) of the popular vote and the other candidate (with almost 75% of the popular vote) loses the election.
  - The election inversion interval is (just short of) 25 percentage points wide.
  - However, if the candidate or party with the favorable vote distribution is also favored by imperfect apportionment, the inversion interval could be could be even greater.

E. E. Schattschneider, *Party Government* (1942), p. 70

Kenneth May, "Probabilities of Certain Election Results," *American Mathematical Monthly* (1948)

Gilbert Laffond and Jean Laine, "Representation in Majority Tournaments," *Mathematical Social Sciences* (2000)

The 25%-75% Rule in 1860 (cont.)

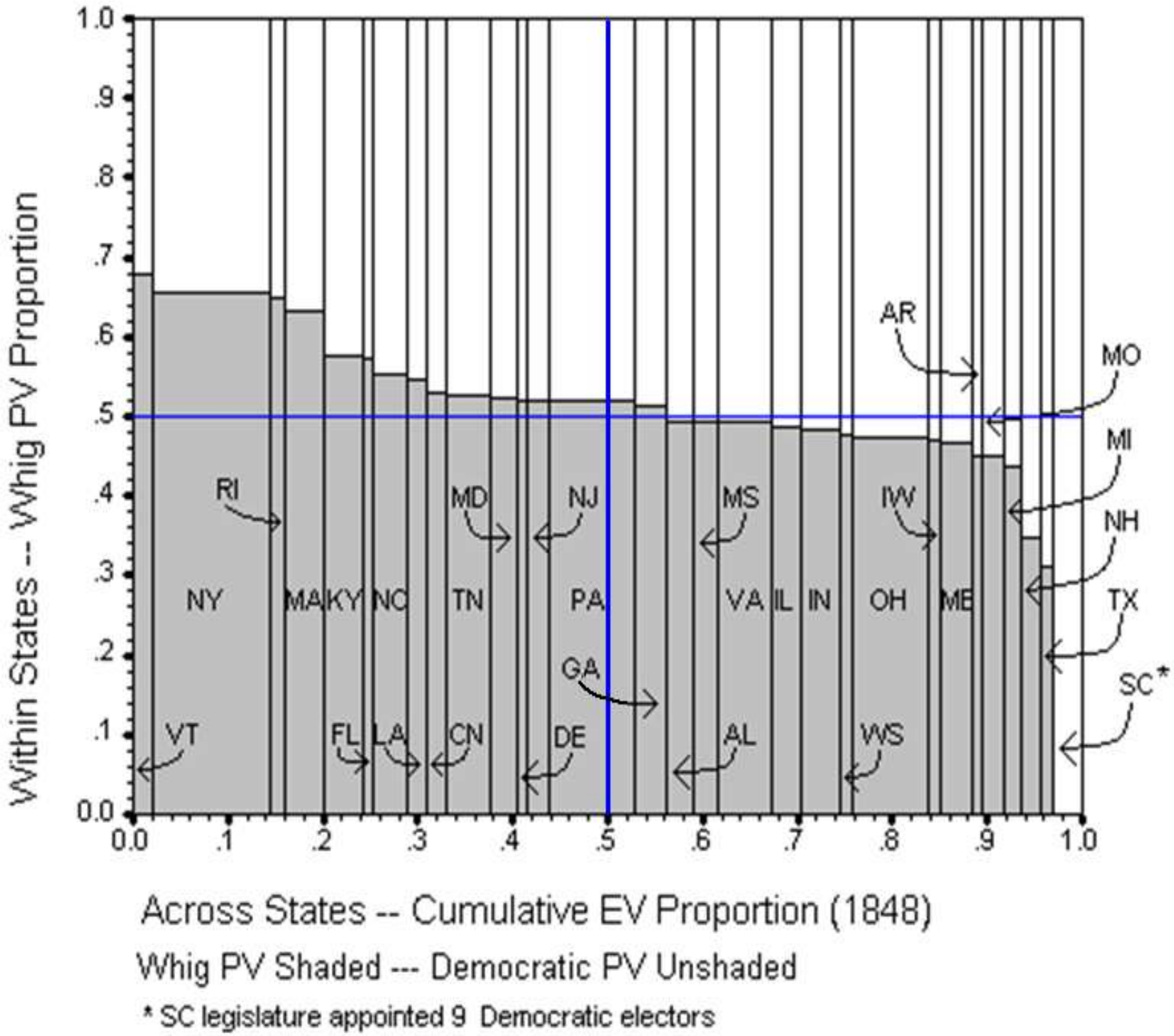
- In the 1860 Lincoln vs. anti-Lincoln scenario, the popular vote distribution over the states closely approximated the logically extreme 25%-75% pattern.
  - Lincoln would have carried all the northern states except NJ, CA, and OR
    - which held a bit more than half the electoral votes (and a larger majority of the [free] population),
    - generally by modest (50-60%) popular vote margins.
  - The anti-Lincoln opposition would have
    - carried all southern states with a bit less than half of the electoral votes (and substantially less than half of the [free] population)
    - by essentially 100% margins; and
    - lost all other states other than NJ, CA, and OR by relatively narrow margins.

Sterling Diagrams: Visualizing Apportionment and Distribution Effects Together

- Carleton Sterling has devised a geometric representation that allows us to visualize (net) apportionment and distribution effects together.
- First, we construct a bar graph of state-by-state popular and electoral vote totals, set up in the following manner.
  - The horizontal axis represents the set of all states:
    - ranked from the strongest to weakest for the winning party; where
    - the thickness of each bar is proportional to the state's electoral vote; and
    - the height of each bar is proportional to the winning party's percent of the popular vote in that state.
  - The next slide shows such a chart for the 1848 election.

Carleton W. Sterling, "Electoral College Misrepresentation: A Geometric Analysis," *Polity*, Spring 1981.

Constructing a Sterling Diagram for 1848



Sterling Diagrams (cont.)

- But the preceding chart is not yet a true Sterling diagram.

- While it is tempting to think that the total shaded and unshaded areas of the diagram represent the *proportions* of the *total* popular vote won by the winning and losing parties respectively, this is not true until we adjust the horizontal scale.

- Having made that adjustment, we need to restore one piece of information lost as a result of the adjustment.

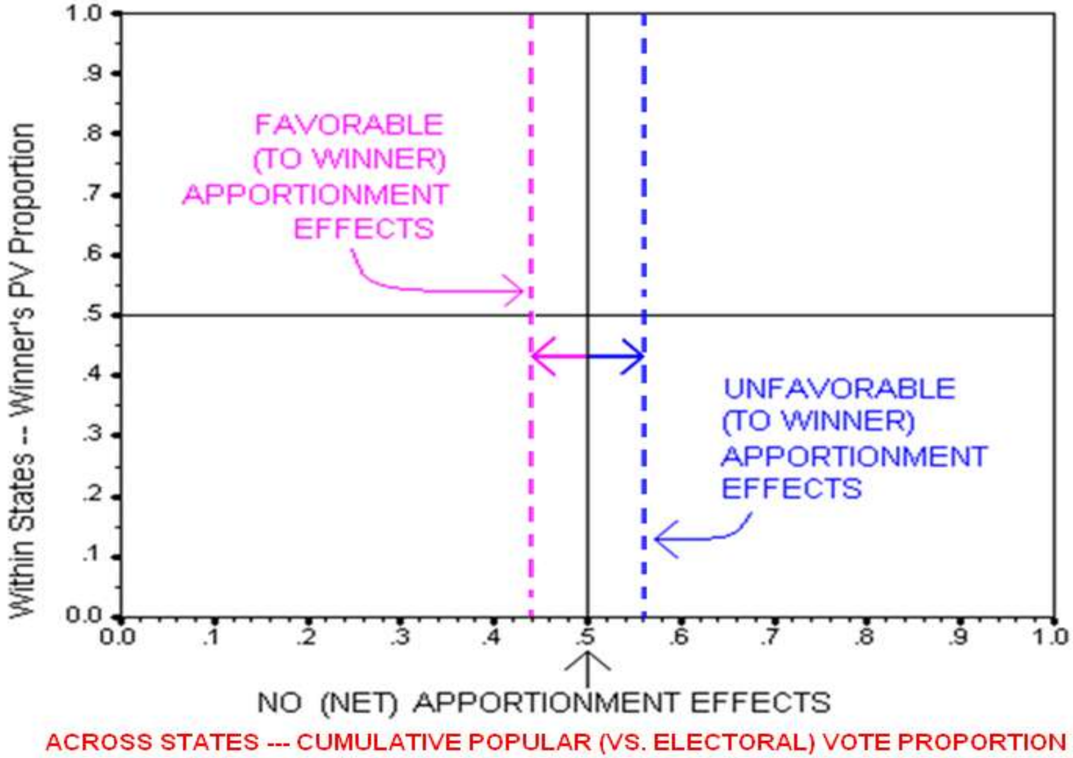
Sterling Diagrams (cont.)

- We need to do two things.

  - Adjust the width of each bar so it is proportional,
    - not to the state's share of electoral votes, but
    - to the state's share of the national popular vote.
    - If electoral votes were perfectly apportioned, no adjustment would be required.

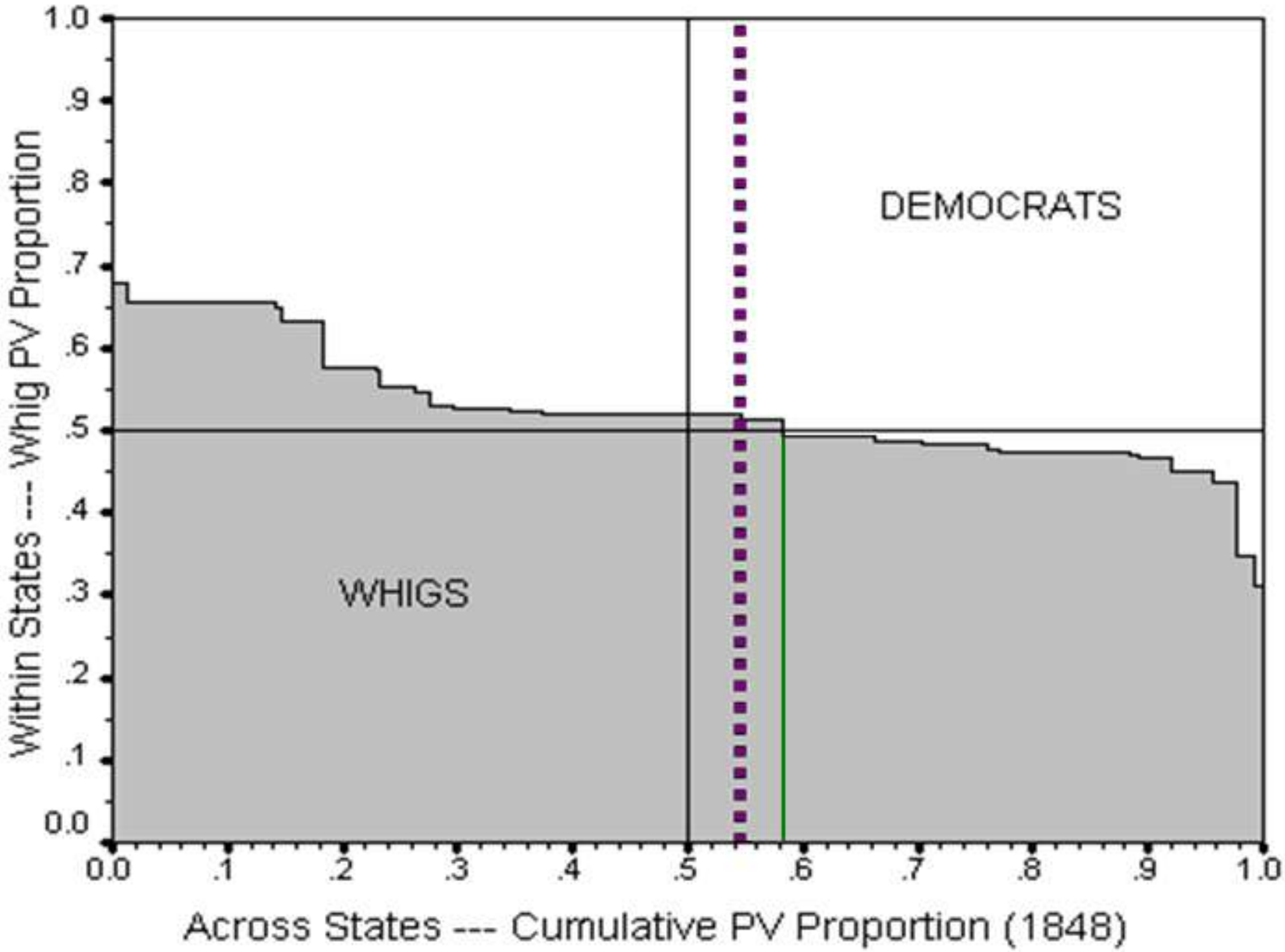
  - Draw a vertical dashed line at the point on the horizontal axis to indicate where the winning party achieves a cumulative electoral vote majority.
    - This will be at (just above) the 50% mark
      - in a perfectly apportioned system, or
      - if there is no *systematic apportionment bias* in the particular landscape.

Sterling Diagrams : Apportionment Effects



- Note that the displacement of the dashed vertical line from .5 does <u>not</u> represent the inversion interval.
  - The inversion interval is the absolute difference between 50% and the smallest popular vote % that produces an electoral vote majority.
  - This interval is the absolute difference between 50% and the share of the popular vote cast by the smallest set of states that produces an electoral vote majority.

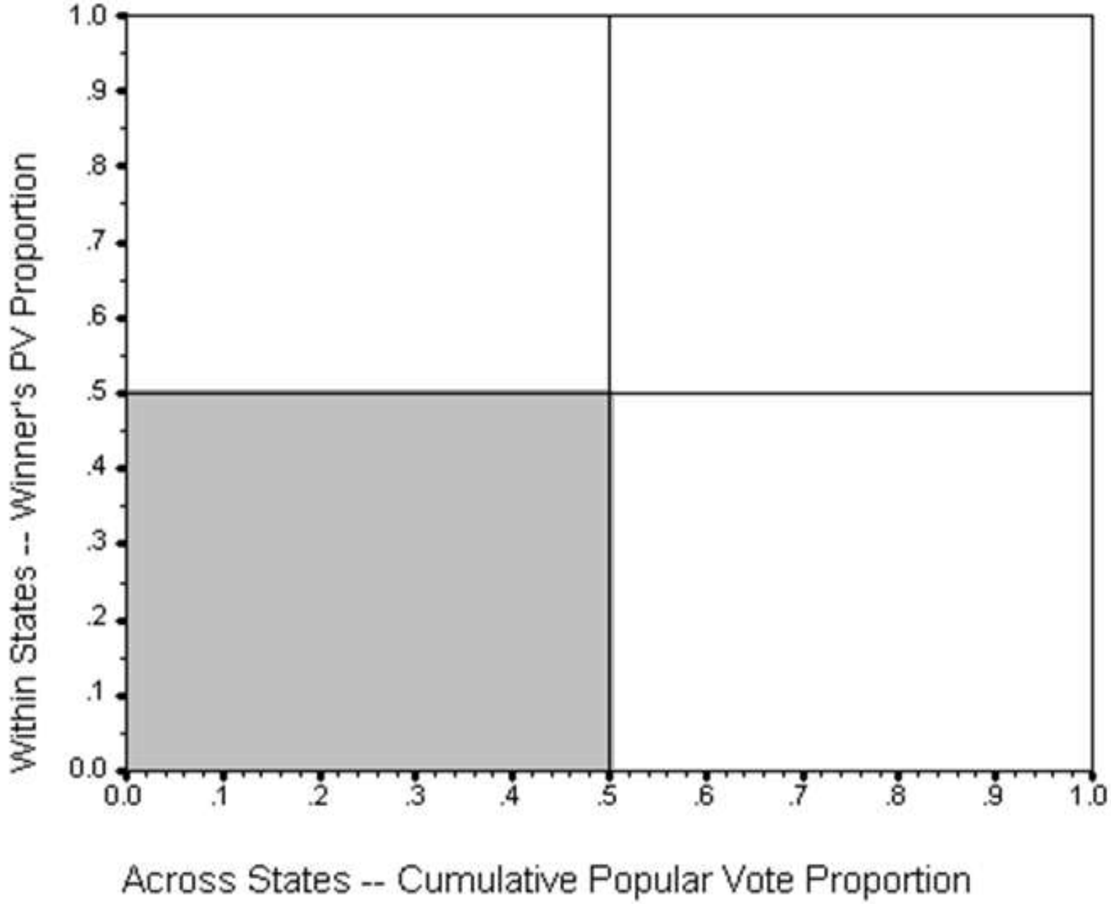
Sterling Diagram for 1848



GIVEN THE 1848 ELECTORAL LANDSCAPE, THE WHIGS NEEDED TO CARRY STATES WITH ABOUT 54.5% OF THE POPULAR IN ORDER TO WIN (I.E., GET MORE THAN HALF OF THE ELECTORAL VOTES)

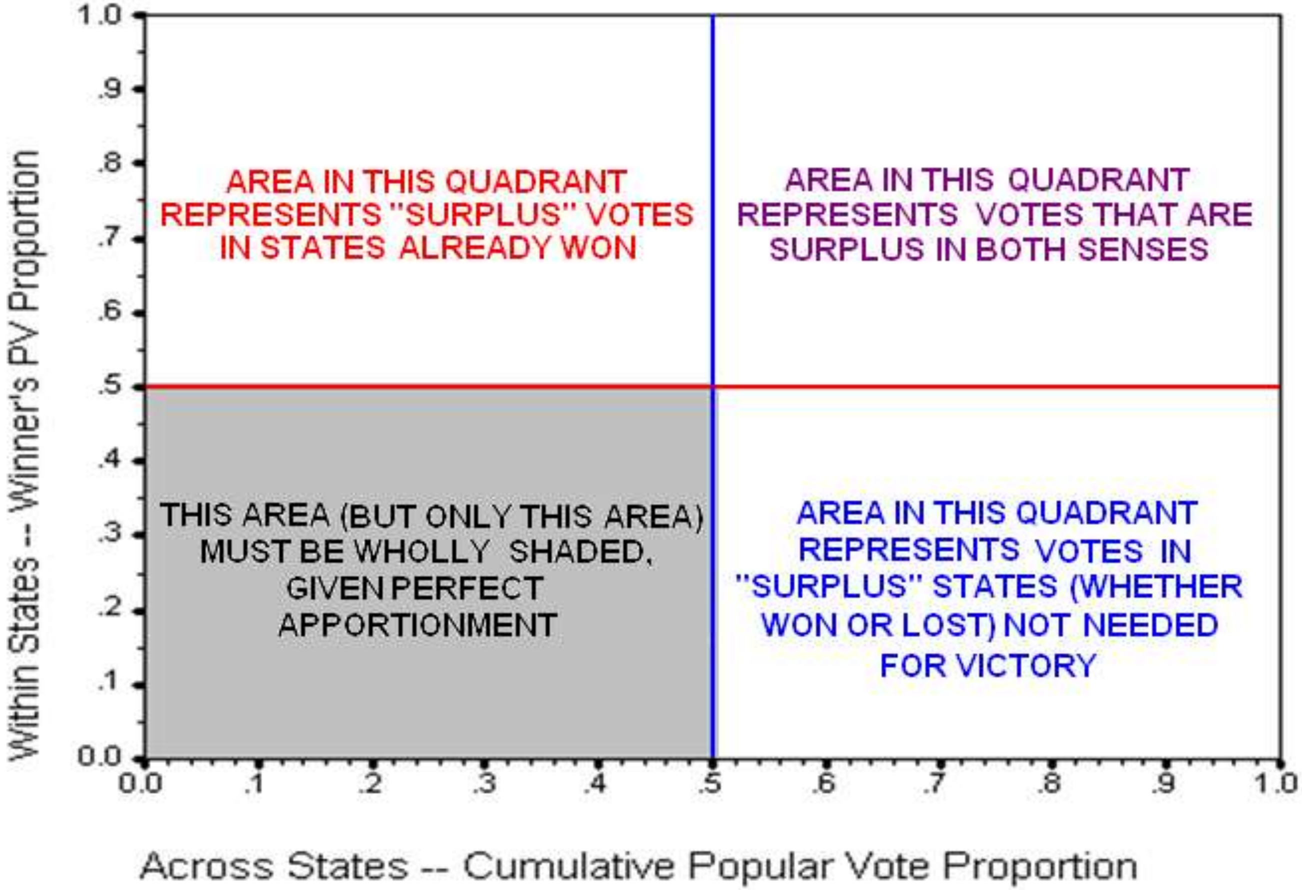
THE WHIGS ACTUALLY CARRIED STATES WITH ABOUT 58.4% OF THE POPULAR VOTE

Sterling Diagrams: The 25%-75% Rule (with Perfect Apportionment)



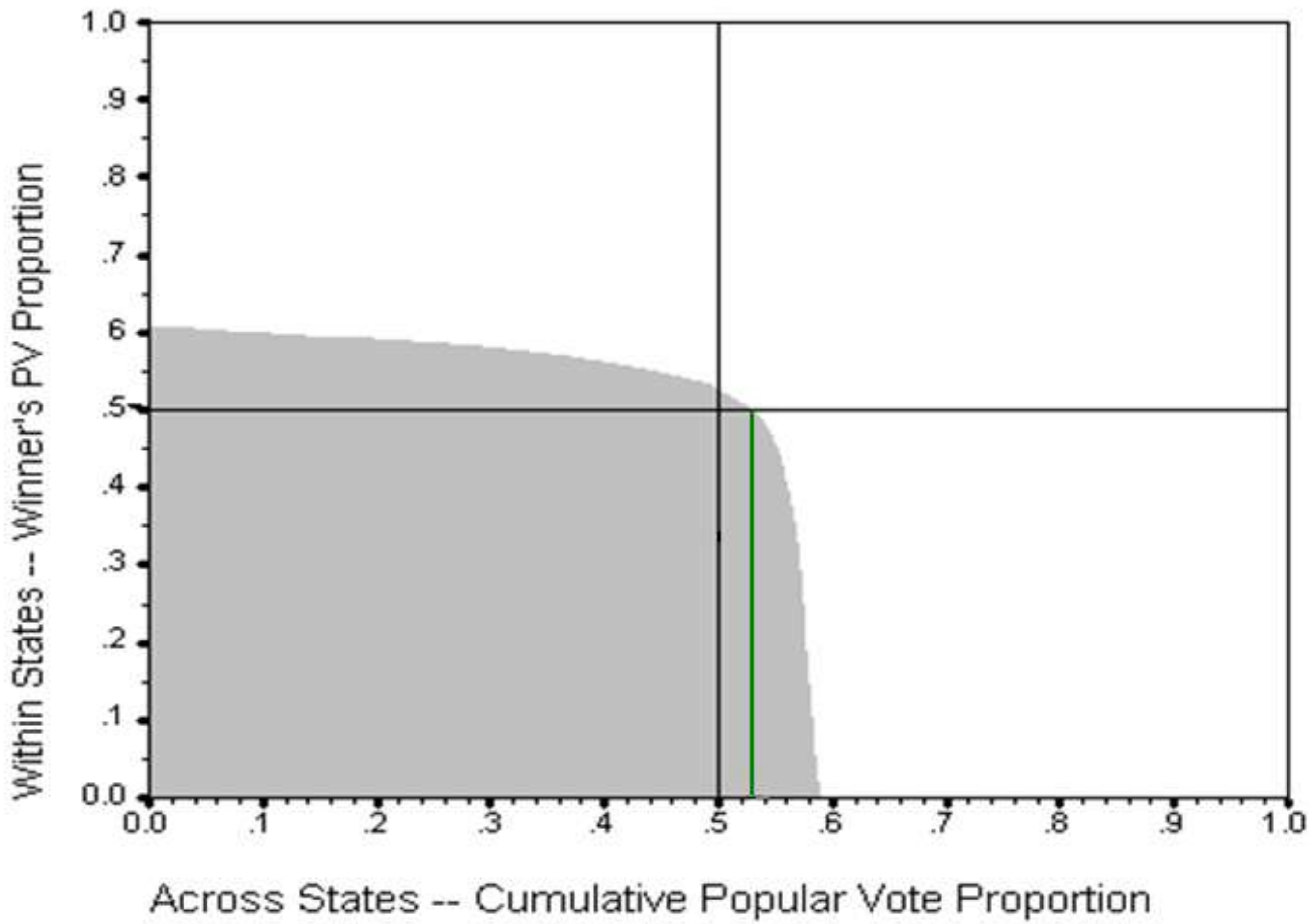
PERFECTLY EFFICIENT DISTRIBUTION (SHADED PARTY WINS 50+ε% OF SEATS/ELECTORAL VOTES WITH 25+ε% OF VOTES)

Sterling Diagrams: A Perfectly Efficient Popular Vote Distribution



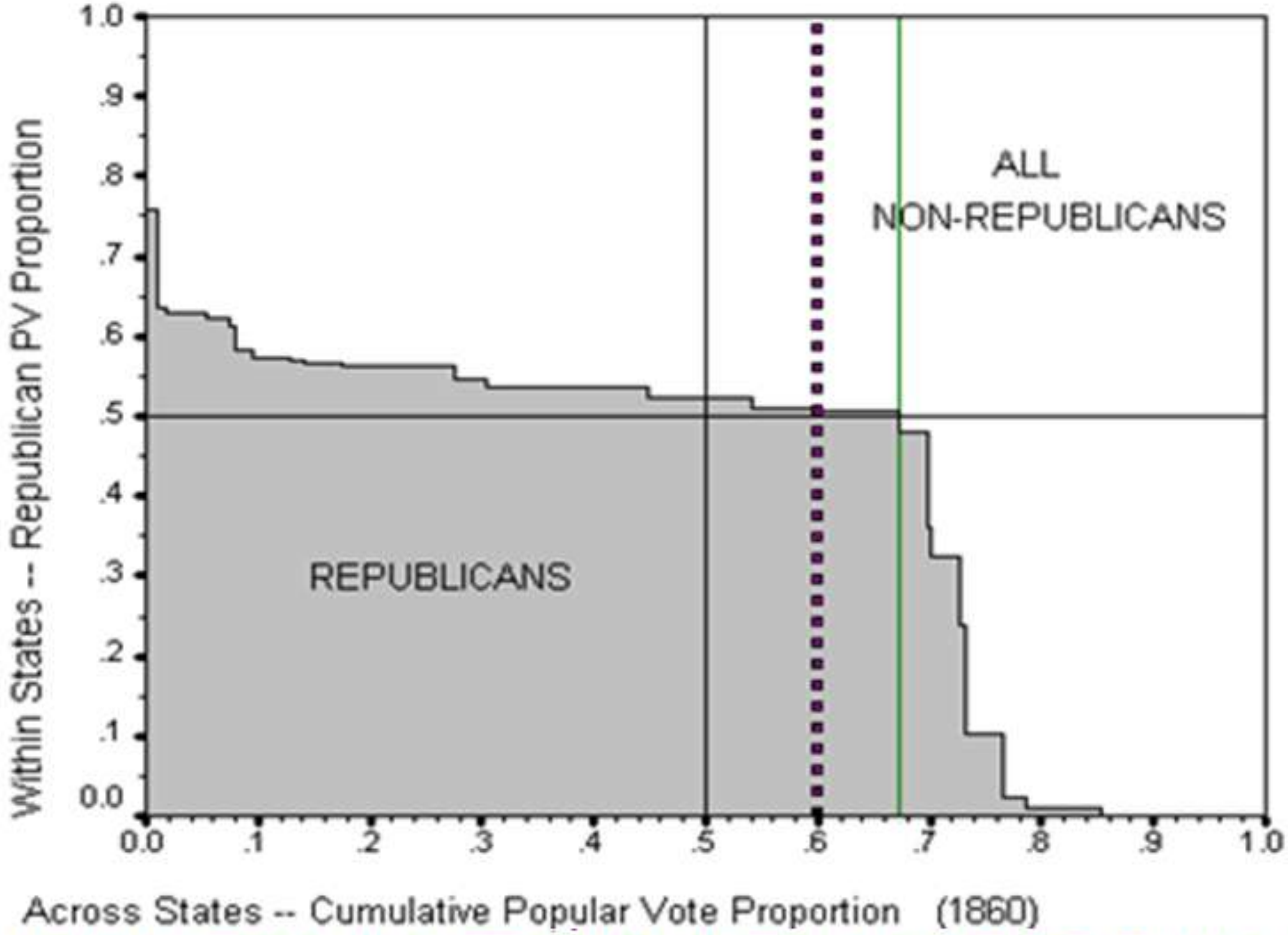
PERFECTLY EFFICIENT DISTRIBUTION (SHADED PARTY WINS 50+ε% OF SEATS/ELECTORAL VOTES WITH 25+ε% OF VOTES)

Sterling Diagrams: The 25%-75% Rule Approximated (with Perfect Apportionment)



HIGHLY EFFICIENT DISTRIBUTION (SHADED PARTY WINS ABOUT 53% OF ELECTORAL VOTES WITH ABOUT 30% OF VOTES)

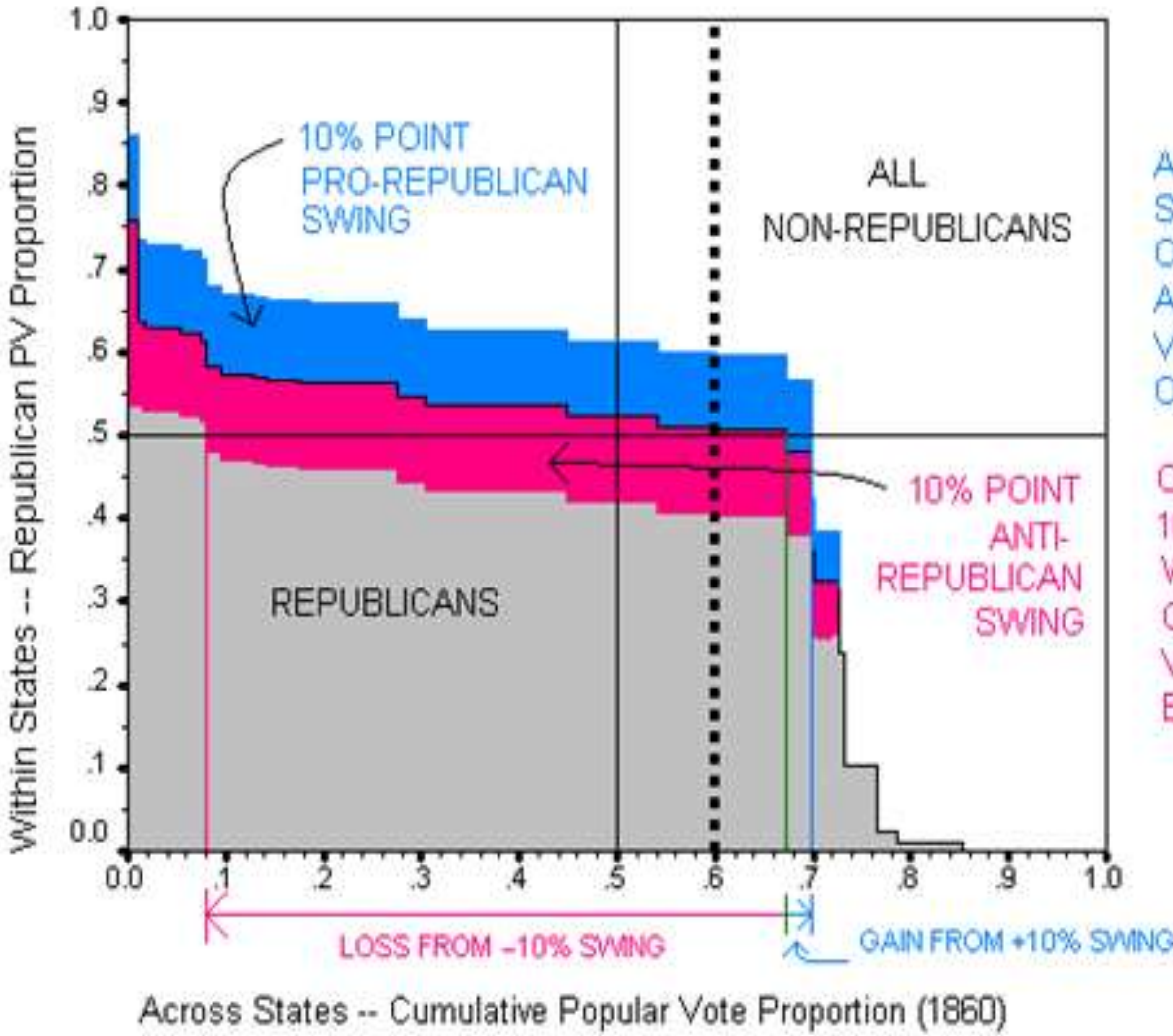
Sterling Diagram: 1860



BECAUSE OF APPORTIONMENT EFFECTS THAT WORKED AGAINST THEM, THE REPUBLICANS NEEDED TO CARRY STATES THAT CAST ABOUT 60% OF THE POPULAR VOTE.

THEY ACTUALLY CARRIED STATES THAT CAST ABOUT 67.6% OF THE VOTE (WINNING ABOUT 56% OF THE ELECTORAL VOTE)

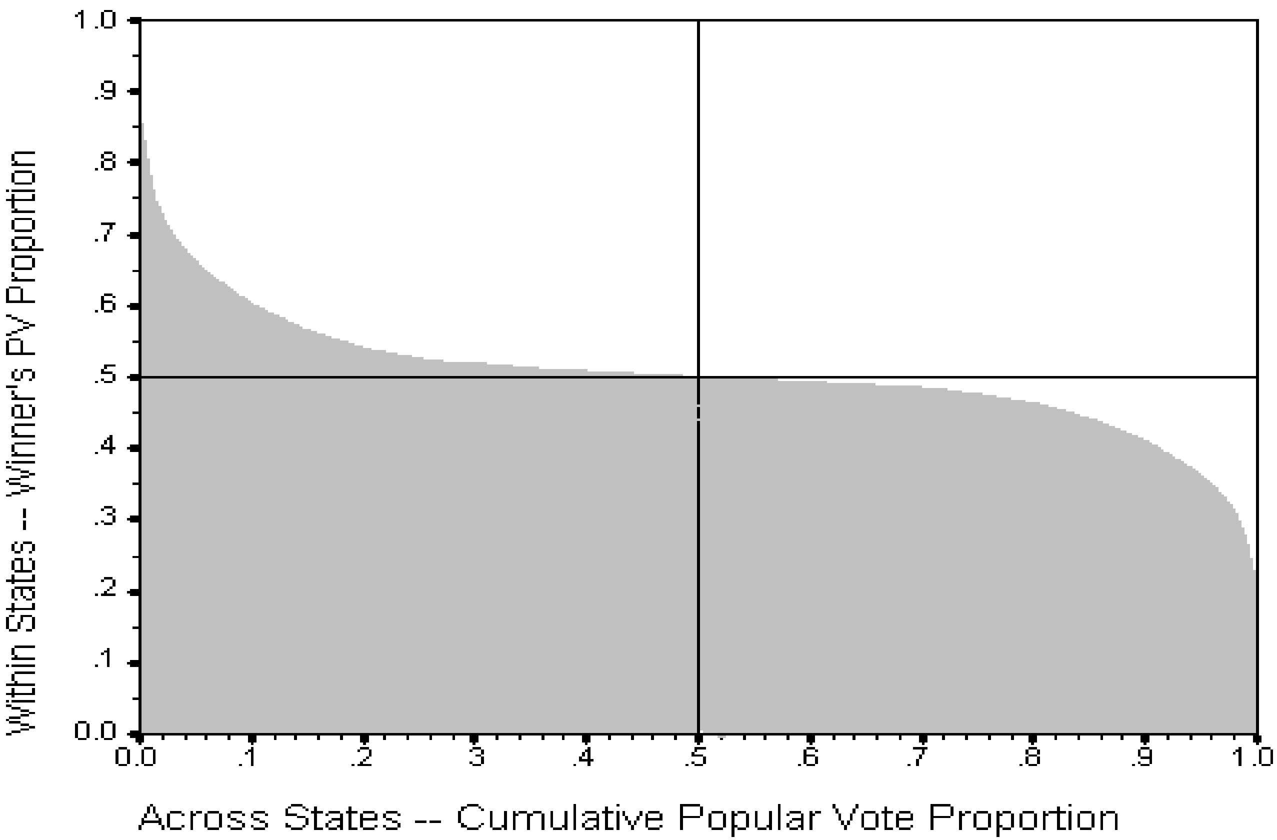
Sterling Diagram: 1860 (cont.)



A UNIFORM 10% PRO-REPUBLICAN SWING WOULD HAVE GAINED THEM ONLY ONE STATE (NJ) CASTING AN ADDITIONAL 2.5% THE POPULAR VOTE (AND AN ADDITIONAL 2.3% OF THE ELECTORAL VOTE)

ON THE OTHER HAND, A UNIFORM 10% ANTI-REPUBLICAN SWING WOULD HAVE COST THEM STATES CASTING 58.5% OF THE POPULAR VOTE (AND 45.9% OF THE ELECTORAL VOTE)

Typical Sterling Diagram (50%-50% Election)



TYPICAL S-CURVE DISTRIBUTION
(PARTY SUPPORT IS NORMALLY DISTRIBUTED OVER DISTRICTS/STATES)

Critiques of the Electoral College

- The tendency of the U.S. Electoral College to produce election inversions is often blamed on
  - the *small state bias* in the apportionment of electoral votes, and more particularly
  - the *winner-take-all* characteristic of the casting of electoral votes.

- Thus two types of reforms of the Electoral College have been or might be proposed.

Electoral College Variants: Apportionment

- To keep the winner-take-all practice but use a different formula for apportioning electoral votes among states:
  - Apportion electoral votes in whole numbers on basis of population only [“House” electoral votes only].
  - Apportion electoral votes fractionally to be precisely proportional to census population.
    - Note that this in practice still does not produce perfect apportionment.

Electoral College Variants: The Casting of Electoral Votes

- Apportion electoral votes as at present but use something other than winner-take-all for casting state electoral votes.
  - *(Pure) Proportional Plan*: casr electoral votes fractionally in precise proportion to state popular vote.
  - *Whole Number Proportional Plan*: cast electoral votes in whole numbers on basis of some apportionment formula applied to state popular vote.
  - *Pure District Plan*: cast electoral votes cast by single-vote districts.
  - *Modified District Plan*: cast two electoral votes cast for the statewide winner; cast the others by (Congressional) district.
  - *National Bonus Plan*: 538 electoral votes are apportioned and cast as at present but an additional 100 (or so) electoral votes are awarded on a winner-take-all basis to the national popular vote winner.

Electoral College Variants (cont.)

- Such “reforms” have been proposed for a variety of reasons, e.g.,
  - to equalize (or redistribute) state or individual (*a priori* or *a posteriori*) voting power;
  - to extend electoral competition beyond “battleground” states; and
  - to eliminate or reduce the likelihood of election inversions.

- We can examine the extent which such variants in fact affect the likelihood of an election inversions.

Electoral College Variants (cont.)

- The proposed reforms in the apportionment of electoral votes may mitigate apportionment effects
  - However, they cannot produce perfect apportionment, because
    - apportionment would still based on populations (as established by a more or less out-of-date census)
      - as opposed to the numbers of popular votes cast.
  - And of course, even perfect apportionment by itself does not preclude election inversions.

Electoral College Variants (cont.)

- The Pure Proportional Plan for casting electoral votes does eliminate distribution effects entirely.
  - But election inversions can still occur under the Pure Proportional Plan due to apportionment effects.
    - The inversions would favor candidates who do exceptionally well in small and/or low turnout states).
  - However, the Pure Proportional Plan combined (somehow) with perfect apportionment would be
    - essentially equivalent to direct national popular vote, so
    - election inversions could not occur.

Electoral College Variants (cont.)

- The Whole Number Proportional and Districts Plans do not eliminate distribution effects, and so
  - they permit election inversions (even with perfect apportionment).
  - Moreover, either District Plan permits election inversions at the state level (except in very small states) as well as national level.

- The rationale for the National Bonus Plan (with a substantial bonus) is clearly to make election inversions almost impossible,
  - by making Presidential elections for all practical purposes direct national popular vote elections,
  - even while nominally keeping the Electoral College in place.

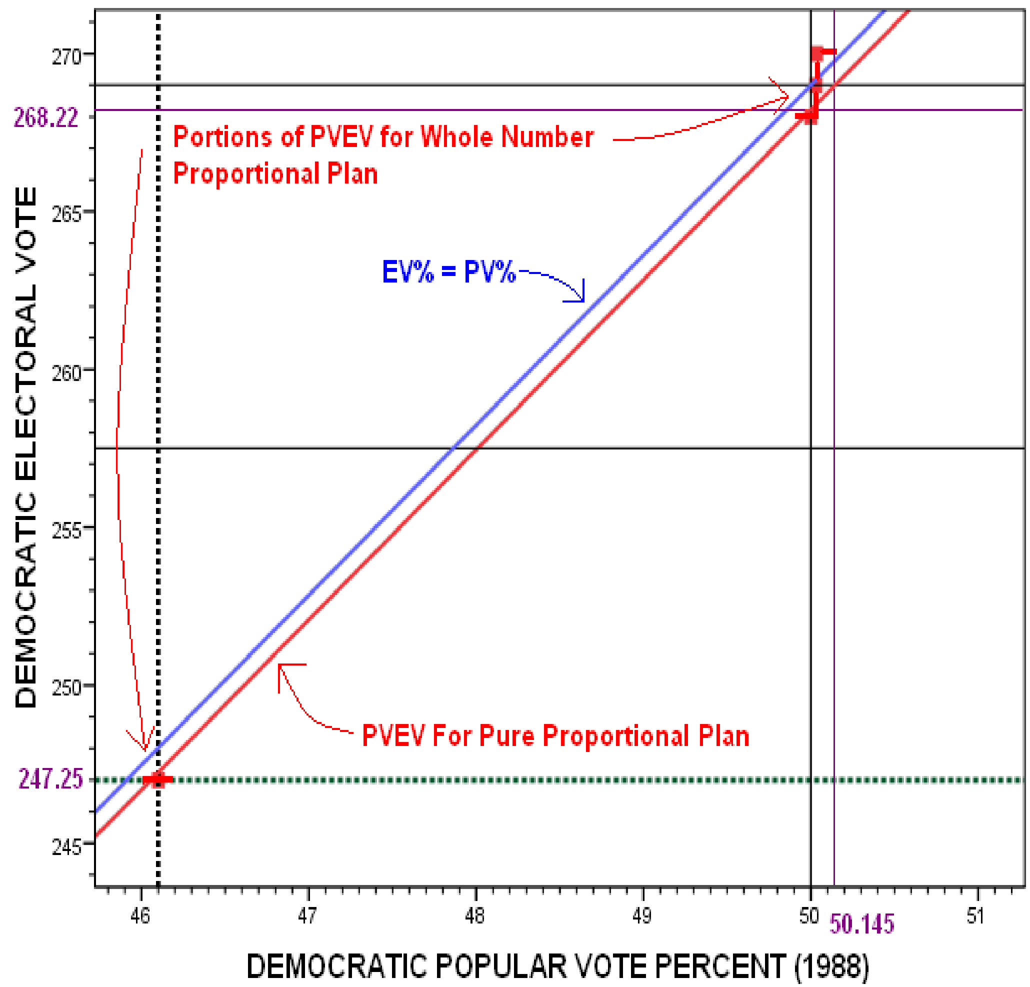
Inversions Under EC Variants: 1988

Electoral College Variant (1980 Census)	Dem. EV at PV = 50%	Inversion Interval	Tie Interval
Existing Electoral College	252	50% - 50.08%	—
Electoral College with House Apportionment	210 (out of 436)	50% – 50.08%	—
Electoral College with Perfect Apportionment	264.053	50% – 50.05%	—
Pure Proportional Plan	268.22	50% – 50.15%	—
Whole Number Proportional Plan	268	50% – 50.03%	50.03% – 50.034%
Pure District Plan (EV = 436)*			
Pure District Plan (EV = 538)*			
Modified District Plan*			

- In 1988, all EC plans have the same small anti-Democratic bias as the existing EC.
- Note that the Whole Number Proportional Plan (and the Pure District Plan) invariably creates a tie interval,
  - and the Modified District Plan typically does the same.

Proportional Plans: PVEV 1988

- The PVEV for the Pure Proportional Plans is a straight line with the same slope as the line EV% = PV%.
  - Its negative intercept (-.7776) reflects bias made possible by imperfect appor-tionment.
- The PVEV for the Whole Number Proportional Plan follows the same line but in small discrete steps.

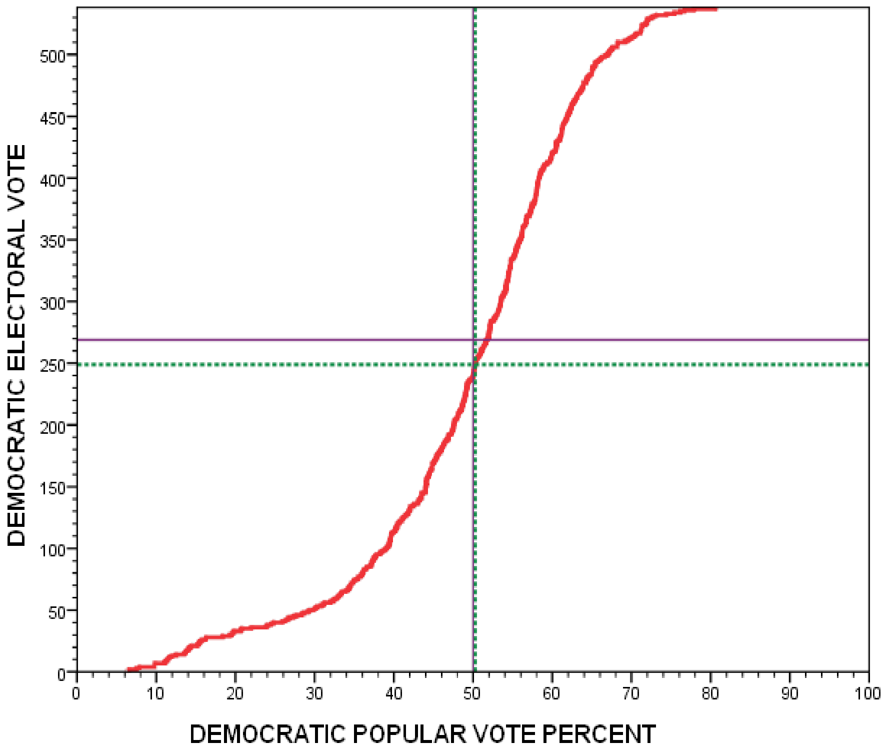


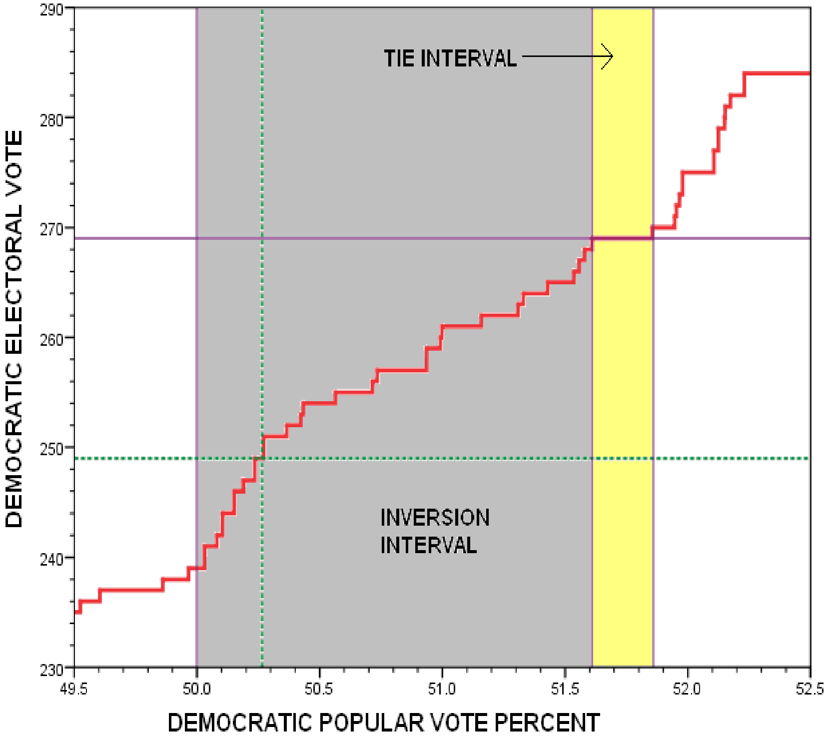
Inversions Under EC Variants: 2000

Electoral College Variant (2000 Census)	Dem. EV at PV =50%	Inversion Interval	Tie Interval
Existing Electoral College	237	50% - 50.27%	—
Electoral College with House Apportionment	203 (out of 436)	50% – 50.15%	—
Electoral College with Perfect Apportionment	244.37	50% – 50.15%	—
Pure Proportional Plan	268.2	50% – 50.15%	—
Whole Number Proportional Plan	268	50% – 50.03%	50.03% – 50.033%
Pure District Plan (EV = 436) (1)	205 (out of 436)	50% – 51.15%	51.15% – 51.25%
Pure District Plan (EV = 436) (2)	206 (out of 436)	50% – 51.33%	51.33% – 51.42%
Pure District Plan (EV = 538)			
Modified District Plan (1)	247	50% – 51.43%	51.43% – 51.68%
Modified District Plan (2)	241	50% – 51.60%	51.60% – 51.85%

- My data for Presidential Vote by Congressional District is not quite consistent with the standard data for Presidential Vote by state.
  - Hence the two separate sets of results (1) and (2).

2000 PVEV under Modified District Plan





Bernoulli Elections

- We next examine the probability that a two-tier *Bernoulli election* (the probability model on which the Banzhaf voting power measure is implictly based) results in an election inversion.
  - Put otherwise, this is the probability that, if everyone votes by flipping coins, a majority of individuals vote “Heads” but the winner based on “electoral votes” is “Tails” or *vice versa.*

Cartoon by George Price, New Yorker, 1937

A Priori Probability of Election Inversions

- Based on very large-scale (*n* = 1,000,000) simulated Bernoulli ("impartial culture") elections, if the number *k* of uniform districts is modestly large (e.g., *k* > 20), about 20.5% of such elections produce reversals.

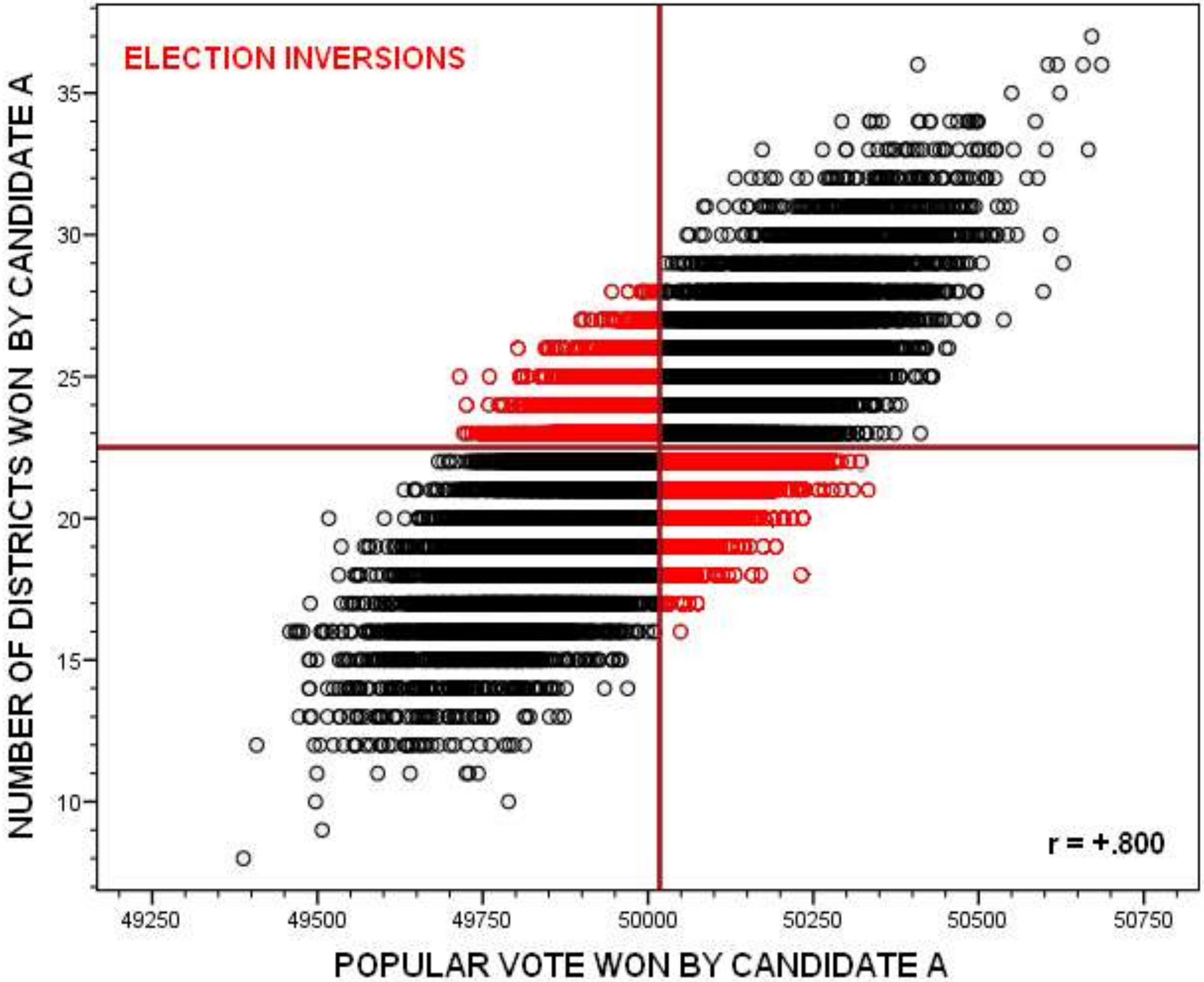
  Feix, Lepelley, Merlin, and Rouet, "The Probability of Conflicts in a U.S. Presidential Type Election," *Economic Theory*, 2004

- My own sample of 120,000 Bernoulli elections with uniform districts produced election inversions 20.36% of the time.

A Priori Probability (cont.)

- The following scattergram shows the pattern for a smaller sample of 30,000 Bernoulli elections, each with
  - 45 districts and
  - 2223 voters per uniform district (100,035 total).
- Any large sample of Bernoulli elections with many voters and "enough" uniform districts produces the same scattergram (once the horizontal and vertical axes are suitably rescaled).
- Notice that
    - the horizontal range (popular votes for the candidate) is very narrow relative to the total number of votes cast),
    - while the vertical range (number of districts carried) is surprisingly wide (relative to the total number of districts).
  - The first fact reflects the statistical fact that essentially all Bernoulli elections are virtual ties with respect to popular votes.
  - The second fact reflects the facts that Bernoulli elections are also virtual ties within districts and there are relatively few districts,

Election Inversions in Bernoulli Elections



A Priori Probability (cont.)

- If the districts are non-uniform (as in the Electoral College), the probability of election inversions evidently increases somewhat.
- In the 120,000 Bernoulli elections, when electoral votes are counted in the manner of the existing Electoral College,
  - 26,241 (21.8675%) produced election inversions, and
  - an additional 952 (0.7933%) produced electoral vote ties.
- The following table shows the rate of inversions and ties for Electoral College variants with respect to the apportionment of electoral votes.

Note: All these simulations take place at the level of states or districts, not individual voters. For each Bernoulli election, the popular vote for the a candidate is generated in each state or district by drawing a random number from the normal distribution that approximates the Bernoulli distribution with p = .5 and n trials.

A Priori Probability: Apportionment Variants

Electoral College Variant (2000 Census)	Inversions	EV Ties
Existing Electoral College	21.8675%	0.7933%
Electoral College with House Apportionment	23.2275%	0.8092%
Electoral College with Perfect Apportionment	23.5808%	0.0000%
Electoral College with State Equality *	24.8283%	0.0000%
Electoral College with "Random" Apportionment	32.7433%	0.7617%

*** Note that this is not a uniform system. While the "districts" have equal weight, they have very unequal numbers of voters.**

- To begin to get a handle on the probability of an electoral vote tie is itself of some interest.
  - Ties became possible when the 23^{rd} Amendment, awarding DC three electoral vote and creating an even number [538] of electoral votes, was ratified prior to the 1964 election.

A Priori Probability: Apportionment Variants (cont.)

- Given that HR apportionment is “better” than the regular EC apportionment and perfect apportionment is the “best” of all, it is perhaps surprising that the latter two produce somewhat more frequent inversions than the regular EC.
- The explanation appears to be as follows:
  - The frequency of inversions in Bernoulli elections is minimized (at about 20.5%) with uniform districts, each having the same weight and the same number of voters.
  - Departures from uniformity in either respect increase the frequency of inversions.
  - Both HR apportionment and perfect apportionment increase state inequality with respect electoral vote weights, relative to the existing EC apportionment, and therefore increase the frequency of inversions.
  - State equality (equal EV weight but highly unequal numbers of voters) produces a still higher frequency of inversions.
- Note also that, given an number of states is odd, no election can produce an EC and HR tie simultaneously,
  - Given a EC tie, both HR and perfect apportionments are considerably more likely to produce an inversion (60.7% / 58.5%) than avoid one (39.3% / 41.5%).
- While full inversions occur more frequently under perfect apportionment than HR apportionment, the latter also produces ties 0.8% of the time.
  - If ties are counted as “half inversions,” perfect apportionment does slightly better than HR apportionment.

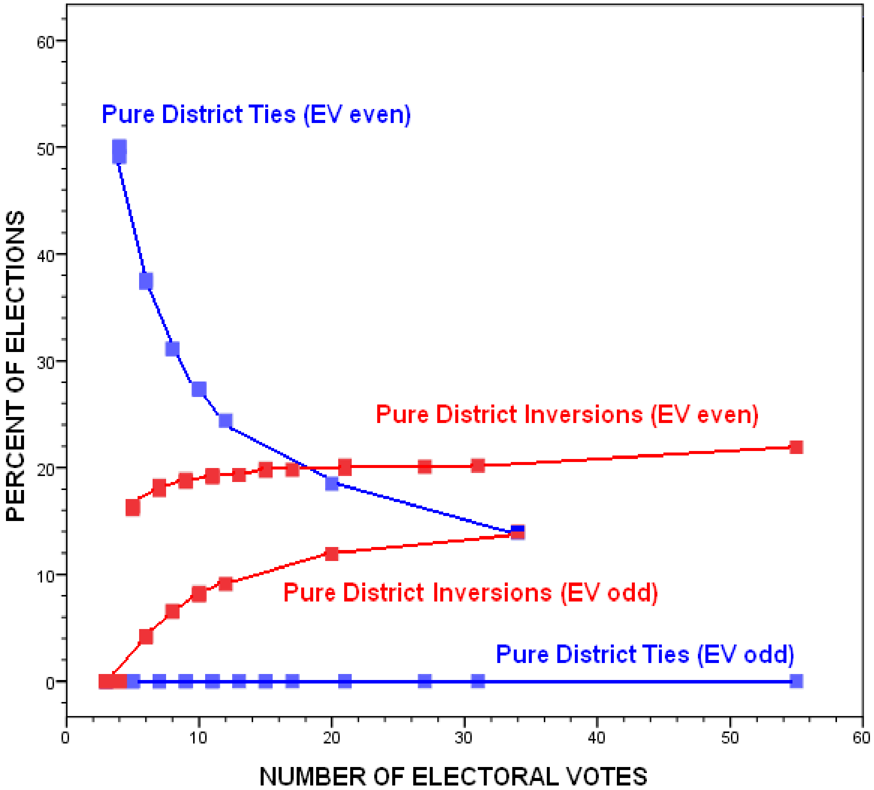
A Priori Probability: EC Reform Plans

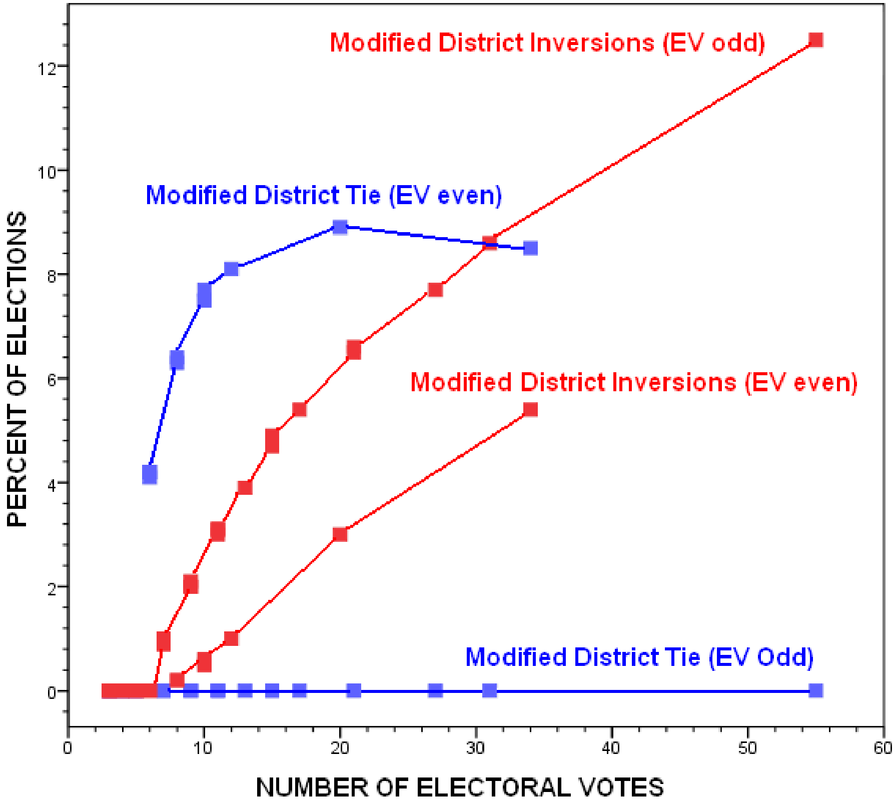
Electoral College Variant (2000 Census)	EC Reversals	EV Ties
Existing Electoral College	21.8675%	0.7933%
Pure District Plan (EV = 436)*	18.8250%	3.8217%
Pure District Plan (EV = 538)*		
Modified District Plan*	16.6625%	2.5292%
Pure Proportional Plan	8.1358%	0.0000%
Whole Number Proportional Plan**	23.2842%	13.7625%

*** District Plans can produce state-level, as well as national, inversions (see next slide).**

**** In essentially every Bernoulli election, the two candidates split electoral votes equally in states with an even number of electoral votes , and one candidate beats the other by a single electoral vote in states with an odd number of electoral votes. Thus, the Whole Number Proportional Plan is essentially the same as the EC with equal apportionment, except that the 17 states with an even number of electoral are in effect removed from the EC and ties are possible because the number of states with an odd number of electoral votes is itself even (34).**

District System State-Level Inversions and Ties





The Majority Deficit

- To this point we have only counted up the frequency of election inversions in Bernoulli elections.
- But an election inversion may be either:
  - a close thing, in that the frustrated popular majority is barely more than 50%, or
  - highly lopsided, in that the frustrated marjority may be considerably larger than 50% (in the limit 75% in a perfectly apportioned system).
- Felsenthal and Machover (1998) define the *majority deficit*
  - given an election inversion, as the size of the frustrated majority minus the successful minority, and
  - as zero otherwise.
- The following table shows the Mean Majority Deficit (MMD1) for these simulated Bernoulli elections.
  - MMD2 is the mean of the (non-zero) majority deficits in the event an election actually occurs.
- The MMDs are small (relative to the simulated electorate of about 122 million voters) because Bernoulli elections in a large electorate (essentially always) extraordinarily close.

Mean Majority Deficit in Bernoulli Elections

Electoral College Variant (2000 Census)	MMD1	MMD2
Existing Electoral College	1037.56	4587.46
Electoral College with House Apportionment	1167.84	4858.58
Electoral College with Perfect Apportionment	1162.32	2464.56
Electoral College with State Equality	1280.24	5156.36
Electoral College with “Random” Apportionment	2196.90	6556.94
Pure District Plan (EV = 436)		
Pure District Plan (EV = 538)		
Modified District Plan		
Pure Proportional Plan	141.26	1736.18
Whole Number Proportional Plan	1840.46	4967.94

The Probability of Election Inversions in the Contemporary Electoral Landscape

- The contemporary electoral landscape is defined as the Democratic two-party popular vote percent averaged over the 2000, 2004, and 2008 Presidential elections.

- The 2000, 2004, and 2008 elections
  - were all relatively close,
  - were all effectively two-party elections, and
  - they had very similar (though certainly not identical) electoral landscapes.

Contemporary Electoral Landscape (cont.)

- In these simulations,
  - the popular vote in each state (or district) is equal its contemporary landscape value,
  - adjusted downward by 0.8114% (to make the Democratic national popular vote percent equal to 50%),
  - ± a normally distributed state-level perturbation with SD = 1.5% ,
  - ± an additional normally distributed national perturbation (or "swing") with SD = 5%.
- Once the simulated data has been generated, electoral votes can be counted up for each of the Electoral College variants to determine the proportion of elections that produce inversions (or ties) under each variant.
  - The Pure District Plan (with 538 districts) cannot be simulated, since no such districts actually exist and therefore there is no landscape data.
  - The Modified District Plan and the Pure District Plans (with 436 districts) simulations are based on the 2000 electoral landscape only (the only year for which I have reasonably good data). [These have not yet been completed]

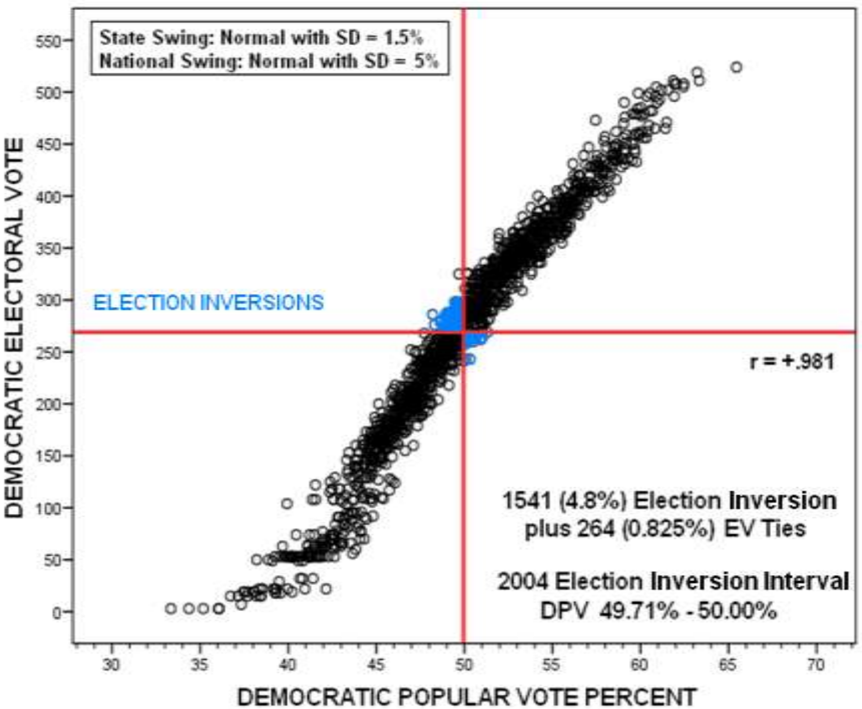
Simulated Contemporary Elections

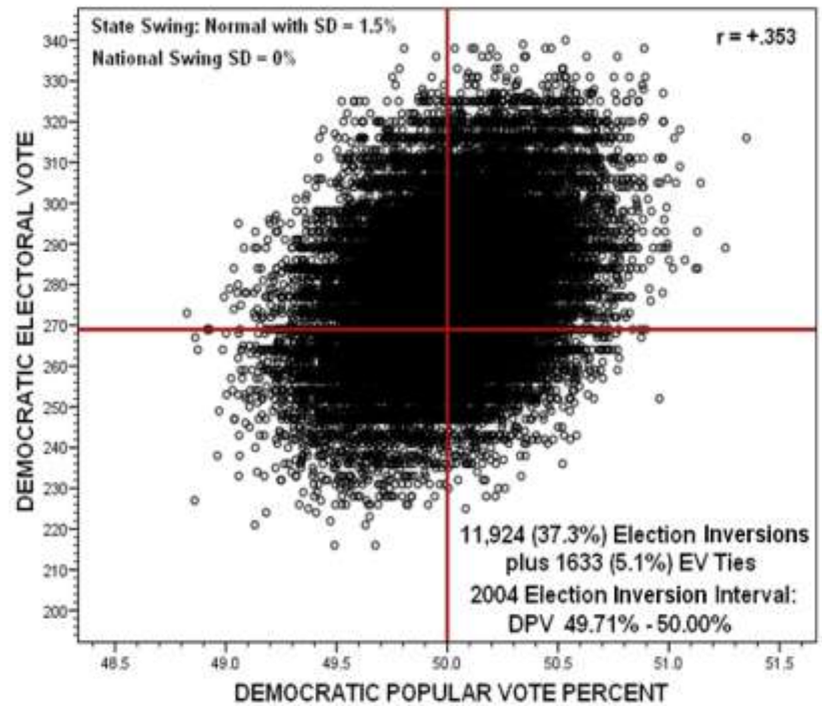
Electoral College Variant (2000 Census)	Inversions	EV Ties
Existing Electoral College	6.4021%	1.4009%
Electoral College with House Apportionment	4.5970%	1.0192%
Electoral College with Perfect Apportionment	5.2979%	0.0000%
Electoral College with State Equality	14.4486%	0.0000%
Electoral College with “Random” Apportionment	5.5337%	0.6359%
Pure Proportional Plan	1.7908%	0.0000%
Whole Number Proportional Plan	2.1775%	1.2933%
Modified District Plan (2000 landscape only)		
Pure District Plan (2000 landscape only)		

Simulated Contemporary Elections (cont.)

- Of these 120,000 simulated elections, nine were *popular vote ties*.
- In this more realistic sample of elections, election inversions (and EV ties) occur substantially less frequently than in Bernoulli elections.
  - This is to be expected because most of these elections are not especially close with respect to popular votes.
    - Only 15.6% were won by less than 51%, and
    - only 30.6% were won by less than 52%.
  - This results from the 5% national swing across elections.
  - Compare the following scattergrams (based on a different set of simulated elections.

Inversions in Simulated Elections with and without a National Swing



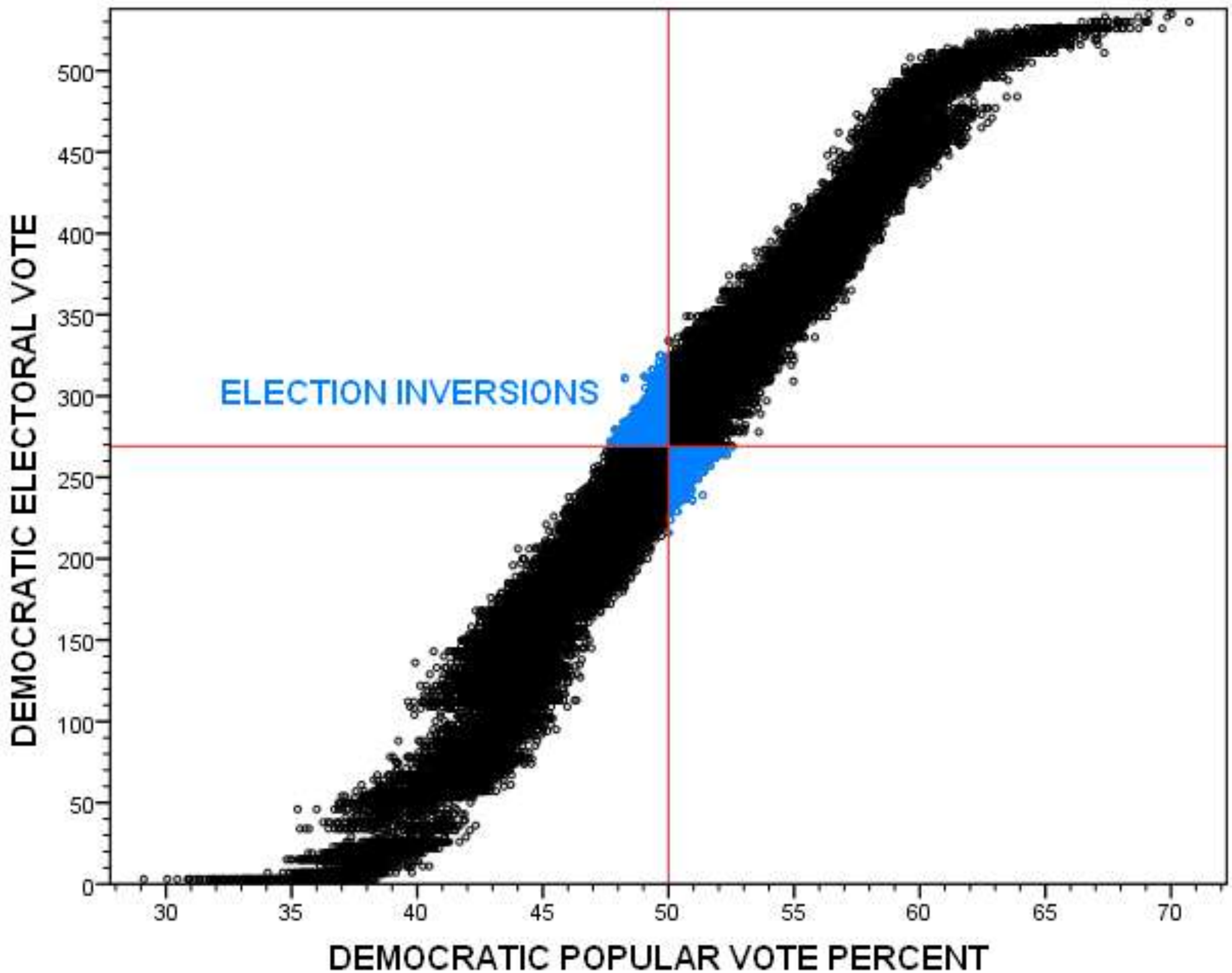


Simulated Contemporary Elections (cont.)

- Bernoulli elections are wholly neutral between candidates or parties.
- The 2000-2008 landscape was adjusted so that each party could be expected to win the popular vote half the time.
  - In fact,
    - Democrats won the popular vote 49.7704% of the time.
    - Republicans won the popular vote 50.2371% of the time.

- Even so, simulated elections based on an electoral landscape are not neutral,
  - and election inversions do not each affect each party in a neutral fashion, and
  - the direction of the bias varies with the Electoral College variant.

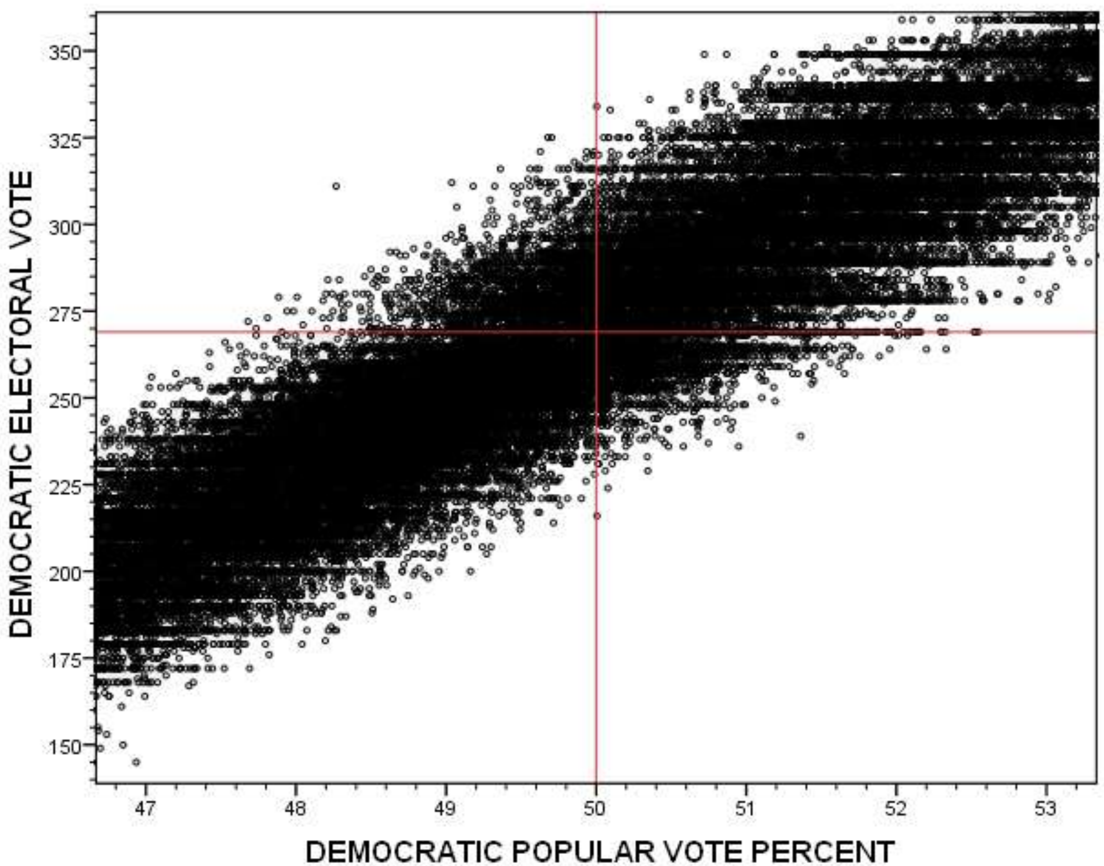
DEV by DPV in 120K Contemporary Elections



Existing EC

erev * popwin Crosstabulation

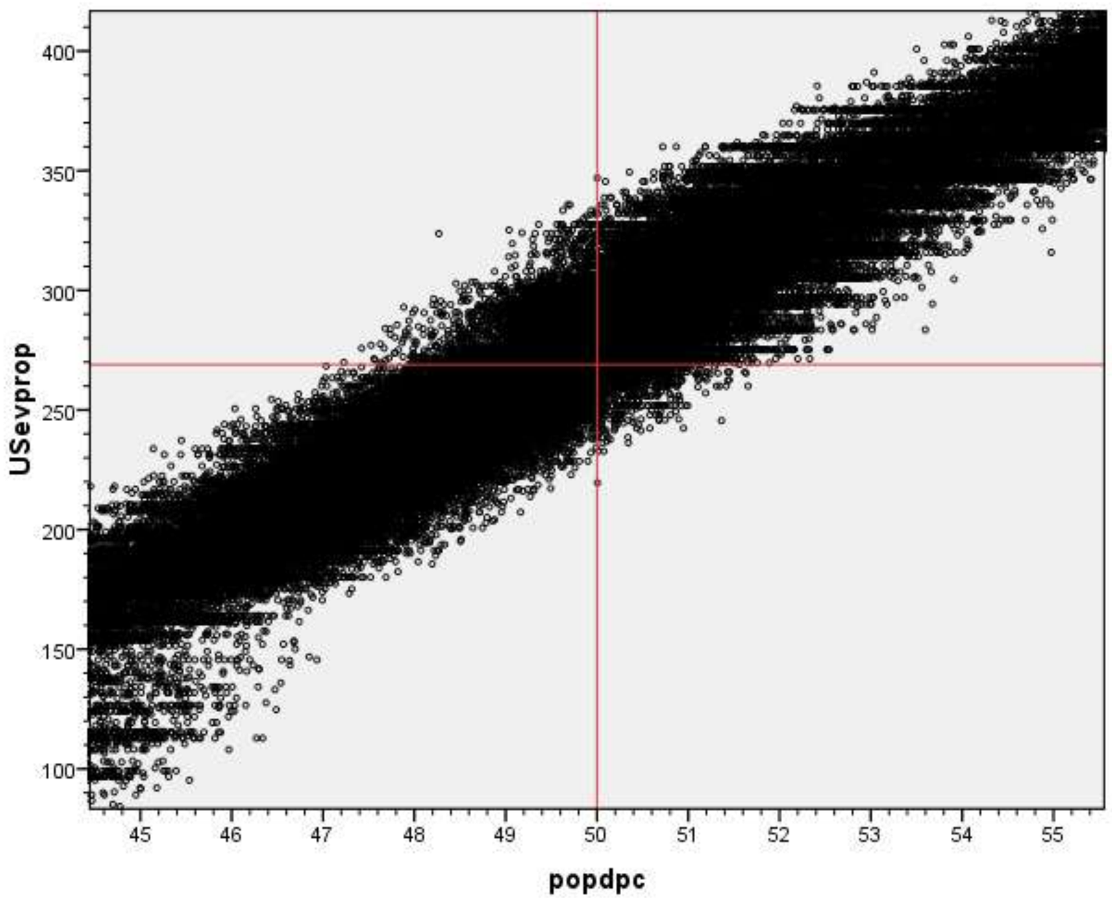
			popwin		
			.00	1.00	Total
erev	.00	Count	57314	53314	110628
		% of Total	47.8%	44.4%	92.2%
	.50	Count	502	1179	1681
		% of Total	.4%	1.0%	1.4%
	1.00	Count	2461	5221	7682
		% of Total	2.1%	4.4%	6.4%
	Total	Count	60277	59714	119991
		% of Total	50.2%	49.8%	100.0%



House Apportionment

hrwin * popwin Crosstabulation

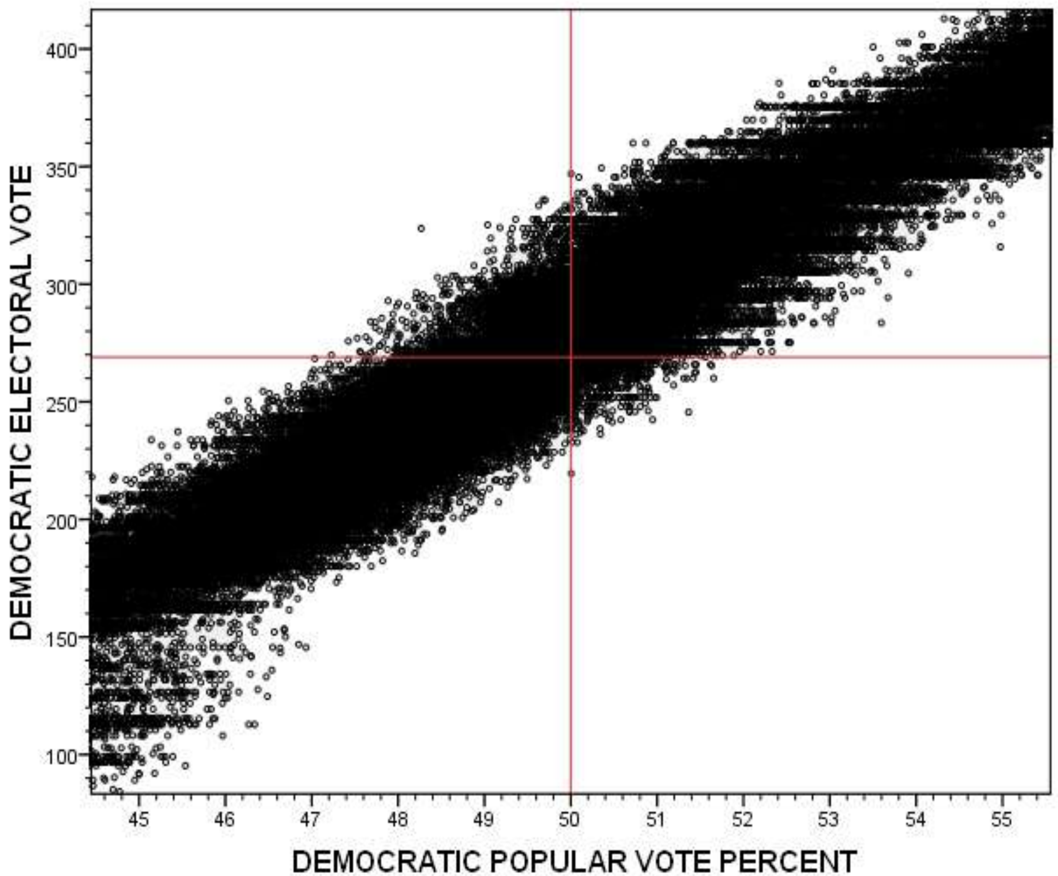
			popwin		
			.00	1.00	Total
hrwin	.00	Count	54821	946	55767
		% of Total	45.7%	.8%	46.5%
	.50	Count	886	337	1223
		% of Total	.7%	.3%	1.0%
	1.00	Count	4570	58431	63001
		% of Total	3.8%	48.7%	52.5%
	Total	Count	60277	59714	119991
		% of Total	50.2%	49.8%	100.0%



Proportional Apportionment

evpropwin * popwin Crosstabulation

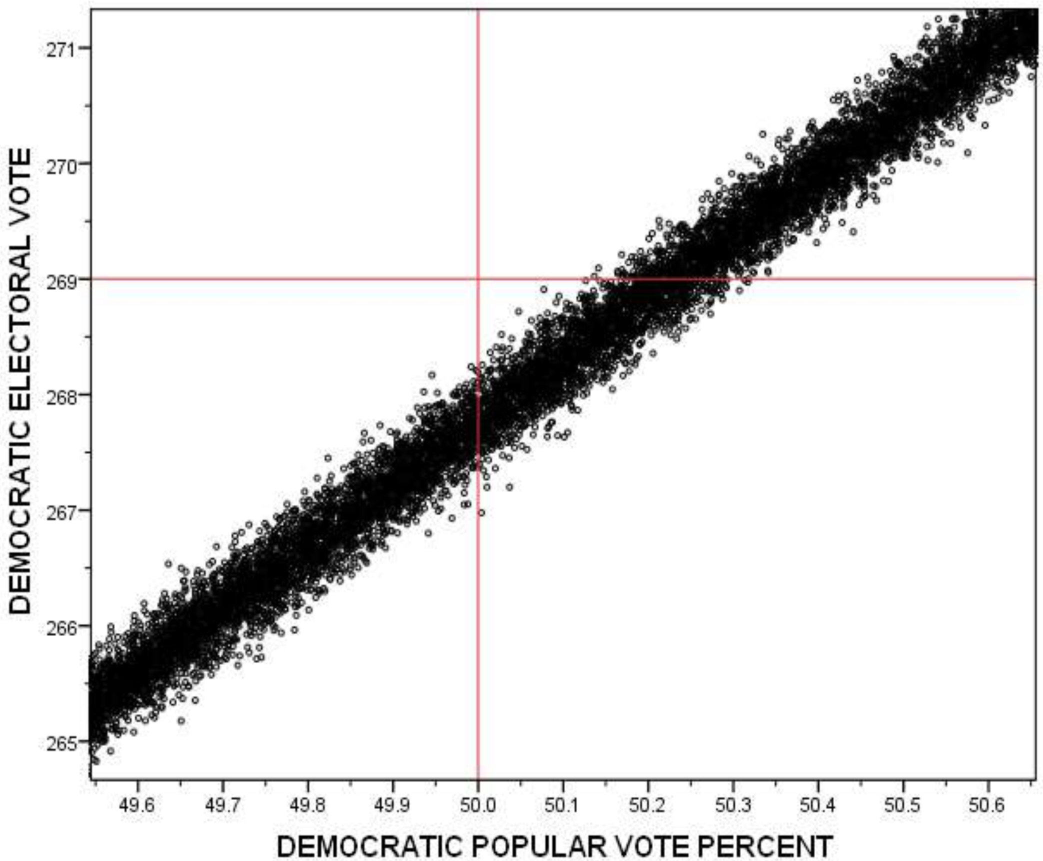
			popwin		
			.00	1.00	Total
evpropwin	.00	Count	54877	957	55834
		% of Total	45.7%	.8%	46.5%
	1.00	Count	5400	58757	64157
		% of Total	4.5%	49.0%	53.5%
	Total	Count	60277	59714	119991
		% of Total	50.2%	49.8%	100.0%



Pure Proportional System

propevwin * popwin Crosstabulation

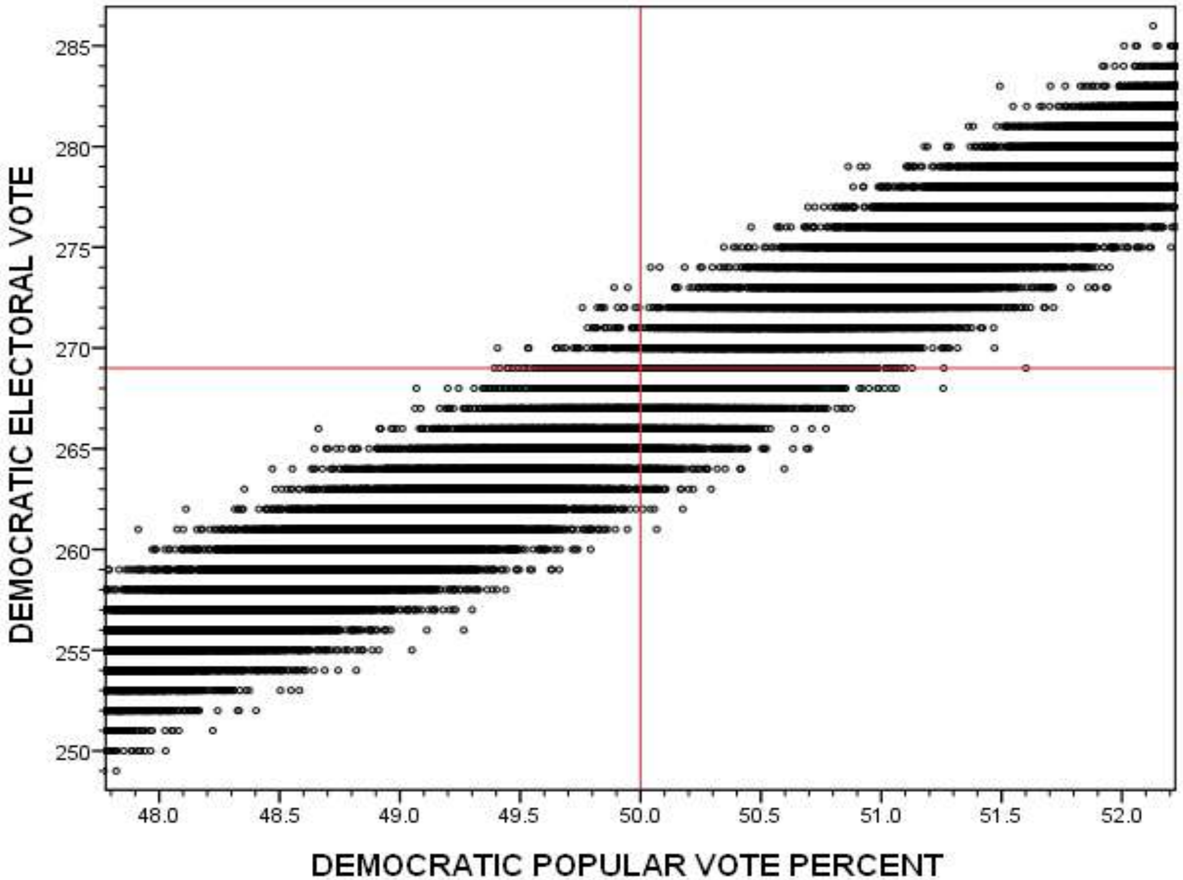
			popwin		
			.00	1.00	Total
propevwin	.00	Count	60280	2149	62429
		% of Total	50.2%	1.8%	52.0%
	1.00	Count	0	57571	57571
		% of Total	.0%	48.0%	48.0%
	Total	Count	60280	59720	120000
		% of Total	50.2%	49.8%	100.0%



Whole Number Proportional System

evcowin * popwin Crosstabulation

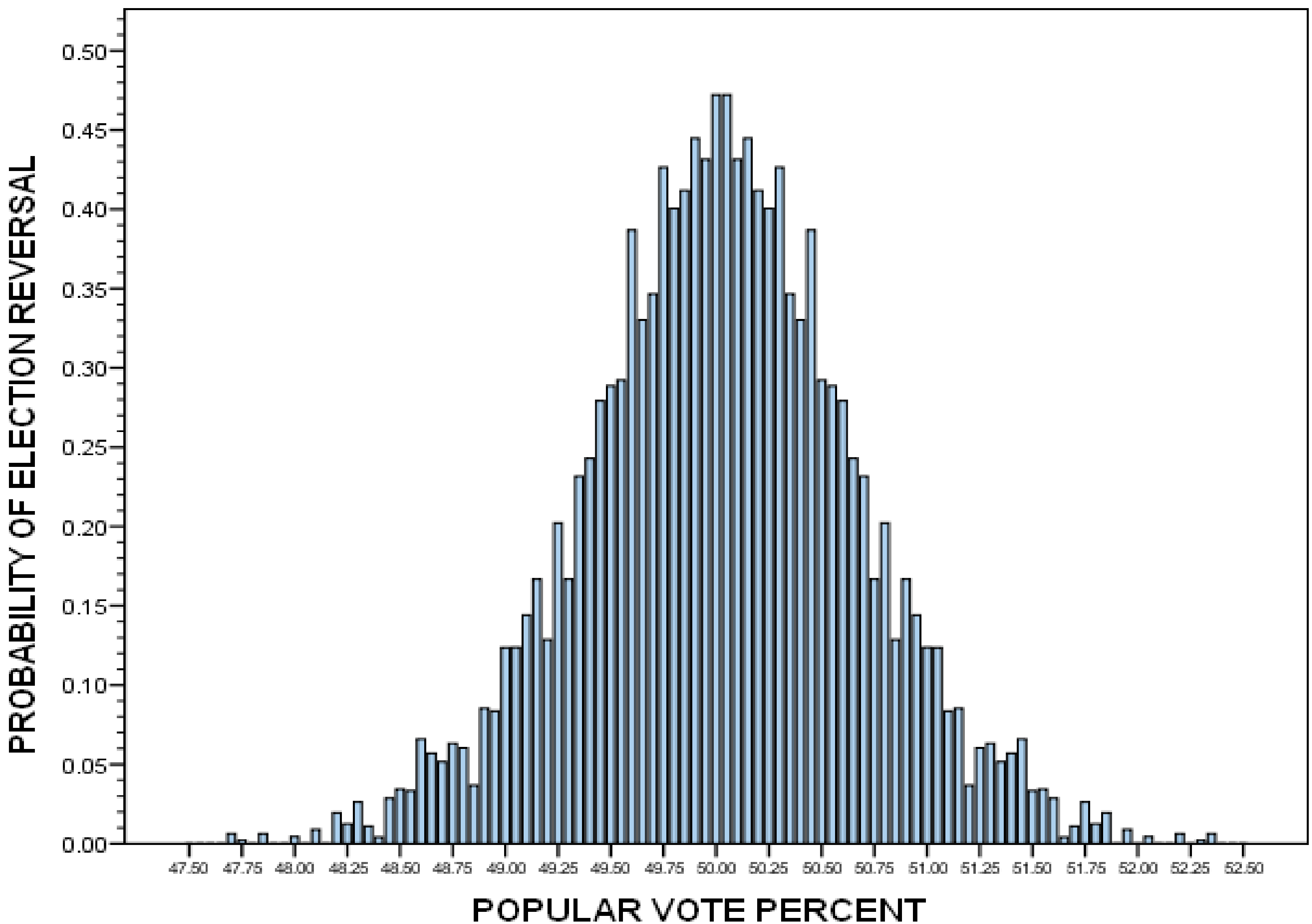
			popwin		
			.00	1.00	Total
evcowin	.00	Count	59931	2500	62431
		% of Total	49.9%	2.1%	52.0%
	.50	Count	236	1316	1552
		% of Total	.2%	1.1%	1.3%
	1.00	Count	113	55904	56017
		% of Total	.1%	46.6%	46.7%
	Total	Count	60280	59720	120000
		% of Total	50.2%	49.8%	100.0%



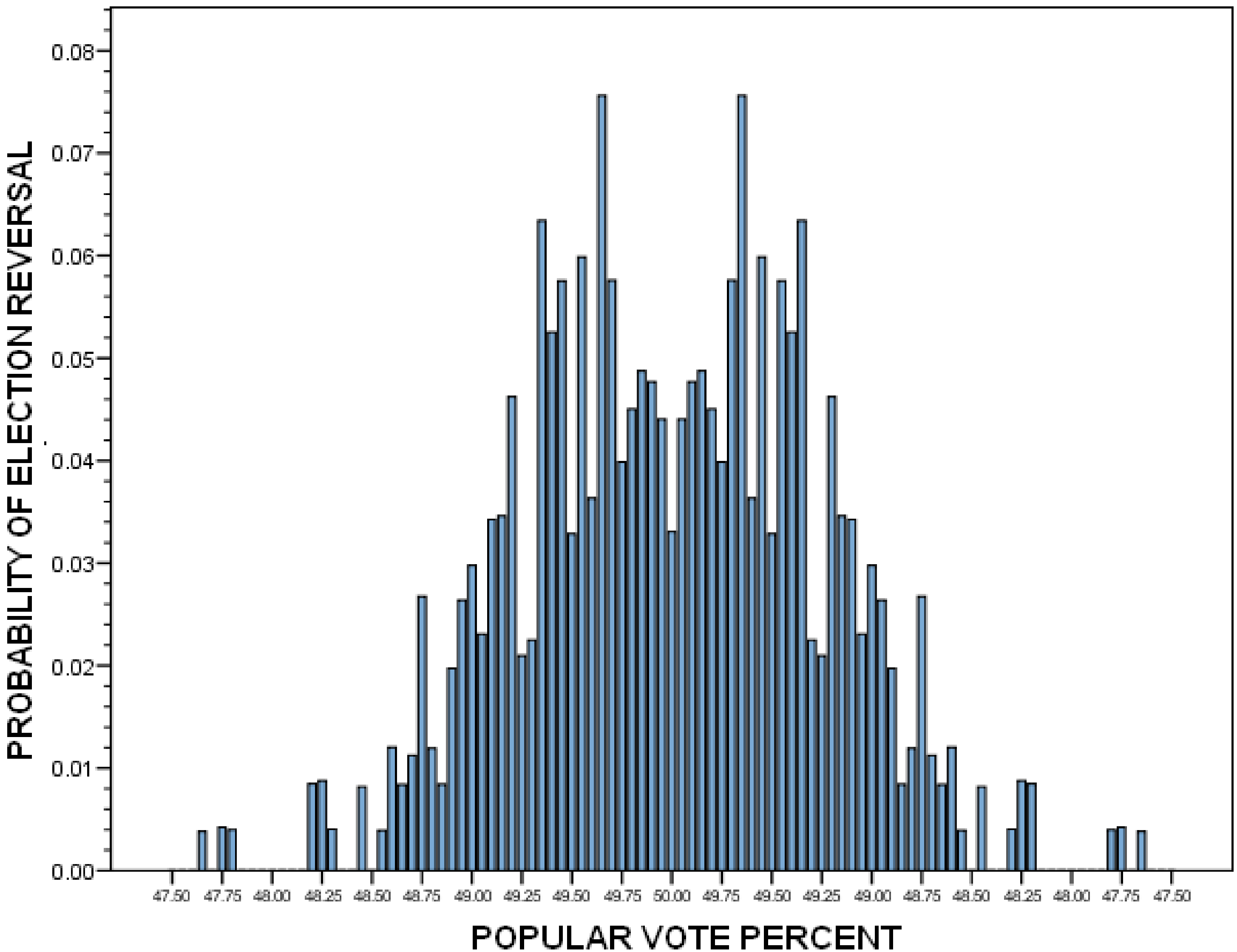
Mean Majority Deficit in Contemporary Elections

Electoral College Variant (2000 Census)	MMD1	MMD2
Existing Electoral College	44,778	573,852
Electoral College with House Apportionment	33,910	603,780
Electoral College with Perfect Apportionment	35,933	678,258
Electoral College with State Equality	196,307	1,358,658
Electoral College with “Random” Apportionment	42,468	688,342
Pure District Plan (EV = 436)		
Pure District Plan (EV = 538)		
Modified District Plan		
Pure Proportional Plan	2,743	153,194
Whole Number Proportional Plan	9,868	284,316

Estimated (Symmetric) Probability of Election Inversions By Popular Vote (Based on 2004 Landscape)



Estimated (Symmetric) Probability of Electoral Vote Ties By Popular Vote (Based on 2004 Landscape)



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