

Assessing the probability of the referendum paradox : the French local election case

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Abstract In an election, the winner in terms of seats, does not have necessarily a majority of the votes. This phenomenon is known in Social Choice Theory as the referendum paradox. Using data for the French “cantonal” elections from 1985 through 2004, first we identify the departments for which such a phenomenon occurs, and determine the occurrence of this paradox. Then, we compare these results with those of the theoretical models.

Key words: Referendum paradox, French “Cantonal” election, Probability.

JEL Classification: D7.

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1 Introduction

In democratic systems, the ideal representation should enable each voter to have equivalent weight and influence. This may mean two very significant things in a system of indirect election. First, the number of elected representatives allocated to each district, shall be proportional to the number of voters enrolled in the electoral register for that electoral district. Secondly, *ceteris paribus*, the party which receives the most votes in the country should control the most seats. The second condition is not always guaranteed. This situation is known in Social Choice Theory as the referendum paradox.

Nurmi (1999) shows that the referendum paradox can be observed in many democracies in which the electoral systems are based on elections in districts. The paradox can appear, as we will see in our examples, in the first-past-the-post system, and also in the French electoral system. For the American presidential elections, an instance of this paradox occurred in the 2000 election (see table 1).

Table 1 The American presidential election of November 2000.

Candidates	Votes	% Votes	Electors
George W. Bush	50,456,002	47.88%	271
Al Gore	50,999,897	48.38%	266
Ralph Nader	2,882,955	2.74%	0
Pat Buchanan	449,895	0.42%	0
Harry Browne	384,431	0.36%	0
Howard Phillips	98,02	0.09%	0
John Hagelin	83,714	0.08%	0
Others	51,186	0.05%	0
Total	105,405,100	100%	538*

*: one elector abstained

Source : <http://www.fec.gov>

George W. Bush was elected President of the United States by winning more States, 30 States (21 to A. Gore), for a total of 271 electors among 538 in the Electoral College (266 to Gore), whereas he obtained less votes than his competitor, 47.9% to 48.4%. This reversed majority phenomenon has happened four times in the American presidential election history, in 1824, 1876, 1888 and 2000. In the same way, during the 20th century, the winning party in the United Kingdom Parliament, in terms of seats, has had less popular support than the losing one (see table 2) in four elections.

France, is not immune to this phenomenon. This article will be devoted specifically to study and illustrate the referendum paradox using the French cantonal electoral data.

Table 2 United Kingdom Parliament election.

Year	Parties	Votes	% Vote	Seats
1910	Conservative party	3,104,407	46.8%	272
	Liberal party	2,866,157	43.5%	274
	Others parties	696,836	9.7%	124
1929	Conservative party	8,656,225	38.1%	260
	Labour party	8,370,417	37.1%	287
	Others parties	5 621 733	24.8%	71
1951	Conservative party	13,718,199	48.0%	321
	Labour party	13,948,883	48.8%	295
	Others parties	929,512	3.2%	9
1974	Conservative party	11,872,180	37.9%	297
	Labour party	11,645,616	37.2%	301
	Others parties	7,822,366	24.9%	37

Source : <http://psephos.adam-carr.net>

The French cantonal elections are based on elections in districts; they are, consequently, likely to encounter this paradox. Let us briefly remind that the purpose of these elections is to elect the members of the district council for the departments: The jurisdiction for this election is the canton, in which a councillor is elected. The district council is renewed by half every three years using a run-off system.

The objectives of this paper are twofold: First, using the data for the cantonal elections in metropolitan France from 1985 to 2004, we want to identify the departments for which such a phenomenon of the reversed majority occurs and try to estimate empirically the occurrence of the paradox. Secondly will compare our results with the theoretical predictions (see Feix *et al.* (2004, 2009)). The interest of the cantonal elections is to present a long series of data, with 93 departments voting every 3 years. Thus, we have 651 (93×7) cases to study.

In this paper, we give first a detailed example of the referendum paradox in the second section. Section 3 is devoted to the estimation of the referendum paradox in cantonal elections. To aggregate the votes within each department, we employ four approaches: (1) votes cast in favour of the winners, (2) votes cast in favour of all the candidates of the two major political camps, (3) taking into account of the presence of third parties and (4) the distinction of the two rounds of elections. Section 4 provides our results. Section 5 is devoted to a discussion about the results of the four approaches, and next compares them with the theoretical models. The last section concludes the paper.

2 The referendum paradox

2.1 The referendum paradox : illustration

Habitually, important legislative decisions are decided in the parliament, but some particularly crucial decisions are subjected to a referendum. This one is a direct vote in which an entire electorate is asked to either accept or reject a particular proposal. Such referendum is an usually optimal democratic solution in a representative democracy. Nevertheless, in some countries where the referenda are consultative and not obligatory, a particular problem can emerge: Which one of the two results can be considered more legitimate; the result of the referendum or the parliamentary result of the vote? It may happen that the majority of the voters favour an opinion and the majority of the representatives opt for its negation. To illustrate this phenomenon, we present Nurmi's example (Nurmi (1999)). Assume that a decision of binary type (Yes/No) must be taken in a country with 62 million register votes. This country is divided into 200 districts, each one composed of 310 000 votes and represented by a unique representative. The jurisdictions are assumed to be of equal size which means that the apportionment is proportional. We display in table 3 the distribution of the opinions on the binary issue and the vote of the representatives, which follows the majority opinion of their districts.

District	$D_1 \dots D_{150}$	$D_{151} \dots D_{200}$
Yes	110,000 . . . 110,000	310,000 . . . 310,000
No	200,000 . . . 200,000	0 . . . 0
MPs	No . . . No	Yes . . . Yes
	150 No	50 Yes

Table 3 The referendum paradox

The new issue, subjected to the popular referendum, is approved because it obtains

$$150 \times 110,000 + 50 \times 310,000 = 32,000,000 \quad \text{Yes}$$

against:

$$150 \times 200,000 = 30,000,000 \quad \text{No}$$

But suppose now that the referendum was only consultative and that the final decision comes to the parliament. The representatives being aware of the opinions of their voters, they would vote according to the majority opinion in their constituency. Then the proposal receives only 50 votes in favor and 150 against. The decision of the MPs goes in opposition to that of the population. Then we notice the occurrence of the referendum paradox.

This phenomenon is inevitable, and it can occur whenever the decision is not taken directly by the people, but by locally elected representatives.

2.2 What is the solution?

Basically, the referendum paradox emphasizes the irreconcilable nature of the direct and indirect votes. Recent works on social choice theory (Laffond and Lainé (2000), Chambers (2008), Perote Peña (2006), Bervoets and Merlin (2005, 2007)) have confirmed that no indirect voting procedure can translate perfectly the results of a direct vote, for any specification of the preferences. Then, both indirect and direct voting procedures can lead to opposite results. A way to circumvent these negative conclusions is to study the probability of the paradox.

In social choice theory, the pioneer work of Arrow (1963) caused a natural reaction which consisted in wondering about the real importance of the “impossibility theorems”. Many works during recent years tried to estimate the probability of the different social choice paradoxes, including the Condorcet effect, the manipulation, etc. One can find a complete panorama on these attempts in Gehrlein (2006).

Concerning the referendum paradox, Feix *et al.* (2004) studied it with two *a priori* models of voting. Indeed, they considered a competition between two parties in N districts of equal size. In a first model (Impartial Culture), one supposes that every voter plays heads or tails to determine his vote. Then, when the number of districts increases, the limit of the paradox tends to about 20.5%. With a second model (Impartial Anonymous Culture), a slight correlation between the voters choices inside the same district makes the paradox probability tend to 16.5% when N tends to infinity. More recently Feix *et al.* (2009) consider district with unequal population. But, are these theoretical estimations realistic ones? An empirical study will allow us to make comparisons between the empirical and theoretical results. To carry out this study, we have to choose a sufficiently rich electoral database. It is the case of the cantonal elections in France.

3 Data and methodology:

3.1 Data:

The cantonal elections allow the population to choose their representatives to district councils of their department. Indeed, every department is divided in cantons (on average 39 cantons per department) that designate a councilor by the means of a plurality run-off system. A councilor is elected for six years, but the renewal of the district council is done partly every three years. To be elected in the first round, the candidate must get the absolute majority of the votes cast, as well as a number of votes at least equal to the quarter of the number of the registered voters. To be qualified to the second round, it is necessary to get more than 10% of the registered voters. If a single candidate satisfies this condition, the

person who is ranked at second place also goes to the second round. During the second round, the candidate who obtains a plurality of votes is elected.

The data in the present study are extracted from the electoral database carried out by the Quetelet center (based upon Home Department data) and the LASMAS (based upon French Statistical institute: INSEE). They are the data of cantonal elections of Metropolitan France, which took place between 1985 and 2004. We excluded from the database Paris, Corsica, and Overseas territories which either have own specific voting rules, or present a much diverse spectrum of political forces. For more detail, we have approximately 1,915 cantons per election year, that is 13,405 election data over a period of 20 years.

From each election arises a lot of discussions and comments. Nevertheless, the central subject remains the modeling of the political scene and the evolution of left/right divide as well as the respective force of the various components of these two blocks. Many factors can influence the balance of power in the cantonal elections¹. The outcome of this balance, in elections held since 1985, is displayed in table 4. The balance of power was rather in favor of the right, as the left during the last 20 years often received less than 45% of the vote. But its position improved since 2001, especially in terms of seats.

Table 4 cantonal election from 1985 (metropolitan France).

		1985	1988	1992	1994	1998	2001	2004
% vote Total, 1 st round	T. Left	41.3	47.8	34.4	40.8	44.6	46.4	48.2
	Others [±]	0.8*	0.2*	10.3*	4.4*	1.2	1.8 ⁺	1.8 ⁺
	T. Right	49.1	45	43	44.8	40.3	44.7	37.5
	FN	8.8	5.4	12.3	9.9	13.9	7.1	12.5
Total Seats	T. Left	642	842	547	804	977	1102	1149
	T. Right	1310	1093	1395	1110	977	813	796
Council Presidencies	T. Left	27	27	20	20	33	37	47
	T. Right	68	68	75	75	62	58	48

[±] This category gathers independent candidates and minor parties, such as the Greens or CPNT (Pro hunting party). * including the Greens, 0.8% in 1985, 1.6% in 1988, 8% in 1992 and 2.6% in 1994. They counted in the left camp from 1995. ⁺ including CPNT, 0.4 in 2001 and 0.1 in 2004.
sources: adapted from Martin (1998, 2004a) and Le Gall (1998, 2001, 2004).

Certainly, since it is a matter of a local ballot, it is easy to note here and there, in several districts, that the ballot is strongly marked by the personality of incumbent candidates whose influence transcends the partisan divides. In-

¹ For more detail, one can consult the papers of Lancelot (1985), Parodi (1985), Guhur (1988), Portelli (1992), Habert, Perrineau et Ysmal (1992), Jaffré (1992, 1998), Van Tuong (1992), Le Gall (1994, 2001, 2004), Martin (1994, 1998, 2004a, 2004b), Chiroux (1994), Guastoni (1998) et Grigny (2004).

dependently of their political orientations, their electorate is often wider than the one of their camp.

Of course equally, all the nuances of the political spectrum are not represented in every district, even in first round, and the choice of the voter is often limited and channeled towards the closest to his opinion candidate. Indeed aggregating the vote totals between the major political coalitions seem to be delicate on the department scale and depends on the “official” labels of the candidates. Let us note, in this respect, that our study will be based on the classification of the candidates proposed by the government services.

3.2 Four methods to estimate the paradox:

Before specifying methodologies adopted here, it is important to reconsider the objective for which several adjustments, on these data, will be operated. This objective is to evaluate the frequency of the paradox. For that, it is necessary for us to build a pertinent method to measure this occurrence while taking into account the specificities of the cantonal ballot in comparison with the idealized structure, presented in the table 3. Two obstacles to overcome to establish an effective measure are:

- The existence of two rounds instead of one
- The frequent presence of a third party in the second round

To answer the above mentioned issues, we circumscribed our methodology to four approaches.

First approach: votes cast in favor of the elected representatives

Firstly, we adopt a method which consists in measuring the occurrence of the paradox according to the votes obtained by the elected councillors only. More precisely, at the level of every district, we keep only the votes that allowed a candidate to be elected, whatever the round it was (first or second), and consider the votes of the other candidates as null. We aggregate, afterwards, these votes by department and by political camps, to examine if the elected candidates of the majority camp in the council are elected with more ballots than the ones of the minority camp.

Second approach: all votes cast in favor of all the candidates

Next, we consider the phenomenon by taking into account, this time all expressed votes in favor of every party. In other terms, we keep all votes collected by the right and left candidates that were elected or not. In fact, this second approach is based on the following principle: We identify the votes received by every political coalition within every district then we aggregate them at the level of the department. After that, we compare the number of seats and

that of votes gathered by every political camp in order to show the existence or the absence of the paradox.

Notice that we do not separate between the votes and the seats obtained in both rounds of balloting. We aggregate the result of the “decisive ballot”, as if the election concerns the first-past-the-post system.

Third approach: the presence of a third party

The score of the extreme right is, over the studied period, sometimes considerable. This party constitutes a third block which can rock the result of a poll. It seems judicious to integrate in our analysis the impact of the presence of a third camp, especially in the second round. Our third step proposes to re-examine the calculation of the paradox with respect to classification left-moderate right-extreme right. Besides the distinction of votes obtained by the left and the right, we distinguish votes collected by the moderate right and the extreme right. We recalculate afterwards the occurrence of the paradox while comparing the seats and the votes for each of the three political forces. We adopt the previous principle concerning the votes, i.e., we incorporate the votes obtained by all the candidates (victorious or not). The objective of this method is to know whether the introduction of an important third political party changes the balance of power.

Fourth approach: the distinction between the rounds

In the previous approaches, we aggregated the electoral data from either the first and the second round, depending on the round where a candidate was declared victorious. Thus, we completely ignored the fact that the participation might rise or decline in the second round, the fact that there are much fewer candidates at the second stage, etc. A robust assessment should then separate the results of the two rounds, analyzing on one hand the cantons that have been declared after the first round, and on the other hand the cantons that have chosen their winner only at the second stage. Therefore, a fourth and last step is suggested. This method is based on the distinction between the rounds as well as on the aggregation of the votes obtained by every competitor according to the classification (right, left and extreme right). The goal is to provide measures of the paradoxes which are not affected by possible modifications of the voting behavior between the two rounds.

Before beginning to present our results, we give some remarks. According to these four ways of defining the referendum paradox, we added the votes of a camp as if it were represented by the same candidates in each canton. But, actually, we are aware that we have aggregated very heterogeneous data, and that each of the above mentioned method is a tentative reconstruction of simple two-party-system.

Another difficulty is due in part to the interpretation of the referendum paradox definition: how to interpret the situations where the two political forces obtain the same number of seats² (and not the same number of votes)? In a lax interpretation, we consider that all the situations of equality are not paradoxical because the popular winner is not beaten with the voting rule. This analysis is then in the logic of the weak version of the paradox. On the other hand, a strong version supposes that all these situations are paradoxical, if it is supposed, in theory, that the popular winner should win with no question.

However, we will show that despite all these problems, the existence of a referendum paradox is clearly highlighted for several departments, whatever the applied method. In next section, the result will be presented in the lax interpretation.

4 Results : assessing the referendum paradox

This section is devoted to the evaluation of the referendum paradox with the four approaches. To complete the analysis, we develop the reason for choosing an approach compared to another, providing and explaining concrete examples.

4.1 Estimating the paradox with the votes cast in favour of only the elected members

The detailed statistics of the first approach appear in the table 5³. The results show that the cantonal elections, since 1985, were far from the absence of the referendum paradox. The department of Aveyron in 2004 is a good example. There are 106 563 registered voters and 23 seats at stake within this department. The right parties succeeded in conquering 14 cantons whereas they collected only 17.64% of the votes of the registered voters. On the other hand, the left winners gathered 9 cantons with 26.43% of the poll.

Table 5: Departments where the paradox appears according to the first approach.

Year	Department	Registered voters	Seats at stake	Left		Right	
				% Vote	C	% Vote	C
	AVEYRON	106,563	23	26.43%	9	17.64%	14
	CALVADOS	202,505	23	19.68%	11	16.09%	12

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² French electoral law states that in case of ties, the candidate elected president of local assembly is the elder.

³ All the results of the four methods of the paradoxical situations in the event of equality appear in appendix.

... table 5 continued

	COTE D'OR	157,377	21	19.80%	10	16.31%	11
	JURA	82,212	17	18.84%	8	16.23%	9
	SARTHE	202,492	19	18.56%	8	16.89%	11
	VIENNE	147,229	20	20.89%	9	15.89%	11
2001	ALPES DE HAUTE PROVENCE	50182	14	18.99%	8	19.33%	6
	CHARENTE	135,987	17	16.40%	7	15.90%	10
	GARD	243,307	23	13.35%	12	21.73%	11
	OISE	259,068	21	15.59%	11	15.81%	10
	SAONE ET LOIRE	220,949	29	17.11%	13	15.99%	16
	TARN	138,956	23	19.26%	12	19.55%	11
1998	CHARENTE	126,877	18	15.38%	8	13.98%	10
	CHER	117,054	18	15.21%	7	14.73%	11
	CORREZE	92,762	19	18.81%	9	18.03%	10
	GERS	64,888	17	19.21%	7	18.80%	10
	INDRE ET LOIRE	173,372	19	14.93%	8	13.72%	11
	MEURTHE ET MOSELLE	261,967	25	15.64%	12	11.57%	13
	PYRENEES ATLANTIQUES	210,259	26	17.06%	11	15.10%	15
	RHONE	449,295	26	13.35%	11	12.43%	15
1994	AISNE	190,775	21	22.06%	10	13.54%	11
	ALLIER	126,474	17	15.86%	9	16.52%	8
	ARDECHE	116,357	17	17.69%	8	17.21%	9
	CHARENTE MARITIME	188,721	25	15.79%	13	17.51%	12
	DOUBS	168,749	17	16.95%	8	16.07%	9
1994	EURE	176,768	21	16.32%	8	15.73%	13
	EURE ET LOIR	131,241	15	15.32%	6	15.24%	8
	NORD	806,162	38	16.70%	18	16.16%	20
	HAUTES PYRENEES	97169	17	18.67%	9	19.01%	8
1992	HAUTE GARONNE	327989	25	13.91%	14	18.17%	11
	LOT	69,466	17	17.95%	10	20.83%	7
	HAUTES PYRENEES	75,018	17	17.53%	9	18.60%	8
1988	AISNE	188,596	21	17.73%	10	13.10%	11
	ARDECHE	111,016	17	15.32%	9	16.89%	8
	CHER	107,058	17	15.43%	8	15.02%	9
	DORDOGNE	136,487	24	19.63%	11	18.64%	13
	SAONE ET LOIRE	217,713	29	15.72%	14	12.68%	15
	SOMME	174,487	23	16.82%	10	16.81%	13
1985	BOUCHES DU RHONE	478,524	23	13.67%	13	15.90%	9
	COTES D'ARMOR	208,366	27	20.25%	14	20.57%	13
	CREUSE	49,454	13	17.35%	7	21.71%	6
	DROME	141,271	19	17.75%	11	18.25%	8
	PUY DE DOME	215,373	30	18.89%	16	20.41%	14
	HAUTE SAONE	90,479	18	20.05%	8	19.89%	10
	VAUCLUSE	141,694	12	18.50%	7	19.98%	5

Although there are less paradoxical cases in 1992, in the other years, the occurrence of the referendum paradox is rather frequent. This paradox appears on average 6.42 times among the 93 departments, for each cantonal election year.

In addition, table 5 also reveals that the paradoxes does not always occur in the same departments, every six years. In other words, the departments which experience this paradoxical phenomenon vary from one electoral year to another. For example, the department of Calvados suffered from the paradox only in 2004, while the department of Aisne was touched by the phenomenon in 1988 and 1994 (same cantons) and the department of Charente encountered the phenomenon in 1998 and 2001, with a different set of cantons at stake.

All in all, the departments coming up, at least only once, against the problem of the reversed majority, account for 41.93% of the departments (39 departments are touched among the 93). Moreover, we count, on the 651 studied cases, 45 paradoxical situations, which means that the frequency of the paradox is 6.91% (45/651). This result seems obvious given the instability of the balance of power between the two major political coalitions, Left and Right, during the period. Another remark which deserves to be mentioned is that the paradoxes does not systematically favor one camp. By way of illustration, in 2004, the right wins more seats with less vote in all the departments suffering from the paradox, while, in 1985, the left gains the majority of the seats with less vote, in all the paradoxical cases. In fact, when one coalition does badly nationwide during an election year, the strength of the incumbent candidates helps it to hold back more seats than expected in some departments, leading then to a referendum paradox.

Not to take into account all the votes of the parties of the same political coalition clearly underlines a limit of this approach⁴. The example of the department of Aveyron (2004) attests to this: The left carried out a score of 60% of the votes - votes only expressed for its elected candidates - against 40% for the right. However, it trailed by 4 points (48% against 52%) when all the votes cast in favour of the two political forces were taken into account. This leads to a reexamination of the probability of the paradox according to the second approach.

4.2 Second approach: all votes cast in favour of all the candidates

The results displayed in table 6 have similarities and divergences compared to the first approach. They confirm the presence of reversed majorities. Moreover, they show that the number of paradoxical cases increased, during every year except for 1994. The cantonal elections of 2004 in which we counted

⁴ The votes of the other parties than the left or right were not taken into account in our analysis for simplification.

11 cases instead of 6 can illustrate our remark. However, the list of departments which suffer from the paradox can drastically change when we compare tables 5 and 6 for a given year. Hautes Alpes (2004) illustrates this fact. The right succeeded in conquering only 6 cantons among the fifteen disputed while collecting 52.50% of the votes, whereas the left got control of 9 seats by obtaining only 47.50% of the votes.

Table 6: Departments where the paradox occurs according to the second approach.

Year	Department	Votes cast	C	Left		Right		Others	
				% Vote	C	% Vote	C	% Vote	C
2004	HAUTES ALPES	34863	15	47.50%	9	52.50%	6	0.00%	0
	CALVADOS	126,833	23	50.27%	11	49.54%	12	0.19%	0
	CHARENTE MARITIME	137,151	26	48.97%	14	51.03%	12	0.00%	0
	MAINE ET LOIRE	151,373	20	47.54%	10	50.75%	9	1.71%	1
	PYRENEES ORIENTALES	105,697	16	47.54%	9	50.00%	6	2.47%	1
	RHONE	291,443	27	48.07%	15	51.14%	11	0.80%	1
	SARTHE	122,986	19	51.52%	8	48.48%	11	0.00%	0
	SEINE ET MARNE	243,037	23	49.56%	14	50.44%	9	0.00%	0
	VAUCLUSE	110,352	12	43.08%	9	56.92%	3	0.00%	0
	VOSGES	91,153	15	43.14%	8	56.73%	7	0.14%	0
	TERRITOIRE DE BELFORT	30,940	8	49.11%	6	50.89%	2	0.00%	0
2001	CHARENTE	78,030	17	50.26%	7	49.74%	10	0.00%	0
	CORREZE	65,035	18	51.31%	7	48.69%	11	0.00%	0
	DOUBS	100,318	17	48.75%	9	51.25%	8	0.00%	0
	ISERE	195,964	29	50.34%	14	49.66%	15	0.00%	0
	OISE	137,360	21	42.97%	11	57.03%	10	0.00%	0
	PYRENEES ORIENTALES	84,170	15	49.79%	9	50.21%	6	0.00%	0
	SAONE ET LOIRE	127,130	29	51.89%	13	47.29%	16	0.82%	0
	SEINE MARITIME	250,447	35	50.50%	14	49.32%	21	0.18%	0
1998	BOUCHES DU RHONE	308,149	27	48.48%	24	51.52%	3	0.00%	0
	CHER	63,301	18	51.40%	7	44.40%	11	4.20%	0
	DROME	86,977	18	49.37%	11	50.63%	7	0.00%	0
	FINISTERE	187,002	28	46.87%	17	52.12%	11	1.02%	0
	HERAULT	192,643	26	47.00%	21	48.56%	5	4.45%	0
	ILLE ET VILAINE	155,439	27	44.32%	14	53.35%	13	2.33%	0
	ISERE	156,067	29	49.14%	16	50.72%	13	0.13%	0
	MEURTHE ET MOSELLE	120,257	25	53.23%	12	46.30%	13	0.47%	0
	PYRENEES ORIENTALES	87,027	16	45.95%	11	52.76%	5	1.29%	0
	SOMME	129,535	23	46.45%	13	52.68%	10	0.87%	0
	VAUCLUSE	90,584	12	39.94%	9	60.06%	3	0.00%	0
	VOSGES	79,501	15	46.07%	8	53.93%	7	0.00%	0
	TERRITOIRE DE BELFORT	25,994	8	49.67%	6	50.33%	2	0.00%	0
VAL D'OISE	137,406	20	42.19%	11	52.77%	9	5.04%	0	
1994	AISNE	118,883	21	53.92%	10	45.74%	11	0.34%	0
	ALLIER	76,200	17	47.95%	9	52.05%	8	0.00%	0
	ALPES DE HAUTE PROVENCE	28,538	14	52.45%	6	47.55%	8	0.00%	0
	BOUCHES DU RHONE	262,479	26	49.09%	18	50.88%	8	0.03%	0
	CHARENTE MARITIME	106,624	25	46.57%	13	53.18%	12	0.25%	0
	DORDOGNE	98,145	24	49.57%	14	50.43%	10	0.00%	0
	INDRE	60,733	13	50.85%	5	49.15%	8	0.00%	0

...Continued on next page...

... table 6 continued

1994	MEURTHE ET MOSELLE	129,529	20	44.20%	12	55.80%	8	0.00%	0
1992	BOUCHES DU RHONE	325,881	29	39.87%	18	59.16%	11	0.97%	0
	HAUTE GARONNE	202,248	25	48.00%	14	48.03%	11	3.97%	0
	HERAULT	179,887	26	43.91%	16	55.87%	10	0.21%	0
	VAL DE MARNE	184,996	25	41.19%	16	55.85%	9	2.96%	0
1988	AISNE	98,613	21	56.28%	10	43.20%	11	0.52%	0
	ALLIER	69,971	17	53.18%	8	46.82%	9	0.00%	0
	ARDECHE	64,366	17	49.11%	9	50.89%	8	0.00%	0
	CHARENTE	66,037	17	52.11%	8	47.89%	9	0.00%	0
	CHARENTE MARITIME	90,309	25	51.48%	12	48.52%	13	0.00%	0
	DORDOGNE	93,046	24	51.21%	11	48.79%	13	0.00%	0
	FINISTERE	163,890	26	48.48%	15	51.12%	11	0.40%	0
	ISERE	155,526	29	49.03%	15	50.76%	14	0.21%	0
	OISE	116,975	21	50.04%	10	49.96%	11	0.00%	0
	SEINE MARITIME	195,827	35	52.27%	14	47.65%	21	0.08%	0
	SOMME	100,203	23	50.39%	10	49.61%	13	0.00%	0
ESSONNE	143,794	21	50.54%	6	48.82%	15	0.64%	0	
1985	BOUCHES DU RHONE	287,234	23	43.51%	13	56.49%	10	0.00%	0
	COTES D'ARMOR	160,237	27	49.48%	14	50.52%	13	0.00%	0
	DROME	94,699	19	48.81%	11	51.19%	8	0.00%	0
	HERAULT	170,314	23	48.37%	14	51.53%	9	0.10%	0
	LANDES	82,362	15	49.48%	10	50.52%	5	0.00%	0
	NORD	553,200	40	49.76%	22	49.77%	18	0.47%	0
	PUY DE DOME	152,349	30	46.35%	16	53.65%	14	0.00%	0
	VAUCLUSE	97,370	12	48.17%	7	51.83%	5	0.00%	0
	TERRITOIRE DE BELFORT	30,315	8	49.09%	5	50.91%	3	0.00%	0
	SEINE SAINT-DENIS	189,631	20	49.90%	11	50.10%	9	0.00%	0
	VAL DE MARNE	251,690	30	46.61%	17	52.20%	13	1.19%	0

C : Councillors.

In general, we have 68 cases out of 651, i.e., the occurrence of the referendum paradox is 10.45%. On the annual average, the paradox appears 9.71 times. We can, moreover, affirm that in all of the 93 departments, 41 encountered the reversed majority at least once.

The remarks advanced in the preceding analysis remain valid. In other words, the departments suffering from the paradox differ from one electoral year to another, and the paradox usually favors the block which is losing nationwide, thanks to its incumbent candidates.

It should be also noted that neither in terms of seats nor in terms of vote, no party dominates the composition of the general councils continuously: sometimes the right controls more seats without having the majority of the votes and thus the left collects more votes without controlling more seats;

sometimes it is the reverse. We quote by way of illustration the department of Charente Maritime. In 1988, the left gleaned 13 seats out of 25 with 48.52% of the popular votes against 12 seats for the right. On the other hand, in 2004 the left wins the election by receiving 14 seats out of the 26 disputed with 48.97% of the votes. But, in general, the left controls more seats without having the majority of the votes, in 44 listed cases among the 68.

A fact ignored in table 6 is the strength of the extreme right in 1998 and 2004, and its capacity to create three-way races in the run-off these years. However, we note that the departments, which have a strong presentation of this political block, encounter paradoxical cases. We find such examples with the departments of the Pyrénées Orientales (2004) and Vaucluse (2004). The left controlled in the first department 9 seats out of 16 with 47.54% of the votes against 6 seats for the right with 50%, including 8% for the extreme right candidates. With regard to Vaucluse, the left obtained 43.08% of the votes against 56.92% for the right including 25% of the votes of the partisans of the extreme right.

Admittedly, other departments also have strong presence of the extreme right vote and do not encounter a reversed majority. It is the case, in 1998, for Indre-et-loire, Oise, Rhône and Seine-et-Marne. But, by taking into account only the votes of the moderate right a paradox appears in these departments. This means that considering the right extreme as a separate block modify all the analysis, at least in 1998 and 2001. Thus, we need to turn to a third method which labels the right extreme votes separately.

4.3 Third approach: the effect of a third block

A drawback of the second approach is that it does not take into account the influence of the Front National in certain departments: Bouches du Rhône is affected by the paradox during four cantonal elections and Pyrénées Orientales during three polls, but the results will be different if one opposes the left to the moderate right only. In other words, in these departments, the sum of the votes of the right according to the second approach was amplified by that of the extreme right without making it possible to win more seats. Indeed, the peril of the right resides, particularly, in the level of the FN's votes and its capacity with being present massively at the second round on a significant level. For instance, in Bouches-du-Rhône (1985), the FN gathered 13.20% of the votes cast (43.51% for the left and 43.29% for the moderate right) and it was present at the second round in 17 cantons out of the 21 renewable ones, but it gained only one seat (against 13 seats for the left and 9 for the moderate right) (see table 7). It is obvious that in cantons 2, 8, 22, 33 and 45, the three-way races (left-moderate right-extreme right), amplified the defeat of the right. The total right largely exceeded the left in terms of votes, however, it is the latter which won these cantons.

Table 7: The result of the renewable cantons in the department of Bouches du Rhone in 1985.

Cantons	Vote cast	Left	Right	FN	S	E
2	17,077	40.72%	40.57%	18.71%	59.28%	L
4	10,107	54.63%	45.37%	0.00%	45.37%	L
6	15,291	49.07%	50.89%	0.04%	50.93%	R
8	24,276	35.50%	32.62%	31.88%	64.50%	L
9	7,142	56.58%	42.47%	0.95%	43.42%	L
12	8,574	53.42%	46.13%	0.45%	46.58%	L
14	11,772	0.00%	48.67%	51.33%	100.00%	FN
18	10,327	35.53%	64.33%	0.15%	64.47%	R
19	12,102	34.35%	65.63%	0.02%	65.65%	R
21	8,351	52.17%	0.00%	47.83%	47.83%	L
22	12,456	37.33%	36.58%	26.08%	62.67%	L
23	14,476	45.51%	54.41%	0.08%	54.49%	R
31	7,261	47.91%	52.09%	0.00%	52.09%	R
33	7,347	46.39%	44.98%	8.63%	53.61%	L
34	9,429	53.37%	46.10%	0.53%	46.63%	L
35	12,333	45.59%	53.89%	0.52%	54.41%	R
36	29,631	49.12%	50.88%	0.00%	50.88%	R
41	17,502	53.77%	0.00%	46.23%	46.23%	L
43	17,272	46.78%	53.20%	0.02%	53.22%	R
45	8,427	43.93%	22.69%	33.38%	56.07%	L
46	16,200	31.15%	68.78%	0.06%	68.85%	R
Total	287,234	43.51%	43.29%	13.20%	56.49%	-

FN : Front national, S : The sum of the votes of the right and extreme right, E : Councillor label.

In several cantons, some candidates withdrew at the last minute, but their names were still proposed to the votes. This explains why some candidates qualified for the second round only got very few votes.

Before analyzing in detail the results, we have to describe the principal changes concerning the extreme right party, mainly the Front National (FN), in the last decades. By clinching 11% votes in the European elections in 1984, the FN list created a surprise, since between 1965 and 1984 the extreme right always received less than 1% of the votes. Henceforth, the FN made the decision to present the candidates in a large number of cantons, modifying, on the right, the range of the choices. Moreover, by regularly exceeding the 10% of the registered voters there, hold to go for the run-off, many classical left-right confrontation in the second round evolved toward three-way races. With this strategy, the FN succeeded to secure the loyalty of a socially stable and geographically heterogeneous electorate. It ended the series of elections which were marked by an affirmed bipolarization, where each candidate was located clearly in the left or the right camp. Consequently, the influence of

the extreme right is felt more and more as a third French political force, starting from the elections of 1984. Notice especially the performance of Jean-Marie Le Pen (FN) in the presidential election of 2002 when it reached in the second round.

Thus, in our calculations, we will try to answer the following question: Does the introduction of a third political force into the analysis change indeed the results? A paradox is now a situation where the camp with the plurality of the votes does not get the plurality of the seats.

Table 8: Departments where the paradox appears according to the third approach.

Year	department	Votes	C	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	HAUTES ALPES	34863	15	47.50%	9	48.89%	6	3.61%	0
	CALVADOS	126,833	23	50.27%	11	46.47%	12	3.06%	0
	COTE D'OR	100,635	21	48.72%	10	48.24%	11	2.86%	0
	INDRE	53,227	13	46.12%	4	42.56%	8	4.03%	0
	LOIRET	118,759	20	41.81%	6	41.19%	12	9.82%	0
	MAINE ET LOIRE	151,373	20	47.54%	10	48.19%	9	2.57%	0
	MARNE	108,668	22	47.38%	9	43.65%	13	8.68%	0
	SARTHE	122,986	19	51.52%	8	47.32%	11	1.16%	0
	VOSGES	91,153	15	43.14%	8	48.52%	7	8.20%	0
	YONNE	78,956	22	46.14%	8	45.08%	14	8.50%	0
2001	ARDENNES	64839	19	48.71%	6	48.55	13	2.58	0
	CHARENTE	78,030	17	50.26%	7	48.30%	10	1.44	0
	CORREZE	65,035	18	51.31%	7	47.97%	11	0.72	0
	DOUBS	100,318	17	48.75%	9	49.39%	8	1.86	0
	FINISTERE	196,763	26	49.34%	11	49.29%	15	1.08	0
	ISERE	195,964	29	50.34%	14	47.58%	15	2.08	0
	NORD	424,584	39	48.94%	17	44.78%	22	6.13	0
	OISE	137,360	21	42.97%	11	54.09%	10	2.93	0
	SAONE ET LOIRE	127,130	29	51.89%	13	43.33%	16	3.96	0
	SEINE MARITIME	250,447	35	50.50%	14	45.17%	21	4.15	0
VAUCLUSE	113,092	12	48.31%	4	44.31%	8	7.29	0	
1998	CHER	63,301	18	51.40%	7	43.68%	11	0.72%	0
	FINISTERE	187,002	28	46.87%	17	50.39%	11	1.72%	0
	ILLE ET VILAINE	155,439	27	44.32%	14	51.95%	13	1.40%	0
	INDRE ET LOIRE	88,619	19	49.74%	8	48.58%	11	1.48%	0
	MEURTHE ET MOSELLE	120,257	25	53.23%	12	43.76%	13	2.55%	0
	OISE	132,213	20	41.59%	8	40.81%	11	16.16%	1
	RHONE	216,403	26	42.33%	11	37.53%	15	17.93%	0
SEINE ET MARNE	178,461	23	45.59%	10	44.80%	13	9.61%	0	

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... table 8 continued

	SOMME	129,535	23	46.45%	13	49.41%	10	3.27%	0
	VOSGES	79,501	15	46.07%	8	51.39%	7	2.54%	0
1994	AISNE	118,883	21	53.92%	10	41.49%	11	4.25%	0
	ALLIER	76,200	17	47.95%	9	50.96%	8	1.09%	0
	ALPES DE H.P	28,538	14	52.45%	6	46.18%	8	1.37%	0
	CHARENTE MARITIME	106,624	25	46.57%	13	51.34%	12	1.83%	0
	EURE ET LOIR	75,760	15	45.64%	6	43.44%	8	10.91%	1
	INDRE	60,733	13	50.85%	5	46.37%	8	2.79%	0
	ISERE	175,800	29	48.86%	12	48.13%	17	1.71%	0
	MEURTHE ET MOSELLE	129,529	20	44.20%	12	55.80%	8	0.00%	0
	NORD	452,093	38	47.60%	18	46.66%	20	5.13%	0
1992	HERAULT	179,887	26	43.91%	16	44.64%	10	11.23%	0
	TARN ET GARONNE	51,707	15	48.30%	6	45.37%	9	4.62%	0
	VAL DE MARNE	184,996	25	41.19%	16	47.11%	9	8.74%	0
1988	AISNE	98,613	21	56.28%	10	41.48%	11	1.73%	0
	ALLIER	69,971	17	53.18%	8	46.64%	9	0.18%	0
	ARDECHE	64,366	17	49.11%	9	49.64%	8	1.25%	0
	CHARENTE	66,037	17	52.11%	8	47.89%	9	0.00%	0
	CHARENTE MARITIME	90,309	25	51.48%	12	47.33%	13	1.19%	0
	DORDOGNE	93,046	24	51.21%	11	48.31%	13	0.48%	0
	FINISTERE	163,890	26	48.48%	15	50.11%	11	1.00%	0
	ISERE	155,526	29	49.03%	15	49.93%	14	0.83%	0
	OISE	116,975	21	50.04%	10	47.88%	11	2.07%	0
	PYRENEES ORIENTALES	66,127	16	47.17%	5	46.86%	10	4.19%	0
	SARTHE	88,796	21	49.69%	8	49.23%	13	1.08%	0
	SAVOIE	52,534	18	49.78%	7	47.73%	11	2.22%	0
	SEINE MARITIME	195,827	35	52.27%	14	46.46%	21	1.19%	0
		SOMME	100,203	23	50.39%	10	48.53%	13	1.08%
	ESSONNE	143,794	21	50.54%	6	48.34%	15	0.48%	0
1985	COTES D'ARMOR	160,237	27	49.48%	14	49.54%	13	0.98%	0
	HERAULT	170,314	23	48.37%	14	48.75%	9	2.78%	0
	PUY DE DOME	152,349	30	46.35%	16	50.82%	14	2.84%	0
	TERRITOIRE DE BELFORT	30,315	8	49.09%	5	50.91%	3	0.00%	0
	SEINE SAINT-DENIS	189,631	20	49.90%	11	50.07%	9	0.03%	0
		VAL DE MARNE	251,690	30	46.61%	17	47.08%	13	5.12%

C : Councillors.

Table 8 provides the overall distribution of the votes and the councillors between the three political forces in the departments suffering from the reversed majority. For simplification purposes, we also do not treat here, the political parties other than the right, the left and the extreme right. The political scene of Marne (2004) is an example showing a paradoxical situation according to this approach. The moderate right enjoys a majority of 13 seats

out of 22 against 9 for the left having obtained only 43.65% of the votes against 47.38% for the left. As for the extreme right, it did not succeed in transferring the 8.68% from the votes which it collected to a seat.

The comparison with former methods provides some different results. Firstly, the paradox emerges in certain departments while it disappears elsewhere, the departments of Yonne (2004) and Loiret (2004) are proofs. Indeed, the former is affected by the referendum paradox according to the third approach and not the second. Therefore, Loiret suffered from the paradox according to the second and not the third approach. Secondly, the number of paradoxical cases slightly fell. On annual average, the paradox emerges in all 93 departments, 9.14 times. Moreover, we count on the whole 64 cases instead of 68, with 45 departments concerned with the phenomenon. Consequently, the frequency of the paradox is 9.83% with 48.39% of the sample suffering, at least only once, from the paradox.

Table 9: The result of the renewable cantons in the department of Vaucluse in 2001.

Cantons	Vote cast	Left	Right	EXR	S	Others	E	R
2 AVIGNON-NORD	11040	39.53%	60.47%	0.00%	60.47%	0.00%	R	2
6 BOLLENE	11226	65.80%	34.20%	0.00%	34.20%	0.00%	L	2
10 CARPENTRAS-SUD	14899	45.26%	54.74%	0.00%	54.74%	0.00%	R	2
11 CAVAILLON	14043	45.18%	54.82%	0.00%	54.82%	0.00%	R	2
12 GORDES	3501	39.85%	51.21%	6.11%	57.33%	2.83%	R	1
17 ORANGE-OUEST	9623	0.00%	50.02%	49.98%	100.00%	0.00%	R	2
18 PERNES-LES-FONTAINES	6854	43.14%	56.86%	0.00%	56.86%	0.00%	R	2
19 PERTUIS	20268	71.62%	16.45%	11.93%	28.38%	0.00%	L	1
20 SAULT	1529	69.52%	26.23%	4.25%	30.48%	0.00%	L	1
21 VAISON-LA-ROMAINE	6591	70.76%	18.04%	11.20%	29.24%	0.00%	L	1
22 VALREAS	4904	42.15%	57.85%	0.00%	57.85%	0.00%	R	2
24 AVIGNON-OUEST	8614	36.45%	63.55%	0.00%	63.55%	0.00%	R	2
Total	113092	48.31%	44.31%	7.29%	51.60%	0.09%	-	-

R : decisive round.

4.4 Fourth approach : the separation of the two rounds

Let us treat another example (see table 9), Vaucluse (2001): the left mobilized 48.31% of the votes for only 4 seats against 8 seats for the right, obtained with a percentage of 44.31% of the votes. If one includes the extreme right, which obtained 7.29% of the votes, the phenomenon of the reversed majority artificially disappears. However, a meticulous reading of the results of the

two rounds separately casts a doubt about the existence of the paradox. In fact, the left obtained 67.85% of the votes and 3 seats in the four cantons that were declared after the first round. For the 8 cantons that needed a second round, the left camp is swept aside by the right parties with only 40,64% of the votes and 1 seat. This result shows that the paradox is absent in each round, even if the score of the extreme right (respectively 10.77% and 5.92% in the first and second rounds) is computed separately for the calculation of the paradox. On the other hand, the result of adding the two rounds revealed the paradox. Can one talk legitimately about the relevance of a paradox when it comes from the aggregation of data of two different rounds when no round shows it? Another legitimate reaction relates to the change of the electoral behaviors which acts especially on the variation of the turnout, and on the electoral behavior of partisans of the candidates that were eliminated after the first round. These facts lead us to remake all calculations by separating the two rounds in order to refine our analysis. The results are exposed in tables 10 and 11.

Table 10: Departments where the paradox happens for seats called at the first round.

Year	Department	Votes	C	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	AIN	20,851	5	45.30%	2	40.61%	3	14.09%	0
	ARDECHE	9,997	5	47.58%	2	45.65%	3	6.50%	0
	CHARENTE MARITIME	32,432	6	45.24%	2	44.70%	4	10.06%	0
	GERS	8,660	3	47.49%	1	43.43%	2	6.54%	0
	ILLE ET VILAINE	43,628	7	52.76%	3	37.54%	4	5.78%	0
	LOIRET	10,890	3	40.73%	1	39.62%	2	19.65%	0
	MAINE ET LOIRE	42,465	6	48.54%	2	36.23%	4	9.15%	0
	DEUX SEVRES	32,557	7	51.05%	3	41.16%	4	7.79%	0
2001	ALPES DE HAUTE PROVENCE	8,542	5	47.60%	3	48.35%	2	4.05%	0
	CHARENTE MARITIME	46,888	9	49.63%	4	42.41%	5	7.96%	0
	EURE	45,715	8	53.61%	3	31.01%	5	14.24%	0
	FINISTERE	52,307	8	51.85%	3	42.99%	5	4.05%	0
	GIRONDE	72,399	11	51.20%	5	36.20%	6	10.56%	0
	NORD	133,595	11	45.88%	4	44.27%	7	9.34%	0
	SEINE MARITIME	110,284	15	47.98%	5	42.19%	10	9.42%	0
1998	ALLIER	19,450	5	51.57%	2	33.13%	3	9.13%	0
	CHARENTE MARITIME	34,612	7	44.35%	3	42.41%	4	10.40%	0
	DORDOGNE	11,772	5	56.17%	2	38.91%	3	4.92%	0
	INDRE ET LOIRE	14,498	4	50.19%	1	39.52%	3	9.02%	0
	LOT ET GARONNE	9,137	3	50.04%	1	32.33%	2	11.75%	0
	NIEVRE	14,981	6	54.54%	2	33.64%	4	10.35%	0
1994	CREUSE	7,654	4	47.97%	1	47.03%	3	3.97%	0
	GERS	13,723	5	47.58%	2	47.08%	3	4.78%	0

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... table 10 continued

	GIRONDE	46,470	7	49.62%	2	38.79%	5	6.77%	0
1992	DORDOGNE	29,291	9	48.42%	4	44.67%	5	4.84%	0
1988	AISNE	44,368	11	55.45%	5	39.55%	6	3.84%	0
	ALLIER	11,216	3	54.22%	1	44.63%	2	1.15%	0
	CHARENTE	32,782	9	50.89%	4	49.11%	5	0.00%	0
	COTE D'OR	18,333	8	50.06%	3	47.57%	5	2.37%	0
1988	MEURTHE ET MOSELLE	12,576	3	48.45%	1	45.11%	2	6.44%	0
	HAUTE SAONE	35,666	12	50.09%	5	48.17%	7	1.74%	0
	SAVOIE	31,442	11	53.57%	5	42.27%	6	3.70%	0
1985	GERS	12,141	5	52.48%	2	47.52%	3	0.00%	0
	NIEVRE	16,285	5	49.28%	1	49.11%	4	1.61%	0

C : Councillors.

Table 11: Departments where the paradox occurs for seat called at the second round.

Year	Department	Vote	C	Left		Right		EXR	
				% Vote	C	% Vote	c	% Vote	C
2004	AVEYRON	54,367	14	50.07%	6	49.93%	8	0.00%	0
	CANTAL	10,378	3	45.81%	2	54.19%	1	0.00%	0
	LOIRET	107,869	15	41.92%	5	41.35%	10	8.82%	0
	MAINE ET LOIRE	108,908	13	47.15%	8	52.85%	5	0.00%	0
	VOSGES	62,019	8	41.99%	5	48.92%	3	9.09%	0
	YONNE	69,623	18	47.58%	7	44.51%	11	7.60%	0
2001	ARDENNES	48,123	14	54.33%	6	45.67%	8	0.00%	0
	BOUCHES DU RHONE	245,176	21	51.94%	10	45.21%	11	2.84%	0
	CHARENTE	59,614	12	51.95%	5	48.05%	7	0.00%	0
	CORREZE	48,717	13	53.64%	6	46.36%	7	0.00%	0
	GARD	140,061	20	49.26%	9	44.19%	11	6.55%	0
	LOIR ET CHER	40,818	8	41.28%	4	48.11%	0	2.61%	0
	MAINE ET LOIRE	56,865	9	48.83%	5	51.17%	4	0.00%	0
	MARNE	40,212	8	42.53%	5	55.24%	3	2.23%	0
	NORD	290,989	28	50.34%	13	45.01%	15	4.65%	0
	OISE	125,507	19	44.39%	11	53.37%	8	2.24%	0
	SEINE MARITIME	140,163	20	52.49%	9	47.51%	11	0.00%	0
TARN	59,543	13	51.71%	4	48.29%	9	0.00%	0	
1998	CHER	58,701	17	52.36%	7	43.12%	10	0.00%	0
	EURE	69,778	14	43.45%	8	44.45%	6	10.12%	0
	FINISTERE	140,788	20	48.31%	14	51.69%	6	0.00%	0
	ILLE ET VILAINE	120,552	20	47.37%	13	50.38%	7	0.00%	0
	INDRE	30,574	7	53.23%	3	46.77%	4	0.00%	0
	MAINE ET LOIRE	76,606	12	46.32%	7	53.68%	5	0.00%	0
	MEURTHE ET MOSELLE	110,430	23	51.96%	10	45.92%	13	1.61%	0
	SEINE MARITIME	159,181	25	56.26%	12	41.36%	13	2.37%	0

...Continued on next page...

... table 11 continued

	SEINE ET MARNE	173,953	22	46.15%	10	44.50%	12	9.35%	0
	TARN ET GARONNE	36,864	11	52.51%	4	45.32%	7	2.17%	0
	VOSGES	73,148	13	47.27%	7	50.78%	6	1.95%	0
	YONNE	42,700	14	45.52%	4	42.11%	9	8.15%	0
1994	AISNE	67,574	12	50.44%	4	49.56%	8	0.00%	0
	ALLIER	59,860	13	48.38%	8	51.62%	5	0.00%	0
	ALPES DE HAUTE PROVENCE	19,956	9	55.86%	4	44.14%	5	0.00%	0
	ARDECHE	52,323	12	49.31%	7	50.69%	5	0.00%	0
1994	CHARENTE MARITIME	77,736	18	47.43%	10	52.57%	8	0.00%	0
	DOUBS	73,976	13	49.17%	7	50.83%	6	0.00%	0
	JURA	56,537	15	40.77%	8	51.50%	6	0.00%	0
	MEURTHE ET MOSELLE	129,529	20	44.20%	12	55.80%	8	0.00%	0
	OISE	122,963	16	45.47%	9	47.69%	7	6.84%	0
	VIENNE	47,840	10	49.27%	8	50.73%	2	0.00%	0
1992	HERAULT	176,548	24	43.37%	14	45.34%	10	11.29%	0
	LOIR ET CHER	37,857	8	49.11%	5	50.89%	3	0.00%	0
	HAUTE SAONE	40,334	11	48.16%	4	47.96%	7	1.27%	0
	SEINE MARITIME	173,340	25	50.45%	11	47.19%	14	1.21%	0
	VAL DE MARNE	137,299	20	41.03%	13	51.37%	7	7.59%	0
1988	ARDENNES	35,897	13	53.45%	6	46.55%	7	0.00%	0
	AVEYRON	11,995	4	49.24%	3	50.76%	1	0.00%	0
	GERS	26,767	8	45.65%	5	54.35%	3	0.00%	0
	LOIRE	91,604	15	50.71%	7	49.29%	8	0.00%	0
	MANCHE	47,220	11	48.48%	4	45.82%	7	0.00%	0
	MEURTHE ET MOSELLE	92,137	17	49.41%	9	50.59%	8	0.00%	0
	MEUSE	20,146	7	49.43%	2	47.28%	5	0.00%	0
	NIEVRE	27,090	10	59.93%	4	40.07%	6	0.00%	0
	SAONE ET LOIRE	67,849	15	49.50%	10	50.50%	5	0.00%	0
	ESSONNE	123,418	18	52.84%	6	47.16%	12	0.00%	0
1995	BOUCHES DU RHONE	277,353	21	42.90%	11	44.11%	9	12.99%	1
	CORREZE	28,863	7	49.01%	4	50.99%	3	0.00%	0
	EURE ET LOIR	65,738	10	48.87%	6	51.13%	4	0.00%	0
	HERAULT	167,100	21	47.90%	12	49.37%	9	2.73%	0
	SEINE MARITIME	149,386	21	53.20%	10	46.80%	11	0.00%	0
	TERRITOIRE DE BELFORT	30,315	8	49.09%	5	50.91%	3	0.00%	0
	SEINE SAINT-DENIS	189,631	20	49.90%	11	50.07%	9	0.03%	0
	VAL DE MARNE	144,917	18	49.40%	11	50.60%	7	0.00%	0

C : Councillors.

As usual, the occurrence of the reversed majority change from one year to another. Moreover, there are departments which have the phenomenon only with the first round like Ain (2004), or only with the second round, Aveyron (2004). If the paradox appears in the first round and also in the

second round, it will not appear inevitably in the two rounds together (see Meurthe-et-Moselle in 1988).

By adopting this fourth approach, the number of paradoxical situations decreases, since we enumerate only 34 and 63 cases respectively in the first and the second round. Consequently, the frequency of the paradox is 5.22% in the first round and 9.68% in the second. However, the number of the departments having the paradox, at least once at the second round, increased to 50%.

After the presentation and the analysis of these four approaches, it appears more relevant to bring together the results of these methods. Also, it appears more judicious to make a first comparison with the theoretical model. Thus, we present in the two following sections a comparative assessment.

5 Discussion

5.1 An analytical comparison of the approaches

The comparative reading of the results with the four ways of measuring the referendum paradox delivers the following remarks. First, according to the all four measurement, we only obtain 7 departments which present robust paradoxical cases: Charente (2001), Oise (2001), Cher(2001), Meurthe-et-Moselle (1998), Aisne (1994), Allier (1994), Charente-Maritime (1994).

Second, whatever the adopted approach, the data confirm the presence of the reversed majority, but the frequency varies. The probability is lower with the fourth approach when the first round is analyzed separately. On the other hand, the highest frequency occurs with the second approach, but this measure does take into account the three way races. Thus, results obtained with the third and fourth approaches, which are very similar, seem more reliable. Moreover, notice that the highest percentage of the departments affected by the paradox, at least once, occurs with the second round according to the fourth method (46/93). In fact, we concentrate here on the most competitive seats, with less influence of the popular incumbents.

According to the four methods and out of the 93 departments studied for the period, we counted 72 departments where the paradox appeared, at least one year with at least one method, i.e. 77.42% of the departments. In the 21 departments free from the phenomenon, the majority is ensured in terms of seats and vote for one camp each year. These departments all are, indeed, always favorable to a political camp except for the Loire Atlantique (see table 12).

Other departments (23 departments) always favor a political family in term of seats but suffer from the paradox (see table 13). The paradox in these departments can be explained by the simple fact that a high tide in terms of vote for one camp is counterbalanced by the resistance of the other

Table 12 The 21 departments where the paradox is absent

departments	L, R or S	departments	L, R or S
6 ALPES MARITIMES	R	61 ORNE	R
9 ARIEGE	L	62 PAS DE CALAIS	L
10 AUBE	R	67 BAS RHIN	R
11 AUDE	L	68 HAUT RHIN	R
43 HAUTE LOIRE	R	74 HAUTE SAVOIE	R
44 LOIRE ATLANTIQUE	S	78 YVELINES	R
48 LOZERE	R	83 VAR	R
52 HAUTE MARNE	R	85 VENDEE	R
53 MAYENNE	R	87 HAUTE VIENNE	L
56 MORBIHAN	R	92 HAUTS DE SEINE	R
57 MOSELLE	R		

L, R or S : always favorable to a camp (left or right) or swing

Table 13 The 23 departments which are always favorable to a camp in terms of seats, and where the paradox appears.

departments	Favorable	departments	Favorable
1 AIN	R	41 LOIR ET CHER	R
8 ARDENNES	R	45 LOIRET	R
12 AVEYRON	R	46 LOT	L
13 BOUCHES DU RHONE	L	50 MANCHE	R
14 CALVADOS	R	51 MARNE	R
15 CANTAL	R	55 MEUSE	R
21 COTE D'OR	R	65 HAUTES PYRENEES	L
30 GARD	L	72 SARTHE	R
31 HAUTE GARONNE	L	73 SAVOIE	R
34 HERAULT	L	86 VIENNE	R
39 JURA	R	89 YONNE	R
40 LANDES	L		

camp in term of seats. For example, the department of Calvados in 2004 had a tilt to the left in terms of vote with a resistance on the right in terms of seats. These figures reflect the traditional dominance of the right candidates in many rural areas.

From tables 12 and 13, 11 departments always give the preference to the left in terms of seats including only 4 not affected the paradox. However, the number of the departments always supporting the right is more important, it quantifies 32 including 16 which do not encounter the phenomenon of the reversed majority.

5.2 Comparison with the theoretical model

Feix *et al.* (2004, 2009) have, in recent papers, estimated the likelihood of the referendum paradox with a priori models. It is then tempting to make a first (and crude) comparison between their theoretical conclusions and our empirical findings. Let us first recall their main findings. When all the

jurisdiction have the same population size, Feix *et al.* (2004) found that the probability of the referendum paradox quickly tends towards limit values of 16.5% under the IC assumption and of 20.5% under the IAC assumption. Using computer simulations, Feix *et al.* (2009) tackled the problem when jurisdictions have different populations. For each federation of N jurisdictions, they draw randomly the populations of each jurisdiction, and then simulate 100,000 elections under IC and IAC in order to estimate the probability of the referendum paradox. Indeed, under IC, each voter flips a fair coin to decide her vote between two parties, A and B . Thus for district i of population n_i , they simulate the results by drawing a random variable ϵ_i from a gaussian distribution with mean $n_i/2$ and standard deviation $\sqrt{n_i}/2$. Then, due to the law of large numbers, all the elections tend to be close. On the contrary, the IAC model allows some district to show a clear tendencies. For a district of population n_i , the number of votes for A is drawn from a uniform distribution on $[0, 1]$ multiplied by n_i . For more details on the models and simulations, see Feix *et al.* (2004, 2009).

Figures 1 and 2 display the average probabilities for the lax, medium and strong version of the paradox, with 1,000 federations which population were drawn randomly from the uniform simplex. The medium definition of the paradox considers that, in case of a tie in term of seats, the winner is selected by flipping a fair coin. This central scenario is less affected than the others by fluctuations of the probabilities between odd and even numbers of jurisdictions and we will consider it as a benchmark. Then, we observe that the average probability of the referendum paradox quickly stabilizes around 25% for the IC model, and slightly below 30% for the IAC case.

As the objective of the empirical analysis was also to derive an average measure of the referendum paradox over federations of different sizes (here, a department is a federations of cantons), we can compare the results of both approaches. Taking into account only the previously obtained results, we note that the differences between the theoretical probabilities and the probabilities observed diverge significantly. The theoretical probabilities, in addition to their nonagreement, are higher. Obviously, the (IC) and (IAC) models describe only extremely tied elections. It is clear for the IC in the sense that each voter chooses to vote with the same probability for one of the two political forces. But, as the IAC assumption supposes that the votes are independent between the jurisdictions, by the law of the great numbers, the total number of the votes cast in favour of a party at the aggregated level will also becomes increasingly gathered around 50% when the number of jurisdictions increases.

Thus, we must be focused on the tightened elections to compare the theoretical and empirical results, and drop from the database the departments whose the inclination for a camp is marked. Examining the results of the can-

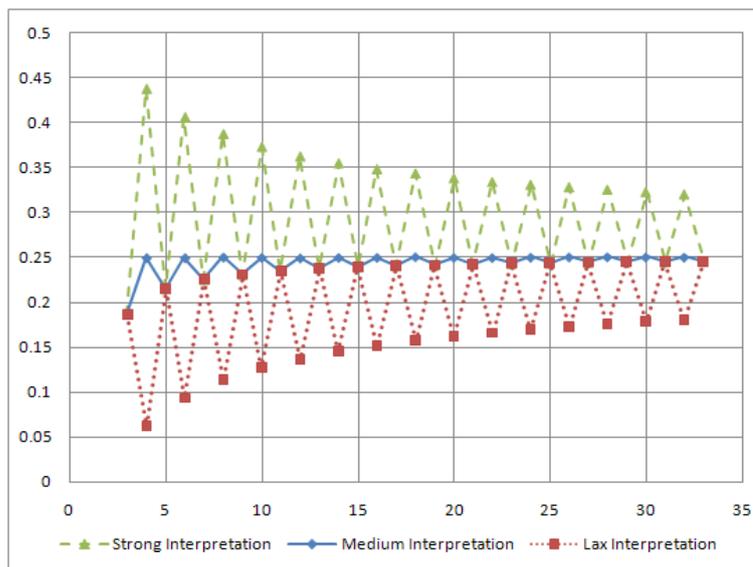


Fig. 1 The average probability of the referendum paradox as a function of N for the lax, medium, and strong definitions, under IC assumption (from Feix et al. (2009)).

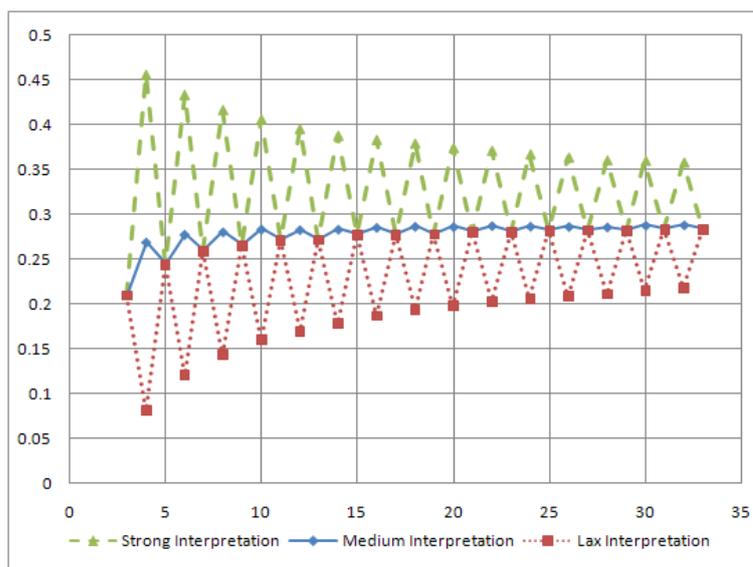


Fig. 2 The average probability of the referendum paradox as a function of N for the lax, medium, and strong definitions, under IAC assumption (from Feix et al. (2009)).

tonal elections with more detail, we can take into account two issues to make a more robust comparison about the frequency of the paradox:

1. Some departments that always favour the same political force should be removed from the database. We consider departments that swung at least once.
2. A tightened election, when the margin between the two political coalitions is less than 10%, is a better base to compute the frequency.

Table 14 summarizes our results for the four methods of measurement, and focusses on the departments where the elections, according to the above mentioned criteria, were close. Each row displays three values: the first corresponds to the weak version of the definition of the paradox, the second corresponds to the medium version -it is the principal value of the comparison- and the last one is the frequency calculated according to the strong version.

Table 14 The frequency of the paradox according to the four methods

	No	Method 1	Method 2	Method 3	Method 4
Raw frequency	651	$\frac{6,91\%}{8,60\%}$ $\frac{10,29\%}{10,29\%}$	$\frac{10,45\%}{12,14\%}$ $\frac{13,82\%}{13,82\%}$	$\frac{9,83\%}{11,52\%}$ $\frac{13,21\%}{13,21\%}$	$\frac{9,68\%}{13,52\%}$ $\frac{17,36\%}{17,36\%}$
The 73 swing departments (in seats and votes)	511	$\frac{8,81\%}{10,96\%}$ $\frac{13,11\%}{13,11\%}$	$\frac{13,31\%}{15,46\%}$ $\frac{17,61\%}{17,61\%}$	$\frac{12,52\%}{14,68\%}$ $\frac{16,83\%}{16,83\%}$	$\frac{12,33\%}{17,22\%}$ $\frac{22,11\%}{22,11\%}$
The 50 swing departments (in seats)	350	$\frac{9,43\%}{12,14\%}$ $\frac{14,86\%}{14,86\%}$	$\frac{16,29\%}{19,00\%}$ $\frac{21,71\%}{21,71\%}$	$\frac{15,14\%}{17,86\%}$ $\frac{20,57\%}{20,57\%}$	$\frac{12,29\%}{16,57\%}$ $\frac{20,86\%}{20,86\%}$
Tightened elections	312*	$\frac{18,67\%}{23,24\%}$ $\frac{27,80\%}{27,80\%}$	$\frac{25,68\%}{28,83\%}$ $\frac{31,98\%}{31,98\%}$	$\frac{24,49\%}{28,37\%}$ $\frac{32,24\%}{32,24\%}$	$\frac{17,95\%}{24,20\%}$ $\frac{30,45\%}{30,45\%}$
Tightened elections and the 73 swing departments	290†	$\frac{19,48\%}{24,03\%}$ $\frac{28,57\%}{28,57\%}$	$\frac{25,79\%}{28,73\%}$ $\frac{31,67\%}{31,67\%}$	$\frac{25,00\%}{28,75\%}$ $\frac{32,50\%}{32,50\%}$	$\frac{19,31\%}{25,00\%}$ $\frac{30,69\%}{30,69\%}$
Tightened elections and the 50 swing departments	215‡	$\frac{22,54\%}{29,48\%}$ $\frac{36,42\%}{36,42\%}$	$\frac{27,84\%}{31,25\%}$ $\frac{34,66\%}{34,66\%}$	$\frac{26,06\%}{30,59\%}$ $\frac{35,11\%}{35,11\%}$	$\frac{17,67\%}{23,26\%}$ $\frac{28,84\%}{28,84\%}$
Limit IC	-	25%	25%	25%	25%
Limit IAC	-	28%	28%	28%	28%

No: The number of observations according to the fourth method

*: The number of observations according to the first, second and third method are respectively: 241, 222 and 245

†: The number of observations according to the first, second and third method are respectively: 231, 221 and 240

‡: The number of observations according to the first, second and third method are respectively: 173, 276 and 188

Even if we take into account the cases of ties, the first line of table 14 shows that occurrence of the paradox remains rather weak when we consider all the departments and all the elections.

We exclude, thereafter, the departments which always support a political coalition in terms of seats and votes. Then there are 73 departments, among the 93, which swing. The result of calculation shows that the frequency

of the paradox, according to the four methods, oscillates between 10,96% and 17,22% according to medium version, and between 13,11% and 22,11% according to strong version (see table 14). On the other hand, when we remove from the database only the departments which always support a political force in terms of seats, one has 350 elections to study (50×7 elections). Consequently, the frequency increases, it is balanced between 12,14% and 19% (according to the median version). However, we are still below the values proposed by *a priori* models.

Secondly, we focused on the assumption according to which the margin between left and right votes does not exceed a certain percentage of vote. Thus, we exclude from the data base the elections which were won with more than ten points at the department level. The interest is to consider only the tightened elections between the left and right parties. At last, we will combine this criteria with the previous one, i.e. we take into account only the tightened elections which took place in the departments that can swing (initially the 73 departments that swing in terms of votes, then the 50 departments that swing in terms of seats and votes).

The results are astounding: the occurrence of the paradox is higher and almost match perfectly the theoretical result. The best fit is probably obtained with the fourth method (second round) and the IC assumption. This could be explained by the fact that the more competitive elections take place at the second round, and then could be well modeled by the IC assumption. On the other hand, the results for methods 2 and 3 seem closer to the results of the IAC simulations. This should not come as a surprise, as the IAC assumption models more homogeneous voting behavior while the two methods take into account less competitive elections from the first round.

However, the choice of the vote margin 10% is arbitrary. Thus, some results with others values of the vote margin can be studied. Then, the previous conclusions must be softened. Only the result corresponding to the one calculated in the 50 departments that swing in terms of seats and votes according to the median definition of the paradox is presented in table 15. The results underline the fact that when the vote margin differ from 10%, the agreement between the empirical and theoretical results disappears. Notice first, that maximal values are obtained for small vote margin (e.g. 73.33% at 1% margin for method 1, 35.21% at 2% margin for method 4). The frequency decreases, for the four methods, as the vote margin increases, and it tends to values that can be half of the theoretical values. At the level of 20% margin all the cases of paradox are taken into account. The theoretical models are good predictor of the order of the magnitudes of the referendum paradoxes, but need to be refined if we wish to come closer to real values. Incidentally, notice that Feix *et al.* (2004, 2009) did not estimate the referendum paradox frequency as function to the variation of the margin of the vote cast in favour of the two political parties.

Table 15 The frequency of the paradox as a function of the vote margin according to the four methods

The vote margin	Method 1	Method 2	Method 3	Method 4
0.01	73.33%	51.79%	60.42%	23.08%
0.02	55.26%	47.54%	46.00%	35.21%
0.03	45.71%	48.08%	48.48%	32.32%
0.04	38.59%	41.20%	42.94%	31.79%
0.05	37.02%	40.48%	40.45%	29.81%
0.06	33.75%	36.55%	40.00%	27.61%
0.07	29.93%	34.71%	36.55%	27.27%
0.08	28.08%	33.06%	34.59%	25.95%
0.09	26.58%	30.14%	32.29%	25.09%
cdef0.10	25.15%	28.83%	30.59%	24.20%
0.11	23.48%	26.95%	29.19%	23.72%
0.12	22.14%	25.84%	28.07%	23.09%
0.13	20.94%	24.31%	26.77%	22.19%
0.14	19.59%	24.02%	25.74%	21.48%
0.15	18.72%	23.45%	25.20%	20.83%
0.16	18.16%	21.70%	24.60%	20.12%
0.17	17.07%	20.57%	23.40%	19.00%
0.18	16.04%	19.76%	22.63%	18.38%
0.19	15.51%	19.07%	21.91%	17.95%
0.20	15.12%	18.61%	21.60%	17.83%
Limit IC	25%	25%	25%	25%
Limit IAC	28%	28%	28%	28%

Broadly, the theoretical and observed probabilities are closer if one eliminates from the database the departments which always give preference to a camps in terms of votes and seats, and only if the tightened elections are considered. These results from the French “cantonal” elections show that the models used by Feix *et al.* (2004, 2009) can be a reasonable approach to estimate the probabilities of paradox when they are correctly used. They suggest that the same methodology can be applied to other voting situations, as long as we can confidently recreate a one round two party competition from the data.

6 Conclusion

Typically, the Social Choice theory focusses on the existence of paradoxical situations, while seldom seeking if such phenomena occur in reality. Our first objective was to find a sufficiently rich electoral database, and a simple paradox which could be clearly identifiable. From our point of view, and in spite of the difficulties of defining precisely the referendum paradox in an election in two rounds gathering more than two parties, this study ended with positive results. According to the paradox measurement method, the frequency oscillates between 6% and 11%, with 40% to 50% of the departments affected.

These results could also be compared with the theoretical study of Feix *et al.* (2004, 2009), which sought the frequency of the paradox with *a priori* models of voting. All in all, the theoretical probabilities and the observed probabilities are closer if one excludes from the database the departments which always give the preference to the same camps in terms of votes and seats, and if one considers only the departments where the margin of vote between the two major coalitions does not exceed 10%.

Though our results clearly identify paradoxical situations, they are silent on their possible causes: the existence of a strong third party, the three way races, the level of the turnout, the dispersion of the unequal population according to cantons, the impact of the drawing of electoral boundaries, or the existence of the two rounds instead of one could be possible sources for the paradoxes. The question of finding a methodology to measure the impact as of these various elements then arises. A path could be to adapt to the case of the two rounds majority system, the methods developed by Anglo-saxon authors (Broockes (1957, 1960), Grofman *et al.* (1997), Johnston *et al.* (1999), Blau (2004)) for the study of the paradox in the United States, New Zealandy and in England (one round system). In particular these authors manage to distinguish three main partisan bias: turnout rate differences, impact of the size differences of population across constituencies (malapportionment) and impact of the drawing of electoral boundaries (gerrymandering), to which are added sometimes the question of the influence of a third block.

Finally, the rather high occurrence of the paradox can cause doubt about the electoral system currently in force for French cantonal elections. Therefore, it seems to us that these results plead for change in the way the councillors are elected. The adoption of a system based on the proportional representation of the political tendencies to the level of the department makes possible to eliminate the multiple factors which disturb the election. Another solution, as for the American presidential elections, would be to attach to each canton a number of mandates proportional to its population and seek for the best apportionment rule.

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Appendix

Table 16: Departments where the result is deadlocked.

Year	Department	Seats at stake	Left	Right	Others
2004	1 AIN	22	11	11	0
	25 DOUBS	18	9	9	0
	57 MOSELLE	26	13	13	0
	60 OISE	20	10	10	0
1998	28 EURE ET LOIR	14	7	7	0
	70 HAUTE SAONE	16	8	8	0
	79 DEUX SEVRES	16	8	8	0
	82 TARN ET GARONNE	14	7	7	0
1994	16 CHARENTE	18	9	9	0
	39 JURA	17	8	8	0
	70 HAUTE SAONE	16	8	8	0
	71 SAONE ET LOIRE	30	15	15	0
1992	4 ALPES DE HAUTE PROVENCE	16	8	8	0
	24 DORDOGNE	26	13	13	0
	84 VAUCLUSE	12	6	6	0
	93 SEINE SAINT-DENIS	20	10	10	0
1988	4 ALPES DE HAUTE PROVENCE	14	7	7	0
	54 MEURTHE ET MOSELLE	20	10	10	0
	58 NIEVRE	16	8	8	0
	84 VAUCLUSE	12	6	6	0
1985	28 EURE ET LOIR	14	7	7	0
	32 GERS	16	8	8	0

Table 17: Departments where the result is deadlocked for seats called at the second round.

Year	Department	Vote	C	Left		Right		EXR	
				% Vote	C	% Vote	C	% Vote	C
2004	23 CREUSE	10471	4	52.78%	2	47.22%	2	0.00%	0
	36 INDRE	34500	6	52.29%	3	36.46%	3	0.00%	0
	51 MARNE	84044	14	51.47%	7	42.97%	7	5.56%	0
	60 OISE	124863	16	43.02%	8	41.95%	8	15.04%	0
2001	15 CANTAL	14302	4	53.63%	2	46.37%	2	0.00%	0
	17 CHARENTE MARITIME	73102	16	46.77%	8	51.51%	8	0.00%	0
	42 LOIRE	114252	16	48.14%	8	51.86%	8	0.00%	0
	48 LOZERE	14950	6	46.05%	3	53.95%	3	0.00%	0
	56 MORBIHAN	95515	14	46.02%	7	53.98%	7	0.00%	0

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... table 17 continued

2001	62 PAS DE CALAIS	143674	22	53.81%	11	45.23%	11	0.00%	0
	77 SEINE ET MARNE	135215	16	47.89%	8	52.11%	8	0.00%	0
	80 SOMME	70528	14	44.00%	6	40.27%	6	0.00%	0
	94 VAL DE MARNE	131071	22	48.92%	11	51.08%	11	0.00%	0
1998	16 CHARENTE	65831	16	47.73%	8	52.27%	8	0.00%	0
	19 CORREZE	41696	12	47.37%	6	52.63%	6	0.00%	0
	41 LOIR ET CHER	48985	12	41.95%	6	55.81%	6	2.24%	0
	43 HAUTE LOIRE	32429	10	43.57%	5	56.43%	5	0.00%	0
	69 RHONE	185184	22	46.70%	11	34.93%	11	18.09%	0
	70 HAUTE SAONE	33477	10	52.79%	5	43.32%	5	3.89%	0
1994	12 AVEYRON	32728	8	50.48%	4	49.52%	4	0.00%	0
	18 CHER	54706	14	44.68%	7	55.32%	7	0.00%	0
	27 EURE	75428	14	45.38%	7	49.33%	7	5.30%	0
	35 ILLE ET VILAINE	128755	20	47.21%	10	50.76%	10	0.00%	0
	37 INDRE ET LOIRE	62465	11	47.74%	5	48.87%	5	0.00%	0
	72 SARTHE	73324	14	49.76%	7	50.24%	7	0.00%	0
	80 SOMME	85748	16	48.08%	8	50.99%	8	0.93%	0
	81 TARN	59141	12	49.20%	6	50.80%	6	0.00%	0
	1992	11 AUDE	53399	10	58.79%	5	41.21%	5	0.00%
40 LANDES		50612	8	52.35%	4	47.65%	4	0.00%	0
46 LOT		39415	12	48.09%	6	51.91%	6	0.00%	0
48 LOZERE		4727	4	53.86%	2	46.14%	2	0.00%	0
84 VAUCLUSE		96238	12	38.49%	6	40.86%	6	19.48%	0
93 SEINE SAINT-DENIS		178218	20	35.39%	10	40.96%	10	23.66%	0
1988	1 AIN	47490	12	45.97%	6	54.03%	6	0.00%	0
	2 AISNE	54245	10	56.95%	5	43.05%	5	0.00%	0
	3 ALLIER	58755	14	52.98%	7	47.02%	7	0.00%	0
	4 ALPES DE HAUTE PROVENCE	15049	8	45.37%	4	54.63%	4	0.00%	0
	7 ARDECHE	45375	10	47.31%	5	52.69%	5	0.00%	0
	16 CHARENTE	33255	8	53.31%	4	46.69%	4	0.00%	0
	41 LOIR ET CHER	25792	6	50.55%	3	49.45%	3	0.00%	0
	44 LOIRE ATLANTIQUE	69539	12	50.58%	6	49.42%	6	0.00%	0
	56 MORBIHAN	70421	10	47.99%	5	52.01%	5	0.00%	0
	65 HAUTES PYRENEES	33333	10	60.12%	5	39.88%	5	0.00%	0
	76 SEINE MARITIME	144576	24	55.23%	12	44.77%	12	0.00%	0
	79 DEUX SEVRES	32795	8	34.87%	4	65.13%	4	0.00%	0
	85 VENDEE	14300	2	51.09%	1	48.91%	1	0.00%	0
	86 VIENNE	49032	12	51.86%	6	48.14%	6	0.00%	0
1985	23 CREUSE	19690	6	49.14%	3	50.86%	3	0.00%	0
	33 GIRONDE	108765	14	48.39%	7	51.61%	7	0.00%	0
	45 LOIRET	56409	10	44.10%	5	55.90%	5	0.00%	0