Maximal Proportionality between Votes and Voting Power: The Case of the Council of the European Union

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Abstract

This article compares the qualified majority thresholds used in the European Union Council to maximally proportional thresholds, where the difference between votes and voting power is the smallest possible. As the author was not aware of an algorithmic solution to find maximally proportional thresholds i.e. the inverse problem of the power indices, the required measurements were carried out through massive power index computations. Surprisingly the qualified majority thresholds used throughout the Council history have been rather close to the most proportional thresholds. Some behavioural patterns regarding the applied classical power indices were also found with respect to the growing number of Member States in the Council.

1 Introduction

The research process for this article started with only one question in mind: Are there vote thresholds (quotas) in the European Union (EU) Council history, where the difference between Member States (MS) vote shares and voting power would be zero (when voting power is measured using the classical power indices)? In other words: are there solutions to the inverse problem of the power indices in the EU Council? The author is not aware of an algorithmic solution to the inverse problem, which means that the problem has to be approached 'the hard way' i.e. through massive index computation. The computation process, in turn, raised some additional research questions, to which the results were able to answer. Taken together, the main research questions are:

1) Are there quotas in the EU Council, which are perfect solution to the inverse problem?

2) How close were (and are) the Council qualified majorities (QMV) to such quotas, where proportionality between votes and voting power are maximal?

3) As 1), however, using square roots of MSs population figures instead of the allocated Council votes?

4) How sensitive is absolute voting power to vote manipulation in a voting body?

5) How close is voting power to votes in voting bodies with different number of voters?

We do not argue here, whether the maximal proportionality quota is desirable or not. Instead, we only want to find out, where this point exists among the proper quotas, and whether there are differences in this respect among the classical power indices i.e. the Shapley-Shubik (SSI) index, the standardized Banzhaf (sBz) index and the absolute Banzhaf (aBz) index.

Since the beginning of the EU in 1958 the allocation of the Council votes has been a more or less 'mystic' procedure of political negotiations shaded with secrecy. In 1958 the system with only six MSs had a rather clear intuition behind it: Germany, France and Italy had four votes each, and these votes together ensured a QMV of 12 (out of 17) votes, if the three MSs acted unanimously. The Benelux countries were also allocated votes: Belgium and the Netherlands two and Luxembourg one. The

MSs population figures reflected the votes poorly. The three large countries had relatively less voting power, than the two middle sized MSs, whose voting power was in turn somewhat boosted. The minor Luxembourg had one vote, however it was not a pivotal voter in any voting configuration. Worth stressing is that the QMV was set to 70.6 % of the votes. In this respect, when the first enlargement and vote reallocation took place in 1973, the votes were completely altered, however the QMV level remained close to 70-71%. This level of QMV actually remained until 2004.

The same rough idea and the level of quota have prevailed until today, however the simplicity of the EU 6 no longer exists. The last enlargement into EU 25 in year 2004 included the same mystic political negotiations with the old idea of maintaining the boost of the middle size MSs voting power. However, the quota level was slightly raised. However, dissatisfaction behind this type of long and difficult ad hoc negotiations without any real and clear systematic rationality has risen. The new constitution proposal takes another path by completely dismissing the Council votes and suggesting to simply use the MSs population figures instead. However, there is a problem if the population figures would be used as such. This would cause the large MSs significantly gaining voting power, while especially the middle sized MSs would suffer a loss.

Numerous scholarly articles have evaluated the nature between MSs population sizes, voting weights and voting power in the Council. Just to mention few studies in the topic, some of the main questions assessed in Felsenthal and Machover (1998; 1999; 2000; 2001; 2003), Hosli and Wolffenbuttel (2001), Laruelle and Widgrén (1998), Moberg (2002), Nurmi et al. (2001), Slomzynski and Zyczkowski (2004), Sutter (2000) and Winkler (1998) are: First, how proportional are the MSs' weighted votes to the MSs' population sizes? Second, how proportional are the MSs votes to the MSs voting power? And third, especially in the works of Felsenthal and Machover, what is the relation between votes, population and voting power? Not turning to this discussion further we note that a common feature among all the previous studies is that they do not investigate votes and quotas systematically, but only refer to one or few quotas. Moreover, the above studies suggest different (perhaps more fair) vote configurations, as our vote configurations below are used only to shed light to aBz index behaviour.

Research setting The Council is treated in two different ways in the analyses: First, the MSs in the Council are considered to be just a set of shareholders of a stock company. Accordingly, we can assume the voters (MSs) having just bought their amounts of shares (votes). Second, we analyse the Council as a two-tier system. In this case we take into account the MSs populations, but set aside the Council votes. This line of study appears since Felsenthal and Machover (2000, 15). The two previous ways relate to 1) and 2) of our research questions. Our tools for the measurement are three of the most common power indices. Two of them, the SSI and sBz indices are relative indices, while the aBz index measures absolute voting power.

To be more specific, the inverse problem can be described in the following way: We fix the power index values of the voters and the quota and determine which vote distribution would result in these power index values. The author is aware of only three articles, which even remotely address the inverse problem. The problem is discussed in Nurmi (1978) and a recipe to solve the inverse problem by randomizing the quota is outlined. Some years later Holler (1985) and Berg and Holler (1986) and provide a mathematical solution to the recipe outlined in Nurmi (1978). How this article relates to the previous discussion is that we do not to try to find an inverse problem solution algorithm, but only to approach the problem the 'hard way', i.e. using powerful power index calculation software. By analyzing a voting body completely, i.e. computing power indices for all voters for all proper quotas, we can empirically find out whether there exists a quota, which would be a solution to the inverse problem. If such a quota does not exist, we are in any case able find the quota(s) where the difference between votes and voting power is at minimum. We shall refer to these quotas as the best fit quotas. Due to the nature of the results in the respective sections, two further analyses are made: regarding to the aBz a sensitivity analysis to vote manipulation and regarding the sBz and SSI indices an effective number of votes (ENV) analysis. These analyses show us how the indices behave in certain conditions.

The analyses are applied to EU Councils from 1953 to present (EU 6, EU 9, EU 10, EU 12, EU 15 and EU 25 Member States), the largest 27 member future Council outlined in the Nice summit in 2000, and some hypothetical scenarios of the 15 member (1995-2004) EU Council[1].

The outline of the paper is as follows: Section 2 introduces the concept of voting power i.e. the power indices together with the best-fit measurement procedure. Section 3 analyses the Councils for the best-fit quotas and provides the aBz sensitivity and ENV analyses. Section 4 finally provides a summary of the findings together with a discussion.

[1]Note that for EU 25 the vote configuration used is not the only possibility.

2 Voting Power and the Best-fit Procedure

2.1 Voting Power

While we have many power indices[2], a common factor among them is that they use swings. A swing for a voter i occurs when i is able to *swing* the vote by withdrawing his/her votes from an otherwise winning set of voters. The differences lie in the ways the swings are manipulated. Before defining the power indices, we have to introduce some general properties. To start with, the set N of actors represents a voting body. The set W includes all winning subsets of N. A Winning subset is a set of voters, which can ensure acceptance of a randomly chosen proposal. A coalition (a set of voters) is denoted by S. If S is winning, it has the value of 1, otherwise it has the value of 0. In game theoretic terms, we have a zero-one normalized simple game (von Neumann and Morgenstern 1944). We denote the characteristic function of a coalition by v. The status of a coalition S is indicated by v in the following way: $v(S) = 1$ if S wins, otherwise $v(S) = 0$. The voters of N might have varying amounts of votes, in which case we say that the simple voting game is weighted. The corresponding weights of N with a vote threshold (quota) q are denoted by $[q; w_1, w_2, \ldots, w_n]$.

The aBz index a.k.a Penrose measure has been suggested twice in the literature. The idea goes back to Lionel Penrose (1946), was later suggested independently by John Banzhaf (1965) and modified to its "absolute" form by Dubey and Shapley (1979). The index values have a probabilistic interpretation. The index values add up to a constant, which rarely is unity.

In practice, the aBz index analyses every possible coalition (there are 2^n of them) for swings. Individual swings are added up and divided by a constant 2^{n-1}. The constant represents individual voters' appearances among the 2^n coalitions. Hence, the probability that the index expresses, is voter i's critical appearances divided by i's total number of appearances. Formally, we have

$$\beta_i = \frac{\sum_{S \subseteq N} \left[v(S) - v(S \setminus \{i\}) \right]}{2^{n-1}}.$$

The sBz index differs from the previous version in only one respect. The denominator is now the sum of every voters' every swing. By doing this we achieve a relative index, but there is a cost. The power index values *do not* have a sensible probabilistic interpretation any more (see Dubey and Shapley 1979; Widgrén 2001). The sBz index values express only relative shares of swings. Formally, the index is

[2] Pajala (2002b) currently lists 25 power indices and measures.

calculated as

$$\overline{\beta_i} = \frac{\sum_{S \subseteq N} \left[v\left(S\right) - v\left(S \setminus \{i\}\right) \right]}{\sum_{j \in N} \sum_{S \subseteq N} \left[v\left(S\right) - v\left(S \setminus \{j\}\right) \right]}.$$

(Banzhaf 1965; Penrose 1946.)

The SSI index is based on a game theoretic notion known as the Shapley Value (Shapley 1953; Shapley and Shubik 1954). In fact, the SSI index is a special case of the Shapley value. While the index has a probabilistic interpretation, it can also be seen as an expected utility measure. A further feature of the index is that it is a relative index; individual voters' index values always add up to unity.

In practice, the SSI index analyses every possible voter permutation (there are n! of them) for swings. Individual swings are added up and divided by the total number of swings, and the result is voters' relative shares of swings. Formally, the index is expressed as

$$\phi_i = \sum_{S \subseteq N} \frac{(s-1)!\,(n-s)!}{n!} \left[v\left(S\right) - v\left(S \setminus \{i\}\right) \right].$$

2.2 The Best-fit Procedure

This section introduces how we calculate the best-fit quotas. We simply measure differences between power index values and relative vote shares or square roots of MS's population figures. If this difference is zero for a voter i at a given quota, then i's voting power is proportional to i's vote share. If the differences are zero for all voters at a given quota, this quota represents maximal proportionality, and is a solution to the inverse problem.

In practice we start from the simple majority quota, compute the power index values for every voter, and measure the absolute difference between relative vote share and the respective power index value and sum up the differences. The same procedure is then repeated for every quota until unanimity. The quota for which the previous sum is the smallest is the best-fit quota. Formally, the most proportional quota Q can be obtained as

$$Q_{best-fit} = min \left(\sum_{q_{smaj}}^{q_{unan}} \sum_{i=1}^{n} \left| b_i - a_i \right| \right),$$

in which n can represent either a single voter, or a voter group (such as the big four in EU 15 Council).

3 Results and Analyses

When we calculate the power index values for all MS groups over all proper quotas and apply the measurement procedure introduced in the previous section, we are able to find out which quotas are the best-fits between votes and voting power. The evolution of the sum of differences is illustrated in Figure 1. Because of the figure scaling, aBz values only equal or lower than 0,3 are shown. The first finding is that the three indices do not agree on the best-fit quotas, since the lowest points of the curves are at different quotas. The sBz index best-fit quota is 201 (sum of differences is 0.00017), regarding aBz 222 (0.0085) and regarding SSI 275 (0.0028). The second finding is that the three curves cross each other at almost the same point, which happens to be very close to the best-fit of the aBz index. Actually, the same phenomenon takes place in every EU Council. The only exceptions are the early EU 6 and EU 9, where the three curves do not cross, but are very close to each other near the aBz best fit point. The difference curves have different shapes, as the index evolution curves above would suggest. The sBz curve starts very low, touches almost zero, but grows very steeply after the best-fit quota. The SSI curve starts similarly rather low, as sBz, however, roughly after quota of 253 gets jumpy. Note that at unanimity the two curves converge. The aBz curve differs considerably from the two previous ones. The curve starts very high (0.73), comes very steeply into the one very distinctive best-fit quota, and grows back at a lowering rate towards unanimity (reaching the level of 0.33), as can be seen from Figure 1.

[FIGURE 1 HERE (EU 25 SSI, ABZ, SBZ DIFFERENCES EVOLUTION)]

Repeating the same measurements to all the historical, current, and one possible future Council[3] the results are reported in the combined Tables 1 and 2, which report the Council's total votes, actual QMVs and suggested best-fit quotas by the SSI, sBz and aBz indices. Reporting these results in detail we shall next consider the Council as a set of shareholders, and then as a two-tier system.

The Council as a set of shareholders To start with the findings, Table 1 shows that the QMV in EU 15 was not a bad choice, if one wants to equalize votes and voting power, as the aBz and SSI best-fits are at the actual QMV of 62 in row four. This is coincidental, since Table 1 also reveals the aBz index having a clear pattern

[3]The current (EU 25) and the future EU 27 Council appear as laid out in the Nice Convention. There is a proposal to use a double majority, however, it is set aside having only a very marginal impact.

Council	Votes	QMV	SSI	SSI'	aBz	aBz'	sBz	sBz'
EU 27	345	255	299	304	237	237	214	240
EU 25	321	232	275	280	222	222	201	223
EU 15	87	62	62	57	62	62	57	45
EU 12	76	54	43	59	55	54	53	39
EU 10	63	45	47	36	46	46	42	36
EU 9	58	41	43	41	43	43	40 or **	36
EU 6	17	12	11or *	10	13	13	9	10

Notes: SSI, aBz and sBz: shareholder scenario. SSI', aBz' and sBz': two-tier scenario
* 9
** 30, 39, 49, 50

Table 1: TABLE 1: EU Councils and respective QMVs compared with best-fit quotas suggested by the SSI, aBz, and sBz indices

regarding the best-fit quotas. The larger the Council becomes, the lower best-fit suggestion the aBz index makes. As the Council quota was roughly 71 % from EU 6 to EU 15, the aBz coincides this at EU 15 as can be seen in Table 2. The fact that also the SSI best-fit is 62 in EU 15 is even more coincidental, as the SSI and sBz best-fits as percentages in Table 2 tend to jump around. This is the case especially for the SSI index, as the best-fit quotas can be found as low as 56.58 (EU 12), or as high as 86.67 (EU 27). The sBz remains rather close to roughly 65%[4] (with the exception of EU 6).

In fact, the aBz best-fit suggestions are all rather close to actual QMVs (except for EU 25 and EU 27, which have a quota greater than 71 %). In some cases the SSI and sBz indices are close too (sBz in EU 9 and EU 12, SSI in EU 6) as can also be seen in Table 2.

In two cases the relative indices suggest multiple best-fits, the SSI in EU 6 (9 and 11) and the sBz in EU 9 (30, 39, 40, 49, 50). This is mainly due to the smallness of the Council and the flatness of the respective best-fit curves. Examples of this are Figures 3 and 4 below.

The Council as a two-tier system When we consider the Council as a two-tier system in the fashion of Felsenthal and Machover (2003) following the Penrose theory, we do not use the allocated votes. Instead, we use square roots of the MSs respective populations instead. The applied population figures here are taken from Felsenthal and Machover (2003, Table 2).

The two-tier values in Tables 1 and 2 reveal some further findings: First, with

[4]Which is in turn very close to the 2/3 majority (66.6%) widely applied in national parliaments, for example.

Council	QMV(%)	SSI(%)	SSI'(%)	aBz(%)	aBz'(%)	sBz(%)	sBz'(%)
EU 27	73.91	86.67	88.12	68.7	68.7	62.03	69.57
EU 25	72.27	85.67	87.23	69.16	69.16	62.62	69.47
EU 15	71.26	71.26	65.52	71.26	71.26	65.52	51.72
EU 12	71.05	56.58	77.63	72.37	71.05	69.74	51.32
EU 10	71.43	74.6	57.14	73.02	73.02	66.67	57.14
EU 9	70.69	74.14	70.69	74.14	74.14	68.97 or **	62.07
EU 6	70.59	64.71or *	58.82	76.47	76.47	52.94	58.82

Notes: SSI, aBz and sBz: shareholder scenario. SSI', aBz' and sBz': two-tier scenario
* 52.94
** 51.72, 67.24, 84.48, 86.21

Table 2: TABLE 2: EU Councils and respective QMVs compared with best-fit quotas suggested by the SSI, aBz, and sBz indices as percentages

the exception of EU 12 the best-fits regarding aBz are identical. It seems that the index could be rather insensitive in one respect, especially in the light of the discussion on the aBz sensitivity to vote changes below. Second, the SSI's best-fit is equal to the actual QMV in EU 9, as can be seen in Tables 1 and 2. This is again purely coincidental, since the relative indices best-fits are not very close to the actual QMVs, when the Council is considered as a two-tier system. Third, the sBz best-fits as percentages in Table 2 are a bit surprising: Regarding EU9-EU15 they are lower compared to the vote-based best-fits, while in EU6, EU25 and EU27 they are somewhat higher than the vote-based best-fits. Worth noting is of course that the population figures have changed since the late 1950s.

Short comparison of the shareholder and two-tier considerations The results reveal that the EU data includes no perfect solutions to the inverse problem, although in some cases the difference to zero is in the level of fourth or fifth decimal. In the shareholder results, only in two cases two indices suggest equal best-fits, namely the aBz and SSI indices in EU 9 and EU 15. In the two-tier results, only SSI and sBz suggest equal best-fits in EU 6 and EU 9. Taken together with the previous exceptions: When relative voting power is proportional to votes, absolute voting power is not. Moreover, the two relative power indices do not agree with each other.

Sensitivity of the aBz index to the vote changes Next we shall test the sensitivity of the aBz index to the best-fit quota through vote manipulation. The respective best-fit curve in Figure 1 would at first sight suggest this index to be the most sensitive to quota changes. Results in Tables 1 and 2 also point to this

MS	Actual	Sc. 1	Sc. 2	Sc. 3	Sc. 4	Sc. 5	Sc. 6
Ger	10	10	*5*	6	15	35	73
UK	10	10	10	6	14	10	1
Fra	10	10	10	6	13	6	1
Ita	10	10	10	6	9	5	1
Spa	8	8	8	6	6	5	1
Net	5	5	5	6	5	4	1
Bel	5	5	5	6	5	4	1
Gre	5	5	5	6	4	4	1
Por	5	*4*	*4*	6	4	3	1
Aus	4	4	4	6	3	3	1
Swe	4	4	4	6	3	3	1
Fin	3	*4*	*4*	6	2	2	1
Den	3	3	3	5	2	1	1
Ire	3	3	3	5	1	1	1
Lux	2	2	*7*	5	1	1	1
Best fit Q	62	62	62	60	63	72	77
$\sum$ of Diffs.	0.028	0.028	0.032	0.063	0.041	0.32	0.216

Table 3: TABLE 3: Actual (EU 15) Council vote distribution and six hypothetical EU 15 scenarios together with best-fit quotas (aBz index)

direction. For the ease of calculation, we shall consider the previous (1995-2004) EU 15 Council. The shape of the best-fit curve is similar to that of EU 25. From above we note that the best-fit is at QMV 62.

We consider six different hypothetical scenarios, in which we increasingly alter the votes of the voters. Within each scenario the original total number of votes (87) is sustained. The results are reported in Table 3.

In the first scenario we only make a very minor change. Portugal is given one vote less and Finland one vote more, so that the size of the four-vote group is now four. As is easy to see from column three in Table 3, this alteration does not change the best-fit suggestion, or even the sum of the differences at the best-fit quota. The next scenario in column four alter the votes further: in addition to scenario one we also cut 5 votes from Germany and give these votes to Luxembourg. As can be seen, the best-fit quota suggestion still does not change, however, the sum of differences increases slightly. Scenario three is completely different: it represents a federalist EU by almost equalizing the number of votes among the MSs. Only the three smallest MSs receive five votes instead of six, as we want to keep the total number of votes fixed at 87. Surprisingly, the best-fit quota changes only marginally from 62 to 60. The sum of differences, however, double to 0.063. Scenario four turns the previous one upside down: now the votes are distributed as unevenly as possible.

The (again) surprising result is that the best-fit quota suggestion is 63, and the sum of differences diminish to 0.041. The only significant changes in the best-fit quota suggestions take place in scenarios five and six. The former has two dominating voters and the latter only one. The resulting quotas are 72 and 77, respectively. Also the sums of differences are at the first decimal level compared to the previous scenarios.

Summing up the six EU 15 scenarios, the aBz index appears to be rather insensitive to the voting body vote distributions. Only in cases, where there are one or two dominating voters there are significant changes in the best-fit suggestion. However, as was reported in Table 2, the number of voters does make a little difference.

Effective Number of Votes We will start by taking a look at how the power indices behave as a function of the quota. Rather many things become apparent after simply seeing how the indices behave between simple majority and unanimity quota. To illustrate this, three figures are drawn in the following. Figure 2 presents the MSs aBz index value evolution over all proper quotas in the EU 25 Council. The x-axis represents the quotas and the y-axis the index values. Comparing the evolution curves to the respective horizontal lines representing the MS group's relative vote share, it is possible to see how close the index values are to the vote share. The highest curve together with the highest horizontal line represent the 29 vote MS group. The curves and lines below represent other MS groups with fewer votes in strict descending order. Thus, the lowest curve and horizontal line represent Malta with three votes. As can be seen, the best-fit quota is nearly perfect, as almost every line and curve pair cross at this quota. The actual QMV is not optimal in this respect.

The aBz curves in Figure 2 have the shape similar to that of half of a parabola. The sBz curves below (rounded by the ENV procedure) in Figure 4 are rather flat, however, for the 29 and 27 vote MSs the shapes would be lightly convex without the ENV rounding. For the other MS groups the non-rounded sBz curves would be lightly concave. The SSI index curves in Figure 3 are almost completely flat, while the high quotas make the curves jump around. The curves would be very similar even without the ENV rounding, as Figure 2 already implies.

Figures 2, 3 and 4 show only half of all possible quotas. The improper quotas from 1 to simple majority would produce exact mirror images in the 'improper side' of simple majority. Hence, a whole aBz index evolution from quota 1 to unanimity would, for example, have the shape of a whole parabola instead of half of it. Regarding the historical EU Councils, the basic shapes of the index value evolution curves are the same, although with fewer voters diminish the curves become coarse as there are fewer quotas.

[FIGURE 2 HERE (ABZ INDEX EVOLUTION IN EU 25)]

Using the sole power index values (the decimal numbers) it is, however, very hard to show anything really 'concrete' as Figure 2 reveals: the power index values and the relative seat shares as such are very abstract numbers. This is the case especially with people, who are not familiar with the concept of voting power. Thus, we shall add a further illustrative aspect to our analysis, which in our opinion is much easier to comprehend. It will also reveal an unexpected result.

The concept of effective number of votes (ENV) is defined in Widgrén (1995): By multiplying voters' power index values (at a given quota) with the total number of votes, and rounding the result to the closest integer, we obtain the ENVs. In contrast, if we multiplied voters' relative vote shares with the total number of votes, we would get the actual numbers of votes. The ENVs tell us, how important a voter strategically is. Note that the same result, again with decimal numbers (without the number rounding regarding ENVs), can be achieved using a power coefficient.[5]

The ENVs provide an answer the following question: when the power index values differ from the relative vote shares, do they differ so much that the ENVs would show another distribution of votes than the actual one? In other words, we can now compare MSs votes to the respective ones created by the ENV rounding. The interesting observation in the case of EU 25 is that the sBz and SSI index values differ from the relative vote shares, however, not much.

Note that the aBz index behaves differently. The index could be applied, however, it would be partly senseless, since the index values usually do not add up to unity. Hence, the aBz ENVs in a particular quota are usually not even close to the actual vote distribution. In fact, an investigation through all EU Councils reveal that the only quota where the ENVs are equal or close to the actual vote distributions are the aBz best-fit quotas.

[FIGURES 3 & 4 HERE (FIG 3. EU 25 SSI, FIG. 4 EU 25 sBz)]

From Figure 4 we can see that the sBz ENVs do not change up to the quota

[5]The power coefficient appears at least in the works of Widgrén (1995) and Felsenthal and Machover (2000; 2001; 2003). The power coefficient for voter i is calculated by dividing i's power index value with i's relative vote share. The resulting values indicate whether $i$has more or less voting power than the relative vote share.

of 229 regarding MS groups having 3,4,7,10,12 or 13 votes. ENVs for the 29 and 27 vote groups diminish slightly. Between quotas 179 and 214 the whole sBz ENV distribution is equal to the actual one. Regarding the Nice quota of 232 the ENVs for the big four MSs group is 27 (at 231 it would have been 28), Spain and Poland have 26 votes, and the 10, 12 and 14 vote groups ENVs are one vote higher compared to the actual votes. For the remaining (small) voter groups the ENVs are equal to the actual votes.

The SSI ENVs in Figure 3 are equal to the sBz ones, and do not change before quota the of 253. Differences appear in the two large voter groups: The SSI shows no change up to quota 263 for the big four, however, Poland and Spain have couple of one vote changes. These changes take place between quotas 246-253. While all other voter groups SSI ENVs are equal the actual votes, the two large voter groups ENVs assign one extra vote to these groups (30 instead of 29 and 28 instead of 27).

In the light of the ENVs it would be rather the same, which quota is applied if it is not set too high, since the values calculated from the sBz or SSI indices change only marginally (if at all) in a wide range of quotas. This, however, shows a feature in the relative indices. Everything remains almost in the same level in relative terms, however, in absolute terms the voting power changes considerably. On the contrast, regarding the aBz index, it appears that only the best-fit points produce ENV distributions close to the actual ones. Also, it needs to be added that the MSs' absolute ability to control the vote outcome (aBz) is maximal at simple majority and diminish rapidly towards unanimity.

Comparison of the above results to some overall performance (synoptic) indices of the Council would be interesting. These synoptics include, for example, the power of the collectivity to act -index (Coleman 1971). The behaviour of these indices needs further investigation, but is beyond the scope of this study. Lastly, the quota has an obvious effect to the power balance between the Council and the Commission: the lower the quota is, the easier it is for the Commission to get its opinions passed in the Council.

Power of the votes and voting power The case of the first Council of EU 6 there still is a simplicity, which enables to observe -let us say- the straight power of the votes rather than the abstract voting power. Having only 6 MSs it is easy to see with practically no mathematics at all what is going on. Even nowadays the MSs nowadays mostly pay attention to the blocking minority. It is much easier to see what is needed for a blocking minority than for the majority. For our last analyse, let us now turn to the EU 6 Council in order to provide an insight to the connection

MS	Votes	Vot.(%)	9	10	11	*12*	13	14	15	16	17
Ger	4	23.53	.233	.267	.233	*.233*	.217	.3	.267	.2	.167
Fra	4	23.53	.233	.267	.233	*.233*	.217	.3	.267	.2	.167
Ita	4	23.53	.233	.267	.233	*.233*	.217	.3	.267	.2	.167
Net	2	11.76	.1	.1	.1	*.15*	.117	.05	.067	.2	.167
Bel	2	11.76	.1	.1	.1	*.15*	.117	.05	.067	.2	.167
Lux	1	5.88	.1	0	.1	*0*	.117	0	.067	0	.167

Table 4: TABLE 4: Votes and Voting Power (SSI index) in the EU 6 Council

between votes and (relative) voting power.

In more than one source, the exact same example is used to show that the connection between votes and voting power is not straightforward (see e.g. Felsenthal and Machover 1998; Pajala 2002a). This example is the EU 6 Council (1958-1973). The first MSs agreed to the vote distribution in the second column in Table 4. At the time almost all of the decisions were made unanimously, despite the QMV, which was set to 12. This particular quota has the property that Luxembourg is a non-pivotal dummy voter, i.e. it cannot contribute anything to any voter combination or permutation. Accordingly, Luxembourg has zero voting power, while the country's relative vote share is still 5.88 %, as can be seen from Table 4. At this point the immediate conclusion usually is that the relationship between votes and voting power is not straightforward. Unfortunately, the analysis in most cases also ends here. As this is not the whole picture, we shall analyse all proper quotas in this voting body.

For the results in Table 4 we have applied the SSI index (for this purpose it is just the same, whether we used SSI or sBz instead), Luxembourg is also a dummy at quotas 10, 14 and 16. The odd-numbered quotas 9, 11, 13, 15, 17 do assign voting power to Luxembourg, as can be seen from Table 4. In fact, Luxembourg now has as much voting power as Belgium and the Netherlands. Regarding unanimity every voter has a veto, and thus all voters have an equal amount of voting power. The QMV 12 was a political decision and carefully set (we can rest assured that the decision makers at the time were not thinking about power index values), probably ensuring that the large three countries could unanimously make a decision without the small ones. This is just bad luck for Luxembourg. In fact, voting power, when the voting body is small, is very dependent on the quota. It is rather true that the connection between votes and relative voting power appear not to be straightforward in the EU 6 case. However, as can be seen from previous sections, the connection is much more straightforward, when the voting body is large (e.g. EU 25). In fact, regarding the relative indices there is a pattern as the more there are voters in the decision making body, the better the correspondence between votes and voting power. Unfortunately the examples in the literature only use the extreme cases of three voters or perhaps the EU 6 thus producing only a limited picture. The absolute voting power is, as we know by now, another story.

4 Discussion

What did the results indicate keeping in mind our research questions set in the introduction? After computing through the whole EU Council history we learned that perfect proportionality between votes or population figures and voting power did not take place in the Council. However, the difference measurement procedure pointed out quotas where the proportionality was almost perfect and the differences were in the level of third or fourth decimal.

Comparing the power indices in the optimization procedure we learned that the behaviour of the indices is almost completely different between the aBz and the relative indices. Only the aBz index creates a pattern, according to which the best-fit quota diminish slightly as the voting body gets larger. From EU 6 to EU 25 the respective best-fit suggestion diminished steadily from 76.47 down to 68.7 percentages of all votes. As EU 6 - EU 15 Council QMV was kept in the level of 71 %, the aBz coincides this at EU 15, where the best-fit quota is 62, as is the actual QMV. The high level quotas of EU 25 and EU 27 seem to be a step away from the best-fit quota. Surprisingly, with only one exception (EU 12) the aBz index produced identical results, whether we used Council votes or the square roots of the MSs populations.

We also saw that that the aBz index always pointed out one very distinctive best-fit quota. A further analysis revealed that the best-fit quotas regarding the aBz index were rather immune to changes where votes from some voters were given to others, unless the changes were so extreme that there were only one or two very dominant voters present. An apparent limitation considering practical Council vote negotiations was that the vote changes were zero-sum in the sense that the total number of votes was not altered.

The relative indices departed substantially from each other and from the aBz index in what comes to the best-fit quotas. Noteworthy at the aBz best-fit points the three indices were all very close to each other. For the rest there were no patterns in what comes to the sBz or SSI best-fit quotas, as the curves representing the index values are very flat except for very high quotas. For a large majority of proper quotas in EU 25 these indices produced practically the same results, especially, if when we rounded things up a bit. This became clear after the ENV investigation.

Moreover, the relative seat shares became practically the same as the respective index values. Actually, the more there were in the voting body, the closer the three values converged. In other words, the relative indices seem to bring us hardly any new information in addition to relative vote shares.

Finally, we saw that when the absolute voting power changed for almost every quota. The relative indices indicated that the relative swing distributions did not change for a large range of quotas. Thus, in practice we could pick almost any quota (not the very high ones) without substantially distorting the relative swing distribution. The question is only which set of absolute voting power values we want to use? It could be a fixed quota such as simple majority or 2/3 qualified majority or the quota could even be randomized following Berg and Holler (1986) in order to equalize votes and voting power. In any case, in order to get an even more detailed picture of a voting body and to shed more light to the question of which vote threshold to choose, a further systematic investigation could be carried out using the synoptic indices.

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Figure 1. Difference between MSs average aBz, sBz and SSI index values and relative vote shares on the proper quotas of the EU 25 Council

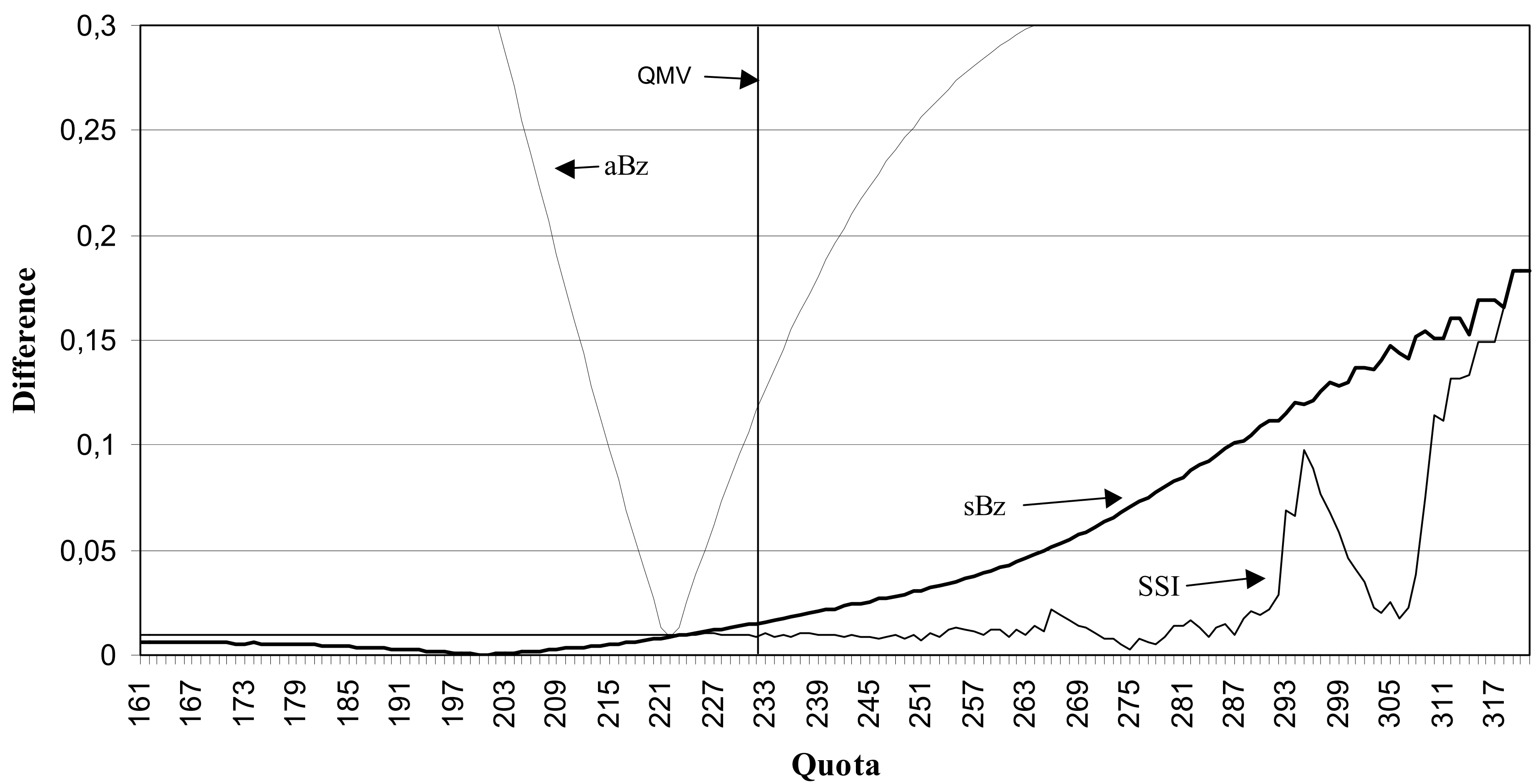

Figure 2. The aBz index value evolution and the relative seat shares of MS groups on the proper quotas of the EU 25 Council

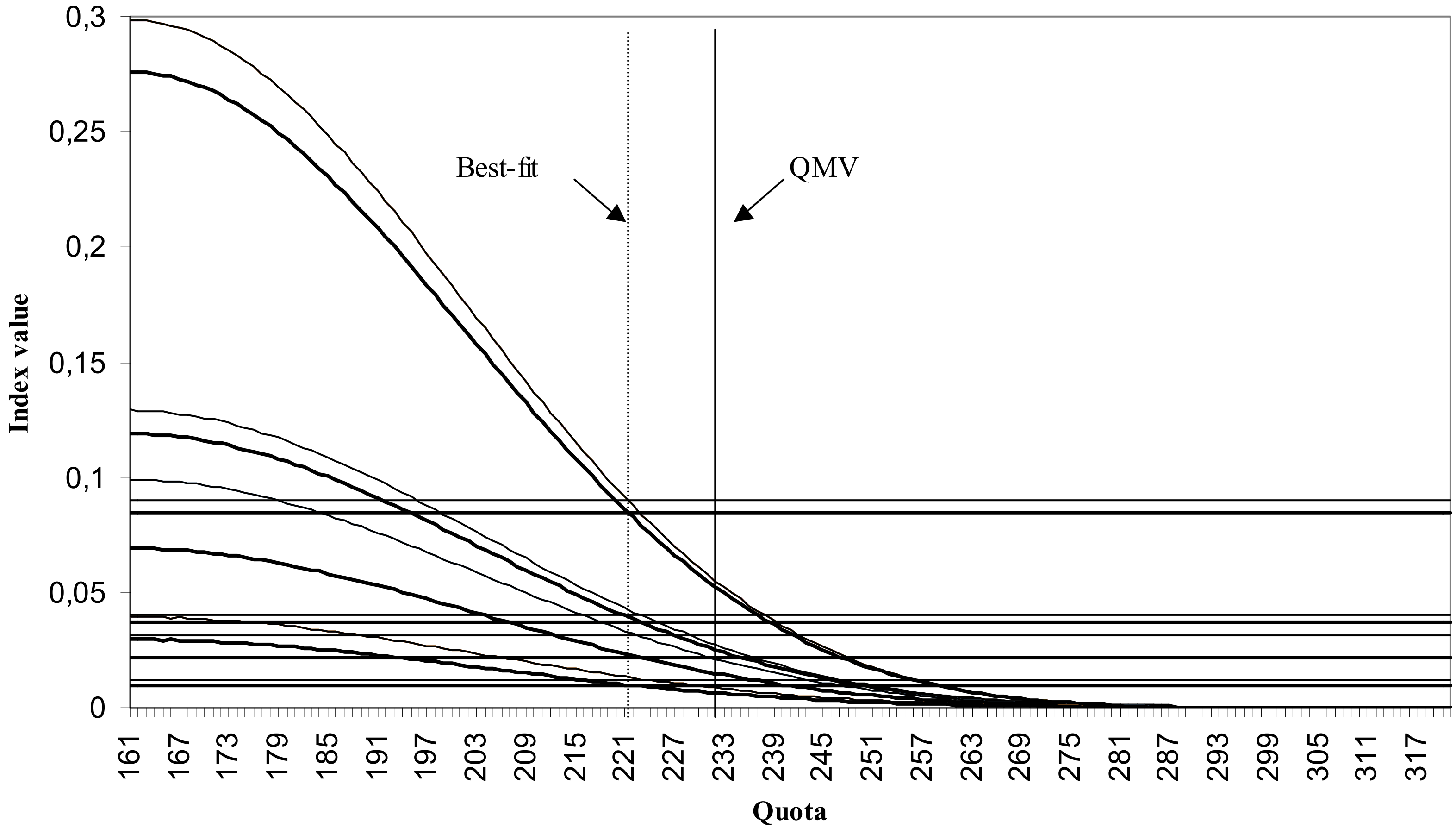

Figure 3. The evolution of SSI index ENVs on the proper quotas of the EU 25 Cuncil

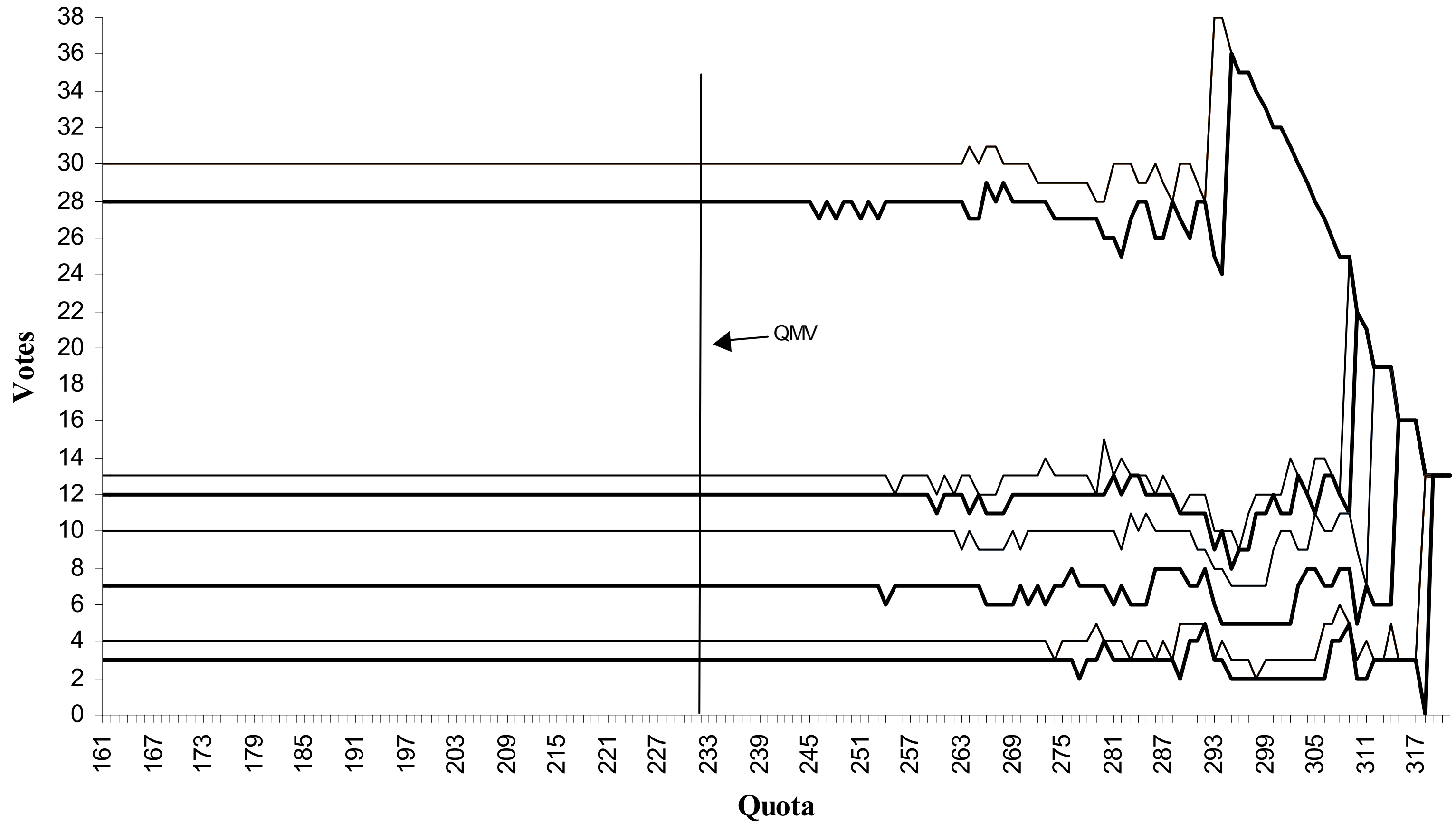

Figure 4. The evolution of sBz index ENVs on the proper quotas of the EU 25 Council