# L S Penrose's limit theorem: Tests by simulation

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Draft 3, July 2004

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ABSTRACT

L S Penrose's limit theorem (PLT) – which is implicit in Penrose [5, p. 72] and for which he gave no rigorous proof – says that, in simple weighted voting games, if the number of voters increases indefinitely while existing voters retain their weights and the relative quota is pegged, then – under certain conditions – the ratio between the voting powers of any two voters converges to the ratio between their weights. Lindner and Machover [3] prove some special cases of PLT; and conjecture that the theorem holds, under rather general conditions, for large classes of weighted voting games, various values of the quota, and with respect to several measures of voting power. We use simulation to test this conjecture. It is corroborated w.r.t. the Penrose–Banzhaf index for a quota of 50% but not for other values; w.r.t. the Shapley–Shubik index the conjecture is corroborated for all values of the quota (short of 100%).

**Keywords:** limit theorems, majority games, simulation, weighted voting games

**JEL classifications:** C71, D71

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## 1 Introduction

Throughout this paper, we shall be concerned with *weighted voting games* (WVGs). Let us recall briefly their definition. A WVG $\mathcal{W}$ consists of a finite set $N$ of together with an assignment of a non-negative real *weight* $w_x$ to each $x \in N$; and a real $q \in (0, 1)$.

For our purposes it will be convenient, and will entail no loss of generality, to assume that all weights are positive. The *relative weight* of voter $a$ in $\mathcal{W}$ is given by

$$\overline{w}_a := \frac{w_a}{\sum_{x \in N} w_x}. \tag{1}$$

A subset $A \subseteq N$ (often referred to as a 'coalition') is said to be *winning* if

$$\sum_{x \in A} \overline{w}_x \geq q. \tag{2}$$

We refer to $N$ as the *assembly* of $\mathcal{W}$, to the members of $N$ as *voters*, and to $q$ as the *quota*.[1]

L S Penrose's limit theorem (PLT) is an assertion about the asymptotic behaviour of the voting power of voters in WVGs with a large number of voters. Here we shall consider the two major indices of voting power: the so-called *Banzhaf* index $\beta$ (which is obtained by normalization from the absolute measure of voting power first proposed by Penrose [4]); and the *Shapley–Shubik* index $\phi$ proposed by these two authors in [6] (which is a special case of the Shapley value for co-operative games). For the definitions of these indices see, for example, Felsenthal and Machover [2].

For a precise statement of PLT we need the concept of a *q-chain* of WVGs, whose definition we borrow from [3].

Let

$$N^{(0)} \subsetneq N^{(1)} \subsetneq N^{(2)} \subsetneq \cdots \tag{3}$$

be an infinite increasing chain of finite non-empty sets, and let

$$N = \bigcup_{n=0}^{\infty} N^{(n)}. \tag{4}$$

[1]In the voting-power literature, $q$ is often referred to as the *relative* quota, as distinct from the *absolute* quota, which equals $q \cdot \sum_{x \in N} w_x$.

Let $w$ be a function that assigns to each $a \in N$ a positive real number $w_a$ as *weight*; and let $q$ be a real $\in (0, 1)$.

For each $n \in \mathbb{N}$ let $\mathcal{W}^{(n)}$ be the WVG whose assembly is $N^{(n)}$ – each voter $a \in N^{(n)}$ being endowed with the pre-assigned weight $w_a$ – and whose quota is $q$.

We shall then say that $\{\mathcal{W}^{(n)}\}_{n=0}^{\infty}$ is a *q-chain* of WVGs.

Further, let $\xi$ be an index of voting power. We shall say that *PLT holds for the q-chain* $\{\mathcal{W}^{(n)}\}_{n=0}^{\infty}$ *with respect to the index* $\xi$ if for any $a, b \in N$

$$\lim_{n\to\infty} \frac{\xi_a[\mathcal{W}^{(n)}]}{\xi_b[\mathcal{W}^{(n)}]} = \frac{w_a}{w_b}. \tag{5}$$

Penrose [5, p. 72] gives an approximation formula for the voting power (as defined by him) of a voter in a WVG $\mathcal{W}$ with quota $\frac{1}{2}$, according to which voters' powers are approximately proportional to their respective weights. He claims that this approximation is valid provided the number of voters in $\mathcal{W}$ is large, and the relative weights of the voters in question are small. He offers no rigorous proof of his claim, merely an outline of an argument, obviously based on some version of the central limit theorem of probability theory.

Penrose's claim implies that PLT holds w.r.t. $\beta$ (the normalized version of his measure of voting power) for any $\frac{1}{2}$-chain, provided the relative weight of each $a \in N^{(n)}$ tends to 0 as $n$ increases; that is,

$$\lim_{n\to\infty} \frac{w_a}{\sum_{x\in N^{(n)}} w_x} = 0.$$

However, Lindner and Machover [3] show by means of a simple counter-example that these conditions are insufficient for Penrose's approximation formula and the version of PLT implied by it. On the other hand, they prove the approximation formula (in a somewhat improved form) as well as PLT w.r.t. $\beta$ for $\frac{1}{2}$-chains satisfying more stringent conditions. (See [3, Theorem 3.6].) They also prove PLT w.r.t. the Shapley–Shubik index $\phi$ for a large class of $q$-chains with arbitrary $q \in (0, 1)$. (See [3, Theorem 2.3].)

Furthermore, they conjecture that PLT holds w.r.t. both $\beta$ and $\phi$ for all $q \in (0, 1)$ and almost all $q$-chains. To be more precise: if a reasonable probability measure is defined on the space of all $q$-chains (or all $q$-chains with integer weights) then the probability that PLT holds equals 1.

In this paper we report the results of simulations designed to test their conjecture. Here is an outline of how we go about it. (A more detailed account will be given in Section 2.)

Obviously, we cannot select at random an entire $q$-chain, which is an infinite object. Instead, we proceed as follows. We fix some 'large' values of $n$, which will be the number of voters. (We allow $n$ to get as large as feasibility of computation allows.) We also fix various values of the quota $q$, spaced at fairly close intervals.

Next, for each of our $n$, we select at random positive weights $w_1, \ldots, w_n$. Replacing these by the corresponding relative weights, we get a random vector $\mathbf{w} = (\overline{w}_1, \ldots, \overline{w}_n)$ in the $(n-1)$-dimensional simplex $\Delta^{(n-1)}$ of all real $n$-vectors with non-negative components that add up to 1:

$$\Delta^{(n-1)} := \left\{ \mathbf{x} \in \mathbb{R}^n : x_i \geq 0,\ i = 1, \ldots, n;\ \sum_{i=1}^{n} x_i = 1 \right\}. \tag{6}$$

This random selection is repeated a large number of times, so that for each of our $n$ we obtain a large random sample of vectors $\mathbf{w} \in \Delta^{(n-1)}$.

For each randomly selected $\mathbf{w}$ and fixed $q$ we have a WVG $\mathcal{W}$, with assembly $N = \{1, \ldots, n\}$, with $\mathbf{w}$ as the vector of weights and $q$ as quota.

For this WVG we compute the vector of values of the Banzhaf index $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ and the Shapley–Shubik index $\boldsymbol{\phi} = (\phi_1, \ldots, \phi_n)$. These vectors also belong to $\Delta^{(n-1)}$.

We are interested in the discrepancy (measured by a suitable metric) between the vector $\boldsymbol{\xi}$ – where $\boldsymbol{\xi}$ is $\boldsymbol{\beta}$ or $\boldsymbol{\phi}$ respectively – and the relative weight vector $\mathbf{w}$. For each of our $n$ and $q$, we compute the mean and standard error of this distance, over our large sample of $\mathbf{w} \in \Delta^{(n-1)}$.

If, for a given value of $q$, the mean and standard deviation of the discrepancy between $\boldsymbol{\xi}$ and $\mathbf{w}$ approach 0 as $n$ increases, then this corroborates the hypothesis that PLT holds for almost all $q$-chains w.r.t. $\boldsymbol{\xi}$. If the mean discrepancy shows no tendency to approach 0 as $n$ increases, this provides evidence against that hypothesis.

In Section 2 we fill in the details of the method outlined above. In Section 4 we present the results of our simulation. These are discussed in the concluding Section 3.

# 2 Description of the method

**2.1. Random selection of weights** In fact, we use two different methods of random selection. The first method selects $n$ positive integer weights $w_i$ independently of one another, with a Poisson probability distribution, shifted so as to avoid 0 weights; thus, for each $i = 1, 2, \dots, n$ we have

$$\mathrm{Prob}\{w_i = k\} = \frac{e^{-1}}{(k-1)!}, \; k = 1, 2, \dots. \tag{7}$$

Our second method selects the random vector $\mathbf{w}$ from an $(n-1)$-dimensional uniform distribution on the simplex $\Delta^{(n-1)}$. There are of course various ways for achieving this. We use the following method, which is very efficient computationally.[2] We select positive real weights $w_i$ independently of one another, each with an exponential probability density

$$f(x) = \begin{cases} e^{-x} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The exponential distribution is a special case of the gamma distribution; and the fact that this probability density for the $w_i$ yields the required uniform distribution for the normalized vector $\mathbf{w}$ follows from a property of the so-called Dirichlet composition. (For details, see [1, pp. 59f].)

Thus we have two separate series of samples. We shall refer to them as the *Poisson* and *uniform* samples, respectively.

In each of these two series, the size of our random sample of vectors $\mathbf{w} \in \Delta^{(n-1)}$ is 1,000 for every chosen value of $n$.

**2.2. Choice of** $n$ As lowest value for $n$ we took $n = 15$, because experience suggests that in cases where the asymptotic behaviour asserted by PLT occurs, it begins to manifest itself at about this value.

The choice of highest value for $n$ was dictated by computational feasibility. The time needed to compute the vectors of values $\boldsymbol{\beta}$ and $\boldsymbol{\phi}$ for given $\mathbf{w}$ and $q$ increases very steeply with $n$. We found that for the Poisson samples $n = 57$ was the highest practicable value. For the uniform samples we were able to go up to $n = 60$.

Fortunately, this range of values of $n$ is sufficient for providing results from which quite firm conclusions can be drawn.

**2.3. Choice of** $q$ Although the conjecture we are testing concerns values of $q$ in the open interval $(0, 1)$, we need only consider values of $q$ in the half-open interval $[0.5, 1)$.

To see this, note that both $\beta$ and $\phi$ are *self-dual* (see [2, p. 180]). Thus, let $\mathcal{W}$ be a WVG with weights $w_x$ ($x \in N$) and quota $q$; and let $\mathcal{W}^*$ be the

[2] We are grateful to Friedrich Pukelsheim for suggesting this method to us.

simple voting game with the same assembly, $N$, whose winning coalitions are those $A \subseteq N$ such that

$$\sum_{x \in A} \overline{w}_x > 1 - q. \tag{9}$$

($\mathcal{W}^*$ is called the *dual* of $\mathcal{W}$. It is easy to see that it is a WVG with the same weights as $\mathcal{W}$ and quota $1 - q + \varepsilon$, for any sufficiently small positive $\varepsilon$.) The self-duality of $\beta$ and $\phi$ implies that

$$\beta_x[\mathcal{W}^*] = \beta_x[\mathcal{W}] \text{ and } \phi_x[\mathcal{W}^*] = \phi_x[\mathcal{W}] \text{ for all } x \in N. \tag{10}$$

Although in our definition (2) of a WVG with quota $q$ we had a 'blunt' inequality ($\geq$), whereas here in (9) we have a sharp inequality ($>$), it is not difficult to see that this makes no difference to the asymptotic behaviour. More precisely, the asymptotic behaviour of a given $q$-chain w.r.t. both $\beta$ and $\phi$ is the same as that of the $(1 - q)$-chain having the same weights.[3]

In our simulation we fix $q$ at the following values:

$$q = 0.50,\ 0.51,\ 0.55,\ 0.60,\ 0.65,\ 0.70,\ 0.75,\ 0.80,\ 0.85,\ 0.90,\ 0.95.$$

**2.4. Measuring the discrepancy** For each $\mathbf{w}$ in our samples and each chosen value of $q$, we compute the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\phi}$ of the values of the Banzhaf and the Shapley–Shubik indices, respectively. In what follows, '$\boldsymbol{\xi}$' stands for either $\boldsymbol{\beta}$ or $\boldsymbol{\phi}$.

PLT w.r.t. $\boldsymbol{\xi}$ claims that asymptotically $\boldsymbol{\xi}$ approaches the normalized vector of weights $\mathbf{w}$. We measure the 'discrepancy' of $\boldsymbol{\xi}$ compared to $\mathbf{w}$ in two ways.

First, we measure the *overall* discrepancy between $\boldsymbol{\xi}$ and $\mathbf{w}$ by the well-known index of distortion $D$, commonly attributed to Loosemore and Hanby:[4]

$$D(\boldsymbol{\xi}, \mathbf{w}) := \frac{1}{2} \sum_{i=1}^{n} |\xi_i - \overline{w}_i|. \tag{11}$$

Second, we measure the *local* (or *componentwise*) discrepancy between $\boldsymbol{\xi}$ and $\mathbf{w}$ by

$$d(\boldsymbol{\xi}, \mathbf{w}) := \max_{1 \leq i \leq n} \left| 1 - \frac{\xi_i}{\overline{w}_i} \right|. \tag{12}$$

Note that $\boldsymbol{\xi}$ is completely determined by $\mathbf{w}$ and $q$. Therefore, if we fix $n$ and $q$, and regard $\mathbf{w}$ as a random variable, then $D(\boldsymbol{\xi}, \mathbf{w})$ and $d(\boldsymbol{\xi}, \mathbf{w})$ are also

[3] In this connection note that the results of Lindner and Machover [3] hold also – with virtually the same proofs – if in the definition (2) of WVG $\geq$ is replaced by $>$.

[4] See however discussion by Taagepera and Grofman [7] of the authorship of this index.

random variables, whose distributions depend on that of $\mathbf{w}$. We are interested in the expected value and standard deviation of these random variable, as functions of $n$ and $q$.

**2.5. Output of computation** In our simulation, we estimate the expected value and standard deviation of $D(\boldsymbol{\xi}, \mathbf{w})$ and $d(\boldsymbol{\xi}, \mathbf{w})$ by computing their mean and standard error for each of our samples. This yields the following outputs for all the chosen values of $n$ and $q$ :

$$\mu D(n, q), \quad \sigma D(n, q), \quad \mu d(n, q), \quad \sigma d(n, q).$$

Here '$\mu$' and '$\sigma$' stand for mean and standard error, respectively.

More specifically, we have a set of four such outputs for each of the two indices and each of our sample series. Thus we have altogether:

$$\begin{array}{llll} \mu_P D(\beta; n, q), & \sigma_P D(\beta; n, q), & \mu_P d(\beta; n, q), & \sigma_P d(\beta; n, q), \\ \mu_U D(\beta; n, q), & \sigma_U D(\beta; n, q), & \mu_U d(\beta; n, q), & \sigma_U d(\beta; n, q), \\ \mu_P D(\phi; n, q), & \sigma_P D(\phi; n, q), & \mu_P d(\phi; n, q), & \sigma_P d(\phi; n, q), \\ \mu_U D(\phi; n, q), & \sigma_U D(\phi; n, q), & \mu_U d(\phi; n, q), & \sigma_U d(\phi; n, q). \end{array}$$

Here '$\beta$' and '$\phi$' and the subscripts '$P$' and '$U$' are labels that refer to the Banzhaf and Shapley–Shubik indices, and the Poisson and uniform samples, respectively.

In Section 4 we present detailed tables (Tables 1&1a–8&8a) as well as 3-D graphs (Figures 1&1a–8&8a) of each of these these sixteen statistics. In each diagram, the values of the statistic in question are plotted along the vertical axis; $n$ is plotted along the $\searrow$ axis and $q$ along the $\nearrow$ axis.

# 3 Conclusions

The tables and diagrams presented in Section 4 show a conspicuous difference between the behaviours of the two indices. We consider these indices in turn.

**3.1. The Banzhaf index** The data for the statistics labelled '$\beta$' do not corroborate the conjecture w.r.t. the Banzhaf index (that is, that PLT holds for almost all $q$-chains w.r.t. this index) except for $q = 0.5$ and perhaps for values of $q$ very close to 0.5. Note that Penrose's original claim concerned only $q = 0.5$. This claim, as we know, does not hold in all cases even for $q = 0.5$, but it does now appear to hold in *almost* all such cases.

One interesting feature of these data ought to be pointed out. For fixed values of $n$ near the bottom of our range, the mean discrepancy between $\boldsymbol{\beta}$

and $\mathbf{w}$ has a dip – a minimum, indicating the closest fit between $\boldsymbol{\beta}$ and $\mathbf{w}$ – at a value of $q$ considerably greater than 0.5. But as $n$ increases the dip edges towards $q = 0.5$.

This general pattern applies to both the Poisson and the uniform samples, and to both measures of discrepancy. But there are some differences of detail.

In the case of $\mu_P D(\beta; n, q)$ (Table 1 and Figure 1), for $n = 15, \ldots, 18$ the dip occurs at $q = 0.65$; for $n = 18, \ldots, 48$ it occurs at $q = 0.60$; and for $n = 50, \ldots, 57$ it occurs at $q = 0.55$.

In the case of $\mu_P d(\beta; n, q)$ (Table 2 and Figure 2), for $n = 15, \ldots, 17$ the dip occurs at $q = 0.65$; for $n = 19, \ldots, 50$ it occurs at $q = 0.60$; and for $n = 49, \ldots, 57$ it occurs at $q = 0.55$.

In the case of $\mu_U D(\beta; n, q)$ (Table 3 and Figure 3), for $n = 15, \ldots, 26$ the dip occurs at $q = 0.65$; and for $n = 27, \ldots, 57$ it occurs at $q = 0.60$.

In the case of $\mu_U d(\beta; n, q)$ (Table 4 and Figure 4), for $n = 15, \ldots, 23$ the dip occurs at $q = 0.65$; and for $n = 24, \ldots, 57$ it occurs at $q = 0.60$.

Extrapolating from these data, it is reasonable to expect that for still greater values on $n$, beyond our range, the dip of all these four quantities – $\mu_P D(\beta; n, q)$, $\mu_P d(\beta; n, q)$, $\mu_U D(\beta; n, q)$ and $\mu_U d(\beta; n, q)$ – should occur at $q = 0.5$.[5]

Our negative findings w.r.t. $\beta$ for $q > 0.5$ of course do not exclude the possibility that PLT holds for large classes of $q$-chains with $q > 0.5$. Finding 'natural' and sufficiently interesting such classes is an open problem.

**3.2. The Shapley–Shubik index** The data for the statistics labelled '$\phi$' corroborate the conjecture w.r.t. the Shapley–Shubik index for all $q \in (0, 1)$. For every chosen value of $q$, the mean discrepancy between $\boldsymbol{\phi}$ and $\mathbf{w}$ – whether measured by $\mu D(\boldsymbol{\phi}, \mathbf{w})$ or $\mu d(\boldsymbol{\phi}, \mathbf{w})$ – seems to approach 0 as $n$ increases. It appears that the PLT w.r.t. $\phi$ does hold almost always.

Let us look at the behaviour of the mean discrepancy between $\boldsymbol{\phi}$ and $\mathbf{w}$ as a function of $q$ and $n$.

Clearly, for any fixed $n$, as $q$ gets very close to 1, we would expect a WVG with $n$ voters to behave somewhat like a unanimity game, in which all voters have the same voting power, irrespective of their weights. Therefore it is reasonable to expect that as $q$ approaches 1, the mean discrepancy between $\boldsymbol{\phi}$ and $\mathbf{w}$ should increase. Also, it is reasonable to expect that as $q$ gets closer

[5]In an unpublished report that came to our attention after completing an earlier draft of this paper, Życzkowski and Słomczyński [8, Section 10] have made essentially the same observations and drew the same conclusion regarding the behaviour of the 'dip' (which they call 'critical point') in the mean discrepancy between $\boldsymbol{\beta}$ and $\mathbf{w}$. Our simulations – using much larger samples and considerably more values of $n$ – provide an independent corroboration of their findings on this matter.

to 1, it would take greater values of $n$ to overcome this 'unanimity effect'. In other words, the closer $q$ is to 1, the slower the convergence to 0 of the mean discrepancy as $n$ increases.

Our data show that this is indeed the case. However, we wish to point out an additional interesting phenomenon concerning the dependence on $n$ of the the mean discrepancy between $\boldsymbol{\phi}$ and $\mathbf{w}$. For every fixed value of $n$ in our range, this mean discrepancy increases monotonically with $q$; but the rate of increase is by no means uniform. For each $n$, as $q$ increases from 0.5 towards 1, we can discern two regimes: at first the increase in the mean discrepancy is very gentle, barely noticeable; then, quite abruptly, it becomes quite steep. In other words, the transition from the 'PLT effect' to the 'unanimity effect' is rather sharp. As may be expected, it seems that the greater the value of $n$, the higher is the value of $q$ at which this transition takes place.

This general pattern applies to both the Poisson and the uniform samples, and to both measures of discrepancy. But again there are some difference of detail as to where (that is, at what value of $q$) the increase becomes steep for a given $n$. For these details we refer the reader to the relevant tables (Tables 5, 6, 7 and 8), in which values that seem to be affected by the 'unanimity effect' are highlighted.

## 4 Results

[See Tables and Figures on separate pages.]

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**Table 1:** $\mu_P D(\beta; n, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $n$ \ $q$ | **0.50** | **0.51** | **0.55** | **0.6** | **0.65** | **0.7** | **0.75** | **0.8** | **0.85** | **0.9** | **0.95** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0122 | 0.0122 | 0.0107 | 0.0064 | 0.0028 | 0.0124 | 0.0259 | 0.0442 | 0.0690 | 0.0989 | 0.1439 |
| **16** | 0.0113 | 0.0113 | 0.0098 | 0.0056 | 0.0031 | 0.0130 | 0.0264 | 0.0444 | 0.0688 | 0.0976 | 0.1442 |
| **17** | 0.0105 | 0.0105 | 0.0089 | 0.0048 | 0.0034 | 0.0136 | 0.0265 | 0.0442 | 0.0686 | 0.0970 | 0.1429 |
| **18** | 0.0098 | 0.0098 | 0.0084 | 0.0042 | 0.0038 | 0.0140 | 0.0269 | 0.0447 | 0.0683 | 0.0976 | 0.1425 |
| **19** | 0.0093 | 0.0093 | 0.0079 | 0.0037 | 0.0042 | 0.0142 | 0.0272 | 0.0447 | 0.0677 | 0.0970 | 0.1404 |
| **20** | 0.0090 | 0.0090 | 0.0076 | 0.0034 | 0.0045 | 0.0149 | 0.0281 | 0.0460 | 0.0693 | 0.0985 | 0.1395 |
| **21** | 0.0085 | 0.0085 | 0.0071 | 0.0029 | 0.0048 | 0.0151 | 0.0284 | 0.0460 | 0.0690 | 0.0981 | 0.1384 |
| **22** | 0.0081 | 0.0081 | 0.0067 | 0.0024 | 0.0051 | 0.0155 | 0.0287 | 0.0464 | 0.0692 | 0.0983 | 0.1373 |
| **23** | 0.0077 | 0.0077 | 0.0063 | 0.0021 | 0.0054 | 0.0158 | 0.0291 | 0.0469 | 0.0701 | 0.0995 | 0.1386 |
| **24** | 0.0072 | 0.0072 | 0.0058 | 0.0017 | 0.0057 | 0.0157 | 0.0289 | 0.0462 | 0.0691 | 0.0979 | 0.1370 |
| **25** | 0.0069 | 0.0068 | 0.0055 | 0.0015 | 0.0059 | 0.0159 | 0.0289 | 0.0462 | 0.0694 | 0.0980 | 0.1383 |
| **26** | 0.0067 | 0.0066 | 0.0053 | 0.0012 | 0.0064 | 0.0164 | 0.0293 | 0.0468 | 0.0699 | 0.0981 | 0.1386 |
| **27** | 0.0064 | 0.0064 | 0.0050 | 0.0010 | 0.0066 | 0.0166 | 0.0298 | 0.0472 | 0.0701 | 0.0985 | 0.1392 |
| **28** | 0.0061 | 0.0060 | 0.0047 | 0.0008 | 0.0066 | 0.0166 | 0.0296 | 0.0469 | 0.0697 | 0.0983 | 0.1388 |
| **29** | 0.0060 | 0.0059 | 0.0046 | 0.0007 | 0.0070 | 0.0170 | 0.0301 | 0.0476 | 0.0706 | 0.0993 | 0.1392 |
| **30** | 0.0057 | 0.0056 | 0.0043 | 0.0006 | 0.0070 | 0.0169 | 0.0300 | 0.0473 | 0.0698 | 0.0985 | 0.1374 |
| **31** | 0.0055 | 0.0055 | 0.0042 | 0.0006 | 0.0071 | 0.0171 | 0.0301 | 0.0475 | 0.0700 | 0.0986 | 0.1373 |
| **32** | 0.0054 | 0.0053 | 0.0040 | 0.0005 | 0.0073 | 0.0173 | 0.0304 | 0.0479 | 0.0703 | 0.0989 | 0.1371 |
| **33** | 0.0052 | 0.0051 | 0.0038 | 0.0006 | 0.0074 | 0.0174 | 0.0305 | 0.0480 | 0.0705 | 0.0990 | 0.1374 |
| **34** | 0.0050 | 0.0050 | 0.0036 | 0.0007 | 0.0076 | 0.0176 | 0.0305 | 0.0478 | 0.0703 | 0.0985 | 0.1371 |
| **35** | 0.0048 | 0.0048 | 0.0035 | 0.0007 | 0.0077 | 0.0176 | 0.0305 | 0.0478 | 0.0705 | 0.0985 | 0.1369 |
| **36** | 0.0048 | 0.0047 | 0.0034 | 0.0008 | 0.0079 | 0.0179 | 0.0309 | 0.0482 | 0.0707 | 0.0988 | 0.1374 |
| **37** | 0.0046 | 0.0046 | 0.0032 | 0.0009 | 0.0080 | 0.0180 | 0.0310 | 0.0484 | 0.0710 | 0.0990 | 0.1375 |
| **38** | 0.0045 | 0.0044 | 0.0031 | 0.0010 | 0.0082 | 0.0181 | 0.0313 | 0.0485 | 0.0711 | 0.0991 | 0.1373 |
| **39** | 0.0044 | 0.0043 | 0.0030 | 0.0011 | 0.0082 | 0.0183 | 0.0314 | 0.0488 | 0.0713 | 0.0995 | 0.1376 |
| **40** | 0.0042 | 0.0042 | 0.0029 | 0.0012 | 0.0083 | 0.0183 | 0.0312 | 0.0484 | 0.0707 | 0.0989 | 0.1365 |
| **41** | 0.0042 | 0.0041 | 0.0028 | 0.0013 | 0.0084 | 0.0183 | 0.0315 | 0.0486 | 0.0711 | 0.0992 | 0.1365 |
| **42** | 0.0041 | 0.0040 | 0.0027 | 0.0014 | 0.0085 | 0.0185 | 0.0316 | 0.0489 | 0.0713 | 0.0995 | 0.1365 |
| **43** | 0.0040 | 0.0039 | 0.0026 | 0.0015 | 0.0086 | 0.0185 | 0.0317 | 0.0490 | 0.0714 | 0.0996 | 0.1370 |
| **44** | 0.0039 | 0.0038 | 0.0025 | 0.0016 | 0.0087 | 0.0187 | 0.0318 | 0.0491 | 0.0715 | 0.0995 | 0.1368 |
| **45** | 0.0038 | 0.0037 | 0.0024 | 0.0017 | 0.0087 | 0.0187 | 0.0318 | 0.0492 | 0.0716 | 0.0998 | 0.1379 |
| **46** | 0.0037 | 0.0037 | 0.0023 | 0.0017 | 0.0089 | 0.0189 | 0.0321 | 0.0494 | 0.0719 | 0.0999 | 0.1375 |
| **47** | 0.0036 | 0.0036 | 0.0022 | 0.0018 | 0.0089 | 0.0189 | 0.0321 | 0.0494 | 0.0719 | 0.1000 | 0.1375 |
| **48** | 0.0035 | 0.0035 | 0.0021 | 0.0019 | 0.0089 | 0.0188 | 0.0319 | 0.0492 | 0.0713 | 0.0994 | 0.1367 |
| **49** | 0.0035 | 0.0034 | 0.0021 | 0.0019 | 0.0090 | 0.0190 | 0.0320 | 0.0493 | 0.0716 | 0.0995 | 0.1367 |
| **50** | 0.0034 | 0.0034 | 0.0020 | 0.0020 | 0.0091 | 0.0191 | 0.0323 | 0.0497 | 0.0721 | 0.1002 | 0.1375 |
| **51** | 0.0033 | 0.0033 | 0.0020 | 0.0021 | 0.0092 | 0.0192 | 0.0324 | 0.0498 | 0.0721 | 0.1003 | 0.1374 |
| **52** | 0.0032 | 0.0032 | 0.0019 | 0.0021 | 0.0091 | 0.0191 | 0.0322 | 0.0494 | 0.0717 | 0.0997 | 0.1365 |
| **53** | 0.0032 | 0.0031 | 0.0018 | 0.0022 | 0.0092 | 0.0192 | 0.0323 | 0.0496 | 0.0719 | 0.1000 | 0.1370 |
| **54** | 0.0032 | 0.0031 | 0.0018 | 0.0023 | 0.0094 | 0.0193 | 0.0324 | 0.0498 | 0.0722 | 0.1001 | 0.1375 |
| **55** | 0.0031 | 0.0030 | 0.0017 | 0.0023 | 0.0094 | 0.0193 | 0.0324 | 0.0497 | 0.0720 | 0.1000 | 0.1374 |
| **56** | 0.0030 | 0.0029 | 0.0017 | 0.0024 | 0.0094 | 0.0193 | 0.0324 | 0.0497 | 0.0720 | 0.1000 | 0.1371 |
| **57** | 0.0030 | 0.0029 | 0.0016 | 0.0024 | 0.0095 | 0.0195 | 0.0326 | 0.0500 | 0.0724 | 0.1005 | 0.1376 |

**Table 1a:** $\sigma_P D(\beta; n, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $n$ \ $q$ | **0.50** | **0.51** | **0.55** | **0.6** | **0.65** | **0.7** | **0.75** | **0.8** | **0.85** | **0.9** | **0.95** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0062 | 0.0062 | 0.0052 | 0.0032 | 0.0023 | 0.0060 | 0.0095 | 0.0143 | 0.0198 | 0.0270 | 0.0350 |
| **16** | 0.0057 | 0.0057 | 0.0047 | 0.0027 | 0.0027 | 0.0059 | 0.0096 | 0.0136 | 0.0189 | 0.0244 | 0.0333 |
| **17** | 0.0047 | 0.0047 | 0.0041 | 0.0023 | 0.0024 | 0.0054 | 0.0084 | 0.0129 | 0.0189 | 0.0250 | 0.0357 |
| **18** | 0.0044 | 0.0044 | 0.0037 | 0.0020 | 0.0025 | 0.0055 | 0.0086 | 0.0128 | 0.0177 | 0.0240 | 0.0332 |
| **19** | 0.0042 | 0.0042 | 0.0035 | 0.0020 | 0.0027 | 0.0052 | 0.0086 | 0.0124 | 0.0170 | 0.0235 | 0.0320 |
| **20** | 0.0036 | 0.0036 | 0.0031 | 0.0018 | 0.0022 | 0.0050 | 0.0084 | 0.0124 | 0.0174 | 0.0239 | 0.0318 |
| **21** | 0.0035 | 0.0035 | 0.0030 | 0.0017 | 0.0024 | 0.0049 | 0.0082 | 0.0121 | 0.0166 | 0.0220 | 0.0294 |
| **22** | 0.0033 | 0.0033 | 0.0026 | 0.0013 | 0.0024 | 0.0050 | 0.0077 | 0.0111 | 0.0152 | 0.0204 | 0.0282 |
| **23** | 0.0026 | 0.0026 | 0.0022 | 0.0012 | 0.0019 | 0.0044 | 0.0072 | 0.0106 | 0.0149 | 0.0201 | 0.0270 |
| **24** | 0.0025 | 0.0025 | 0.0021 | 0.0010 | 0.0020 | 0.0045 | 0.0071 | 0.0106 | 0.0150 | 0.0204 | 0.0276 |
| **25** | 0.0025 | 0.0025 | 0.0020 | 0.0010 | 0.0020 | 0.0043 | 0.0071 | 0.0105 | 0.0148 | 0.0198 | 0.0273 |
| **26** | 0.0024 | 0.0024 | 0.0018 | 0.0008 | 0.0021 | 0.0042 | 0.0067 | 0.0099 | 0.0138 | 0.0186 | 0.0257 |
| **27** | 0.0021 | 0.0021 | 0.0016 | 0.0007 | 0.0020 | 0.0043 | 0.0067 | 0.0099 | 0.0137 | 0.0180 | 0.0250 |
| **28** | 0.0019 | 0.0018 | 0.0015 | 0.0006 | 0.0019 | 0.0039 | 0.0064 | 0.0094 | 0.0132 | 0.0178 | 0.0246 |
| **29** | 0.0019 | 0.0019 | 0.0014 | 0.0005 | 0.0020 | 0.0039 | 0.0062 | 0.0091 | 0.0127 | 0.0170 | 0.0232 |
| **30** | 0.0017 | 0.0017 | 0.0013 | 0.0004 | 0.0018 | 0.0039 | 0.0063 | 0.0093 | 0.0130 | 0.0174 | 0.0231 |
| **31** | 0.0018 | 0.0017 | 0.0013 | 0.0004 | 0.0020 | 0.0039 | 0.0061 | 0.0090 | 0.0125 | 0.0169 | 0.0223 |
| **32** | 0.0015 | 0.0015 | 0.0012 | 0.0004 | 0.0018 | 0.0036 | 0.0060 | 0.0087 | 0.0121 | 0.0167 | 0.0220 |
| **33** | 0.0014 | 0.0014 | 0.0011 | 0.0003 | 0.0017 | 0.0036 | 0.0058 | 0.0084 | 0.0118 | 0.0158 | 0.0212 |
| **34** | 0.0014 | 0.0014 | 0.0011 | 0.0004 | 0.0018 | 0.0037 | 0.0059 | 0.0086 | 0.0121 | 0.0162 | 0.0221 |
| **35** | 0.0013 | 0.0012 | 0.0010 | 0.0004 | 0.0017 | 0.0035 | 0.0057 | 0.0083 | 0.0116 | 0.0155 | 0.0218 |
| **36** | 0.0013 | 0.0013 | 0.0010 | 0.0005 | 0.0018 | 0.0037 | 0.0059 | 0.0088 | 0.0121 | 0.0162 | 0.0223 |
| **37** | 0.0013 | 0.0013 | 0.0009 | 0.0005 | 0.0018 | 0.0036 | 0.0057 | 0.0084 | 0.0117 | 0.0155 | 0.0211 |
| **38** | 0.0011 | 0.0011 | 0.0008 | 0.0005 | 0.0017 | 0.0034 | 0.0054 | 0.0079 | 0.0109 | 0.0150 | 0.0204 |
| **39** | 0.0012 | 0.0012 | 0.0009 | 0.0005 | 0.0018 | 0.0036 | 0.0056 | 0.0083 | 0.0115 | 0.0153 | 0.0205 |
| **40** | 0.0011 | 0.0011 | 0.0008 | 0.0005 | 0.0017 | 0.0035 | 0.0056 | 0.0081 | 0.0110 | 0.0146 | 0.0194 |
| **41** | 0.0010 | 0.0010 | 0.0007 | 0.0005 | 0.0016 | 0.0032 | 0.0051 | 0.0075 | 0.0105 | 0.0146 | 0.0194 |
| **42** | 0.0010 | 0.0010 | 0.0007 | 0.0004 | 0.0016 | 0.0032 | 0.0051 | 0.0074 | 0.0104 | 0.0142 | 0.0191 |
| **43** | 0.0010 | 0.0010 | 0.0007 | 0.0005 | 0.0017 | 0.0033 | 0.0052 | 0.0077 | 0.0106 | 0.0142 | 0.0192 |
| **44** | 0.0010 | 0.0010 | 0.0007 | 0.0005 | 0.0017 | 0.0034 | 0.0052 | 0.0077 | 0.0106 | 0.0144 | 0.0193 |
| **45** | 0.0009 | 0.0008 | 0.0006 | 0.0004 | 0.0016 | 0.0031 | 0.0050 | 0.0072 | 0.0101 | 0.0138 | 0.0194 |
| **46** | 0.0008 | 0.0008 | 0.0006 | 0.0004 | 0.0016 | 0.0031 | 0.0049 | 0.0070 | 0.0098 | 0.0130 | 0.0180 |
| **47** | 0.0009 | 0.0009 | 0.0006 | 0.0005 | 0.0017 | 0.0033 | 0.0052 | 0.0074 | 0.0103 | 0.0137 | 0.0184 |
| **48** | 0.0008 | 0.0007 | 0.0005 | 0.0004 | 0.0015 | 0.0030 | 0.0047 | 0.0069 | 0.0097 | 0.0135 | 0.0188 |
| **49** | 0.0008 | 0.0008 | 0.0005 | 0.0005 | 0.0016 | 0.0031 | 0.0048 | 0.0071 | 0.0099 | 0.0133 | 0.0180 |
| **50** | 0.0008 | 0.0007 | 0.0005 | 0.0004 | 0.0016 | 0.0031 | 0.0050 | 0.0073 | 0.0103 | 0.0138 | 0.0184 |
| **51** | 0.0007 | 0.0007 | 0.0005 | 0.0005 | 0.0015 | 0.0030 | 0.0047 | 0.0069 | 0.0095 | 0.0128 | 0.0172 |
| **52** | 0.0007 | 0.0007 | 0.0004 | 0.0004 | 0.0015 | 0.0029 | 0.0047 | 0.0068 | 0.0094 | 0.0128 | 0.0173 |
| **53** | 0.0007 | 0.0007 | 0.0004 | 0.0004 | 0.0015 | 0.0029 | 0.0046 | 0.0067 | 0.0094 | 0.0128 | 0.0174 |
| **54** | 0.0007 | 0.0007 | 0.0004 | 0.0005 | 0.0016 | 0.0030 | 0.0046 | 0.0067 | 0.0094 | 0.0126 | 0.0172 |
| **55** | 0.0006 | 0.0006 | 0.0004 | 0.0004 | 0.0015 | 0.0028 | 0.0045 | 0.0066 | 0.0092 | 0.0124 | 0.0169 |
| **56** | 0.0006 | 0.0006 | 0.0004 | 0.0004 | 0.0014 | 0.0028 | 0.0044 | 0.0066 | 0.0092 | 0.0125 | 0.0170 |
| **57** | 0.0006 | 0.0006 | 0.0004 | 0.0004 | 0.0015 | 0.0028 | 0.0045 | 0.0066 | 0.0092 | 0.0126 | 0.0172 |

**Table 2:** $\mu_P d(\beta; n, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $q$ / $n$ | **0.50** | **0.51** | **0.55** | **0.6** | **0.65** | **0.7** | **0.75** | **0.8** | **0.85** | **0.9** | **0.95** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0578 | 0.0578 | 0.0498 | 0.0284 | 0.0155 | 0.0615 | 0.1197 | 0.1899 | 0.2755 | 0.3700 | 0.5370 |
| **16** | 0.0545 | 0.0545 | 0.0464 | 0.0246 | 0.0173 | 0.0651 | 0.1230 | 0.1930 | 0.2759 | 0.3669 | 0.5544 |
| **17** | 0.0524 | 0.0524 | 0.0443 | 0.0220 | 0.0201 | 0.0695 | 0.1272 | 0.1985 | 0.2832 | 0.3731 | 0.5674 |
| **18** | 0.0505 | 0.0505 | 0.0424 | 0.0190 | 0.0223 | 0.0728 | 0.1312 | 0.2026 | 0.2847 | 0.3771 | 0.5784 |
| **19** | 0.0480 | 0.0480 | 0.0401 | 0.0168 | 0.0245 | 0.0739 | 0.1331 | 0.2031 | 0.2838 | 0.3759 | 0.5696 |
| **20** | 0.0482 | 0.0482 | 0.0400 | 0.0154 | 0.0265 | 0.0798 | 0.1405 | 0.2129 | 0.2943 | 0.3851 | 0.5632 |
| **21** | 0.0460 | 0.0460 | 0.0377 | 0.0131 | 0.0286 | 0.0813 | 0.1425 | 0.2141 | 0.2940 | 0.3851 | 0.5504 |
| **22** | 0.0448 | 0.0447 | 0.0361 | 0.0110 | 0.0305 | 0.0848 | 0.1462 | 0.2187 | 0.2983 | 0.3889 | 0.5487 |
| **23** | 0.0428 | 0.0427 | 0.0342 | 0.0095 | 0.0323 | 0.0862 | 0.1485 | 0.2209 | 0.3016 | 0.3908 | 0.5541 |
| **24** | 0.0404 | 0.0401 | 0.0318 | 0.0079 | 0.0337 | 0.0862 | 0.1474 | 0.2185 | 0.2991 | 0.3874 | 0.5571 |
| **25** | 0.0388 | 0.0385 | 0.0304 | 0.0068 | 0.0353 | 0.0873 | 0.1486 | 0.2197 | 0.3009 | 0.3892 | 0.5630 |
| **26** | 0.0387 | 0.0383 | 0.0297 | 0.0055 | 0.0386 | 0.0919 | 0.1534 | 0.2258 | 0.3064 | 0.3937 | 0.5684 |
| **27** | 0.0382 | 0.0377 | 0.0290 | 0.0046 | 0.0408 | 0.0946 | 0.1580 | 0.2306 | 0.3111 | 0.3982 | 0.5711 |
| **28** | 0.0355 | 0.0350 | 0.0268 | 0.0038 | 0.0400 | 0.0926 | 0.1545 | 0.2259 | 0.3059 | 0.3943 | 0.5669 |
| **29** | 0.0368 | 0.0363 | 0.0274 | 0.0036 | 0.0442 | 0.0995 | 0.1639 | 0.2375 | 0.3179 | 0.4041 | 0.5674 |
| **30** | 0.0342 | 0.0337 | 0.0254 | 0.0033 | 0.0430 | 0.0970 | 0.1604 | 0.2332 | 0.3122 | 0.3988 | 0.5584 |
| **31** | 0.0336 | 0.0331 | 0.0246 | 0.0035 | 0.0440 | 0.0986 | 0.1620 | 0.2345 | 0.3138 | 0.4003 | 0.5584 |
| **32** | 0.0331 | 0.0327 | 0.0239 | 0.0036 | 0.0458 | 0.1014 | 0.1656 | 0.2389 | 0.3180 | 0.4043 | 0.5557 |
| **33** | 0.0322 | 0.0317 | 0.0230 | 0.0039 | 0.0467 | 0.1020 | 0.1661 | 0.2394 | 0.3186 | 0.4050 | 0.5568 |
| **34** | 0.0320 | 0.0316 | 0.0225 | 0.0050 | 0.0487 | 0.1050 | 0.1697 | 0.2433 | 0.3231 | 0.4082 | 0.5613 |
| **35** | 0.0305 | 0.0301 | 0.0213 | 0.0051 | 0.0488 | 0.1044 | 0.1681 | 0.2414 | 0.3214 | 0.4065 | 0.5621 |
| **36** | 0.0310 | 0.0306 | 0.0214 | 0.0063 | 0.0516 | 0.1086 | 0.1744 | 0.2488 | 0.3286 | 0.4126 | 0.5670 |
| **37** | 0.0301 | 0.0297 | 0.0205 | 0.0070 | 0.0520 | 0.1091 | 0.1743 | 0.2483 | 0.3284 | 0.4125 | 0.5677 |
| **38** | 0.0294 | 0.0291 | 0.0198 | 0.0075 | 0.0531 | 0.1101 | 0.1762 | 0.2501 | 0.3302 | 0.4146 | 0.5670 |
| **39** | 0.0290 | 0.0287 | 0.0193 | 0.0085 | 0.0540 | 0.1119 | 0.1779 | 0.2522 | 0.3316 | 0.4157 | 0.5667 |
| **40** | 0.0289 | 0.0286 | 0.0190 | 0.0094 | 0.0557 | 0.1146 | 0.1812 | 0.2561 | 0.3358 | 0.4196 | 0.5638 |
| **41** | 0.0286 | 0.0283 | 0.0187 | 0.0102 | 0.0569 | 0.1161 | 0.1839 | 0.2586 | 0.3385 | 0.4222 | 0.5635 |
| **42** | 0.0275 | 0.0272 | 0.0177 | 0.0106 | 0.0565 | 0.1155 | 0.1821 | 0.2568 | 0.3362 | 0.4197 | 0.5620 |
| **43** | 0.0271 | 0.0268 | 0.0171 | 0.0114 | 0.0577 | 0.1164 | 0.1838 | 0.2590 | 0.3388 | 0.4226 | 0.5639 |
| **44** | 0.0271 | 0.0268 | 0.0168 | 0.0120 | 0.0594 | 0.1188 | 0.1865 | 0.2616 | 0.3414 | 0.4241 | 0.5653 |
| **45** | 0.0258 | 0.0255 | 0.0159 | 0.0126 | 0.0584 | 0.1169 | 0.1838 | 0.2589 | 0.3385 | 0.4219 | 0.5693 |
| **46** | 0.0262 | 0.0259 | 0.0158 | 0.0134 | 0.0613 | 0.1211 | 0.1898 | 0.2654 | 0.3455 | 0.4282 | 0.5703 |
| **47** | 0.0259 | 0.0257 | 0.0154 | 0.0142 | 0.0621 | 0.1223 | 0.1910 | 0.2669 | 0.3470 | 0.4290 | 0.5704 |
| **48** | 0.0249 | 0.0246 | 0.0146 | 0.0144 | 0.0619 | 0.1214 | 0.1900 | 0.2663 | 0.3458 | 0.4280 | 0.5678 |
| **49** | 0.0245 | 0.0241 | 0.0140 | 0.0149 | 0.0624 | 0.1221 | 0.1903 | 0.2660 | 0.3458 | 0.4278 | 0.5674 |
| **50** | 0.0241 | 0.0237 | 0.0136 | 0.0152 | 0.0625 | 0.1223 | 0.1907 | 0.2666 | 0.3463 | 0.4285 | 0.5685 |
| **51** | 0.0237 | 0.0233 | 0.0133 | 0.0159 | 0.0634 | 0.1231 | 0.1918 | 0.2675 | 0.3470 | 0.4298 | 0.5678 |
| **52** | 0.0232 | 0.0228 | 0.0128 | 0.0162 | 0.0635 | 0.1237 | 0.1923 | 0.2682 | 0.3480 | 0.4294 | 0.5656 |
| **53** | 0.0234 | 0.0229 | 0.0127 | 0.0171 | 0.0654 | 0.1263 | 0.1955 | 0.2719 | 0.3516 | 0.4335 | 0.5684 |
| **54** | 0.0235 | 0.0230 | 0.0126 | 0.0179 | 0.0673 | 0.1290 | 0.1991 | 0.2762 | 0.3563 | 0.4372 | 0.5707 |
| **55** | 0.0227 | 0.0222 | 0.0121 | 0.0180 | 0.0669 | 0.1278 | 0.1975 | 0.2747 | 0.3545 | 0.4360 | 0.5690 |
| **56** | 0.0220 | 0.0215 | 0.0116 | 0.0183 | 0.0662 | 0.1268 | 0.1959 | 0.2726 | 0.3520 | 0.4338 | 0.5689 |
| **57** | 0.0221 | 0.0216 | 0.0114 | 0.0189 | 0.0680 | 0.1291 | 0.1991 | 0.2760 | 0.3558 | 0.4375 | 0.5713 |

**Table 2a:** $\sigma_P d(\beta;\, n,\, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0340 | 0.0340 | 0.0286 | 0.0158 | 0.0145 | 0.0353 | 0.0562 | 0.0738 | 0.0842 | 0.0882 | 0.1465 |
| **16** | 0.0334 | 0.0334 | 0.0275 | 0.0142 | 0.0157 | 0.0377 | 0.0592 | 0.0744 | 0.0848 | 0.0832 | 0.1372 |
| **17** | 0.0302 | 0.0302 | 0.0254 | 0.0123 | 0.0158 | 0.0368 | 0.0560 | 0.0744 | 0.0844 | 0.0874 | 0.1393 |
| **18** | 0.0296 | 0.0296 | 0.0244 | 0.0105 | 0.0172 | 0.0398 | 0.0593 | 0.0768 | 0.0845 | 0.0859 | 0.1202 |
| **19** | 0.0289 | 0.0289 | 0.0235 | 0.0100 | 0.0186 | 0.0389 | 0.0596 | 0.0758 | 0.0842 | 0.0826 | 0.1058 |
| **20** | 0.0271 | 0.0271 | 0.0220 | 0.0098 | 0.0171 | 0.0402 | 0.0608 | 0.0765 | 0.0843 | 0.0782 | 0.0926 |
| **21** | 0.0266 | 0.0266 | 0.0216 | 0.0087 | 0.0188 | 0.0402 | 0.0607 | 0.0772 | 0.0844 | 0.0780 | 0.0827 |
| **22** | 0.0260 | 0.0260 | 0.0202 | 0.0072 | 0.0196 | 0.0418 | 0.0611 | 0.0762 | 0.0835 | 0.0765 | 0.0837 |
| **23** | 0.0223 | 0.0223 | 0.0175 | 0.0066 | 0.0178 | 0.0388 | 0.0577 | 0.0726 | 0.0799 | 0.0728 | 0.0830 |
| **24** | 0.0220 | 0.0219 | 0.0170 | 0.0056 | 0.0187 | 0.0406 | 0.0587 | 0.0746 | 0.0820 | 0.0752 | 0.0864 |
| **25** | 0.0209 | 0.0208 | 0.0162 | 0.0052 | 0.0187 | 0.0387 | 0.0573 | 0.0719 | 0.0789 | 0.0720 | 0.0803 |
| **26** | 0.0215 | 0.0214 | 0.0161 | 0.0041 | 0.0213 | 0.0422 | 0.0609 | 0.0761 | 0.0821 | 0.0747 | 0.0721 |
| **27** | 0.0206 | 0.0203 | 0.0151 | 0.0037 | 0.0212 | 0.0425 | 0.0616 | 0.0761 | 0.0826 | 0.0745 | 0.0701 |
| **28** | 0.0186 | 0.0183 | 0.0140 | 0.0033 | 0.0204 | 0.0406 | 0.0591 | 0.0734 | 0.0798 | 0.0716 | 0.0660 |
| **29** | 0.0192 | 0.0189 | 0.0139 | 0.0029 | 0.0221 | 0.0428 | 0.0605 | 0.0748 | 0.0812 | 0.0731 | 0.0590 |
| **30** | 0.0169 | 0.0167 | 0.0124 | 0.0027 | 0.0203 | 0.0401 | 0.0578 | 0.0715 | 0.0785 | 0.0717 | 0.0587 |
| **31** | 0.0182 | 0.0179 | 0.0128 | 0.0034 | 0.0225 | 0.0420 | 0.0603 | 0.0738 | 0.0797 | 0.0725 | 0.0563 |
| **32** | 0.0170 | 0.0167 | 0.0121 | 0.0034 | 0.0220 | 0.0415 | 0.0597 | 0.0728 | 0.0791 | 0.0716 | 0.0560 |
| **33** | 0.0159 | 0.0157 | 0.0113 | 0.0034 | 0.0214 | 0.0417 | 0.0593 | 0.0730 | 0.0796 | 0.0718 | 0.0566 |
| **34** | 0.0164 | 0.0161 | 0.0111 | 0.0044 | 0.0230 | 0.0434 | 0.0612 | 0.0746 | 0.0808 | 0.0734 | 0.0606 |
| **35** | 0.0151 | 0.0148 | 0.0105 | 0.0043 | 0.0221 | 0.0423 | 0.0598 | 0.0733 | 0.0792 | 0.0715 | 0.0589 |
| **36** | 0.0154 | 0.0152 | 0.0104 | 0.0049 | 0.0235 | 0.0431 | 0.0605 | 0.0741 | 0.0799 | 0.0738 | 0.0595 |
| **37** | 0.0155 | 0.0152 | 0.0104 | 0.0053 | 0.0240 | 0.0444 | 0.0619 | 0.0747 | 0.0811 | 0.0738 | 0.0541 |
| **38** | 0.0144 | 0.0143 | 0.0096 | 0.0050 | 0.0239 | 0.0429 | 0.0601 | 0.0726 | 0.0784 | 0.0710 | 0.0504 |
| **39** | 0.0148 | 0.0145 | 0.0097 | 0.0056 | 0.0246 | 0.0450 | 0.0621 | 0.0751 | 0.0812 | 0.0737 | 0.0513 |
| **40** | 0.0140 | 0.0138 | 0.0090 | 0.0058 | 0.0242 | 0.0440 | 0.0615 | 0.0745 | 0.0801 | 0.0733 | 0.0486 |
| **41** | 0.0138 | 0.0136 | 0.0089 | 0.0059 | 0.0245 | 0.0447 | 0.0620 | 0.0749 | 0.0804 | 0.0738 | 0.0470 |
| **42** | 0.0135 | 0.0134 | 0.0085 | 0.0059 | 0.0249 | 0.0447 | 0.0616 | 0.0740 | 0.0797 | 0.0734 | 0.0491 |
| **43** | 0.0128 | 0.0127 | 0.0081 | 0.0062 | 0.0246 | 0.0437 | 0.0607 | 0.0730 | 0.0786 | 0.0716 | 0.0521 |
| **44** | 0.0134 | 0.0133 | 0.0082 | 0.0064 | 0.0262 | 0.0470 | 0.0642 | 0.0775 | 0.0832 | 0.0768 | 0.0525 |
| **45** | 0.0122 | 0.0121 | 0.0075 | 0.0066 | 0.0249 | 0.0444 | 0.0614 | 0.0735 | 0.0786 | 0.0721 | 0.0533 |
| **46** | 0.0122 | 0.0120 | 0.0072 | 0.0069 | 0.0257 | 0.0449 | 0.0619 | 0.0742 | 0.0795 | 0.0729 | 0.0479 |
| **47** | 0.0127 | 0.0126 | 0.0075 | 0.0072 | 0.0265 | 0.0457 | 0.0622 | 0.0745 | 0.0796 | 0.0740 | 0.0485 |
| **48** | 0.0110 | 0.0108 | 0.0063 | 0.0068 | 0.0243 | 0.0423 | 0.0575 | 0.0690 | 0.0739 | 0.0688 | 0.0466 |
| **49** | 0.0113 | 0.0111 | 0.0063 | 0.0071 | 0.0255 | 0.0446 | 0.0602 | 0.0720 | 0.0776 | 0.0713 | 0.0444 |
| **50** | 0.0109 | 0.0107 | 0.0061 | 0.0071 | 0.0252 | 0.0440 | 0.0605 | 0.0728 | 0.0785 | 0.0728 | 0.0458 |
| **51** | 0.0109 | 0.0107 | 0.0061 | 0.0078 | 0.0259 | 0.0449 | 0.0613 | 0.0732 | 0.0786 | 0.0721 | 0.0445 |
| **52** | 0.0101 | 0.0100 | 0.0057 | 0.0075 | 0.0248 | 0.0432 | 0.0593 | 0.0714 | 0.0765 | 0.0716 | 0.0449 |
| **53** | 0.0107 | 0.0105 | 0.0059 | 0.0079 | 0.0264 | 0.0456 | 0.0622 | 0.0740 | 0.0791 | 0.0733 | 0.0473 |
| **54** | 0.0111 | 0.0108 | 0.0059 | 0.0085 | 0.0276 | 0.0466 | 0.0627 | 0.0744 | 0.0790 | 0.0734 | 0.0477 |
| **55** | 0.0100 | 0.0098 | 0.0053 | 0.0080 | 0.0261 | 0.0442 | 0.0600 | 0.0718 | 0.0768 | 0.0710 | 0.0458 |
| **56** | 0.0097 | 0.0095 | 0.0052 | 0.0081 | 0.0252 | 0.0438 | 0.0594 | 0.0711 | 0.0766 | 0.0709 | 0.0449 |
| **57** | 0.0099 | 0.0097 | 0.0051 | 0.0084 | 0.0267 | 0.0450 | 0.0608 | 0.0726 | 0.0775 | 0.0717 | 0.0448 |

**Table 3:** $\mu_U D(\beta; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0507 | 0.0500 | 0.0420 | 0.0275 | 0.0160 | 0.0278 | 0.0499 | 0.0792 | 0.1173 | 0.1664 | 0.2320 |
| **16** | 0.0467 | 0.0461 | 0.0390 | 0.0249 | 0.0140 | 0.0271 | 0.0497 | 0.0804 | 0.1178 | 0.1656 | 0.2311 |
| **17** | 0.0434 | 0.0429 | 0.0362 | 0.0225 | 0.0121 | 0.0263 | 0.0498 | 0.0798 | 0.1178 | 0.1661 | 0.2319 |
| **18** | 0.0406 | 0.0402 | 0.0338 | 0.0204 | 0.0105 | 0.0260 | 0.0502 | 0.0800 | 0.1169 | 0.1644 | 0.2299 |
| **19** | 0.0359 | 0.0356 | 0.0301 | 0.0183 | 0.0099 | 0.0248 | 0.0487 | 0.0789 | 0.1158 | 0.1625 | 0.2278 |
| **20** | 0.0347 | 0.0345 | 0.0288 | 0.0164 | 0.0093 | 0.0260 | 0.0499 | 0.0794 | 0.1162 | 0.1632 | 0.2277 |
| **21** | 0.0304 | 0.0302 | 0.0253 | 0.0147 | 0.0091 | 0.0251 | 0.0487 | 0.0784 | 0.1152 | 0.1618 | 0.2265 |
| **22** | 0.0299 | 0.0296 | 0.0247 | 0.0134 | 0.0087 | 0.0259 | 0.0494 | 0.0793 | 0.1163 | 0.1634 | 0.2278 |
| **23** | 0.0295 | 0.0293 | 0.0241 | 0.0124 | 0.0088 | 0.0267 | 0.0508 | 0.0804 | 0.1167 | 0.1632 | 0.2275 |
| **24** | 0.0260 | 0.0258 | 0.0214 | 0.0112 | 0.0086 | 0.0261 | 0.0499 | 0.0798 | 0.1167 | 0.1632 | 0.2267 |
| **25** | 0.0247 | 0.0245 | 0.0203 | 0.0103 | 0.0087 | 0.0263 | 0.0500 | 0.0793 | 0.1159 | 0.1626 | 0.2265 |
| **26** | 0.0232 | 0.0230 | 0.0190 | 0.0093 | 0.0089 | 0.0265 | 0.0501 | 0.0795 | 0.1162 | 0.1628 | 0.2270 |
| **27** | 0.0217 | 0.0215 | 0.0178 | 0.0086 | 0.0090 | 0.0264 | 0.0501 | 0.0795 | 0.1160 | 0.1626 | 0.2270 |
| **28** | 0.0212 | 0.0211 | 0.0172 | 0.0077 | 0.0093 | 0.0269 | 0.0504 | 0.0797 | 0.1162 | 0.1630 | 0.2275 |
| **29** | 0.0209 | 0.0207 | 0.0168 | 0.0072 | 0.0096 | 0.0277 | 0.0512 | 0.0804 | 0.1166 | 0.1629 | 0.2267 |
| **30** | 0.0196 | 0.0194 | 0.0157 | 0.0064 | 0.0096 | 0.0276 | 0.0507 | 0.0797 | 0.1159 | 0.1621 | 0.2259 |
| **31** | 0.0190 | 0.0189 | 0.0152 | 0.0058 | 0.0101 | 0.0280 | 0.0511 | 0.0800 | 0.1164 | 0.1629 | 0.2267 |
| **32** | 0.0182 | 0.0180 | 0.0145 | 0.0053 | 0.0102 | 0.0282 | 0.0513 | 0.0800 | 0.1160 | 0.1622 | 0.2259 |
| **33** | 0.0179 | 0.0177 | 0.0141 | 0.0048 | 0.0107 | 0.0288 | 0.0517 | 0.0804 | 0.1165 | 0.1627 | 0.2265 |
| **34** | 0.0171 | 0.0170 | 0.0134 | 0.0043 | 0.0109 | 0.0288 | 0.0518 | 0.0804 | 0.1162 | 0.1623 | 0.2259 |
| **35** | 0.0163 | 0.0161 | 0.0128 | 0.0041 | 0.0110 | 0.0290 | 0.0520 | 0.0807 | 0.1166 | 0.1628 | 0.2262 |
| **36** | 0.0155 | 0.0154 | 0.0122 | 0.0036 | 0.0112 | 0.0292 | 0.0521 | 0.0806 | 0.1163 | 0.1623 | 0.2259 |
| **37** | 0.0155 | 0.0154 | 0.0121 | 0.0034 | 0.0116 | 0.0296 | 0.0526 | 0.0813 | 0.1170 | 0.1630 | 0.2265 |
| **38** | 0.0146 | 0.0145 | 0.0113 | 0.0029 | 0.0117 | 0.0296 | 0.0523 | 0.0808 | 0.1166 | 0.1626 | 0.2262 |
| **39** | 0.0142 | 0.0140 | 0.0109 | 0.0027 | 0.0117 | 0.0295 | 0.0523 | 0.0808 | 0.1166 | 0.1626 | 0.2257 |
| **40** | 0.0141 | 0.0140 | 0.0107 | 0.0024 | 0.0122 | 0.0299 | 0.0525 | 0.0807 | 0.1163 | 0.1621 | 0.2255 |
| **41** | 0.0140 | 0.0139 | 0.0106 | 0.0022 | 0.0126 | 0.0303 | 0.0528 | 0.0811 | 0.1166 | 0.1624 | 0.2258 |
| **42** | 0.0128 | 0.0127 | 0.0098 | 0.0021 | 0.0124 | 0.0300 | 0.0526 | 0.0810 | 0.1168 | 0.1628 | 0.2260 |
| **43** | 0.0129 | 0.0127 | 0.0096 | 0.0019 | 0.0127 | 0.0303 | 0.0527 | 0.0809 | 0.1164 | 0.1623 | 0.2259 |
| **44** | 0.0125 | 0.0124 | 0.0093 | 0.0018 | 0.0129 | 0.0305 | 0.0530 | 0.0813 | 0.1169 | 0.1630 | 0.2264 |
| **45** | 0.0123 | 0.0122 | 0.0091 | 0.0018 | 0.0131 | 0.0307 | 0.0532 | 0.0813 | 0.1169 | 0.1627 | 0.2259 |
| **46** | 0.0119 | 0.0118 | 0.0087 | 0.0017 | 0.0133 | 0.0308 | 0.0532 | 0.0813 | 0.1168 | 0.1626 | 0.2259 |
| **47** | 0.0114 | 0.0113 | 0.0084 | 0.0016 | 0.0133 | 0.0307 | 0.0531 | 0.0813 | 0.1169 | 0.1628 | 0.2262 |
| **48** | 0.0112 | 0.0111 | 0.0082 | 0.0016 | 0.0135 | 0.0309 | 0.0532 | 0.0813 | 0.1168 | 0.1626 | 0.2260 |
| **49** | 0.0111 | 0.0109 | 0.0080 | 0.0017 | 0.0137 | 0.0311 | 0.0535 | 0.0815 | 0.1169 | 0.1627 | 0.2261 |
| **50** | 0.0107 | 0.0106 | 0.0077 | 0.0017 | 0.0138 | 0.0312 | 0.0536 | 0.0817 | 0.1170 | 0.1627 | 0.2258 |
| **51** | 0.0105 | 0.0104 | 0.0075 | 0.0017 | 0.0139 | 0.0313 | 0.0535 | 0.0816 | 0.1171 | 0.1628 | 0.2259 |
| **52** | 0.0102 | 0.0101 | 0.0073 | 0.0018 | 0.0141 | 0.0315 | 0.0539 | 0.0821 | 0.1175 | 0.1632 | 0.2262 |
| **53** | 0.0100 | 0.0099 | 0.0071 | 0.0019 | 0.0142 | 0.0315 | 0.0538 | 0.0818 | 0.1170 | 0.1627 | 0.2259 |
| **54** | 0.0097 | 0.0096 | 0.0068 | 0.0019 | 0.0143 | 0.0316 | 0.0538 | 0.0819 | 0.1172 | 0.1628 | 0.2258 |
| **55** | 0.0096 | 0.0095 | 0.0067 | 0.0020 | 0.0145 | 0.0319 | 0.0542 | 0.0822 | 0.1175 | 0.1631 | 0.2261 |
| **56** | 0.0092 | 0.0091 | 0.0064 | 0.0020 | 0.0145 | 0.0318 | 0.0540 | 0.0820 | 0.1173 | 0.1627 | 0.2258 |
| **57** | 0.0091 | 0.0090 | 0.0063 | 0.0022 | 0.0145 | 0.0317 | 0.0539 | 0.0818 | 0.1170 | 0.1624 | 0.2254 |
| **58** | 0.0089 | 0.0088 | 0.0061 | 0.0023 | 0.0147 | 0.0320 | 0.0542 | 0.0821 | 0.1173 | 0.1627 | 0.2255 |
| **59** | 0.0088 | 0.0087 | 0.0060 | 0.0024 | 0.0148 | 0.0320 | 0.0541 | 0.0819 | 0.1170 | 0.1625 | 0.2255 |
| **60** | 0.0086 | 0.0085 | 0.0058 | 0.0025 | 0.0149 | 0.0322 | 0.0543 | 0.0822 | 0.1173 | 0.1627 | 0.2256 |

**Table 3a:** $\sigma_U D(\beta; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $n$ \ $q$ | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0649 | 0.0645 | 0.0525 | 0.0283 | 0.0246 | 0.0258 | 0.0342 | 0.0416 | 0.0480 | 0.0531 | 0.0553 |
| **16** | 0.0591 | 0.0586 | 0.0476 | 0.0248 | 0.0204 | 0.0240 | 0.0335 | 0.0409 | 0.0471 | 0.0513 | 0.0533 |
| **17** | 0.0580 | 0.0575 | 0.0469 | 0.0258 | 0.0182 | 0.0226 | 0.0324 | 0.0392 | 0.0449 | 0.0495 | 0.0520 |
| **18** | 0.0568 | 0.0564 | 0.0453 | 0.0229 | 0.0142 | 0.0221 | 0.0325 | 0.0392 | 0.0447 | 0.0490 | 0.0510 |
| **19** | 0.0513 | 0.0509 | 0.0410 | 0.0227 | 0.0146 | 0.0206 | 0.0300 | 0.0375 | 0.0444 | 0.0495 | 0.0508 |
| **20** | 0.0489 | 0.0484 | 0.0379 | 0.0177 | 0.0125 | 0.0211 | 0.0295 | 0.0359 | 0.0414 | 0.0465 | 0.0492 |
| **21** | 0.0408 | 0.0405 | 0.0320 | 0.0161 | 0.0131 | 0.0190 | 0.0270 | 0.0338 | 0.0394 | 0.0445 | 0.0479 |
| **22** | 0.0407 | 0.0402 | 0.0308 | 0.0136 | 0.0116 | 0.0196 | 0.0272 | 0.0331 | 0.0392 | 0.0450 | 0.0473 |
| **23** | 0.0425 | 0.0420 | 0.0317 | 0.0131 | 0.0104 | 0.0199 | 0.0272 | 0.0329 | 0.0382 | 0.0429 | 0.0459 |
| **24** | 0.0338 | 0.0334 | 0.0253 | 0.0114 | 0.0101 | 0.0180 | 0.0249 | 0.0312 | 0.0367 | 0.0414 | 0.0444 |
| **25** | 0.0342 | 0.0338 | 0.0252 | 0.0110 | 0.0096 | 0.0177 | 0.0248 | 0.0312 | 0.0368 | 0.0419 | 0.0451 |
| **26** | 0.0296 | 0.0293 | 0.0216 | 0.0098 | 0.0102 | 0.0171 | 0.0236 | 0.0296 | 0.0356 | 0.0408 | 0.0440 |
| **27** | 0.0269 | 0.0265 | 0.0197 | 0.0093 | 0.0100 | 0.0160 | 0.0224 | 0.0288 | 0.0346 | 0.0394 | 0.0427 |
| **28** | 0.0266 | 0.0263 | 0.0190 | 0.0082 | 0.0093 | 0.0159 | 0.0218 | 0.0277 | 0.0336 | 0.0388 | 0.0428 |
| **29** | 0.0278 | 0.0274 | 0.0196 | 0.0077 | 0.0093 | 0.0164 | 0.0222 | 0.0275 | 0.0330 | 0.0378 | 0.0421 |
| **30** | 0.0252 | 0.0249 | 0.0178 | 0.0068 | 0.0087 | 0.0157 | 0.0214 | 0.0268 | 0.0325 | 0.0375 | 0.0411 |
| **31** | 0.0232 | 0.0229 | 0.0162 | 0.0059 | 0.0089 | 0.0156 | 0.0208 | 0.0257 | 0.0311 | 0.0366 | 0.0408 |
| **32** | 0.0220 | 0.0217 | 0.0153 | 0.0053 | 0.0084 | 0.0151 | 0.0204 | 0.0253 | 0.0302 | 0.0352 | 0.0394 |
| **33** | 0.0225 | 0.0221 | 0.0154 | 0.0049 | 0.0086 | 0.0152 | 0.0201 | 0.0246 | 0.0296 | 0.0346 | 0.0388 |
| **34** | 0.0216 | 0.0212 | 0.0145 | 0.0047 | 0.0086 | 0.0148 | 0.0198 | 0.0245 | 0.0291 | 0.0336 | 0.0373 |
| **35** | 0.0202 | 0.0199 | 0.0134 | 0.0052 | 0.0082 | 0.0139 | 0.0192 | 0.0239 | 0.0285 | 0.0334 | 0.0378 |
| **36** | 0.0178 | 0.0175 | 0.0119 | 0.0047 | 0.0079 | 0.0135 | 0.0187 | 0.0235 | 0.0280 | 0.0325 | 0.0372 |
| **37** | 0.0195 | 0.0192 | 0.0128 | 0.0044 | 0.0081 | 0.0139 | 0.0190 | 0.0239 | 0.0283 | 0.0328 | 0.0368 |
| **38** | 0.0149 | 0.0146 | 0.0101 | 0.0034 | 0.0077 | 0.0130 | 0.0176 | 0.0221 | 0.0269 | 0.0315 | 0.0357 |
| **39** | 0.0171 | 0.0168 | 0.0111 | 0.0036 | 0.0076 | 0.0129 | 0.0178 | 0.0225 | 0.0274 | 0.0324 | 0.0362 |
| **40** | 0.0162 | 0.0159 | 0.0104 | 0.0029 | 0.0080 | 0.0133 | 0.0178 | 0.0222 | 0.0266 | 0.0308 | 0.0347 |
| **41** | 0.0170 | 0.0167 | 0.0108 | 0.0023 | 0.0083 | 0.0134 | 0.0175 | 0.0215 | 0.0258 | 0.0306 | 0.0351 |
| **42** | 0.0115 | 0.0113 | 0.0077 | 0.0031 | 0.0070 | 0.0119 | 0.0165 | 0.0211 | 0.0257 | 0.0304 | 0.0348 |
| **43** | 0.0151 | 0.0148 | 0.0094 | 0.0022 | 0.0076 | 0.0124 | 0.0167 | 0.0208 | 0.0250 | 0.0293 | 0.0335 |
| **44** | 0.0127 | 0.0124 | 0.0080 | 0.0025 | 0.0074 | 0.0119 | 0.0161 | 0.0203 | 0.0246 | 0.0290 | 0.0327 |
| **45** | 0.0133 | 0.0131 | 0.0082 | 0.0026 | 0.0076 | 0.0123 | 0.0164 | 0.0203 | 0.0244 | 0.0285 | 0.0323 |
| **46** | 0.0124 | 0.0121 | 0.0075 | 0.0024 | 0.0073 | 0.0117 | 0.0157 | 0.0198 | 0.0239 | 0.0281 | 0.0320 |
| **47** | 0.0093 | 0.0091 | 0.0063 | 0.0019 | 0.0067 | 0.0112 | 0.0153 | 0.0197 | 0.0243 | 0.0289 | 0.0327 |
| **48** | 0.0110 | 0.0108 | 0.0067 | 0.0020 | 0.0071 | 0.0113 | 0.0152 | 0.0191 | 0.0230 | 0.0271 | 0.0310 |
| **49** | 0.0099 | 0.0097 | 0.0062 | 0.0020 | 0.0069 | 0.0111 | 0.0152 | 0.0192 | 0.0233 | 0.0274 | 0.0316 |
| **50** | 0.0094 | 0.0092 | 0.0059 | 0.0018 | 0.0068 | 0.0110 | 0.0150 | 0.0192 | 0.0235 | 0.0277 | 0.0313 |
| **51** | 0.0097 | 0.0096 | 0.0060 | 0.0019 | 0.0069 | 0.0112 | 0.0150 | 0.0190 | 0.0230 | 0.0270 | 0.0307 |
| **52** | 0.0086 | 0.0085 | 0.0055 | 0.0019 | 0.0065 | 0.0108 | 0.0148 | 0.0187 | 0.0227 | 0.0267 | 0.0304 |
| **53** | 0.0080 | 0.0078 | 0.0050 | 0.0020 | 0.0065 | 0.0105 | 0.0144 | 0.0182 | 0.0219 | 0.0258 | 0.0298 |
| **54** | 0.0070 | 0.0069 | 0.0045 | 0.0018 | 0.0062 | 0.0101 | 0.0139 | 0.0178 | 0.0220 | 0.0261 | 0.0300 |
| **55** | 0.0072 | 0.0071 | 0.0045 | 0.0019 | 0.0062 | 0.0102 | 0.0138 | 0.0174 | 0.0212 | 0.0252 | 0.0294 |
| **56** | 0.0063 | 0.0062 | 0.0040 | 0.0017 | 0.0057 | 0.0095 | 0.0133 | 0.0170 | 0.0209 | 0.0248 | 0.0286 |
| **57** | 0.0067 | 0.0066 | 0.0042 | 0.0018 | 0.0060 | 0.0100 | 0.0140 | 0.0179 | 0.0218 | 0.0255 | 0.0288 |
| **58** | 0.0060 | 0.0059 | 0.0038 | 0.0019 | 0.0057 | 0.0096 | 0.0134 | 0.0171 | 0.0210 | 0.0250 | 0.0287 |
| **59** | 0.0064 | 0.0063 | 0.0038 | 0.0019 | 0.0059 | 0.0096 | 0.0133 | 0.0169 | 0.0205 | 0.0243 | 0.0280 |
| **60** | 0.0060 | 0.0059 | 0.0037 | 0.0019 | 0.0058 | 0.0096 | 0.0133 | 0.0169 | 0.0205 | 0.0242 | 0.0279 |

**Table 4:** $\mu_U d(\beta; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.2380 | 0.2116 | 0.1842 | 0.1488 | 0.1370 | 0.1793 | 0.2639 | 0.3667 | 0.5451 | 0.9039 | 1.8070 |
| **16** | 0.2036 | 0.1885 | 0.1635 | 0.1239 | 0.1064 | 0.1559 | 0.2356 | 0.3493 | 0.4914 | 0.8225 | 1.6838 |
| **17** | 0.1841 | 0.1758 | 0.1507 | 0.1086 | 0.0906 | 0.1379 | 0.2234 | 0.3293 | 0.4857 | 0.7601 | 1.6640 |
| **18** | 0.1684 | 0.1607 | 0.1380 | 0.0894 | 0.0734 | 0.1268 | 0.2192 | 0.3297 | 0.4533 | 0.7120 | 1.5392 |
| **19** | 0.1496 | 0.1449 | 0.1240 | 0.0768 | 0.0618 | 0.1197 | 0.2122 | 0.3172 | 0.4405 | 0.6879 | 1.5328 |
| **20** | 0.1446 | 0.1414 | 0.1198 | 0.0678 | 0.0533 | 0.1207 | 0.2172 | 0.3170 | 0.4394 | 0.6644 | 1.4250 |
| **21** | 0.1319 | 0.1303 | 0.1088 | 0.0591 | 0.0503 | 0.1202 | 0.2159 | 0.3178 | 0.4296 | 0.6508 | 1.4118 |
| **22** | 0.1298 | 0.1278 | 0.1065 | 0.0535 | 0.0470 | 0.1223 | 0.2181 | 0.3195 | 0.4310 | 0.6317 | 1.3786 |
| **23** | 0.1292 | 0.1280 | 0.1058 | 0.0498 | 0.0470 | 0.1276 | 0.2267 | 0.3275 | 0.4312 | 0.6185 | 1.3122 |
| **24** | 0.1191 | 0.1180 | 0.0974 | 0.0437 | 0.0470 | 0.1280 | 0.2264 | 0.3274 | 0.4302 | 0.6116 | 1.2766 |
| **25** | 0.1146 | 0.1136 | 0.0936 | 0.0404 | 0.0471 | 0.1300 | 0.2280 | 0.3282 | 0.4305 | 0.5985 | 1.2572 |
| **26** | 0.1110 | 0.1099 | 0.0899 | 0.0367 | 0.0485 | 0.1332 | 0.2316 | 0.3323 | 0.4324 | 0.5918 | 1.2416 |
| **27** | 0.1061 | 0.1052 | 0.0858 | 0.0335 | 0.0494 | 0.1353 | 0.2341 | 0.3349 | 0.4341 | 0.5899 | 1.2168 |
| **28** | 0.1060 | 0.1051 | 0.0848 | 0.0306 | 0.0523 | 0.1404 | 0.2398 | 0.3406 | 0.4396 | 0.5898 | 1.2008 |
| **29** | 0.1051 | 0.1042 | 0.0836 | 0.0282 | 0.0549 | 0.1450 | 0.2451 | 0.3461 | 0.4444 | 0.5880 | 1.1841 |
| **30** | 0.1010 | 0.1001 | 0.0800 | 0.0252 | 0.0562 | 0.1471 | 0.2463 | 0.3467 | 0.4446 | 0.5852 | 1.1791 |
| **31** | 0.0998 | 0.0989 | 0.0784 | 0.0226 | 0.0591 | 0.1505 | 0.2502 | 0.3508 | 0.4486 | 0.5865 | 1.1642 |
| **32** | 0.0971 | 0.0963 | 0.0761 | 0.0207 | 0.0611 | 0.1537 | 0.2540 | 0.3547 | 0.4522 | 0.5849 | 1.1578 |
| **33** | 0.0968 | 0.0959 | 0.0751 | 0.0189 | 0.0641 | 0.1579 | 0.2581 | 0.3588 | 0.4561 | 0.5876 | 1.1458 |
| **34** | 0.0945 | 0.0936 | 0.0728 | 0.0173 | 0.0665 | 0.1603 | 0.2607 | 0.3610 | 0.4580 | 0.5884 | 1.1423 |
| **35** | 0.0917 | 0.0908 | 0.0704 | 0.0163 | 0.0684 | 0.1630 | 0.2645 | 0.3655 | 0.4623 | 0.5900 | 1.1391 |
| **36** | 0.0894 | 0.0885 | 0.0684 | 0.0150 | 0.0700 | 0.1651 | 0.2666 | 0.3675 | 0.4640 | 0.5904 | 1.1379 |
| **37** | 0.0895 | 0.0886 | 0.0679 | 0.0143 | 0.0728 | 0.1690 | 0.2710 | 0.3722 | 0.4685 | 0.5936 | 1.1347 |
| **38** | 0.0863 | 0.0854 | 0.0650 | 0.0127 | 0.0744 | 0.1700 | 0.2715 | 0.3724 | 0.4689 | 0.5937 | 1.1302 |
| **39** | 0.0837 | 0.0828 | 0.0627 | 0.0121 | 0.0750 | 0.1702 | 0.2717 | 0.3727 | 0.4691 | 0.5944 | 1.1257 |
| **40** | 0.0841 | 0.0831 | 0.0622 | 0.0112 | 0.0780 | 0.1738 | 0.2752 | 0.3757 | 0.4716 | 0.5946 | 1.1257 |
| **41** | 0.0849 | 0.0840 | 0.0623 | 0.0110 | 0.0815 | 0.1788 | 0.2809 | 0.3818 | 0.4775 | 0.5985 | 1.1284 |
| **42** | 0.0795 | 0.0787 | 0.0585 | 0.0109 | 0.0809 | 0.1772 | 0.2792 | 0.3802 | 0.4761 | 0.5988 | 1.1279 |
| **43** | 0.0795 | 0.0786 | 0.0577 | 0.0103 | 0.0829 | 0.1793 | 0.2810 | 0.3818 | 0.4777 | 0.5990 | 1.1280 |
| **44** | 0.0790 | 0.0781 | 0.0570 | 0.0105 | 0.0858 | 0.1833 | 0.2860 | 0.3871 | 0.4828 | 0.6018 | 1.1279 |
| **45** | 0.0785 | 0.0775 | 0.0561 | 0.0108 | 0.0881 | 0.1861 | 0.2888 | 0.3899 | 0.4851 | 0.6032 | 1.1272 |
| **46** | 0.0766 | 0.0756 | 0.0544 | 0.0106 | 0.0889 | 0.1868 | 0.2895 | 0.3904 | 0.4855 | 0.6036 | 1.1285 |
| **47** | 0.0750 | 0.0741 | 0.0532 | 0.0113 | 0.0906 | 0.1888 | 0.2916 | 0.3926 | 0.4878 | 0.6055 | 1.1304 |
| **48** | 0.0738 | 0.0729 | 0.0518 | 0.0116 | 0.0918 | 0.1898 | 0.2926 | 0.3936 | 0.4885 | 0.6062 | 1.1310 |
| **49** | 0.0740 | 0.0731 | 0.0515 | 0.0125 | 0.0946 | 0.1937 | 0.2971 | 0.3981 | 0.4927 | 0.6088 | 1.1322 |
| **50** | 0.0722 | 0.0713 | 0.0499 | 0.0132 | 0.0953 | 0.1942 | 0.2976 | 0.3988 | 0.4936 | 0.6091 | 1.1321 |
| **51** | 0.0710 | 0.0701 | 0.0487 | 0.0137 | 0.0963 | 0.1948 | 0.2978 | 0.3987 | 0.4935 | 0.6097 | 1.1333 |
| **52** | 0.0700 | 0.0691 | 0.0479 | 0.0144 | 0.0979 | 0.1974 | 0.3011 | 0.4023 | 0.4969 | 0.6118 | 1.1358 |
| **53** | 0.0689 | 0.0680 | 0.0467 | 0.0152 | 0.0992 | 0.1985 | 0.3022 | 0.4032 | 0.4977 | 0.6121 | 1.1355 |
| **54** | 0.0683 | 0.0674 | 0.0459 | 0.0163 | 0.1013 | 0.2012 | 0.3051 | 0.4063 | 0.5006 | 0.6135 | 1.1364 |
| **55** | 0.0682 | 0.0673 | 0.0455 | 0.0172 | 0.1035 | 0.2042 | 0.3085 | 0.4097 | 0.5039 | 0.6162 | 1.1380 |
| **56** | 0.0659 | 0.0650 | 0.0438 | 0.0176 | 0.1033 | 0.2037 | 0.3080 | 0.4094 | 0.5036 | 0.6158 | 1.1388 |
| **57** | 0.0650 | 0.0641 | 0.0428 | 0.0186 | 0.1041 | 0.2042 | 0.3083 | 0.4093 | 0.5034 | 0.6149 | 1.1386 |
| **58** | 0.0649 | 0.0640 | 0.0424 | 0.0198 | 0.1066 | 0.2077 | 0.3125 | 0.4140 | 0.5078 | 0.6175 | 1.1395 |
| **59** | 0.0642 | 0.0633 | 0.0415 | 0.0206 | 0.1077 | 0.2085 | 0.3128 | 0.4138 | 0.5075 | 0.6184 | 1.1408 |
| **60** | 0.0634 | 0.0624 | 0.0408 | 0.0215 | 0.1089 | 0.2104 | 0.3155 | 0.4168 | 0.5104 | 0.6195 | 1.1420 |

**Table 4a:** $\sigma_U d(\beta; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.1965 | 0.1838 | 0.1507 | 0.1296 | 0.1419 | 0.1553 | 0.1804 | 0.2163 | 0.3275 | 0.7686 | 1.4704 |
| **16** | 0.1656 | 0.1634 | 0.1329 | 0.1026 | 0.1026 | 0.1333 | 0.1388 | 0.1705 | 0.2245 | 0.5433 | 1.0038 |
| **17** | 0.1616 | 0.1564 | 0.1273 | 0.0953 | 0.0933 | 0.0942 | 0.1134 | 0.1354 | 0.2399 | 0.3923 | 0.9997 |
| **18** | 0.1551 | 0.1531 | 0.1211 | 0.0709 | 0.0691 | 0.0782 | 0.1090 | 0.1301 | 0.1698 | 0.2665 | 0.7931 |
| **19** | 0.1429 | 0.1405 | 0.1123 | 0.0631 | 0.0609 | 0.0784 | 0.0999 | 0.1161 | 0.1473 | 0.2496 | 1.1170 |
| **20** | 0.1414 | 0.1396 | 0.1088 | 0.0514 | 0.0507 | 0.0731 | 0.0997 | 0.1140 | 0.1393 | 0.2415 | 0.6141 |
| **21** | 0.1233 | 0.1223 | 0.0958 | 0.0448 | 0.0494 | 0.0710 | 0.0972 | 0.1109 | 0.1148 | 0.2239 | 0.6701 |
| **22** | 0.1269 | 0.1258 | 0.0964 | 0.0399 | 0.0453 | 0.0729 | 0.0993 | 0.1103 | 0.1108 | 0.1637 | 0.5684 |
| **23** | 0.1314 | 0.1299 | 0.0983 | 0.0390 | 0.0432 | 0.0751 | 0.1000 | 0.1113 | 0.1101 | 0.1429 | 0.4647 |
| **24** | 0.1129 | 0.1117 | 0.0848 | 0.0335 | 0.0433 | 0.0725 | 0.0985 | 0.1097 | 0.1054 | 0.1310 | 0.3730 |
| **25** | 0.1137 | 0.1125 | 0.0844 | 0.0325 | 0.0422 | 0.0728 | 0.0989 | 0.1115 | 0.1089 | 0.1150 | 0.3517 |
| **26** | 0.1048 | 0.1036 | 0.0773 | 0.0295 | 0.0431 | 0.0742 | 0.0985 | 0.1096 | 0.1063 | 0.1011 | 0.3609 |
| **27** | 0.0978 | 0.0966 | 0.0723 | 0.0278 | 0.0419 | 0.0725 | 0.0969 | 0.1085 | 0.1055 | 0.0947 | 0.2832 |
| **28** | 0.0982 | 0.0969 | 0.0711 | 0.0257 | 0.0416 | 0.0740 | 0.0978 | 0.1084 | 0.1049 | 0.0931 | 0.2731 |
| **29** | 0.1018 | 0.1004 | 0.0729 | 0.0240 | 0.0428 | 0.0753 | 0.0987 | 0.1089 | 0.1052 | 0.0905 | 0.2286 |
| **30** | 0.0955 | 0.0942 | 0.0686 | 0.0222 | 0.0412 | 0.0750 | 0.0986 | 0.1090 | 0.1057 | 0.0850 | 0.2544 |
| **31** | 0.0935 | 0.0923 | 0.0660 | 0.0191 | 0.0434 | 0.0775 | 0.0998 | 0.1091 | 0.1054 | 0.0826 | 0.1959 |
| **32** | 0.0893 | 0.0881 | 0.0632 | 0.0180 | 0.0416 | 0.0758 | 0.0977 | 0.1065 | 0.1026 | 0.0783 | 0.1851 |
| **33** | 0.0908 | 0.0895 | 0.0635 | 0.0170 | 0.0438 | 0.0787 | 0.1004 | 0.1086 | 0.1040 | 0.0792 | 0.1588 |
| **34** | 0.0889 | 0.0876 | 0.0611 | 0.0167 | 0.0447 | 0.0795 | 0.1014 | 0.1100 | 0.1054 | 0.0765 | 0.1523 |
| **35** | 0.0840 | 0.0827 | 0.0572 | 0.0176 | 0.0436 | 0.0760 | 0.0972 | 0.1055 | 0.1015 | 0.0755 | 0.1532 |
| **36** | 0.0778 | 0.0767 | 0.0534 | 0.0165 | 0.0436 | 0.0760 | 0.0972 | 0.1057 | 0.1016 | 0.0749 | 0.1474 |
| **37** | 0.0823 | 0.0811 | 0.0557 | 0.0162 | 0.0450 | 0.0781 | 0.0987 | 0.1065 | 0.1022 | 0.0756 | 0.1381 |
| **38** | 0.0722 | 0.0711 | 0.0495 | 0.0134 | 0.0451 | 0.0780 | 0.0983 | 0.1057 | 0.1011 | 0.0732 | 0.1332 |
| **39** | 0.0759 | 0.0747 | 0.0506 | 0.0139 | 0.0446 | 0.0769 | 0.0971 | 0.1047 | 0.1006 | 0.0741 | 0.1305 |
| **40** | 0.0763 | 0.0751 | 0.0506 | 0.0127 | 0.0474 | 0.0801 | 0.0998 | 0.1070 | 0.1025 | 0.0726 | 0.1290 |
| **41** | 0.0789 | 0.0776 | 0.0514 | 0.0118 | 0.0482 | 0.0804 | 0.0991 | 0.1051 | 0.1001 | 0.0730 | 0.1291 |
| **42** | 0.0628 | 0.0618 | 0.0424 | 0.0135 | 0.0460 | 0.0778 | 0.0971 | 0.1039 | 0.0994 | 0.0721 | 0.1279 |
| **43** | 0.0722 | 0.0710 | 0.0464 | 0.0116 | 0.0479 | 0.0795 | 0.0984 | 0.1046 | 0.0993 | 0.0703 | 0.1224 |
| **44** | 0.0666 | 0.0655 | 0.0431 | 0.0119 | 0.0480 | 0.0789 | 0.0970 | 0.1028 | 0.0975 | 0.0695 | 0.1186 |
| **45** | 0.0690 | 0.0677 | 0.0436 | 0.0134 | 0.0491 | 0.0801 | 0.0981 | 0.1036 | 0.0984 | 0.0697 | 0.1174 |
| **46** | 0.0656 | 0.0645 | 0.0413 | 0.0118 | 0.0490 | 0.0797 | 0.0979 | 0.1037 | 0.0988 | 0.0694 | 0.1156 |
| **47** | 0.0568 | 0.0560 | 0.0379 | 0.0115 | 0.0483 | 0.0793 | 0.0973 | 0.1028 | 0.0975 | 0.0692 | 0.1181 |
| **48** | 0.0619 | 0.0608 | 0.0384 | 0.0117 | 0.0498 | 0.0799 | 0.0975 | 0.1030 | 0.0978 | 0.0673 | 0.1124 |
| **49** | 0.0590 | 0.0580 | 0.0373 | 0.0128 | 0.0504 | 0.0806 | 0.0980 | 0.1031 | 0.0977 | 0.0676 | 0.1149 |
| **50** | 0.0571 | 0.0562 | 0.0361 | 0.0129 | 0.0507 | 0.0804 | 0.0974 | 0.1023 | 0.0967 | 0.0677 | 0.1122 |
| **51** | 0.0584 | 0.0574 | 0.0362 | 0.0130 | 0.0515 | 0.0816 | 0.0989 | 0.1040 | 0.0981 | 0.0679 | 0.1115 |
| **52** | 0.0541 | 0.0532 | 0.0341 | 0.0130 | 0.0504 | 0.0803 | 0.0974 | 0.1022 | 0.0968 | 0.0671 | 0.1109 |
| **53** | 0.0525 | 0.0516 | 0.0328 | 0.0139 | 0.0514 | 0.0808 | 0.0975 | 0.1022 | 0.0965 | 0.0656 | 0.1089 |
| **54** | 0.0495 | 0.0486 | 0.0309 | 0.0141 | 0.0516 | 0.0810 | 0.0973 | 0.1015 | 0.0956 | 0.0669 | 0.1087 |
| **55** | 0.0500 | 0.0492 | 0.0309 | 0.0145 | 0.0521 | 0.0816 | 0.0980 | 0.1021 | 0.0960 | 0.0653 | 0.1078 |
| **56** | 0.0459 | 0.0451 | 0.0284 | 0.0138 | 0.0505 | 0.0793 | 0.0956 | 0.0999 | 0.0943 | 0.0635 | 0.1049 |
| **57** | 0.0473 | 0.0465 | 0.0292 | 0.0147 | 0.0510 | 0.0799 | 0.0963 | 0.1008 | 0.0951 | 0.0653 | 0.1051 |
| **58** | 0.0450 | 0.0442 | 0.0277 | 0.0151 | 0.0516 | 0.0798 | 0.0954 | 0.0990 | 0.0931 | 0.0641 | 0.1045 |
| **59** | 0.0467 | 0.0459 | 0.0277 | 0.0158 | 0.0532 | 0.0820 | 0.0980 | 0.1019 | 0.0955 | 0.0644 | 0.1025 |
| **60** | 0.0447 | 0.0439 | 0.0268 | 0.0160 | 0.0520 | 0.0800 | 0.0955 | 0.0993 | 0.0934 | 0.0637 | 0.1015 |

**Table 5:** $\mu_P D(\phi;\, n,\, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $n$ \ $q$ | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0160 | 0.0160 | 0.0160 | 0.0160 | 0.0160 | 0.0161 | 0.0163 | 0.0181 | 0.0241 | 0.0598 | 0.1333 |
| **16** | 0.0148 | 0.0148 | 0.0148 | 0.0148 | 0.0148 | 0.0148 | 0.0151 | 0.0160 | 0.0207 | 0.0504 | 0.1320 |
| **17** | 0.0137 | 0.0137 | 0.0137 | 0.0137 | 0.0137 | 0.0138 | 0.0139 | 0.0147 | 0.0190 | 0.0429 | 0.1285 |
| **18** | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0135 | 0.0164 | 0.0358 | 0.1250 |
| **19** | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0126 | 0.0150 | 0.0314 | 0.1166 |
| **20** | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0119 | 0.0120 | 0.0137 | 0.0276 | 0.1086 |
| **21** | 0.0112 | 0.0112 | 0.0112 | 0.0112 | 0.0112 | 0.0112 | 0.0112 | 0.0114 | 0.0126 | 0.0231 | 0.0969 |
| **22** | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0108 | 0.0116 | 0.0205 | 0.0891 |
| **23** | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0103 | 0.0109 | 0.0180 | 0.0836 |
| **24** | 0.0096 | 0.0096 | 0.0096 | 0.0096 | 0.0096 | 0.0096 | 0.0096 | 0.0097 | 0.0102 | 0.0159 | 0.0804 |
| **25** | 0.0092 | 0.0092 | 0.0092 | 0.0092 | 0.0092 | 0.0092 | 0.0092 | 0.0092 | 0.0095 | 0.0142 | 0.0778 |
| **26** | 0.0089 | 0.0089 | 0.0089 | 0.0089 | 0.0089 | 0.0089 | 0.0089 | 0.0089 | 0.0091 | 0.0130 | 0.0758 |
| **27** | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0088 | 0.0122 | 0.0724 |
| **28** | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0083 | 0.0110 | 0.0669 |
| **29** | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0102 | 0.0615 |
| **30** | 0.0077 | 0.0077 | 0.0077 | 0.0077 | 0.0077 | 0.0077 | 0.0077 | 0.0077 | 0.0077 | 0.0093 | 0.0538 |
| **31** | 0.0074 | 0.0074 | 0.0074 | 0.0074 | 0.0074 | 0.0074 | 0.0074 | 0.0074 | 0.0075 | 0.0087 | 0.0485 |
| **32** | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0083 | 0.0434 |
| **33** | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0078 | 0.0389 |
| **34** | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0074 | 0.0367 |
| **35** | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0066 | 0.0069 | 0.0352 |
| **36** | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0067 | 0.0326 |
| **37** | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0064 | 0.0310 |
| **38** | 0.0061 | 0.0061 | 0.0061 | 0.0061 | 0.0061 | 0.0061 | 0.0061 | 0.0061 | 0.0061 | 0.0062 | 0.0280 |
| **39** | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0061 | 0.0258 |
| **40** | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0059 | 0.0223 |
| **41** | 0.0056 | 0.0056 | 0.0056 | 0.0056 | 0.0056 | 0.0056 | 0.0056 | 0.0056 | 0.0056 | 0.0057 | 0.0201 |
| **42** | 0.0055 | 0.0055 | 0.0055 | 0.0055 | 0.0055 | 0.0055 | 0.0055 | 0.0055 | 0.0055 | 0.0056 | 0.0186 |
| **43** | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0173 |
| **44** | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0053 | 0.0158 |
| **45** | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0150 |
| **46** | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0050 | 0.0141 |
| **47** | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0050 | 0.0132 |
| **48** | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0048 | 0.0123 |
| **49** | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0047 | 0.0111 |
| **50** | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0046 | 0.0104 |
| **51** | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0046 | 0.0098 |
| **52** | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0089 |
| **53** | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0086 |
| **54** | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0082 |
| **55** | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0076 |
| **56** | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0071 |
| **57** | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0067 |

**Table 5a:** $\sigma_P D(\phi;\ n,\ q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.0052 | 0.0053 | 0.0056 | 0.0068 | 0.0143 | 0.0229 | 0.0272 |
| **16** | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0046 | 0.0048 | 0.0059 | 0.0120 | 0.0198 | 0.0255 |
| **17** | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0042 | 0.0053 | 0.0105 | 0.0179 | 0.0237 |
| **18** | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0039 | 0.0046 | 0.0077 | 0.0160 | 0.0245 |
| **19** | 0.0035 | 0.0035 | 0.0035 | 0.0035 | 0.0035 | 0.0036 | 0.0036 | 0.0040 | 0.0073 | 0.0148 | 0.0251 |
| **20** | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0034 | 0.0034 | 0.0034 | 0.0036 | 0.0061 | 0.0129 | 0.0261 |
| **21** | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0034 | 0.0050 | 0.0116 | 0.0243 |
| **22** | 0.0027 | 0.0027 | 0.0027 | 0.0027 | 0.0027 | 0.0027 | 0.0027 | 0.0029 | 0.0041 | 0.0105 | 0.0230 |
| **23** | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0024 | 0.0025 | 0.0025 | 0.0025 | 0.0035 | 0.0099 | 0.0218 |
| **24** | 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0023 | 0.0024 | 0.0032 | 0.0086 | 0.0223 |
| **25** | 0.0022 | 0.0022 | 0.0022 | 0.0022 | 0.0022 | 0.0022 | 0.0022 | 0.0023 | 0.0031 | 0.0075 | 0.0215 |
| **26** | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0020 | 0.0026 | 0.0066 | 0.0202 |
| **27** | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.0023 | 0.0063 | 0.0195 |
| **28** | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0018 | 0.0020 | 0.0050 | 0.0191 |
| **29** | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0018 | 0.0046 | 0.0186 |
| **30** | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0016 | 0.0017 | 0.0040 | 0.0180 |
| **31** | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0016 | 0.0033 | 0.0162 |
| **32** | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0015 | 0.0031 | 0.0149 |
| **33** | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0014 | 0.0026 | 0.0134 |
| **34** | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0014 | 0.0027 | 0.0126 |
| **35** | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0013 | 0.0021 | 0.0120 |
| **36** | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0023 | 0.0115 |
| **37** | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0019 | 0.0106 |
| **38** | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0017 | 0.0103 |
| **39** | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0016 | 0.0099 |
| **40** | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0015 | 0.0082 |
| **41** | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0013 | 0.0079 |
| **42** | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0012 | 0.0075 |
| **43** | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0011 | 0.0070 |
| **44** | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0011 | 0.0065 |
| **45** | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0010 | 0.0064 |
| **46** | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0056 |
| **47** | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0055 |
| **48** | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0008 | 0.0054 |
| **49** | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0008 | 0.0050 |
| **50** | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0047 |
| **51** | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0045 |
| **52** | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0041 |
| **53** | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0041 |
| **54** | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0039 |
| **55** | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0036 |
| **56** | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0033 |
| **57** | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0030 |

**Table 6:** $\mu_P d(\phi; n, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $n$ \ $q$ | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0673 | 0.0673 | 0.0672 | 0.0674 | 0.0678 | 0.0691 | 0.0721 | 0.0803 | 0.1203 | 0.2491 | 0.4919 |
| **16** | 0.0629 | 0.0629 | 0.0630 | 0.0632 | 0.0633 | 0.0638 | 0.0659 | 0.0709 | 0.1006 | 0.2269 | 0.4744 |
| **17** | 0.0602 | 0.0602 | 0.0604 | 0.0603 | 0.0605 | 0.0609 | 0.0618 | 0.0671 | 0.0911 | 0.2082 | 0.4532 |
| **18** | 0.0576 | 0.0576 | 0.0575 | 0.0575 | 0.0575 | 0.0577 | 0.0586 | 0.0615 | 0.0757 | 0.1844 | 0.4194 |
| **19** | 0.0544 | 0.0544 | 0.0544 | 0.0544 | 0.0545 | 0.0548 | 0.0553 | 0.0573 | 0.0695 | 0.1659 | 0.3881 |
| **20** | 0.0539 | 0.0539 | 0.0539 | 0.0539 | 0.0539 | 0.0540 | 0.0542 | 0.0552 | 0.0641 | 0.1524 | 0.3571 |
| **21** | 0.0513 | 0.0513 | 0.0513 | 0.0513 | 0.0513 | 0.0513 | 0.0516 | 0.0526 | 0.0591 | 0.1276 | 0.3215 |
| **22** | 0.0494 | 0.0494 | 0.0494 | 0.0494 | 0.0494 | 0.0494 | 0.0496 | 0.0506 | 0.0554 | 0.1152 | 0.3029 |
| **23** | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0475 | 0.0478 | 0.0514 | 0.0978 | 0.2893 |
| **24** | 0.0449 | 0.0449 | 0.0449 | 0.0449 | 0.0449 | 0.0449 | 0.0450 | 0.0453 | 0.0488 | 0.0858 | 0.2797 |
| **25** | 0.0431 | 0.0431 | 0.0431 | 0.0431 | 0.0431 | 0.0431 | 0.0432 | 0.0431 | 0.0455 | 0.0728 | 0.2730 |
| **26** | 0.0423 | 0.0423 | 0.0423 | 0.0423 | 0.0423 | 0.0423 | 0.0424 | 0.0424 | 0.0435 | 0.0660 | 0.2691 |
| **27** | 0.0414 | 0.0414 | 0.0414 | 0.0414 | 0.0414 | 0.0414 | 0.0414 | 0.0415 | 0.0425 | 0.0621 | 0.2647 |
| **28** | 0.0390 | 0.0390 | 0.0390 | 0.0390 | 0.0390 | 0.0390 | 0.0390 | 0.0390 | 0.0398 | 0.0537 | 0.2525 |
| **29** | 0.0395 | 0.0395 | 0.0395 | 0.0395 | 0.0395 | 0.0395 | 0.0395 | 0.0395 | 0.0395 | 0.0511 | 0.2469 |
| **30** | 0.0372 | 0.0372 | 0.0372 | 0.0372 | 0.0372 | 0.0372 | 0.0372 | 0.0372 | 0.0372 | 0.0464 | 0.2330 |
| **31** | 0.0363 | 0.0363 | 0.0363 | 0.0363 | 0.0363 | 0.0363 | 0.0363 | 0.0363 | 0.0364 | 0.0435 | 0.2257 |
| **32** | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0358 | 0.0421 | 0.2163 |
| **33** | 0.0346 | 0.0346 | 0.0346 | 0.0346 | 0.0346 | 0.0346 | 0.0346 | 0.0346 | 0.0347 | 0.0392 | 0.2085 |
| **34** | 0.0341 | 0.0341 | 0.0341 | 0.0341 | 0.0341 | 0.0341 | 0.0341 | 0.0341 | 0.0342 | 0.0388 | 0.2036 |
| **35** | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0328 | 0.0356 | 0.1989 |
| **36** | 0.0328 | 0.0328 | 0.0328 | 0.0328 | 0.0328 | 0.0328 | 0.0328 | 0.0328 | 0.0328 | 0.0354 | 0.1944 |
| **37** | 0.0318 | 0.0318 | 0.0318 | 0.0318 | 0.0318 | 0.0318 | 0.0318 | 0.0318 | 0.0318 | 0.0334 | 0.1891 |
| **38** | 0.0311 | 0.0311 | 0.0311 | 0.0311 | 0.0311 | 0.0311 | 0.0311 | 0.0311 | 0.0311 | 0.0323 | 0.1781 |
| **39** | 0.0305 | 0.0305 | 0.0305 | 0.0305 | 0.0305 | 0.0305 | 0.0305 | 0.0305 | 0.0305 | 0.0316 | 0.1682 |
| **40** | 0.0302 | 0.0302 | 0.0302 | 0.0302 | 0.0302 | 0.0302 | 0.0302 | 0.0302 | 0.0302 | 0.0312 | 0.1578 |
| **41** | 0.0297 | 0.0297 | 0.0297 | 0.0297 | 0.0297 | 0.0297 | 0.0297 | 0.0297 | 0.0298 | 0.0302 | 0.1457 |
| **42** | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0292 | 0.1354 |
| **43** | 0.0283 | 0.0283 | 0.0283 | 0.0283 | 0.0283 | 0.0283 | 0.0283 | 0.0283 | 0.0283 | 0.0284 | 0.1314 |
| **44** | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.0280 | 0.1198 |
| **45** | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.0269 | 0.1150 |
| **46** | 0.0270 | 0.0270 | 0.0270 | 0.0270 | 0.0270 | 0.0270 | 0.0270 | 0.0270 | 0.0270 | 0.0271 | 0.1115 |
| **47** | 0.0266 | 0.0266 | 0.0266 | 0.0266 | 0.0266 | 0.0266 | 0.0266 | 0.0266 | 0.0266 | 0.0267 | 0.1023 |
| **48** | 0.0258 | 0.0258 | 0.0258 | 0.0258 | 0.0258 | 0.0258 | 0.0258 | 0.0258 | 0.0258 | 0.0259 | 0.0934 |
| **49** | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0254 | 0.0858 |
| **50** | 0.0249 | 0.0249 | 0.0249 | 0.0249 | 0.0249 | 0.0249 | 0.0249 | 0.0249 | 0.0249 | 0.0250 | 0.0797 |
| **51** | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0245 | 0.0711 |
| **52** | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0240 | 0.0632 |
| **53** | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0615 |
| **54** | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0240 | 0.0561 |
| **55** | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0232 | 0.0512 |
| **56** | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0226 | 0.0465 |
| **57** | 0.0225 | 0.0225 | 0.0225 | 0.0225 | 0.0225 | 0.0225 | 0.0225 | 0.0225 | 0.0225 | 0.0226 | 0.0433 |

**Table 6a:** $\sigma_P d(\phi; n, q)$ **for** $n$ **from 15 to 57 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0246 | 0.0246 | 0.0247 | 0.0248 | 0.0252 | 0.0259 | 0.0299 | 0.0404 | 0.0816 | 0.0786 | 0.1397 |
| **16** | 0.0237 | 0.0237 | 0.0236 | 0.0236 | 0.0237 | 0.0239 | 0.0258 | 0.0307 | 0.0688 | 0.0760 | 0.1274 |
| **17** | 0.0218 | 0.0218 | 0.0218 | 0.0220 | 0.0221 | 0.0227 | 0.0240 | 0.0314 | 0.0606 | 0.0797 | 0.1134 |
| **18** | 0.0208 | 0.0208 | 0.0208 | 0.0207 | 0.0207 | 0.0207 | 0.0221 | 0.0268 | 0.0465 | 0.0816 | 0.0929 |
| **19** | 0.0205 | 0.0205 | 0.0206 | 0.0206 | 0.0205 | 0.0206 | 0.0210 | 0.0230 | 0.0405 | 0.0813 | 0.0879 |
| **20** | 0.0196 | 0.0196 | 0.0196 | 0.0196 | 0.0197 | 0.0196 | 0.0200 | 0.0209 | 0.0353 | 0.0796 | 0.0775 |
| **21** | 0.0190 | 0.0190 | 0.0190 | 0.0190 | 0.0190 | 0.0190 | 0.0190 | 0.0205 | 0.0293 | 0.0772 | 0.0697 |
| **22** | 0.0180 | 0.0180 | 0.0180 | 0.0180 | 0.0180 | 0.0181 | 0.0180 | 0.0192 | 0.0256 | 0.0750 | 0.0646 |
| **23** | 0.0162 | 0.0162 | 0.0162 | 0.0162 | 0.0162 | 0.0162 | 0.0161 | 0.0161 | 0.0193 | 0.0699 | 0.0551 |
| **24** | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0159 | 0.0163 | 0.0206 | 0.0642 | 0.0571 |
| **25** | 0.0149 | 0.0149 | 0.0149 | 0.0149 | 0.0149 | 0.0149 | 0.0149 | 0.0150 | 0.0182 | 0.0541 | 0.0541 |
| **26** | 0.0150 | 0.0150 | 0.0150 | 0.0150 | 0.0150 | 0.0150 | 0.0150 | 0.0150 | 0.0168 | 0.0479 | 0.0494 |
| **27** | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0146 | 0.0146 | 0.0160 | 0.0449 | 0.0484 |
| **28** | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0146 | 0.0364 | 0.0450 |
| **29** | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0134 | 0.0138 | 0.0336 | 0.0420 |
| **30** | 0.0123 | 0.0123 | 0.0123 | 0.0123 | 0.0123 | 0.0123 | 0.0123 | 0.0122 | 0.0124 | 0.0270 | 0.0408 |
| **31** | 0.0126 | 0.0126 | 0.0126 | 0.0126 | 0.0126 | 0.0126 | 0.0126 | 0.0126 | 0.0125 | 0.0230 | 0.0415 |
| **32** | 0.0120 | 0.0120 | 0.0120 | 0.0120 | 0.0120 | 0.0120 | 0.0120 | 0.0120 | 0.0121 | 0.0215 | 0.0426 |
| **33** | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0116 | 0.0180 | 0.0418 |
| **34** | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0115 | 0.0116 | 0.0197 | 0.0474 |
| **35** | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0109 | 0.0145 | 0.0437 |
| **36** | 0.0109 | 0.0109 | 0.0109 | 0.0109 | 0.0109 | 0.0109 | 0.0109 | 0.0109 | 0.0110 | 0.0169 | 0.0489 |
| **37** | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0108 | 0.0139 | 0.0474 |
| **38** | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0124 | 0.0503 |
| **39** | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0124 | 0.0571 |
| **40** | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0098 | 0.0099 | 0.0130 | 0.0576 |
| **41** | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0096 | 0.0108 | 0.0606 |
| **42** | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0095 | 0.0112 | 0.0636 |
| **43** | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0097 | 0.0635 |
| **44** | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0104 | 0.0650 |
| **45** | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0092 | 0.0649 |
| **46** | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0090 | 0.0641 |
| **47** | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0641 |
| **48** | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0077 | 0.0079 | 0.0622 |
| **49** | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0082 | 0.0608 |
| **50** | 0.0078 | 0.0078 | 0.0078 | 0.0078 | 0.0078 | 0.0078 | 0.0078 | 0.0078 | 0.0077 | 0.0079 | 0.0597 |
| **51** | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0079 | 0.0565 |
| **52** | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0072 | 0.0534 |
| **53** | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0076 | 0.0525 |
| **54** | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0075 | 0.0077 | 0.0492 |
| **55** | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0070 | 0.0467 |
| **56** | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0429 |
| **57** | 0.0069 | 0.0069 | 0.0069 | 0.0069 | 0.0069 | 0.0069 | 0.0069 | 0.0069 | 0.0069 | 0.0070 | 0.0401 |

**Table 7:** $\mu_U D(\phi;\, n,\, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0452 | 0.0449 | 0.0444 | 0.0444 | 0.0447 | 0.0454 | 0.0472 | 0.0510 | 0.0708 | 0.1090 | 0.1863 |
| **16** | 0.0416 | 0.0415 | 0.0416 | 0.0413 | 0.0418 | 0.0429 | 0.0435 | 0.0471 | 0.0632 | 0.1018 | 0.1794 |
| **17** | 0.0392 | 0.0392 | 0.0391 | 0.0386 | 0.0389 | 0.0388 | 0.0400 | 0.0439 | 0.0582 | 0.0942 | 0.1717 |
| **18** | 0.0367 | 0.0367 | 0.0366 | 0.0363 | 0.0360 | 0.0357 | 0.0367 | 0.0407 | 0.0523 | 0.0898 | 0.1651 |
| **19** | 0.0336 | 0.0335 | 0.0335 | 0.0334 | 0.0331 | 0.0328 | 0.0343 | 0.0371 | 0.0488 | 0.0816 | 0.1577 |
| **20** | 0.0319 | 0.0319 | 0.0319 | 0.0317 | 0.0317 | 0.0317 | 0.0319 | 0.0342 | 0.0437 | 0.0760 | 0.1525 |
| **21** | 0.0291 | 0.0291 | 0.0291 | 0.0291 | 0.0291 | 0.0291 | 0.0297 | 0.0316 | 0.0401 | 0.0702 | 0.1470 |
| **22** | 0.0282 | 0.0282 | 0.0282 | 0.0282 | 0.0282 | 0.0283 | 0.0280 | 0.0299 | 0.0382 | 0.0658 | 0.1399 |
| **23** | 0.0273 | 0.0273 | 0.0273 | 0.0273 | 0.0273 | 0.0272 | 0.0266 | 0.0283 | 0.0355 | 0.0600 | 0.1351 |
| **24** | 0.0253 | 0.0253 | 0.0253 | 0.0254 | 0.0254 | 0.0253 | 0.0254 | 0.0267 | 0.0338 | 0.0562 | 0.1303 |
| **25** | 0.0242 | 0.0242 | 0.0242 | 0.0242 | 0.0242 | 0.0243 | 0.0241 | 0.0249 | 0.0310 | 0.0526 | 0.1240 |
| **26** | 0.0230 | 0.0230 | 0.0230 | 0.0230 | 0.0231 | 0.0231 | 0.0232 | 0.0239 | 0.0293 | 0.0490 | 0.1197 |
| **27** | 0.0220 | 0.0220 | 0.0220 | 0.0220 | 0.0220 | 0.0220 | 0.0220 | 0.0231 | 0.0265 | 0.0454 | 0.1144 |
| **28** | 0.0212 | 0.0212 | 0.0212 | 0.0212 | 0.0212 | 0.0213 | 0.0212 | 0.0219 | 0.0255 | 0.0423 | 0.1109 |
| **29** | 0.0206 | 0.0206 | 0.0206 | 0.0206 | 0.0207 | 0.0206 | 0.0206 | 0.0206 | 0.0243 | 0.0401 | 0.1070 |
| **30** | 0.0196 | 0.0196 | 0.0196 | 0.0196 | 0.0196 | 0.0196 | 0.0197 | 0.0194 | 0.0227 | 0.0361 | 0.1011 |
| **31** | 0.0191 | 0.0191 | 0.0191 | 0.0191 | 0.0191 | 0.0191 | 0.0191 | 0.0188 | 0.0214 | 0.0358 | 0.0970 |
| **32** | 0.0184 | 0.0184 | 0.0184 | 0.0184 | 0.0184 | 0.0184 | 0.0184 | 0.0182 | 0.0199 | 0.0331 | 0.0932 |
| **33** | 0.0179 | 0.0179 | 0.0179 | 0.0179 | 0.0179 | 0.0178 | 0.0178 | 0.0178 | 0.0192 | 0.0315 | 0.0899 |
| **34** | 0.0172 | 0.0172 | 0.0172 | 0.0172 | 0.0172 | 0.0172 | 0.0172 | 0.0172 | 0.0184 | 0.0295 | 0.0871 |
| **35** | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0166 | 0.0178 | 0.0276 | 0.0834 |
| **36** | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0170 | 0.0262 | 0.0806 |
| **37** | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0158 | 0.0159 | 0.0166 | 0.0250 | 0.0772 |
| **38** | 0.0152 | 0.0152 | 0.0152 | 0.0152 | 0.0152 | 0.0152 | 0.0153 | 0.0152 | 0.0158 | 0.0237 | 0.0739 |
| **39** | 0.0148 | 0.0148 | 0.0148 | 0.0148 | 0.0148 | 0.0148 | 0.0147 | 0.0148 | 0.0152 | 0.0227 | 0.0703 |
| **40** | 0.0144 | 0.0144 | 0.0144 | 0.0144 | 0.0144 | 0.0144 | 0.0144 | 0.0144 | 0.0144 | 0.0215 | 0.0683 |
| **41** | 0.0142 | 0.0142 | 0.0142 | 0.0142 | 0.0142 | 0.0142 | 0.0141 | 0.0141 | 0.0140 | 0.0207 | 0.0657 |
| **42** | 0.0136 | 0.0136 | 0.0136 | 0.0136 | 0.0136 | 0.0136 | 0.0137 | 0.0138 | 0.0139 | 0.0193 | 0.0634 |
| **43** | 0.0133 | 0.0133 | 0.0133 | 0.0133 | 0.0133 | 0.0133 | 0.0133 | 0.0133 | 0.0134 | 0.0185 | 0.0606 |
| **44** | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0130 | 0.0131 | 0.0130 | 0.0130 | 0.0131 | 0.0172 | 0.0585 |
| **45** | 0.0127 | 0.0127 | 0.0127 | 0.0127 | 0.0127 | 0.0127 | 0.0127 | 0.0128 | 0.0128 | 0.0165 | 0.0560 |
| **46** | 0.0124 | 0.0124 | 0.0124 | 0.0124 | 0.0124 | 0.0124 | 0.0124 | 0.0124 | 0.0126 | 0.0159 | 0.0539 |
| **47** | 0.0121 | 0.0121 | 0.0121 | 0.0121 | 0.0121 | 0.0121 | 0.0121 | 0.0122 | 0.0122 | 0.0151 | 0.0520 |
| **48** | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0147 | 0.0499 |
| **49** | 0.0116 | 0.0116 | 0.0116 | 0.0116 | 0.0116 | 0.0116 | 0.0116 | 0.0116 | 0.0116 | 0.0143 | 0.0483 |
| **50** | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0114 | 0.0134 | 0.0460 |
| **51** | 0.0111 | 0.0111 | 0.0111 | 0.0111 | 0.0111 | 0.0111 | 0.0111 | 0.0111 | 0.0111 | 0.0131 | 0.0441 |
| **52** | 0.0110 | 0.0110 | 0.0110 | 0.0110 | 0.0110 | 0.0110 | 0.0110 | 0.0110 | 0.0110 | 0.0128 | 0.0426 |
| **53** | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0123 | 0.0406 |
| **54** | 0.0105 | 0.0105 | 0.0105 | 0.0105 | 0.0105 | 0.0105 | 0.0105 | 0.0105 | 0.0105 | 0.0117 | 0.0390 |
| **55** | 0.0103 | 0.0103 | 0.0103 | 0.0103 | 0.0103 | 0.0103 | 0.0103 | 0.0103 | 0.0103 | 0.0114 | 0.0373 |
| **56** | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0101 | 0.0111 | 0.0358 |
| **57** | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0099 | 0.0109 | 0.0346 |
| **58** | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0098 | 0.0105 | 0.0330 |
| **59** | 0.0095 | 0.0095 | 0.0095 | 0.0095 | 0.0095 | 0.0095 | 0.0095 | 0.0095 | 0.0095 | 0.0102 | 0.0319 |
| **60** | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0100 | 0.0307 |

**Table 7a:** $\sigma_U D(\phi; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| $n$ \ $q$ | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.0402 | 0.0394 | 0.0375 | 0.0365 | 0.0343 | 0.0329 | 0.0327 | 0.0316 | 0.0343 | 0.0382 | 0.0466 |
| **16** | 0.0339 | 0.0339 | 0.0328 | 0.0314 | 0.0313 | 0.0315 | 0.0298 | 0.0313 | 0.0311 | 0.0375 | 0.0435 |
| **17** | 0.0331 | 0.0331 | 0.0341 | 0.0301 | 0.0288 | 0.0266 | 0.0278 | 0.0285 | 0.0325 | 0.0367 | 0.0431 |
| **18** | 0.0307 | 0.0312 | 0.0306 | 0.0282 | 0.0257 | 0.0233 | 0.0247 | 0.0272 | 0.0298 | 0.0349 | 0.0415 |
| **19** | 0.0287 | 0.0286 | 0.0288 | 0.0263 | 0.0240 | 0.0213 | 0.0238 | 0.0249 | 0.0293 | 0.0335 | 0.0409 |
| **20** | 0.0243 | 0.0244 | 0.0247 | 0.0239 | 0.0227 | 0.0209 | 0.0207 | 0.0236 | 0.0270 | 0.0329 | 0.0386 |
| **21** | 0.0206 | 0.0205 | 0.0210 | 0.0201 | 0.0195 | 0.0181 | 0.0199 | 0.0220 | 0.0248 | 0.0295 | 0.0375 |
| **22** | 0.0195 | 0.0195 | 0.0195 | 0.0195 | 0.0191 | 0.0185 | 0.0179 | 0.0199 | 0.0245 | 0.0298 | 0.0357 |
| **23** | 0.0192 | 0.0192 | 0.0192 | 0.0191 | 0.0186 | 0.0174 | 0.0160 | 0.0186 | 0.0232 | 0.0290 | 0.0352 |
| **24** | 0.0157 | 0.0157 | 0.0157 | 0.0162 | 0.0155 | 0.0150 | 0.0154 | 0.0176 | 0.0228 | 0.0270 | 0.0346 |
| **25** | 0.0155 | 0.0155 | 0.0155 | 0.0155 | 0.0152 | 0.0154 | 0.0140 | 0.0163 | 0.0208 | 0.0273 | 0.0334 |
| **26** | 0.0136 | 0.0136 | 0.0136 | 0.0135 | 0.0139 | 0.0139 | 0.0135 | 0.0157 | 0.0193 | 0.0250 | 0.0316 |
| **27** | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0123 | 0.0121 | 0.0124 | 0.0146 | 0.0179 | 0.0244 | 0.0310 |
| **28** | 0.0117 | 0.0117 | 0.0116 | 0.0116 | 0.0119 | 0.0116 | 0.0113 | 0.0137 | 0.0173 | 0.0235 | 0.0304 |
| **29** | 0.0118 | 0.0118 | 0.0118 | 0.0118 | 0.0121 | 0.0113 | 0.0115 | 0.0118 | 0.0164 | 0.0232 | 0.0296 |
| **30** | 0.0108 | 0.0108 | 0.0108 | 0.0109 | 0.0108 | 0.0107 | 0.0109 | 0.0104 | 0.0149 | 0.0223 | 0.0287 |
| **31** | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0100 | 0.0101 | 0.0099 | 0.0139 | 0.0222 | 0.0281 |
| **32** | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0094 | 0.0096 | 0.0094 | 0.0092 | 0.0125 | 0.0207 | 0.0273 |
| **33** | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0088 | 0.0086 | 0.0087 | 0.0123 | 0.0202 | 0.0265 |
| **34** | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0086 | 0.0083 | 0.0087 | 0.0117 | 0.0197 | 0.0263 |
| **35** | 0.0080 | 0.0080 | 0.0080 | 0.0080 | 0.0081 | 0.0084 | 0.0079 | 0.0080 | 0.0111 | 0.0185 | 0.0261 |
| **36** | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0073 | 0.0075 | 0.0070 | 0.0078 | 0.0100 | 0.0167 | 0.0249 |
| **37** | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0077 | 0.0072 | 0.0080 | 0.0098 | 0.0174 | 0.0251 |
| **38** | 0.0066 | 0.0066 | 0.0066 | 0.0066 | 0.0066 | 0.0066 | 0.0069 | 0.0071 | 0.0088 | 0.0152 | 0.0236 |
| **39** | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0068 | 0.0067 | 0.0065 | 0.0068 | 0.0084 | 0.0159 | 0.0237 |
| **40** | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0065 | 0.0064 | 0.0063 | 0.0070 | 0.0152 | 0.0228 |
| **41** | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0064 | 0.0062 | 0.0061 | 0.0065 | 0.0144 | 0.0225 |
| **42** | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.0060 | 0.0071 | 0.0133 | 0.0224 |
| **43** | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0057 | 0.0056 | 0.0052 | 0.0064 | 0.0129 | 0.0216 |
| **44** | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0059 | 0.0121 | 0.0213 |
| **45** | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0052 | 0.0053 | 0.0060 | 0.0110 | 0.0212 |
| **46** | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0049 | 0.0047 | 0.0059 | 0.0105 | 0.0204 |
| **47** | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0045 | 0.0049 | 0.0056 | 0.0101 | 0.0200 |
| **48** | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0044 | 0.0047 | 0.0100 | 0.0192 |
| **49** | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0043 | 0.0044 | 0.0046 | 0.0096 | 0.0196 |
| **50** | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0042 | 0.0045 | 0.0046 | 0.0086 | 0.0189 |
| **51** | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0043 | 0.0043 | 0.0087 | 0.0185 |
| **52** | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0041 | 0.0045 | 0.0083 | 0.0189 |
| **53** | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0037 | 0.0042 | 0.0075 | 0.0176 |
| **54** | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0039 | 0.0071 | 0.0170 |
| **55** | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0038 | 0.0065 | 0.0167 |
| **56** | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0035 | 0.0063 | 0.0159 |
| **57** | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0036 | 0.0067 | 0.0166 |
| **58** | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0031 | 0.0037 | 0.0056 | 0.0160 |
| **59** | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0032 | 0.0053 | 0.0150 |
| **60** | 0.0029 | 0.0029 | 0.0029 | 0.0029 | 0.0029 | 0.0029 | 0.0029 | 0.0030 | 0.0034 | 0.0054 | 0.0155 |

**Table 8:** $\mu_U d(\phi; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.2407 | 0.2094 | 0.2095 | 0.2196 | 0.2479 | 0.2790 | 0.3403 | 0.4011 | 0.5585 | 0.7495 | 1.2201 |
| **16** | 0.2002 | 0.1817 | 0.1874 | 0.1992 | 0.2140 | 0.2352 | 0.2882 | 0.3516 | 0.4921 | 0.7216 | 1.1095 |
| **17** | 0.1802 | 0.1724 | 0.1696 | 0.1788 | 0.1889 | 0.2158 | 0.2566 | 0.3416 | 0.4821 | 0.6873 | 1.1229 |
| **18** | 0.1665 | 0.1529 | 0.1552 | 0.1573 | 0.1739 | 0.1951 | 0.2300 | 0.3032 | 0.4359 | 0.6703 | 1.0603 |
| **19** | 0.1451 | 0.1415 | 0.1421 | 0.1449 | 0.1515 | 0.1709 | 0.2079 | 0.2705 | 0.3766 | 0.5923 | 1.0307 |
| **20** | 0.1362 | 0.1320 | 0.1327 | 0.1359 | 0.1435 | 0.1554 | 0.1844 | 0.2530 | 0.3508 | 0.5662 | 0.9862 |
| **21** | 0.1243 | 0.1231 | 0.1230 | 0.1249 | 0.1325 | 0.1488 | 0.1710 | 0.2235 | 0.3574 | 0.5501 | 0.9806 |
| **22** | 0.1202 | 0.1181 | 0.1176 | 0.1199 | 0.1276 | 0.1310 | 0.1596 | 0.1979 | 0.3015 | 0.5051 | 0.9217 |
| **23** | 0.1152 | 0.1148 | 0.1150 | 0.1167 | 0.1191 | 0.1247 | 0.1372 | 0.1855 | 0.2796 | 0.5054 | 0.8908 |
| **24** | 0.1088 | 0.1081 | 0.1085 | 0.1088 | 0.1112 | 0.1161 | 0.1282 | 0.1682 | 0.3029 | 0.4680 | 0.8859 |
| **25** | 0.1044 | 0.1039 | 0.1041 | 0.1046 | 0.1056 | 0.1111 | 0.1200 | 0.1495 | 0.2525 | 0.4181 | 0.8434 |
| **26** | 0.1005 | 0.1002 | 0.1003 | 0.1005 | 0.1019 | 0.1050 | 0.1153 | 0.1447 | 0.2304 | 0.3876 | 0.8192 |
| **27** | 0.0965 | 0.0964 | 0.0964 | 0.0966 | 0.0988 | 0.0999 | 0.1136 | 0.1344 | 0.2043 | 0.3821 | 0.8134 |
| **28** | 0.0947 | 0.0946 | 0.0946 | 0.0948 | 0.0953 | 0.0974 | 0.1021 | 0.1257 | 0.2030 | 0.3532 | 0.7760 |
| **29** | 0.0925 | 0.0925 | 0.0925 | 0.0926 | 0.0930 | 0.0940 | 0.0985 | 0.1162 | 0.1873 | 0.3530 | 0.7559 |
| **30** | 0.0892 | 0.0892 | 0.0892 | 0.0892 | 0.0895 | 0.0902 | 0.0935 | 0.1094 | 0.1677 | 0.3202 | 0.7358 |
| **31** | 0.0873 | 0.0873 | 0.0873 | 0.0873 | 0.0875 | 0.0882 | 0.0917 | 0.1045 | 0.1546 | 0.3084 | 0.7065 |
| **32** | 0.0849 | 0.0849 | 0.0849 | 0.0850 | 0.0850 | 0.0855 | 0.0876 | 0.0975 | 0.1389 | 0.2977 | 0.7110 |
| **33** | 0.0835 | 0.0835 | 0.0835 | 0.0836 | 0.0836 | 0.0838 | 0.0853 | 0.0923 | 0.1369 | 0.2639 | 0.6576 |
| **34** | 0.0815 | 0.0815 | 0.0815 | 0.0815 | 0.0815 | 0.0818 | 0.0831 | 0.0885 | 0.1252 | 0.2762 | 0.6228 |
| **35** | 0.0795 | 0.0795 | 0.0795 | 0.0795 | 0.0795 | 0.0798 | 0.0805 | 0.0848 | 0.1162 | 0.2602 | 0.6364 |
| **36** | 0.0775 | 0.0775 | 0.0775 | 0.0775 | 0.0776 | 0.0777 | 0.0782 | 0.0811 | 0.1071 | 0.2442 | 0.6489 |
| **37** | 0.0766 | 0.0766 | 0.0766 | 0.0766 | 0.0766 | 0.0767 | 0.0769 | 0.0804 | 0.1022 | 0.2147 | 0.6086 |
| **38** | 0.0742 | 0.0742 | 0.0742 | 0.0742 | 0.0742 | 0.0743 | 0.0768 | 0.0775 | 0.1018 | 0.2018 | 0.5601 |
| **39** | 0.0722 | 0.0722 | 0.0722 | 0.0722 | 0.0722 | 0.0722 | 0.0724 | 0.0754 | 0.0925 | 0.2186 | 0.5754 |
| **40** | 0.0713 | 0.0713 | 0.0713 | 0.0713 | 0.0713 | 0.0713 | 0.0714 | 0.0730 | 0.0899 | 0.1870 | 0.5483 |
| **41** | 0.0708 | 0.0708 | 0.0708 | 0.0708 | 0.0708 | 0.0708 | 0.0709 | 0.0729 | 0.0808 | 0.1704 | 0.5342 |
| **42** | 0.0683 | 0.0683 | 0.0683 | 0.0683 | 0.0683 | 0.0683 | 0.0684 | 0.0694 | 0.0806 | 0.1711 | 0.5117 |
| **43** | 0.0671 | 0.0671 | 0.0671 | 0.0671 | 0.0671 | 0.0671 | 0.0672 | 0.0675 | 0.0732 | 0.1570 | 0.5201 |
| **44** | 0.0664 | 0.0664 | 0.0664 | 0.0664 | 0.0664 | 0.0665 | 0.0665 | 0.0681 | 0.0720 | 0.1441 | 0.4814 |
| **45** | 0.0654 | 0.0654 | 0.0654 | 0.0654 | 0.0654 | 0.0654 | 0.0655 | 0.0663 | 0.0710 | 0.1419 | 0.4865 |
| **46** | 0.0641 | 0.0641 | 0.0641 | 0.0641 | 0.0641 | 0.0641 | 0.0642 | 0.0642 | 0.0697 | 0.1332 | 0.4780 |
| **47** | 0.0631 | 0.0631 | 0.0631 | 0.0631 | 0.0631 | 0.0631 | 0.0631 | 0.0635 | 0.0665 | 0.1295 | 0.4466 |
| **48** | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0617 | 0.0618 | 0.0637 | 0.1312 | 0.4142 |
| **49** | 0.0614 | 0.0614 | 0.0614 | 0.0614 | 0.0614 | 0.0614 | 0.0614 | 0.0615 | 0.0639 | 0.1276 | 0.4402 |
| **50** | 0.0600 | 0.0600 | 0.0600 | 0.0600 | 0.0600 | 0.0600 | 0.0600 | 0.0602 | 0.0619 | 0.1078 | 0.4389 |
| **51** | 0.0589 | 0.0589 | 0.0589 | 0.0589 | 0.0589 | 0.0589 | 0.0589 | 0.0591 | 0.0598 | 0.1060 | 0.4258 |
| **52** | 0.0583 | 0.0583 | 0.0583 | 0.0583 | 0.0583 | 0.0583 | 0.0583 | 0.0584 | 0.0603 | 0.1008 | 0.3879 |
| **53** | 0.0573 | 0.0573 | 0.0573 | 0.0573 | 0.0573 | 0.0573 | 0.0573 | 0.0573 | 0.0577 | 0.0912 | 0.4008 |
| **54** | 0.0566 | 0.0566 | 0.0566 | 0.0566 | 0.0566 | 0.0566 | 0.0566 | 0.0566 | 0.0571 | 0.0895 | 0.3707 |
| **55** | 0.0562 | 0.0562 | 0.0562 | 0.0562 | 0.0562 | 0.0562 | 0.0562 | 0.0563 | 0.0572 | 0.0863 | 0.3709 |
| **56** | 0.0548 | 0.0548 | 0.0548 | 0.0548 | 0.0548 | 0.0548 | 0.0548 | 0.0549 | 0.0552 | 0.0811 | 0.3634 |
| **57** | 0.0539 | 0.0539 | 0.0539 | 0.0539 | 0.0539 | 0.0539 | 0.0539 | 0.0540 | 0.0540 | 0.0839 | 0.3668 |
| **58** | 0.0536 | 0.0536 | 0.0536 | 0.0536 | 0.0536 | 0.0536 | 0.0536 | 0.0536 | 0.0542 | 0.0732 | 0.3388 |
| **59** | 0.0528 | 0.0528 | 0.0528 | 0.0528 | 0.0528 | 0.0528 | 0.0528 | 0.0528 | 0.0530 | 0.0750 | 0.3419 |
| **60** | 0.0522 | 0.0522 | 0.0522 | 0.0522 | 0.0522 | 0.0522 | 0.0522 | 0.0522 | 0.0531 | 0.0692 | 0.3148 |

**Table 8a:** $\sigma_U d(\phi; n, q)$ **for** $n$ **from 15 to 60 and selected values of** $q$ **from 0.50 to 0.95 (1000 trials)**

| n \ q | 0.50 | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **15** | 0.1724 | 0.1425 | 0.1339 | 0.1450 | 0.2648 | 0.2195 | 0.3275 | 0.4654 | 0.5520 | 0.9951 | 0.8078 |
| **16** | 0.1261 | 0.1071 | 0.1183 | 0.1705 | 0.1712 | 0.1456 | 0.2315 | 0.2321 | 0.5023 | 0.6545 | 0.5795 |
| **17** | 0.1090 | 0.1070 | 0.1009 | 0.1099 | 0.1165 | 0.1760 | 0.1776 | 0.3247 | 0.5394 | 0.8358 | 0.7214 |
| **18** | 0.1270 | 0.0869 | 0.0837 | 0.0815 | 0.1454 | 0.1672 | 0.1776 | 0.3430 | 0.7665 | 0.6283 | 0.6627 |
| **19** | 0.0805 | 0.0818 | 0.0849 | 0.0789 | 0.0759 | 0.1018 | 0.2078 | 0.2180 | 0.3276 | 0.4633 | 0.7902 |
| **20** | 0.0721 | 0.0686 | 0.0694 | 0.0711 | 0.0835 | 0.0922 | 0.1351 | 0.3122 | 0.3230 | 0.5143 | 0.6757 |
| **21** | 0.0617 | 0.0608 | 0.0612 | 0.0604 | 0.1258 | 0.1590 | 0.1852 | 0.2473 | 0.8614 | 0.7472 | 0.8092 |
| **22** | 0.0602 | 0.0599 | 0.0595 | 0.0597 | 0.1184 | 0.0673 | 0.2216 | 0.1358 | 0.4277 | 0.5048 | 0.6281 |
| **23** | 0.0593 | 0.0594 | 0.0593 | 0.0594 | 0.0595 | 0.0624 | 0.0761 | 0.1802 | 0.2815 | 0.6583 | 0.6522 |
| **24** | 0.0525 | 0.0525 | 0.0534 | 0.0532 | 0.0522 | 0.0541 | 0.0647 | 0.1232 | 0.7564 | 0.6885 | 1.1029 |
| **25** | 0.0518 | 0.0519 | 0.0519 | 0.0527 | 0.0513 | 0.0616 | 0.0633 | 0.1110 | 0.3553 | 0.3306 | 0.8810 |
| **26** | 0.0483 | 0.0483 | 0.0482 | 0.0481 | 0.0488 | 0.0502 | 0.0684 | 0.1549 | 0.3856 | 0.2816 | 0.8072 |
| **27** | 0.0454 | 0.0454 | 0.0454 | 0.0453 | 0.0609 | 0.0450 | 0.2258 | 0.1078 | 0.2064 | 0.3721 | 1.0214 |
| **28** | 0.0444 | 0.0444 | 0.0444 | 0.0443 | 0.0448 | 0.0443 | 0.0460 | 0.0854 | 0.2210 | 0.3741 | 0.8024 |
| **29** | 0.0447 | 0.0447 | 0.0447 | 0.0446 | 0.0452 | 0.0433 | 0.0479 | 0.0904 | 0.1738 | 0.3997 | 0.6185 |
| **30** | 0.0427 | 0.0427 | 0.0427 | 0.0427 | 0.0426 | 0.0426 | 0.0431 | 0.1080 | 0.1514 | 0.4064 | 1.2762 |
| **31** | 0.0413 | 0.0413 | 0.0413 | 0.0412 | 0.0412 | 0.0408 | 0.0432 | 0.0924 | 0.2042 | 0.3534 | 0.6941 |
| **32** | 0.0397 | 0.0397 | 0.0397 | 0.0397 | 0.0397 | 0.0399 | 0.0397 | 0.0781 | 0.1293 | 0.5796 | 1.0411 |
| **33** | 0.0393 | 0.0393 | 0.0393 | 0.0393 | 0.0393 | 0.0386 | 0.0410 | 0.0514 | 0.1892 | 0.2097 | 0.5634 |
| **34** | 0.0377 | 0.0377 | 0.0377 | 0.0377 | 0.0377 | 0.0377 | 0.0401 | 0.0473 | 0.1020 | 0.8151 | 0.5774 |
| **35** | 0.0358 | 0.0358 | 0.0358 | 0.0358 | 0.0358 | 0.0364 | 0.0355 | 0.0496 | 0.1130 | 0.3714 | 1.0673 |
| **36** | 0.0345 | 0.0345 | 0.0345 | 0.0345 | 0.0345 | 0.0349 | 0.0336 | 0.0386 | 0.0820 | 0.4236 | 1.0397 |
| **37** | 0.0350 | 0.0350 | 0.0350 | 0.0349 | 0.0349 | 0.0351 | 0.0339 | 0.0474 | 0.0803 | 0.1738 | 1.0334 |
| **38** | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0327 | 0.0326 | 0.0767 | 0.0432 | 0.0952 | 0.1568 | 0.6252 |
| **39** | 0.0323 | 0.0323 | 0.0323 | 0.0323 | 0.0323 | 0.0323 | 0.0316 | 0.0458 | 0.0755 | 0.7461 | 0.7249 |
| **40** | 0.0325 | 0.0325 | 0.0325 | 0.0325 | 0.0325 | 0.0324 | 0.0321 | 0.0355 | 0.1658 | 0.1747 | 0.8461 |
| **41** | 0.0325 | 0.0325 | 0.0325 | 0.0325 | 0.0325 | 0.0325 | 0.0319 | 0.0546 | 0.0547 | 0.1345 | 0.4989 |
| **42** | 0.0294 | 0.0294 | 0.0294 | 0.0294 | 0.0294 | 0.0294 | 0.0294 | 0.0313 | 0.0637 | 0.2319 | 0.6402 |
| **43** | 0.0300 | 0.0300 | 0.0300 | 0.0300 | 0.0300 | 0.0299 | 0.0296 | 0.0289 | 0.0349 | 0.1366 | 0.5385 |
| **44** | 0.0290 | 0.0290 | 0.0290 | 0.0290 | 0.0290 | 0.0290 | 0.0289 | 0.0564 | 0.0386 | 0.1016 | 0.3652 |
| **45** | 0.0288 | 0.0288 | 0.0288 | 0.0288 | 0.0288 | 0.0288 | 0.0287 | 0.0337 | 0.0383 | 0.1557 | 0.5110 |
| **46** | 0.0278 | 0.0278 | 0.0278 | 0.0278 | 0.0278 | 0.0278 | 0.0277 | 0.0269 | 0.0472 | 0.1216 | 0.5385 |
| **47** | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0270 | 0.0288 | 0.0346 | 0.1614 | 0.4342 |
| **48** | 0.0263 | 0.0263 | 0.0263 | 0.0263 | 0.0263 | 0.0263 | 0.0263 | 0.0260 | 0.0290 | 0.2219 | 0.2569 |
| **49** | 0.0262 | 0.0262 | 0.0262 | 0.0262 | 0.0262 | 0.0262 | 0.0262 | 0.0263 | 0.0386 | 0.2276 | 0.6092 |
| **50** | 0.0255 | 0.0255 | 0.0255 | 0.0255 | 0.0255 | 0.0255 | 0.0255 | 0.0269 | 0.0300 | 0.0920 | 0.4657 |
| **51** | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0253 | 0.0259 | 0.0265 | 0.1124 | 0.4584 |
| **52** | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0244 | 0.0250 | 0.0374 | 0.1100 | 0.3822 |
| **53** | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0239 | 0.0238 | 0.0244 | 0.0765 | 0.6349 |
| **54** | 0.0231 | 0.0231 | 0.0231 | 0.0231 | 0.0231 | 0.0231 | 0.0231 | 0.0229 | 0.0242 | 0.1010 | 0.3162 |
| **55** | 0.0228 | 0.0228 | 0.0228 | 0.0228 | 0.0228 | 0.0228 | 0.0228 | 0.0229 | 0.0273 | 0.0922 | 0.4384 |
| **56** | 0.0217 | 0.0217 | 0.0217 | 0.0217 | 0.0217 | 0.0217 | 0.0217 | 0.0217 | 0.0230 | 0.0956 | 0.3686 |
| **57** | 0.0219 | 0.0219 | 0.0219 | 0.0219 | 0.0219 | 0.0219 | 0.0219 | 0.0220 | 0.0218 | 0.2453 | 0.5197 |
| **58** | 0.0213 | 0.0213 | 0.0213 | 0.0213 | 0.0213 | 0.0213 | 0.0213 | 0.0213 | 0.0230 | 0.0555 | 0.3310 |
| **59** | 0.0211 | 0.0211 | 0.0211 | 0.0211 | 0.0211 | 0.0211 | 0.0211 | 0.0211 | 0.0219 | 0.1073 | 0.3364 |
| **60** | 0.0206 | 0.0206 | 0.0206 | 0.0206 | 0.0206 | 0.0206 | 0.0205 | 0.0206 | 0.0287 | 0.0562 | 0.2659 |