Maintaining OPOV:
Some Problems Associated with the Periodic Need to Re-Draw Parliamentary Boundaries in the UK and the US

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by

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ABSTRACT

The presentation will describe and analyze briefly the following six problems that are associated with the periodic need to re-draw the boundaries of districts (constituencies) in political systems based on single-member districts with emphasis on the UK and the US. Most of these problems can be characterized as involving 'decision making under conflict' and are looked at in-depth during the current year by both theoreticians and practitioners invited by the VPP group at CPNSS to a workshop in July at University of Caen. The problems have also featured in Public Lectures at LSE in November 2007 and two days ago.

- Historical overview of the OPOV principle in the UK and US.

- Two alternative methods to satisfy OPOV in political systems based on single-member districts.

- Problem 1: How many representatives?

- Problem 2: What should be the frequency of general redistricting?

- Problem 3: Which kind of population register should serve as the basis for redistricting?

- Problem 4: Should there be a uniform quota for all states in a federation or for all ethnic regions in a multi-ethnic nation?

- Problem 5: Apportioning representatives to states or regions: How should remainders be rounded?
  - Five seemingly reasonable methods for rounding remainders.
  - Four paradoxes associated with rounding methods.

- Problem 6: How can gerrymandering be minimized in drawing the boundaries of single-representative equally-sized districts?

• **Single-Member Districts**

Both the UK and the US belong to the class of countries whose national legislatures consist of representatives each of whom represents one district (constituency). This representation system is known in the literature as '*single-member districts'*. Currently there are 646 constituencies in the UK (and hence 646 MPs in the UK's House of Commons) and 435 Congressional districts in the US (and hence 435 Representatives in the US's House of Representatives).

• There are more than 50 countries in the world whose legislature (lower house) is based on single-member districts. The currently most well known, apart from the UK and US, include India (currently 543 districts), Canada (currently 308 districts), and France (currently 577 constituencies). These countries elect the representatives employing different election procedures, e.g., fist-past-the-post, runoff, alternative vote.

• **The Principle of OPOV**

The principle of democratic representation requires that, a priori, each citizen should be able – through his representative(s) – to exert the same extent of influence on the political process as that of any other citizen. This principle is currently known as the principle of '*one person, one vote*' (OPOV) but, as explained below, in order to provide for every person's vote the same value, it should in fact be called '*one person, one value*', or for the sake of better English '*Every Person, Equal Value*' (EPEV).

• **The Origins of OPOV in the UK and the US**

UK

As far as I know, the slogan 'one man, one vote' (OMOV) originated in the UK. It was traditionally used in the context of demands for suffrage reform. When the House of Commons was originally founded the emphasis was on representing *areas* – counties, boroughs, and later on universities. The entitlement to vote for Members of Parliament representing the constituencies varied widely, with different qualifications – such as owning property of a certain value, holding an apprenticeship, qualifying for paying the local-government rates, or holding a degree from the university in question. Those who qualified for the vote in more than one constituency were entitled to vote in each constituency, while many adults did not qualify for the vote at all. Plural voting was also present in local government, whereby the owners of business property qualified for votes in the relevant wards.

Over time reformers argued that Members of Parliament and other elected officials should represent citizens equally – interpreting the term "equally" to mean that each voter should only be entitled to exercise the vote once in an election. Successive *Reform Acts* both extended the franchise eventually to almost all adults (barring convicts, lunatics and members of the House of Lords) and also reduced and finally eliminated most of the plural voting by 1950 for both House of Commons and local-government elections.

US

As far as I know, the slogan ‘one person, one vote’ (OPOV) was first uttered in the US by Supreme Court Justice William O. Douglas who delivered in 1963 the Court's opinion in *Gray v. Sanders*. He said: “The conception of political equality from the Declaration of Independence to Lincoln's Gettysburg Address, to the Fifteenth, Seventeenth, and Nineteenth Amendments can only mean one thing -- one person, one vote.”

Unlike the situation in the UK, this slogan was not employed in the US in the context of demands for broadening the suffrage or for abolishing plural voting, but in the context of demands for equalizing the sizes of the various electoral districts. Once the practice developed in the US to elect each state’s representatives from districts within the state rather than at-large, the question arose as to whether the state legislature (which had responsibility for drawing these congressional districts) was required to see that these districts were equal in population. Some states redrew their US House districts every ten years, many did not. Some never redrew them, except when it was mandated by a change in the number of representatives to which that state was entitled in the House of Representatives. This led to disproportionality in the influence of voters across the states.

Additionally, in most US states, electoral districts for seats in the state’s upper house or Senate were ostensibly created at least partially on the basis of geography, rather than population. Whereas a state’s lower house seats might or might not be reapportioned on a decennial basis, such as those of the US House of Representatives, in most states, state senate district boundaries were *never* redrawn. As the US became more urban, this led to the dilution of votes of urban voters when casting ballots for state senate seats: a city dweller’s vote had less influence on the makeup of the state legislature than did a rural inhabitant’s.

In a series of four US Supreme Court decisions in the early 1960s, notably *Baker v. Carr* (1962), *Gray v. Sanders* (1963), *Wesberry v. Sanders* (1964), and *Reynolds v. Sims* (1964), it was ruled that districts for the US House of Representatives and for the legislative districts of both houses of state legislatures had to be roughly equal in population. (The US Senate was not affected by these rulings, as its makeup is explicitly established in the US Constitution). The Supreme Court's decisions concerning malapportionment ended the pattern of gross rural over-representation and urban under-representation in the US House and state legislatures. Eventually the US Supreme Court extended its rulings also to local (city) districts in its decision in *Morris et al v. Board of Estimate of the City of New York* (1989).

A more detailed survey of the legal history of OPOV in the US can be found in the 4th chapter of Moshé Machover and my book *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes* (Cheltenham UK:Edward Elgar, 1998).

- **Alternative Methods for Satisfying the Principle of OPOV**

There are, in principle, two alternative methods to satisfy OPOV in political systems based on single-member districts:

a) To equalize the number of citizens in each district and to award equal weight (number of votes) in the national legislative assembly to each district’s representative.

b) To leave the number of citizens in the various districts unequal and award the various representatives unequal weight (number of votes) in the national legislative assembly so as to equalize the (indirect) *a priori voting power* (influence) of all citizens across the various districts. (According to Lionel S. Penrose's [1946] *Square Root Rule* this can be obtained if the a priori voting power of every representative in the second tier of a two-tier voting system will be approximately proportional to the square root of the number of his/her constituents).

• In multi-nation legislative assemblies, such as the Council of Ministers of the European Union or the International Monetary Fund – where the number of citizens of the represented countries cannot be equalized and every country can be regarded as one district represented by a single representative – the only method that can be employed in order to satisfy OPOV is method (b) above.

The first 3-year grant of VPP awarded by the Leverhulme Trust was dedicated to further exploring the theory of a priori voting power and the way it is (or should be) implemented in various international organizations, mainly in the Council of Ministers of the European Union.

• We decided to dedicate part of the current (second) 3-year grant of VPP – the period from November 2007 through November 2008 – to exploring the problems associated with method (a) above which, due to demographic changes, requires the periodic re-drawing of at least some districts' boundaries so as to equalize their population with that of all the other districts.

In order to explore these difficulties we decided to conduct three public lectures at LSE (one in November 2007, one in May 2008, and one in the autumn 2008) and one workshop at the University of Caen in July 2008. The participants in these four events will be both theoreticians as well as UK and US practitioners involved in redistricting in these two countries.

Maintaining OPOV, particularly in political systems based on single-member districts, must tackle the following six problems – some of which involve decision making under conflict. Most of these problems will be discussed and looked at in-depth in the aforementioned public lectures and workshop.

• **Problem 1: How Many Representatives?**

US

The Constitution set the number of representatives at 65 from 1787 until the first enumeration in 1790 and stated that the "the number of representatives shall not exceed one for every thirty thousand, but each state shall have at least one representative". The first apportionment, based on the 1790 census resulted in 105 members. From 1800 through 1840, the number of representatives was determined by a revised maximal number of persons each representative could represent ("fixed ratio"), although the way to handle fractional remainders changed. Therefore, the number of representatives changed with that ratio, which was in turn affected by the growth of the population and with the admission of new states to the union.[1]

[1] Following the censuses of 1790, 1800, 1810, 1820, and 1830, the number of representatives apportioned was 105, 141, 181, 213, and 240, respectively.

For the 1850 census and later apportionments, the number of seats was determined prior to the final apportionment ("fixed house size"); and thus, the ratio of persons each was to represent was the result of the calculations.[2] In 1911, the House size was fixed at 433 with provision for the addition of one seat each for Arizona and New Mexico when they became states (U.S. Statutes at Large, 37 Stat 13, 14 (1911)). The House size, 435 members, has not been changed since, except for a temporary increase to 437 during the 3-year period between the time of admission of Alaska and Hawaii as states in 1959 and the redistricting of 1962 (that was based on the 1960 census).

The Constitution also requires that each State will have at least one representative in the House of Representative regardless of its population size. Currently there are seven states having only one representative (Alaska, Delaware, Montana, North Dakota, South Dakota Vermont, Wyoming).

UK

The number of MPs in the House of Commons is not fixed and has been increasing steadily with every general redistribution (except for a single downwards blip caused by the one-off reduction of 13 seats in Scotland in the 2005 elections consequent on the *Scotland Act 1998* 3). (The number of MPs increased from 625 in 1950, to 630 in 1955, to 630 in 1974, to 650 in 1983, to 651 in 1992, to 659 in 1997; it decreased to 646 in 2005 and is likely to increase again to about 650 prior to the next elections).

This tendency of constant increase is mainly due to the minimum number of MPs set out in *Schedule 2 of the Parliamentary Constituencies Act [1986 c. 56]* for the four countries of the UK. According to this Act the number of constituencies in Great Britain “shall not be substantially greater or less than 613”, the number of constituencies in Wales “shall not be less than 35”, and the number of constituencies in Northern Ireland “shall not be greater than 18 or less than 16”. Following the *Scotland Act 1998* the current number of constituencies in Scotland is 59 (down from a minimum of 71).

The UK must therefore decide whether to amend its laws so as to prevent any increase in the number of MPs.

- **Problem 2: What should be the Frequency of General Redistricting?**

Ideally, redistricting, if needed, should take place prior to every national election and be based on the exact size of the population (or eligible voters) existing prior to these elections. This ideal is not attained anywhere.

US

The Constitution (Article I.2.3) states that redistricting, if needed, should be implemented in the nearest Congressional election following a census that must be conducted every 10 years.

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[2] Following the censuses of 1840, 1850, 1860, 1870, 1880, 1890, and 1900, the number of representatives apportioned was 223, 234, 241, 292, 325, 356, and 386, respectively. Note that due to the change in determining the total number of representatives, the number of representatives apportioned following the censuses of 1840 and 1850 was smaller than the number of representatives apportioned following the 1830 census.

The first census was conducted in 1790 and the last occurred in 2000. The only census which was not followed by reapportionment was the census of 1920.

Currently, all levels of government in the United States using single-member districts must redraw their districts after each decennial census. The census is taken on April 1st of the last year of each decade (e.g. 1980, 1990, 2000, 2010). Its results are released around January of the following year (e.g. 2011) . Every government must use this new data to redraw its districts before its next election (usually in the next even numbered year [e.g. 2012]). If it fails to act timely, a court will redraw the districts in time for the election because the old districts are prima facie unconstitutional.

UK

The *House of Commons (Redistribution of Seats) Act, 1944* provided that all parliamentary boundaries should be revised at intervals of between 3 and 7 years after the first general redistribution. That took place by statute in 1948 and was the basis of the 1950 general election. The intention was plainly for a redistribution to take place once every parliament yet to allow for the uncertain length of parliaments.

When the recommendations of the first routine redistribution came before Parliament late in 1954, there were cross-party protests against the proposed upheaval. By the *Redistribution of Seats Act, 1958* the periodicity was lengthened to 10-15 years (i.e., approximately once every three normal parliaments).

The *Redistribution of Seats Act 1992* reduced the time to 8-12 years. The Conservative Government plainly wanted to accelerate the current redistribution so that new boundaries would be in place for the next general elections (which took place in 1997).

The case against frequent revisions turns on the inconvenience caused to citizens and to MPs. However, whatever the interval, there is a case for having a firmly guaranteed timetable. The Labour Government in 1969 and the Conservative Government in 1992 unilaterally varied the arrangements for party advantage.

Moreover, unlike the situation in the US, there is nothing in British legislation which prescribes how quickly the Boundary Commissions should complete their review, and how quickly must Parliament act on these proposals. Hence it must be stressed that in any decision on frequency of redistricting in the UK, the length of time consumed by the actual process of redistricting must be a major consideration.

Other Countries

In comparable countries with single-member constituencies, ten years has been the maximum allowable period between redistributions.

In Canada and in New Zealand redistricting normally occurs every five years, after quinquennial census. In Australia boundaries are normally reviewed every seven years. In Germany boundaries are reviewed in the first year of each four-year Bundestdag. In France there is no fixed period. In India boundary revision has been suspended since 1975.

• Problem 3: Which Kind of Population Register Should Serve as the Basis for Redistricting?

In principle, the population on which redistricting should be based is the population of eligible voters. However, in most countries citizens who wish to be eligible to vote must perform an active act of registration – which is usually not mandatory. This implies that citizens who could register but chose not to do so are not considered as eligible voters.

So in most countries there are two types of registers on which redistricting could be based: the periodic census or the electoral register.

According to the US Constitution, redistricting decisions, i.e., the number of (single-member) districts allotted to every state, are based on the decennial censuses which count the resident population of each of the 50 states plus overseas federal employees (military and civilian) and their dependents living with them who were not included in their home states.[3] This implies that redistricting decisions may be skewed in favor of those states that have a relatively large population that is not eligible to vote (e.g., children under 18, aliens, felons, persons who did not register) but which is nevertheless counted in the census.[4]

In the UK, in contrast, redistricting decisions are based on electoral registers – which are currently complied and run by the Electoral Commission. The *Representation of the People Act 1983* requires electoral registration officers (ERO's) to prepare and publish a register of electors for their area to include the names of everyone who appears to them to be eligible taking reasonable steps to obtain the required information. A canvas form is sent to each household in the UK every autumn for completion and return by the householder. The form asks for the details of all those eligible to vote (or eligible to vote in the near future) who are resident on 15 October. If an ERO considers someone is not entitled to be registered as an elector, they have no discretion to omit that person's name from the register. Although registration is not in itself compulsory, an ERO has the power to require information for the purpose of maintaining the register of electors. The current penalty for failing to complete and return the electoral registration is a fine not exceeding £1,000.

Since 1985, British citizens resident abroad have been permitted to register and vote at parliamentary and European Parliamentary elections. Applications to register in the constituency where electors were last registered before they left the UK must be renewed annually. Voters can register for up to 15 years after they last lived in the UK. Initial applications by overseas voters, Crown servants and British Council employees must be witnessed (attested) by a British passport holder.

Members of the British armed forces serving abroad and their families can now register at their home address as ordinary electors or by making an annual service declaration.

A person's name may appear on the electoral register only if they reside at an address within the electoral area. Residence is not defined by law, but it has been held by the courts to entail a 'considerable degree of permanence'. For example, a student who has a permanent home

[3] The population of the District of Columbia is excluded from the apportionment population.
[4] The US Constitution stated that every slave (euphemistically referred to as 'all other persons') – who of course was not eligible to vote – should count as 3/5 of a free person. In this way the number of representatives allotted to Southern states was inflated. This provision was repealed by section 2 of the 14th Amendment ratified in 1868.

address and a term-time address, and spends about the same amount of time in each, can be registered lawfully in both addresses. It is for the local ERO to decide in the light of the relevant circumstances, whether a person may be said to be resident at a particular address. Although a person may appear on two or more electoral registers, it is an offence to vote more than once in parliamentary elections or in the same local council election. It is, however, legal to vote in elections for different local councils.

The *Representation of the People Act 2000* introduced a new facility to allow certain people to make a 'declaration of local connection', i.e., a statement that they have a significant link with a locality. Previous electoral legislation did not prevent the homeless, remand prisoners or those in mental institutions (other than the criminally insane) from registering, but the requirement to establish residence on a qualifying date effectively did. The 2000 Act empowers ERO's to register applicants by accepting a declaration providing a contact address, e.g., that of an institution in which remand prisoners or mental patients are resident, or the address at which they were living or would otherwise be living. The homeless can register at the address of a place they have spent a substantial part of their time during the previous three months.

The *Representation of the People Act 2000* introduced voluntary 'rolling' electoral registration to enable people to be added to or deleted from the electoral register at any time of the year rather than on a single date. To do so the interested person should approach his ERO after the annual publication date of the electoral register (1 December since 2001) asking the s/he be registered at his/her residential address at that time. If the ERO finds the applicant eligible to vote and if the application was made by about the middle of the month, the applicant's name should be added to the register on the first working day of the following month. The only exception to this is between September and November of each year when the EROs are carrying out the comprehensive canvass upon which is based the new revised register published on 1 December. This arrangement enables people to register closer to polling day when an election is about to take place, thereby increasing the number of people eligible to participate in the election.

Finally, it should also be noted that any attempt to move from electorate to population as the basis for drawing constituency boundaries in the UK would be hotly controversial in Northern Ireland, where the ratio of electorate to population is smaller in one community than in the other. Such a move would therefore have partisan consequences. On grounds of uniformity throughout the UK, that may rule it out for Great Britain as well.

Problem 4: Should there be a Uniform Quota for All States in a Federation or for All Ethnic Regions in a Multi-Ethnic Nation?

The principle of OPOV requires that the number of representatives allotted to each state in a federation or to any ethnic region in a multi-ethnic nation should be, as nearly as possible, proportional to the state's/region's population.

In the US the quota is indeed uniform for all states – except that each state is entitled to at least one representative in the House of Representatives regardless of the size of its population.

In the UK the quota is not uniform for all four countries that constitute the UK. According to *Schedule 2 of the Parliamentary Constituencies Act 1986* [c. 56] :

–The number of constituencies in Great Britain shall not be substantially greater or less than 613.

– The number of constituencies in Scotland shall not be less than 71 [this was repealed by the *Scotland Act 1998* which determined that the quota for all Scottish constituencies will be the same as the quota for England, except that the Orkney Islands and the Shetland Islands will constitute separate constituencies.]

– The number of constituencies in Wales shall not be less than 35.

–The number of constituencies in Northern Ireland shall not be greater than 18 or less than 16.

The table below depicts the size of the total electorate, as well as the average electorates per seat, in the four countries of the UK following the 2005 parliamentary elections.

	(1) Total Electorate	(2) Average Electorate per MP	(3) Actual No. of Seats in Parliament	(4) Seats Due at Equal Representation*	(5) (3) – (4)
UK	44,245,939	68,492	646	646	--
England	37,041,396	70,022	529	541	-12
Scotland	3,839,900	65,083	59	56	+3
Wales	2,224,650	55,616	40	32	+8
N. Ireland	1,139,993	63,333	18	17	+1

Source: *British National Commission* official results for the 2005 parliamentary elections (downloadable from http://tinyurl.com/292x46 and from http://tinyurl.com/35bpya); author's calculations.

* The numbers in this column are obtained by dividing each of the four countries' electorates by the average national electorate (68,492) and rounding the result according to every rounding method listed in Problem 5 below except Jefferson's method which assigns to England, Scotland, Wales and N. Ireland 542, 56, 32, and 16 seats, respectively.

Problem 5: How Should Quotas be Fixed or How Should Remainders be Rounded?

Ideally each constituency electorate (or population) within a fixed unit (e.g., a state or an ethnic region) should be as close as possible to the *electoral quota*, where the electoral quota is the electorate (or population) of the whole nation divided by the number of seats in the legislature. But whereas the number of representatives allotted to each unit must be an integer number, the division of each unit's electorate (or population) by the quota does not usually result in an integer number. The problem therefore is which method of rounding remainders ought to be used.

There are five seemingly reasonable methods for rounding remainders. Under some of these methods the (same) outcome can be obtained by employing different arithmetical formulae and hence, depending on the formula used, the same method has been called in different countries by different names. As all these methods have been first proposed in the US, we list below the names by which they were known in the US and the periods in which they were used there.

1790 to 1830 – The *Jefferson method* of greatest divisors (fixed ratio with rejected fractional remainders). Under this method, one first chooses the size of the House to be apportioned. Then one finds a divisor *X* so that the whole numbers contained in the quotients of the states sum to the required total. One assigns to each state its whole number. Fractional remainders were not considered, no matter how large. Thus a state with a quotient of 3.99 received three representatives, the same number as a state with a quotient of 3.01.

Jefferson's method is currently used, inter alia, in Argentina, Austria and Switzerland (where it is called "the Hagenbach-Bischoff method"), Belgium, Chile, Columbia, Croatia, Czech Republic, Denmark, East Timor, Ecuador, Finland, Hungary, Iceland, Israel, Italy, Japan, Republic of Macedonia, the Netherlands, Paraguay, Poland, Portugal, Romania, Scotland, Serbia, Slovenia, Spain, Turkey, and Wales (where it is called "the D'Hondt method"),[5] in Brazil (where it is called "the method of highest averages"), as well as (under one of these names) in the Federal Republic of Germany and in Liechtenstein.

1832 -- The mathematician James Dean, of Vermont, proposed in 1832 a method identical to Webster's, but it rounded off entitlements not at 1/2 but at the harmonic mean of the two nearest integers.[6] The Dean method would have given an advantage to small states (such as Vermont) and it was not adopted.

1840--The *Webster method* of major fractions (fixed ratio with retained major fractional remainders). This method was applied in the same way as the Jefferson method, except if a fractional remainder were greater than one-half, another seat would be assigned. Thus a state with a quotient of 3.51 received four representatives, while a state with a quotient of 3.49 received three.

Webster's method also has several aliases: it is alternately referred to as "Sainte Laguë method" and the "method of odd numbers".[7] It is currently used for pure PR in Bosnia, Denmark, Latvia, Kosovo, Sweden, Norway, New Zealand, and in the elections to the Palestinian Legislative Council in 2006. The UK Electoral Commission used the Webster method to assign European Parliament seats to the 12 regional constituencies in the UK in 2003.

1850-1900--The *Vinton* or *Hamilton method* established a predetermined number of representatives for each apportionment, and divided the population of each state by a ratio determined by dividing the apportionment population of the United States by the total number of representatives. The resulting whole number was assigned to each state, with an additional seat assigned, one at a time, to the states with the largest fractional remainders, up to the predetermined size of the House of Representatives.

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[5] Under D'Hondt's method one assigns, one seat at a time, to the recipient (state, party) who obtains the largest quotient in the formula $v / (s+1)$, where v denotes the number of votes received by the recipient in an election (or census) and s denotes the number of seats in the legislature that the recipient had already received. At the beginning of the apportionment process $s = 0$ for all recipients.

[6] The harmonic mean of two integers a and b is equal to $2ab / (a+b) = 1 / [(1/a + 1/b) / 2]$. It is always smaller than both the geometric and arithmetic mean of a and b.

[7] Similar to D'Hondt's method, according to St. Lagüe's method one assigns, one seat at a time, to the recipient whose quotient under the formula $v / 2(s+1)$ is largest.

This method, which is usually referred to as the "method of greatest remainders", has been used in Israel for pure PR during the period 1948-73, and is currently used for the pure PR part in each of Costa Rica's provinces, in Russia, Ukraine, Namibia, and for the federal parts of Sweden's single House.

1910, 1930--*The method of major fractions* assigned seats similarly to the Webster method of 1840 by rounding fractional remainders using the arithmetic mean. The ratio was selected so that the result would be the predetermined size of the House of Representatives. In 1910, the House size was fixed at 433 with provision for the addition of one seat each for Arizona and New Mexico when they became states.[8]

Since 1940--*The Hill method of equal proportions* assigns seats similarly to the Jefferson and Webster methods, except it rounds fractional remainders of the quotient of the state population divided by the ratio differently. With this method, an additional seat is assigned if the fraction exceeds the difference obtained by subtracting the integer part of the quotient from the geometric mean of this integer and the next consecutive integer. The size of the House of Representatives remained fixed at 435 (except when Alaska and Hawaii became states, there was a temporary addition of one seat for each until the apportionment following the 1960 census).

Following the 1990 census, two lawsuits concerning apportionment issues were filed in federal courts. The U.S. Supreme Court held that the method of equal proportions was constitutional; that the Congress had properly exercised its apportionment authority; and that the inclusion of U.S. federal military and civilian personnel, and their dependents, in the apportionment populations of the states was constitutional. These cases were United States Department of Commerce v. Montana 112 S.Ct. 1415 (1992) and Franklin v. Massachusetts 112 S.Ct. 2767 (1992).

No rounding method is or can be paradox-free.

Hamilton's method suffers from the following three paradoxes:

• The *Alabama Paradox*: This is a monotonicity paradox. Ceteris paribus, a state (or a country) may be allotted fewer representatives in the national legislature if the total number of representatives apportioned is increased. The US Congress ceased using Hamilton's method soon after the chief statistician of the US Bureau of the Census (CW Seaton) informed the US Congress (in 1881) that the Hamilton method is defective because he discovered that, ceteris paribus, Alabama would be entitled according to this method to 8 representatives in the US Congress if the total number of representatives is 299, but to only 7 representatives if the total number of representatives is increased to 300.

Example demonstrating the Alabama Paradox.

Suppose 25 representatives have to be allotted among 5 states, A-F, whose population is 1,500, 1,500, 900, 500, 500, 200, respectively.

[8] The number 433 was chosen because it was the smallest number guaranteeing that no state's representation in the House of Representatives will be smaller than its representation following the 1900 census.

In this case the (Hare) quota per representative is 204 (5,100 : 25). The two-round allocations and remainders are as follows:

State	A	B	C	D	E	F	Total
Population	1500	1500	900	500	500	200	5100
Representatives							25
Quota							204
Quotas Received	7.35	7.35	4.41	2.45	2.45	0.98	
1st Round Allocation	7	7	4	2	2	0	22
Remainder	0.35	0.35	0.41	0.45	0.45	0.98	
Surplus Allocation				1	1	1	3
Total Representatives	7	7	4	3	3	1	25

Now suppose that, ceteris paribus, 26 representatives are to be allotted instead of 25. In this case the (Hare) quota per representative is 196 (5,100 : 26) and we obtain the following table:

State	A	B	C	D	E	F	Total
Population	1500	1500	900	500	500	200	5100
Representatives							26
Quota							196
Quotas Received	7.65	7.65	4.59	2.55	2.55	1.02	
1st Round Allocation	7	7	4	2	2	1	23
Remainder	0.65	0.65	0.59	0.55	0.55	0.02	
Surplus Allocation	1	1	1				3
Total Representatives	8	8	5	2	2	1	26

As we can see, states D and E now get only two representatives each (instead of 3).

• The *Population Paradox*: The population paradox violates the fundamental idea that changes in apportionment ought to reflect correctly changes in population. According to this paradox state B may gain seats at the expense of state A between two successive censuses even though the rate of growth of the population of state A was larger than that of state B.

Example demonstrating the Population Paradox

Suppose that in the Singsing Federation there are three states, A,B, and C, and that its House of Representatives is composed of 386 representatives. A population census is conducted in Singsing every five years and the number of representatives allotted to each of its three states is proportional to the states' population using Hamilton's method for rounding remainders. The table below depicts the situation following the censuses conducted in 2000 and 2005.

State	Population 2000	Exact Seats 2000	Rounded Seats 2000	Population 2005	Exact Seats 2005	Rounded Seats 2005
A	1,854,184	9.5997	10	1,873,951	9.5091	9
B	694,466	3.5955	3	699,114	3.5476	4
C	72,006,958	372.8048	373	73,495,708	372.9433	373
Total	74,555,608	386.0000	386	76,068,773	386.0000	386

As can be seen from this table, the population of state A was 2.67 times larger than that of state B in 2000 and 2.68 times larger than that of state B in 2005. Yet according to Hamilton's rounding method state B gained one seat at the expense of state A in 2005.

• The *New States Paradox*: If the number of representatives in the legislature is increased in order to accommodate a new state such that the new state gets a number of (additional) representatives which is approximately equal to its quota, it can nevertheless happen under Hamilton's method that state B may gain a representative at the expense of state A although the population sizes of states A and B did not change.

Example demonstrating the New States Paradox

Suppose that in the Federation of Liliput there were at the beginning of the year 2000 three states, A,B, and C, whose population is depicted in the table below and that its House of Representatives consisted of 386 delegates apportioned proportionally according to Hamilton's method among the three states. It thus turns out that, on average, each of the 386 delegates represented 193,167 person (= 74,562,608 : 386).

State	Population Before D Joined	Exact Seats Before D Joined	Rounded Seats Before D Joined	Population After D Joined	Exact Seats After D Joined	Rounded Seats After D Joined
A	7,264,183	37.6056	38	7,264,183	37.5886	37
B	694,466	3.5952	3	694,466	3.5935	4
C	66,603,959	344.7992	345	66,603,959	344.6433	345
D	–	–	–	1,000,000	5.1745	5
Total	74,562,608	386.0000	386	75,562,608	391.0000	391

Now suppose that a new state, D, joins the Federation of Liliput. As D's population is 1,000,000 it should be entitled to about 5 delegates (1,000,000 : 193,167). In fact, as can be seen from the last column in above table, it receives this number, bringing the total in Liliput's House of Representatives to 391 delegates. As the population of the other three states has not changed, one would have expected that the distribution of the remaining 386 delegates among the other three (veteran) states should not be affected. But under Hamilton's method this is not necessarily so. As can be seen from the last column in the above table, state B gains one delegate at the expense of state A.

The three "divisor" methods (Jefferson, Webster and Hill) suffer from the following paradox:

• *Not Staying Within the Quota Paradox*: If the size of the legislature is predetermined then it is possible that a state (or country) may receive more representatives in the national legislature than its quota rounded up, or fewer representatives than its quota rounded down. Of the three methods which suffer from this paradox Webster's method stays more closely to the quota than the other two.

Following is an example showing how the Webster, Hill, and Jefferson methods do not stay within the quota vis-à-vis State D: this state gets less than its quota rounded down according to Webster's and Hill's methods, while it gets more than its quota rounded up according to Jeffesrson's method.

State	Population	Quota	W and H Apport.	J Apport. (Divisor = 42,200)
A	70,653	1.552	2	1
B	117,404	2.579	3	2
C	210,923	4.633	5	4
D	1,194,456	26.236	25	28
Total	1,593,436	35	35	35

Problem 6: Who Should Determine the Boundaries of Districts and How Can it be Done Fairly?

The fact that all districts have the same population size is no guarantee in itself against gerrymandering, as the US experience has shown**.** For ensuring OPOV under the above-mentioned method (a), equal electorates in all districts are a necessary but not a sufficient condition for fairness.

• Gerrymandering is a form of redistricting in which electoral district or constituency boundaries are manipulated for electoral advantage or disadvantage of particular constituents, such as members of a racial, linguistic, religious or a class group, often in the favor of ruling incumbents or a specific political party. The word "gerrymander" is named for the Governor of Massachusetts Elbridge Gerry (1744–1814) and is a portmanteau of his name with the word "salamander" which was used to describe the appearance of a tortuous electoral district pressed through the Massachusetts legislature in 1812 by Jeffersonian Democrats, in order to disadvantage their electoral opponents in the upcoming senatorial election – and reluctantly signed into law by Gerry.

• The possibility to gerrymander equally-sized districts is demonstrated in the two hypothetical grid maps below. Suppose each of the 25 squares in each of the grid maps constitutes a 5000-person ward, and that every 5 contiguous wards must constitute one district (constituency) to be represented in parliament by a single representative – the one who obtained the majority of the votes in the district. Suppose further that the majority of voters in each ward is denoted by the letter R (Republicans) or D (Democrats).

The Republicans constitute the majority in 13 (52%) of the 25 wards, while the Democrats constitute the majority in 12 (48%) wards. So if the (integer) number of representatives in the 5-member parliament is to be as close as possible to the distribution of Democrats and Republicans in the entire 25 wards, there should be two districts represented by a Democratic representative (i.e., two districts in each of which there are at least 3 wards with a Democratic majority) and three districts represented by a Republican representative.

R	D	R	D	R
D	R	R	D	R
D	R	R	R	D
D	D	R	R	D
D	R	D	R	D

Map 1

R	D	R	D	R
D	R	R	D	R
D	R	R	R	D
D	D	R	R	D
D	R	D	R	D

Map 2

Source: Z. Landau, O. Reid, I. Yershov, "A fair division solution to the problem of redistricting", mimeographed, March 2006, p. 7. Downloadable from http://tinyurl.com/2lsawq

However, if the Republicans were to draw the district lines then, as can be seen in Map 1, they could draw the lines such that they control four districts and the Democrats only one district. But if the Democrats were to draw the district lines then, as can be seen from Map 2, they could draw the lines such that they control four districts and the Republicans only one.

So Map 1 depicts the possibility that gerrymandering may enable the (Republican) majority to obtain in parliament considerably more seats than its fair share, while Map 2 depicts the even worse possibility where gerrymandering enables the (Democratic) minority to obtain a large majority of seats in the parliament.

• It should be noted that, in general, in a single party districting protocol, without geometric constraints, a party with X% of support of the voters, can win just under min (2X%, 100%) of the districts by adopting the strategy of 'packing and cracking' the supporters of the rival party, i.e., drawing the district lines such that it barely wins in the districts it wins and badly loses in any district it loses. In reality, the geometric constraints of the layout of the voting map usually mean that this ideal outcome cannot be achieved. However, in most cases the party involved in a single party districting protocol, with even partial knowledge of the voting map, can win significantly larger percentage than X% of the districts – as has been often demonstrated in the US when the party in control of the districting maps changes.

• So who should determine the boundaries of districts so that the result of redistricting is considered to be fair?

US

In 36 of the 50 US states the state legislature has primary responsibility for creating a redistricting plan,[9] usually subject to the approval of the state's governor. Hence the US is a

[9] Of the remaining 14 states seven are small states where the entire state constitutes a single district (Alaska, Delaware, Montana, North Dakota, South Dakota, Vermont, Wyoming), five states (Arizona, Hawaii, Idaho,

single-district country where episodes of gerrymandering are very likely to occur. The most recent example of gerrymandering in the US happened when Republicans captured control of the Texas legislature in 2002 and redrew the state districts in mid-decade. The result was that the Texas representation in the US House of Representatives changed from 15 Republicans and 17 Democrats following the 2002 congressional elections, to 22 Republicans and 10 Democrats following the 2004 congressional elections. The Democrats gained back three of the five redrawn districts following the 2006 congressional elections, so the current representation of Texas in the House of Representatives consists of 13 Democrats and 19 Republicans.

In a decision on June 28, 2006, the US Supreme Court upheld most of the Texas congressional map engineered in 2003 by former House majority leader Tom DeLay. The seven-to-two decision now allows politicians in all of the USA to redraw and gerrymander districts as often as they like (not just after census-mandated reapportionment and redistricting) to protect their political parties and seats, so long as they do not harm racial and ethnic minority groups. A 5-4 majority threw out one Congressional district in the case for this reason. In other words, whereas purely racial gerrymandering is constitutionally prohibited, partisan gerrymandering is constitutionally permissible.

In response, some states have considered steps to revoke and separate redistricting authority from politicians and give it to other, more neutral, commissions.

The recent Texas gerrymandering episode was analyzed two days ago by Steve Bickerstaff (who wrote a book about it), and will be re-analyzed, in more detail, in the upcoming VPP workshop in July at the University of Caen.

UK

In the UK there were several famous gerrymandering episodes, particularly in Northern Ireland in the 1920s and 1930s.

Currently in the UK four non-partisan Boundary Commissions (one each for England, Scotland, Wales and Northern Ireland) are responsible for reviewing the boundaries of parliamentary constituencies within guidelines set by Parliament. In the upcoming VPP workshop, the recent redistricting of Scotland (from 72 to 59 districts) will be described and discussed, as well as novel game-theoretic and other methods that may be considered as more fair than current methods in determining districts' boundaries.

• It may be argued that:

1) The strict adherence to equal district sizes – which often requires that districts' boundaries in the US have to be re-drawn – may contribute to gerrymandering;

2) Once the responsibility for drawing district boundaries is shouldered by politicians, they have the right to manipulate it, so that it gives them an advantage (as long as an unpopular incumbent can still be unseated).

New Jersey and Washington), carry out congressional redistricting by an independent, bipartisan commission, and two states (Iowa and Maine) give independent bodies authority to propose redistricting plans, but preserve the role of their legislatures to approve them.

3) Gerrymandering might not necessarily be 'bad'. Gerrymandering can be used to create districts that better represent certain minorities. Such districts exist commonly in the US, particularly in California, Texas, and some Southern states.

An example of 'beneficial' gerrymandering is the case of Arizona's Native American reservations. Here it was thought inappropriate that the Hopi and Navajo nations should both be represented by the same House member because of historic conflicts between these two tribes. Since the Hopi reservation is completely surrounded by the Navajo reservation, this required an unusual (gerrymandered?) district configuration which features a fine filament along a river course several hundred miles in length attaching two regions.

4) Except for clear discrimination on racial or religious lines prohibited by the *Voting Rights Act of 1965*,[10] there is no 'universally' accepted definition of what constitutes a 'gerrymander' or an 'inappropriate' redistricting (of equally sized districts).[11]

• So in discussing how to maintain OPOV in political systems based on single-member districts one has to distinguish between the issue of malapportionment and the issue of gerrymandering -- and both must be resolved. To correct malapportionment does not necessarily imply that districts have to be of equal size, and inasmuch as (1) one continues to maintain equally-sized political systems based on single-member districts, and (2) there arises a need to re-draw districts' boundaries (e.g., because a state has been apportioned fewer/additional representatives in the legislature following a census) -- then one must devise a *process* for re-drawing boundaries that will be considered fair by reasonable persons.

• Except for assigning the responsibility for re-drawing boundaries to a neutral or cross-party agency, several apparently fair mathematical or game-theoretical procedures have been proposed.

Thus, for example, the Center for Range Voting has proposed a way to draw districts by simple algorithm. Because the algorithm uses as inputs only the shape of the state, the number N of districts needed, and the population distribution – and does not know the party loyalties of those people – the result cannot be biased. The algorithm (slightly simplified) is:

1. Start with the boundary outline of the state.
2. Let N = A+B where A and B are nearly equal whole numbers as possible. (For example, 7 = 4 + 3).
3. Among all possible dividing lines that split the state into two parts with population ratio A:B, choose the *shortest*.

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[10] In 1973, the Supreme Court held certain legislative multi-member districts unconstitutional under the 14th Amendment on the ground that they systematically diluted the voting strength of minority citizens in Bexar County, Texas. This decision in *White v. Regester*, 412 U.S. 755 (1973), strongly shaped litigation through the 1970s against at-large systems and gerrymandered redistricting plans. In *Mobile v. Bolden*, 446 U.S. 55 (1980), however, the Supreme Court required that any constitutional claim of minority vote dilution must include proof of a racially discriminatory purpose, a requirement that was widely seen as making such claims far more difficult to prove.

[11] In *Karcher v. Daggett* [466 U.S. 910 (1984)] US Supreme Court Justice Brenan, joined in his dissent by Justices White and Marshall, stated that although states may justify deviations from the ideal of district population equality based on the decennial census in order to '…making districts compact, preserving municipal boundaries, preserving cores of prior districts, avoiding contests between incumbents, and inhibiting gerrymandering' … We have never concluded, nor in my view should we conclude, that the existence of noncompact or gerrymandered districts is by itself a constitutional violation.'

4. We now have two hemi-states, each to contain a specified number (namely A and B) of districts. Handle them recursively via the same splitting procedure.

This algorithm has the advantage of simplicity, ultra-low cost, clear unbiasedness, and it produces simpler boundaries that do not meander needlessly. But it has the disadvantage that it ignores geographic features such as rivers, cliffs, and highways. As of July 2007, shortest-splitline redistricting pictures are available for all US states. (These are available at http://rangevoting.org/SplitLR.html).

A similar idea is to constitutionally define a specific minimum isoprerimetric quotient, or minimum ratio, between the area and perimeter of any given congressional voting district. (See mathematical explanation in http://mathworld.wolfram.com/IsoperimetricQuotient.html). Computer algorithms could ensure that population districts were drawn in such a way as to minimize isoperimetric inequality and effectively eliminate gerrymandering. Although technologies presently exist to define districts in this manner, there exists no national movement anywhere to implement such a policy.

• In this connection it should be noted that the introduction of modern computers and the development of elaborate voter databases alongside special districting software has made gerrymandering a far more precise science. Using these databases, politicians can obtain detailed information about every household including political party registration, previous campaign donations, and the number of times residents voted in previous elections. Using this information alongside other predictors of voting behavior such as age, income, race, or education level, drawers of a new electoral map can predict the voting behavior of each potential district with an astonishing degree of precision, greatly increasing the efficiency of gerrymandering and reducing the chance of accidentally making a district competitive.

This, I think, is something we should focus on in the upcoming Caen workshop in July 2008.