LSE Philosophy, Logic & Scientific Method

Department of Philosophy, Logic and Scientific Method public lecture

Lakatos Award Lectures

Professor Gordon Belot

Professor, Department of Philosophy, University of Michigan

Professor David Malament

Distinguished Professor Emeritus, Department of Logic and Philosophy of Science, University of California, Irvine

Professor John Worrall

Chair, LSE

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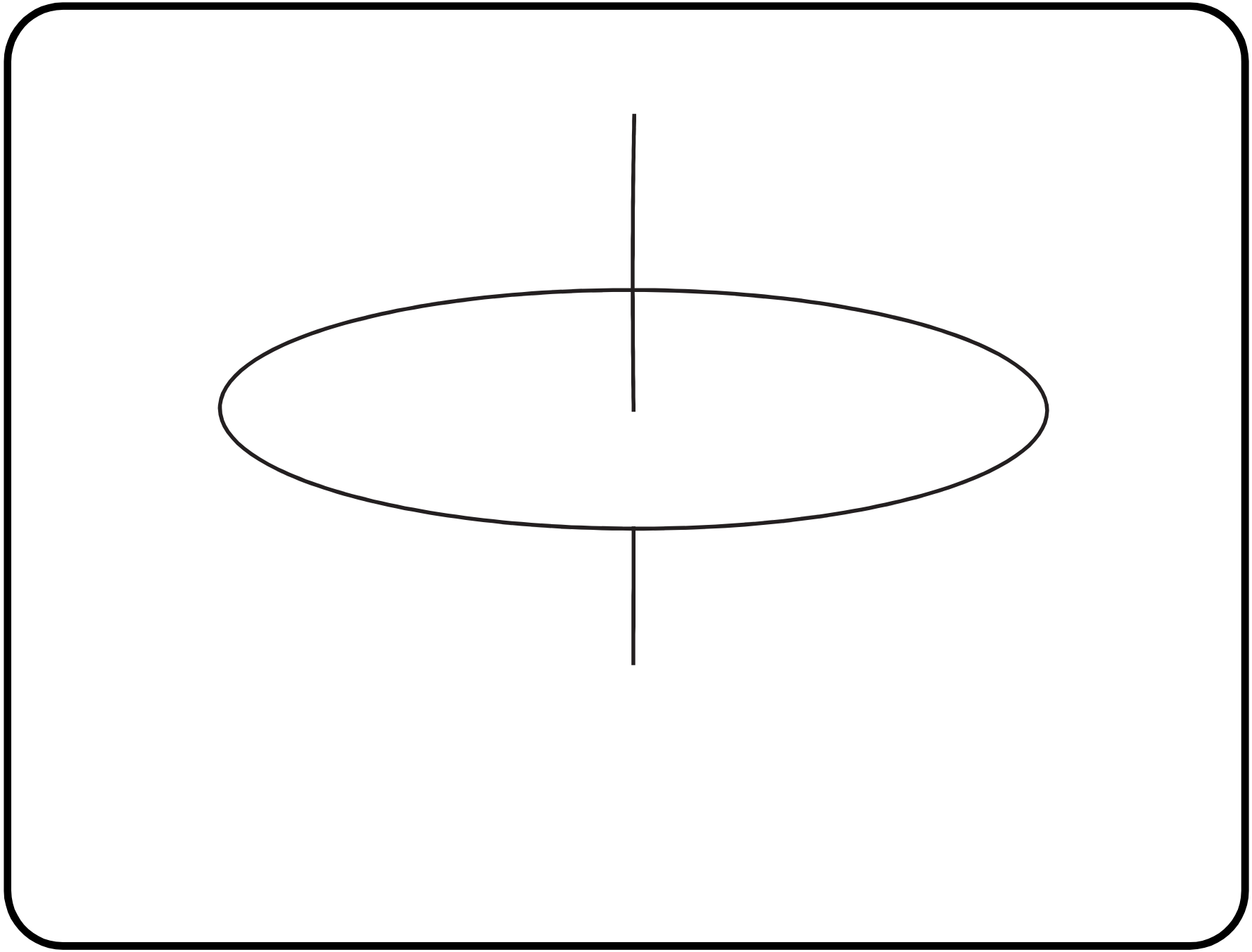
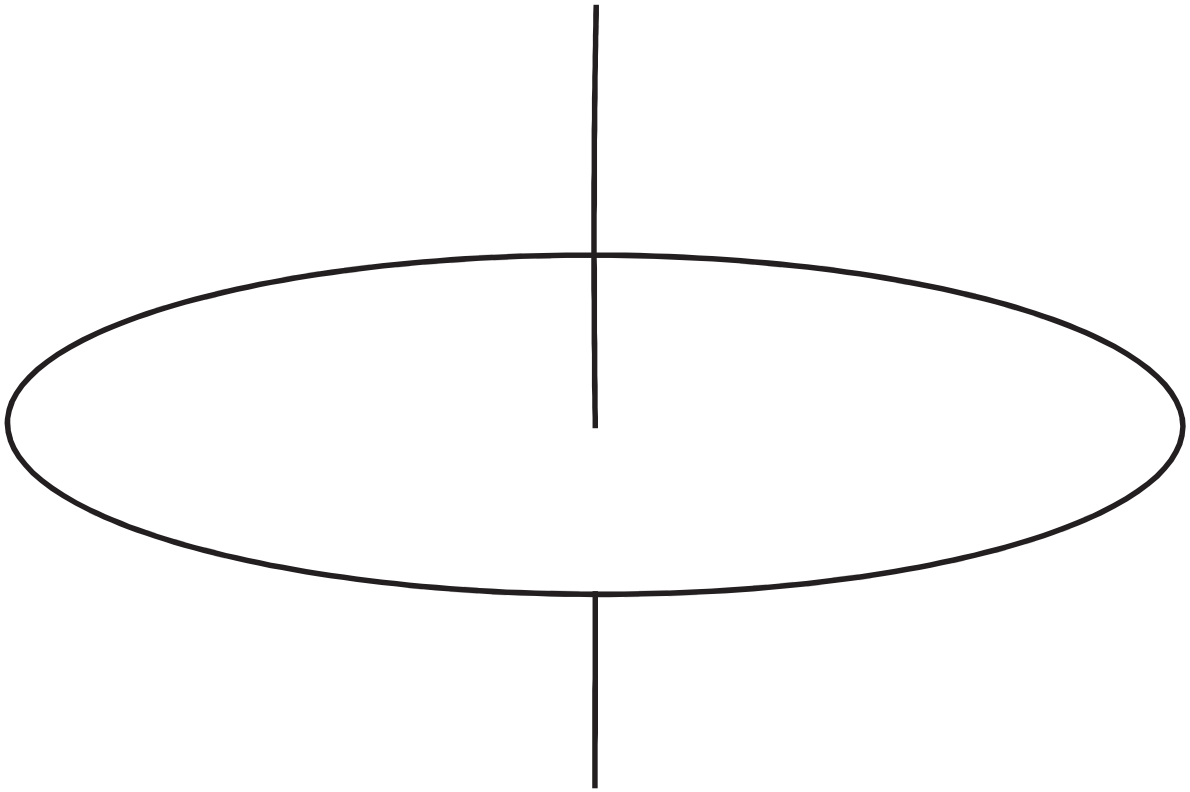
On the Concept of "Rotation" in Relativity Theory

David B. Malament





What does it mean to say that the ring is ***not-rotating***

about the axis?

Principal Claims:

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In some circumstances allowed by relativity theory (not all) ...

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(a) The question has no simple (unique) answer. One has many inequivalent criteria of rotation.

(b) None of these criteria fully answers to our classical intuitions.

(c) It is possible to capture (b) in the form of a "no-go theorem".

Three criteria of non-rotation:

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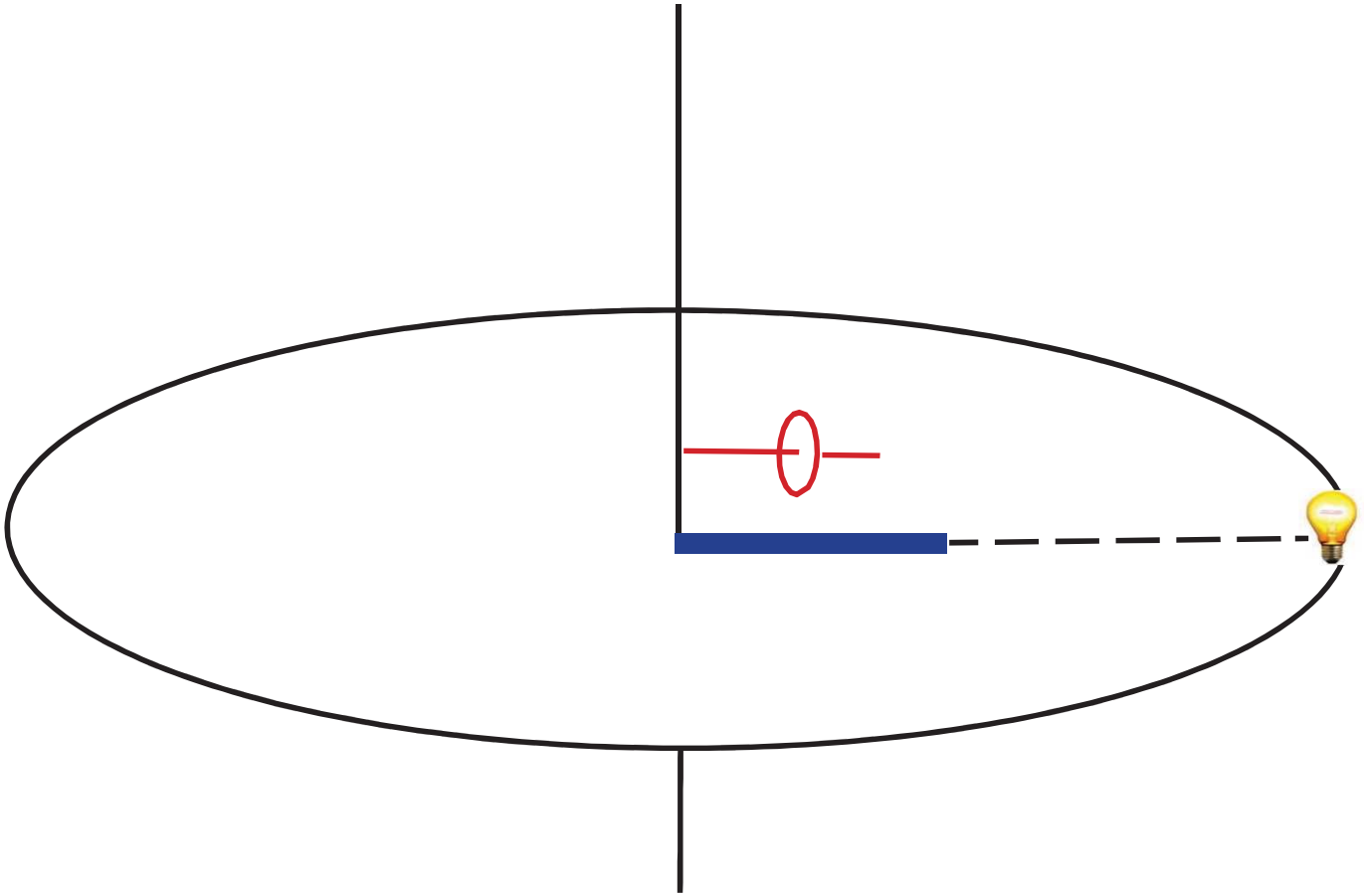
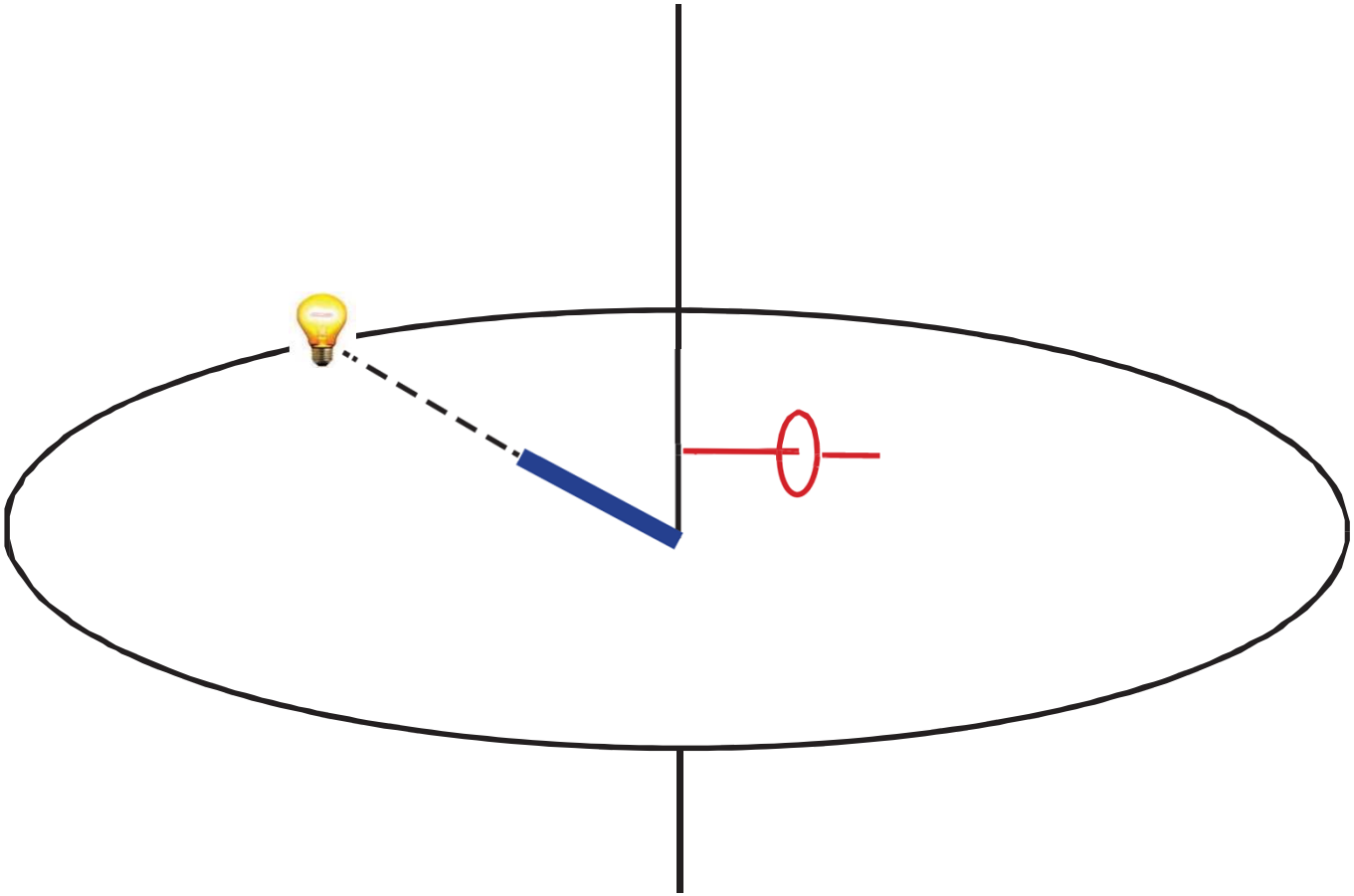
(3) zero angular momentum (ZAM)



CIA criterion of non-rotation



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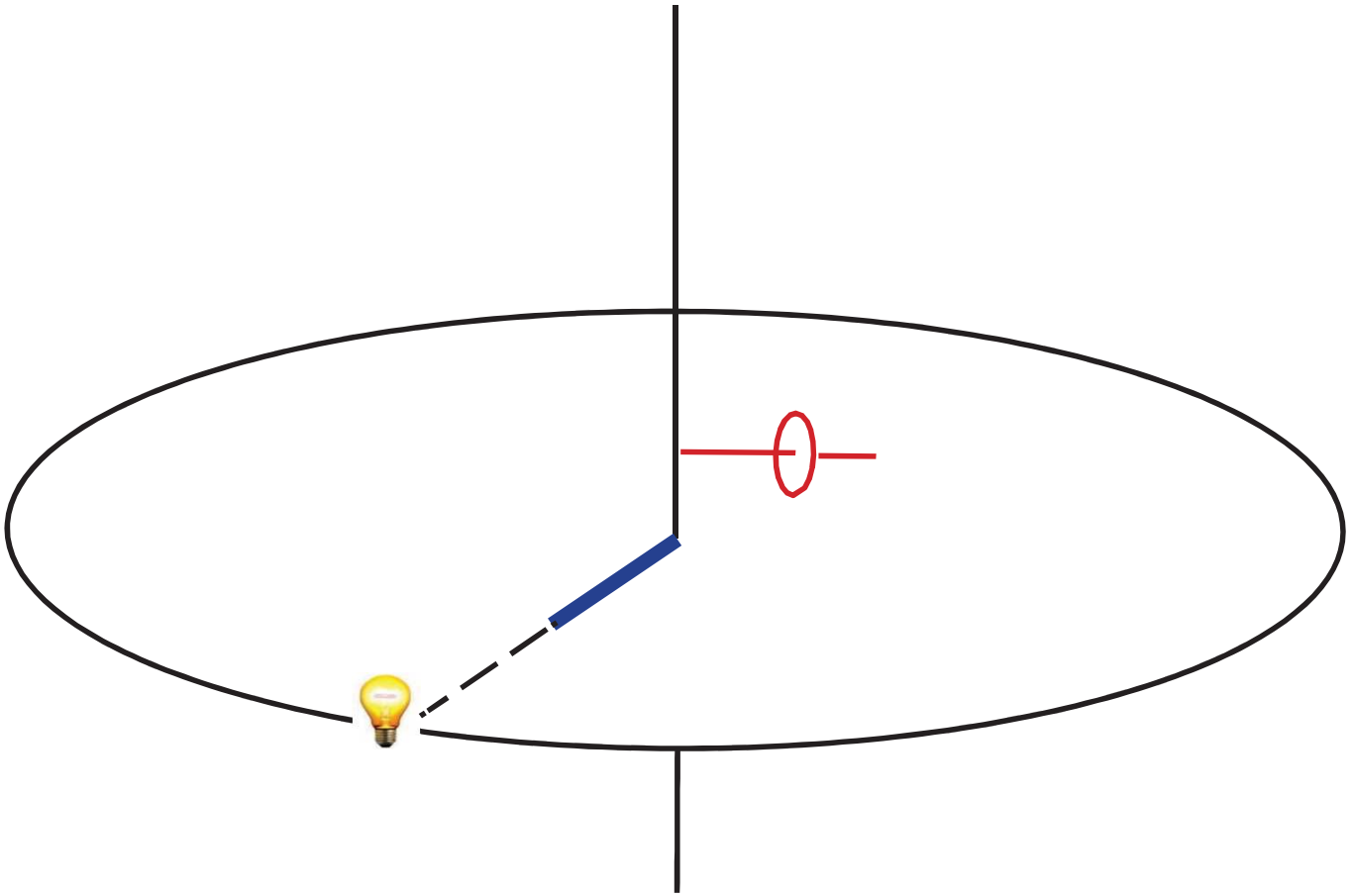

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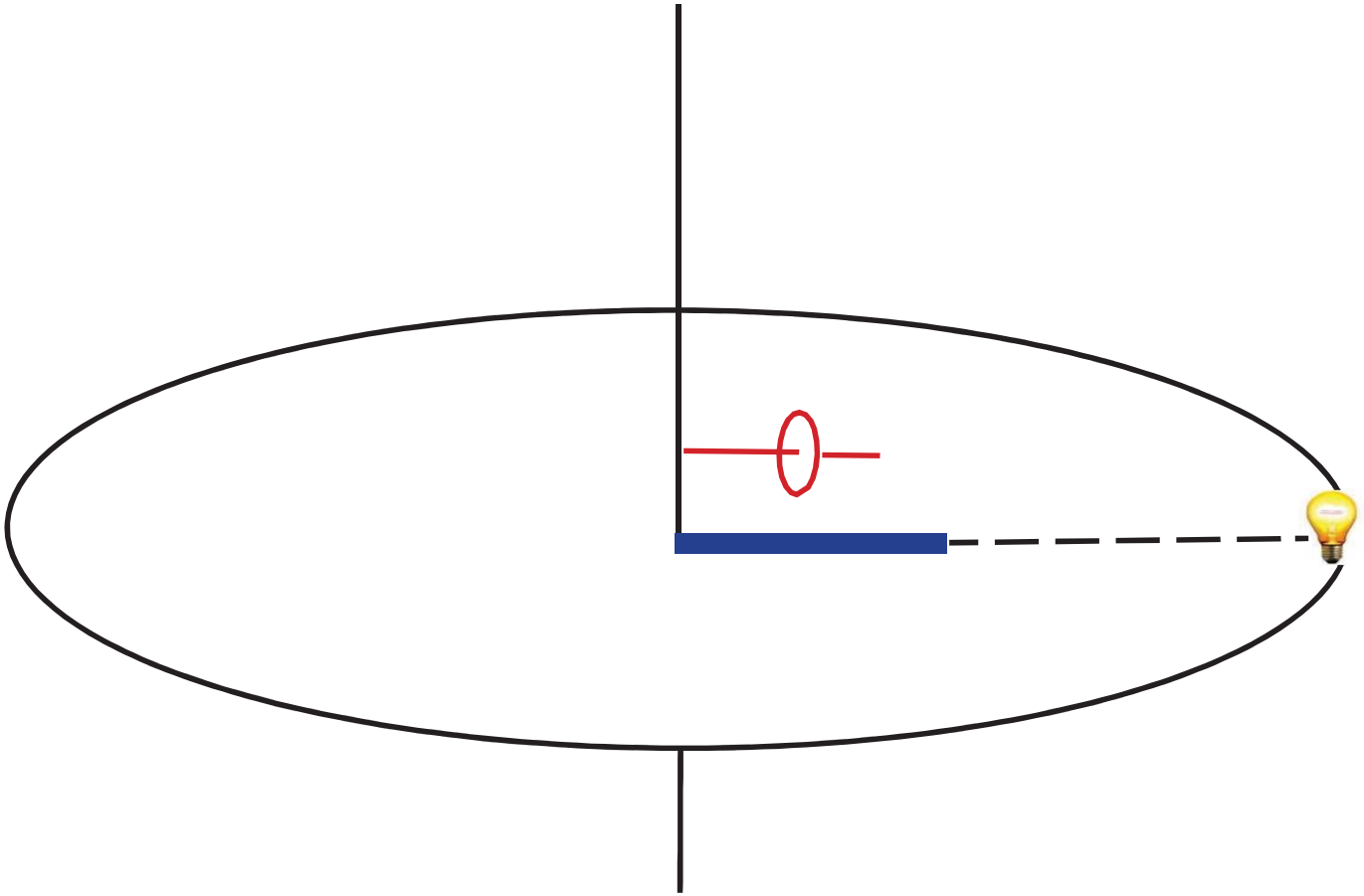

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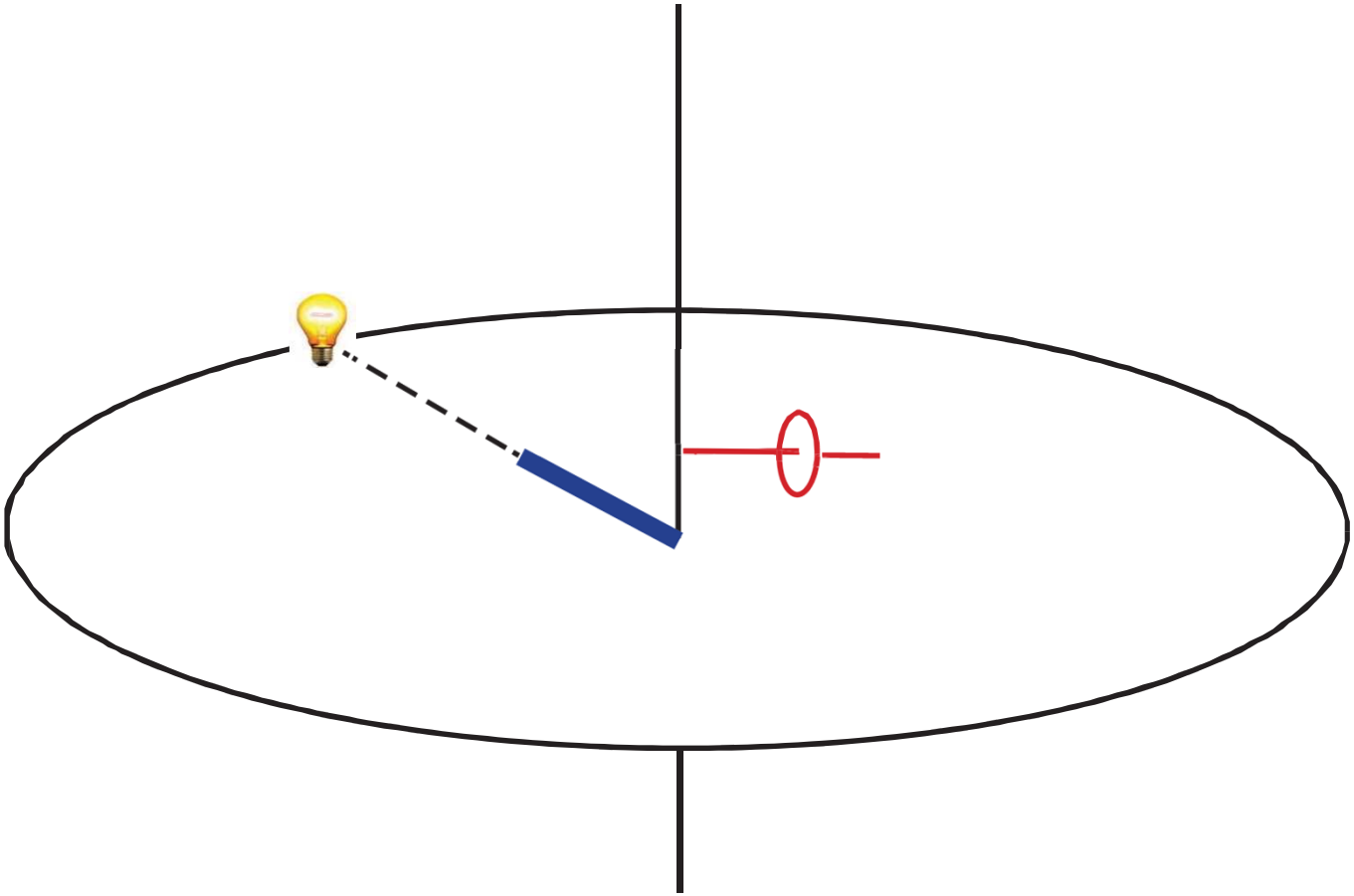

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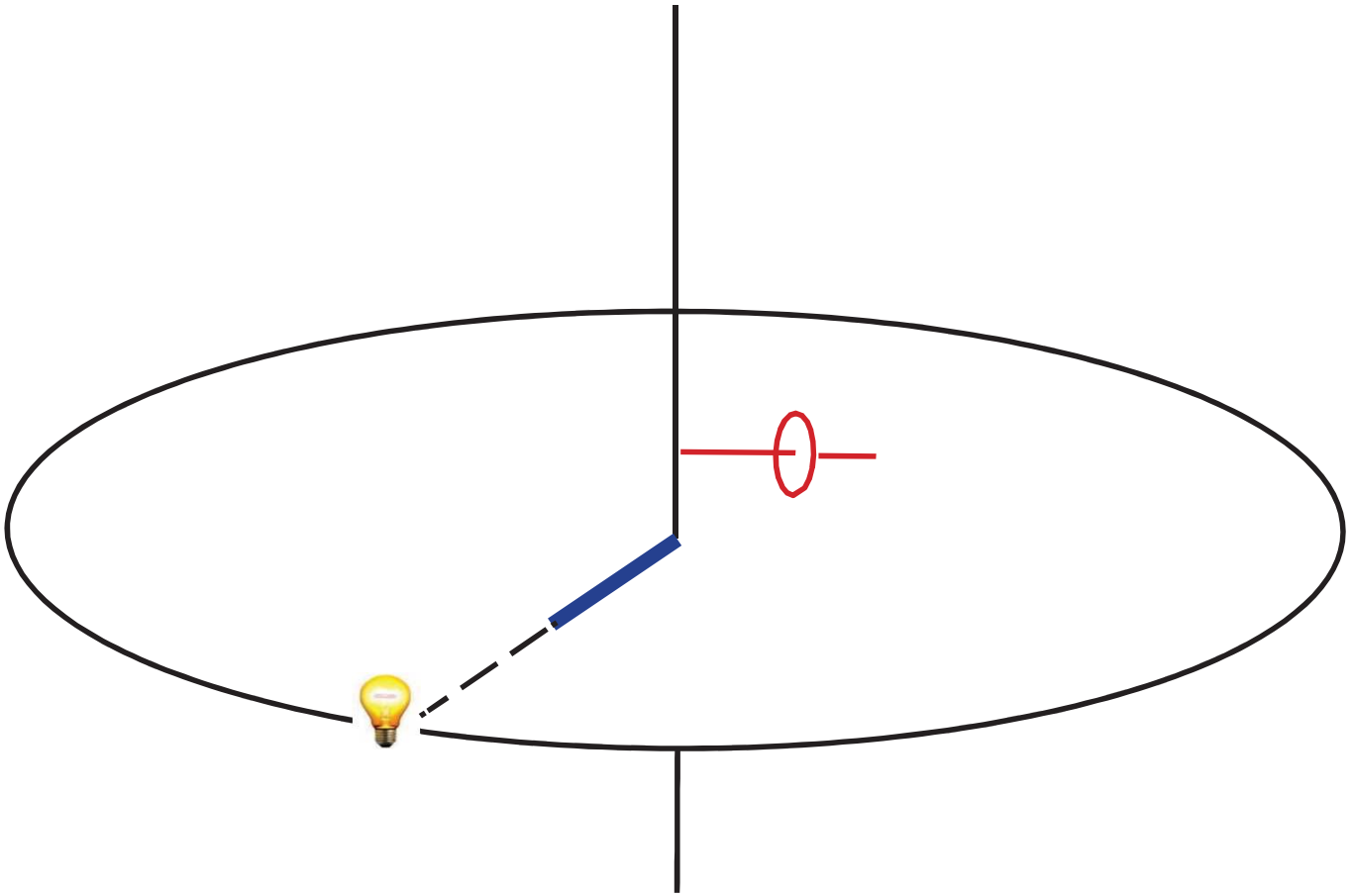

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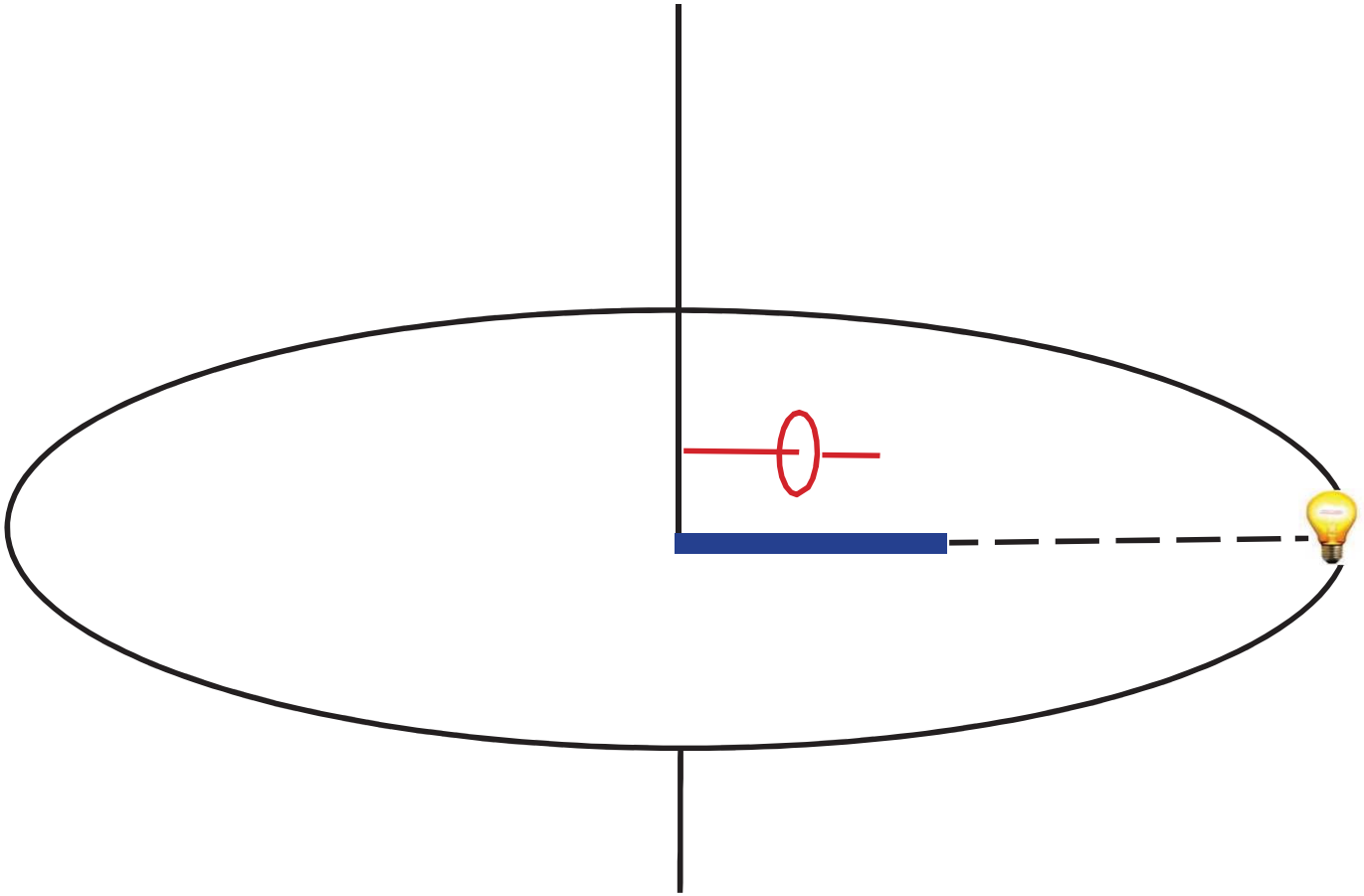
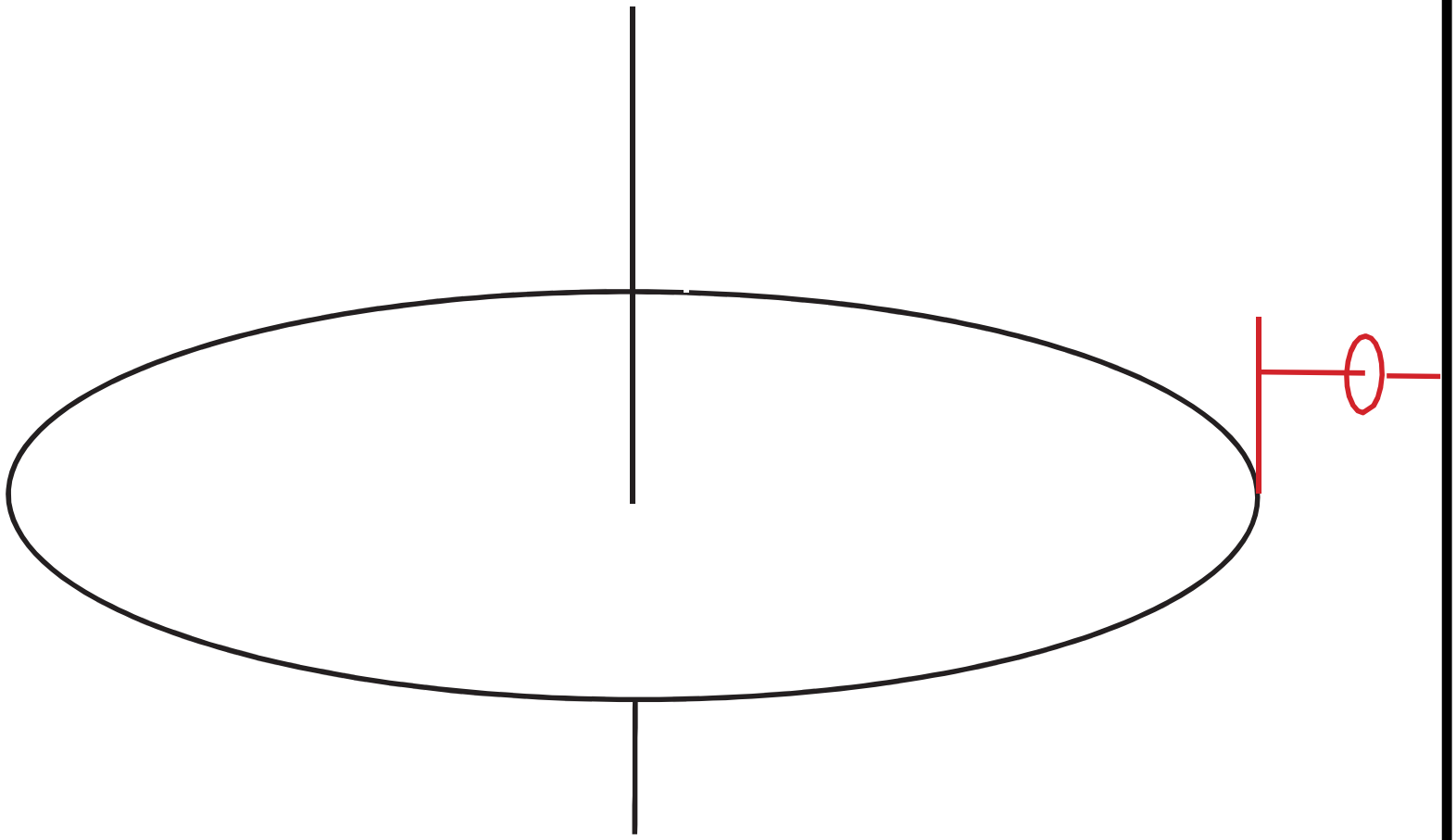
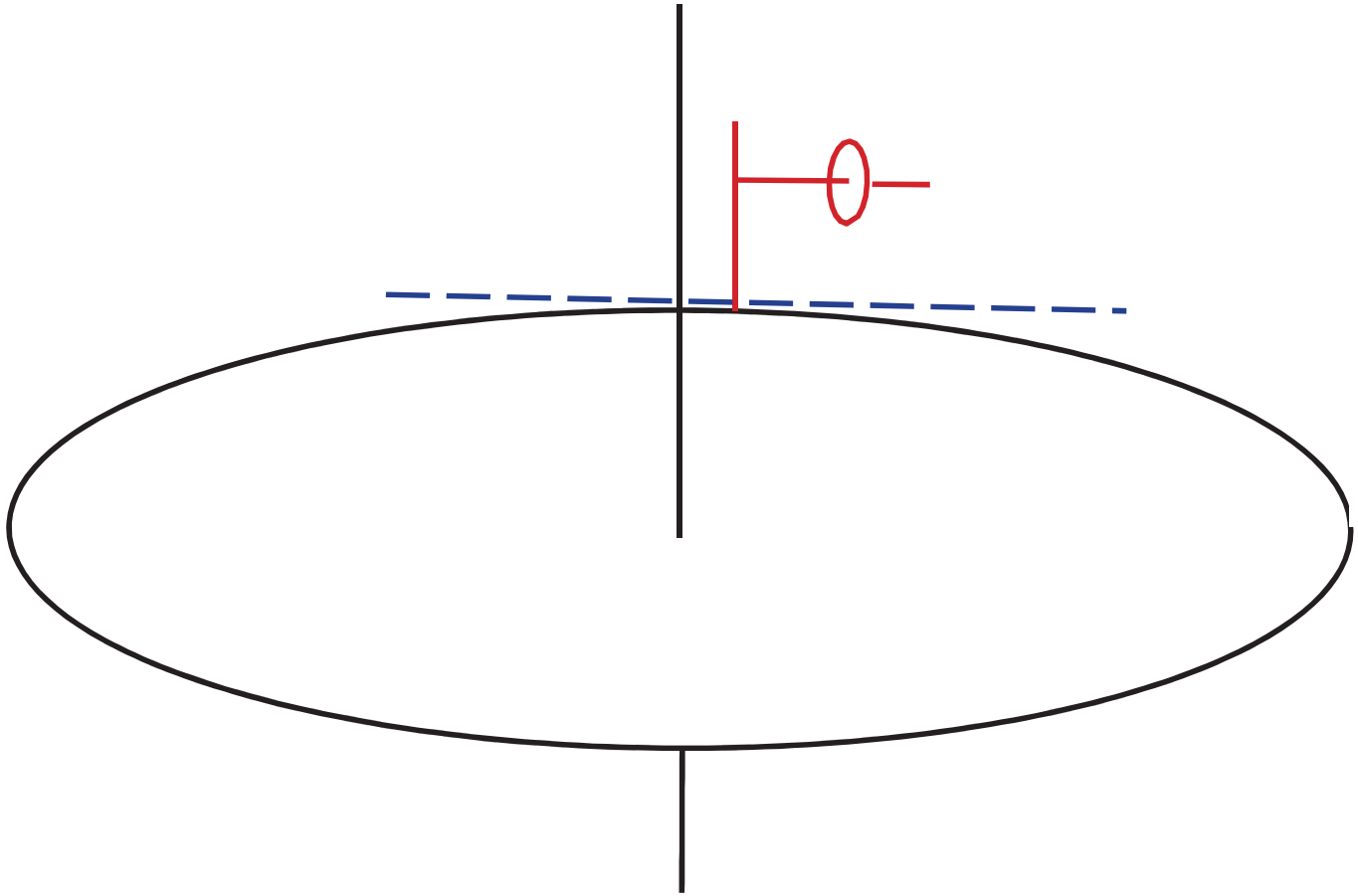


CIA criterion of non-rotation

We could also set this up with a water bucket.



CIR criterion of non-rotation



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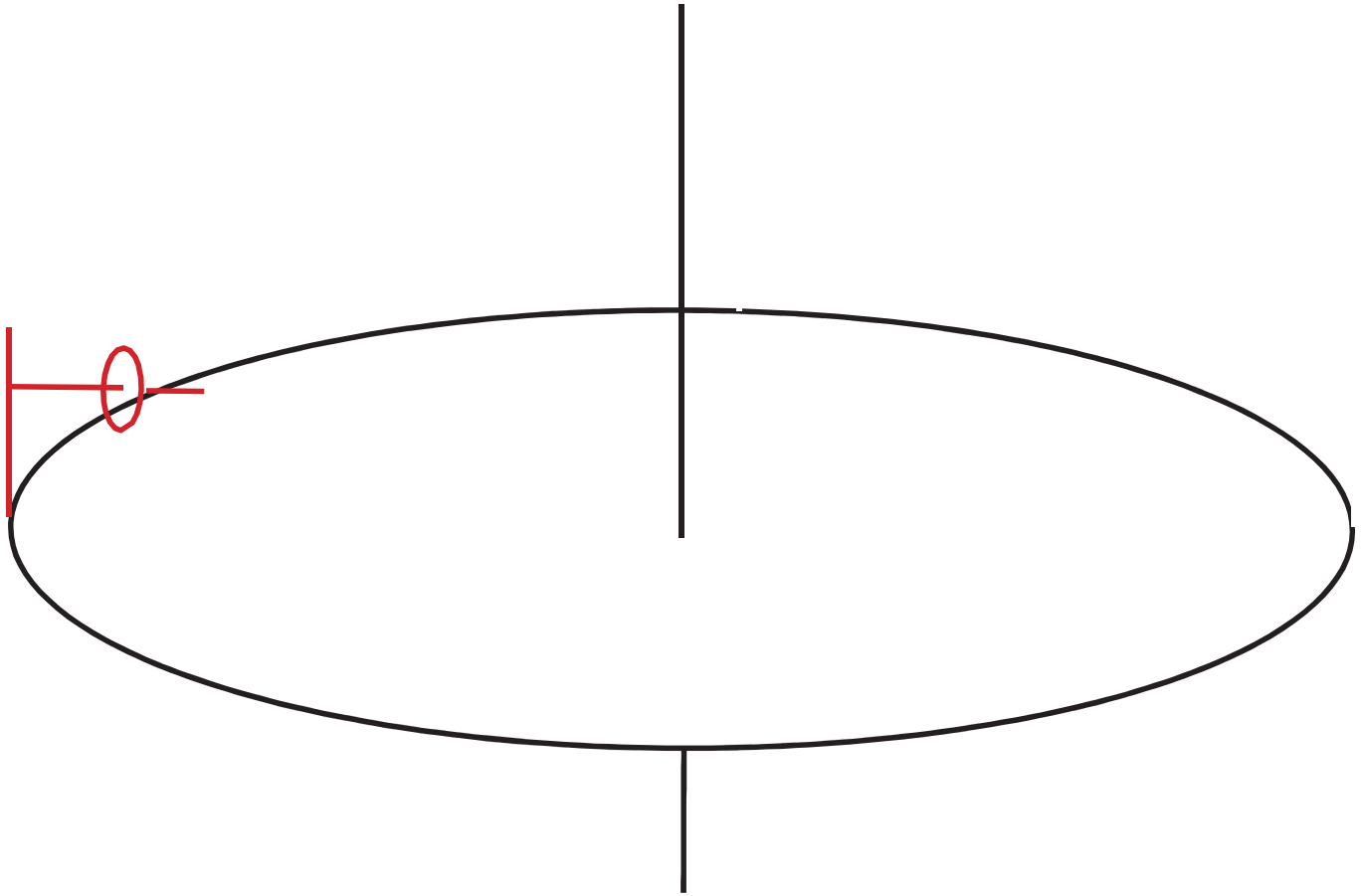

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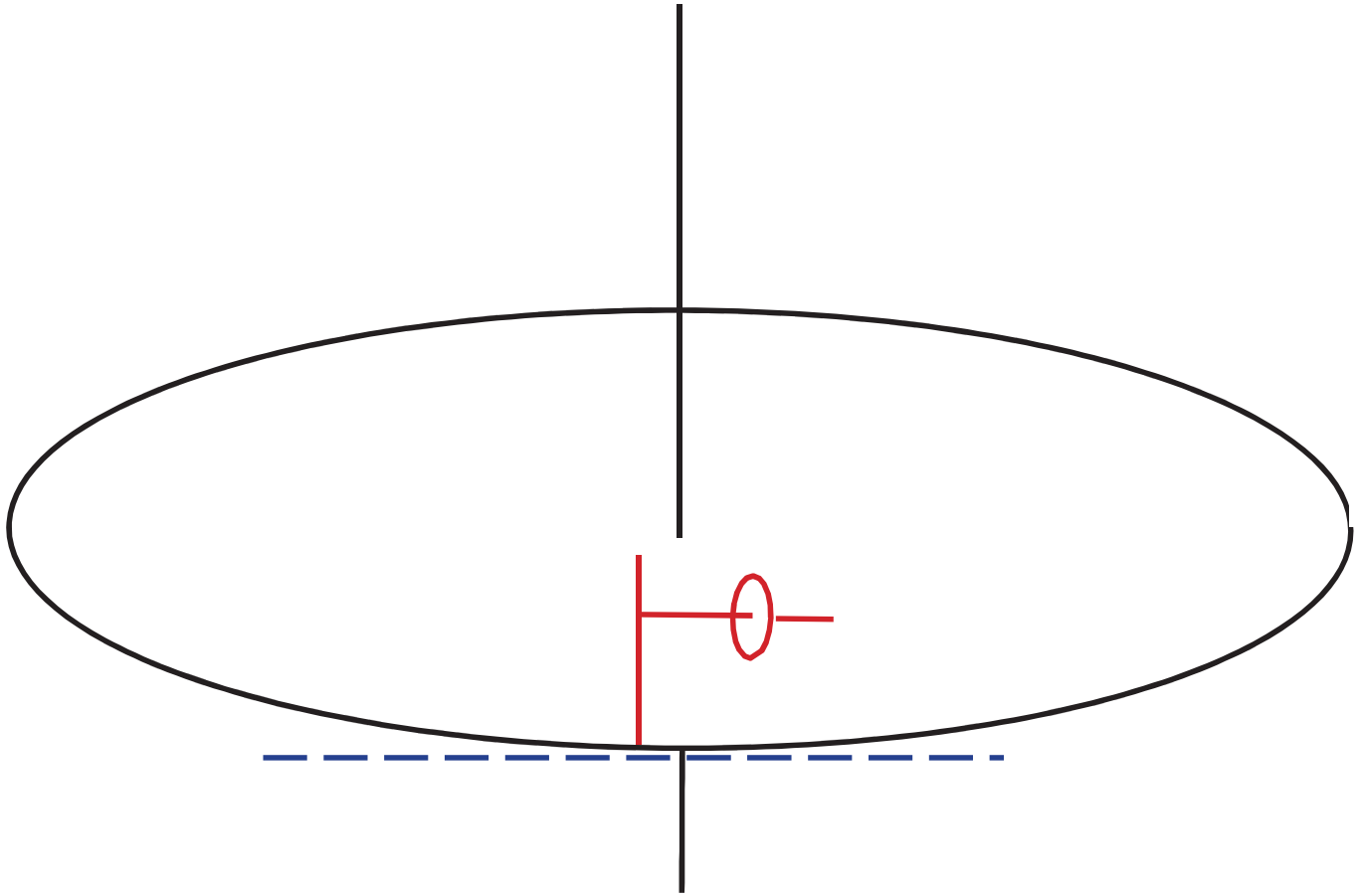

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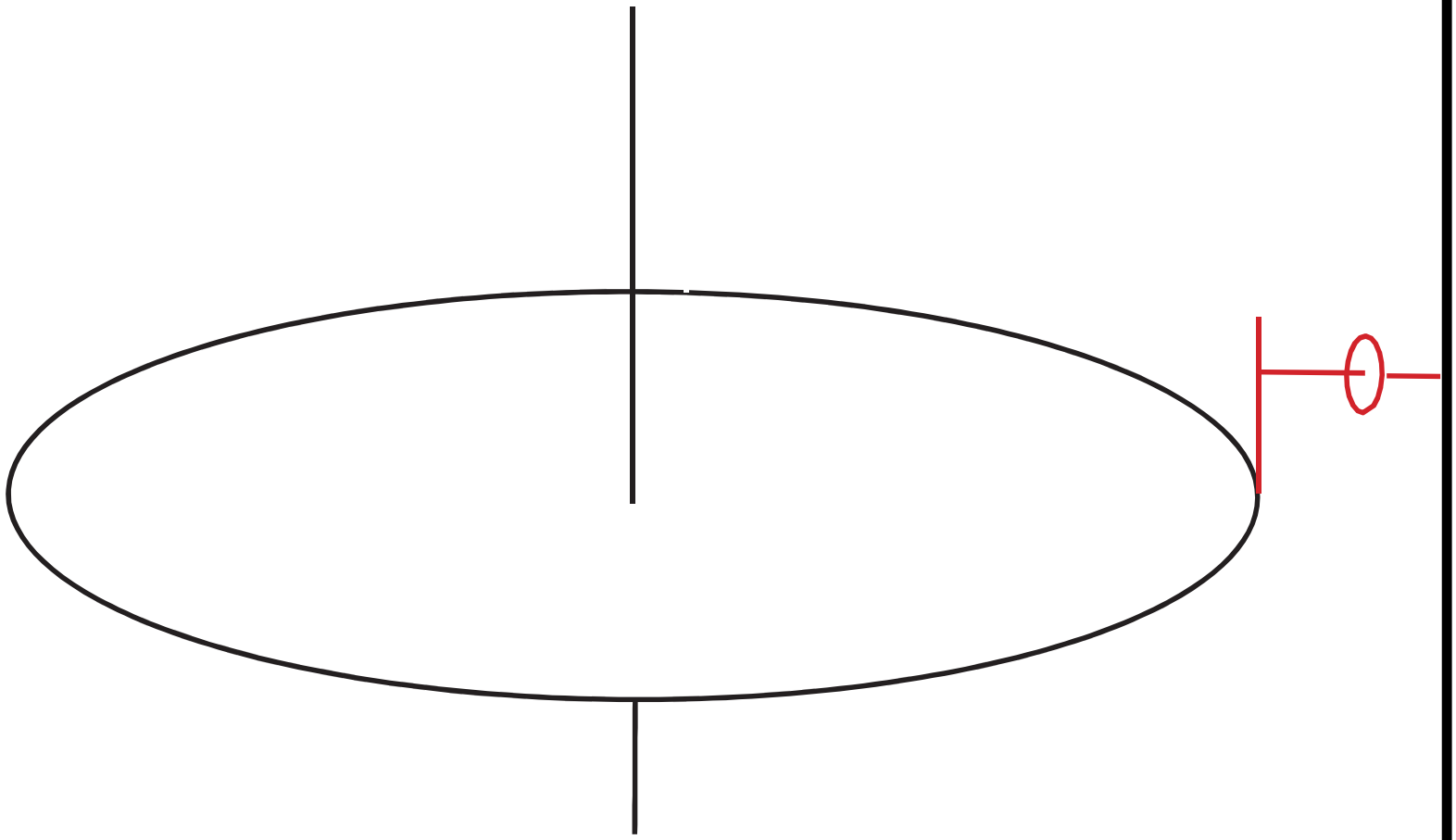

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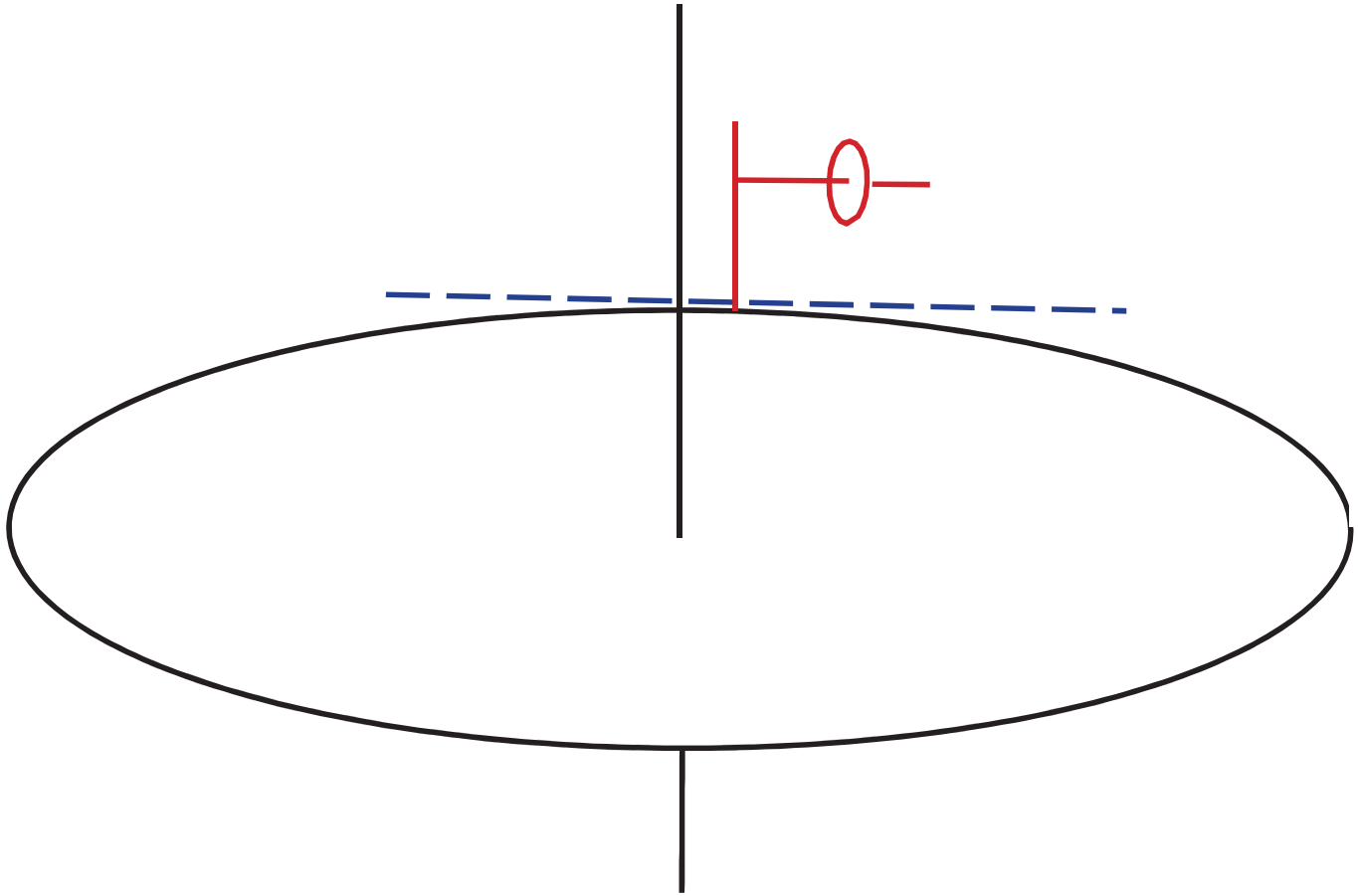

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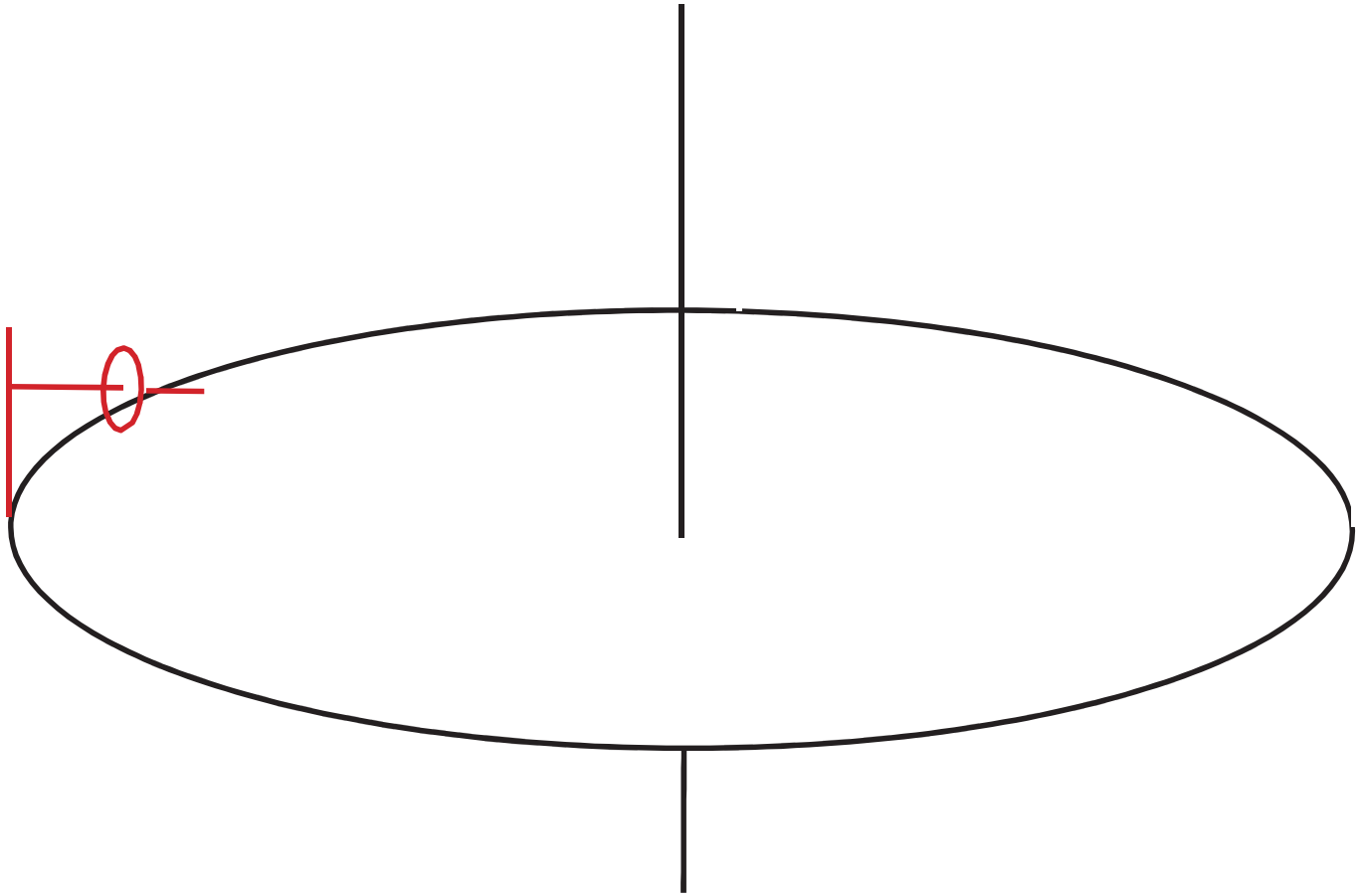

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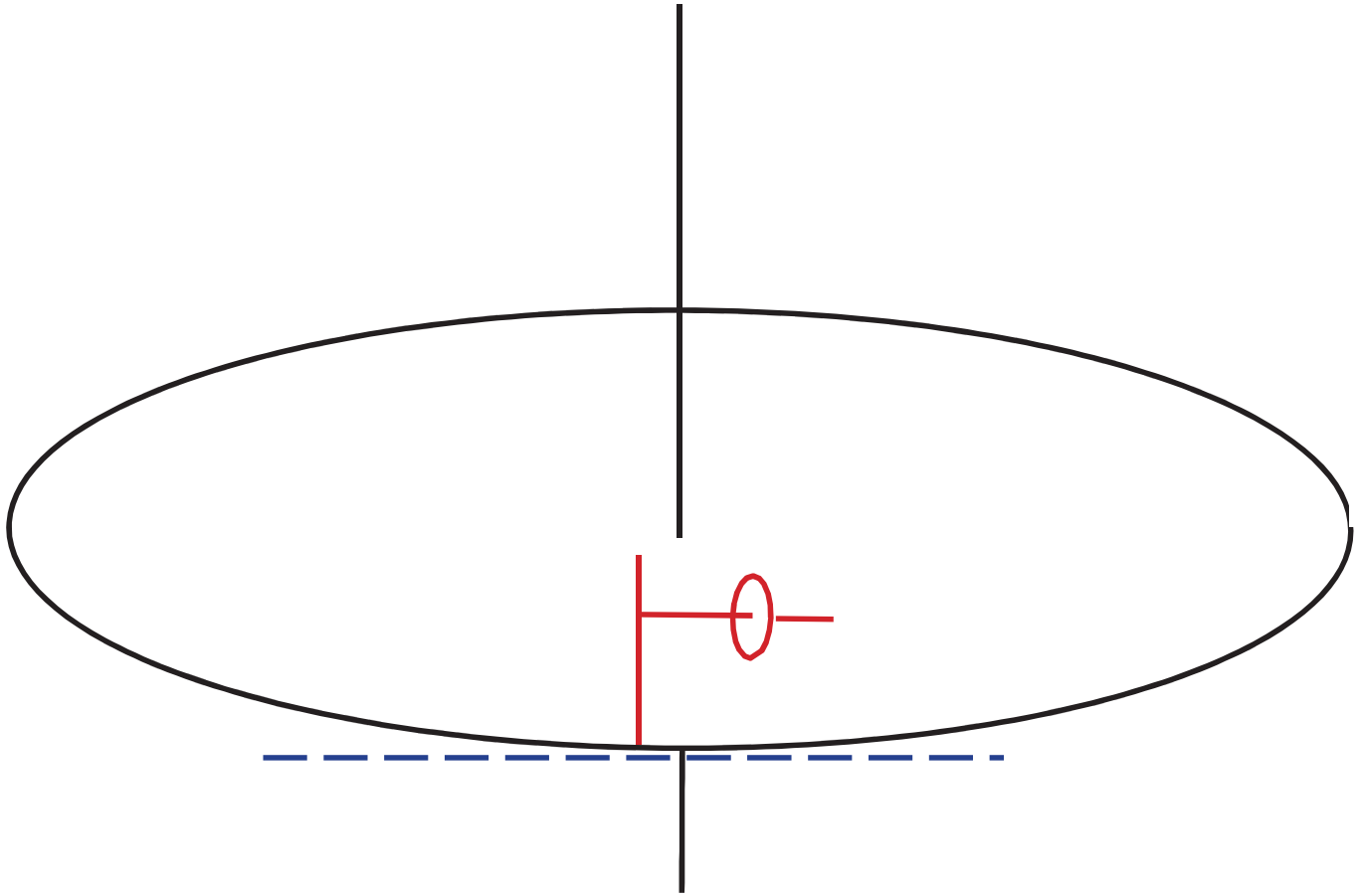

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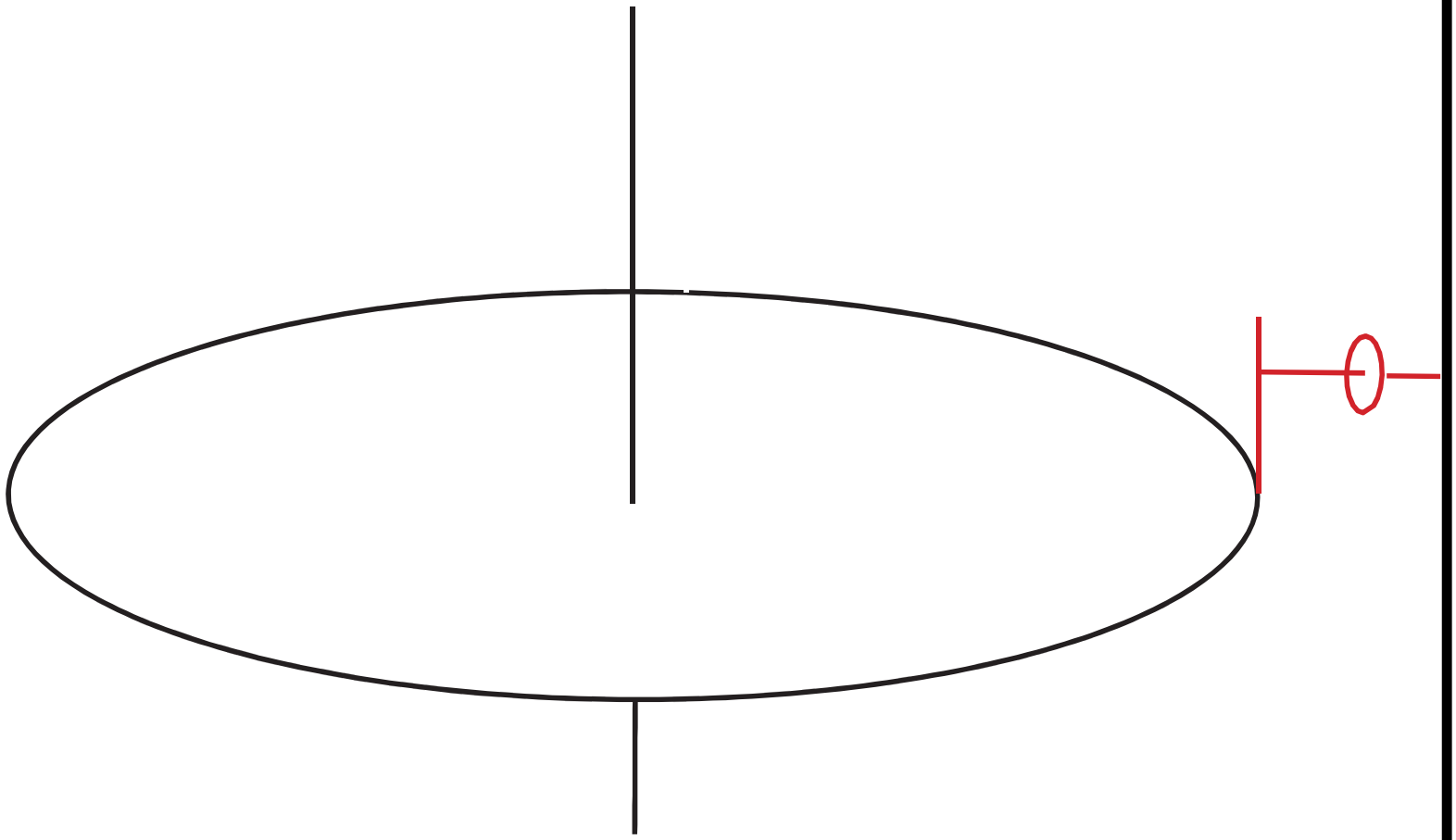

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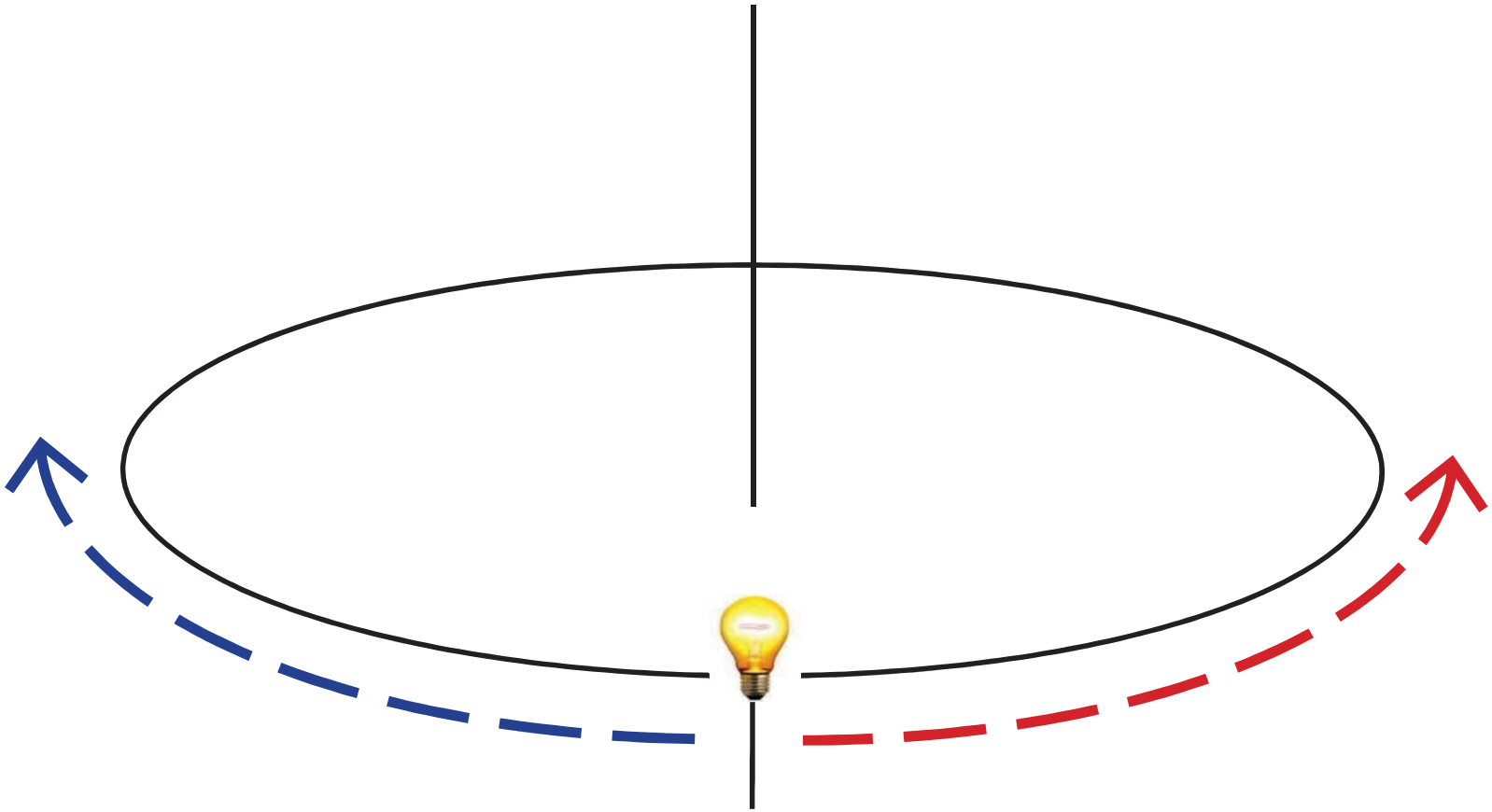
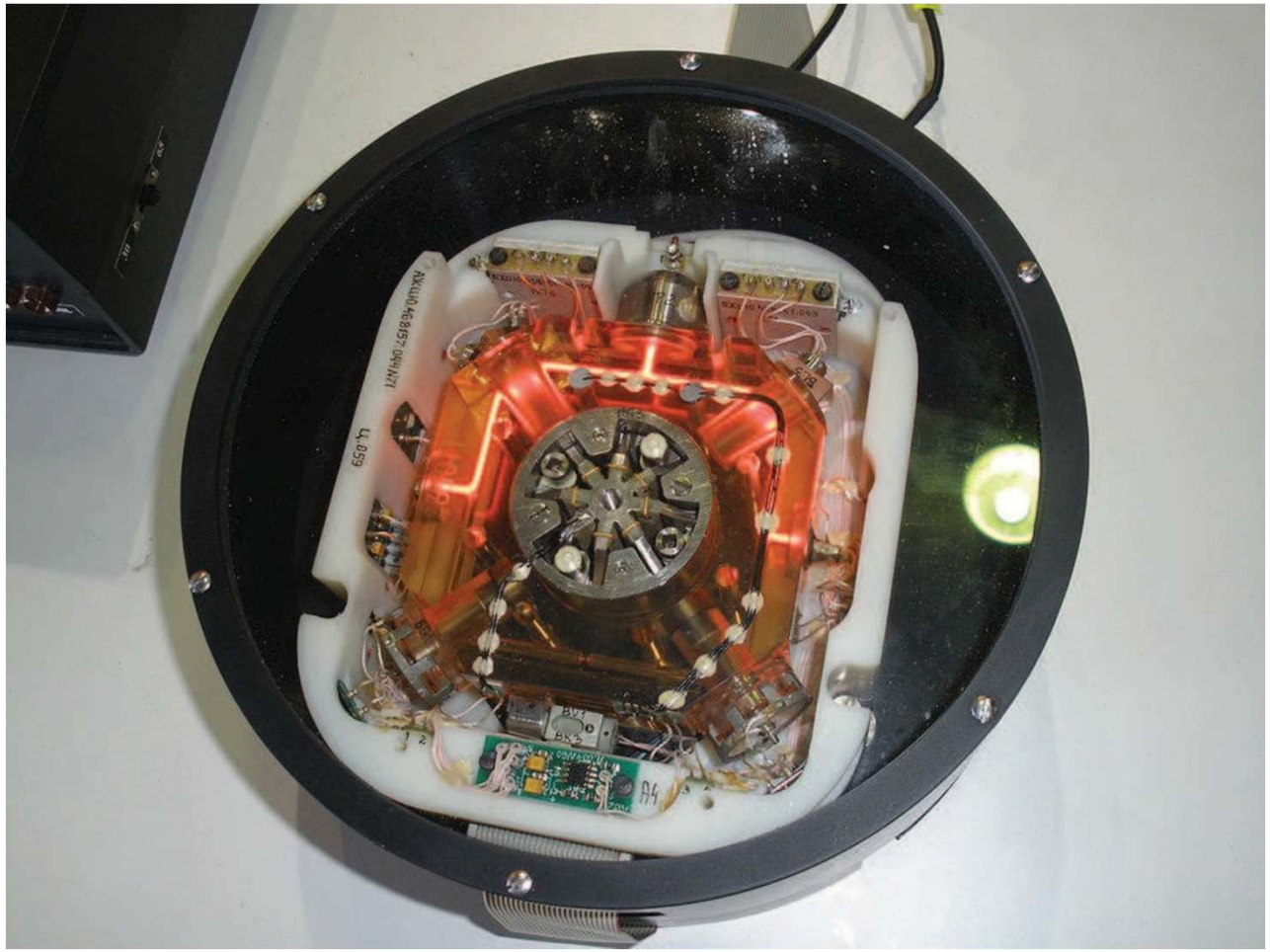


ZAM criterion of non-rotation



Ring Laser Gyroscope (courtesy of Wikipedia)

Do the three criteria (CIA, CIR, ZAM) agree?

First Point:

In some relativistic spacetime models -including ones that may well describe regions of our universe, e.g., the Kerr solution -no two of the three criteria agree.

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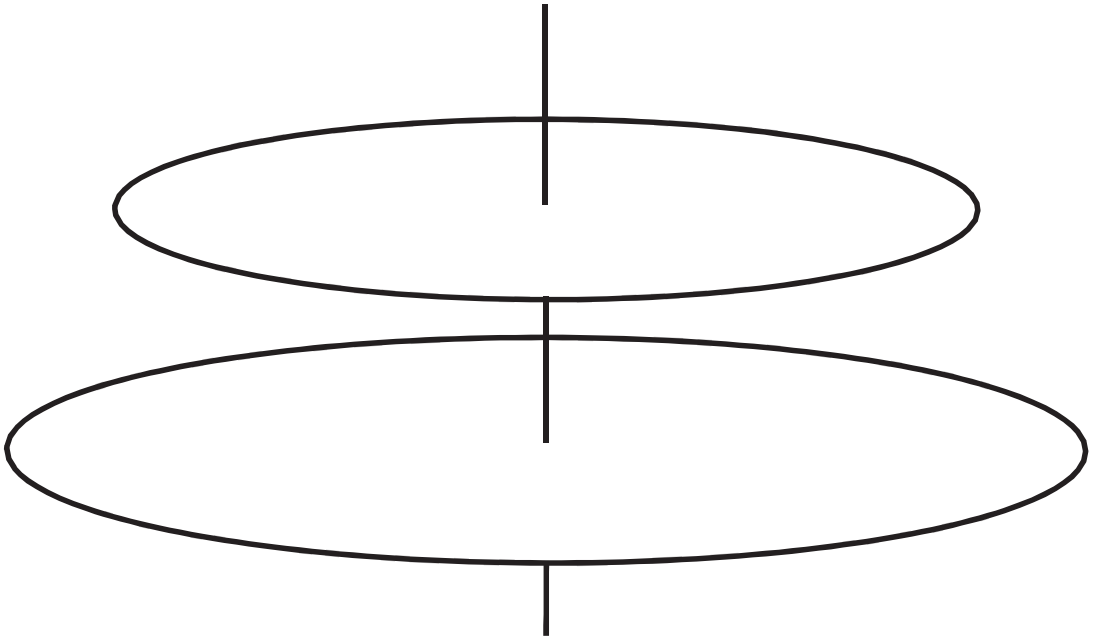
criteria of
non-rotation

conditions on
criteria of
non-rotation



Relative Rotation
Condition

Relative Rotation Condition:

For all rings R_1 and R_2 (with the same axis), if

(1)R_1 is "non-rotating,"

and

(2)R_2 is non-rotating relative to R_1,

then

R_2 is "non-rotating."

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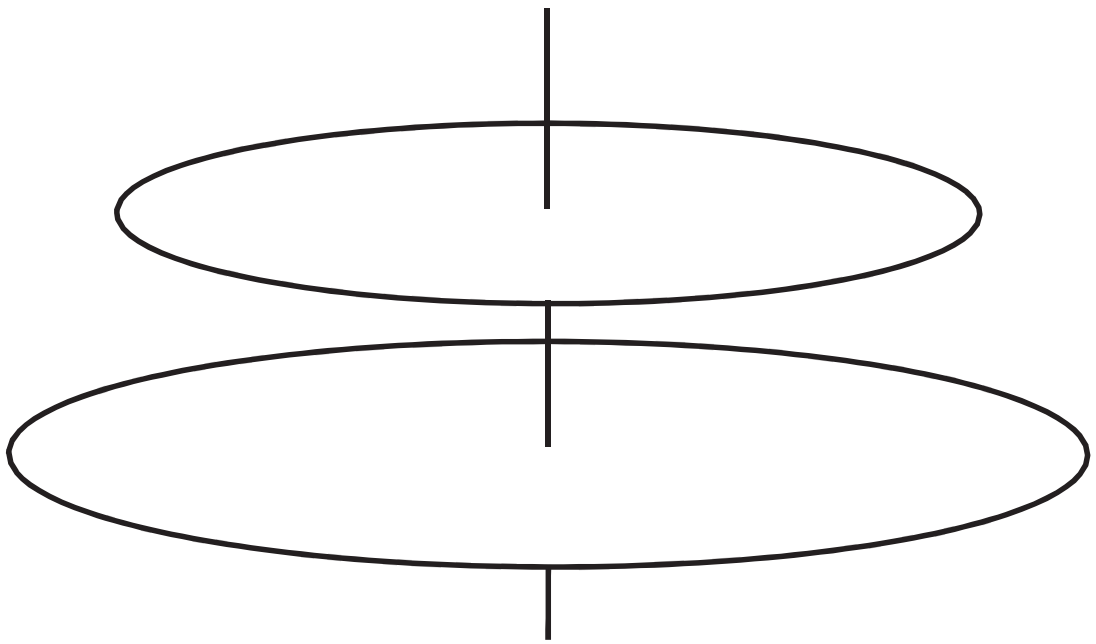

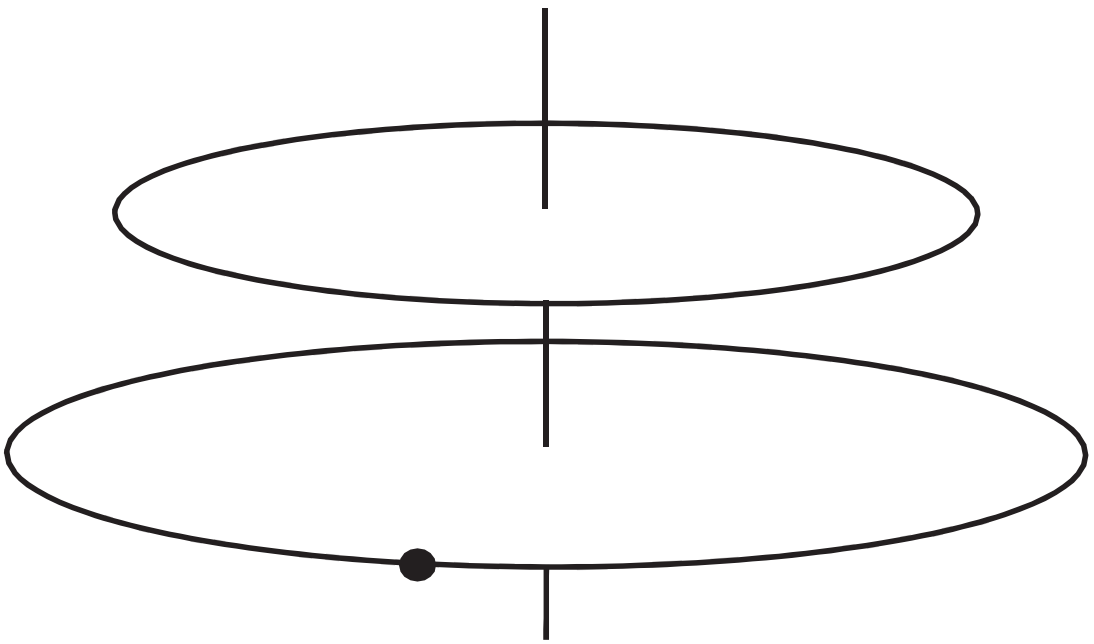

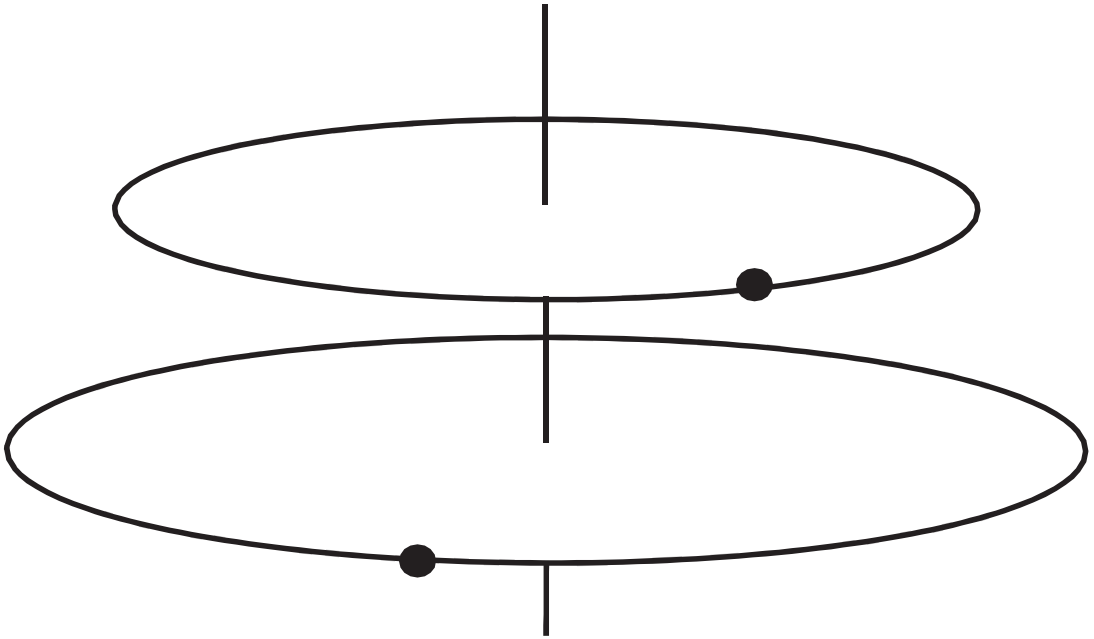

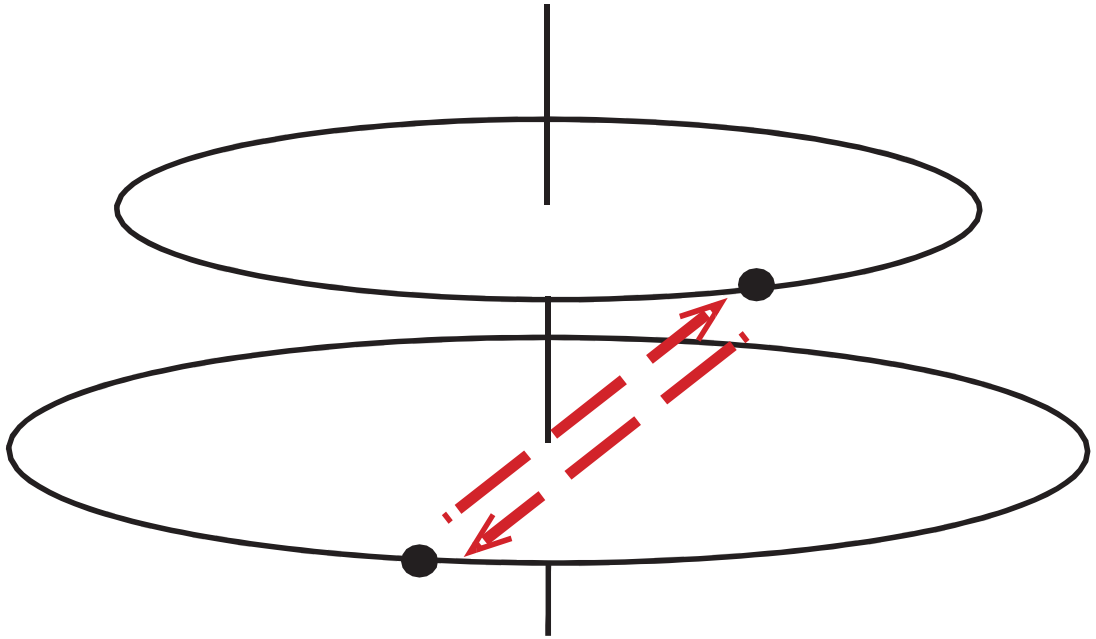

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Do the three criteria (CIA, CIR, ZAM) satisfy the relative rotation condition?

Second Point:

In the Kerr solution, for example, none of them satisfy the relative rotation condition.

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Are there ***any*** criteria of non-rotation that satisfy the relative rotation condition in the Kerr solution?

Are there *any* criteria of non-rotation that satisfy the relative rotation condition in the Kerr solution?

Yes, but none are reasonable candidates.

Now we turn to two other conditions (that one might want a criterion of non-rotation to satisfy).

[relative rotation

condition] limit condition

non-vacuity condition

The three criteria do not agree in general, but they (always) agree "in the limit for infinitely small rings".

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The claim requires proof, but it is what we should expect.

rotation
at a point

rotation over
extended regions

Limit Condition:

Let $R_1, R_2, R_3, \ldots$ be a sequence of rings, each "non-rotating," that converges to a point on the axis. For all i, let ring R_i have angular velocity ω_i with respect to the CIA criterion. Then $\omega_i \to 0$.

Third Point:

In *all* relativistic spacetimes, including the Kerr solution, the CIR and ZAM criteria (and the CIA criterion) satisfy the limit condition.

Are there ***any*** criteria of non-rotation that satisfy both the relative rotation condition and the limit condition in the Kerr solution?

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Exactly one – the vacuous criterion according to which no ring ever qualifies as "non-rotating".

Non-Vacuity Condition:

Some ring, in some state of motion (or non-motion), qualifies as "non-rotating."

Fourth Point:

No-Go Theorem. There is no criterion of non-rotation that satisfies the following three conditions in the Kerr solution:

(1) the relative rotation condition

(2) the limit condition

(3) the non-vacuity condition.

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Given any candidate criterion of "non-rotation" in the Kerr solution,

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Given any candidate criterion of "non-rotation" in the Kerr solution, if it makes correct determinations of non-rotation in the "limit for infinitely small rings",

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Given any candidate criterion of "non-rotation" in the Kerr solution, if it makes correct determinations of non-rotation in the "limit for infinitely small rings", and if it is non-vacuous,

Think about it this way:

Given any candidate criterion of "non-rotation" in the Kerr solution, if it makes correct determinations of non-rotation in the "limit for infinitely small rings", and if it is non-vacuous, then it *must* violate the relative rotation condition.

Does this mean we cannot talk about rotation in relativity theory?

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Not at all.

The End

Thank you for awarding me this wonderful prize.

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