

Pacioli's Lens: Through a Glass, Darkly

Richard Macve

R.Macve@lse.ac.uk

Primary reference:

https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3671724

AHR online conference

25 June 2021

A 'paradigm shift'?

- Sangster (2018) [*The Accounting Review*, 93(2): 299-314] argues that, in the first printed manual on double-entry bookkeeping ('DEB') in 1494, Pacioli presented a novel 'axiomatic' approach that requires a corresponding 'paradigmatic shift' in our view of his contribution.
- This paper challenges Sangster's interpretation

- It calls for deeper understanding both of the historical development of DEB in the West and of the comparative accounting developments in the East, particularly in China.
- It tentatively concludes that, although indigenous imperial Chinese accounting practice differed in *form* from Western DEB, nevertheless despite its variety of forms it had in some cases captured the structural essentials of DEB's *content* and *functions*.

DEB

- Examples of accounts kept in DEB from around Italy (including in particular Florence and Genoa) have been traced back as far as about another 200 years before Pacioli's 1494 treatise (de Roover 1955, 1956; Goldthwaite 2015). Today, DEB still underlies the FASB's and IASB's conceptual frameworks that adopt a 'balance sheet approach' to the determination of enterprise income (e.g. Bromwich, Macve, and Sunder 2010) and it is the hallmark of the accounting and auditing profession around the world.

Pacioli

- Sangster (2018) goes very significantly beyond well-known evaluations of Pacioli's contribution (e.g. Yamey 1994a; Macve, 1996) and argues that 'Pacioli reveals a simplicity in the then-unrecognized axiomatic foundation of double entry that has been largely overlooked. The findings represent a paradigm shift in how we perceive Pacioli, his treatise, and double entry.'
- i.e. it's not just a series of (often confusing) rules

Pacioli (cont.)

- The bulk of Sangster's 2018 paper sets out a view of Pacioli's life, motivation, and approach in publishing his treatise that is in most respects broadly consistent with previous work (e.g. Macve 1996) while refining some of the minor historical details of Pacioli's life and work (c.f. Vollmers 2015) His argument is reiterated in Sangster's chapter 'Al nome de Dio e bon guadagno – Pacioli, Venice, Education and Bookkeeping', Chapter 15 in *The Origins of Accounting Culture: The Venetian Connection*, edited by M. [Sargiacomo](#), S. [Coronella](#), C. Mio, U. [Sostero](#), and R. [Di Pietra](#). 2018. Routledge New Works in Accounting History. New York / Abingdon: Taylor & Francis.

Pacioli's 'keys'

- It has been recognised by historians of mathematics that Pacioli's treatment of algebra in the *Summa* was innovative in generalising from individual problems and their numerical solutions to providing 'keys' that showed how to approach a class of similar problems (Heeffer 2012).
- But, while innovative, these 'keys' are arrived at from repeated numerical examples that establish a generalizable pattern (as given in the example in Figure 1 on p.307 of Sangster 2018), rather than exhibiting a 'paradigm shift' of being derived from first principles (or 'axioms') as Sangster claims.

De Scripturis?

- Moreover Sangster then further argues that Pacioli also adopted this approach based on ‘keys’ in his section of the *Summa* on DEB (the *de Scripturis*) and that his explanation of DEB was correspondingly ‘axiomatic’.
- But I argue here that the illustrations in Sangster’s Figures 1 and 2 do not bear this out.

Structure of my argument

Drawing on generally accepted understandings of the nature of 'axioms', 'postulates' and 'theorems' and using simple logic, together with reviewing relevant literature on Pacioli and on the history of mathematics, I consider:

- first the nature and role of axioms etc., then
- the nature and role of 'keys' in Pacioli's algebra, noting their absence in his treatise on DEB;
- concluding with an outline discussion of the historical significance of Pacioli's treatise and of DEB in world history.

Axioms / Postulates / Theorems

- Euclid's axioms (or 'common notions'), on which he built his geometry, can be expressed as:
- Things are equal to one another if those things are equal to the same thing.
- The wholes are equal if equals are added to equals.
- The remainders are equal if equals are subtracted from equals.
- Things are equal to one another if they coincide with one another.
- The whole is greater than the part.

These axioms are ‘self-evident’ truths applicable to *any field* of scientific knowledge and they are sufficient to show how to derive *all* the rules of DEB from the ‘Balance Sheet Equation’.

- While ‘axioms’ and ‘postulates’ are often regarded as interchangeable concepts, the stricter distinction is that postulates are generally related to *particular fields* of knowledge (so Euclid’s postulates for geometry relate to what can be said about lines, angles and circles).

The Balance Sheet Equation: $A-L=E$

- The Balance Sheet Equation itself might therefore be regarded as the fundamental *postulate* of DEB, so that, together with Euclid's axioms, that is sufficient to generate all the rules of the system, showing how changes in these elements are interlinked.
- However, formal exposition of DEB through the 'Balance Sheet Equation' was not developed until the 19th century (Gentili and Giacomello 2017).

$$A-L=E$$

- Nevertheless, it has been argued (e.g. Macve 1996) that Pacioli had an *implicit* understanding of this logic as he begins his treatise by showing how to prepare an opening 'inventory' (i.e. effectively a balance sheet of (net) assets and the equivalent amount of owner's equity capital); then explains how transactions are to be both 'debited and 'credited' so that the total of the debits is always equal to the total of the credits (consistent with Euclid's axioms); and concludes by explaining how to prepare a closing P&L and Balance Sheet from the resulting balances in the ledger.
- So the underlying logic seems clear and Pacioli was familiar with Euclid and translated him into Latin in 1509.

Figure 2 from Sangster (2018) p.309: Double entry using axioms

- All transactions involve two elements: an item exchanged and a form of settlement {Axiom 1} (Chapter 9)
- One element is debit and the other is credit {Axiom 3} (Chapter 11)
- Debit = credit {Axiom 4} (Chapter 12)
- ⇒ The amount of the form of settlement = the amount of the item exchanged [Theorem 1]
- The amount of the form of settlement = the amount of the item exchanged [Theorem 1]
- All forms of settlement can substitute for each other {Axiom 2} (Chapter 9)
- Cash received is a debit <Postulate 1> (Chapter 12)
- Cash is a form of settlement (Chapter 9)
- ⇒ If a form of settlement is received, it is a debit [Theorem 2]
 - ⇒ And the item exchanged is a credit [Theorem 3]
- The amount of the form of settlement = the amount of the item exchanged [Theorem 1]
- All forms of settlement can substitute for each other {Axiom 2} (Chapter 9)
- Cash given is a credit <Postulate 3> (Chapter 18)
- Cash is a form of settlement (Chapter 9)
- ⇒ If a form of settlement is given, it is a credit [Theorem 4]
 - ⇒ And the item exchanged is a debit [Theorem 5]
- The entries in the money column are to be in one currency only {Axiom 5} (Chapter 12)

Sangster's so-called 'axioms', 'postulates' and 'theorems'.

Is this 'pedagogically effective' when Axiom 1 doesn't come till Chapter 9; and Axioms 4 and 5 till Chapter 12? And they are not collected together in the 'Summary' (Chapter 36)?

- They are a bewildering mixture of the intuitively obvious (his 'Axiom 5'), some definitions, and several plausible but not necessary propositions.
- It is also unclear how it has been decided which are 'axioms', which are 'postulates', and which are 'theorems'.

‘Dr /Cr’

- For example, ‘Axiom 3’ (“one element is debit and the other is credit”) and ‘Postulates 1 and 3’ (or more generally ‘Theorem 2’ and ‘Theorem 4’) are simply a naming convention (as the convention could as well have been ‘an item on the left’ and ‘an item on the right’ or indeed ‘called dog’ and ‘called cat’ or have adopted any other arbitrary labels as long as they are to be consistently applied).
- The choice of ‘debit’ and ‘credit’ as the labels of course simply reflects the historical evolution of Italian bookkeeping wherein debtors were originally the main recorded assets and creditors the main recorded liabilities.

A further crucial limitation of Pacioli's DEB as it is described by Sangster is that it only deals with *transactions* (e.g. sales, purchases, current expenses). By contrast the 'Balance Sheet Equation' enables one to deduce the correct bookkeeping treatment of *any* item one wishes to record, including accruals such as provisions for depreciation, for bad and doubtful debts, for pensions, for deferred taxes, and for all the other (often controversial) items that constitute the major problems in modern financial accounting.

And outside Venice actual Italian DEB practice reflected such accruals.

The role of 'keys'

- Sangster (2018, Figure 1) illustrates how Pacioli deployed 'keys' (*chiave*) in his exposition of algebra in the *Summa*. That example is taken from Heeffer (2012, 39).
- However, Heeffer comments:
'These "general principles" are presented without any argumentation except for numerical examples as a test.... His restructuring of abacus problem solving methods is undoubtedly inspired by [t]his teaching experience'

‘Keys’ (cont.)

- So, as argued above, Pacioli does *not* rely on axiomatic algebra in formulating these so-called ‘general principles’.
- And although he uses ‘keys’ in the *Summa*, when one turns to the *de Scripturis* itself one does not find any instances where a similar exposition is applied to developing illustrative examples of DEB.
- Sangster however argues that examples were unnecessary given the clear ‘axiomatic’ structure of Pacioli’s exposition.

Summary

- Sangster's argument fails in the following respects:
- Pacioli's exposition of algebra in the *Summa* through 'keys' is not axiomatic.
- Pacioli does not actually use similar 'keys' in the *de Scripturis* to explain DEB.
- So even if the 'keys' in the *Summa* were axiomatic, Pacioli does not explain the rules for DEB in the *de Scripturis* by deriving them from axioms.

So what?

- Pacioli's 'game changing' printed, vernacular exposition of DEB (utilising Arabic numerals) is a reasonably useful mercantile appendage to the *Summa* rather than a derivation from its mathematics and in this respect is therefore similar to the (plagiarised) section on weights and measures, currencies etc. ('Tariffs') that he also included (Yamey 1994a)
- The lens through which to appreciate it is one that scans its location within the longer history of European and then North American accounting development and institutionalization; and one that contrasts this history with that of other cultures that developed significant mercantile economies.

How important is DEB?

- The financial statements DEB produces are widely interpreted as summarising a business's progress and the state of its capital.
- But these can be produced without processing within the fully integrated, cross-referenced structure of DEB.
- Indigenous Chinese accounting, as recently found in surviving examples from the 16th century onwards (e.g. the records analysed in Yuan et al. 2017 and Yuan and Macve 2019), like other Eastern systems using traditional Chinese characters, did not have the indexed and page-numbered books that Pacioli recommends but did track assets, liabilities, equity and income.

‘CIHAR’

- Historical understanding can only be achieved by considering the context of the emergence, dissemination and institutionalization of DEB in the West and by putting it in comparative perspective with developments in the East, where further research can now be most valuably focussed (Dobie and McCollum-Oldroyd 2020).
- Rather than further poring over Pacioli, the research importance of further collaborative study of China and its accounting history, alongside that of other Eastern economies, is clear in order to illuminate and inform the mainstream of ‘comparative international historical accounting research’ (Carnegie and Napier, 2012).

谢谢, 请多批评指正!

Xiè xiè, qǐng duō pīpíng zhǐzhèng!

*Thank you, please
criticise and correct as
much as possible!*

Some Appendices

The Balance Sheet Equation and the Articulation of Financial Statements

- Resources = Claims
- Assets = Claims by Outsiders ('Liabilities') + Claims by Owners ('Equity capital')
- $A = L + E$
- $A - L = E$
- $\Delta(A - L) = \Delta E$
- $\Delta(A - L) = \Delta[E1 + E2]$
- $\Delta(A - L) = \Delta[E1a + E1b] + \Delta E2$
- $\Delta E1$ = 'Comprehensive income'
- $\Delta E1 = \Delta E1a + \Delta E1b$
- $\Delta E1a$ = 'net income' (US) / 'net profit' (UK)
- $\Delta E1a$ = 'income - expenditure' or 'revenue - costs' (as in 'income statement' / 'Profit & Loss Account')
- $\Delta E1b$ = 'other comprehensive income'
- $\Delta E2$ = transactions with owners (e.g. new share issues; dividends)

Note: 'A' and 'L' can be subdivided into 'accounts' for any number of component assets and liabilities (or groups of these) as required. If these are presented so that increases in assets are entered on their left side, and increases in liabilities and equity on their right side, they form the familiar pattern of 'T' accounts.

If $\Delta E2$ is typically dividends it can equivalently be written as $-D$ (and any new capital contributions are 'negative dividends'). The 'clean surplus equation' (Feltham and Ohlson 1995) is then:

$$BV_t = BV_{t-1} + Y_t - D_t \text{ where:}$$

- BV_t is ending book value (i.e. A-L) and BV_{t-1} is opening book value for the period ended at time t ,
- Y_t is the period's 'clean surplus' income (i.e. $\Delta E1$),
- and D_t is the period's dividends (i.e. $\Delta E2$).
- The 'clean surplus equation' in its simple algebra therefore also reflects Euclid's fundamental axioms (in particular that if $a=b$, then $a+c = b+c$; or equivalently, $a-c = b-c$; $a+c-c = b$; and $a = b+c-c$ (the possible variant entries within DEB).

- Note however that Hicks argued that this bookkeeping approach to *measuring capital and income* (which still underpins the Conceptual Frameworks of modern accounting standard setting bodies—e.g. Macve 1997—as well as modern financial analysis—e.g. Penman 2010) does not necessarily supply the most relevant information for business and investment decisions (Bromwich et al., 2010).
- The ‘value of the firm’ at time t can be expressed as $(BV_t + \text{‘the present value of expected future residual earnings’})$ (e.g. Feltham and Ohlson 1995).