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# An Omitted Variable Bias Framework for Sensitivity Analysis of Instrumental Variables

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# Motivating Example: Estimating the Returns to Schooling

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A first approach: run OLS of log earnings (Y) on education (D), adjusting for  $X = \{race, experience, regional factors\}$ . Here reproduce Card (1993).

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Our restricted OLS estimate is not causal, and may suffer from what is known as "omitted variable bias" (OVB). What can we do now?

If we can find a variable Z that (i) changes the incentives to schooling (D); and (ii) is "otherwise" unrelated to earnings (Y), then we can obtain a valid estimate\* of the causal effect of schooling on earings, even without measuring unobserved confounders U.



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If these assumptions hold, we can use IV regression adjusting for **X** to estimate the "true" returns to schooling, and we obtain the value of 13.2% (details next).

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But proximity is not randomized... couldn't we have the same problem as with OLS?

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Although we proposed IV as a solution to the OVB problem, it may itself suffer from OVB! How much can we trust the 13.2% estimate?

# The two main approaches to IV estimation: just different flavors of OLS

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$$Y = \hat{\lambda}_{res} Z + \mathbf{X} \hat{\beta}_{res} + \hat{\epsilon}_{y,res} \implies \hat{\lambda}_{res} \approx 4.2 \%$$

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Indirect Least Squares (ILS):

$$\hat{\tau}_{\text{ILS,res}} := \frac{\text{Effect of Z on Y (RF)}}{\text{Effect of Z on D (FS)}} = \frac{\hat{\lambda}_{\text{res}}}{\hat{\theta}_{\text{res}}} \approx \frac{0.042}{0.319} \approx 13.2\%$$
(Standard errors obtained with the delta-method)

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Two-Stage Least Squares (2SLS):

**Second stage:** 
$$Y = \hat{\tau}_{2SLS, res} \widehat{D}_{res} + \mathbf{X}\hat{\beta}_{2SLS, res} + \hat{\epsilon}_{2SLS, res}$$

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#### (Identical to Fieller's (1954) proposal for the confidence interval of a ratio.)

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Suppose the true causal effect of D on Y has some specific value, say,  $\tau_{0.}$ 

**Anderson-Rubin:** 
$$Y - \tau_0 D = \hat{\phi}_{\tau_0, \text{res}} Z + \mathbf{X} \hat{\beta}_{\tau_0, \text{res}} + \hat{\epsilon}_{\tau_0, \text{res}}$$

Note that, if  $\tau_0$  is the true causal effect  $\phi_{\tau_0, \text{res}} = 0$ 

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Confidence interval: 
$$\operatorname{CI}_{1-\alpha}(\tau) = \left\{ \tau_0; \ t_{\hat{\phi}_{\tau_0, \operatorname{res}}}^2 \le t_{df, \alpha}^{*2} \right\} \approx [2.4\%, 28.5\%]$$

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**Two important facts:** 1) the confidence interval includes zero, if, and only if, we cannot reject the RF is zero; and 2) the confidence interval is unbounded, if and only if, we cannot reject the FS is zero.

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This approach has correct test size regardless of instrument strength.

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## **OVB for IV**— the problem statement

	What we have	What we want		
First stage:	$D = \hat{\theta}_{res} Z + \mathbf{X} \hat{\psi}_{res} + \hat{\epsilon}_{d,res}$			
Reduced form:	$Y = \hat{\lambda}_{res} Z + \mathbf{X} \hat{\beta}_{res} + \hat{\epsilon}_{y,res}$			
Anderson-Rubin:	$Y_{\tau_0} = \hat{\phi}_{\tau_0, res} Z + \mathbf{X} \hat{\beta}_{\tau_0, res} + \hat{\epsilon}_{\tau_0, res}$			
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At their core, IV estimates are OLS estimates. So we can leverage all sensitivity tools for OLS for the sensitivity of IV.

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*Point estimate:* this formally holds for *all estimators* discussed here. *Confidence intervals:* this formally holds for the *AR/Fieller approach*.

(significance testing using ILS/2SLS can lead to logically incoherent conclusions)

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Characterize the strength of omitted variables W with two sensitivity parameters:

- (i) how much residual variance W explains of the the instrument  $R^2_{Z \sim W|\mathbf{X}}$
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#### 1. Worst possible inferences given a postulated strength of W:

Construct confidence interval replacing the usual critical value t\* (e.g 1.96) with an *adjusted critical value* t<sup>+</sup>:

$$\operatorname{CI}_{1-\alpha,\mathbf{R}^{2}}^{\max}(\lambda) = \left[\hat{\lambda}_{\operatorname{res}} - t_{\alpha,\operatorname{df}-1,\mathbf{R}^{2}}^{\dagger\max} \times \widehat{\operatorname{se}}(\widehat{\lambda}_{\operatorname{res}}), \ \hat{\lambda}_{\operatorname{res}} + t_{\alpha,\operatorname{df}-1,\mathbf{R}^{2}}^{\dagger\max} \times \widehat{\operatorname{se}}(\widehat{\lambda}_{\operatorname{res}})\right]$$

(using the reduced form as the example)

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- (i) how much residual variance W explains of the the instrument  $R_{Z\sim W|\mathbf{X}}^2$
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#### 1. Worst possible inferences given a postulated strength of W:

Construct confidence interval replacing the usual critical value t\* (e.g 1.96) with an *adjusted critical value* t<sup>+</sup>:

$$\operatorname{CI}_{1-\alpha,\mathbf{R}^{2}}^{\max}(\lambda) = \left[\hat{\lambda}_{\operatorname{res}} - t_{\alpha,\operatorname{df}-1,\mathbf{R}^{2}}^{\dagger\max} \times \widehat{\operatorname{se}}(\hat{\lambda}_{\operatorname{res}}), \quad \hat{\lambda}_{\operatorname{res}} + t_{\alpha,\operatorname{df}-1,\mathbf{R}^{2}}^{\dagger\max} \times \widehat{\operatorname{se}}(\hat{\lambda}_{\operatorname{res}})\right]$$

 $t_{\alpha,\mathrm{df}-1,\mathbf{R}^2}^{\dagger \max}$  is a function of the two sensitivity parameters  $\mathbf{R}^2 := (R_{Y \sim W|Z,\mathbf{X}}^2, R_{Z \sim W|\mathbf{X}}^2)$ 

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#### 2. Sensitivity statistics for routine reporting:

- Minimal strength of W needed to be problematic: (i) robustness value (RV); (ii) extreme robustness values (XRV).
- 3. Formal bounds on the strength of W if it were as strong as observed covariates
  - Leverage claims of relative importance of variables.

For a choice of  $\tau_0$  create the "putative potential outcome"  $Y_{\tau_0} := Y - \tau_0 D$  and run the AR regression:

$$Y_{\tau_0} = \hat{\phi}_{\tau_0, \text{res}} Z + \mathbf{X} \hat{\beta}_{\tau_0, \text{res}} + \hat{\epsilon}_{\tau_0, \text{res}}$$

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To recover all possible inferences given any strength of W, simply invert the Anderson-Rubin test with an *ovb-adjusted* critical threshold.

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 With this, we can get all usual OLS sensitivity results for IV estimates, such defining (extreme) robustness values, contour plots, etc.

## **Back to schooling**

Outcome: <i>Earnings</i> (log)						
Instrument	Estimate	S.E.	t-value	$R^2_{Y \sim Z \mid \mathbf{X}}$	$XRV_{q^*, \alpha}$	$RV_{q^*, \alpha}$
Proximity	0.042	0.018	2.33	0.18%	0.05%	0.67%
Bound (1x smsa): $R^2_{Y \sim W Z, \mathbf{X}} = 2\%$ , $R^2_{Z \sim W \mathbf{X}} = 0.6\%$ , $t^{\dagger}_{\alpha, df - 1, \mathbf{R}^2} = 2.55$						
<b>Note:</b> df = 2994, $q^* = 1$ , $\alpha = 0.05$						

Table 1: minimal sensitivity reporting of the reduced-form

Outcome: Earnings (log)InstrumentEstimateS.E.t-value $R_{Y \sim Z | \mathbf{X}}^2$ XRV $_{q^*, \alpha}$ RV $_{q^*, \alpha}$ Proximity0.0420.0182.330.18%0.05%0.67%Bound (1x smsa): $R_{Y \sim W | Z, \mathbf{X}}^2$ = 2%,  $R_{Z \sim W | \mathbf{X}}^2$ = 0.6%,  $t_{\alpha, df - 1, \mathbf{R}^2}^{\dagger}$ = 2.55Note:df= 2994,  $q^* = 1$ ,  $\alpha = 0.05$  $\alpha = 0.05$  $\alpha = 0.05$  $\alpha = 0.05$ 

Table 1: minimal sensitivity reporting of the reduced-form

**Robustness value (RV):** if confounders/side-effects explained only 0.67% both of the residual variation of the outcome and of the instrument, this is already sufficient to explain away the reduced-form, and hence the IV estimate (at the 5% significance level).

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**Bounds:** Note the point estimate is <u>not</u> robust to confounders/side-effects as strong as an *smsa*, an indicator of whether the individual lived in a metropolitan area. It is not hard to imagine unobserved variables as strong as those in this scenario.

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## Sensitivity contours — AR lower and upper limits



(a) Sensitivity contours: lower limit

(b) Sensitivity contours: upper limit

Figure: Sensitivity contour plots for the lower (a) and upper (b) limits of the 95% confidence interval for the IV estimate.

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## Conclusions

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- Sensitivity of the reduced form, first stage or a specific null hypothesis using the Anderson-Rubin regression can already be easily performed with sensemakr. Software for the full-fledged IV sensitivity will be available soon (for R and Stata).

## Learn more!

- Watch the presentation on youtube (link: https://tinyurl.com/ovb4iv)
- An Omitted Variable Bias Framework for Sensitivity Analysis of Instrumental Variables (preliminary draft) (link: <u>https://tinyurl.com/</u> ovb4iv-draft)
- Making Sense of Sensitivity: Extending Omitted Variable Bias (link: <u>https://tinyurl.com/jrssb</u>)
- sensemakr: Sensitivity Analysis Tools for OLS in R and Stata (link: <u>https://tinyurl.com/jss-sensemakr</u>)

## THANK YOU!