Population allocation of scarce binary treatments

Estimands for Settings with Limited Resources

Aaron Sarvet
asarvet@g.harvard.edu

2 July 2021

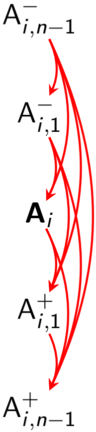
Motivation

Clinical and public health decision makers commonly contend with the challenge of resource limitations.

Motivation

Likewise, resource constraints present special challenges for causal inference:

Special challenge 1: Causal connections between patients



Implication: iid implausible in most settings with limited resources

Motivation

Likewise, resource constraints present special challenges for causal inference:

Special challenge 1: Causal connections between patients

Special challenge 2: Counterfactual regimes depend on features of all patients

Color Code and Level of Access	Assessment of Mortality Risk/Organ Failure
Blue No ventilator provided. Use alternative forms of medical intervention and/or palliative care or discharge. Reassess if ventilators become available.	Exclusion criterion OR SOFA > 11
Red Highest Use ventilators as available	SOFA < 7 OR Single organ failure[2]
Yellow Intermediate Use ventilators as available	SOFA 8 – 11
Green Use alternative forms of medical intervention or defer or discharge. Reassess as needed.	No significant organ failure AND/OR No requirement for lifesaving resources

Table 9-11: Allocation of Livers from Non-DCD Deceased Donors at Least 18 Years Old and Less than 70 Years Old

Classification	Candidates with a MELD or PELD score of at least	And registered at a transplant hospital that is at or within this distance from a donor hospital	Donor blood type	Candidate blood type
1	Status 1A	500NM	Any	Any
2	Status 1B	500NM	Any	Any
3	Status 1A	2,400NM and candidate is registered in Hawaii or 1,100NM and candidate is registered in Puerto Rico	Any	Any
4	Status 1B	2,400NM and candidate is registered in Hawaii or 1,100NM and candidate is registered in Puerto Rico	Any	Any
5	37	150NM	O	O or B
6	37	150NM	Non-O	Any
7	37	250NM	O	O or B
8	37	250NM	Non-O	Any
9	37	500NM	O	O or B
10	37	500NM	Non-O	Any

Implication: Individualized dynamic treatment rules are insufficient

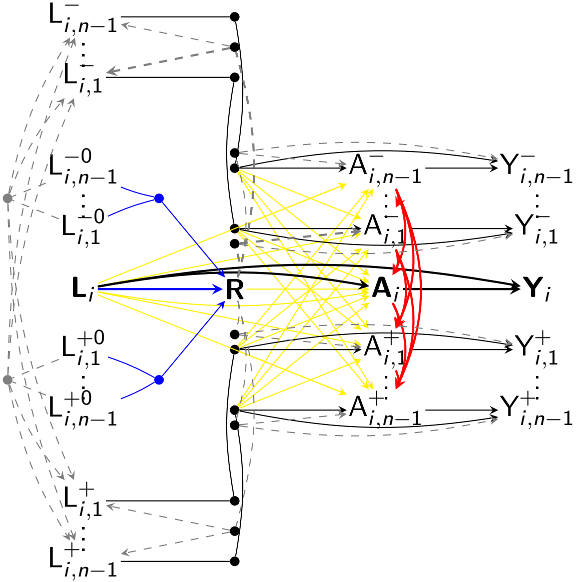
Principles of Approach

Implication of limited resource settings require modifications to our usual approaches to recover the familiar and useful elements in our toolbox

	Usual	**Modified**
Unit of superpop.	Patient	Cluster (of patients)
# of obs. units	n	1
Relation btw units	iid	iid
# of obs. patients	n	n
Relation btw patients	iid	causal cnxs.[1]

1 With regularity conditions weaker than iid

Principles of Approach



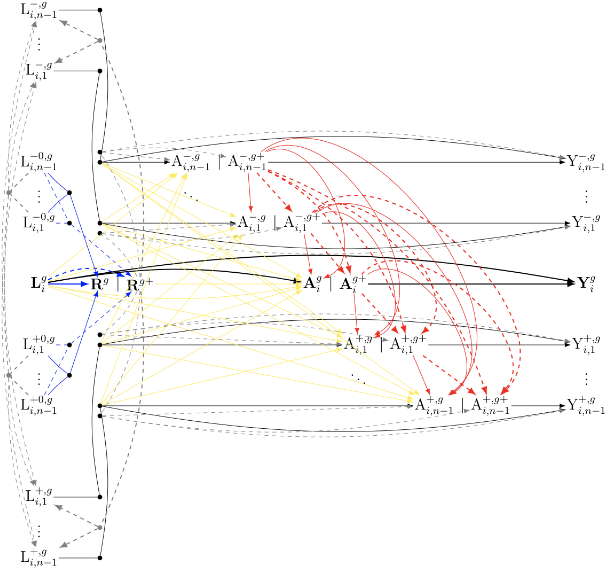
Principles of Approach

Implication of limited resource settings require modifications to our usual approaches to recover the familiar and useful elements in our toolbox

	Usual	**Modified**
Unit of superpop.	Patient	Cluster (of patients)
# of obs. units	n	1
Relation btw units	iid	iid
# of obs. patients	n	n
Relation btw patients	iid	causal cnxs.[1]
Dyn. regimes, g:	$\mathcal{L} \mapsto \{0,1\}$	e.g. $\{\mathcal{L}\}^n \mapsto \{0,1\}^n$ $\{\mathcal{L}\}^n \mapsto S(\{0,\ldots,n\})$

1 With regularity conditions weaker than iid

Principles of Approach



Principles of Approach

Implication of limited resource settings require modifications to our usual approaches to recover the familiar and useful elements in our toolbox

	Usual	**Modified**
Unit of superpop.	Patient	Cluster (of patients)
# of obs. units	n	1
Relation btw units	iid	iid
# of obs. patients	n	n
Relation btw patients	iid	causal cnxs.[1]
Dyn. regimes, g:	$\mathcal{L} \mapsto \{0,1\}$	e.g. $\{\mathcal{L}\}^n \mapsto \{0,1\}^n$ $\{\mathcal{L}\}^n \mapsto S(\{0,\ldots,n\})$
Example Estimand	$\mathbb{E}[Y^g]$	$\mathbb{E}[\frac{1}{n}\sum_{i=1}^n Y_i^g]$

[1] With regularity conditions weaker than iid

$\frac{1}{n}\sum_{i=1}^n Y_i^g$: Cluster average outcome

Principles of Approach

Theorem

Under Model $\mathcal{M}$ (conditions in supp.), $\mathbb{E}[\frac{1}{n}\sum_{i=1}^{n} Y_i^g]$ is identified by

$$\Psi_n^g = \sum_{a_i \in \{0,1\}, \mathcal{L}_i} Q_Y(1, a_i, l_i) g_n^*(a_i, l_i) Q_L(l_i),$$

where $g_n^*(a_i, l_i)$ is identified from the observed data by

$$g_n^*(a_i, l_i) = \sum_{\mathcal{P}_L(l_i)} g^*_{n, \mathbb{l}=\{\mathbb{l}_{n-1}, l_i\}}(a_i, l_i) P(\mathbb{L}_{n-1} = \mathbb{l}_{n-1}),$$

Principles of Approach

Theorem

(cont.) and

$$g^*_{n,\mathbb{l}_n}(1, l_i) = \begin{cases} \frac{\kappa_n - nP^*_{n,\mathbb{l}_n}(\Lambda(L_i) > \omega_{n,\mathbb{l}_n})}{nP^*_{n,\mathbb{l}_n}(\Lambda(L_i) = \omega_{n,\mathbb{l}_n})}, & : \Lambda(l_i) = \omega_{n,\mathbb{l}_n}, \\ I(\Lambda(l_i) > \omega_{n,\mathbb{l}_n}), & : otherwise, \end{cases}$$

where

$$\omega_{n,\mathbb{l}_n} \inf\{c \in \mathbb{R} : nP^*_{n,\mathbb{l}_n}(\Lambda(L_i) > c) \leq \kappa_n\},$$

Conclusion

- Approach introduced here presents opportunities to expand the scope of causal questions we seek to answer
- Bolder estimands for a complex social world
- Stay tuned for a series of forthcoming papers

Acknowledgements

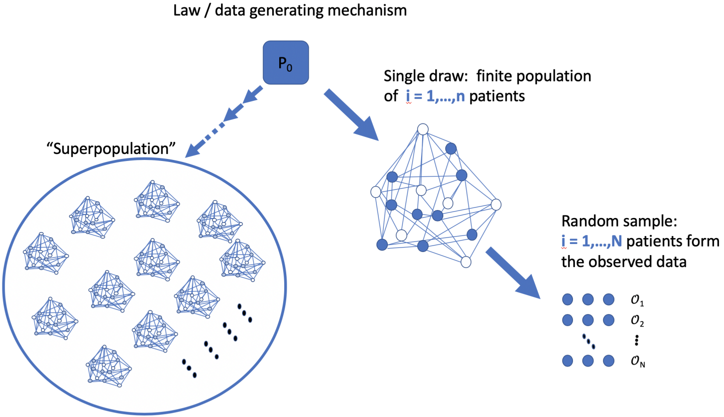
Thank you

especially to my mentors and collaborators: **Mats Stensrud**, *James Robins, Thomas Richardson, Betsy Ogburn, Jessica Young*

Section 1

Supplementary material

Supplementary material



Supplementary material

A1. **Structural invariance:** For each k and for for all i, j: $f_{V_{k,i}} = f_{V_{k,j}}$.

A2. **Conditional identically-distributed errors:** For each pair of variables $(V_{k,i}, V_{k,j})$, for all i, j, and for all $pa(v_k)$, we have that: $\left(\epsilon_{V_{k,i}} \middle| Pa(V_{k,i}) = pa(v_k)\right) \sim \left(\epsilon_{V_{k,j}} \middle| Pa(V_{k,j}) = pa(v_k)\right)$.

A3. **Conditional noninterference:** For all i, and all k, $Pa(L_{k,i}) \subseteq \mathcal{O}_i$, and for each k, m, for all $i \neq j$, $\epsilon_{V_{k,i}} \perp\!\!\!\perp \epsilon_{V_{m,j}}$.

A4. **Strict exogeneity of the randomizer:** For all k, $Z_{k,i} = \epsilon_{Z_{k,i}}$.

Supplementary material

B1. **No unit-level confounding for covariates and outcomes**: $\underline{L}^g_{k+1,i} \perp\!\!\!\perp A^g_{k,i}, R^g_{k,i} \mid X^g_{k,i} = x_{k,i}, \overline{Z}^g_{k,i} = \overline{z}_{k,i}, \overline{R}^g_{k-1,i} = \overline{r}_{k-1}$ for all $x_{k,i} \in \mathcal{X}_{k,i}, \overline{z}_{k,i} \in \overline{\mathcal{Z}}_{k,i}, \overline{r}_{k-1} \in \overline{\mathcal{R}}_{k,i}$, $t \in \{0, \dots, K\}$, $i \in \{1, \dots, n\}$

B2. **Rank and randomizer irrelevance**: $\underline{L}_{k+1,i} \perp\!\!\!\perp R_{k,i}, Z_{k,i} \mid A_{k,i} = a_{k,i}, X_{k,i} = x_{k,i}, \overline{R}_{k-1,i} = \overline{r}_{k-1,i}, \overline{Z}_{k-1,i} = \overline{z}_{k-1,i}$ for $a_{k,i} \in \mathcal{A}_{k,i}, x_{k,i} \in \mathcal{X}_{k,i}, \overline{r}_{k-1,i} \in \overline{\mathcal{R}}_{k-1,i}, \overline{z}_{k-1,i} \in \overline{\mathcal{Z}}_{k-1,i}$, $t \in \{0, \dots, K\}$, $i \in \{1, \dots, n\}$.

B3. **Positivity** For each $i \in \{1, \dots, n\}$, it holds that: $g_k(A^{g+}_{k,i}, X^{g+}_{k,i}) > 0$, w.p.1., for all $t \in \{0, \dots, K\}$.

B4. Λ-**stability**: There exists a function $\Lambda_k : \mathcal{X}_{k,i} \mapsto \{0, \dots, n^*\}$, with $n^* < n$, such that
$\Lambda_k(X^{g+}_{k,i}) > \Lambda_k(X^{g+}_{k,j}) \implies \Lambda^*_{k,i}(X^{g+}_k, Z^g_k) > \Lambda^*_{k,j}(X^{g+}_k, Z^g_k)$, w.p.1.
for all $i, j \in \{1, \dots, n\}$ and all $k \in \{0, \dots, K\}$.