2022 Colloquia in Combinatorics

Queen Mary day (11 May 2022)

Schedule

10:00 Tea/Coffee

10:30 Mathilde Bouvel
11:20 Nóra Frankl

Lunch break

13:40 Julian Sahasrabudhe
14:30 Florian Frick

Coffee break

15:50 Laura Sanità
16:40 Alexander Holroyd

All lectures will be in the Maths Lecture Theatre located in Mathematical Sciences building (building 4 on this map).

Abstracts

10:30 Mathilde Bouvel (Loria & CNRS)

Limits of permutations and graphs avoiding substructures

In this talk, I would like to present a survey of a series of papers, describing limits of random graphs or random permutations, taken uniformly at random conditioned to avoid certain substructures.

Our first results concern families of pattern-avoiding permutations. Our approach is to use the co-called substitution decomposition of permutations, thus encoding permutations as trees. Using analytic combinatorics, we are then able to compute the expected densities of patterns in our permutations. This result can be interpreted in the framework of permutons, thus providing limit shape results for random pattern-avoiding permutations.

Analoguous results can be obtained for hereditary classes of graphs (defined by the avoidance of induced subgraphs), following a similar methodology. The obtained results are limit shape results in the framework of graphons.

11:20 Nóra Frankl (Alfréd Rényi Institute of Mathematics)

Max norm Ramsey theory

The chromatic number of $\mathbb{R}^n$ with respect to a metric space S is the minimum number of colours needed to colour all points of $\mathbb{R}^n$ without a monochromatic isometric copy of S. In Euclidean Ramsey theory a set S is called Ramsey, if the chromatic number tends to infinity with n. Classifying Euclidean Ramsey sets is a very difficult open problem, of which little is known. Recently, Kupavskii and Sagdeev showed that if we replace the Euclidean norm with the max norm, then every finite set is Ramsey. In this talk, we discuss several related results. We find essentially tight bounds for the max norm chromatic number for several families of sets, and discuss related results about covering the integers with translates of given sets. We also prove that, unlike in the Euclidean case, there exist infinite max norm Ramsey sets.

Joint work with Andrey Kupavskii and Arsenii Sagdeev.

13:40 Julian Sahasrabudhe (University of Cambridge)

The singularity probability of random symmetric matrices

Let $\mathbb{A}$ be a n by n matrix, uniformly drawn from all symmetric random matrices with entries in $\{1, -1\}$. We show that the probability that $\mathbb{A}$ is singular is exponentially small.

14:30 Florian Frick (Carnegie Mellon University & Free University Berlin)

Hyperplane partitions and transversals

Topological combinatorics approaches discrete problems by augmenting them to have geometric content that make them amenable to the tools of topology. An early success story of this approach is the Ham Sandwich theorem, which states that any d finite point sets in $\mathbb{R}^d$ can simultaneously be cut in half by an affine hyperplane. I will discuss generalizations to multiple hyperplanes and unbalanced cuts. I will also explain some elements of the interconnected theory that has emerged, placing these problems in the broader context of topological combinatorics, such as relations to intersection combinatorics of convex hulls of point sets and topological lower bounds for chromatic numbers of (hyper-)graphs.

This includes joint work with S. Murray, S. Simon, and L. Stemmler as well as with P. Blagojević, A. Haase, and G.M. Ziegler.

15:50 Laura Sanità (Eindhoven University of Technology)

On the Simplex method for 0/1 polytopes

The Simplex method is one of the most popular algorithms for solving linear programs, but despite decades of study, it is still not known whether there exists a pivot rule that guarantees it will always reach an optimal solution with a polynomial number of steps. In fact, a polynomial pivot rule is not even known for linear programs over 0/1 polytopes (0/1-LPs), despite the fact that the diameter of a 0/1 polytope is bounded by its dimension.

This talk will focus on the behavior of the Simplex method for 0/1-LPs, and discuss pivot rules that are guaranteed to require only a polynomial number of non-degenerate pivots.

Joint work with Alexander Black, Jesus De Loera, Sean Kafer.

16:40 Alexander E. Holroyd (University of Bristol)

Finitely dependent colouring

A central concept of probability and ergodic theory is mixing in its various forms. The strongest and simplest mixing condition is finite dependence, which states that variables at sufficiently well separated locations are independent. A 50-year old conundrum is to understand the relationship between finitely dependent processes and block factors (a block factor is a finite-range function of an independent family). The issue takes a surprising new turn if we in addition impose a local constraint (such as proper colouring) on the process. In particular, this has led to the discovery of a beautiful yet mysterious stochastic process that seemingly has no right to exist.

LSE day (12 May 2022)

Schedule

10:00 Tea/Coffee

10:30 Matthew Kwan
11:20 Maura Paterson

Lunch break

13:40 Angelika Steger
14:30 Alex Scott

Coffee break

15:50 Vera Traub
16:40 Amin Coja-Oghlan

All lectures will be in the Sheikh Zayed Theatre located on the lower ground floor of the New Academic Building (NAB building on this map).

Abstracts

10:30 Matthew Kwan (Institute of Science and Technology Austria)

High-girth Steiner triple systems

In this talk I'll discuss our recent resolution of a conjecture due to Erdős on the existence of Steiner triple systems with arbitrarily high girth. I'll start by giving a brief overview of random greedy processes and the method of iterative absorption, and discussing the relevant challenges in the high-girth setting. Then, I'll outline some ideas that help overcome these challenges, related to sparsification, efficient absorption, and "retrospective" analysis of random processes.

This is joint work with Ashwin Sah, Mehtaab Sawhney, and Michael Simkin.

11:20 Maura Paterson (Birkbeck)

The combinatorics of authentication with perfect secrecy

For many uses of cryptography it is vital to provide authentication: some sort of assurance that messages are legitimately sent by the person who you think is sending them, rather than an imposter. Authentication codes are a combinatorial tool that was introduced by Simmons as a way to analyse how efficiently/effectively authentication can be provided in an unconditionally secure setting, where you do

not make any assumptions about the adversary's computational power. These have been extensively studied since the '80s. Splitting authentication codes are nondeterministic authentication codes that can offer stronger protection than deterministic authentication codes with the same parameters. In this talk we will explore the design of splitting authentication codes that offer confidentiality as well as authentication, and we will discuss their connections to objects that can be seen as generalisations of more familiar design-theoretic structures.

This talk is based on joint work with Doug Stinson.

13:40 Angelika Steger (ETH Zürich)

The multi-arm bandit problem

Consider a machine with K arms, where each arm provides a random reward from an unknown probability distribution that potentially can change over time. The objective of the player is to maximize the sum of rewards by using an appropriate strategy. The crucial tradeoff that the player faces at each step is the tradeoff between „exploitation" of the arm that has the highest expected payoff and "exploration" to get more information on the underlying probability distributions. In this talk we provide new optimal algorithms for some versions of the problem.

14:30 Alex Scott (University of Oxford)

Induced subgraphs of induced subgraphs of large chromatic number

Given a graph G with very large chromatic number, must it contain an induced subgraph that has (fairly) large chromatic number and is "simple" in some way? One possibility is for G to contain a large clique, but what if it does not? Can we drop to a simply structure subgraph with large chromatic number?

The aim of this talk is to show some limitations to this approach. We prove that for every graph F with at least one edge there are graphs H of arbitrarily large chromatic number and the same clique number as F such that every induced subgraph of H with chromatic number at least $c = c(F)$ contains an induced copy of F. This generalizes theorems of Briański, Davies and Walczak, and of Carbonero, Hompe, Moore and Spirkl. An analogous statement holds when clique number is replaced by odd girth.

Joint work with Antonio Girão, Freddie Illingworth, Emil Powierski, Michael Savery, Youri Tamitegama and Jane Tan.

15:50 Vera Traub (ETH Zürich)

Better-Than-2 Approximations for Weighted Tree Augmentation

Augmentation problems ask about the cheapest way to increase the (edge-)connectivity of a graph by adding edges among a given set of options. One of the most elementary and intensely studied augmentation problems is (Weighted) Tree Augmentation. Here, a spanning tree has to be augmented into a 2-edge-connected graph. We give the first approximation algorithms for Weighted Tree Augmentation that beat the longstanding approximation factor of 2, which can be achieved through many standard techniques. More precisely, we give a relative greedy approach and a local search procedure that can be interpreted as a refinement thereof, leading to an approximation ratio of $1.5 + \epsilon$. The technical backbone of our approach is a new decomposition theorem for tree augmentation solutions.

This is joint work with Rico Zenklusen.

16:40 Amin Coja-Oghlan (Dortmund)

The full rank condition for sparse random matrices

We derive a sufficient condition for a sparse random matrix with given numbers of non-zero entries in the rows and columns having full row rank. The result covers both matrices over finite fields with independent non-zero entries and $\{0,1\}$-matrices over the rationals. The sufficient condition is generally necessary as well.