2023 Colloquia in Combinatorics

Queen Mary day (10 May 2023)

Schedule

10:00 Tea/Coffee

10:30 Carla Groenland
11:20 Louis Theran

Lunch break

13:40 Cécile Mailer
14:30 Thomas Bloom

Coffee break

15:50 Mihyun Kang
16:40 Torsten Mütze

All lectures will be in the Maths Lecture Theatre located in the Mathematical Sciences building (building 4 on this map).

Abstracts

10:30 Carla Groenland (Utrecht University)

Skipless chain decompositions and antichain saturation

The Boolean lattice is the poset $\{0,1\}^n$ ordered via set inclusion. We show that given disjoint chains $C_1, \ldots, C_m$ in the Boolean lattice, we can create m disjoint skipless chains that cover the elements in $\cup_{i=1}^m C_i$ (where we call a chain skipless if any two consecutive elements differ in size by exactly one). This strengthens a result of Lehman-Ron [JCT-A, 2001]. We find a simple proof for the asymptotics of antichain saturation numbers as corollary (resolving various conjectures), and with additional work can even pinpoint most values exactly. This is based on joint work with Paul Bastide, Hugo Jacob and Tom Johnston.

11:20 Louis Theran (University of St Andrews)

Globally rigid graphs and maximum likelihood inference in Gaussian graphical models

The Gaussian graphical model associated with a graph G with n vertices is the family of n-variate Gaussian distributions with mean zero and inverse covariance matrix that has zeros in the off-diagonal entries corresponding to the non-edges of G. A question first raised by Dempster in the 1970's and popularised more recently by Steffen Lauritzen is: for a fixed G, what is the minimum number of datapoints required for a maximum likelihood estimate of the model to exist almost surely? This quantity is called the maximum likelihood threshold (MLT) of G, and combinatorial properties that bound it have received quite a bit of attention.

In breakthrough work, Uhler gave an upper bound based on a new algebraic graph parameter that Blekherman and Sinn, who provided examples showing Uhler's upper bound is not tight, have called the "generic completion rank" (GCR) of G. I'll describe a reformulation of the MLT via a connection to the rigidity and flexibility of frameworks that leads to non-trivial combinatorial lower bounds on the MLT of a graph. The new lower bounds are powerful enough to classify the graphs with MLT at most 3 or GCR at most 4.

13:40 Cécile Mailer (University of Bath)

Scaling limit of critical random trees in random environment

It has been well-known since Aldous' seminal work in the 90s that a critical Galton-Watson tree conditioned to survive until generation n converges, as n goes to infinity, to a continuous random tree (CRT). This is the equivalent, in the world of random trees, of the fact that a random walk with finite-variance i.i.d. increments converges to the Brownian motion.

In this is joint work with Guillaume Conchon-Kerjan and Daniel Kious, we prove that a Galton-Watson tree "in random environment" (GWRE) also converges to the CRT.

GWREs are Galton-Watson trees in which the offspring distribution of an individual depends on its generation, and the sequence of offspring distributions (indexed by the generation) is sampled in an i.i.d. way. I will review known results on GWREs before stating our main result and giving ideas of the proof.

14:30 Thomas Bloom (University of Oxford)

Three-term arithmetic progressions: past, present, and future

The study of sets of integers without three-term arithmetic progressions, in particular the question of how large such a set can be, is one of the central topics of additive combinatorics. In this talk I will review some of the history of this problem and the new ideas involved, leading to the spectacular recent bounds of Kelley and Meka, and discuss some open problems that remain.

15:50 Mihyun Kang (Graz University of Technology)

Isoperimetric inequalities and supercritical percolation on product graphs

We derive isoperimetric inequalities for high-dimensional product graphs, from which we investigate isoperimetric and structural properties of the giant component in supercritical bond-percolation on product graphs. We obtain upper bounds on the diameter of the giant component and the mixing time of the lazy simple random walk on the giant component. These results not only generalise, but also improve substantially upon the known bounds in the case of the hypercube. This talk is based on joint work with Sahar Diskin, Joshua Erde, and Michael Krivelevich.

16:40 Torsten Mütze (University of Warwick)

On Hamilton cycles in highly symmetric graphs

In 1970 Lovász conjectured that every connected vertex-transitive graph has a Hamilton cycle, apart from five exceptional graphs, one of them being the infamous Petersen graph. This problem received a lot of attention, and it has far-ranging connections to algebra, algorithms, geometry, etc. The question turns out to be surprisingly difficult even for vertex-transitive graphs defined by explicit constructions, e.g., Cayley graphs for various groups and generators. Another plentiful source of vertex-transitive graphs are intersecting set systems. One example for this are Kneser graphs $K(n, k)$, whose vertices are all k-element subsets of an n-element set, and the edges connect disjoint sets. In this talk I present our line of work that settles Lovász' conjecture for all known graphs defined by intersecting set systems. In particular, we show that all Kneser graphs $K(n, k)$ admit a Hamilton cycle, except the Petersen graph $K(5, 2)$.

LSE day (11 May 2023)

Schedule

09:30 Tea/Coffee

10:00 Annika Heckel
10:50 Robin Wilson
11:35 Norman Biggs

Lunch break

13:15 Paul Seymour
14:05 Nina Kamčev

Coffee break

15:20 Guus Regts
16:10 Martin Anthony

All lectures will be in the Sheikh Zayed Theatre located on the lower ground floor of the New Academic Building (NAB building on this map).

Abstracts

10:00 Annika Heckel (Uppsala Universitet)

The chromatic number of $G(n, 1/2)$: bounds, concentration and conjectures

I will discuss the chromatic number of the random graph $G(n, p)$, which is the minimum number of colours needed for a vertex colouring where neighbours are always coloured differently. In particular I will talk about the concentration (or lack thereof) of the chromatic number of $G(n, 1/2)$, and how the behaviour of the required number of colours changes drastically if we restrict the possible colouring types to exclude very large colour classes. For example, the equitable chromatic number of $G(n, 1/2)$ turns out to be extremely tightly concentrated. This leads to a number of conjectures about the limiting distribution of the chromatic number of $G(n, 1/2)$.

Based on joint work with Oliver Riordan and with Konstantinos Panagiotou.

10:50 Robin Wilson (Open University)

A century of graph theory

It can be interesting and instructive to see how a subject develops historically, and who the people were who created it. This talk covers the period from the 1890s, when graph theory was largely a collection of isolated results, to the 1990s when it had become part of mainstream mathematics. Among the people we meet on our journey are Heawood, Petersen, Birkhoff, König, Kuratowski, Whitney, Tutte, Ringel, Appel & Haken, and Robertson & Seymour, while the topics covered include map colouring, factorization, enumeration, planarity, trees, graph structure and graph algorithms.

13:15 Paul Seymour (Princeton University)

Bounded diameter tree-decompositions

When does a graph$G$ admit a tree-decomposition in which every bag has diameter at most d? One necessary condition is that there is no "geodesic" cycle of length more than $3d$; but this is not sufficient, even qualitatively, because one can make graphs in which every geodesic cycle has length at most four, and yet every tree-decomposition has a bag with large diameter. Nevertheless, it turns out that this is close to the right answer, as we will explain.

Joint with Eli Berger.

14:05 Nina Kamčev (University of Zagreb)

Canonical colourings in random graphs

Rödl and Ruciński have extended Ramsey's Theorem to random graphs, showing that there is a large constant C such that with high probability, any two-colouring of the edges of the random graph $G_{n,p}$ with $p = Cn^{-\frac{2}{\ell+1}}$ contains a monochromatic copy of K_ℓ. We investigate how this result extends to arbitrary colourings of $G_{n,p}$. Namely, when no assumptions are made on the edge colouring of $G_{n,p}$, one can only hope to find one of the four *canonical colourings* of K_ℓ, as in the well-known canonical version of Ramsey's Theorem due to Erdős and Rado.

We show that indeed, with high probability, any colouring of $G_{n,p}$ with $p = Cn^{-\frac{2}{\ell+1}}$ contains a canonically coloured copy of K_ℓ. As a consequence, the proof yields $K_{\ell+1}$-free graphs G for which every edge colouring contains a canonically coloured K_ℓ. A crucial tool in our proof is the transference principle developed by Conlon and Gowers. (Joint work with Mathias Schacht.)

15:20 Guus Regts (University of Amsterdam)

The chromatic polynomial: complex zeros and computational complexity

The chromatic polynomial is a central concept in combinatorics and has been studied in the literature from a large range of perspectives. In this talk I want to focus on the location of the complex zeros and the computational complexity of approximately evaluating the chromatic polynomial at algebraic numbers for planar graphs. While at first these two topics may seem unrelated, I will argue that they are in fact deeply connected.

By a celebrated result of Sokal it is known that the zeros of the chromatic polynomial of planar graphs (series-parallel graphs in fact) are dense in the complement of the disk of radius one centered at one. I will cover results describing zeros inside this disk and explain why approximating the chromatic polynomial at a non-real number in the collection of these zeros is computationally hard.

Based on joint work with Ferenc Bencs and Jeroen Huijben

16:10 Martin Anthony (London School of Economics)

Probabilistic modelling in machine learning: combinatorial parameters, margin and width

This talk gives an overview of some approaches to the probabilistic modelling of supervised machine learning. This begins with a discussion of the now classical framework introduced by Valiant, in which the VC-dimension plays an important role. Extensions to models in which classifiers arise from real-valued functions are then discussed. (This is an appropriate framework for studying many practical machine learning systems such as artificial neural networks and support vector machines.) When using real-valued functions for classification, the concept of ‘classification margin’ is central, and generalisations of the VC-dimension are useful in quantifying learning performance. The classification margin provides a way of measuring how definitive is a classification by real-valued functions, but it is not applicable if the classifiers are binary-valued. For such circumstances, we have introduced the notion of ’sample width’ and I will discuss this and some related results (obtained with Joel Ratsaby).