BSDES, DECOUPLED MILD SOLUTIONS OF ASSOCIATED DETERMINISTIC EQUATIONS AND APPLICATIONS TO HEDGING UNDER BASIS RISK.

Joint Risk & Stochastics and Financial Mathematics Seminar,
November 21st 2019
London School of Economics (LSE)

Francesco RUSSO
ENSTA Paris, Institut Polytechnique de Paris
http://uma.ensta-paristech.fr/∼russo

The talk is based on joint work with Adrien Barrasso (CMAP Polytechnique) and Ismail Laachir (ZELIADE).

The aim of this talk consists in introducing a new formalism for the deterministic analysis associated with backward stochastic differential equations driven by general càdlàg martingales, coupled with a forward process.

When the martingale is a standard Brownian motion, the natural deterministic analysis is provided by the solution u of a semilinear PDE of parabolic type coupled with a function v which is associated with the gradient ∇u, when u is of class C^1 in space. When u is only a viscosity solution of the PDE, the link associating v to u is not completely clear: sometimes in the literature it is called the *identification* problem. We introduce in particular the notion of a *decoupled mild solution* of a PDE, a IPDE, a path-dependent PDE or more generally a deterministic problem associated with a BSDE.

The idea is to introduce a suitable analysis to investigate the equivalent of the identification problem first in a general Markovian setting with a class of examples. An interesting application concerns the hedging problem under basis risk of a contingent claim $g(X_T, S_T)$, where S (resp. X) is an underlying price of a traded (resp. non-traded but observable) asset, via the celebrated Föllmer-Schweizer decomposition. We revisit the case when the couple of price processes (X, S) is a diffusion and we provide explicit expressions when (X, S) is an exponential of additive processes. Extensions to non-Markovian (path-dependent) cases are discussed.