The talk is based on joint work with Adrien Barrasso (CMAP Polytechnique) and Ismail Laachir (ZELIADE).

The aim of this talk consists in introducing a new formalism for the deterministic analysis associated with backward stochastic differential equations driven by general càdlàg martingales, coupled with a forward process.

When the martingale is a standard Brownian motion, the natural deterministic analysis is provided by the solution $u$ of a semilinear PDE of parabolic type coupled with a function $v$ which is associated with the gradient $\nabla u$, when $u$ is of class $C^1$ in space. When $u$ is only a viscosity solution of the PDE, the link associating $v$ to $u$ is not completely clear: sometimes in the literature it is called the identification problem. We introduce in particular the notion of a decoupled mild solution of a PDE, a IPDE, a path-dependent PDE or more generally a deterministic problem associated with a BSDE.

The idea is to introduce a suitable analysis to investigate the equivalent of the identification problem first in a general Markovian setting with a class of examples. An interesting application concerns the hedging problem under basis risk of a contingent claim $g(X_T, S_T)$, where $S$ (resp. $X$) is an underlying price of a traded (resp. non-traded but observable) asset, via the celebrated Föllmer-Schweizer decomposition. We revisit the case when the couple of price processes $(X, S)$ is a diffusion and we provide explicit expressions when $(X, S)$ is an exponential of additive processes. Extensions to non-Markovian (path-dependent) cases are discussed.