Risk & Stochastics and Financial Mathematics Joint Seminar in 2017

Seminars are listed in reverse chronological order, most recent first.

Thursday 7 December - Thomas Kruse (Duisburg-Essen)
Multilevel Picard approximations for high-dimensional nonlinear parabolic partial differential equations

In this talk we present a family of new approximation methods for high-dimensional PDEs and BSDEs.

A key idea of our methods is to combine multilevel approximations with Picard fixed-point approximations. Thereby we obtain a class of multilevel Picard approximations.
Our error analysis proves that for semi-linear heat equations, the computational complexity of one of the proposed methods is bounded by $O(d\,\eps^{-(4+\delta)})$ for any $\delta > 0$, where d is the dimensionality of the problem and $\eps\in(0,\infty)$ is the prescribed accuracy.

We illustrate the efficiency of one of the proposed approximation methods by means of numerical simulations presenting approximation accuracy against runtime for
several nonlinear PDEs from physics (such as the Allen-Cahn equation) and financial engineering (such as derivative pricing incorporating default risks) in the case of $d=100$ space dimensions.

The talk is based on joint work with W. E, M. Hutzenthaler, and A. Jentzen.

Thursday 23 November - Charles-Albert Lehalle (Capital Fund Management)
Closing The Loop of Optimal Trading: a Mean Field Game of Controls

This talk explains how to formulate the now classical problem of optimal liquidation (or optimal trading) inside a Mean Field Game (MFG). This is a noticeable change since usually mathematical frameworks focus on one large trader in front of a " background noise " (or " mean field "). In standard frameworks, the interactions between the large trader and the price are a temporary and a permanent market impact terms, the latter influencing the public price. Here the trader faces the uncertainty of fair price changes too but not only. He has to deal with price changes generated by other similar market participants, impacting the prices permanently too, and acting strategically. Our MFG formulation of this problem belongs to the class of " extended MFG ", we hence provide generic results to address these " MFG of controls ", before solving the one generated by the cost function of

optimal trading. We provide a closed form formula of its solution, and address the case of " heterogenous preferences " (when each participant has a different risk aversion). Last but not least we give conditions under which participants do not need to instantaneously know the state of the whole system, but can " learn " it day after day, observing others' behaviors.

Thursday 9 November - **Thibaut Mastrolia (CMAP)**
An overview of contract theory: from the Holmström-Milgrom paradigm to multi-agent systems

In this talk, we investigate a situation in which two economical entities interact to maximize their own payoff. One of them is called the Principal (she) and proposes to the second one, named the Agent (he), some incentives to modify her wealth. The main difficulty comes from the fact that the Principal observes the result of the work of her Agent without observing his work directly. This kind of situation coincides exactly with a moral hazard problem in which the Principal has to design an employment contract given to her Agent to maximize her utility without observing directly his work. We identify this paradigm with a Stackelberg equilibrium that can be explicitly solved in several examples. We begin to study the example of Holmström and Milgrom dealing with one Principal and one Agent in the continuous case. We then extend this investigation to N-interacting Agents hired by one Principal and to a model with one Agent hired by several Principals.

26 October - **Joaquin Narro (Alcazar Investment Management Ltd / Bainbridge Partners LLP)**
Forecasting Prices of Electricity Futures: Practice vs. Theory

Electricity plays a major role in people’s lives. The reliability and affordability of electricity, together with the need to address climate change, are major issues for our society at large. The world-wide drive to support low-carbon electricity generation is based on a myriad of legislative initiatives with implications far beyond traditional power systems. In this complex environment, it is vital for all stakeholders involved in electricity generation, from consumers to producers, from investors to regulators, to understand the pricing of electricity. This presentation explores several examples of forecasting of electricity prices using different mathematical models, highlighting the practical benefits and hindrances of different approaches.

12 October - **Saul Jacka (Warwick)**
Optimal stopping, the dual problem and smooth pasting

Let G be a semimartingale, and S its Snell envelope. Under the assumption that $G\in\mathcal{H}^1$, we show that the finite-variation part of S is absolutely continuous with respect to the decreasing part of the finite-variation part of G. In

the Markovian setting, this enables us to identify sufficient conditions for the value function of the optimal stopping problem to belong to the domain of the extended (martingale) generator of the underlying Markov process. We then show that the dual of the optimal stopping problem is a stochastic control problem for a controlled Markov process, and the optimal control is characterised by a function belonging to the domain of the martingale generator. Finally, we give an application to the smooth pasting condition.

28 September - Christoph Belak (Trier)
Utility Maximization with Constant Costs

We study the problem of maximizing expected utility of terminal wealth for an investor facing a mix of constant and proportional transaction costs. While the case of purely proportional transaction costs is by now well understood and existence of optimal strategies is known to hold for very general class of price processes, the case of constant costs remains a challenge since the existence of optimal strategies is not even known in tractable models (such as, e.g., the Black-Scholes model). In this talk, we present a novel approach which allows us to construct optimal strategies in a multidimensional diffusion market with price processes driven by a factor process and for general lower-bounded utility functions.

The main idea is to characterize the value function associated with the optimization problem as the pointwise infimum V of a suitable set of superharmonic functions. The advantage of this approach is that the pointwise infimum inherits the superharmonicity property, which in turn allows us to prove a verification theorem for candidate optimal strategies under mild regularity assumptions on V. Indeed, for the verification procedure based on superharmonic functions to be applicable, it suffices that the pointwise infimum V is continuous.

In order to establish the continuity of V, we adapt the stochastic Perron's method to our situation to show that V is a discontinuous viscosity solution of the associated quasi-variational inequalities. A comparison principle for discontinuous viscosity solutions then closes the argument and shows that V is continuous. With this, the verification theorem becomes applicable and it follows that the pointwise infimum V coincides with the value function and that the candidate optimal strategies are indeed optimal.

This talk is based on joint work with Sören Christensen (University of Hamburg) and Frank T. Seifried (University of Trier).

20 September - Keita Owari (Ritsumeikan University)
A Komlós-Type Theorem in Dual Orlicz Spaces

We give a Komlós type result for bounded sequences in dual Orlicz spaces (i.e. Orlicz spaces which are the duals of (Δ_2) Orlicz spaces; e.g. the space of random variables with some exponential moments). Its "utility grade" version asserts that any bounded sequence in such a space has an (a.s. convergent) sequence of forward convex combinations whose supremum remains in the same space. If the probability space is atomless, this type of Komlós theorem characterises the class of dual Orlicz spaces, or their preduals, the Δ_2-Orlicz spaces. Some consequences in convex duality in finance are also given.

28 September - Chiara Donnini (Naples)
Strictly fair allocations in economies with atoms

We investigate the fairness property of Walrasian allocations in mixed exchange markets that are economies having both atoms (large or non-negligible traders) and an atomless sector (small or negligible traders). We provide two sufficient conditions under which equal-income Walrasian equilibria are the only efficient allocations which are also strictly equitable as defined by Zhou. A further characterization is obtained without extra assumptions via a weaker notion of envy-freeness based on the Aubin approach. Our analysis proceeds with a discussion on the problems arising in differential information economies.

27 April - Peter Kort (Tilburg)
Capacity Choice in (Strategic) Real Options Models

This paper considers investment decisions within an uncertain dynamic and competitive framework. Each investment decision involves to determine the timing and the capacity level. In this way we extend the main bulk of the real options theory where the capacity level is given. We consider a monopoly setting as well as a duopoly setting. In the duopoly setting we provide a fully dynamic analysis of entry deterrence / accommodation strategies. We find that the first investor overinvests in capacity in order to delay entry of the second investor. In very uncertain economic environments the first investor always ends up being the largest firm in the market. If uncertainty is moderately present, a reduced value of waiting implies that the preemption mechanism forces the first investor to invest so soon that a large capacity cannot be afforded. Then it will eventually end up with a capacity level being lower than the second investor.

Then we extend the above setting to explicitly consider an incumbent-entrant framework. We find that the incumbent invests earlier than the entrant and that entry deterrence is achieved through timing rather than through overinvestment. This is because the incumbent invests earlier and in a smaller amount compared to a scenario without potential entry. If, on the other hand, the capacity size is exogenously given, the investment order changes and the entrant invests before the incumbent does.

Finally we consider an innovative incumbent that has an option to perform a product innovation. Question is whether, after introducing the new product on the market, the firm keeps on producing its old product. We provide scenarios under which this is optimal or when the firm will abandon the old product.

16 March - Sergio Pulido (ENSIIE)
Density of probability measures with the martingale representation property

Using the theory of analytic maps, we prove density results for measures satisfying a backward formulation of the martingale representation property. These results are useful to study equilibrium-based mechanisms of pricing. This is joint work with Dmitry Kramkov.

13 March - Yuri Imamura (Tokyo University of Science)
Asymptotic Static Hedge via Symmetrization

In the talk, we are interested in the risk to cover (some portion of) the price of the option at a default time. The risk, which we call timing risk, is a risk of uncertain dividend, especially of its payment time. Credit derivatives typically are exposed to the risk. We will discuss how it could be hedged by a static position of European path-independent options, generalizing P. Carr and J. Picron (1999) where they applied the semi-static hedging formula of barrier options to hedge a payment at a stopping time in a Black-Scholes environment. We will give an exact hedging formula in an multi-dimensional general diffusion setting.

9 March - Romain Blanchard
Robust optimal investment in discrete time for unbounded utility function

We investigates the problem of maximising worst case expected terminal utility in a discrete time financial model with a finite horizon under non-dominated model uncertainty. We use dynamic programming framework together with measurable elections arguments to prove that under mild integrability assumption, an optimal portfolio exists for unbounded utility function defined on the half-real line. We revisit also the non-arbitrage condition in the robust framework.

2 March - Dirk Becherer (HU Berlin)
Stochastic illiquidity

In classical models from math. finance, dynamic trading strategies are executed against price processes which are exogenously given, and are not affected by the strategies. Economically, this means to assume that liquidity is unlimited or

investors are 'small'. We discuss optimal control problems from mathematical finance in models for large investors, whose trading strategies have an intertemporal effect on the prices, against which they are executed. An original feature of the model, that we discuss, is that the transient price impact due to illiquidity in our model is stochastic and multiplicative, instead of being additive and deterministic (as a function of the strategy).

Joint work with Todor Bilarev, Peter Frentrup, HU Berlin, some related papers are on arxiv.

24 February - Agostino Capponi (Columbia)
Bail-ins and Bail-outs: Incentives, Connectivity, and Systemic Stability

We analyze the stability of an interbank network, in which rescues in the form of subsidized bail-ins or public bailouts can be coordinated to stop financial contagion. The coordination of a rescue consortium between a benevolent social planner and the banks is modeled as a sequential game. We show that the equilibrium welfare losses are generically unique, depending heavily on the network structure, which influences whether or not the social planner's threat to not intervene is credible. We provide conditions under which the threat is credible and characterize the optimal intervention plan.

Our analysis shows that sparsely connected networks may enhance financial stability in two ways: (i) a smaller amplification of the shock without intervention may enhance credibility of the social planner's threat and (ii) because default resolution costs are concentrated, the creditors of defaulting banks can be incentivized to make large contributions to a subsidized bail-in. This may make a sparsely connected network socially preferable over a more densely connected network, even if the densely connected network is financially more stable in the absence of any intervention.

Based on joint work with Benjamin Bernard and Joseph Stiglitz

16 February - Peter Johnson (Manchester)
Sequential Testing problems for Bessel Processes

Consider the motion of a Brownian particle that takes place either in a two-dimensional plane or in the three-dimensional space. Given that only the distance of the particle to the origin is being observed, the problem is to detect the true dimension as soon as possible and with minimal probabilities of the wrong terminal decisions. This talk will discuss the solution to this problem in the Bayesian formulation under any prior probability of the true dimension when the passage of time is penalised linearly.

This is nice example of tackling an optimal stopping problem for a 2-dimensional coupled Markov process. The solution uses a measure change, a stochastic time-change, Mayer and Lagrange reformulations, and allowing for negative initial times, which could help provide ideas for solving other 2-dimensional optimal stopping problems.

13 February - Zbyszek Palmowski (Wroclaw University of Science and Technology)
Ruin probabilities: exact and asymptotic results

Ruin theory concerns the study of stochastic processes that represent the time evolution of the surplus of a stylized non-life insurance company. The initial goal of early researchers of the field, Lundberg (1903) and Cramer (1930), was to determine the probability for the surplus to become negative. In those pioneer works, the authors show that the ruin probability decreases exponentially fast to zero with initial reserve tending to infinity when the net profit condition is satisfied and clam sizes are light-tailed.

During the lecture we explain when and why we can observe this phenomenon and discuss also the heavy-tailed case. We demonstrate main techniques and results related with the asymptotics of the ruin probabilities: Pollaczek-Khinchin formula, Lundberg bounds, change of measure, Wiener-Hopf factorization, principle of one big jump and theory of scale functions of Levy processes.

2 February - Julio Backhoff (TU Vienna)
Existence of extremal diffusions matching a continuum of marginal and applications

Given the law of a diffusion process, we consider the problem of adjusting its drift via change of measure in a cost-optimal way so as to meet a prescribed continuum of marginals. When the cost criterion is the relative entropy, the optimizer is a singular diffusion (the so called critical Nelson process), extensively studied in the literature typically through approximation or large deviation techniques. In this talk we will consider different optimality criteria, and using convex duality as well as stochastic control techniques, obtain the existence of a singular optimal diffusion. As an application, we will discuss the link between this problem and imperfect hedging with static portfolios.

This is work in progress with J. Fontbona.

19 January - Athena Picarelli (Oxford)
High-order filtered schemes for time-dependent second order HJB equations

In this work, we present and analyse a class of "filtered" numerical schemes for second order Hamilton-Jacobi-Bellman equations.

Our approach follows the ideas recently introduced in B.D. Froese and A.M. Oberman, "Convergent filtered schemes for the Monge-Ampère partial differential equation" (SIAM J. Numer. Anal., 2013) and more recently applied by other authors to stationary or time-dependent first order Hamilton-Jacobi equations.

For high order approximation schemes (where "high" stands for greater than one), the inevitable loss of monotonicity prevents the use of the classical theoretical results for convergence to viscosity solutions.

The work introduces a suitable local modification of these schemes by "filtering" them with a monotone scheme, such that they can be proven convergent and still show an overall high order behaviour for smooth enough solutions.

We give theoretical proofs of these claims and illustrate the behavior with numerical tests from mathematical finance, focusing also on the use of backward differencing formulae for constructing the high order schemes.