Risk & Stochastics and Financial Mathematics Joint Seminar in 2010

Seminars are listed in reverse chronological order, most recent first.

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8 November - Luitgard Veraart (LSE)
The relaxed investor with partial information

We consider an investor in a financial market consisting of a riskless bond and several risky assets. The price processes of the risky assets are geometric Brownian motions where either the drifts are modelled as random variables assuming a constant volatility matrix or the volatility matrix is considered random and drifts are assumed to be constant. The investor is only able to observe the asset prices but not all the model parameters and hence information is only partial. A Bayesian approach is used with known prior distributions for the random model parameters.

We assume that the investor can only trade at discrete time points which are multiples of h>0 and investigate the loss in expected utility of terminal wealth which is due to the fact that the investor cannot trade and observe continuously. It turns out that in general a discretization gap appears, i.e for $h \to 0$ the expected utility of the h-investor does not converge to the expected utility of the continuous investor. This is in contrast to results under full information in(Rogers, L.C.G. 2001. The relaxed investor and parameter uncertainty. Finance and Stochastics, 5 (2), 131-154).

We also present simple asymptotically optimal portfolio strategies for the discrete-time problem. Our results are illustrated by some numerical examples.

This is joint work with Nicole Bäuerle and Sebastian Urban.

29 April - Yuliya Mishura (Kyiv)
Financial applications of the models with long-range dependence

In our work we move away from the semimartingale model of financial market and consider the models with so called long-range dependence. The arbitrage problems are discussed in the most general setting.

In particular, we consider financial market with risky asset governed by both the Wiener process and fractional Brownian motion with Hurst parameter $H>3/4$. Using Hitsuda and Cheridito representations for the mixed Brownian--fractional

Brownian process, we present the solution of the problem of quantile hedging and clarify in
this case the dependence of maximal possible success probability on the available initial capital $\nu <H_0 $. More general problem of efficient hedging is also solved.

11 March - Hans Rudolf Lerche (Freiburg)
Blackwell Prediction

Let $x_1 , x_2 , . . .$ be a (not necessarily random) infinite 0-1 sequence. We wish to sequentially predict the sequence. This means that, for each $n \geq 1$, we will guess the value of x_{n+1}, basing our guess on knowledge of $x_1 , . . . , x_n$. Of interest are algorithms which predict well for *all* 0-1 sequences. An example is the algorithm of Blackwell. It can be deduced from Blackwell's generalization of the von Neumann minimax theorem on games. We shall discuss this and the generalization of Blackwell's algorithm to three and more categories. The three category algorithm will be explained using a geometric model (the so-called prediction prism). The Blackwell algorithm has interesting properties. It predicts arbitrary 0-1 sequences as well or better than independent, identically distributed Bernoulli variables, for which it is optimal. Similar results hold for the three and more category generalizations of Blackwell's algorithm.

5 Feb - Tomas Björk (Stockholm School of Economics)
Time inconsistent stochastic control

We present a theory for stochastic control problems which, in various ways, are time inconsistent in the sense that they do not admit a Bellman optimality principle. We attach these problems by viewing them within a game theoretic framework, and we look for subgame perfect Nash equilibrium points.

For a general controlled Markov process and a fairly general objective functional we derive an extension of the standard Hamilton-Jacobi-Bellman equation, in the form of a system of non-linear equations, for the determination for the equilibrium strategy as well as the equilibrium value function. All known examples of time inconsistency in the literature are easily seen to be special cases of the present theory. We also prove that for every time inconsistent problem, there exists an associated time consistent problem such that the optimal control and the optimal value function for the consistent problem coincides with the equilibrium control and value function respectively for the time inconsistent problem. We also study some concrete examples.

Risk & Stochastics and Financial Mathematics Joint Seminar in 2011

Seminars are listed in reverse chronological order, most recent first.

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8 December - Alex Miljatovic (Warwick)

No title or abstract available

24 November - Kees van Schaik (Manchester)

No title or abstract available

17 November - Mete Soner (ETH)

No title or abstract available

10 November - Kevin Warner (Tower Research Capital)

No title or abstract available

3 November - Jordan Stoyanov (Newcastle)
Moment Analysis of Distributions: Classical and Recent Results

The main discussion will be on characterization/properties of distributions in terms of the moments. This turns to be important for stochastic models in many areas, including in finance and risk modelling. Some distributions are uniquely determined by the moments (M-determinate), others are non-unique (M-indeterminate). Along classical criteria, some recent developments will be presented and used to analyze the moment determinacy of distributions of random variables or stochastic processes. All statements and criteria will be well illustrated by examples involving popular distributions (N, LogN, SN, Exp, Po, IG, etc.) Several facts will be reported, sme of them are not so well-known, they are surprising and even shocking. It will be shown that the moment determinacy of the distributions is essential in inference problems. Some challenging open questions will be outlined.

27 October - Huyên Pham (University Paris Diderot)
Optimal High Frequency Trading with Limit and Market Orders

We propose a framework for studying optimal market making policies in a limit order book (LOB). The bid-ask spread of the LOB is modelled by a Markov chain with finite values, multiple of the tick size, and subordinated by the Poisson process of

the tick-time clock. We consider a small agent who continuously submits limit buy/sell orders at best bid/ask quotes, and may also set limit orders at best bid (resp. ask) plus (resp. minus) a tick for getting the execution order priority, which is a crucial issue in high frequency trading. By trading with limit orders, the agent faces an execution risk since her orders are executed only when they meet counterpart market orders, which are modelled by Cox processes with intensities depending on the spread and on her limit prices. By holding non-zero positions on the risky asset, the agent is also subject to the inventory risk related to price volatility. Then the agent can also choose to trade with market orders, and therefore get immediate execution, but at a least favourable price because she has to cross the bid-ask spread.

The objective of the market maker is to maximize her expected utility from revenue over a short term horizon by a tradeoff between limit and market orders, while controlling her inventory position. This is formulated as a mixed regime switching regular/impulse control problem that we characterize in terms of quasi-variational system by dynamic programming methods. In the case of a mean-variance criterion with martingale reference price or when the asset price follows a Levy process and with exponential utility criterion, the dynamic programming system can be reduced to a system of simple equations involving only the inventory and spread variables.

Calibration procedures are derived for estimating the transition matrix and intensity parameters for the spread and for Cox processes modelling the execution of limit orders. Several computational tests are performed both on simulated and real data, and illustrate the impact and profit when considering execution priority in limit orders and market orders.

20 October - **Ragnar Norberg (LSE and Lyon)**

No title or abstract available

13 October - **Almut Veraart (Imperial)**
Ambit Stochastics with Applications to Energy Markets

This talk gives a brief introduction into the new area of ambit stochastics, which constitutes a general probabilistic framework for tempo-spatial modelling. Certain classes of random fields and stochastic processes within the framework of ambit stochastics will be presented and their applicability to modelling spot, forward and futures prices from energy markets will be discussed.

This is joint work with Ole. E. Barndorff-Nielsen (Aarhus) and Fred Espen Benth (Oslo)

6 October - **Roman Muraviev (ETH)**

No title or abstract available

9 June - Guilia Di Nunno (Oslo)
Dynamic no-good-deal bounds and no-good-deal pricing measures

We study price systems consistent with no-good-deal pricing measures for given bounds on the Sharpe ratio and we introduce the definition of dynamic no-good-deal bounds and pricing measure. The development of the theory requires a sandwich preserving extension theorem for linear operators, which we present in some generality. We then show how this result can be applied to obtain static and dynamic no-good-deal pricing measures. If time permits, we can also provide other examples of reasonably restricted classes of equivalent martingale measures that can be obtained.

This presentation is based on a paper with Dr. Jocelyn Bion-Nadal (CNRS-Ecole Polytechnique, France).

19 May - Bernt Oksendal (Oslo)
Optimal pricing strategies and Stackelberg equilibria in time- delayed stochastic differential games

In the classical newsvendor problem there are two agents: (i) The manufacturer, who today (i.e. at time t-\delta) decides the unit price to sell the manufactured goods for to the retailer, with delivery tomorrow (at time t); (ii) The retailer, who then today (at time t-\delta) decides the quantity to order from the manufacturer and the price to sell each item for to the public the next day. What is the optimal price set by the manufacturer and the optimal quantity to order and the optimal retailer price? The problem is that neither of these agents know what the demand will be the next day, only its probabilistic distribution. This is a problem that occurs in many situations, for example in the pricing of electricity in a liberated electricity market. We generalize this classical newsvendor problem to continuous time and a jump diffusion setting, and formulate it as a problem to find the Stackelberg equilibrium of a stochastic differential game with delayed information flow. We find a maximum principle for this type of control problem, and use it to solve the optimal pricing problem in some specific cases.

Presentation is based on recent joint work with Leif Sandal and Jan Ubøe, both at NHH, Bergen, Norway.

24 March - Peter Bank (Technische Universität Berlin and Quantitative Products Laboratory)
Market indifference prices

We discuss the pricing and wealth dynamics in a market where a large trader's orders are filled at indifference prices. As we will see, this indifference principle is mathematically best described by a nonlinear SDE for the market makers' utility process. We will derive this SDE and discuss its solvability in terms of Malliavin derivatives and Sobolev embedding results for stochastic integrals.

10 March - Lioudmila Vostrikova (Université d'Angers)
F-divergence minimal martingale measures and optimal portfolios for exponential Levy models with a change-point

We study exponential Levy models with change-point which is a random variable, independent from initial Levy processes. On canonical space with initially enlarged ltration we describe all equivalent martingale measures for change-point model and we give the conditions for the existence of f-minimal equivalent martingale measure. Using the connection between utility maximisation and f-divergence minimisation, we obtain a general formula for optimal strategy inchange-point case for initially enlarged ltration and also for progressively en-larged ltration when the utility is exponential. We illustrate our results consid-ering the Black-Scholes model with change-point.

3 March - Carlos G. Pacheco González (CINVESTAV)
The Kac semi-group and applications to stochastic control

In this talk we present the Kac semi-group within the context of Markov processes, and we show applications in stochastic control problems with non-constant discounted criteria. In particular we set a Hamilton-Jacobi-Bellman equation for a problem where the discounted process is the Cox-Ingersol-Ross model.

24 February - Sergey Nadtochiy (Oxford)
An approximation scheme for the optimal investment strategy in incomplete market

Characterizing and constructing the solutions to stochastic optimization problems of optimal portfolio choice is a long standing problem. In this talk, I will discuss a new method based on a splitting scheme for the associated Hamilton-Jacobi-Bellman equation in a two-factor stochastic volatility model for the stock price. The scheme converges to a solution of the corresponding PDE, and yields an explicit uniform approximation of the optimal investment strategy. This solution approach offers, among others, insightful observations on how market incompleteness is

processed and how it affects the 'infinitesimal' investment preferences. This is joint work with Thaleia Zariphopoulou.

3 February - Chris Rogers (Cambridge)
Diverse beliefs and market selection

This talk presents the basic framework for equilibrium pricing where agent heterogeneity is characterized by diverse beliefs. This turns out to be a tractable and sensible modelling framework in which to study various phenomena, which we will illustrate with several examples, drawn in the main from the literature on market selection. The Market Selection Hypothesis loosely speaking proposes that agents with `inferior' beliefs will eventually be `eliminated' from the market, but these terms need to be defined. Once they are, we are able to prove some results about when agents are indeed eliminated from the market; these results only partly confirm the intuition of the Market Selection Hypothesis. We have some surprising examples which show that some very unexpected phenomena may occur.

Risk & Stochastics and Financial Mathematics Joint Seminar in 2012

Seminars are listed in reverse chronological order, most recent first.

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10 May - Brenda Lopez Cabrera (Humboldt-Universität zu Berlin)
State Price Densities implied from Weather Derivatives

A State Price Density (SPD) is the density function of a risk neutral equivalent martingale measure for option pricing, and is indispensible for exotic option pricing and portfolio risk management. Many approaches have been proposed in the last two decades to calibrate a SPD using financial options from the bond and equity markets. Among these, non and semi parametric methods were preferred because they can avoid model mis-specification of the underlying and thus give insight into complex portfolio propelling. However, these methods usually require a large data set to achieve desired convergence properties. Despite recent innovations in financial and insurance markets, many markets remain incomplete, and there exists an illiquidity issue. One faces the problem in estimation by e.g. kernel techniques that there are not enough observations locally available. For this situation, we employ a Bayesian quadrature method because it allows us to incorporate prior assumptions on the model parameters and hence avoids problems with data sparsity. It is able to compute the SPD of both call and put options simultaneously, and is particularly robust when the market faces the illiquidity issue. As illustration, we calibrate the SPD for weather derivatives, a classical example of incomplete markets with financial contracts payoffs linked to nontradable assets, namely, weather indices.

3 May - Larbi Alili (Warwick)
On some involutive inversions of one dimensional diffusions

No title or abstract available

15 March - Josef Teichmann (ETH Zürich)
Finite dimensional realizations for the CNKK-volatility surface model

We show that parametrizations of volatility surfaces (and even more involved multivariate objects) by time-dependent Lévy processes (as proposed by Carmona-Nadtochiy-Kallsen-Krühner) lead to quite tractable term structure problems. In this context we can then ask whether the corresponding term structure equations allow for (regular) finite dimensional realization, which necessarily leads to models driven by an affine factor process. This is another confirmation that affine processes play a particular role in mathematical finance. The analysis is based on a careful

geometric analysis of the term structure equations by methods from foliation theory.

8 March - Johan Tysk (Uppsala)

No title or abstract available

23 February - Vicky Henderson (Oxford)
Executive Stock Options: Portfolio Effects

Executives compensated with stock options generally receive grants periodically and so on any given date, may have a portfolio of options of differing strikes and maturities on their company's stock. Non-transferability and trading restrictions in the company stock result in the executive facing unhedgeable risk. We employ exponential utility indifference pricing to analyse the optimal exercise thresholds for each option, option values and cost of the options to shareholders. Portfolio interaction effects mean that each of these differ, depending on the composition of the remainder of the portfolio. We demonstrate that the exercise threshold for a particular option can be discontinuous at the time that the option's position in the exercise order changes.

The cost to shareholders of an option portfolio is lowered relative to its cost computed on a per-option basis. The model can explain a number of empirical observations - which options are attractive to exercise first, how exercise changes following a new grant, and early exercise.

Joint work with Jia Sun and Elizabeth Whalley (WBS).

16 February - Mike Tehranchi (Cambridge)
Put-call symmetry and self-duality

We discuss generalisations of the notions of put-call symmetry and self-duality. These notions have found applications in the pricing and hedging of certain path-dependent contingent claims. Our results include a classification of the possible forms of self-duality in one-dimension: in addition to the arithmetic and geometric duality already appearing the literature, there exists exactly one other type among continuous models. We also give a description of the possible forms of put-call symmetry for common models: in dimension greater than two, interesting new symmetries appear.

9 February - Daniel Hernández (CIMAT)
Dynamic risk measures for exponential Levy market models

The study of robust utility maximization problems for Levy processes is closely related with risk measures. In this talk we shall present recent results on the form of the penalization function associated with risk measures defined in a proper set of absolutely continuous measures, for a Levy market model.

9 February - Markus Riedle (King's College London)

No title or abstract available

2 February - **Curdin Ott (Bath)**

No title or abstract available

19 January - Damiano Brigo (King's College London)
Arbitrage-free valuation of counterparty credit risk

Although explicit pricing of counterparty credit risk goes way back to 1994 in the financial modelling literature, only after the eight credit events that happened in one month of 2008 the research environment has become increasingly active in modelling credit valuation adjustments (CVA). Basel III is also imposing heavy capital requirements on CVA after noticing that about 2/3 of the losses during the crisis are due to CVA mark to market volatility rather than to actual defaults.
In this talk we introduce the mathematics of CVA and explain why it is a difficult hybrid derivatives valuation problem. Subtleties on payoff and modelling mathematics including wrong way risk, closeout conventions, first to default risk, collateral modelling, re-hypothecation and gap risk are investigated with quantitative case studies from a few asset classes. General conclusions on the mathematical difficulties involved in CVA pricing and risk management are presented.

12 January - Johannes Ruf (Oxford)
On the Hedging of Options on Exploding Exchange Rates

Recently strict local martingales have been used to model exchange rates. In such models, put-call parity does not hold if one assumes minimal superreplicating costs as contingent claim prices. I will illustrate how put-call parity can be restored by changing the definition of a contingent claim price. More precisely, I will discuss a change of numeraire technique when the underlying is only a local martingale. Then, the new measure is not necessarily equivalent to the old measure. If one now defines the price of a contingent claim as the minimal superreplicating costs under both measures, then put-call parity holds. I will discuss properties of this new

pricing operator. To illustrate this techniques, I will discuss the class of "Quadratic Normal Volatility" models, which have drawn much attention in the financial industry due to their analytic tractability and flexibility.

This talk is based on joint work with Peter Carr and Travis Fisher.

Risk & Stochastics and Financial Mathematics Joint Seminar in 2013

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9 December - Igor Evstigneev (Manchester)
Modelling Dynamics and Equilibrium of Asset Markets: A Behavioral Approach

Conventional models of dynamic equilibrium in asset markets are based on the principles of General Equilibrium theory (Walras, Arrow, Debreu, Radner and others). This theory in its classical form assumes that market participants are fully rational and their goals can be described in terms of the maximization of utilities subject to budget constraints. The objective of this work is to develop an alternative modelling approach admitting that market participants may have a whole variety of other patterns of behavior determined by their individual psychology, not necessarily reducible to fully rational utility maximization. The models developed do not rely upon restrictive hypotheses (perfect foresight) and avoid using unobservable agents' characteristics such as individual utilities and beliefs, which makes them amenable to quantitative practical applications. The results obtained are concerned with fundamental questions and problems in Financial Economics such as equilibrium asset pricing and portfolio selection. The modelling frameworks combine stochastic dynamic games and evolutionary game theory. The methods employed are based on the stochastic stability analysis of nonlinear random dynamical systems.

Joint work with Rabah Amir (University of Iowa), Thorsten Hens (University of Zurich) and Klaus R. Schenk-Hoppé (University of Leeds).

2 December - Johannes Muhle-Karbe (ETH Zurich)
Optimal Liquidity Provision in Limit Order Markets

A small investor provides liquidity at the best bid and ask prices of a limit order market. For small spreads and frequent orders of other market participants, we explicitly determine the investor's optimal policy and welfare. In doing so, we allow for general dynamics of the mid price, the spread, and the order flow, as well as arbitrary preferences of the liquidity provider under consideration.
Joint work with Christoph Kühn.

25 November - Giorgia Callegaro (Universita Degli Studi di Padova)
An application to credit risk of a hybrid Monte Carlo–optimal quantization method

In this paper, we use a hybrid Monte Carlo-Optimal quantization method to approximate the conditional survival probabilities of a firm, given a structural model for its credit default, under partial information.

We consider the case when the firm's value is a non-observable stochastic process ${(V_t)}_{t \ge 0}$ and investors in the market have access to a process ${(S_t)}_{t \ge 0}$, whose value at each time t is related to $(V_s, 0 \le s \le t)$.
We are interested in the computation of the conditional survival probabilities of the firm given the ``investor's information''.

As an application, we analyze the shape of the credit spread curve for zero coupon bonds in two examples. Calibration to available market data is also analysed.

18 November - Martin Larsson (EPFL)
Polynomial preserving diffusions and models of the term structure

Polynomial preserving processes are multivariate Markov processes that extend the important class of affine processes. They are defined by the property that the semigroup leaves the space of polynomials of degree at most n invariant, for each n, which lends significant tractability to models based on these processes. In this talk I will discuss existence and uniqueness of polynomial preserving diffusions, a task which is made nontrivial due to degenerate and non-Lipschitz diffusion coefficients, as well as a complicated geometric structure of the state space. I will then discuss how polynomial preserving processes can be used to build term structure models that accommodate three features that are otherwise difficult to combine: nonnegative short rates, tractable swaption pricing, and unspanned factors affecting volatility and risk premia.

11 November 2013 -Claudio Fontana (INRIA Paris)
On honest times and arbitrage possibilities

In the context of a general continuous financial market model, we study whether the additional information associated with an honest time T gives rise to arbitrage profits. By relying on the theory of progressive enlargement of filtrations, we explicitly show that arbitrage profits can never be realised strictly before T, while classical arbitrage opportunities can be realised exactly at T as well as after T. Moreover, arbitrages of the first kind can only be obtained by starting to trade as soon as T occurs. We carefully study the behavior of local martingale deflators and consider no-arbitrage-type conditions weaker than no free lunch with vanishing risk. Finally, we discuss extensions of the theory to the case of general semi martingale models.

7 November 2013 - Kostas Kardaras (LSE)
Equilibrium in risk-sharing games

A market is considered with several acting financial agents, whose aim is to increase their utility by efficiently sharing their random endowments. Given the endogenously derived optimal sharing rules, we consider the situation where agents do not reveal their true endowments, but instead report as endowments the random quantities that maximise their utility when the sharing rules are applied. Under exponential utilities (coinciding with entropic risk measures), an analysis of Nash equilibrium is carried out, where it is shown in particular that the optimal contract of each agent possesses endogenous bounds only depending on the agents' risk tolerance, and not on their random endowment. Existence and uniqueness of Nash equilibrium for the 2-player game is obtained. Furthermore, it is discussed how such an equilibrium benefits extremely high risk tolerance agents and results in risk-sharing inefficiency.

Joint work with M. Antropelos.

24 October - Budhi Arta Surya (SBM ITB)
Optimal Capital Structure with Scale Effects under Spectrally Negative Levy Models

The optimal capital structure model with endogenous bankruptcy was first studied by Leland (1994) and Leland and Toft (1996), and was later extended to the spectrally negative Levy model by Hilberink and Rogers (2002) and Kyprianou and Surya (2007). This paper incorporates the scale effects by allowing the values of bankruptcy costs and tax benefits dependent on the firm's asset value. These effects have been empirically shown, among others, in Warner (1976), Ang et al. (1982), and Graham and Smith (1999). By using the fluctuation identities for the spectrally negative Levy process, we obtain a candidate bankruptcy level as well as a sufficient condition for optimality. The optimality holds in particular when, monotonically in the asset value, the value of tax benefits is increasing, the loss amount at bankruptcy is increasing, and its proportion relative to the asset value is decreasing. The solution admits a semi-explicit form, and this allows for instant computation of the optimal bankruptcy levels, equity/debt/firm values and optimal leverage ratios. A series of numerical studies are given to analyze the impacts of scale effects on the default strategy and the optimal capital structure.

Joint work with Kazutoshi Yamazaki

30 May - Mikhail Urusov (Duisburg-Essen)
On The Boundary Behaviour of Diffusions and the Martingale Property of the Associated Local Martingales

Abstract. Let ψ_r denote the increasing r-harmonic function ($r > 0$) associated with a regular one-dimensional diffusion X with the state space $[\alpha, \beta] \subseteq [-\infty, \infty]$. We give several different necessary and sufficient conditions for the local martingale $(e^{-rt}\psi_r(X_t))_{t\geq 0}$ to be a true martingale when the boundary β is inaccessible (if β is accessible, there is nothing to study). One of them is that β should be a natural boundary. In particular, the property of interest does not depend on $r > 0$, and the boundary behaviour of X at α does not play any role. Clearly, the result has its analogue for the decreasing r-harmonic function φ_r. The functions ψ_r and φ_r and their properties are important in optimal stopping. The counterpart of the property of interest for $r = 0$, namely the property of the local martingale $(p(X_{t\wedge\zeta}))_{t\geq 0}$ to be a true martingale (p is the scale function, ζ is the hitting time of $\{\alpha, \beta\}$), turns out to have a different characterisation, where the boundary behaviour of X both at α and at β plays a role.

This is a joint work with Mihail Zervos.

15 May - Michael Schröder (Vrije Universiteit Amsterdam)
Mechanisms for no-arbitrage term-structure modelling, with applications to interest-rates and realized-variance

Suppose that the sentiment is changing in some financial market, or that conditions have changed recently. Examples include volatility levels which are expected to change, or interest-rates expected to be adjusted.
How do we quantify the effects of these changes on derivatives positions?
We will discuss mechanisms for the construction of `no-arbitrage' term structures which enable this; these retain tractability in valuing derivatives and comply with stylized facts like mean-reversion and positivity of rates. This will be illustrated in a paradigm valuation of typical fixed-income derivatives.

20 March - Mikhail Zhitlukhin (Manchester and Steklov Mathematical Institute Moscow)
General Bayesian changepoint detection problems for Brownian motion

We consider changepoint detection problems for Brownian motion in a general Bayesian setting. A changepoint (another name - "disorder") is an unknown moment of time when the drift of an observable Brownian motion changes. The goal is to find a stopping time which is close as possible to the moment of disorder. We solve the problem for a wide class of penalty functions and prior distributions of the moment of disorder. Using the results obtained we also study optimal stopping problems for a Brownian motion and a geometric Brownian motion with a

changepoint. The latter result can be used to model asset price processes with changing trend. In the last part of the talk we apply the method to real financial data and show it gives comparatively good results.

Risk & Stochastics and Financial Mathematics Joint Seminar in 2014

Seminars are listed in reverse chronological order, most recent first.

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27 November - Jan-Henrik Steg (Bielefeld)
Symmetric Equilibria in Stochastic Timing Games

We construct subgame-perfect equilibria with mixed strategies for symmetric stochastic timing games with arbitrary strategic incentives. The strategies are qualitatively different for local first- or second-mover advantages, which we analyze in turn. When there is a local second-mover advantage, the players may conduct a war of attrition with stopping rates that we characterize in terms of the Snell envelope from the general theory of optimal stopping, which is very general but provides a clear interpretation. With a local first-mover advantage, stopping typically results from preemption and is abrupt. Equilibria may differ in the degree of preemption, precisely at which points it is triggered. We provide an algorithm to characterize where preemption is inevitable and to establish the existence of corresponding payoff-maximal symmetric equilibria.

13 November - Igor Makarov (LSE)
Marking-to-market and price impact

The paper studies incentives and trading decisions of money managers who trade in markets with price impact. I show that in markets with price impact the practice of marking-to-market funds' assets creates incentives for managers to accumulate excessively large positions. This trading behaviour may force prices away from their fundamental levels for a long time and may result in large losses for investors.

30 October – Samuel Drapeau (Humboldt Universität zu Berlin)
Numerical representation of convex preferences on Anscombe–Aumann acts

We study the preferences of agents for diversification and better outcomes when they are facing both, in Frank Knight's formulation, measurable as well as unmeasurable uncertainty.

Following Anscombe and Aumann, such a situation can be modeled by preferences expressed on stochastic kernels, that is scenario dependent lotteries.
By means of automatic continuity methods based on Banach-Dieudonné's Theorem on Fréchet spaces, we provide a robust representation.

This gives us some insight into the nature of uncertainty aversion these preferences are expressing.

We further investigate under which conditions these two intricate dimensions of uncertainty can be disentangle into a distributional uncertainty, in the direction of von Neumann and Morgenstern's theory, and a probability model uncertainty, in the spirit of risk measures.

These results allow in particular to address both Allais as well as Elsberg's paradox. Joint work with P. Cheridito, F. Delbaen, and M. Kupper.

16 October - Scott Robertson (Carnegie Mellon)
Indifference pricing for contingent claims: large deviations effects

In this talk, we consider utility indifference prices and optimal purchasing quantities for a non-traded contingent claim in an incomplete semi-martingale market with vanishing hedging errors, making connections with the theory of large deviations. This work is motivated by the recent explosive growth in the derivatives market; in particular we seek to explain why such positions are being taken and what the effects are in terms of pricing. To make the analysis tractable, we concentrate on sequences of semi-complete markets where for each n the claim h_n admits the decomposition h_n = D_n+Y_n where D_n is replicable and Y_n is completely unhedgeable in that the indifference price of Y_n for an exponential investor is its certainty equivalent. Under broad conditions, we may assume that Y_n vanishes in accordance with a large deviations principle as n grows. In this setting, we identify limiting indifference prices as the position size becomes large, and show the prices typically are not the unique arbitrage free price in the limiting market. Furthermore, we show that optimal purchase quantities occur at the large deviations scaling, and hence large positions endogenously arise in this setting.

Joint work with Konstantinos Spiliopoulos, Boston University.

19 May - Elena Boguslavskaya (Brunel)
An effective method to solve optimal stopping problems for Lévy processes in infinite horizon or how to avoid differential or integro-differential equations while solving an optimal stopping problem for a Lévy problem

We present a method to solve optimal stopping problems in infinite horizon for a Levy process when the reward function g can be non-monotone.

To solve the problem we introduce two new objects. Firstly, we define a random variable $\eta(x)$ which corresponds to the $\argmax$ of the reward function. Secondly, we propose a certain integral transform which can be built on any suitable random variable. It turns out that this integral transform constructed from

$\eta(x)$ and applied to the reward function produces an easy and straightforward description of the optimal stopping rule. We illustrate our results with several examples.

The method we propose allows to avoid complicated differential or integro-differential equations which arise if the standard methodology is used.

15 May - Michael Schroeder (VU Amsterdam)
Volatility smiles and derivatives: a direct route to new kinds of high-dimensionality?

FX-rates provide the textbook example for financial instruments that are not just traded in a 24/7 fashion but quoted in real time over the entire year. FX-derivatives are similar in their liquidity, but completely different in character. FX-options, for example, are famously liquidly traded at at most 5 strikes and 10 maturities; their design is subordinated to the daily 4pm-GMT-fixings. We will outline methods to handle such institutional discetenesses in model-based approaches to derivatives. Highlighting recent stochastic volatility models of Ornstein-Uhlenbeck type, this will include methods for model calibration based on discretely-sampled generalized BS-formulas. We will also report on exact valuation methods for discretely-sampled Asian options (as part of an LSE-project with M.Frentz (Bank of Sweden)).

17 March - Sergio Pulido (EPFL)
Markovian cubature rules for polynomial preserving processes

Polynomial preserving processes are defined as time-homogeneous Markov jump-diffusions whose generator leaves the space of polynomials of any fixed degree invariant. Polynomial preserving processes include affine processes, whose transition functions admit an exponential-affine characteristic function. These processes are attractive for financial modeling because of their tractability and robustness. In this work we study Markovian cubature rules for polynomial preserving processes. These rules aim to exploit the defining property of polynomial preserving processes in order to reduce the complexity of the implementation of such models. More precisely, we study conditions guaranteeing the existence of finite-state Markov processes that match the moments of a given polynomial preserving process. The states of these processes together with their transition probabilities can be interpreted as Markovian cubature rules. We first give a characterization theorem for the existence of Markovian cubature rules in continuous time. This theorem illustrates the complexity of the problem by combining algebraic and geometric considerations. We show that for polynomial preserving diffusions, there are no continuous-time Markovian cubature rules for high order moments. We provide a positive result by showing that the construction is possible when one considers finite-state Markov chains on lifted versions of the state space. Additionally, by relaxing the continuous-time cubature problem, we can

construct discrete time finite-state Markov processes that match moments of arbitrary order. This discrete time construction relies on the existence of long-run moments for the polynomial process and cubature rules over these moments.

This is joint work with Damir Filipovic and Martin Larsson.

10 March - Giorgio Ferrari (Bielefeld)
A Stochastic Partially Reversible Investment Problem on a Finite Time-Horizon: Free-Boundary Analysis

We study a continuous-time, finite horizon, stochastic partially reversible investment problem for a firm producing a single good in a market with frictions. The production capacity is modelled as a one-dimensional, time-homogeneous, linear diffusion controlled by a bounded variation process which represents the cumulative investment-disinvestment strategy. We associate to the investment-disinvestment problem a zero-sum optimal stopping game and characterize its value function through a free-boundary problem with two moving boundaries. These are continuous, bounded and monotone curves that solve a system of non-linear integral equations of Volterra type. The optimal investment-disinvestment strategy is then shown to be a diffusion reflected at the two boundaries.

This is joint work with Tiziano De Angelis (University of Manchester)

3 March - Yu-Jui Huang (Dublin City)
Model-independent Superhedging under Portfolio Constraints

In a discrete-time market, we study the problem of model-independent superhedging of exotic options under portfolio constraints. The superhedging portfolio consists of static positions in liquidly traded vanilla options, and a dynamic trading strategy, subject to certain constraints, on the risky asset. By the theory of Monge-Kantorovich optimal transport, we establish a superhedging duality, which admits a natural connection to convex risk measures. With the aid of this duality, we derive a model-independent version of the fundamental theorem of asset pricing under portfolio constraints. It is worth noting that our method covers a large class of Delta constraints as well as Gamma constraint.

17 February - Antoine Jacquier (Imperial)
Asymptotics of forward implied volatility

We study the asymptotic behaviour of the forward implied volatility (namely the implied volatility corresponding to forward-start European options). Our tools rely on (finite-dimensional) large deviations and saddlepoint analysis, albeit not necessarily relying on standard convexity arguments. We shall also relate this to the Freidlin-Wentzell approach for sample paths. From a practical point of view, this

sheds light on the dynamics of forward implied volatilities, which we highlight numerically in the Heston model.

10 February - Anna Aksamit (Université d'Evry Val d'Essonne)
Optional semimartingale decomposition and non-arbitrage condition in enlarged filtration

Our study addresses the question of how an arbitrage-free semimartingale model is affected when stopped at a random horizon or when a random variable satisfying Jacod's hypothesis is incorporated. Precisely, we focus on the No-Unbounded-Profit-with-Bounded-Risk condition, which is also known in the literature as the first kind of non-arbitrage. In the general semimartingale setting, we provide a necessary and sufficient condition on the random time for which the non-arbitrage is preserved for any process. Analogous result is formulated for initial enlargement with random variable satisfying Jacod's hypothesis. Moreover we give an answer to a stability of non-arbitrage question for fixed process. The crucial intermediate results in enlargement of filtration theory are obtained. For local martingales from the reference filtration we provide special optional semimartingale decomposition up to random time and in initially enlarged filtration under Jacod's hypothesis. An interesting link to absolutlety continuous change of measure problem is observed.

5 February - Walter Schachermayer (Vienna)
Duality Theory for Portfolio Optimisation under Transaction Costs

In this talk, we develop a dynamic duality theory for portfolio optimisation under proportional transaction costs with cadlag price processes. In particular, we provide examples that illustrate the new effects arising from the combination of the transaction costs and jumps of the underlying price process.

The talk is based on joint work with Christoph Czichowsky.

3 February - Claude Martini (Zeliade Systems)
Calibration of the SSVI model and applications to model free option pricing bounds

Gatheral and Jacquier achieved in 2012 a consistent (arbitrage-free) extension of the parametric SVI model in the maturity dimension. This Surface SVI (SSVI) model is parameterized by a correlation coefficient and a function which corresponds to the curvature of the smile at each maturity. We go through a re-parameterization of the SSVI model that lends itself to a nice 2-stages calibration procedure. Calibration examples on CBOE SPX delayed quotes are provided. Since the SSVI model does calibrate very well, we eventually get an explicit arbitrage-free parameterization of the market implied volatility surface. We compute the model-free Beiglböck-Juillet-Touzi-Henry Labordère optimal transport bounds of an exotic option in this setting. An executable version of this work is available on the Zanadu platform (joint work

with I.Laachir, ENSTA and Zeliade Systems). (Keywords: implied volatility, SVI, calibration, Optimal Transport option bounds).

27 January - René Aid (EDF)
A high-dimensional investment model in electricity generation

In this talk, we will show how the progresses made in the recent decade in numerical methods for optimal stopping time problems and optimal switching problems allow to design efficient and yet realistic models to study the dynamic of investment in electricity generation. We will give the example of a high-dimensional electricity generation investment model. This model takes into account electricity demand, cointegrated fuel prices, carbon price and random outages of power plants. The evolution of the optimal generation mix is illustrated on a realistic numerical problem in dimension 8, i.e. with 2 different technologies and 6 random processes.

This talk is based on a joint work with Luciano Campi, Nicolas Langrene and Huyen Pham.

20 January - Hansjoerg Albrecher (HEC Lausanne)
On theoretical and practical aspects of catastrophe insurance

In this talk we give an overview of some recent results and developments in the modelling of insurance risk related to natural catastrophes. In addition to some theoretical results on the statistics of such extremal events, we present a study of flood and storm risk in Austria. The feasibility of the general principle of time diversification in this context is also discussed.

13 January - Agostino Capponi (John Hopkins)
Optimal Investment in Credit Derivatives Portfolio under Contagion Risk

We consider the optimal portfolio problem of a power investor who wishes to allocate her wealth between several credit default swaps (CDSs) and a money market account. We model contagion risk among the reference entities in the portfolio using a reduced form Markovian model with interacting default intensities. Using the dynamic programming principle, we establish a lattice dependence structure between the Hamiltonian-Jacobi-Bellman equations associated with the default states of the portfolio.

We show existence and uniqueness of a classical solution to each equation and characterize them in terms of solutions to inhomogeneous Bernoulli's type ODEs. We provide a precise characterization for the directionality of the CDS investment strategy and perform a numerical analysis to assess the impact of default contagion. We find that the increased intensity triggered by default of a very risky

entity strongly impacts size and directionality of the investor strategy. Such findings outline the key role played by default contagion when investing in portfolios subject to multiple sources of default risk

Risk & Stochastics and Financial Mathematics Joint Seminar in 2015

Seminars are listed in reverse chronological order, most recent first.

Thursday 3 December - Kristoffer Glover (University of Technology, Sydney)
The optimal time to close an open-ended mutual fund

In this talk I will attempt to shed some light onto two intimately linked questions about empirically observed mutual fund behaviour: (1) Why do open-ended mutual funds decide to close their doors to new investors? and (2) Why do these funds underperform after closing? A theoretical model for the optimal closure of an open-ended mutual fund is developed in which the fund is subject to performance sensitive fund flows and a decreasing return-to-scale on its investment portfolio. Such funds are found to optimally close 'too late' from the perspective of the fund investors, since the optimal fund size that maximises the fund manager's expected fee income is larger than the size at which the fund's 'alpha' is expected to becomes negative. In other words, the fund closes at a point where the decreasing returns-to-scale have already started to negatively affect fund performance; thus explaining the empirically observed underperformance after closure. Additional empirical predictions generated from the model will also be discussed.

19 November - Rémy Praz (Copenhagen Business School)
Equilibrium asset pricing with both liquid and illiquid markets

I study a general equilibrium model in which investors face endowment risk and trade two correlated assets; one asset is traded on a liquid market whereas the other is traded on an illiquid over-the-counter (OTC) market. Endowment shocks not only make prices drop, they also make the OTC asset more difficult to sell, creating an endogenous liquidity risk. This liquidity risk increases the risk premium of both the OTC asset and liquid asset. Furthermore, the OTC market frictions increase the trading volume and the cross-sectional dispersion of ownership in the liquid market. Finally, if the economy starts with only the OTC market, then I explain how opening a correlated liquid market can increase or decrease the OTC price depending on the illiquidity level. The model's predictions can help explain several empirical findings.

5 November - Yaroslav Melnyk (Swiss Financial Institute @ École polytechnique fédérale de Lausanne)
Portfolio Optimization with Recursive Utility under Small Transaction Costs

In this article we investigate the portfolio problem of an investor with Epstein-Zin recursive utility under proportional transaction costs. We characterize the solution via variational inequalities and prove existence of classical solutions for small cost parameters. We also provide a suitable verification theorem. This allows us to derive rigorous asymptotic expansions for optimal no-trading regions and consumption strategies and to investigate the effects of the investor's relative risk aversion and the elasticity of intertemporal substituion (EIS) ψ on the optimal strategies. Our main findings are: (a) At the leading order, the no-trading region is the same as with expected additive utility; in particular, it is determined solely by the relative risk aversion. The no-trading region depends on the investor's EIS only at the next-to-leading order, and only indirectly (via optimal consumption). (b) The investor's optimal consumption depends on his EIS also at the leading order. The optimal consumption rate is higher, as a percentage of current wealth, than in the frictionless case if and only if $\psi>1$.

Based on joint work with Johannes Muhle-Karbe and Frank Thomas Seifried.

22 October - Johannes Ruf (University College London and an Associate Member at the Oxford-Man Institute of Quantitative Finance)
Föllmer's Measure and Novikov/Kazamaki-Type Conditions

In the first part of the talk, I will discuss the construction of Föllmer's measure on the canonical path space. In the second part, I will discuss sharpened Novikov/Kazamaki-type conditions that provide sufficient and necessary conditions for the martingale property of a nonnegative local martingale.

This talk is based on joined papers with Nicolas Perkowski and Martin Larsson.

8 October - Sebastian Herrmann (ETH Zürich)
Hedging with Small Uncertainty Aversion

We study the pricing and hedging of derivative securities with uncertainty about the volatility of the underlying asset. Rather than taking all models from a prespecified class equally seriously, we penalise less plausible ones based on their "distance" to a reference local volatility model. In the limit for small uncertainty aversion, this leads to explicit formulas for prices and hedging strategies in terms of the security's cash gamma.

6 October - Nevroz Sen (McGill)
Estimation theory for non-linear major-minor mean field games

In the Mean Field Games (MFG) framework where there is an agent (so-called Major) which has asymptotically non vanishing influence on any other Minor agent, the best response control process of each Minor agent depends upon its own state, the Major agent's state and the conditional distribution of the generic minor agent, namely the system's stochastic mean field; this is in contrast to the basic MFG setup where the mean field is deterministic. The theory of MFG with a Major agent (MM-MFG) is well understood when the observations of the Minor agents are complete.

In this talk we analyze the non-linear MM-MFG problem where each Minor agent partially observes the Major agent's state. We employ non-linear filtering theory derived for McKean-Vlasov type state equations and the Separation Principle in order to analyze the game in the infinite population limit. The main results are the existence and uniqueness of the solutions to the stochastic MFG system equations and the epsilon-Nash equilibrium property where the best response control process of each Minor agent depends upon the conditional density generated by that agent's non-linear filter together with the system's mean field and its own state.

Joint work with Peter E. Caines

28 May - Martin Herdegen (ETH Zürich)
Sensitivity of optimal consumption streams

We study the sensitivity of optimal consumption streams with respect to perturbations of the random endowment. We show that to the leading order, any consumption correction for the perturbed endowment is still optimal as long as the budget constraint is binding. More importantly, we also establish the optimal correction at the next-to leading order. This can be computed in two steps. First, one has to find the optimal correction for a deterministic perturbation. This only involves the risk-tolerance process of the unperturbed problem and yields a risk-tolerance martingale and a corresponding risk-tolerance measure. If the risk-tolerance process and the interest rate are deterministic, the latter is constant. In a second step, one can then calculate the optimal correction for any random perturbation. This is given by an explicit formula whose key ingredients are the conditional expectations of the terminal cumulative perturbation and the integrated risk-tolerance process under the risk-tolerance measure.
Joint work with Johannes Muhle-Karbe.

19 March - Dylan Possamaï (CMAP École polytechnique)
A primer on Principal/Agent models and their recent extensions

We will present the main ideas and intuitions behind the modelization of the so-called principal/agent models, which are at the heart of the contracting theory. The theory emerged in the 70s from the acknowledgment that almost everything in economics was to a certain degree a matter of incentives (incentives to work hard, to produce, to study, to invest, to consume reasonably…) and the fact that such situations could not be reproduced using the general equilibrium theory. While a great number of studies were devoted to quite comprehensive models in discrete time, their continuous-time counterparts have only recently received a lot attention from the economics literature, starting with the breakthrough works of Holmstrom and Milgrom or later on of Sannikov. We will review the modelization of these problems, both in the cases of moral-hazard and adverse selection and see the type of mathematical tools that can be used to treat them. Moreover, if time permits, we will try to look at some recent generalizations of the theory, as well as still open problems.

5 March - Fausto Gozzi (Luiss University)
Impact of time illiquidity in a mixed market without full observation

In this talk we present and study a class of optimal portfolio problems in a two-assets market where one asset is illiquid in the sense that it can be traded only at given random times (of exponential law) and it cannot be fully observed. This feature arises in many cases in real markets and it clearly modifies the optimal policies with respect to the benchmark given by the standard Merton model. We first recall the Merton model, then introduce a model of Pham and Tankov, where only one illiquid asset is present: we show how to solve this model by the dynamic programming approach. Then we consider the more difficult case of two correlated assets with partial observation and show how the dynamic programming approach also applies to this case in a satisfactory way. We also give some numerical experiment to evaluate the impact of the illiquidity and of the lack of full observation.

26 February - Anastasia Ellanskaya (LAREMA, Departement de Mathematiques, Universite d'Angers)
Utility Maximisation and Utility Indifference Price for Exponential Semi-martingale Models and HARA Utilities

We consider the utility maximisation problem for semi-martingale models and HARA (hyperbolic absolute risk aversion) utilities. Using specific properties of HARA utilities, we reduce the initial maximisation problem to the conditional one, which we solve by applying a dual approach. Then we express the solution of the conditional maximisation problem via conditional information quantities related to

HARA utilities, like the Kullback–Leibler information and Hellinger-type integrals. In turn, we express the information quantities in terms of information processes, which is helpful in indifference price calculus. Finally, we give equations for indifference prices. We apply the results to Black–Scholes model with correlated Brownian motions, jump-diffusion model and Lévy model and give an explicit expression for information quantities. Then the previous formulas for the indifference price can be applied.

19 February - Pietro Siorpaes (Vienna)
Optimal investment and price dependence in a semi-static market

We study the problem of maximizing expected utility from terminal wealth in a semi-static market composed of derivative securities, which we assume can be traded only at time zero, and of stocks, which can be traded continuously in time. Using a general utility function defined on the positive real line, we first study existence and uniqueness of the solution, and then we consider the dependence of the outputs of the utility maximization problem on the price of the derivatives, investigating not only stability but also differentiability, monotonicity, convexity and limiting properties.

5 February - Michael Schmutz (Bern and Swiss Financial Market Supervisory Authority (FINMA))
Challenges in risk based solvency frameworks

Risk-based solvency frameworks such as Solvency II to be introduced in the EU or the Swiss Solvency Test (SST) in force since 2011 in Switzerland seek to assess the financial health of insurance companies by quantifying the capital adequacy through calculating the solvency capital requirement (SCR). Companies can use their own economic capital models (internal models) for this calculation, provided the internal model is approved by the insurance supervisor. The Swiss supervisor has essentially completed the first round of internal model approvals. This has provided the supervisor and the industry with many insights into the challenges of designing, assessing, and supervising such models and has shown that there is a considerable number of challenges, in particular modelling challenges, that have not yet been solved in a completely satisfactory way. Some of the most important challenges and problems will be discussed along with some approaches to solutions.

22 January - Georgy Chabakauri (LSE)
Multi-Asset Noisy Rational Expectations Equilibrium with Contingent Claims

We consider a noisy rational expectations equilibrium in a multi-asset economy populated by informed and uninformed investors, and noise traders. Informed investors privately observe an aggregate risk factor affecting the probabilities of

different states of the economy. Uninformed investors attempt to extract that information from asset prices, but full revelation is prevented by noise traders. We relax the usual assumption of normally distributed asset payoffs and allow for assets with more general payoff distributions, including contingent claims, such as options and other derivatives. We show that assets reveal information about the risk factor only if they help span the exposure of probabilities of states to the risk factor. When the market is complete, we provide equilibrium asset prices and optimal portfolios of investors in closed form. In incomplete markets, we derive prices and portfolios in terms of easily computable inverse functions.

Joint work with Kathy Yuan (LSE) and Konstantinos E. Zachariadis (LSE)