Mini-Conference on Infinite Combinatorics 2012

LSE Mathematics Department 25th July 2012

Conference theme


Hugo Steinhaus

Steinhaus's sum-theorem for `large' sets is at the heart of several strands of infinite combinatorics and of related finitary combinatorics, particularly Fraïssé direct and inverse limits. These strands include additive combinatorics, e.g. Ruziewicz-van der Waerden theorems, automatic continuity theorems (for measurable homomorphisms), and reduction of continuous to sequential properties. Probability theory is an end-user (e.g. regular variation, Kingman's discrete skeleton and extreme value theories). Generalizations of Steinhaus' result range from continuous groups to discrete semigroups involving notions such as `amenability at 1', idempotent ultrafilters, and recently shift-compactness of group actions and refinement topologies. (The latter notions may be viewed as strengthened-form Baire Category theorems and can explain en route some of the measure-category duality via density topologies.) A model-theoretic approach to group action relies on generic and mutually generic automorphisms.

This Mini-Conference in Infinite Combinatorics was aimed at bringing together mathematicians researching in fields where a common underpinning may be seen to be a theorem of Stainhaus. The organiser (Prof. Adam Ostaszewski) came to this field several years ago, motivated by research questions on regular variation arising in Probability Theory, and established the connection with several fields: infinite combinatorics (Ramsey Theory), general topology, group theory and descriptive set theory. Speakers were invited to represent these different aspects and the organiser was very pleased to create a platform for further research.

Support from the London Mathematical Society and from the British Combinatorial Committee is gratefully acknowledged.

Where possible, slides and recordings of the day's presentations are made available below.

Speakers

Nick Bingham, Imperial College / LSE
Title: Topological Regular Variation
Abstract: This talk aims to motivate first the classical theory of regular variation (see e.g. the book of that title, Bingham-Goldie-Teugels, 1987/89), and its more recent topological counterpart (see e.g. the 17 Bingham-Ostaszewski papers, 2008-11, all on MathSciNet, plus Ostaszewski papers, most on Math-SciNet). The contents are illustrated by the section headings:
1. Why regular variation?
2. Extreme-value theory (EVT)
3. Regular variation; BGT
4. Bitopology [Euclidean and density topologies]
5. Measure-category duality
6. Analytic sets
7. Groups and actions
8. Innite dombinatorics
9. Logical assumptions

Adam Ostaszewski, LSE

Title: Steinhaus' Theorem and its descendants

Abstract: The classical real-line Steinhaus sum-theorem that A+A contains an open set when A is large in the sense of measure or category, has generated a corpus of work which will be recalled and continues to inspire, among them results on automatic continuity in group theory and open mapping principles in analysis. Such results may be deduced from the notion of shift-compactness associated with group action. In the presence of almost completeness this property follows from the separation of points from closed nowhere dense sets by appropriate group members. Variant forms of separation may be deduced by passing to a refinement topology; thus the density topology permits measure results to follow from category results. Analyticity provides a useful mild form of completeness. Mention will be made of the relationship between this approach and the use of generic automorphisms in the sense of Truss.

Dona Strauss, Hull

Title: Chains of idempotents in beta-N (Joint work with N. Hindman and Y. Zelenyuk)

Abstract: The properties of idempotents in βS play a significant role in combinatorics. They have often provided surprisingly short proofs of important theorems - Hindman's Theorem and the Hales-Jewett Theorem are examples - and they have suggested new theorems. I shall describe what we know about idempotents in βN and talk about a new result obtained by N. Hindman, Y. Zelenyuk and myself about the existence of decreasing chains of idempotents in βN. We showed that, for every non-minimal idempotent p in βN and every countable ordinal α, there is a decreasing chain of idempotents in βN indexed by α, with p as the maximum element. Whether there are any uncountable chains of idempotents in βN is an open question.

An audio recording (with accompanying screenshots) of this presentation is available at **http://richmedia.lse.ac.uk/maths/20121107_SlawomirSolecki.mp4**.

Sławomir Solecki, University of Illinois at Urbana-Champaign

Title: An abstract approach to Ramsey theory with applications

Abstract: We give an abstract approach to finite Ramsey theory and prove a general Ramsey-type theorem. We deduce from it a self-dual Ramsey theorem, which is a new result naturally generalizing both the classical Ramsey theorem and the dual Ramsey theorem of Graham and Rothschild. In fact, we recover the pure finite Ramsey theory from our general Ramsey-type result in the sense that the classical Ramsey theorem, the Hales-Jewett theorem (with Shelah's bounds), the Graham-Rothschild theorem, the versions of these results for partial rigid surjections due to Voigt, and the new self-dual Ramsey theorem are all obtained as iterative applications of the general result.

An audio recording (with accompanying screenshots) of this presentation is available at ***http://richmedia.lse.ac.uk/maths/20121107_SlawomirSolecki.mp4***.

Imre Leader, Cambridge

Title: Sparse Pairwise Sums (no slides available)

Abstract: Hindman's celebrated theorem states that, whenever the natural numbers are finitely coloured, some colour class contains all the finite sums from a sequence. What would a `sparse' version of this be? One obvious question would be: does there exist a subset S of the natural numbers that is so rich that whenever it is finitely coloured then some colour class contains all the singleton and pairwise sums from a sequence, and yet so sparse that it does not contain all the finite sums from any sequence?

This is the `simplest possible' sparse infinitary question, and it has remained open through the years. Amusingly, a published conjecture of Nesetril and Rodl would imply that it is true while a published conjecture of Hindman would imply that it is false.

In this talk, we will look at some background and recent developments on this problem.

An audio recording of this presentation is available at **http://richmedia.lse.ac.uk/maths/20121107_ImreLeader.mp3*.***