Department of Mathematics

Seminar on Combinatorics, Games and Optimisation in 2015/16

Seminars are listed in reverse chronological order, most recent first

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15 June - Marc Renault (Paris)

The bijective ratio of online algorithms
Bijective analysis is an intuitive technique for evaluating the performance of online algorithms that is based on pairwise comparison of the costs incurred by two algorithms on sets of request sequences of the same size. Despite its success in providing a clear separation between algorithms for problems such as paging and list update, bijective analysis is not readily applicable to all online problems and algorithms since it stipulates a very strong relation between the compared algorithms that either may be difficult to establish or, worse, may not even exist.

In this work, we address these two deficiencies of bijective analysis. First, we generalize previous techniques that allow us to show optimality of certain greedy- like algorithms for a much wider class of online problems. Second, to account for situations in which an online algorithm is not bijectively optimal, we introduce the bijective ratio as a natural extension of exact bijective analysis. We demonstrate the applicability of the bijective ratio to one of the canonical online problems, namely the continuous k-server problem on metrics such as the line, the circle, and the star. Among our results, we show that the greedy algorithm attains bijective ratios O(k) consistently across these metrics; this is in stark contrast to competitive analysis, according to which it has an unbounded competitive ratio, even for the line.

This is a joint work with Spyros Angelopoulos and Pascal Schweitzer.

9 June - Gergely Ambrus (British Columbia/Alfréd Rényi Institute of Mathematics)

Vector sum estimates in normed spaces
Consider a set V of n vectors in a d-dimensional real normed space whose norm is at most one. Assume that the vectors sum to 0. We consider several questions.

First, what is the best possible bound S so that there always exists an ordering of the elements of V, according to which the initial partial sums have norms bounded by S? This question is due to Steinitz. Second, what is the best bound R so that for any given k<n, one choose a subset of V with cardinality k, so that the sum of the elements of this subset has norm at most R? Somewhat surprisingly, there exist such bounds depending only on d, but not on n. We are going to use linear algebraic methods for giving estimates, which are sharp in some cases.

This is a joint work with I. Barany and V. Grinberg.

2 June - János Pach (EPFL)

Chromatic number vs. clique number
Given a set S of geometric objects, their disjointness graph is the graph on the vertex set S, in which two vertices are connected by an edge if and only if they are disjoint. It is shown, among other things, that the chromatic number of the disjointness graph of segments in d-dimensional space is bounded from above by the 4-th power of its clique number. Joint work with G. Tardos and G. Toth.

31 May - Jon Lee (Michigan)

Relaxing efficiently and kindly
In the context of global optimization, "spatial branch and bound" is the workhorse general-purpose algorithm for so-called factorable formulations. I will present some results on making a key algorithmic aspect efficient, and a way to handle some limited type of non-differentiability. The first involves calculating some volume formulae for certain parametric families of low-dimensional polytopes, and surprisingly needs arguments about continuity and determining when various multivariate polynomials are nonnegative. The second starts with calculus and curve fitting, and in the end looks at when certain univariate polynomials are nonnegative.

27 May - Dömötör Pálvölgyi (Eötvös /Cambridge)

Polychromatic coloring and cover-decomposition problems in the plane
Is it true that given a finite point set on a sphere and a set of halfspheres, such that the set system that they induce on the point set is a Sperner family, we can select a subset of the points that meet every halfsphere in at least one but at most two points?

I don't know the answer to this question (waiting to be solved by YOU!), but I know that the above holds in the plane if instead of halfspheres we take (pseudo)halfplanes.

I will talk about consequences of similar results in polychromatic coloring and cover-decomposition, and also mention several other open problems.

9 May - Jochen Koenemann (Waterloo/Bonn)

Network Bargaining - Where Bargaining & Matching Theory Meet
Bargaining is a central topic of study in economics and in the social sciences. In the most basic setting, two agents A & B negotiate how to split a dollar. Assume the agents have monetary outside options a & b, respectively, that they receive, should negotiations fail; Nash's famous bargaining solution postulates that in an equilibrium, the agents each receive their outside options, and that the remainder is split evenly; i.e., the agents receive x_A and x_B such that x_A-a=x_B-b.

In this talk we will look at a natural generalization of Nash bargaining to agents interacting in social networks. We will present a natural equilibrium concept extending Nash's condition, and present Kleinberg & Tardos' recent characterization of graphs that admit equilibria. We will present connections between bargaining theory, cooperative games, and matching

theory and use these to derive elegant algorithms for the computation of equilibria. We will also discuss several ways in which unstable instances of network bargaining can be stabilized.

27 April - Neil Olver (VU Amsterdam)

On integrality gaps for Steiner problems
The Steiner tree, Steiner forest, and related problems are among the most classical in network design. I will discuss two results related to natural cut-based relaxations for such problems.

1) The integrality gap of the bidirected cut relaxation for the Steiner tree problem is suspected to be better than 2, but this has remained open for many years. I will discuss how better bounds can (constructively) be obtained for a certain class of instances, via a connection with a much stronger "hypergraphic" LP. (Joint work with A. Feldmann, J. Koenemann and L. Sanita.)

2) We show that the natural cut LP for the prize-collecting Steiner forest problem has an integrality gap strictly worse than 2. This is in contrast to the situation for prize-collecting Steiner tree as well as (non-prize collecting) Steiner forest. (Joint work with J. Koenemann, R. Ravi, G. Schaefer and C. Swamy.)