

# PhD Seminar on Combinatorics, Games and Optimisation in 2018

Seminars are listed in reverse chronological order, most recent first

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Friday 14 December - [Bento Natura](#) (LSE)

## The Shallow-Light Steiner Tree Problem

In a central step in the design of modern VLSI-chips, millions of pins have to be connected. The scale of this task requires fast algorithms with provably good performance.

The problem can be approximated by the Shallow-Light Steiner Arborescence problem, in which we minimize the total length of a Steiner Arborescence that propagates a signal from a source  $s$  to a set of sinks  $T$ , such that the delay on the paths obeys given constraints. In our model, the delay depends on the length of the path as well as on the number of edges on the path.

We present algorithms and inapproximability results for the general case and based on this a quasilinear time algorithm with improved approximation guarantee for the special case, where the metric space is  $(\mathbb{R}^2, l^1)$ .

Friday 30 November - [Benny Sudakov](#) (ETH)

## Subgraph statistics

Consider integers  $k, \ell$  such that  $0 \leq \ell \leq \binom{k}{2}$ . Given a large graph  $G$ , what is the fraction of  $k$ -vertex subsets of  $G$  which span exactly  $\ell$  edges? When  $G$  is empty or complete, and  $\ell$  is zero or  $\binom{k}{2}$ , this fraction can be exactly 1. On the other hand if  $\ell$  is not one of these extreme values, then by Ramsey's theorem, this fraction is strictly smaller than 1.

The systematic study of the above question was recently initiated by Alon, Hefetz, Krivelevich and Tyomkyn who proposed several natural conjectures. In this talk we discuss a theorem which proves one of their conjectures and implies an

Asymptotic version of another. We also make some first steps towards analogous question for hypergraphs. Our proofs involve some Ramsey-type arguments, and a number of different probabilistic tools, such as polynomial anticoncentration inequalities and hypercontractivity.

Joint work with M. Kwan and T. Tran

## Friday 23 November - Zaniar Ghadernezhad (Imperial College London) Part II

### Convex Ramsey property and non-amenability of automorphism groups of generic structures

In this talk we start by presenting some correspondences between (extreme) amenability of automorphism groups of Fraïssé–Hrushovski generic structures, and Ramsey type properties of their smooth classes, similar to results of Kechris, Pestov and Todorcevic (2005), and Moore (2013). Then we focus on the combinatorial part and especially on the convex Ramsey property. The convex Ramsey property can be translated to some conditions on matrices. We will then consider automorphism groups of certain Hrushovski's generic graphs and show that they are not amenable using certain matrices by exhibiting a combinatorial/geometrical criterion which forbids amenability.

## Friday 16 November - Rachel Kirsch (LSE)

### Many cliques with few edges

The problem of maximizing the number of cliques has been studied within several classes of graphs. For example, among graphs on  $n$  vertices with clique number at most  $r$ , the Turán graph  $T_r(n)$  maximizes the number of copies of  $K_t$  for each size  $t$ . Among graphs on  $m$  edges, the colex graph  $\mathcal{C}(m)$  maximizes the number of  $K_t$ 's for each size  $t$ .

In recent years, much progress has been made on the problem of maximizing the number of cliques among graphs with  $n$  vertices and maximum degree at most  $r$ . The graph  $aK_{r+1} \cup bK_r$ , where  $n = a(r+1) + b$  and  $0 \leq b \leq r$ , was shown to maximize the total number of cliques, and is conjectured to maximize the number of  $K_t$ 's for  $t \geq 3$ . This conjecture has been proven in significant cases.

In this talk, we discuss the edge analogue of this problem: which graphs with  $m$

edges and maximum degree at most  $r$  have the maximum number of cliques? We prove in some cases that the extremal graphs again contain as many disjoint copies of  $K_{r+1}$  as can fit, with the leftovers in another component. In the edge analogue, these remaining edges form a colex graph.

## Friday 9 November - Zaniar Ghadernezhad (Imperial College London) Part I

### Convex Ramsey property and non-amenability of automorphism groups of generic structures

In this talk we start by presenting some correspondences between (extreme) amenability of automorphism groups of Fraïssé–Hrushovski generic structures, and Ramsey type properties of their smooth classes, similar to results of Kechris, Pestov and Todorcevic (2005), and Moore (2013). Then we focus on the combinatorial part and especially on the convex Ramsey property. The convex Ramsey property can be translated to some conditions on matrices. We will then consider automorphism groups of certain Hrushovski’s generic graphs and show that they are not amenable using certain matrices by exhibiting a combinatorial/geometrical criterion which forbids amenability

## Friday 12 October - Patrick Morris (FU Berlin)

Venue: 32L.B.09 from 12:00 - 13:00

### Finding any given 2-factor in sparse pseudorandom graphs

Given an  $n$ -vertex pseudorandom graph  $G$  and a potential 2-factor  $H$  (that is, a union of disjoint cycles whose lengths sum to  $n$ ), we wish to find a copy of  $H$  in  $G$ , i.e. an embedding  $\varphi: V(H) \rightarrow V(G)$  so that  $\varphi(u)\varphi(v) \in E(G)$  for all  $uv \in E(H)$ . Particular instances of this problem include finding a triangle-factor and finding a Hamilton cycle in  $G$ .

In this talk, after giving the relevant context and history in this field, we will sketch how to find a given  $H$  in any suitably pseudorandom graph  $G$ . The graphs we will consider will be  $(n, d, \lambda)$  graphs i.e.  $n$  vertex  $d$ -regular graphs whose second eigenvalue in absolute value is  $\lambda$ .

Our condition  $\lambda = O(d^2/(n \log n))$  is within a log factor from being tight and provides a positive answer to a recent question of Nenadov (arXiv:1805.09710).

This represents joint work with Jie Han, Yoshiharu Kohayakawa and Yury Person.

Friday 5 October - [Matthew Jenssen](#) (University of Oxford)

### **Kissing Numbers and Spherical Codes in High Dimension**

We prove a lower bound of  $\Omega(d^{3/2} \cdot (2/\sqrt{3})^d)$  on the kissing number in dimension  $d$ . This improves the classical lower bound of Chabauty, Shannon, and Wyner by a linear factor in the dimension. We obtain a similar linear factor improvement to the best known lower bound on the maximal size of a spherical code of acute angle  $\theta$  in high dimensions.

Joint work with Felix Joos and Will Perkins.

Friday 11 May - [Shagnik Das](#) (FU Berlin)

### **Colourings without monochromatic chains**

In 1974, Erdős and Rothschild introduced a new kind of extremal problem, asking which  $n$ -vertex graph has the maximum number of monochromatic-triangle-free red/blue edge-colourings. While this original problem strengthens Mantel's theorem, recent years have witnessed the study of the Erdős-Rothschild extension of several classic combinatorial theorems. In this talk, we seek the Erdős-Rothschild extension of Sperner's Theorem. More precisely, we search for the set families in  $2^{[n]}$  with the most monochromatic- $k$ -chain-free  $r$ -colourings. Time and interest permitting, we shall present some results, sketch some proofs, and offer open problems.

This is joint work with Roman Glebov, Benny Sudakov and Tuan Tran.

Friday 2 March - [Adrian Vetta](#) (McGill)

### **Ascending Combinatorial Auctions: Theory and Practice**

I will explain the workings of ascending combinatorial auctions. In particular, I will examine the combinatorial clock auction (CCA) and its variants used in bandwidth auctions. Examples and theoretical results will be given that explain how minor modifications in the auction design can lead to dramatic improvements in the quantitative performance guarantees associated with the CCA. This talk will be based upon recent work with Max Dupre le Tour and prior work with Nicolas Bousquet, Yang Cai, and Christoph Hunkenschröder.

Friday 23 February - [Cedric Koh](#) (Waterloo)

### **Stabilizing Weighted Graphs**

An edge-weighted graph  $G=(V,E)$  is called stable if the value of a maximum-weight matching equals the value of a maximum-weight fractional matching. Stable graphs play an important role in some interesting game theory problems, such as network

bargaining games and cooperative matching games, because they characterize instances which admit stable outcomes. Motivated by this, in the last few years many researchers have investigated the algorithmic problem of turning a given graph into a stable one, via edge- and vertex-removal operations. However, all the algorithmic results developed in the literature so far only hold for unweighted instances, i.e., assuming unit weights on the edges of  $G$ .

We give the first polynomial-time algorithm to find a minimum cardinality subset of vertices whose removal from  $G$  yields a stable graph, for any weighted graph  $G$ . The algorithm is combinatorial and exploits new structural properties of basic fractional matchings, which are of independent interest. In particular, one of the main ingredients of our result is the development of a polynomial-time algorithm to compute a basic maximum-weight fractional matching with minimum number of odd cycles in its support. This generalizes a fundamental and classical result on unweighted matchings given by Balas more than 30 years ago, which we expect to prove useful beyond this particular application.

In contrast, we show that the problem of finding a minimum cardinality subset of edges whose removal from a weighted graph  $G$  yields a stable graph, does not admit any constant-factor approximation algorithm, unless  $P=NP$ . In this setting, we develop an  $O(\Delta)$ -approximation algorithm for the problem, where  $\Delta$  is the maximum degree of a node in  $G$ . This is joint work with Laura Sanità.

**Friday 16 February - [Olaf Parczyk](#) (Frankfurt)**

### **Randomly perturbed graphs**

We study the model of randomly perturbed dense graphs, that is, for any constant  $\alpha > 0$ , the union of some  $n$ -vertex graph  $G_\alpha$  with minimum degree at least  $\alpha n$  and the binomial random graph  $G(n, p)$ .

We introduce a general approach for studying the appearance of spanning subgraphs in this model. Using this, we can give simpler proofs of several results in the literature concerning the appearance of different spanning subgraphs in this model and obtain new results for bounded degree graphs, powers of Hamilton cycles and universality for bounded degree trees. This addresses two questions of Krivelevich, Kwan, and Sudakov.

This is joint work with Julia Böttcher, Jie Han, Yoshiharu Kohayakawa, Richard Montgomery, and Yury Person.

**Friday 2 February - [George Barmpalias](#) (Institute of Software, Chinese Academy of Sciences, Beijing)**

### **From algorithmic learning of languages to learning probability distributions (and back).**

Algorithmic learning theory traditionally studies the learnability of grammars given sufficiently long texts, while recent work by (Vitanyi and Chater 2017) and (Bienvenu

et al. 2014) has adapted this framework to the study of learnability of probability distributions from random data.

In this study, one is given a sufficiently long stream of random data and the task is to guess a probability distribution with respect to which the data is algorithmically random.

We show certain equivalences between algorithmic learning of languages and probability distributions, that allow to transfer much of the classic theory to the study of algorithmic learning of probability distributions. In particular, we prove that for certain families of probability measures that are parametrized by reals (texts), learnability of a subclass of probability measures is equivalent to learnability of the class of the corresponding real parameters. Based on these equivalences, we present a number of applications, providing many new results regarding explanatory and behaviorally correct learnability of classes of measures, thus drawing parallels between the two learning theories.

**Friday 12 January - [Stanislav Kučera](#) (LSE)**

### **Simultaneous Minimum Spanning Trees**

Simultaneous Embedding with Fixed Edges (SEFE) is a problem where given  $k$  planar graphs we ask whether they can be simultaneously embedded so that the embedding of each graph is planar and common edges are drawn the same. Problems of SEFE type have inspired questions of Simultaneous Geometrical Representations and further derivations. Based on this motivation we investigate the generalisation of the simultaneous paradigm on the classical combinatorial problem of minimum spanning trees. Given  $k$  graphs with weighted edges, such that they have a common intersection, are there minimum spanning trees of the respective graphs such that they agree on the intersection? We show that the unweighted case is polynomial-time solvable while the weighted case is only polynomial-time solvable for  $k=2$  and it is NP-complete for  $k>2$ .