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# The Price vs Quantity debate: climate policy and the role of business cycles[1]

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**Abstract**

What is the optimal instrument design and choice for a regulator attempting to control emissions by private agents in face of uncertainty arising from business cycles? In applying Weitzman's result [Prices vs. quantities, *Review of Economic Studies*, 41 (1974), 477-491] to the problem of greenhouse gas emissions, the price-quantity literature has shown that, under uncertainty about abatement costs, price instruments (carbon taxes) are preferred to quantity restrictions (caps on emission), since the damages from climate change are relatively flat. On the other hand, another recent piece of academic literature has highlighted the importance of adjusting carbon taxes to business cycle fluctuations in a procyclical manner. In this paper, we analyze the optimal design and the relative performance of price versus quantity instruments in the face of uncertainty stemming from business cycles. Our theoretical framework is a general equilibrium real business cycle model with a climate change externality and distortionary fiscal policy. First, we find that in an infinitely flexible control environment, the carbon tax fluctuates very little and is approximately constant, whilst emissions fluctuate a great deal in response to a productivity shock. Second, we find that a *fixed* price instrument is advantageous over a *fixed* quantity instrument due to the cyclical behavior of abatement costs, which tend to increase during expansions and decline during economic downturns. Our results suggest that the carbon tax is approximately constant over business cycles due to "flat" damages in the short-run and thus procyclical behavior as suggested by other studies cannot be justified merely on the grounds of targeting the climate externality.

**Keywords**: carbon tax, cap-and-trade, business cycles, distortionary taxes, climate change
**JEL Classifications**: E32, H23, Q54, Q58

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# 1 Introduction

Two classic alternatives for regulating pollutants are a cap-and-trade or a tax; the former is a quantity control and the latter is a price control. In the face of uncertainty stemming from unexpected changes in economic circumstances, what is the optimal instrument design and choice for a regulator attempting to control emissions by private agents? In this paper, we aim to provide an answer to this question.

The literature that compares the relative performance of price and quantity instruments under uncertainty started with a seminal contribution of Weitzman (1974), who analyzed the optimal instrument choice under a static partial equilibrium framework, consisting of a reduced form specification of abatement costs and benefits from abatement. The important character of his setup is that, a regulator issues either a single price order (fixed price) or a single quantity order (fixed quantity) before uncertainty is resolved and these fixed policies result in different expected social welfare outcomes under uncertainty. Specifically, Weitzman shows that under uncertainty about the abatement costs, the relative slopes of the marginal benefit (damage) function and the marginal cost functions determine whether one instrument is preferred to another. If the expected marginal benefit function from reducing emissions is flat relative to the marginal cost of abatement, then a price control is preferred. If, however, the marginal benefit function is steeper, then a quantity control is preferred[2].

By applying the static Weitzman's analysis to a problem of controlling pollution caused by emissions of greenhouse gases, the literature has extended his framework to a dynamic (but still partial equilibrium) setting (e.g., Hoel and Karp (2002), Newell and Pizer (2003), Karp and Zhang (2005))[3]. This literature emphasizes that, for stock pollutants, such as greenhouse gases, the total stock of pollution changes little from one year to another, so that the marginal benefit function is basically flat in the short-run. Applying the Weitzman's relative-slopes result, the stock pollutant nature of the CO2 appears to make preference for prices over quantities.

In this paper, we extend this line of research and analyze the design and compare the relative performance of price and quantity instruments within a general equilibrium framework in case of specific uncertainty - business cycles. This analysis is an important extension of the early studies of the price-quantity argument because of the following reasons. Given the time profile of business cycles - short-run - the intuition of the price-quantity studies holds within the business cycle setting, implying the regulator favors a price instrument over a quantity instrument, over the business cycle. These studies however describe situations when a regulator issues a single-order instrument, which is fixed for at least some period of time, before he can review it. Contrary to such an approach, a number of recent papers (Heutel (2012), Lintunen and Vilmi, 2013) consider policy settings in which regulators select instruments after the uncertainty is resolved and investigate the optimal design of carbon pricing instruments over business cycles. This class of models finds that the optimal carbon taxes and emissions vary over the business cycle in a procyclical way. In addition, Heutel (2012) finds that carbon taxes fluctuate more than emissions under a carbon tax policy in response

[2]Another implication of the Weitzman's result is that benefit uncertainty, unless it is correlated with cost uncertainty, does not affect the net benefit under both price and quantity controls and thus the optimal choice between carbon taxes and emissions caps. As a result, the many followers of Weitzman (1974) have mostly focused on uncertainty arising from shocks to abatement costs of firms, with a key exception of Stavins (1996) who shows that under reasonable conditions, correlation between costs and benefits can reverse the conclusions drawn on the basis of the relative-slope rule

[3]The original Weitzman's analysis has been also extended to analyze performance of hybrid policies that combine elements of taxes and cap-and-trade schemes (e.g., Roberts and Spence (1976)) and to indexed-instruments (e.g., Newell and Pizer (2003))

to a productivity shock.

These are two classes of studies that consider different types of policies: fixed versus state-contingent policies and they yield different policy recommendations. The first class, starting from a premise that complex policies that are state-contingent are hard to implement in practice, suggests choosing a fixed price instrument to address the climate externality in the short-run. The second class suggests making the stringency of regulation responsive to economic fluctuations. In the case of price regulation, this implies the carbon tax is increased during economic expansions and is reduced during recessions. In the case of quantity based regulation, this implies the cap on emissions is relaxed during booms and is tightened during recessions (Doda (2014)). Better understanding on how climate policies interact with economic fluctuations and whether business cycles have any bearing on the relative merits of the price over quantity instruments in the short-run will help in drawing correct and realistic policy implications. This also suggests that the question of the price-quantity literature needs to be analyzed within a more realistic setting by means of the real business cycle model. In this way, it would be also possible to link two strands of the literature and provide additional insight on the design and operation of carbon pricing mechanisms over the business cycle.

Our theoretical framework is an extension of the model in Heutel (2012) by incorporating distortionary fiscal policy. We calibrate the model to the US economy and use it to investigate the design and dynamics of optimal carbon tax and cap-and-trade policies. As in Heutel (2012) optimal climate policies appear to be contingent policies due to the way uncertainty is modeled within the framework: After the shock is realized, the regulator chooses the instruments to reflect the new contingency of the state of the economy and to facilitate the adjustment to the shock. State-contingent policies are ideal instruments and thus serve the role of benchmarks in our analysis in assessing the relative merits of single-order instruments regulators typically choose in practice. Single-order or "basic" instruments in our model are either a carbon tax or a cap on emissions fixed at their corresponding steady-state values. We perform welfare analysis and compare welfare losses from implementing basic regulatory policies instead of baseline policies. In that way, we draw comparisons with the existing price versus quantity literature and analyze which instrument is preferred under business cycles, if regulators issue a single order instrument (as in reality). In our model, uncertainty arises from productivity shocks which indirectly affect abatement costs by firms; the model can also include the shocks directly to abatement costs. The general equilibrium nature of our theoretical framework allows modeling and calibrating these shocks explicitly. This is an advantage of our model over the early studies of the price-quantity literature that have analyzed shocks to abatement costs within a reduced form specification (typically quadratic) of costs and benefits of pollution.

Simulations of the model produce several results. First, we demonstrate that in an idealized world, in which planners can continually adjust instruments to reflect current contingencies of state of economy, considerations about uncertainty in economic shocks are irrelevant. Specifically, the expected welfare outcome and stock of pollution of greenhouse gases is the same irrespective whether a regulator uses a baseline price or a quantity instrument.

Second, under such idealized conditions, we show that if a regulator chooses a carbon tax as an instrument, it remains approximately constant over the business cycle. If a quantity restriction is chosen, then the optimal quantity varies over the business cycle. These results are explained as follows. A carbon tax fixes the price of emissions (price of carbon) with an equilibrium quantity of emissions determined by the market. In contrast, the cap-and-trade fixes the quantity of emissions and leaves it to the market to determine the price of permits, or shadow price of a unit of emissions. Following Pigou's principle, the private sector's cost of emissions (i.e. the carbon tax in case of price instruments and the price of permits, in the case of a cap-and-trade) must correspond to the marginal damages of pollution. Thus, the optimal carbon tax must be essentially constant during that period according to the idea (the same as studies in the price versus quantity literature) that the damages from climate change are essentially constant in the short-run. In the case of a quantity based policy, in each period, before uncertainty is resolved, the level of emissions mandated by the regulator deduces the price of permits by the marginal abatement cost meeting the emissions constraint. In our analysis, uncertainty comes from business cycles which drive the marginal costs. During a period of economic expansion the cost of abatement tends to increase as more firms conduct abatement, whereas recessions reduce the demand for polluting goods, which in turn tend to lower the expected costs of abatement. With the price of permits corresponding to the (approximately) constant level of damages in the short-run, and with marginal costs varying over business cycles, the optimal quantity restriction must vary as well.

Finally, we observe that if a regulator chooses either a fixed price instrument or fixed quantity instrument, then taxes are a more efficient instrument than a quantity instrument. Intuitively, if under idealized conditions, as discussed above, the carbon tax is approximately constant and emissions vary more than welfare costs associated with fixing the instrument at its steady-state value. This should generate smaller losses under the former than under the latter policy. Our estimates of such welfare losses confirm this intuition: we find a fixed tax policy leads to a welfare loss of USD 232.83 per capita per annum, as opposed to the fixed quantity instrument that generates a loss of USD 247.31 per capita per annum. We also note that even though both instruments are fixed, firms as well as the rest of the economy can continually adjust to the shocks. For instance, by comparing impulse responses under both baseline and fixed quantity policies, we show that pronounced differences in the responses of the variables under these two policies appear only at the firm's level and specifically in abatement spending and respectively in the fraction of emissions abated. All other variables, and specifically consumption of private and public goods exhibit almost identical responses, suggesting that the welfare costs of fixing quantity are not significantly higher than the welfare costs that involve fixed price policies.

Some of our analysis assumed state-contingent policies under which a regulator can continually readjust the instruments. It is apparent that, in reality, it is not feasible for a regulator to quantify shocks affecting the economy and to continually readjust instruments to reflect changes in economic conditions. This means that whether a regulator would choose fixed price or fixed quantity instrument will matter for the welfare outcome. Our results have important implications for a regulator selecting a fixed price or a fixed quantity instrument in the face of uncertainty coming from unexpected changes in economic circumstances. We argue that a price instrument is preferred to a quantity instrument due to the cyclical behavior of abatement costs that fluctuate with business cycles. The dynamics of abatement costs make price controls superior for controlling emissions when faced with unexpected fluctuations in economic conditions. In reality, if a regulator chooses a price instrument, then he has to estimate only the level of marginal damages to guide the level of carbon tax. In the short-run, the level of damages is constant. If, however, he decides to choose a quantity instrument, then within the context of our framework, it means that the planner has to re-estimate the marginal costs of abatement by firms every period to make an optimal choice on quantity restriction. Such *one*-dimensional uncertainty associated with setting carbon tax and *two*-dimensional uncertainty associated with setting the quantity restriction favors the former over the latter. In addition, in practice, regulators likely face an information gap on their side[4]: firms likely possess better information about abatement costs than the regulator because they are closer to the actual production process. This further reinforces our argument in favor of prices.

Our main finding that the carbon tax is almost constant and fluctuates less than emissions in response to a TFP shock contrasts the procyclicality and higher volatility of carbon taxes than emissions found by Heutel (2012). In this paper, we make progress in understanding this fundamental difference in our results and argue that carbon taxes play a macroeconomic stabilization role as well as correcting the climate change externality in Heutel's model. Drawing the parallel with the finding of the optimal commodity taxation theory that tells us that energy taxation is unlikely to be justified merely on the grounds of raising public revenues and targeting externalities, we argue that it would be less appropriate to use carbon taxes over and above that needed to correct externalities over the business cycle.

The rest of the paper is organized as follows. Section 2 describes the core model. Section 3 discusses the model's calibration. Section 4 presents and discusses the results under price and quantity instruments when the regulator can continuously adjust instruments to reflect current states of nature. The same section presents results when the economy is hit by two correlated shocks - productivity and shock to abatement technology; it also discusses policy implications of our main results. Section 5 discusses what drives divergence in our results when compared to those of Heutel (2012). Section 6 concludes.

# 2 Real business cycle model with distortionary taxes and climate externality

The baseline model used in this paper is an extension of the real business cycle model with climate externalities of Heutel (2012) by introducing distortionary fiscal policy. The economy consists of households, firms, and the government. Households obtain utility from consumption of both public and private goods, as well as from leisure. Goods are produced using private capital and labor. Following Heutel (2012), production causes greenhouse gas emissions, which accumulate in the atmosphere and lead to climate change that causes damages by reducing output according to a damage function. As in Heutel (2012), we assume that firms can counteract the adverse productivity effect of climate change by increasing spending on abatement. The government levies emissions, corporate and labor taxes on firms. The raised revenue from these taxes are used to finance the public good provision and the public debt.

## 2.1 Households

A representative household maximizes:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \tag{1}$$

In this utility function $c_t$ and $g_t$ represent private and public consumption, $l_t$ is the number of hours worked by the household. The representative household faces the following budget constraint:

$$c_t + i_t + \rho_{Bt} b_{t+1} = w_t l_t + r_t k_{t-1} + \pi_t + b_t \tag{2}$$

[4]As in the original Weitzman's analysis and in his many extensions

where $i_t$ is private investment, $\pi_t$ is firm profits, $b_{t+1}$ denotes one-period government bond purchases, $\rho_{Bt}$ is the price of one-period bonds. Households derive income by supplying labor and capital to firms at rental rates $w_t$ and $r_t$. The private capital stock is accumulated according to:

$$k_t = (1 - \delta)k_{t-1} + i_t \tag{3}$$

First-order conditions of the household maximization problem imply:

$$w_t = -\frac{u'_L(t)}{u'_c(t)} \tag{4}$$

$$u'_c(t) = \beta E_t u'_c(t+1)[1 - \delta + r_{t+1}] \tag{5}$$

$$u'_c(t)\rho_{Bt} = \beta E_t u'_c(t+1) \tag{6}$$

Equation (4) equates the marginal rate of substitution of leisure for consumption to real wages and defines household's labor supply. Condition (5) is a standard stochastic Euler equation, which determines intertemporal allocation: it equates the intertemporal marginal rate of substitution in consumption to the real rate of return on private capital. Condition (6) is the counterpart of equation (5) for domestic bonds.

## 2.2 Final goods production

Output $y_t$ is produced by identical firms, and then can be used for consumption, investment, abatement or government spending:

$$y_t = (1 - d(x_t))f(k_{t-1}, l_t; a_t) \tag{7}$$

where $a_t$ represents an exogenous productivity shock that follows a stationary stochastic process:

$$\ln a_t = \rho \ln a_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_\varepsilon^2), \mid \rho \mid < 1, \tag{8}$$

We assume that the stock of pollution in the atmosphere, denoted by $x_t$, adversely affects output through damage function $d(x_t)$ specified in the parametrization section 3. The formulation of climate damages as a fraction of output lost as in (7) was introduced by Nordhaus (1991) and since then has been extensively used in the literature. The mapping of emissions to economic damage can be thought as comprising of two steps: first, emissions increase the concentration of greenhouse gases leading to climate change (represented by change in the global mean temperature), and second, changes in temperature cause economic damages. Some papers, e.g., Barrage (2014) follow Nordhaus's approach and model two steps of mapping from carbon concentration to damages. We follow equally common specification and map CO2 concentration to damages in one step (Heutel (2012), Golosov et al. (2014))[5]. The specification (7) assumes that climate change (or concentration of greenhouse gas emissions in our set up) affects output directly. Such specification is standard

[5]The two stage mapping would have introduced a set of lags in the effect of current-emissions on output, but would have not changed our results. This is because cyclical changes in emissions levels have very little effect on the pollution stock because of the long-lived nature of CO2, and thus it is relatively immaterial whether a ton of carbon dioxide emitted today or a few periods later. We demonstrate in the Online appendix that the damages from pollution do not change significantly with business cycles.

in climate change modeling analysis and it represents in aggregate form the dependence of production of many goods on climate change conditions, such as production of agricultural goods, forestry, fisheries etc.

Profits of firms are defined as:

$$\pi_t = (1 - \tau_{ct})y_t - w_t(1 + \tau_{Lt})l_t - \tau_{Et}e_t - r_t k_{t-1} - z_t \tag{9}$$

where $\tau_{Lt}$ is payroll (labor) tax, $\tau_{ct}$ is corporate tax, $\tau_{Et}$ is tax on emissions, $e_t$ are emissions, which are by-product of production, and $z_t$ is spending on abatement by firms; private abatement spending is assumed to abate the $\mu_t$ fraction of emissions via the following relation:

$$\frac{z_t}{y_t} = m(\mu_t) \tag{10}$$

so that firms face the emissions constraint given by:

$$e_t = (1 - \mu_t)h(y_t) \tag{11}$$

where $h(y_t)$ determines total emissions from producing $y_t$ output. Following Heutel (2012), we assume that a climate change externality arises because firms do not take into account their emission's impact on the pollution stock and thus on productivity. In other words, firms take $x_t$ as a given. Optimality conditions of the firm imply:

$$r_t = (1 - d(x_t))f'_k[1 - \tau_{ct} - \tau_{Et}(1 - \mu_t)h'(y_t) - m(\mu_t)] \tag{12}$$

$$w_t(1 + \tau_{Lt}) = (1 - d(x_t))f'_L[1 - \tau_{ct} - \tau_{Et}(1 - \mu_t)h'(y_t) - m(\mu_t)] \tag{13}$$

$$\tau_{Et} = \frac{y_t m'(\mu_t)}{h(y_t)} \tag{14}$$

Equation (12) is an optimal condition of demand for capital, which implies that the return associated with an increase in capital stock by one unit is equal to the marginal product of capital, net of additional tax payments on increased emissions associated with an increase in output and net of additional spending on abatement to clean a given fraction $\mu$ of extra emissions stemming from an increase in output. Equation (13) is the counterpart of equation (12) for labor demand. Finally, equation (14) says that the firm reacts to the carbon tax by choosing the level of abatement (equivalently the level of emissions) such that the tax on emissions would be equal to the marginal cost of emissions reduction.

## 2.3 Government

The government budget constraint is balanced according to:

$$g_t + b_t = w_t \tau_{Lt} l_t + \tau_{Et} e_t + \tau_{ct} y_t + \rho_{Bt} b_{t+1} \tag{15}$$

where the government raises revenues by taxing labor income and emissions and levying corporate tax to finance public debt $b_t$ and provision of public goods, $g_t$. The government can issue new one-period bonds $b_{t+1}$. The government budget constraint (15) incorporates market clearing for bonds which requires that households demand for bonds and government supply for bonds are equated.

## 2.4 Carbon cycle

Following Heutel (2012), we assume that each period new industrial domestic and foreign carbon dioxide emissions increase the existing pollution stock that decays at a linear rate $\eta$:

$$x_t = \eta x_{t-1} + e_t + e_t^{row} \tag{16}$$

where $e_t$ is current-period domestic emissions that are related to the output produced and fraction $\mu_t$ that is abated, while $e_t^{row}$ is current-period emissions from the rest of the world and $\eta$ is the fraction of the pollution stock that remains in the atmosphere.

Atmosphere, however, is not the only reservoir of the carbon dioxide. Even without industrial emissions there exists a natural carbon cycle encompassing flows of carbon dioxide among different reservoirs, such as atmosphere, oceans etc. Nordhaus (2008) distinguishes in his model between three of them: the atmosphere, biosphere including the upper oceans, and the deep oceans. The flows between the first two reservoirs are relatively quick, but the carbon dioxide from deep oceans, that is the largest carbon reservoir, interacts only at a very slow pace with the remaining two reservoirs. In Nordhaus (2008) only the atmosphere is directly influenced by industrial activity. The mass of carbon accumulated in the atmosphere has in turn impact on the radiative forcing that enters the damage function adversely affecting the production process. The carbon accumulated in each of the reservoirs is a function of past carbon values, depreciated by a parameter analogous to our decay rate $\eta$. So, in this paper, we leave the flows between atmosphere and other carbon reservoirs aside and model the effect of industrial activity on atmosphere. One-dimensional representation of the carbon cycle based on the stock of pollution in atmosphere only has been also utilized in Golosov et al. (2014).

## 2.5 Characterizing equilibrium

To construct the Ramsey problem, we reorganize some of the constraints in order to reduce the number of choice variables and to obtain a compact expression for the household budget constraint. In particular, combining (2),(9) and (15) gives the following resource constraint for the economy:

$$c_t + k_t - (1-\delta)k_{t-1} + z_t + g_t = y_t \tag{17}$$

Next, by adding and substituting for $w_t$ from (4), we rewrite the government's budget constraint as follows:

$$g_t + b_t = -\frac{u'_L(t)}{u'_c(t)}\tau_{Lt} l_t + \tau_{Et}(1-\mu_t)h(y_t) + \tau_{ct} y_t + \rho_{Bt} b_{t+1} \tag{18}$$

Substituting (4) into (13) gives:

$$-\frac{u'_L(t)}{u'_c(t)}(1+\tau_{Lt}) = (1-d(x_t)) f'_L [1 - \tau_{ct} - \tau_{Et}(1-\mu_t)h'(y_t) - m(\mu_t)] \tag{19}$$

## 2.6 Ramsey problem

Ramsey planner maximizes the utility of households:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \tag{20}$$

subject to (5), (14), (17), (18), (19) and

$$y_t = (1 - d(x_t))f(k_{t-1}, l_t; a_t) \tag{21}$$

$$x_t = \eta x_{t-1} + (1 - \mu_t)h(y_t) + e_t^{row} \tag{22}$$

where we also use the function (12) for definition of $r_t$.

The government chooses $c_t$, $\mu_t$, $k_t$, $y_t$, $x_t$, $l_t$, $\tau_{Lt}$, $\tau_{Et}$, $\tau_{ct}$, $g_t$ and $b_{t+1}$ to maximize (3) subject to the constraints specified above.

The Lagrangian for this problem is given by:

$$\begin{aligned}
L \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \{u(t) + \lambda_t \left[-u'_c(t) + \beta u'_c(t+1)(1 - \delta + r_{t+1})\right] + \\
& +\Omega_t[c_t + k_t - (1-\delta)k_{t-1} + m(\mu_t)y_t + g_t - y_t] + \\
& +\chi_t[\tau_{Et}h(y_t) - y_t m'(\mu_t)] + \\
& +\Lambda_t[-g_t - b_t - \frac{u'_L(t)}{u'_c(t)}\tau_{Lt}l_t + \tau_{Et}(1-\mu_t)h(y_t) + \tau_{ct}y_t + \rho_{Bt}b_{t+1}] \\
& +\lambda_{pt}[y_t - (1 - d(x_t))f(k_{t-1}, l_t, k_{Gt-1})] + \\
& +\varsigma_t \left[-\frac{u'_L(t)}{u'_c(t)}(1 + \tau_{Lt}) - (1 - d(x_t))f'_L(t)(1 - \tau_{ct} - \tau_{Et}(1-\mu_t)h'(y_t) - m(\mu_t))\right] + \\
& +\Phi_t[x_t - \eta x_{t-1} - e_t^{row} - (1-\mu_t)h(y_t)]
\end{aligned}$$

The first-order conditions of the Ramsey problem are given in Appendix 8.3.

# 3 Parametrization

In calibrating the model, we select parameter values that enable the theoretical model to generate features that are (as closely as possible) consistent with the main features of the US economy. We assign values to structural parameters using values that are common in business cycle studies of fiscal policy and macroeconomic models with climate change externalities. In calibrating the climate part of the model, we draw strongly on estimates and parameter values used in Heutel (2012). Baseline parameter values of the model are summarized in Table 1, while Table 2 reports macroeconomic ratios implied by the theoretical model as well as the corresponding values for the US data. Data sources employed in these calculations are summarized in Appendix 8.2.

In calibrating the model, a time period represents one quarter. The production function is given by

$$f(k_{t-1}, l_t; a_t) = a_t k_{t-1}^{\alpha} l_t^{1-\alpha} \tag{23}$$

We set $\alpha$ at 0.36, which is a value commonly used in the standard RBC literature. For the TFP process, we assume that $\rho = 0.95$ and $\sigma_\varepsilon = 0.007$, where the value of the standard deviation is as in Heutel (2012) and is similar to the value 0.0056 as in Schmitt-Grohe and Uribe (2007). We show below that our results are not sensitive to changes in the value of $\rho$. The private capital depreciation rate, $\delta$, is set at 0.025 (Heutel, 2012). We set the discount factor $\beta$ at 0.98.

For the quantitative analysis, we consider the following form of the households utility function:

$$u(c_t, l_t, g_t) = \frac{c_t^{1-\kappa} - 1}{1 - \kappa} + \theta \frac{g_t^{1-\kappa} - 1}{1 - \kappa} - \frac{l_t^{1+\psi}}{1 + \psi} \tag{24}$$

| Parameter | Value | Definition |
|---|---|---|
| $\alpha$ | 0.36 | private capital share in the production function |
| $\rho$ | 0.95 | persistence of the TFP shock |
| $\sigma_\varepsilon$ | 0.007 | standard deviation of the TFP shock |
| $\delta$ | 0.025 | private capital depreciation rate (quarterly) |
| $\beta$ | 0.98 | subjective discount factor (quarterly) |
| $\kappa$ | 1.6 | coefficient of relative risk aversion |
| $\theta$ | 0.236 | weight of public consumption in utility |
| $1/\psi$ | 0.4 | Frisch elasticity of labor supply |
| $\eta$ | 0.9979 | pollution decay |
| $d_2$ | 5.2096e-10 | damage function parameter |
| $d_1$ | -1.2583e-06 | damage function parameter |
| $d_0$ | 1.3950e-3 | damage function parameter |
| $\theta_1$ | 0.05607 | abatement cost equation parameter |
| $\theta_2$ | 2.8 | abatement cost equation parameter |
| $1-\nu$ | 0.696 | elasticity of emissions with respect to output |

Table 1: Baseline parameter values

with the coefficient of relative risk aversion, $\kappa$, set to 1.6, which implies that the value of the intertemporal elasticity of substitution (EIS) is 0.625 in the model. The standard value of $\kappa$ in the literature is 1 (see, e.g., Golosov et al., 2013). We set the value of $\psi$ such that a Frisch elasticity of labor supply is 0.4, in line with macroeconomic estimates reported by Rogerson and Wallenius (2009). The weight of public consumption in the utility function, $\theta$ is set at 0.236.

Following Heutel (2012), the pollution stock in the atmosphere evolves according to the following equation: $x_t = \eta x_{t-1} + e_t + e_t^{row}$. We set the value of $\eta$ at 0.9979 as in Heutel (2012), who calibrated this parameter assuming that 83 years represent the half-life of atmospheric carbon dioxide. In actuality, there is no single number that describes the lifetime of carbon dioxide in the atmosphere because it is weighted sum of exponential decays at different rates. Carbon dioxide is not destroyed in the air, but is instead exchanged between the atmosphere, the ocean, and land. For other greenhouse gases, lifecycle estimation is possible, see the report of Intergovernmental Panel on Climate Change (2001). Thus, the values used by different studies vary. Following Archer (2005), Golosov et al. (2014) we calibrate the half-life of CO2 to 150 years. Assuming a half-life of 150 years would lead to $\eta = 0.9988$ in our model. Our model's result are not sensitive to changes in $\eta$ - the decay parameter influences quantitative responses of only three variables (emissions, stock of pollution and fraction of emissions abated), while the responses of all other variables remain the same.

The emissions produced by the rest of the world, $e_t^{row}$ are set to 4 times the steady state of domestic emissions, which is guided by the following considerations. According to data by the U.S. Environmental Protection Agency (EPA), the USA accounted for 19% of global CO2 emissions from fossil fuel combustion in 2008, which means that global emissions were four times higher than those in the US[6].

[6] `http://www.epa.gov/climatechange/ghgemissions/global.html`

The loss of potential output due to pollution is governed by the function $d(x) = d_2x^2 + d_1x + d_0$. We set the values of $d_2$, $d_1$, $d_0$ respectively to 5.2096e-10, -1.2583e-06, 1.3950e-3, following Heutel (2012), who calibrates these values to match the damages from carbon dioxide in the atmosphere estimated by papers in the environmental literature. Specifically, Heutel (2012) bases this estimation on Nordhaus (2008). The DICE model includes a damage function, expressing climate damages as a fraction of world output. In the DICE model, the damages are calculated for the whole world, but the supplementary material[7] explains how the impact of each larger region in the world was taken into account, so the percentage impact of the US in the world damages is given. Nordhaus's model (Nordhaus, 2008) contains equations linking the pollution stock to its radioactive force and its impact on the temperature of oceans and the atmosphere. Having the pollution stock, one can thus compute damages as a fraction of output, using the model's equation. Heutel (2012) does this exercise for 100 different pollution stocks ranging from 600GtC to 1200GtC and plots the pollution stock against damages on a graph. The resulting functional relationship is fitted to a quadratic function, which is the $d(x)$ function in the model[8]. Our baseline calibration gives damages of 0.59%[9].

The abatement cost function is taken directly from Nordhaus (2008) and has the form $m(\mu) = \theta_1\mu^{\theta_2}$. We set $\theta_1 = 0.05607$ and $\theta_2 = 2.8$, following Heutel (2012).

| Symbol | Variable | Model | Data |
|---|---|---|---|
| $c/y$ | personal consumption/output | 0.58 | 0.68 |
| $g/y$ | government consumption/output | 0.26 | 0.15 |
| $i/y$ | private domestic investment/output | 0.16 | 0.16 |
| $e/y$ | emissions/output | 0.76 | 0.60 |
| $b/y$ | public debt/output | 0.77 | 0.77 |
| $\mu$ | fraction of emissions abated, % | 0.54 | 1.85 |
| $\tau_E$ | tax on emissions, % | 0.002 | - |
| $\tau_L$ | labor tax, % | 15.4 | 15.4 |
| $\tau_C$ | corporate tax, % | 21.25 | 35 |
| $\tau_E e/y$ | revenue from carbon tax, % of GDP | 0.0013 | 0.7 (estimate) |

Table 2: Structure of the theoretical economy and the data

Given our baseline parametrization, the theoretical model implies very low level of carbon taxes in the steady-state, 0.002% (for comparison Heutel's model implies 0.0487%), and respectively the very low share of carbon tax revenues in GDP, only 0.0013% of GDP. We think this is not problematic and does not drive our results, since the estimates suggest that in reality the share of carbon tax revenues in GDP is a negligible number anyway. Specifically, different estimates for the US evaluate the possible net revenue in the range 0.51-0.8% of US GDP (Table 2 in Gale et al., 2013). As Gale et al. (2013) report, in 2007 the carbon tax raised revenues equivalent to 0.3% of GDP in Finland and Denmark and 0.8% of GDP in Sweden. In

[7] `http://www.econ.yale.edu/~nordhaus/homepage/Accom_Notes_100507.pdf`

[8] It is important to note that the damage function was calibrated using the point estimates of the equilibrium climate sensitivity, which is highly uncertain with a "likely" range between 1.5 and 4.5°C (Intergovernmental Panel on Climate Change (2013)). In fact, investigations started with analysis in Weitzman (2009) suggests that the climate sensitivity parameter is better thought as the distribution with fat tails.

[9] Given the small values of the parameter values in the damage function and for the relevant values of concentration of stock of pollution, the damage function would not give damages greater than 100%.

Australia carbon tax revenue 2012-2013 accounted to 1.2% of GDP[10].

Finally, output is mapped into emissions through $h(y_t) = y^{1-\nu}$, with $e_t = (1 - \mu_t)h(y_t)$, where $1 - \nu$ represents the elasticity of emissions with respect to output. We set the value of $1 - \nu$ at 0.696, which is the estimate from a (seasonally adjusted) ARIMA regression of the log of emissions of CO2 on the log of GDP for US data in years 1981-2003. As in Heutel (2012), we solve the model by log-linearizing around the steady-state.

# 4 Simulation results

## 4.1 Results under baseline price instrument policy

Figure 1 shows the impulse responses (IR) of the key variables to a 1% increase in productivity under both carbon tax and cap-and-trade policies. All variables are expressed in terms of percentage deviations from the steady state, except for the tax rates, for which responses are expressed as absolute deviations from their steady-state values[11]. Given the objectives of the paper, we report plots of the impulse response functions only for key variables related to our analysis, but results for the remaining variables are available upon request. The continuous line represents the baseline model, the dashed line represents the model with alternative tax policy. We start with discussing the results under baseline carbon tax policy.

Impulse responses obtained from simulations of the baseline model result in the following key qualitative results. First, consistent with the findings of other studies on optimal carbon tax over the business cycle (e.g., Heutel, 2012), emissions increase in the periods following a positive productivity shock. Given the long-lived nature of carbon dioxide, increased emissions result in a higher pollution stock over the medium term, and increase by around 0.008% in 25 years time. We demonstrate in Online appendix 8.1 that this number is quite small, by estimating corresponding increases in mean global temperature and sea levels associated with this rise in atmospheric greenhouse gases. Second, the labor tax increases by 1.66 percentage points (10.77 percent), the corporate tax decreases by 1.22 percentage points (5.75 percent) while tax on emissions is raised by only 0.000014 percentage points, corresponding to 0.7 percent relative to the steady-state value in response to the shock.

Very small fluctuations in carbon tax can be explained drawing on the intuition of the "price versus quantity" literature. Given the long-lived nature of greenhouse gases, the additional damage from each additional ton of carbon emissions is constant in the short-run. In terms of the model presented in section 2, concentration of CO2 emissions in the atmosphere $x_t = \eta x_{t-1} + e_t + e_t^{rpw}$, $x_t \sim x_{t-1}$ as well as damages $d(x_t) = d_2 x^2 + d_1 x + d_0$ remain essentially constant over the business cycle. Following Pigou's principle, the private sector's marginal cost - carbon tax under baseline policy - must correspond to the level of the marginal damages, which are "flat" in the short-run. This explains why the optimal carbon tax is essentially constant over the business cycles[12].

[10] `http://www.treasury.gov.au`

[11] Please note that for each time period $t$, we plot the values of those stock variables which enter current production process, namely $x_t$ and $k_{t-1}$. Since $e_t$affects $x_t$ contemporaneously, $x_t$ jumps in response to the shock, while $k_{t-1}$ does not.

[12] Abstracting from business cycles, Golosov et al. (2014) propose a tractable Ramsey growth model and show that an optimal carbon tax is proportional to output. Rezai and van der Ploeg (2014) generalize their result to allow for some elements such as population growth, a temperature lag and general degrees of intergenerational inequality aversion and show that the global carbon tax rises in proportion with GDP if marginal climate damages are proportional to GDP.

## 4.2 Results under baseline quantity instrument policy

The baseline model assumes that tax on emissions is an instrument to combat climate change. An alternative policy to control emissions is a cap-and-trade or emissions trading scheme, in which governments restrict the emissions (by imposing cap on emissions) firms produce. Following Heutel (2012), we introduce a cap-and-trade scheme into our framework, by assuming that the government mandates the level of emissions a firm can produce, $q_t$. In other words, the government allocates permits to each firm (one representative firm) for free, so that it does not generate revenue. The setting features the simplest cap-and-trade scheme that does not allow for policies similar to a "safety valve", in which firms are allowed to purchase an unlimited number of permits at a set price, which equivalently sets a ceiling on the price of permits (see e.g., Pizer (2002) for more details); we also abstract from incorporating active banking, which allows regulated firms to shift obligations across time in response to periods of unexpectedly high or low marginal costs (see, e.g., Fell et al. (2012))[13]. And since the theoretical framework features one representative firm, the quantity constraint is equivalent to a cap-and-trade scheme.

An individual's budget constraint and FOC in this setting remain as in the baseline model. There are only changes in the firm's problem and in the government's budget constraint. Specifically, firms do not pay taxes and respectively the government budget omits revenues from taxing emissions. Profits of the firms are defined as:

$$\pi_t = (1-\tau_{ct})y_t - w_t(1+\tau_{Lt})l_t - r_t k_{t-1} - z_t \tag{25}$$

subject to emissions constraint $q_t = (1-\mu_t)h(y_t)$ and abatement spending $z_t = m(\mu_t)y_t$. The government budget constraint is balanced according to:

$$g_t + b_t = w_t\tau_{Lt}l_t + \tau_{ct}y_t + \rho_{Bt}b_{t+1} \tag{26}$$

Optimality conditions of the firm imply:

$$r_t = (1-d(x_t))f'_k[1-\tau_{ct} - m(\mu_t) - \frac{m'(\mu)y_t}{h(y_t)}(1-\mu_t)h'(y)] \tag{27}$$

$$w_t(1+\tau_{Lt}) = (1-d(x_t))f'_L[1-\tau_{ct} - m(\mu_t) - \frac{m'(\mu)y}{h(y)}(1-\mu_t)h'(y)] \tag{28}$$

$$q_t = (1-\mu_t)h(y_t) \tag{29}$$

Equation (29) is just a constraint on the quantity of emissions produced. Equations (27)-(28) are analogous to the equations (12)-(13) under tax policy, and they are optimal conditions of demand for capital and labor, respectively. They also demonstrate that the price of permits - the shadow price of a unit of emissions under quantity policy - is $p_{Et} \equiv m'(\mu)y_t/h(y_t)$. For comparison, under price instrument, as shown in equation (14), firms reduce emissions until the marginal cost of reductions equal to the tax - price of carbon. In other words, carbon tax fixes the price of emissions, so that the equilibrium quantity is determined in the market; in contrast, the cap-and-trade fixes the quantity of emissions and leaves it to the market to determine the price. In a deterministic world, the carbon tax under priced policy would be equal to the shadow price induced from

[13]Active banking can make cap-and-trade scheme more flexible in terms of intertemporal allocation of abatement decisions by firms. As a result, in face of temporary uncertainty in costs, under cap-and-trade with banking and borrowing, emissions fluctuate period-by-period and prices are relatively constant (Parsons and Taschini (2013)).

cap on emissions and two instruments lead to the same emissions outcome. In such world, there is simple equivalence between policies: a given price yields a specific quantity of emissions and vice-versa. Under uncertainty, however, and if policies must be fixed before the uncertainty is resolved as in the framework of the price-quantity literature, two policies lead to different outcomes (see, e.g., Weitzman (1974)), and the price of carbon under priced policy would not be equal to the induced price from cap-and-trade.

In our framework, under *uncertainty* in business cycles driven by the *same* productivity shock, both policies lead to the same expected welfare outcome and optimal quantity under cap-and-trade varies with business cycles. Plots of impulse responses of key variables, 1, demonstrate that both policies lead to the same expected welfare and emissions outcome. All variables, except tax on labor, wages, government bonds, spending on abatement and fraction of emissions abated, exhibit identical responses. The above-mentioned variables respond differently because the government does not generate any revenue from a cap-and-trade and thus these variables need to adjust accordingly to generate the expected welfare outcome as under price instrument.

Since in our model regulators can continually readjust instruments to reflect changes in economic circumstances, both lead to the same expected welfare outcome. Another result worth mentioning is that under cap-and-trade policy, the optimal restriction of emissions varies with business cycles. The intuition for this result becomes clear and simple after understanding the instrument's mechanism. In each period before the uncertainty is resolved, the regulator mandates the level of emissions, which will deduce the shadow price of carbon by the marginal cost meeting the emissions constraint. In our framework uncertainty comes only from business cycles, which will then affect the level of marginal costs, which tend to increase during booms and to fall during recessions. Thus, every period when the uncertainty is resolved, the state of the nature will be associated with a different marginal cost. Following Pigou's principle (Pigou (1920)), private sector's cost - the shadow price of carbon - must correspond to the marginal damages of pollution. Thus, with essentially constant level of damages in the short-run, and with varying over the business cycle marginal cost, the optimal quantity restriction must vary with business cycle to deduce a shadow price that is not only consistent with the target for emissions, but also internalizes externality.

### 4.3 Fixed priced and fixed quantity based policies and welfare

As discussed above, when the regulator can continually readjust the policies, the choice of the optimal instrument - price or quantity - becomes irrelevant as both policies lead to the same expected welfare outcome. Feasibility of such complex policies can be doubted in the practice and we discuss the implications of our results for policy analysis in section below. In contrast, the price-quantity literature, initiated with analysis in Weitzman (1974), focuses on the consequences of "basic" policies, those when the regulator chooses either a fixed price or fixed quantity policy before any uncertainty is resolved. To follow this convention, in this section we investigate the relative performance of fixed price and fixed quantity policies (fixed at corresponding steady-state values), by comparing welfare losses from fixing policies compared to the baseline policies.

Our measure of welfare is the amount of baseline steady-state policy consumption a household would be willing to give up to be as well off under the alternative specification as under the baseline policy, following the procedure of Schmitt-Grohe and Uribe (2007). The results are shown in Table 3. For the consumption-equivalence, a number of, e.g., 0.64 means that the alternative environmental tax policy reduces welfare by

0.64% of consumption on average.

| Welfare in consumption-equivalents, % | |
|---|---|
| Model with fixed emissions tax | 0.64 |
| Model with fixed quantity | 0.68 |

Table 3: Welfare effects of alternative tax policies

We express welfare costs associated with single order instruments in monetary value, using the 2013 US annual personal consumption expenditure[14], which stood at USD 11,496.2 bn. By using this data, and converting this to per capita terms[15], we find a fixed tax instrument does in fact lead to a lower welfare loss compared to the fixed quantity instrument: USD 232.83 per person with taxes vs. USD 247.31 under quantity controls[16] .

These relatively small differences in the welfare losses under tax instrument compared to the quantity instrument can be explained as follows. Even though, both instruments are fixed, firms as well as the rest of the economy can continually adjust to the shocks. For instance, the impulse responses under both baseline and fixed quantity policies demonstrate (figures 2 and 3), pronounced differences in the responses of the variables under these two policies appear only at the firm;s level and specifically in abatement spending and respectively in the fraction of emissions abated. And as welfare comprises consumption of both private and public goods, it is not surprising to see small differences in welfare costs under fixed tax and fixed cap-and-trade are justified. In line with that, we will show in the next section that responses to the shock to abatement technology also occur primarily at the firm's level. Such adjustment occurring at the firm's level will have implications for policy conducted in reality as we will discuss policy implications of our results in section 4.5.

Finally, some other studies also find very small differences in the welfare gains from contrasting different policy instruments, even though those estimates are not directly comparable with ours. In particular, Pizer (1999) investigates the relative performance of taxes with rate controls (fractional reduction CO2 emissions at a given time) in an integrated climate-economy model under uncertainty which is modeled allowing thousands of different states of nature. He finds that uncertainty leads to a preference for taxes over control rates, with the optimal rate control generating welfare gains[17] equivalent to a USD 73 increase in current per capita consumption, whilst the optimal tax policy generates an USD 86 increase.

[14]Data source is the NIPA table, see Appendix 8.2 for more details.

[15]Population in the US in 2013 stood at 316.1 million people.

[16]The uncertainty in our paper arises from temporary shocks and our results are not sensitive to changes in the persistence of the shocks. See figure 5 in the Online appendix that presents the IRFs under different values of the persistence of the shock under carbon tax policy. The welfare ranking of the instruments also remain unchanged and the results are available upon request. But in general, dynamic structure of cost uncertainty can affect the choice between a price or quantity control, as shown in Parsons and Taschini (2013). Specifically, by using reduced form specification in tradition of the early price-quantity literature, they show that temporary shocks to abatement cost favor the use of a price control, whilst the permanent shocks favor a quantity control.

[17]The source of such gain is due to those states of the nature in which the marginal costs of reduction of emissions are low, while the marginal benefits are high, which favor more stringent policies. While opposing states of nature that favor less stringent policies and thus generate losses from more stringent policies, but such losses are not as significant as the gains, resulting in overall improvement in welfare. In other words, more stringent than the optimal control rate policy ignoring uncertainty improves welfare.

## 4.4 Associated shocks to abatement technology

The one of the key underpinnings of the argument we proposed for comparing the relative merits of alternative price and quantity mechanisms is that price instrument gives flexibility to firms to find their own most efficient solutions in controlling emissions. To provide further evidence for that, we perform next experiment, in which we assume that the economy is hit by two, correlated shocks, productivity shock and shock to abatement technology. Such experiment has been motivated by the following considerations.

In our baseline model, uncertainty comes from the productivity shock. The existing "price versus quantity" literature, however, models a reduced form of the abatement cost function with mean-zero random shocks to marginal abatement costs. The shocks to the reduced form of abatement costs may originate (indirectly) from productivity shocks or directly from business cycles. In our framework, we can differentiate between these two types of shocks to abatement costs, by considering productivity shock and an abatement shock. We introduce an abatement shock as a shock to abatement technology $\varepsilon_{ab,t}$:

$$\frac{z_t}{y_t} = m(\mu_t)\varepsilon_{ab,t} \tag{30}$$

which assumed follows AR(1) process, defined as:

$$\ln \varepsilon_{ab,t} = \rho_{\varepsilon_{ab,t}} \ln \varepsilon_{ab,t-1} + \rho_{ab}\varepsilon_t, \tag{31}$$

where $\varepsilon_t$ is the shock to productivity, and $\rho_{ab}\varepsilon_t$ is a shock to abatement technology. Following the discussion above, we assume $\rho_{ab} > 0$. Note that we have defined the shock to abatement technology such that a positive value of $\rho_{ab}\varepsilon_t$ increases abatement costs, that is, abatement of a given fraction of emissions $\mu$ associated with a given output becomes more costly. As mentioned earlier, there are two new values in this extension that we need to parametrize: the value of the persistence of the shock to abatement technology, and the value of correlation between shocks to productivity and abatement. Since we assume that abatement costs vary with business cycles, we can set the value of persistence of the shock to abatement technology equal to the one of productivity shock. And since the value of the correlation between productivity and abatement a priori is unknown, we experiment with two values of $\rho_{ab}$: 0.4 and 0.7.

Comparison of impulse responses under the baseline policy and under correlated shock case (figure 4) reveals that adjustment to the shock to abatement technology happens through changing the total spending on abatement, without any notable effects on the behavior of the remaining variables. As a result, the firm produces the same level of emissions and abate the same fraction of emissions. To sum up, firms find their own most efficient solutions to controlling emissions.

## 4.5 Policy implications

State-contingent policies[18] considered in the model are difficult, if not impossible, to implement in practice because they involve continual readjustment of policies and require complete knowledge about distribution of shocks affecting economy. Despite these arguments, our baseline results provides important policy implications and insights for a policymaker seeking a policy regulation - fixed price or fixed quantity restriction - to control CO2 emissions in face of uncertainty stemming from business cycles.

[18]Newell and Pizer (2003) extended the original analysis of Weitzman's to indexed policies, where quantities are proportional to an index, such as economic output. They find that a general indexed quantity policy improves the ex post performance of fixed quantities, but comparison to a fixed price policy is more complex. But they point out that identifying the proper economic activity indicator is a complex task: the indicator must capture the direction and the right intensity of the shock.

Specifically, under priced policy, our results suggest that in practice the regulator has to estimate the level of marginal damages to inform the level of carbon tax. As seen in the model, firms react to a carbon tax, by reducing emissions until the marginal cost of abatement equals to the tax. Conversely, if regulator selects quantity based policy, he must estimate both the level of the marginal damages and the marginal cost of abatement to deduce the target for emissions, which induce the shadow price of emissions that internalizes the externality. But, the level of marginal costs vary with business cycles and to be able to set a target that yields the economically efficient outcome, he must re-estimate marginal costs every period. Thus, *one-dimensional* uncertainty associated with setting the carbon price compared with *two-dimensional* uncertainty associated with setting quantity target argues in favor of the former over the latter instrument. In drawing this policy implication, we were referring to a genuine uncertainty stemming from business cycles and uncertainty in estimating the marginal damages (so-called social cost of carbon) that exist not only for regulator but also for producers. However, in reality, another type of uncertainty may be present - information gap - randomness that is certain to a producer but is unknown to a regulator or vice-versa. Specifically, it is plausible to assume, as in the original Weitzman's analysis (Weitzman (1974) and as in most of studies that have followed) that uncertainty in the marginal costs function is an information gap on the side of the regulator[19]. That is, firms possess better information about costs than the regulator because they are actually closer to the actual production process. The presence in reality of the type of information gap as described above reinforces our argument in favor of prices. Carbon taxes, helping in controlling emissions, likely provide firms and businesses with flexibility to innovate and find their most efficient solutions, whilst not requiring for a regulator to face a difficult task of estimating marginal costs of abatement by firms. As we discussed, for instance in the previous section, even under the idealized circumstances when regulator can continually adjust instruments, the adjustment to the shock to abatement technology happens at the firm's level.

This reasoning in superiority of price over quantity echoes an argument of Pizer (2003) in favor of price, but without formal analysis of this paper[20]:

> Rather than attempting to hit a fixed quantity target at any cost, we should instead price emissions at our best guess concerning their rate of marginal damage. Since there is a real risk that the costs of hitting a fixed quantity target can be extremely high - depending on growth and technology - such targets make little sense.

Finally, we find duality in our argument - one-layer of uncertainty vs two-layer of uncertainty in face of business cycle shocks- with another idea of Weitzman, laid out in his recent paper Weitzman (2014). Weitzman contrasts the properties of an idealized binding harmonized price with an idealized binding cap-and-trade system within the context of international negotiations that aim solving global warming externality problem. He argues that setting an internationally-harmonized carbon price involves only *one* layer of negotiations as opposed to *two* on quantity side. His basic intuition is as follows. Under a quantity-based system, $n$ countries

[19] Laffont (1977) provides a detailed discussion of the information structure present in policy choice problems by regulator choosing between prices or quantities in tradition of the original Weitzman's analysis.

[20] Pizer (2003) tests the robustness of the claim that under possibility of catastrophic damages, quantity instrument is preferred instrument. The existence of some thresholds of climate change is one of the few arguments for quantity-based regulations, as most analyses, including the current paper, overwhelmingly argue in favor of price rather than quantity instrument. Other arguments in favor of quantity may include political economy and many administrative challenges and for comprehensive discussion of the advantages of carbon tax vs cap-and-trade, see, e.g., Hepburn (2006) and Goulder and Schein (2013).

participating in negotiations must agree on the single aggregate level of emissions and on the distribution of aggregate emissions among $n$ parties. By contrast, a price-based system of negotiations focus on agreeing to a single one-dimensional uniform price.

# 5 Carbon taxes and business cycles

We have shown that carbon tax is approximately constant over the business cycle and our results are extension of the findings of the "price versus quantity" literature to a general equilibrium framework. Impulse response function results also demonstrate that emissions exhibit larger volatility than taxes. But our results are in contrast with the findings of the Heutel (2012) who points out to procyclical behavior of carbon taxes in response to business cycle shocks. Moreover, he finds that carbon taxes fluctuate by more than emissions in response to a productivity shock. In this section we attempt to understand what drives divergence in our results.

The procyclicality result of carbon taxes in the Heutel's model can be explained, by referring to the optimal conditions of the firms and household's Euler equations:

$$r_t = (1 - d(x_t))f'_k[1 - \tau_{Et}(1 - \mu_t)h'(y_t) - m(\mu_t)] \tag{32}$$

$$\tau_{Et} = \frac{y_t m'(\mu_t)}{h(y_t)} \tag{33}$$

$$u_{c,t} = \beta E_t u_{c,t+1}[1 - \delta + r_t] \tag{34}$$

The equation (33) is identical to the equation in our model (14) and represents the role of carbon taxes internalizing the climate externality. The setting of the theoretical framework in the Heutel's model however implies that carbon taxes also distort capital accumulation and thus return on capital (32) and thus affect intertemporal reallocation of consumption, through Euler equation (34). This means that Ramsey planner uses carbon taxes to facilitate consumption smoothing across periods. Intuitively, as abatement is costlier during economic expansions, the carbon tax must to rise to prompt firms avoid producing more emissions during expansions; opposite is true during declines in economic activity. This facilitates intertemporal reallocation of emissions across periods: emit less today than otherwise during boom but be compensated for that with higher than otherwise emissions during recessions. As emissions are by-product of output, such trade-off in emissions creates intertemporal reallocation of consumption. In line with this, Heutel points out that: "It is variance in consumption, not in pollution stock, that leads to the variance in the emissions tax". In such way, carbon taxes end up playing role that it is initially not subscribed to, and in particular macroeconomic stabilization role. Such "non-standard" outcome usually appears in the optimal taxation literature when the tax system is *incomplete*. The tax system in the Heutel's model is indeed incomplete in the sense that there are more competitive equilibrium conditions in which taxes are involved than tax instruments (Chari and Kehoe (1998)). These equations are (32) and (33).

Completeness of the tax system is important for at least two reasons. First, as shown by Chari and Kehoe (1998), Correia (1996), Aruoba and Chugh (2010) and many others, an incomplete tax system requires that new constraints reflecting this incompleteness to be added to the Ramsey problem. Second, incomplete tax systems can lead to"non-standard" policy prescriptions because some instruments end up serving as imperfect proxies for other, unavailable instruments.[21]. Thus to ensure completeness of the tax system in the framework similar to one in Heutel, there is need of introducing one additional *distortionary* tax, but since we also incorporate labor into his original model, we need to introduce two distortionary taxes: on labor and corporate income, and under such setting carbon tax would only play the role it is introduced originally for - correction of climate externality.

The conclusion that emerges out of the above discussion has important policy implications. It suggests that taxation of emissions cannot be justified on the grounds of macroeconomic stabilization tool and other than to target climate change externalities. This is similar logic to the conclusion of the optimal taxation theory applied to the taxation of energy and energy related products, that pure revenue raising is best done with wide-base taxes, such as VAT or taxes on labor, rather than carbon taxes[22].

# 6 Conclusion

The relevance and importance of the analysis of an optimal policy instrument for a regulator seeking to control CO2 emissions in the face of unexpected fluctuations in economic activity has increased very recently, particularly in the aftermath of the financial and economic crisis of 2008. In the wake of the global financial crisis, knowing whether environmental policies should be accommodative to unexpected changes in economic conditions has received intense interest. Such interest has been partly prompted by a marked and persistent drop in the price of permits within the largest cap-and-trade system, the EU's Emissions Trading Scheme (EU ETS). As argued by many observers, this was mainly driven by a combination of low demand for emissions permits caused by the recession and inflexibility of the caps on emissions to changes in economic conditions. The debate is under way on how the EU ETS system needs to be reformed to make the system more resilient to unanticipated shocks, in particular stemming from changes in economic circumstances. Another recent study (Heutel (2012)) also finds that optimal carbon taxes and emissions are procyclical with business cycles, implying that carbon pricing mechanisms should respond accordingly to economic fluctuations and cycles.

This paper seeks to contribute to this debate by analyzing the optimal design of and contrasting the relative performance of two polar instruments - price and quantities - over the business cycles. By doing so, this paper also links the price-quantity literature with the recent emerging literature that investigates the optimal design of environmental policies over business cycles. We focus on price-based and quantity-based policies, most frequently contrasted in the literature (Weitzman (1974) and his many extensions), but we acknowledge that it is possible to form hybrid instruments, which are a combination of price and quantity

[21] Correia (1996) provide examples in which an incomplete tax system results in non-zero capital-income taxation. See also discussion in Aruoba and Chugh (2010). de Miguel and Manzano (2006), for instance, show that governments use oil taxes to accomodate business cycle shocks, if it does not have enough available fiscal instruments (that is under incomplete tax system) in a small open economy that imports oil.

[22] Diamond and Mirrlees, 1971) points out the desirability of undistorted production decisions. The theorem suggests that pure revenue raising is best done with low rates on large-base taxes, such as VAT or labor taxes. This has important implications for the potential of the "double dividend" phenomenon associated with environmental policies - benefits additional to the correction of an environmental market failure - e.g., lower unemployment and/or higher GDP. The second benefit is understood to arise from the use of energy tax revenues to reduce distortionary taxes elsewehere in the economy. But since increases in energy taxes lead to tax erosion of the bases of pre-existing labor or capital taxes, in order to raise the same revenue, a higher tax burden is paid. The benefits from cutting distortionary taxes do not normally overweight the distortion created by the tax erosion effect and the double dividend arises only in specific circumstances only. For more discussion of the conditions under which a double dividend arises, see, e.g., Goulder (2013).

mechanisms, which are superior to the sole use of either policies considered. We focus on the simplest form of a cap-and-trade mechanism when considered quantity based regulation, and in particular, abstracted from so-called banking or borrowing, which in a dynamic setting, can make quantity policies more flexible. We have analyzed both state-contingent and "basic" fixed priced and fixed based quantity regulation. We find that the dynamics of the marginal costs are such that they tend to increase during booms and to decline during recessions making a price instrument preferred to a quantity instrument. We also find that a carbon tax is essentially constant over the business cycle. Our results thus provide an additional argument and lend further support to the findings of Pizer (1999), Hoel and Karp (2002) and others who argue in favor of a price rather than quantity instrument in controlling CO2 emissions in the short-run, when damages from climate changes remains relatively "flat".

# 7 Appendix: graphs

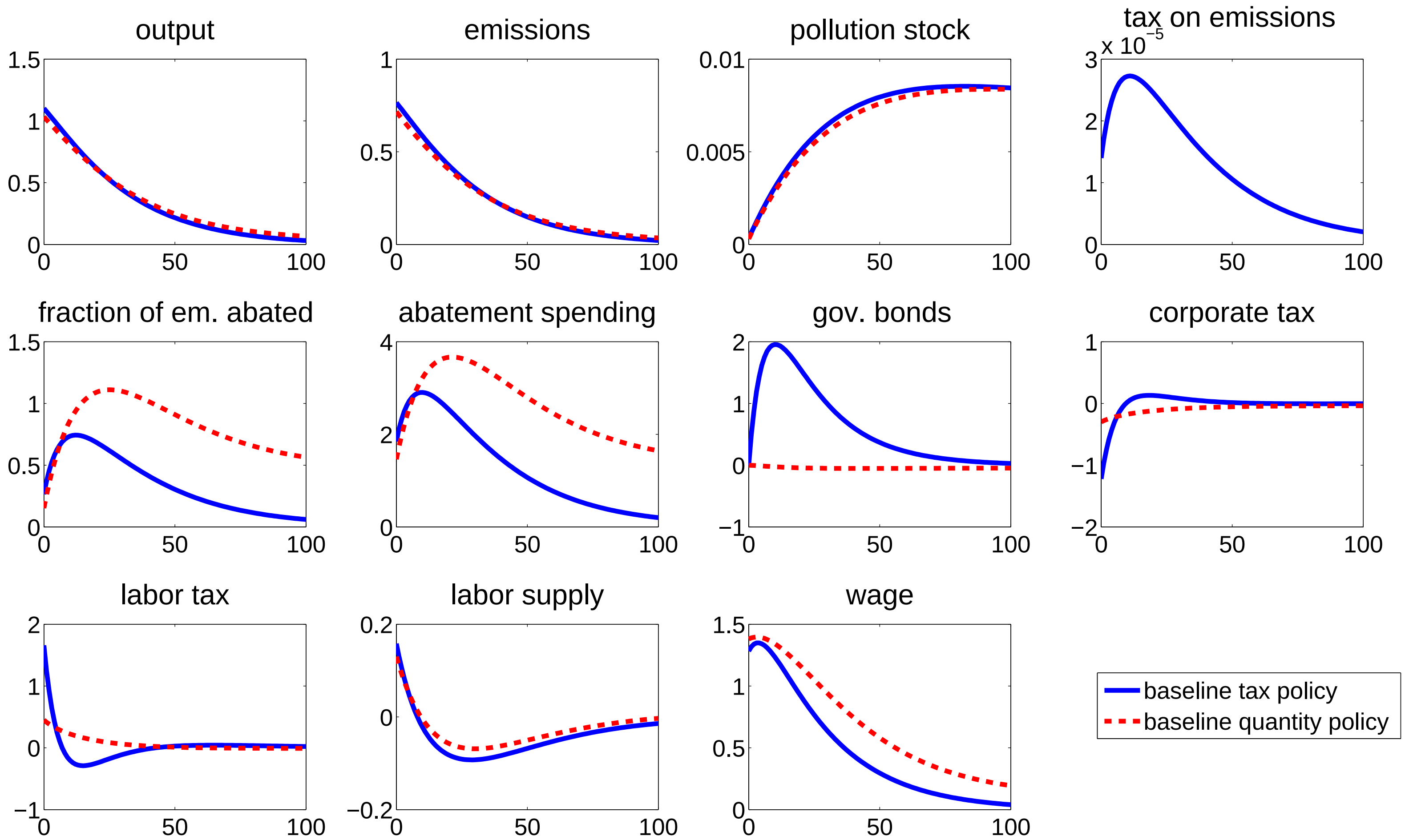


Figure 1: Impulse response under baseline carbon tax and cap-and-trade policies to a positive TFP shock

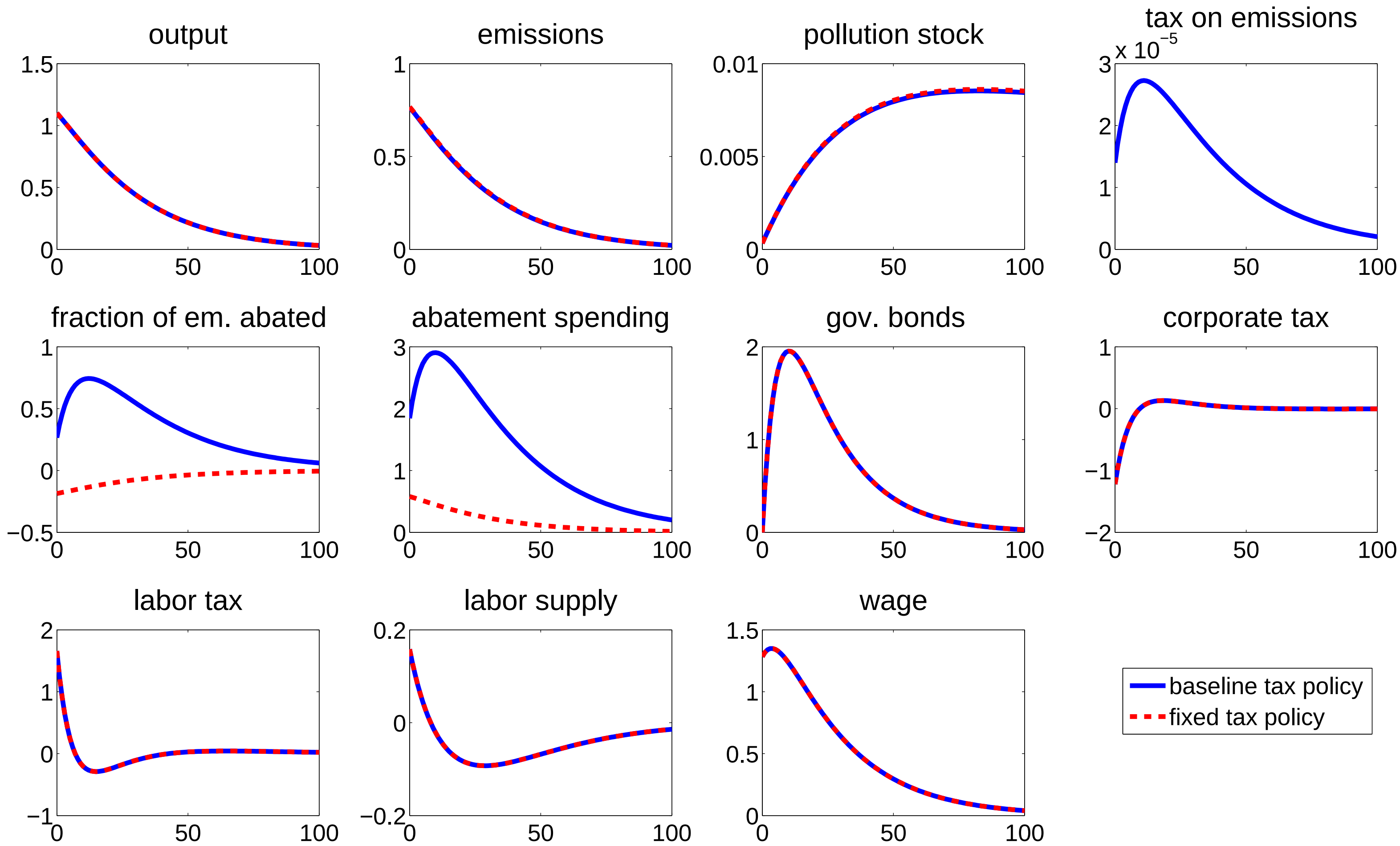


Figure 2: Responses under baseline carbon tax and fixed carbon tax policies to a positive TFP shock

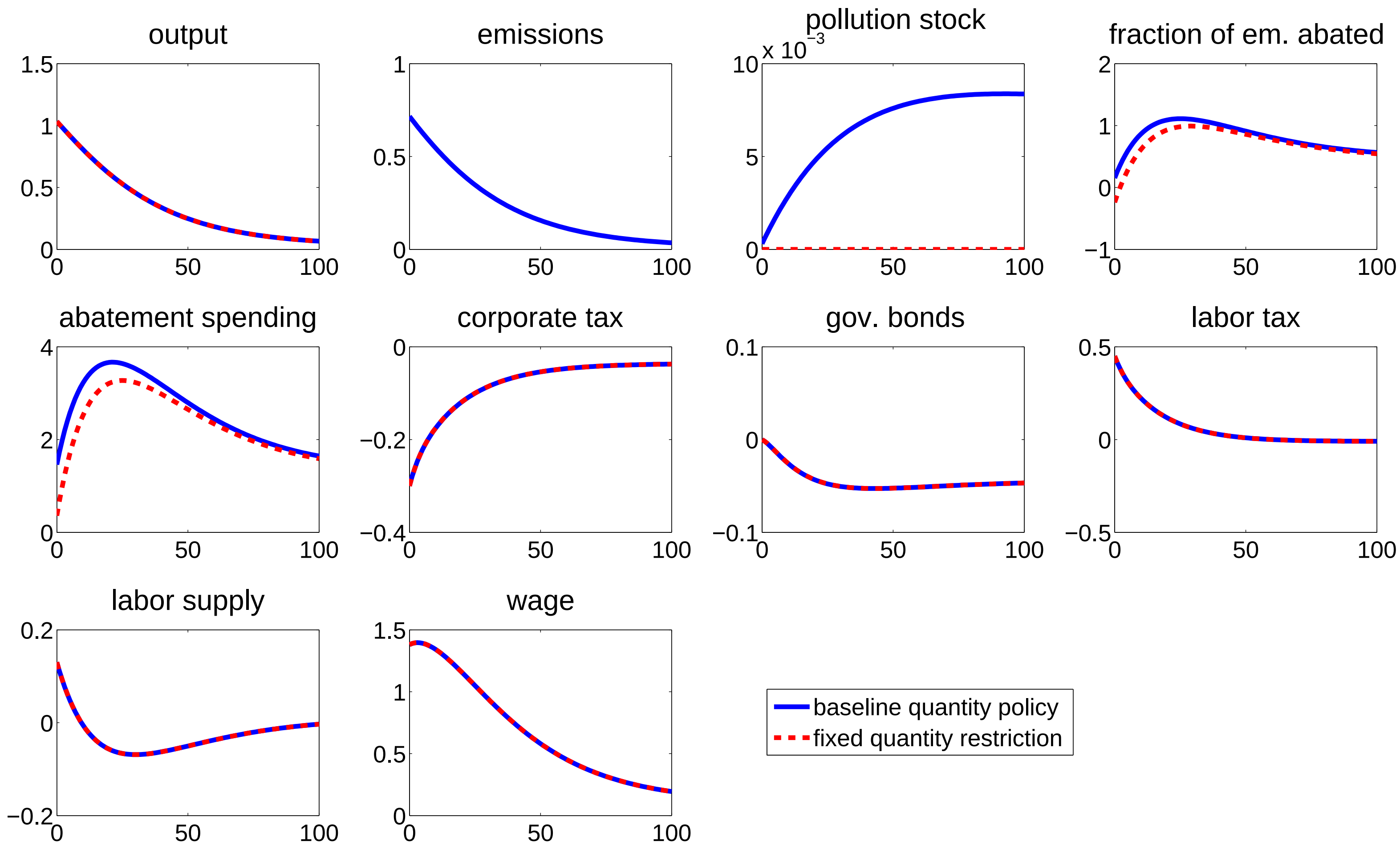


Figure 3: Responses under baseline cap-and-trade and fixed quantity restriction policies to a positive TFP shock

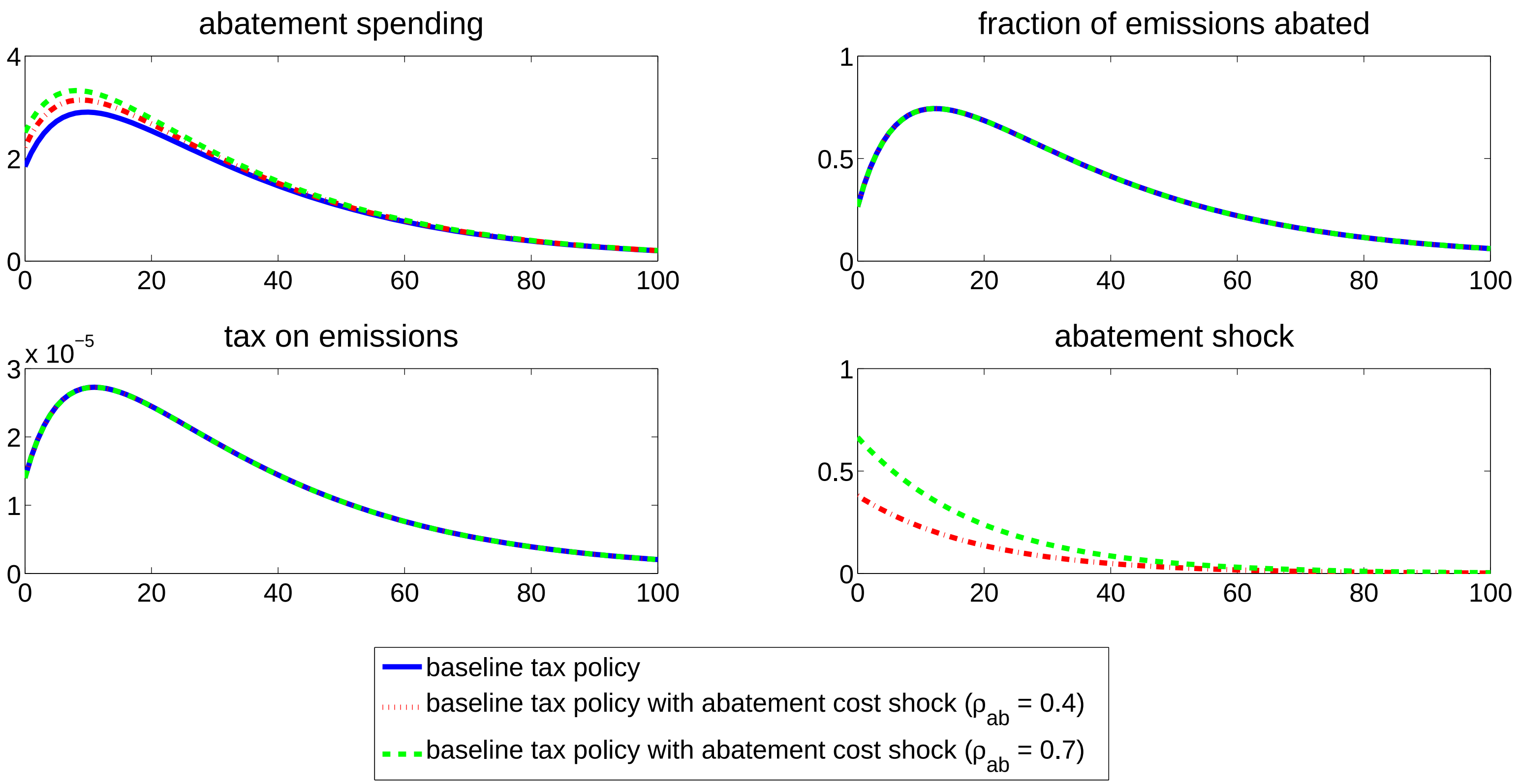


Figure 4: Responses to a TFP shock and to a TFP shock correlated with a shock to abatement technology

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# 8 Online Appendix: not for publication

## 8.1 Productivity shock and associated increase in the stock of pollution

In our baseline model a 1% TFP shock results in an increase in the pollution stock of about 0.008% over 25 years. How does this number relate to reality? The Mauna Loa Observatory[23] provides monthly information on the concentration of the atmospheric carbon dioxide. The concentrations are expressed in parts per million (ppm), which give the ratio of the number of greenhouse gas molecules to the total number of molecules of dry air. The Carbon Dioxide Information Analysis Center[24] provides conversion tables that enable us to convert this measure of atmospheric CO2 concentration into gigatons of carbon. 1 ppm by volume of atmosphere CO2 equals to 2.13 GtC. This measure does not count the mass of oxygen in the CO2 molecule, but since the atom weight of carbon (12 units) and of CO2 (44 units), one unit of GtC is equivalent to 44/12=3.67 GtCO2 (see Dessler and Parson, 2010, p.201), and 1ppm is therefore equivalent to 2.13 GtC and 7.82 GtCO2.

As of July 2014, the concentration of CO2 in atmosphere stood at 399.00 ppm or equivalently at 849.87 GtC or 3059.53 GtCO2. If we treat this value as our steady state, an additional increase of 0.008% in the pollution stock over 25 years time period, as suggested by impulse response function, corresponds to 0.032 ppm, or 0.068 GtC and 0.24 GtCO2 increase in the concentration of CO2 in atmosphere. World CO2 emissions in 2012 stood at 34.5 GtCO2. Assuming that the level of yearly emissions does not change, over a period of 25 years the world will emit 862.5 GtCO2 meaning that an additional increase in the CO2 stock due to a TFP shock constitutes only 0.028% of all emissions over a 25-year period.

Intergovernmental Panel on Climate Change (2007) reports a table (Table 5.1, p. 67) that relates CO2 concentration in the atmosphere to the global temperature and average sea level increase above pre-industrial levels. At CO2 concentrations of 350-400 ppm (current level), global temperature increase above pre-industrial levels ranges from 2.0-2.5°C, and the global average sea level rises above the pre-industrial level from 0.4-1.4 m. For the 400-440 ppm range the corresponding numbers are: 2.4-2.8°C and 0.5-1.7°C. Thus an increase in CO2 concentration from the current level to 440 ppm (by 40 ppm) could lead to a maximal increase in the temperature above pre-industrial levels of 0.4°C (2.8°C-2.4°C) and the maximum sea rise level of 0.3 m (1.7m-1.4m). Treating these estimates as our reference, we can conclude that an additional increase in CO2 concentrations of 0.032 ppm would correspond to an increase in temperature by 0.0003192°C and an additional increase in the sea level by 0.0002394 m over 25 years interval following the productivity shock.

[23] http://co2now.org/

[24] http://cdiac.ornl.gov/pns/convert.html

## 8.2 Appendix A: Data sources

In this section we describe data sources and USA data entry components into table 2.
Data from the NIPA tables are for year 2013.

- GDP - from the NIPA Table 1.5.5. Gross Domestic Product, Expanded Detail, line 1.
- Personal consumption expenditure - from the NIPA Table 1.5.5. Gross Domestic Product, Expanded Detail, line 2.
- Government consumption expenditure - from the NIPA Table 1.5.5. Gross Domestic Product, Expanded Detail, line 55+line 58+line 61.
- Government gross investment - from the NIPA Table 1.5.5. Gross Domestic Product, Expanded Detail, line 56+line 59+line 62.
- Gross private domestic investment - from the NIPA Table 1.5.5. Gross Domestic Product, Expanded Detail, line 26.
- Emissions per unit of total GDP - for 2012 for the US from the United States Environmental Protection Agency (2013), p.ES-24, Table ES-9.
- Fraction of emissions abated: derived from author's calculations with original data from Creyts et al. (2007), who provide estimates of potential abatement projections for greenhouse gases in the US. They estimate that the US would potentially abate cumulative 3GtCO2 of emissions for the period 2005-2030. Assuming the same amount of emissions abated every year during 25 years time period, from 2005 to 2030, and given that total greenhouse gas emissions amounted to 6.5GtCO2 by the US in 2012 (United States Environmental Protection Agency (2013)), we obtain 1.85%, an estimate of the fraction of emissions abated in 2012.
- Abatement Spending - from the U.S.Census Bureau (2008), Table 1 (Pollution Abatement Operating Costs) and Table 2 (Pollution Abatement Capital Expenditures). U.S.Census Bureau (2008) is a survey of a sample of 20000 manufacturing plants, which, according this survey, spent 20677.6 mln USD on pollution abatement operating costs and 5907.8 mln USD on pollution abatement capital expenditures in 2005. By combining these data with the US GDP data for 2005, USD 13095.4 bln, we obtain estimate of the fraction of abatement spending in GDP, 0.2%, reported in the main part of the paper.
- Labor tax - OECD, Taxing Wages 2014 (May 2014), `http://www.oecd-ilibrary.org/taxation/taxing-wages-2014_tax_wages-2014-en`
- Central government corporate income tax rate - OECD, Taxation of Corporate and Capital Income, Corporate Income Tax
- Revenue from environmental taxes - Congressional Budget Office (2013) estimates potential tax revenues from carbon taxes at 1.2 trillion USD in a 10 years period. Assuming a yearly revenue of 0.12 trillion USD, we calculate it as a fraction of US GDP in 2013 and obtain the estimate 0.7% of GDP.
- Steady state value of government bonds as relation to output - based on Table B79 (federal debt held by public as percent of gross domestic product) from Council of Economic Advisers (2013).

### 8.3 Appendix B: First-order conditions of the Ramsey problem

The first-order conditions of the Ramsey problem outlined in section 2.6 are given by:

$$u_c'(t) - \lambda_t u_{cc}''(t) + \lambda_{t-1} u_{cc}''(t)(1-\delta+r_t) + \Omega_t - (\Lambda_t \tau_{Lt} l_t - \varsigma_t(1+\tau_{Lt}))\frac{u_{Lc}''(t)u_c'(t) - u_L'(t)u_{cc}''(t)}{(u_c'(t))^2} = 0 \quad (35)$$

$$\lambda_{t-1} u_c'(t)\frac{\partial r_t}{\partial \mu_t} + \Omega_t m'(\mu) y_t - \chi_t y_t m''(\mu) - \Lambda_t \tau_{Et} h(y_t) + \varsigma_t(1-d(x_t)) f_L'[\tau_{Et} h'(y_t) - m'(\mu_t)] + \Phi_t h(y_t) = 0 \quad (36)$$

$$\begin{aligned} &\lambda_t \beta u_c'(t+1)\frac{\partial r_{t+1}}{\partial k_t} + \Omega_t - \beta(1-\delta)\Omega_{t+1} - \lambda_{pt+1}\beta(1-d(x_{t+1})) f_k'(t+1) + \\ &+\beta\varsigma_{t+1}(1-d(x_{t+1})) f_{kL}(t+1)[1-\tau_{ct+1} - \tau_{Et+1}(1-\mu_{t+1} h'(y_{t+1}) - m(\mu_{t+1})] = 0 \end{aligned} \quad (37)$$

$$\begin{aligned} &\lambda_{t-1} u_c' \frac{\partial r_t}{\partial y_t} + \Omega_t(m(\mu_t)-1) + \chi_t[\tau_{Et} h'(y_t) - m'(\mu_t)] + \Lambda_t[\tau_{Et}(1-\mu_t)h'(y_t) + \tau_{ct}] + \\ &+\lambda_{pt} + \varsigma_t(1-d(x_t)) f_L'[-\tau_{Et}(1-\mu_t)h''(y_t)] - \Phi_t(1-\mu_t)h'(y_t) = 0 \end{aligned} \quad (38)$$

$$\lambda_{t-1} u_c' \frac{\partial r_t}{\partial x_t} + \lambda_{pt} d'(x_t) f(t) - \varsigma_t d'(x_t) f_L'[1-\tau_{ct} - \tau_{Et}(1-\mu_t)h'(y_t) - m(\mu_t)] + \Phi_t - \beta\eta\Phi_{t+1} = 0 \quad (39)$$

$$\begin{aligned} &u_L' - \lambda_t u_{cL}'' + \lambda_{t-1} u_{cL}''(1-\delta+r_t) + \lambda_{t-1} u_c' \frac{\partial r_t}{\partial l_t} + \\ &+\Lambda_t[-\frac{u_L'}{u_c'}\tau_{Lt} - \tau_{Lt} l_t \frac{u_{Lc}'' u_c' - u_L' u_{cc}''}{(u_c')^2}] - \lambda_{pt}(1-d(x_t)) f_L' + \varsigma_t(1+\tau_{Lt})\frac{u_{Lc}'' u_c' - u_L' u_{cc}''}{(u_c')^2} + \\ &+\varsigma_t(1-d(x_t)) f_{LL}''[1-\tau_{ct} - \tau_{Et}(1-\mu_t)h'(y_t) - m(\mu_t)] = 0 \end{aligned} \quad (40)$$

$$-\Lambda_t \frac{u_L'}{u_c'} l_t + \varsigma_t \frac{u_L'}{u_c'} = 0 \quad (41)$$

$$\lambda_{t-1} u_c' \frac{\partial r_t}{\partial \tau_{Et}} + \chi_t h(y_t) + \Lambda_t(1-\mu_t)h(y_t) - \varsigma_t(1-d(x_t)) f_L'(1-\mu_t)h'(y_t) = 0 \quad (42)$$

$$\lambda_{t-1} u_c' \frac{\partial r_t}{\partial \tau_{ct}} + \Lambda_t y_t - \varsigma_t(1-d(x_t)) f_L' = 0 \quad (43)$$

$$\Lambda_t \rho_{Bt} - \beta\Lambda_{t+1} = 0 \quad (44)$$

$$u_g' - \lambda_t u_{cg}'' + \lambda_{t-1} u_{cg}''(1-\delta+r_t) - \Lambda_t + \Omega_t + (\varsigma_t(1+\tau_{Lt}) - \Lambda_t \tau_{Lt} l_t)\frac{u_{Lg}'' u_c' - u_L' u_{cg}''}{(u_c')^2} = 0 \quad (45)$$

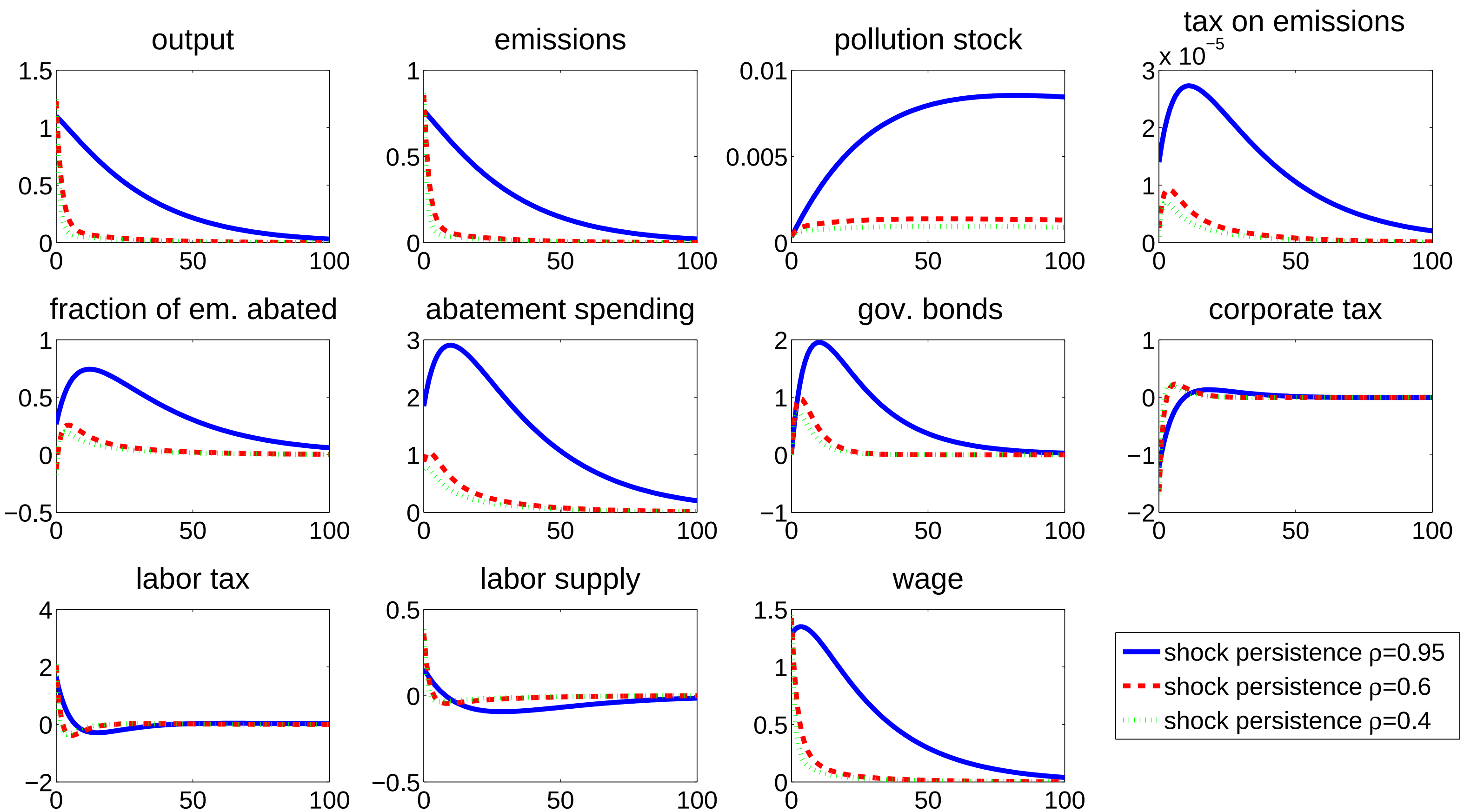


Figure 5: Responses to a TFP shock under different values of the persistence of the shock $\rho$