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Policy Interactions and the Transition to Clean Technology

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Abstract

In light of increased willingness to address climate change challenges and the growing role and interest of central banks among other institutions in mitigation strategies, we develop a macro-financial-environmental DSGE model to assess different types of fiscal, monetary, and macroprudential policies aimed at reducing CO2 emissions. Our model includes financial intermediaries facing endogenous balance-sheet constraints and is comprised of a two-sector economy (i.e. green and dirty sectors), where each sector faces different emissions intensities. Banks fund both green and dirty sectors, which otherwise operate separately. Green firms operate under a technology that enables them to emit less than dirty firms. We then use the model to assess the effect of above policies, namely fiscal, monetary, and macroprudential. First, we present the baseline fluctuations under the linear and the ZLB environments, before moving to the policy scenario analyses. We show that a substantial carbon tax is needed in the Euro Area to be aligned with the Paris Agreement. However, in terms of welfare, the optimal tax is of a much smaller magnitude. Thus, in order to avoid further distorting the welfare, macroprudential and monetary policies could be used and could play a major role. In particular, we find that a macroprudential policy favorable to the green sector boosts green capital and output. In respect to QE, we find that a carbon tax improves the benefits of both green and dirty asset purchases. However, we find that macroprudential policy is needed to provide an incentive to central banks to engage in green QE. This work aims to provide central banks and similar institutions with the tools to contribute to climate change mitigation, and demonstrates the importance of including these institutions in the push to reduce global emission levels.

Keywords: Climate Change, Two-Sector Economy, Optimal Fiscal Policy, Zero-Lower-Bound, Macroprudential Policy, Quantitative Easing.

JEL: Q58, E32, E52.

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1 Introduction

Climate change has shifted from a fringe issue to a worldwide emergency. Our understanding of the phenomena and our willingness to act have developed significantly, in part paralleling the ways in which climate change is being experienced around the globe. It has become a hot topic where academics, industry, and lay people alike are finding common ground. As such, growing academic awareness is leading to important literature in the domain. The implementation of a strategy for the substantial reduction of greenhouse gases (GHG) at the global level has become a major priority. Since the Rio Conference in 1992, a debate has raged in academic and political circles over the growth-environmental trade-off. Discussions focus on the means by which economic activities could align with environmental concerns instead of being hindered by assumed mutual exclusivity. In practice, especially in the short and medium terms, however, financial and economic activity on one side, and environmental policy on the other, are in tension. A need for short-term policies aimed at bridging environmental quality and economic efficiency, as well as financial stability, in order to foster economic sustainability are in dire need. Of special concern are climate actions that may strongly impact macroeconomic activity[1] given the potentially high added cost of GHG offsetting. With the substantial effects of climate actions on the overall economy, a growing body of research from the field of macroeconomics and macro-finance, among others, are now tackling these issues.

A growing interest in a "Green Financial System"—as outlined in the Paris "One Planet Summit" held in December 2017 where "[E]ight central banks and supervisors established a Network of Central Banks and Supervisors for Greening the Financial System (NGFS)"—is putting climate change challenges at the heart of the macro financial system. NGFS [2019] recently published a call for action in which it outlined the role central banks can play in monitoring and mitigating climate change, considering the adverse impact it could have on financial stability. Integrating climate change challenges within the macro-monetary and

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[1]At least in the short to medium term.

macro-prudential frameworks is increasingly gaining momentum within institutions such as the European Central Bank (ECB), thus making research combining macroeconomics and climate change environmental concerns extremely relevant to policy makers. Earlier this year, Bolton, Despres, Pereira Da Silva, Samama, and Svartzman [2020] advocated in a joint publication from the BIS and *Banque de France* for "better coordination of fiscal, monetary and prudential and carbon regulations", which is perfectly in line with the findings in our article.

Tackling climate change challenges requires innovating classic research paths, which tend to favor the use of models that capture only one of the following: environmental variables, macroeconomic behavior, or monetary and financial policy. However, as underlined by Rudebusch and Swanson [2012], this limited modeling approach is reductive, and indicates that macroeconomic modeling suffers from theoretical incompleteness. Policy recommendations (based on such models) that aim to mitigate GHG effects, should be able to capture macroeconomic variations, monetary and financial policy, as well as environmental constraints, as these are tightly linked. In addition, policymakers were and are more than ever in need of models that capture the most interlinked image of the economy in order to take the best course of action. In the face of climate change, being able to capture both the variations of the macroeconomy and the effect of monetary policy is of high importance, as investment and labor, among others, rely on outcomes of such models (e.g. The ECB usage of Dynamic Stochastic General Equilibrium (DSGE) models).

Given this gap in the environmental-macroeconomic-monetary-macroprudential approach, our paper seeks to assess the interactions among environmental policies: i) fiscal, ii) monetary, and iii) macroprudential, each of which is aimed at reducing CO2 emissions, using a heterogeneous macroeconomic production economy. To the best of our knowledge, this is the first article to look at the interaction between environmental, monetary, and macroprudential policies in a DSGE model under both a non-zero-lower-bound (non-ZLB) environment and a ZLB environment[2]. Going forward, it will be an important component to

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[2]The ZLB environment corresponds to an environment where nominal interest rates are close to zero and can't be further lowered by central banks.

successfully fight climate change. Our paper falls within at least three strands of literature. We first build on the canonical versions of New Keynesian (NK) models such as Woodford [2003], Christiano, Eichenbaum, and Evans [2005] or Smets and Wouters [2003] to derive the core of our economy[3]. Second, we add environmental components as in Heutel [2012] among others to introduce the environmental components, which allows for the analysis of the dynamics of the economy under the presence of the CO2 externality. However, as opposed to Annicchiarico and Di Dio [2015], we differentiate between green and dirty firms as opposed to using one sole representation, thus borrowing from the multi-sector literature of Woodford [2003] and Carvalho and Nechio [2016] among others. Finally, we include balance sheet constrained financial intermediaries as in Gertler and Karadi [2011]. Because we introduce a macroprudential authority that can alter this constraint, we also draw on Pietrunti [2017] and Gertler, Kiyotaki, and Queralto [2012].

One of our main findings is that an environmental tax efficiency on emission reduction heavily depends on the abatement efficiency (i.e. low transition cost). Moreover, the optimal Ramsey tax policy from a welfare perspective is found to be of a small magnitude, suggesting that a tax policy isn't enough for the climate change mitigation strategy. Thus in order to allow for more flexibility, and to ease the welfare burden, other policies are greatly needed. Monetary and macroprudential policies could therefore play a major role in offsetting climate change. In particular, we find that a macroprudential policy favorable to the green sector boosts green capital and output, meaning that there is a lower emissions to output ratio. In respect to quantitative easing (QE), we find that a carbon tax improves the benefits of both green and dirty asset purchases. However, we find that macroprudential policy is needed to provide an incentive to central banks to engage in green QE. This means that the choice between dirty and green QE implies a trade-off between higher output and lower emissions. Our actual findings could be further reinforced if we were to see a transition to a greener economy favoring the green sector over the dirty one, as illustrated in our simulated transition in Figure 1 and Figure 2, and as argued in the work carried by Acemoglu, Akcigit,

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[3]Note that for simplicity we abstract from wages rigidities.

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Hanley, and Kerr [2016] where the focus in on the long-term transition strategies.

Merging these different sets of policy tools will not only help contribute to this burgeoning field of research and address the gaps identified above, but will also set the path for new analysis in macroeconomics, environmental policy, and monetary policy. The proposed approach can help shape policy making, and empower central banks among other institutions address one the most pressing issues of our time.

The paper is organized as follow: section 2 presents the model, section 3 explains the calibration, section 4 displays the results and section 5 concludes.

2   The Model

Using the NK-DSGE framework as a foundation, the present paper investigates the potential role of fiscal policy, central bank monetary policy, and macroprudential policy, in mitigating climate change challenges. We first model our two-sector economy following Woodford [2003] for the labor specific component within the household, and the two-sector production economy following Carvalho and Nechio [2016]. Then, we model the environmental component following Nordhaus [2008] and Heutel [2012], among others. Finally, drawing from Gertler and Karadi [2011], we model the financial intermediaries and the banking sector.

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \ldots \infty)$. The production sectors produce two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms via financial intermediaries. Public authorities decide on the fiscal and environmental policy, while the central bank decides on the monetary and macroprudential policy.

2.1   The Household

At each period, the representative households supply two types of labor to the sectors of which our economy is comprised (i.e 'green' and 'dirty' sectors denoted by $k \in \{g, d\}$[4]), while

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[4]where g refers to the green sector and d to the dirty sector.

they also consume and save. Households have two choices to save: lending their money either to the government or to financial intermediaries that will finance firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households cannot lend their money to a financial intermediary owned by one of their members. Household members who are workers supply labor and return their salaries to the household to which they belong.

Agents can switch between the two occupations over time. There is a fraction f of agents who are bankers and a probability θ_B that a banker remains a banker in the next period. Thus, $(1-f)\theta_B$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to the household, which will use them as start-up funds for the new banker.

Households solve the following maximization problem:

$$\max_{\{C_t, L_{t,k}, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \frac{\varepsilon_{t+i}^B}{\varepsilon_t^B} \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L_{k,t+i}^{1+\varphi} \right] \tag{1}$$

s.t.

$$C_t + B_{t+1} = \sum_k \left(\frac{W_{t,k}}{P_t} L_{t,k} + \Pi_{t,k} \right) + \frac{T_t}{P_t} + R_t B_t, \tag{2}$$

where $\beta \in (0,1)$ is the discount factor, parameters σ, $\varphi > 0$ shape the utility function of the representative household associated with risk consumption C_t, and labor in each sector k is $L_{t,k}$. The consumption index C_t is subject to external habits with degree $h \in [0;1)$ while $\chi_k > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked for each sector k. Labor supply $L_{t,k}$ in each sector is remunerated at nominal wage $W_{t,k}$. $\Pi_{t,k}$ is profits from the ownership of firms (both financial and non-financial) that will serve as start-up funds for the new banker and T_t is lump sum taxes. As we assume that intermediary deposits and government bonds are one period bonds, $R_t B_t$ is interest received on bonds held. B_{t+1} is bonds acquired. Finally, ε_t^B is a preference shock that follows an AR(1) process: $\varepsilon_t^B = \rho_B \varepsilon_{t-1}^B + \sigma_B \eta_t^B$, with $\eta_t^B \sim \mathcal{N}(0,1)$..

Solving the first order conditions and denoting ϱ_t as the marginal utility of consumption, the labor/supply and consumption/savings equations are:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{3}$$

$$\varrho_t = \chi_k \frac{L_{t,k}^{\varphi}}{W_{t,k}/P_t}, \tag{4}$$

$$1 = \beta E_t \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \Lambda_{t,t+1} R_{t+1}, \tag{5}$$

where the stochastic discount factor is the expected variation in marginal utility of consumption: $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$.

2.2 The Firms

2.2.1 The Final Firms

Using the multi-sector framework from Carvalho and Nechio [2016], and under non-perfect competition, we assume that production is comprised of two sectors as outlined above: green and dirty, indexed by $k \in \{g, d\}$, where representative firms produce a final good $Y_{t,k}$ in these two competitive sectors, using no more than capital and labor to produce the intermediate good Y_{jt} where $j \in (0, 1)$ is the continuum of intermediate goods firms. The "bundling" of intermediate goods within the two sectors leads to a final good. Goods are symmetric and act under perfect competition. The final economy good is a constant elasticity of substitution aggregate of the two sectors:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1 - \varkappa)^{\frac{1}{\theta}} Y_{t,d}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{6}$$

with $\theta \in (1, \infty)$ the elasticity of substitution between the two sectors, and $\varkappa$ the weight of each sector. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j in each of the two sectors k

at prices $P_{jt,k}$:

$$\max_{Y_{jt}} \Pi_t^{\text{Final}} = P_t Y_t - \varkappa \int_0^1 P_{jt,g} Y_{jt,g} dj - (1 - \varkappa) \int_0^1 P_{jt,d} Y_{jt,d} dj, \tag{7}$$

where the aggregation of green and dirty firms reads as:

$$Y_{t,k} = \int_0^1 \left(Y_{jt,k}^{1-\frac{1}{\theta_k}} \right)^{\frac{1}{1-\frac{1}{\theta_k}}}. \tag{8}$$

However, while we assume a constant elasticity of substitution between the final sectors, we consider a different elasticity of substitution θ_k between differentiated intermediate goods of the two sectors. As the goods of the two sectors entail different costs, a different elasticity of substitution is considered. This assumption, which shapes the marginal cost structure, is based both on theoretical work of Tucker [2010] as well as on the empirical findings of Chegut, Eichholtz, and Kok [2019] and Chan, Li, and Zhang [2013], where it is found that green projects entail higher marginal cost (7-13 percent higher costs for green projects in the construction industry compared to non green projects depending on the 'greenness' of the project, and 5-7 percent higher costs in the cement and iron & steel sectors, respectively).

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}} \right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} Y_t. \tag{9}$$

Under perfect competition and free entry, the price of the final good is denoted P_t, while the price $P_{t,k}$ is the price index of sector-k intermediate goods. Finally, the price $P_{jt,k}$ is the price charged by firm j from sector k.

The prices of final aggregate goods and for each sector are given by:

$$P_t = \left(\varkappa P_{t,g}^{1-\theta} + (1 - \varkappa) P_{t,d}^{1-\theta} \right)^{\frac{1}{1-\theta}}, \tag{10}$$

$$P_{t,k} = \left(\int_0^1 P_{jt,k}^{1-\theta_k} dj \right)^{\frac{1}{1-\theta_k}}. \tag{11}$$

2.2.2 The Intermediate Firms

As our economy is comprised of two categories of firms green corresponding to environmentally-friendly firms with a stock of capital k_g and dirty with higher emissions rate of a stock of capital k_d relying in CO2 intensive components.

The representative firms j in each sector k of the modeled economy seek profit maximization by making a trade-off between the desired level of capital and labor. Furthermore, the firms will incur externality costs and choose the level of abatement to maximize their profit. As presented in Heutel [2012] RBC model, the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externalities deteriorate the environment and the stock of pollutant alters production possibilities of firms as follows:

$$Y_{jt,k} = (1 - d(X_t))\varepsilon_t^{A_k} K_{jt-1,k}^{\alpha} L_{jt,k}^{1-\alpha}, \ \alpha \in (0,1), \tag{12}$$

where $d(X_t)$ is a convex polynomial function of order 2 displaying the stock of pollution ($d(X) = a + bX + cX^2$, with $(a,b,c) \in \mathbb{R}^3$, which is borrowed from Nordhaus [2008]).

In addition, α is the classical elasticity of output with respect to capital, and $\varepsilon_t^{A_k}$ is a sector-specific technology shock that follows an $AR(1)$ process: $\varepsilon_t^{A_k} = \rho_{A_k}\varepsilon_{t-1}^{A_k} + \sigma_{A_k}\eta_t^{A_k}$, with $\eta_{lt}^{A_k} \sim \mathcal{N}(0,1)$. Furthermore, the carbon emissions stock X follows a law of motion:

$$X_t = (1 - \gamma_d)X_{t-1} + E_{jt} + E^*, \tag{13}$$

where E_{jt} is the flow of emissions from both the green and dirty firms ($E_{jt} = \varkappa E_{jt,g} + (1 - \varkappa)E_{jt,d}$) at time t and γ_d is the decay rate. E^* represents the rest of the world emissions.

The amount of emissions is modeled by a nonlinear technology such as abatement costs that would reduce the inflow of emissions:

$$E_{jt,k} = (1 - \mu_{jt,k})\varphi_t Y_{jt,k}. \tag{14}$$

The emissions E at firm level are proportional to the production Y with φ_t the proportion of emissions to output.[5] Also, emissions could be reduced through an abatement effort μ. The firms are allowed to invest in an abatement effort, which is assumed to be different between the green and dirty sectors, thus incurring the firms' direct costs.

We model the direct abatement effort costs following Heutel [2012]:

$$Z_{jt,k} = f(\mu_{jt,k})Y_{jt,k}, \ \theta_1 > 0, \ \theta_2 > 1, \tag{15}$$

where

$$f(\mu_{jt,k}) = \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}}, \tag{16}$$

with $\theta_{1,k}$ and $\theta_{2,k}$ representing the cost efficiency of abatement parameters for each sector.

Thus the profits of our representative intermediate firms in each sector Π_{jt} will correspond to the difference between the revenues of the intermediate firms and their costs. The revenues are the real value of intermediate goods $Y_{jt,k}$, while the costs generate from wages $W_{t,k}$ (paid to the labor force $l_{jt,k}$), investment in capital $K_{jt,k}$ (with returns $R_{t,k}^K$), abatement $\mu_{jt,k}$ (the firms are enduring), and any environmental damages captured by emissions $E_{jt,k}$.

$$
\begin{aligned}
\Pi_{jt,k} &= \frac{P_{jt,k}}{P_t}Y_{jt,k} - \frac{W_{t,k}}{P_t}L_{jt} - \frac{R_{t,k}^K}{P_t}K_{jt,k} - \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}}Y_{jt,k} - \frac{\tau_{et,k}}{P_t}E_{jt,k} \\
&= \left(\frac{P_{jt,k}}{P_t} - MC_{t,k}\right)Y_{jt,k}.
\end{aligned}
\tag{17}
$$

As firms are not free to update prices each period, they first choose inputs so as to minimize cost, given a price, subject to the demand constraint.

The cost-minimization problem yields the real marginal cost which can be expressed following first-order conditions with respect to the firm's optimal choice of output and abatement as follows:

$$\Psi_{jt,k} = \Psi_{t,k} = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{1}{\varepsilon_t^{A,k}(1-d(X_t))} \left(\frac{W_{t,k}}{P_t}\right)^{1-\alpha} \left(\frac{R_{t,k}^K}{P_t}\right)^\alpha, \qquad (18)$$

$$\varphi_t \frac{\tau_{et,k}}{P_t} Y_{jt,k} - \theta_{1,k}\theta_{2,k}\mu_{jt,k}^{\theta_{2,k}-1} Y_{jt,k} = 0, \qquad (19)$$

$$MC_{jt,k} = MC_{t,k} = \Psi_{t,k} + \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \frac{\tau_{et,k}}{P_t}(1-\mu_{t,k})\varphi_t, \qquad (20)$$

where $\Psi_{jt,k} = \Psi_{t,k}$ is the marginal cost component related to the same capital-labor ratio all firms of each sector choose (18). This marginal cost component is common to all intermediate firms, however it is different across sectors.

The equation (19) is the optimal condition on abatement: abating CO2 emissions is optimal when its marginal gain equals its marginal cost. This highlights the key role of emissions in shaping price dynamics where the production of one additional unit of goods reduces the profits of firms, which in turn is partially compensated for by the marginal gain from emitting GHGs in the atmosphere.

In addition, as abatement effort μ is common to all firms of the same sector, as is the cost of abatement to which firms of the same sector are subject, the total marginal cost captures both abatement costs and emission costs (20).

Also, we note that in the case of the laissez-faire scenario, $MC_{t,k} = \Psi_{t,k}$ as the firms are not subject to emissions and abatement constraints.

In addition, the monopolistic firms engage in infrequent price setting à la Calvo. We assume that intermediate goods producers for each sector re-optimize their prices $P_{jt,k}$ only at the time when a price change signal is received. The probability (density) of receiving such a signal h periods from today is assumed to be independent from the last time the firm received the signal. A number of firms ξ will receive the price-change signal per unit of time. All other firms keep their old prices. Thus, the profit maximization of our intermediate firms reads as follows:

$$\max_{P_{jt,k},\mu_{jt}} \mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \sum_k \Pi_{jt+i,k} \tag{21}$$

$$s.t. \ Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t, \tag{22}$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor, or as commonly called in the macro-finance literature, the pricing kernel (for $i=1$ we note $M_{t,t+1} = \beta \Lambda_{t,t+1}$ as in Jermann [1998]).

The NK Philips Curve pricing equations[6] are as follows:

$$p_{t,k}^* = \frac{P_{t,k}^*}{P_t} = = \frac{\theta_k}{\theta_k - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \mathrm{MC}_{t,k+i} \Im_{t,k+i}}{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \Im_{t,k+i}}, \tag{23}$$

where

$$\begin{aligned}\Im_{t,k+i} &= \left(\frac{1}{P_{t,k+i}}\right)^{-\theta_k} \left(\frac{P_{t,k+i}}{P_{t+i}}\right)^{-\theta} P_t^\theta Y_{t+i}\\ &= P_{t,k+i}{}^{\theta_k - \theta} \left(\frac{P_{t+i}}{P_t}\right)^\theta Y_{t+i},\end{aligned} \tag{24}$$

or equivalently:

$$p_{t,k}^* = \frac{P_{t,k}^*}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}}, \tag{25}$$

$$\text{with:} S_{t,k} = P_{t,k}^{\theta_k - \theta} \Psi_{t,k} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^\theta S_{t+1,k}, \tag{26}$$

$$\text{and:} \Theta_{t,k} = P_{t,k}^{\theta_k - \theta} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^{\theta-1} \Theta_{t+1,k}, \tag{27}$$

$$\text{and:} \Upsilon_{t,k} = P_{t,k}^{\theta_k - \theta} \left[\theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \frac{\tau_{et,k}}{P_t}(1 - \mu_{t,k})\varphi_t\right] Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi_{t+1}^\theta \Upsilon_{t+1,k}, \tag{28}$$

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[6]For the full mathematical derivations and algebra, please refer to the online appendix.

with inflation $\pi_t = P_t/P_{t-1}$.

The pricing equation below is obtained simply by equating the dynamic marginal revenues to the dynamic marginal costs, thus, yielding an optimal pricing condition p^*. As in each period a fraction ξ of the intermediate firms of each sector choose their optimal price P_k^*, we can rewrite the final firms goods price P_k as a weighted average of the last period's price level and the price set by firms adjusting in the current period: $P_{t,k} = (\xi P_{t-1,k}^{1-\theta_k} + (1-\xi) P_{t,k}^{*1-\theta_k})^{\frac{1}{1-\theta_k}}$. In addition, please note that the j-index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t, will make the same decisions.

2.2.3   Capital Producing Firms

We assume that households own capital producing firms and receive profits. Green and dirty firms buy specific types of capital from intermediate goods firms at the end of period t and then repair depreciated capital and create new capital. They then sell both the new and re-furbished capital. As discussed above, the relative price of a unit of new capital is either $Q_{t,g}$ or $Q_{t,d}$. We suppose that there are flow adjustment costs associated with producing new capital. Then, capital producing firms face the following maximization problem:

$$\max_{\{I_{t,k}^n\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s,k} - 1) I_{t+s,k}^n - f_k(.) (I_{t+s,k}^n + \bar{I}_k) \right\} \tag{29}$$

$$\text{with} \quad I_{t,k}^n = I_{t,k} - \delta K_{t,k}, \tag{30}$$

$$K_{t,k} = K_{t-1,k} + I_{t,k}^n, \tag{31}$$

$$\text{and} \quad f_k(.) = \frac{\eta_i}{2} \left(\frac{I_{t+s,k}^n + \bar{I}_k}{I_{t+s-1,k}^n + \bar{I}_k} - 1 \right)^2, \tag{32}$$

where $I_{t,k}^n$ and $I_{t,k}$ are net and gross capital created, respectively, $\bar{I}_k$ is the steady state investment for each kind of firm, $\delta K_{t,k}$ is the quantity of re-furbished capital, and η_i the inverse elasticity of net investment to the price of capital. Thus, we get the following value

for $Q_{t,k}$:

$$Q_{t,k} = 1 + f_k(.) + f_k'(.) \left(\frac{I_{t,k}^n + \bar{I}_k}{I_{t-1,k}^n + \bar{I}_k} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f_k'(.) \left(\frac{I_{t+1,k}^n + \bar{I}_k}{I_{t,k}^n + \bar{I}_k} \right)^2 \right\}. \quad (33)$$

2.3   Financial Intermediaries

We modify the setup of Gertler and Karadi [2011] to allow financial intermediaries to invest in both green and carbon-intensive ('dirty') firms. We model the incentive constraint as in Pietrunti [2017] instead of Gertler et al. [2012] as this specification allows for a more realistic implementation of macroprudential policy through regulatory weights on loans.[7]

A representative bank's balance sheet can be depicted as:

$$Q_{t,g}S_{t,g} + Q_{t,d}S_{t,d} = N_t + B_t, \quad (34)$$

where $S_{t,g}$ and $S_{t,d}$ are financial claims on green and dirty firms and $Q_{t,g}$ and $Q_{t,d}$ their respective relative price. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, the banks' equity capital evolves as follows :

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,d}Q_{t-1,d}S_{t-1,d} - R_tB_{t-1}, \quad (35)$$

$$N_t = (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d} + R_tN_{t-1}, \quad (36)$$

where $R_{t,k} = \frac{R_{k,t}^K/P_t - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denote the gross rate of return on a unit of the bank's assets from $t - 1$ to t for sector k.[8]

The goal of a financial intermediary is to maximize its equity over time. Thus, we can

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[7]As a robustness exercise (see Appendix C), we use Gertler et al. [2012] modeling specification and show that our results regarding the model dynamics remain unchanged.

[8]Note that the depreciated capital has a value of one as adjustment costs only apply to net investment.

write the following objective function:

$$V_t = E_t \left\{ \sum_{i=1}^{\infty} \Delta \beta^i \Lambda_{t,t+i} (1 - \theta_B) \theta_B^{i-1} N_{t+1} \right\}, \tag{37}$$

where Δ is a parameter allowing to adjust the bankers' discount factor. We introduce a regulator in charge of the supervision of financial intermediaries. Drawing on Pietrunti [2017], we assume that this regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t \geq \lambda (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}), \tag{38}$$

with λ the risk weight on loans and λ_g and λ_d specific weights that can be applied on loans to green or dirty firms. As will be made clear below, the regulator can modify these weights, altering the constraint weighing on banks and thus the financial frictions in our economy. In our baseline version of the model however, we will only calibrate λ to match the steady state capital ratio and leave λ_g and λ_d equal to one. We guess that the value function is linear of the form $V_t = \Gamma_t N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_{t,g}, S_{t,d}} E_t \left\{ \Delta \beta \Lambda_{t,t+1} \Omega_{t+1} N_{t+1} \right\}, \tag{39}$$

where $\Omega_t \equiv 1 - \theta_B + \theta_B \Gamma_t$. Maximization subject to constraint (38) yields the following first order and slackness conditions:

$$\Delta \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,g} - R_{t+1}) \right\} = \nu_t \lambda_g \lambda, \tag{40}$$

$$\Delta \beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1} (R_{t+1,d} - R_{t+1}) \right\} = \nu_t \lambda_d \lambda, \tag{41}$$

$$\nu_t \left[\Gamma_t N_t - \lambda (\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}) \right] = 0, \tag{42}$$

where ν_t is the multiplier for constraint (38). One interesting result is that we get:

$$N_t \geq \Xi_t(\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}), \tag{43}$$

where $\Xi_t = \lambda/\Gamma_t$ is the capital ratio for banks and λ_g and λ_d represent potential rewards or penalties on the weights required by the regulator on green and dirty loans, respectively.[9] Finally, we rewrite the value function to find Γ_t:

$$
\begin{aligned}
V_t &= \lambda\nu_t(\lambda_g Q_{t,g} S_{t,g} + \lambda_d Q_{t,d} S_{t,d}) + \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}N_t\right\} \\
\Gamma_t N_t &= \nu_t\Gamma_t N_t + \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_t N_t\right\} \\
\Gamma_t &= \frac{1}{1-\nu_t}\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}\right\}.
\end{aligned}
\tag{44}
$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \theta_B[(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d}] + (\theta_B R_t + \omega)N_{t-1}, \tag{45}$$

with $\omega \in [0;1)$ the proportion of funds transferred to entering bankers.

2.4 Public Authorities

2.4.1 Central Bank

Policy Rate Setting

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t - \bar{\imath} = \rho_c\left(i_{t-1} - \bar{\imath}\right) + \left(1 - \rho_c\right)\left[\phi_\pi\left(\pi_t - \bar{\pi}\right) + \phi_y\left(Y_t - Y_{t-1}\right)\right], \tag{46}$$

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[9]For instance, if $\lambda_g < 1$ banks will need to hold less capital for loans they grant to green firms compared to dirty firms.

where $\bar{\imath}$ is the steady state of the nominal rate i_t, $\rho_c \in [0, 1)$ is the smoothing coefficient, $\phi_\pi \geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is the output gap stance penalizing deviations of output from its steady state $\bar{Y}$. Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t \{\pi_{t+1}\}. \tag{47}$$

Because we want to replicate the current economic conditions as closely as possible, we will calibrate our model such that the nominal rate would be extremely low by historical standards (1 percent at the steady state). This drastically limits the scope of conventional monetary policy, as the central bank can not set its nominal interest rate below zero. The Zero Lower Bound (ZLB) implies non linear responses to shocks that affect the path of the nominal rate and we must take it into account. To do so, we will use the non-linear technique of simulation developed by Guerrieri and Iacoviello [2015].

Quantitative Easing

The ZLB also implies that central banks must prove innovative to keep fulfilling their mandates in a liquidity trap environment. A common alternative to nominal interest rate setting is the use of assets purchase programs, also referred to as Quantitative Easing (QE). In the previous section, we showed how the value of loans to both dirty and green firms are determined. We now introduce a central bank that can substitute for financial intermediaries in financing these firms. Much like the Corporate Sector Purchase Program (CSPP) in the Euro Area, the central bank has the ability to fund non-financial firms in order to reduce corporate spread, steer private investment, and ultimately keep inflation in range with its target. Then for each type of firm k we have:

$$Q_{t,k} S_{t,k} = Q_{pt,k} S_{pt,k} + Q_{gt,k} S_{gt,k}, \tag{48}$$

17

with $Q_{gt,k}S_{gt,k}$ the total real value of loans to firms of type k held by the central bank. Note that $S_{t,k} = K_{t,k}$, as firms from either sector, do not face frictions when requesting financing. As in Gertler and Karadi [2011], we model this intervention by assuming that the central bank holds a portion $\psi_{t,k}$ of total loans to non-financial firms belonging to each sector:

$$Q_{gt,k}S_{gt,k} = \psi_{t,k}Q_{t,k}S_{t,k}. \tag{49}$$

For simplicity, we abstract from monitoring costs. We assume that the central bank follows a counter-cyclical credit policy rule to decide the share of assets $\psi_{t,k}$ it holds. This rule is defined as follows:

$$\psi_{t,k} = \rho_{u_k}\psi_{t-1,k} + \varepsilon_t^{\psi_k}, \tag{50}$$

where $\rho_{u_k} \in [0,1)$ is the rule smoothing coefficient and $\varepsilon_t^{\psi_k}$ represents a shock to the credit policy following an $AR(1)$ shock process: $\varepsilon_t^{\psi_k} = \rho_{\psi_k}\varepsilon_{t-1}^{\psi_k} + \sigma_{\psi_k}\eta_t^{\psi_k}$, with $\eta_t^{\psi_k} \sim \mathcal{N}(0,1)$. The latter is motivated by credit policy shocks which are not directly motivated by spreads gaps. Note that in our baseline model $\psi_{t,k} = 0$ so that the central bank lets financial intermediaries be the sole source of funding for firms.

2.4.2 Macroprudential Authority

As briefly explained before, there is a macroprudential regulator with the ability to modify weights on dirty and green loans in the regulatory constraint. For the purpose of this article, we only need to simulate a one-time increase/decrease in these weights. This is why, as opposed to Pietrunti [2017] or Angelini, Neri, and Panetta [2014], we do not specify a macroprudential rule. Even though it could be an interesting exercise to have a macroprudential authority responding to changes in total emissions to set the weight on green loans, it seems to us that it goes beyond the scope of this article and could be left for further research.

2.4.3 Government

The government sets a budget constraint according to the following rule[10]:

$$T_t + \tau_{et}E_t + s_{t,g}\psi_{t,g}K_{t,g} + s_{t,g}\psi_{t,d}K_{t,d} = G_t, \tag{51}$$

with the public expenditure G_t finding its source from taxes T_t, revenue from emissions $\tau_{et}E_t$ and from public financial intermediation on both green and dirty firms $s_{t,g}\psi_{t,g}K_{t,g}$ and $s_{t,d}\psi_{t,d}K_{t,d}$. The government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}}Y_t. \tag{52}$$

Environmental Policy

The government decides to either ratify or not ratify (or renege on) the Paris Agreement. When the government is not operating an environmental policy (i.e the laissez-faire equilibrium) the tax τ_{et} is set equal to 0. Otherwise, when the government tries to hold to the COP 21 Agreement (i.e. a GHG emission reduction) $\tau_{et} > 0$.

This is explained further in the results section.

2.5 Normalization and Aggregation

It is also common in most NK classical models that in equilibrium, factors and goods markets clear as shown below.

First, the market-clearing conditions for aggregate capital, investment, labor, and wages, in the two sector economy reads as[11]: $K_t = \sum_k g(\varkappa) \int_0^1 K_{jt,k}dj$, $I_t = \sum_k g(\varkappa) \int_0^1 I_{jt,k}dj$, $L_t = \sum_k g(\varkappa) \int_0^1 L_{jt,k}dj$, and $W_t = \sum_k g(\varkappa) \int_0^1 W_{jt,k}dj$.

Similarly global aggregate emissions and aggregate emissions cost are two weighted sums of sectoral emissions $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k}dj$, and sectoral emissions cost $Z_t =$

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[10]In the baseline version of the model (without tax and QE), the budget constrain collapses to $T_t = G_t$.
[11]Where $g(\varkappa) = \varkappa$ for sector the green sector g and $(1 - \varkappa)$ for the dirty sector d.

$\sum_k g(\varkappa) \int_0^1 Z_{jt,k} dj$, respectively.

As presented in Gali and Monacelli [2008], the Calvo $D_{pt,k}$ price dispersion is essentially a measure of distortion introduced by dispersion in relative prices. This shows that there is an additional distortion associated with relative price fluctuations owing to price stickiness. The Calvo $D_{pt,k}$ price dispersion is bounded below at 1, where 1 would be the value in the case of flexible prices, where all firms choose the same price. The price dispersion in our two-sector economy reads as:

$$\int_0^1 Y_{jt,k} \mathrm{dj} = \int_0^1 \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_{t,k} \mathrm{dj} = D_{pt,k} Y_{t,k}, \tag{53}$$

with $D_{pt,k}$ the aggregate loss of efficiency induced by price dispersion of the intermediate goods[12]. In other words, it also reads as $D_{pt,k} = (1 - \xi) \left(\frac{P_{t,k}}{P_t}\right)^{(\theta_k - \theta)} \left(p_{t,k}^*\right)^{-\theta_k} + \xi \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} \pi_{t,k}^{\theta_k} D_{pt-1,k}$.

Furthermore, as outlined in Annicchiarico and Di Dio [2015], in the addition to the classical changes operating in an NK model[13], the two-sector environmental economy we introduce, is also affected at the aggregate level by the price dispersion. The emissions as well as the abatement cost reads as:

$$E_{t,k} = (1 - \mu_t) D_{pt,k} Y_{t,k}, \tag{54}$$

$$Z_{t,k} = \theta_{1,k} \mu_{t,k}^{\theta_{2,k}} D_{pt,k} Y_{t,k}. \tag{55}$$

In addition, the resource constraint of the economy reads as follows:

$$Y_t = C_t + G_t + I_t + \sum_k f_k(.)(I_{t+s,k}^n + \bar{I}_k) + Z_t. \tag{56}$$

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[12] Please refer to the online appendix for the full computation.
[13] Where: $Y_{t,k} = (1 - d(X_t)) \varepsilon_t^{A_k} K_{t-1,k}^\alpha L_t^{1-\alpha} D_{pt,k}^{-1}$ and $\Pi_t = (1 - MC_{t,k} D_{pt,k}) Y_{t,k}$.

3 Calibration

Calibrated parameters are reported in Table 1. For parameters related to business cycle theory, their calibration is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio at 40 percent[14], the share of hours worked per day at one third in each sector, and the capital intensity in the production function α at 0.33. The inverse elasticity of net investment to the price of capital η_i is set at 1.728 as in Gertler and Karadi [2011] and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2, as argued by Stern [2008] and Weitzman [2007]. We set the discount factor at 0.99751 to get a steady state real interest rate of 1 percent. This choice is motivated by the low interest rate environment we have witnessed in recent years.

The environmental component parameters, and specifically the damage function parameters d_0, d_1, and d_2 of the model are set as in Nordhaus [2008] and Heutel [2012]. The global level of the rest of the world's emissions E^* is set at 1.32 in order to replicate the pre-industrial steady state level of the stock of emissions $X_t = 800$ GTons CO2 emissions. To calibrate the share of the green firms/sector, what we consider green in our model is a sector with a carbon performance allowing for an emission target aligned with the Paris Agreement of 2 degrees Celsius or below. We use sectoral data made available by Transition Pathway Initiative[15] to set the share of green firms $\varkappa$ to 30 percent. Furthermore, for the intensity of emissions to GDP for each sector, as argued by De Haas and Popov [2019], CO2 intensity differs largely between sectors and industries. Using the European Environmental Agency CO2 emissions intensity data[16] as well as the OECD GDP data, we observe a carbon intensity level of 35-40 percent for the last few of years. Thus the carbon intensity for each sector should satisfy the following equation $\varkappa\varphi_g + (1 - \varkappa)\varphi_d = 0.4$. We set φ_d to ensure the observed CO2 to GDP ratio of about 50 percent (in the energy and industrial services).

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[14]We match the level of the Euro Area.

[15]https://www.transitionpathwayinitiative.org/tpi/sectors

[16]https://www.eea.europa.eu/data-and-maps/figures/ghg-emission-intensity-of-european

Setting a value for the dirty sector carbon intensity automatically then yields a value for φ_g of 15 percent for the green firms as their level of emissions is much lower. The abatement parameters $\theta_{k,1}$ and $\theta_{k,2}$, which pin down the abatement costs for each sector are set as in Heutel [2012] for the dirty sector, and are assumed to be higher for the green sector. As highlighted in the McKinsey cost curve for GHG abatement[17], the cost for abating an additional unit increases steadily (or even arguably exponentially) as cheaper technologies are used first. As our green firms are considered to have already benefited from these technologies, they incur higher abatement costs than the dirty firms. The decay rate of emissions δ_x is set at 0.21 percent. Finally, θ_d the dirty firms' marginal cost parameter is calibrated as in Smets and Wouters [2007] to replicate the mean markup and marginal cost levels observed in the economy, while θ_g as highlighted in the final firm section of the model is calibrated such that the difference in the marginal cost between the two sectors is 6 percent higher as argued by Chan et al. [2013] and Chegut et al. [2019].

As for the financial parameters, we set the probability of staying a banker θ_B at 0.98, which is a slightly more conservative value compared to Gertler and Karadi [2011]. λ is calibrated at 0.1754 to generate a spread of 80 basis points between risky and risk-less assets. This value is taken from Fender, McMorrow, Sahakyan, and Zulaica [2019]. The authors also find that the spread between green and dirty bonds recently disappeared. Thus, we target the same steady state for R_g and R_d. Δ is a parameter allowing the introduction of a different discount factor in the bankers' objective function relative to households and is set to 0.99. The proportional transfer to the entering banker ω is set to 0.004 in order to match a capital ratio of approximately 14.4 percent in the Euro Area. Finally, the monetary rule parameters are set as in Smets and Wouters [2003].

The AR(1) parameters for all the shocks, namely the two technology, preference, and capital quality shocks are calibrated as in Smets and Wouters [2003], Gertler and Karadi [2011] or Christiano, Motto, and Rostagno [2014].

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[17]https://www.mckinsey.com/business-functions/sustainability/our-insights/a-cost-curve-for-greenhouse-gas-reduction

4 Quantitative Analysis

4.1 Dynamics of the Model at the ZLB

Before moving to the analyses of each of the policies and their interactions, this section highlights the model's dynamics under the non-ZLB environment (i.e linear) versus the ZLB environment (i.e. non-linear). Shocks are calibrated so that the ZLB would bind for a few periods and start at period one.

Figure 3 and Figure 4 present the responses to a 15 percent positive green technology shock and dirty technology shock, respectively. The autoregressive parameter is set to 0.82 in line with Smets and Wouters [2003]. As our economy is comprised of two sectors, we allow for the possibility of different technology shocks, which affect each sector differently.

As expected following any technology shock, inflation decreases significantly more under the ZLB environment than the under non-ZLB environment, while the interest rate falls less in the ZLB than in the non-ZLB environment, as the central bank is unable to significantly lower lower its policy rate to counter balance the decreasing inflation rates resulting from the positive technology shocks.

Under the linear model, both the green and dirty TFP shocks raise more the aggregate output as well as the consumption as compared to the non-linear model. The small magnitude of the distortionary effect introduced by the ZLB environment (red line as compared to the blue line) is not persistent as the ZLB does not bind for a long period in the chosen example. Likewise, under the non-ZLB environment, the aggregate emissions both fall and rise more than in the non-linear model following the green and dirty technology shocks, respectively. This emissions dynamics are mainly driven by the sectoral shock. The green TFP shock increases the emission in the green sector while decreasing it in the dirty sector and vise versa.

Figure 5 presents the responses to a -6.5 percent negative preference shock with an autoregressive parameter of 0.95 as in Christiano et al. [2014]. Similar to the technology

shocks, the aggregate output and the aggregate emissions increase more under the linear environment than under the non-linear, while consumption and interest rate decrease more under the non-linear than under the linear environment, as the ZLB acts as constraint to the central bank. The same dynamics are observed in each sector regarding output and emissions. The ZLB by halting investment, further decrease the output and slow the recovery in each sector leading to a higher decrease in emissions.

Figure 6 plots responses to -1.4 percent negative capital quality shock where the autoregressive parameter is set to 0.66 as in Gertler and Karadi [2011]. On the financial side, this drop in the quality of capital leads to a decrease in banks' capital ratio, triggering a negative chain of events for the economy. Investment drops sharply, and as capital declines, the aggregate output starts falling. Because production is moving away from its steady state, hours worked and wages also fall. Ultimately, consumption decreases as both revenues from labor and capital are affected. The central bank being constrained by the ZLB, it can not lower its policy rate as much as it would otherwise to steer investment. This leads to a deteriorated situation as compared to the linear model simulation.

For all the following sections we use the ZLB environment as the baseline model, and we contrast it to the fiscal, macroprudential, and monetary policies. This is motivated by the fact that current nominal rates are at or near the ZLB in most developed countries, and likely to stay at this level for a prolonged period of time. We will also use the exact same calibrations for shocks to allow for a precise comparison between different specifications of the model.

4.2   Fiscal Environmental Policy Scenario

4.2.1   A Fiscal Policy To Meet the Paris Agreement

To compare the economic variations, we contrast a laissez-faire scenario where no environmental policy is implemented with a scenario where the government is inline with the COP 21 Agreement (i.e. a GHG emission reduction target of 20 percent), and thus implements

an environmental policy.

Technically, this means that the business-as-usual policy would set $\tau_{e,t} = \mu_t = 0$ indicating that firms are not investing in any abatement technology to reduce emissions nor is there an enforced policy controlling for emissions production; while the environmental policy regime sets a tax on emissions at a fixed level aiming at reducing by 20 percent the emissions level. As we have two types of sectors, we allow the green firms to emit less, however they incur a higher abatement cost for each extra unit than the dirty firms as former is already using green technology, thus making it more difficult for them to abate an extra unit for at cheaper cost.

The steady state level of abatement is therefore determined in such way that the total amount of emissions abated from both sectors—while accounting for their heterogenaities in abatement possibilities, costs, and emission intensity to GDP—totals the Paris Agreement target. Moreover, setting cost parameters at levels such as those found by Heutel [2012], Annicchiarico and Di Dio [2015], and Benmir, Jaccard, and Vermandel [2020] yields an aggregate environmental tax close to 10 percent of the GDP.

Figure 7 and Figure 8 compare the responses of both aggregate and sectoral production Y, Y_g, and Y_d, emissions E (also both aggregate and sectoral), consumption C, inflation π, and real interest R in the case of a positive technology green shock and a positive technology dirty shock similar to the simulation described in the previous section.

On one hand, in our baseline model, a green TFP shock raises the green output as the green sector finds itself more productive compared to the dirty sector, which sees its output fall. However, as the impact of the shock is more significant in the green sector (even though it represents only 30 percent of our economy) relative to the dirty sector, the aggregate output increases.

On the other hand, the aggregate emissions fall driven mainly by the dirty sector production drop. Although a rise of the green sector production increases the emissions level as green firms are more productive, the fact that they are CO2 friendly (i.e. a low emission to GDP intensity), makes this emissions increase less pronounced proportionally to the

dirty sector emissions decrease as the firms in the former sector are experiencing a slow in productivity.

In turn, this rise in production both in the case of green and dirty TFP shocks contributes to an increase in household consumption as they see their wealth increase.

Turning to the effect of an introduction of an environmental policy (i.e. an environmental tax) as shown in green in Figure 7 and Figure 8, in the case of the green shock, the decline in emissions in the green sector provoked by the environmental policy is characterized by a slowdown in the output as compared to the laissez-faire equilibrium. In other words, it is beneficial to increase the stock of emissions during periods of recessions, and reduce them in booms. The firms from the dirty sector take advantage of this drop in the green sector to produce slightly more, which increases the emissions streaming from this sector. As the intensity of emissions to GDP of the dirty sector is far more significant than the green sector, the aggregate emissions increase under the tax policy (although are still negative) as compared to the laissez-faire.

Conversely, a dirty TFP shock has the opposite effects on global emissions than that of a green technology shock. As argued by Heutel [2012] among others, the emissions decrease when an environmental policy is introduced, thus retrieving the pro-cyclicality aspect of an environmental tax. Thus, it is optimal to increase the tax during booms, and to lower it during recessions, as a consumption sacrifice is very costly.

Relative to the cases of the green and dirty technology shocks, the preference shock in Figure 9 has a similar impact on the policy. In the laissez-faire equilibrium, the negative shock generates a decrease in consumption almost similar in magnitude to the environmental tax scenario, as households are less impatient and therefore prefer to postpone consumption to the next periods. The shock also leads to an initial decrease in the level of emissions as firms' production fall driven by the consumption drop. However, as the household inter-temporal trade-off increases savings, and thus investments, firms' output rises consequently in both sectors rapidly thereafter. The policy helps reduce the quantity of emissions by a considerable percentage relative to the laissez-faire scenario without distorting the economy

driven mainly by the efficiency of abatement costs. As the abatement costs rise it is expected that the tax would distort the output and household consumption.

Finally, we also note as in the case of the technology shock, the preference shock generates a fall in the policy rate as well as inflation as firms are more productive.

4.2.2   The Optimal Environmental Fiscal Policy

In this section we investigate the role abatement costs play in the effectiveness of an optimal policy à la Ramsey, where a benevolent government (Ramsey planner) maximizes the expected discounted utility of households, given the constraints of the decentralized economy. We consider the case of a Ramsey planner controlling optimally the tax rate on emissions. As a common practice, we assume that the government is able to commit to the contingent policy rule it announces at time 0 (i.e. ex-ante commitment to a feedback policy, so as to have the ability to dynamically adapt the policy to the changed economic conditions). In what follows we first consider the case of a Ramsey planner choosing environmental regulation for different levels of environmental abatement costs Z_t. We start from the optimality conditions for households and firms and then reduce the number of constraints to the Ramsey planner's optimal problem by substitution. The dynamic responses of the Ramsey plans are computed by taking second order approximations of the set of first order conditions around the steady state.

Ramsey Optimal Response Under Environmental Externality Policy

The Ramsey optimal tax rate on emissions, which maximizes welfare, namely τ_{et}, only appears in the abatement function u. Thus, the Lagragian problem reads as follows:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_{jt+i} - hC_{jt+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L_{k,jt+i}^{1+\varphi} \right.$$
$$\left. + \sum_i \lambda_{i,t} \text{FOC}_{i,t} \right),$$

with $\{\lambda_{i,t}\}$ representing the sequences of the Lagrange multipliers on the constraints.

The planner will choose the sequences

$\left\{Q_{k,t}, C_t, Y_{k,t}, L_{k,t}, I_{k,t}, K_{k,t+1}, E_{k,t}, X_{k,t}, \Theta_{k,t}, S_{k,t}, \Upsilon_{t,k}, D_{k,pt}, \Pi_{t,k}, p_{t,k}^* \right\}_{t=0}^{\infty}$ and $\{\tau_{et,k}\}_{t=0}^{\infty}$ given the exogenous processes. The first-order conditions as usually outlined are optimal from a "timeless perspective" in such a way as to prevent the Ramsey planner from reneging on prior announcements.

Findings

The model is solved through Dynare using the Ramsey setup and applying perturbation methods. As seen in Annicchiarico and Di Dio [2015] the optimal level of environmental tax policy is found to be of small order.

As the fraction of dirty firms is much higher in the economy than that of green firms, and as the former have higher intensity of emissions to output, the Ramsey social planner will optimize over the dirty sector and then set the same level of the tax to both sectors.

The level of the optimal tax, which maximizes the welfare is found to be of a small magnitude (as mentioned), as the household welfare tends to deteriorate when a tax policy is introduced since the utility of consumption does not capture the effects of climate change directly (Benmir et al. [2020] show how the welfare improves if the marginal utility of consumption to emissions $u_{ec} \neq 0$.[18]). The negative effects of the environmental externality are captured through the production of the firms and then impact the household via the potentially shrinking profits. However as the magnitude of the latter is of a small proportion, the optimal solution is found to be of the order of .02 percent.

The optimal tax is found to be sensitive to the abatement efficiency and as presented in Table 3) (and in Benmir et al. [2020]).

Since the optimal tax is shown to be of a small magnitude, increasing the 10 percent baseline tax could further distort the welfare, indicating a need to seek other policy instruments in addition to fiscal ones. In order to achieve higher targets of CO2 emissions

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[18]This utility specification could be explored in future research.

reduction—which are otherwise necessary to offset climate change—calls for innovative approaches and policies that could ease the burden on the tax payers should be sounded. Thus macroprudential and monetary policies could play an important role in achieving such goals.

4.3 Introducing Macroprudential Policy

We introduce macroprudential policy through a simple drop in the weight on green loans in the regulatory constraint. The idea is that the regulator wants to give an incentive to banks to invest in green loans rather than dirty ones. For financial intermediaries, it means they have to hold less net worth to maintain the same level of loans to the green sector. In other words, we expect this shift in λ_g to increase K_g at the steady state and hence lead to a greener economy. To perform the following exercises, we now set λ_g to 0.7, maintaining λ_d unchanged at 1^{19}. We first show the impact on steady state values in Table 4. In particular, we see that a decrease in the green loans weight of 30 percent leads to an increase of the green capital stock of more than 3 percent. However, this goes hand in hand with a decrease in the rate on green loans, resulting in a spread between dirty and green rates. In our setup, it will have consequences on behavior of banks having to maximize their objective function. We then simulate a preference shock to show how the drop in λ_g affects the usual dynamic of the model. Ultimately, we simulate a capital quality shock to illustrate the trade-off between improving steady-state values and deteriorating responses to negative financial shocks.

Figure 10 plots the responses to a negative preference shock in the baseline tax model and in the tax model augmented with macroprudential policy. The introduction of an incentive for banks to fund green firms leads to a higher increase in the green output compared to the dirty output. However, because the studied policy relax the overall constraint on banks, it also leads to an increase in dirty output. As green firms have cleaner production technology by definition, the emission to GDP ratio is further reduced compared to the tax only model.

Figure 11 plots the responses to a negative capital quality shock as presented in the first subsection of the quantitative analysis. We assume that the shock uniformly affects both

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[19]Note that it does not impact the steady state level of the capital ratio.

sectors. The first-round effect is a sharp drop in both green and dirty interest rates. Because we introduced heterogeneity in the regulatory constraint, the dirty rate is more impacted by the shock when macroprudential policy is active, translating into a positive green spread[20]. This encourages financial intermediaries to extend their loans to green firms compared to dirty firms in a second-round effect. Overall, it induces a decrease of the emissions to GDP ratio in the recovery from the shock, but also a lower aggregate output in the first periods. This is mainly due to the fact the dirty sector is dominating in our economy. If the green sector were bigger than the dirty one, we would see macroprudential policy improving both the emission to output ratio and the aggregate output. Also note that the heterogenous effect exhibited here would be inverted in the case of a positive capital quality shock. Thus, it clearly appears that introducing variance in macroprudential weights imply a trade-off between boosting the steady state level of output and the reaction of the economy to financial shocks, when the dirty sector is dominating. We might be able to mitigate this effect by introducing a macroprudential rule such as in Angelini et al. [2014] or Pietrunti [2017], but this would be out of the scope of this paper.

4.4   Quantitative Easing and the Policy Mix

We now introduce quantitative easing. As defined above, the central bank has the ability to substitute to financial intermediaries in financing either green or dirty firms. The scenario studied here is a series of four positive 2 percent shocks in ψ_t^k. This is akin to a purchase program decided by a monetary authority and results in the central bank holding a bit more than 12 percent of either green or dirty assets at the peak of the program. We calibrate the autoregressive parameter to 0.66 so that the assets bought slowly exit the central bank's balance sheet.

Figure 12 and Figure 13 display the reaction of selected variables to a series of positive dirty and green QE shock, respectively. We plot the responses when only the QE is active (blue line), when both the QE and the tax are active (red dotted line), and when the QE,

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[20]The green spread is defined as the difference between the green real rate and the dirty real rate.

the tax, and the macroprudential policy are active (green dashed line).

A first interesting finding and a crucial one for a central bank is that dirty and green QE both induce a rise in the inflation rate. These programs both lead to an increase in the inflation rate of roughly 1.6 percent to 2 percent at an annual rate, absent any other shock. It is a prerequisite that green QE has a positive impact on inflation in order to become a potential monetary policy tool, and these results indicate that a green QE could also be justified on the ground of low inflation expectations.

A second result is that the introduction of a carbon tax has a positive environmental effect on the impact of QE. It keeps exactly the same effect on output and inflation, but reduces total emissions. However, without introducing macroprudential policy, there is no apparent reason for a central bank to implement green QE rather than the dirty QE. This can be explained by the fact that both assets have the exact same yields at the steady state, meaning that they are completely interchangeable for financial intermediaries. As a consequence, their yield react exactly the same way and the spread stays at zero.

When introducing a macroprudential reward on the loans to green firms, however, public authorities can alter this mechanism. In this case, a trade-off appears between higher GDP growth and lower emissions. With both types of QE, the introduction of a tax and macro-prudential policy allows the reduction of emissions relative to output. However, opting for green QE leads to a greater drop in emissions relative to output, at the cost of a slightly lower boost to GDP and inflation. Once again, this trade-off would disappear in the event that the green sector grows enough to be as big or bigger than the dirty one. Policy makers could then achieve both higher output and lower emissions with the above mentioned policy coordination.

Figure 1 and Figure 2 represent the transition paths where the weights of the greener sector is gradually increasing, thus making the greener sector predominant. Moving toward a greener economy not only decreases substantially emissions, which in turn decreases the environmental policy (i.e. the tax), it also helps achieve the so sought after decoupling of emissions and output. The emissions to output $E_Y = E/Y$ falls almost linearly with an

increase in the weights of the green sector and drive the level of the tax to a low level equivalent to the optimal tax level found under the Ramsey equilibrium.

5   Conclusion

We developed a macro-environmental-financial DSGE model with both endogenously constrained financial intermediaries and heterogenous firms. We then used the model to assess the effects of various policies and their interactions on carbon emissions.

Within our framework there are trade-offs between maximizing output and consumption and reducing the impact of the economy on climate change. In particular we find that a 10 percent environmental tax, demanding a level of abatement of 10 percent and 20 percent from the green and dirty sectors, respectively, is needed in order to be aligned with the Paris Agreement target. However, these tax and abatement levels heavily depend on the abatement efficiency (i.e. low transition cost), and are found to be of smaller magnitude under the optimal Ramsey setup. As mitigation efforts needed to offset the negative effects of CO_2 emissions exceed those of 20 percent reduction used as a baseline policy in our model and pledged in the Paris Agreement, and as the short/medium term tax effects on the welfare are shown to be distortionary, a fiscal policy alone is not sufficient. Thus, there is a strong need for additional tools. Looking at the role monetary and macroprudential policies can play, we find that a macroprudential policy favorable to the green sector boosts green capital and output, implying a lower emissions to output ratio. Regarding QE, we find that a carbon tax improves the benefits of both green and dirty asset purchases. However, macroprudential policy is needed to provide an incentive to central banks to engage in green QE. Choosing between dirty and green QE then implies a trade-off between higher output and lower emissions. This trade-off would disappear in the event that the green sector grows enough to be as big as or bigger than the dirty one.

We hope that this article will pave the way for more research on the interaction between environmental, monetary, and macroprudential policies. Many exercises could be conducted

using our framework. In particular, we think that further research could be devoted to the impact of non-linearities within the financial sector on the dynamics of the model and on the role that endogenous TFP could play in fostering the emergence of greener output growth. We also believe it could be fruitful to examine how to capture the environmental quality on the welfare of households in more direct ways than in existing models.

6   Acknowledgment

7   Bibliography

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A   Appendix: Tables

	Calibrated parameters	Values
Standard Parameters		
β	Discount factor	0.9975
α	Capital intensity	0.33
δ	Depreciation rate of capital	0.025
h	Habits formation parameter	0.8
σ	Risk aversion	2
φ	Disutility of labor	1
η_I	Capital adjustment cost	1.728
$\varkappa$	% of Green firms in the economy	30%
θ	Price elasticity	5
θ_g	Price elasticity in sector G	11
θ_d	Price elasticity in sector D	7
ξ	Price stickiness (Calvo parameter)	3/4
$\bar{L}$	Labor supply	1/3
$\bar{g}/\bar{y}$	Public spending share in output	0.4
Environmental Parameters		
$\bar{e}_d/\bar{y}_d = \varphi_d$	Emissions-to-output ratio in sector D	0.15
$\bar{e}_g/\bar{y}_g = \varphi_g$	Emissions-to-output ratio in sector G	0.5
γ_d	CO_2 natural abatement	1-0.9979
$\theta_{1,g}$	Abatement cost parameter for sector G	2.41
$\theta_{2,g}$	Abatement cost parameter for sector G	2.7
$\theta_{1,d}$	Abatement cost parameter for sector D	.8
$\theta_{2,d}$	Abatement cost parameter for sector D	2.7
a	Damage function parameter	1.3950e-3
b	Damage function parameter	-6.6722e-6
c	Damage function parameter	1.4647e-8
Banking Parameters		
ω	Proportional transfer to the entering bankers	0.004
Δ	Parameter impacting the discount factor of bankers	0.99
λ	Risk weight on green firms	0.1754
θ_B	Probability of staying a banker	0.98
ρ_c	Smoothing monetary rule coefficient	0.8
ϕ_y	Output policy parameter	0.2
ϕ_Π	Inflation policy parameter	1.5

Table 1

Calibrated parameter values (quarterly basis)

	Steady state values		
	Baseline	Tax	% Change
Aggregate Output	2.0492	2..0511	0.1%
Green Output	1.0591	1.0601	0.1%
Dirty Output	1.0284	1.0294	0.1%
Aggregate Emissions	0.4076	0.3311	-19%
Green Sector Emissions	0.1588	0.1431	-10%
Dirty Sector Emissions	0.5142	0.4117	-20%
Consumption	0..9734	0.9653	-1%
Green Sector Abatement	-	0.1	N/A
Dirty Sector	-	0.2	N/A
Aggregate Tax as % of GDP	-	0.1113	N/A
Tax as % of GDP in Green	-	0.1203	N/A
Tax as % of GDP in Dirty	-	0.1120	N/A

Table 2

Steady state values –Baseline versus Tax Policy

|	Baseline (No abatement)	Efficient Abatement	Costly Abatement
	-	$\theta_{1,d} = .8$	$\theta_{1,d} = 10.8$
Tax as % of GDP in Green	-	0.0029	0.0029
Tax as % of GDP in Dirty	-	0.0023	0.0024
Green Abatement Level	-	0.01	0.0072
Dirty Abatement Level	-	0.01	0.0039
Aggregate Emissions	0.407	0.398	0.403
Green Sector Emissions	0.158	0.156	0.156
Dirty Sector Emissions	0.514	0.502	0.510

Table 3

Ramsey optimal tax sensitivity to the abatement costs in the dirty sector

	Steady state values		
	Tax	Tax & MacroPru	% Change
Aggregate Output	2.0511	2.0609	0.4777
Green Output	1.0601	1.0710	1.0282
Dirty Output	1.0294	1.0294	0
Aggregate Emissions	0.3311	0.3316	0.1510
Green Sector Emissions	0.1431	0.1445	0.9783
Dirty Sector Emissions	0.4117	0.4117	0
Consumption	0.9653	0.9686	0.3418
Green Capital Stock	10.8965	11.2348	3.1046
Dirty Capital Stock	9.9759	9.9759	0
Green Real Rate	1.0044	1.0038	-0.0597
Dirty Real Rate	1.0044	1.0044	0
Aggregate Tax as % of GDP	0.1113	0.1110	-0.2695
Tax as % of GDP in Green	0.1215	0.1202	-1.0699
Tax as % of GDP in Dirty	0.1120	0.1120	0

Table 4

Steady state values –Tax versus Tax and Macroprudential Policy

B   Appendix: Figures

Figure 1: Sectoral weights, carbon intensity, and the environmental policy
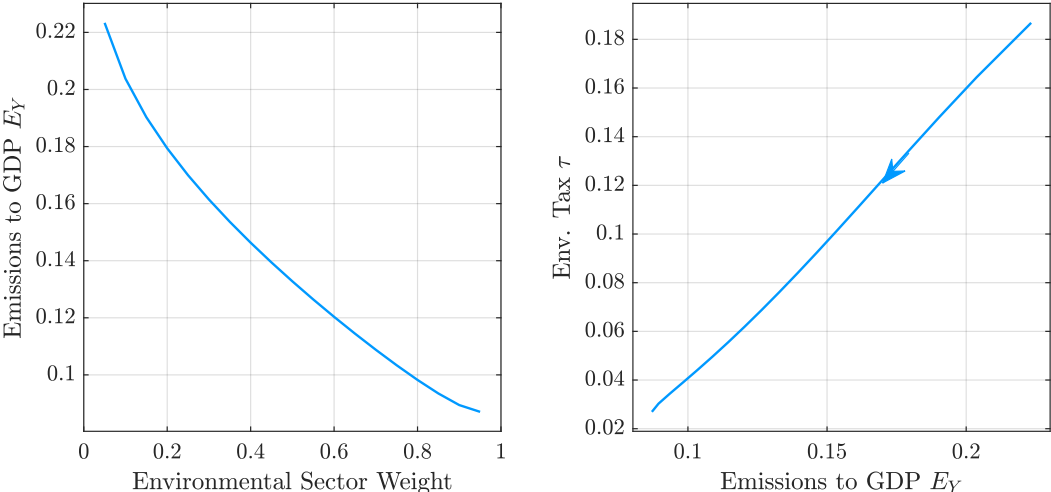
Notes: The graph on the left reports the interaction between emissions to output and sectoral weights. The right graph reports how sectoral weight through emissions to output drives the carbon tax.

Figure 2: Sectoral weights, emission levels (normalized to one), and the environmental policy
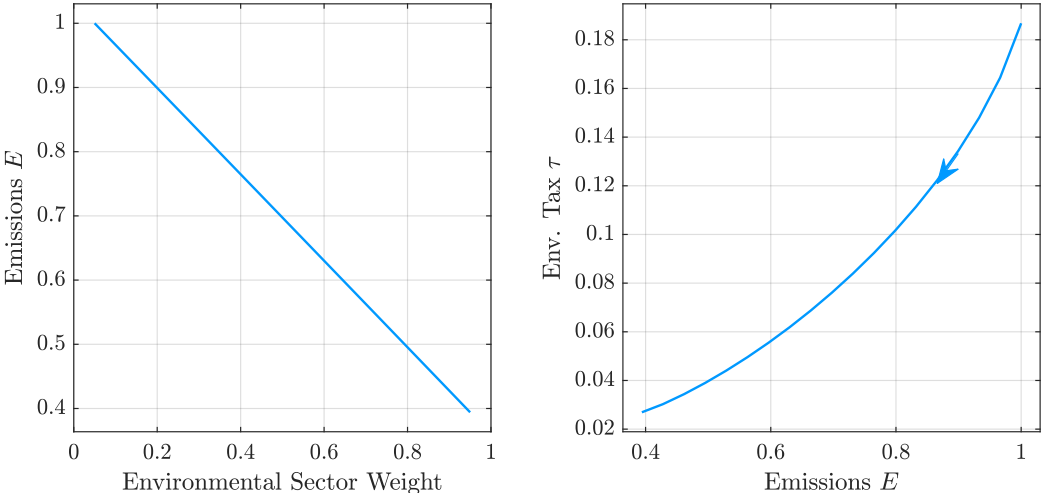
Notes: The graph on the left reports the interaction between emissions and sectoral weight. The right graph reports how sectoral weights shape the carbon tax.

Figure 3: Effect of a positive green technology shock $(\varepsilon_t^{A_g})$ on selected variables between the linear and non-linear models - percentage deviations from steady state.
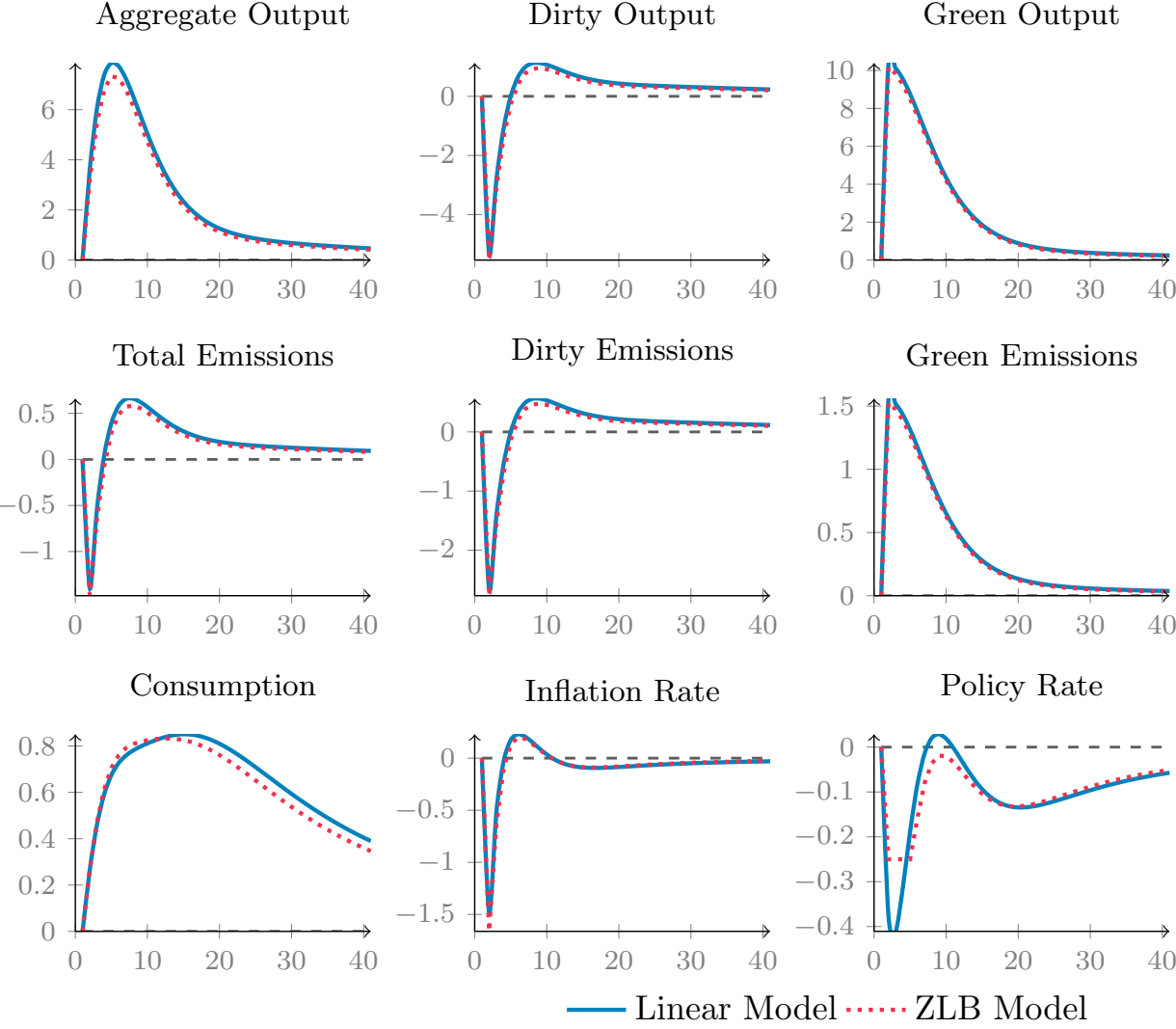
Figure 4: Effect of a positive dirty technology shock $(\varepsilon_t^{A_d})$ on selected variables between the linear and non-linear models - percentage deviations from steady state.
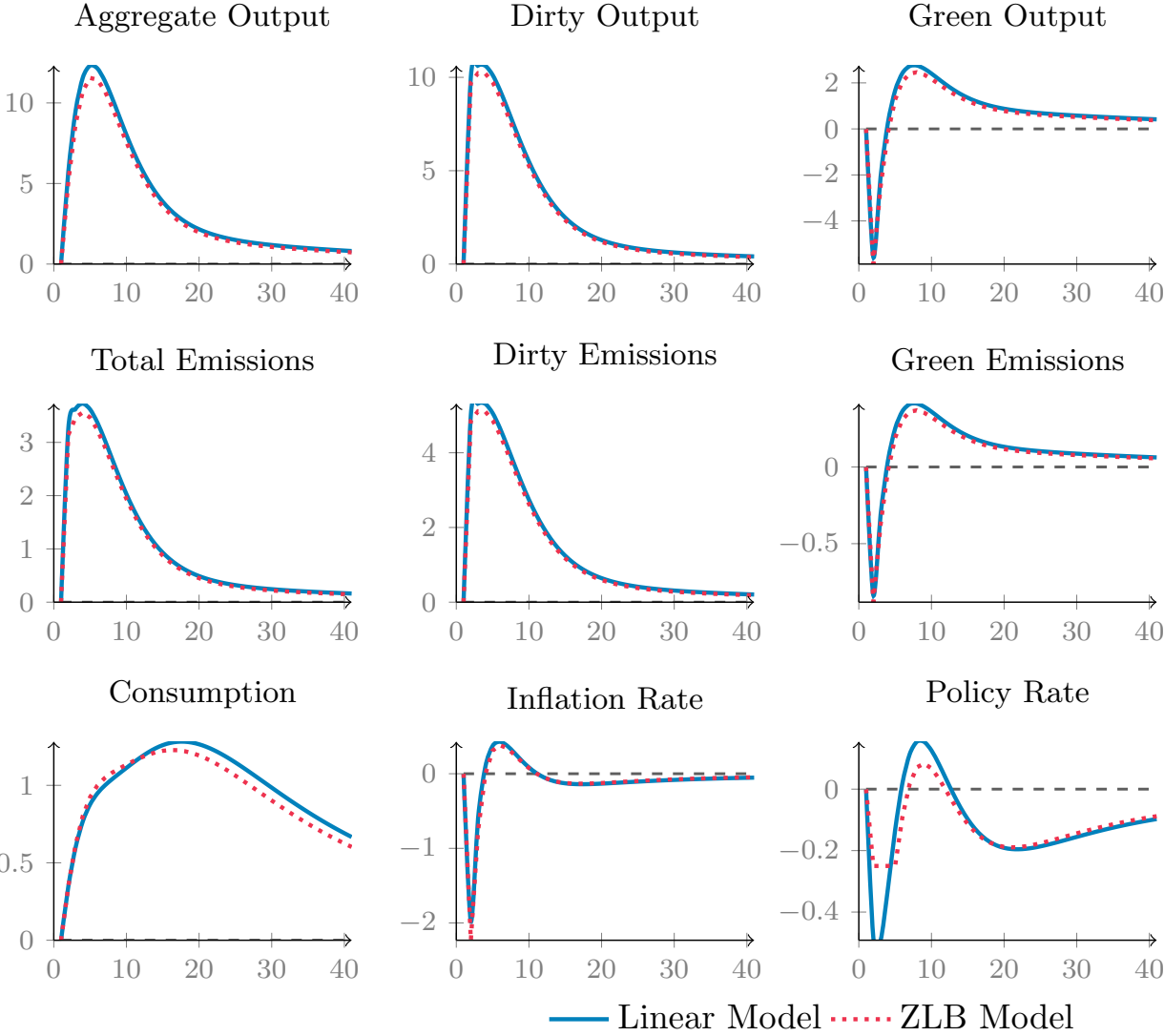
Figure 5: Effect of a negative preference shock (ε_t^B) on selected variables between the linear and non-linear models - percentage deviations from steady state.
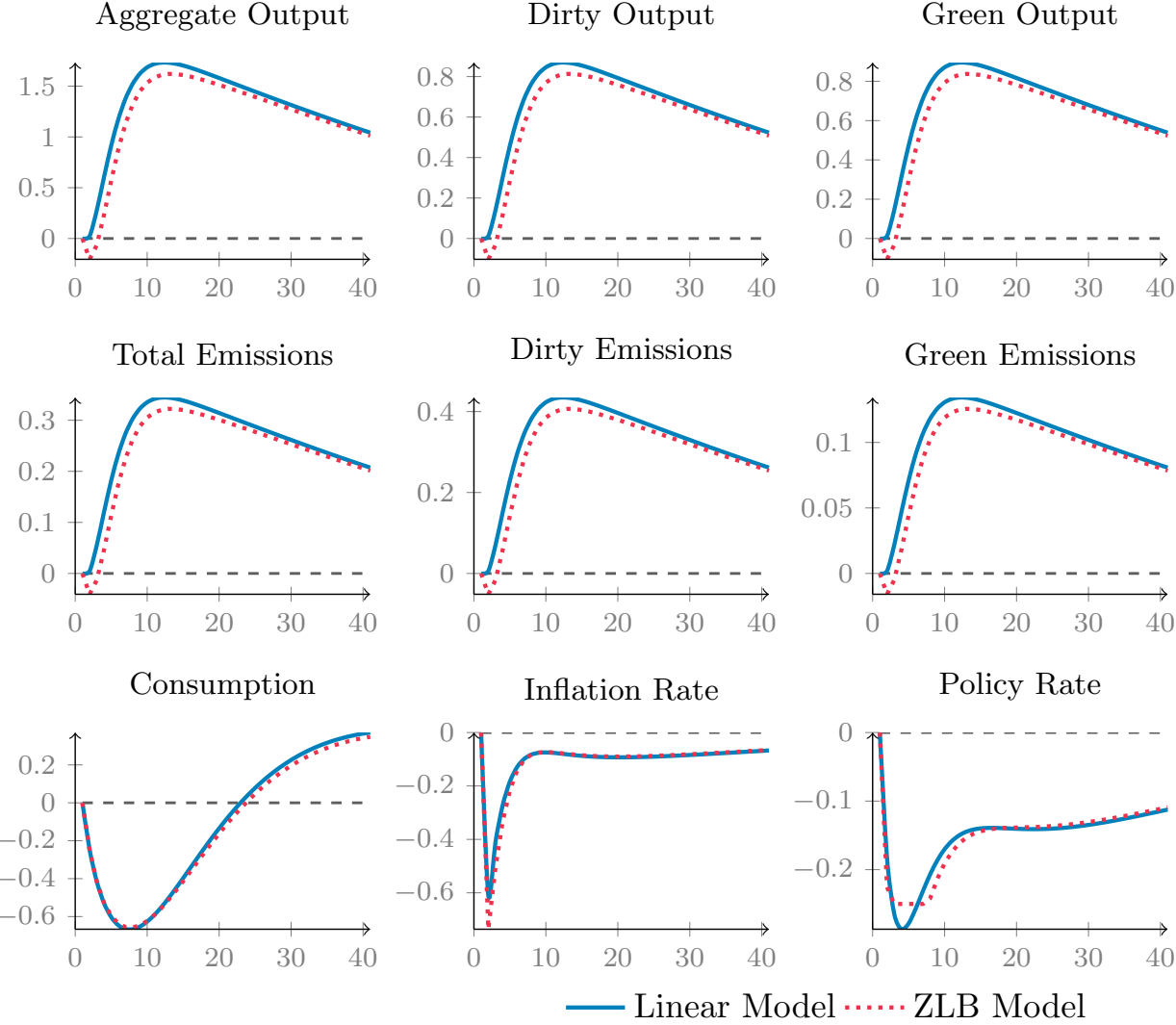
Figure 6: Effect of a negative capital shock (ε_t^K) on selected variables between the linear and non-linear models - percentage deviations from steady state.
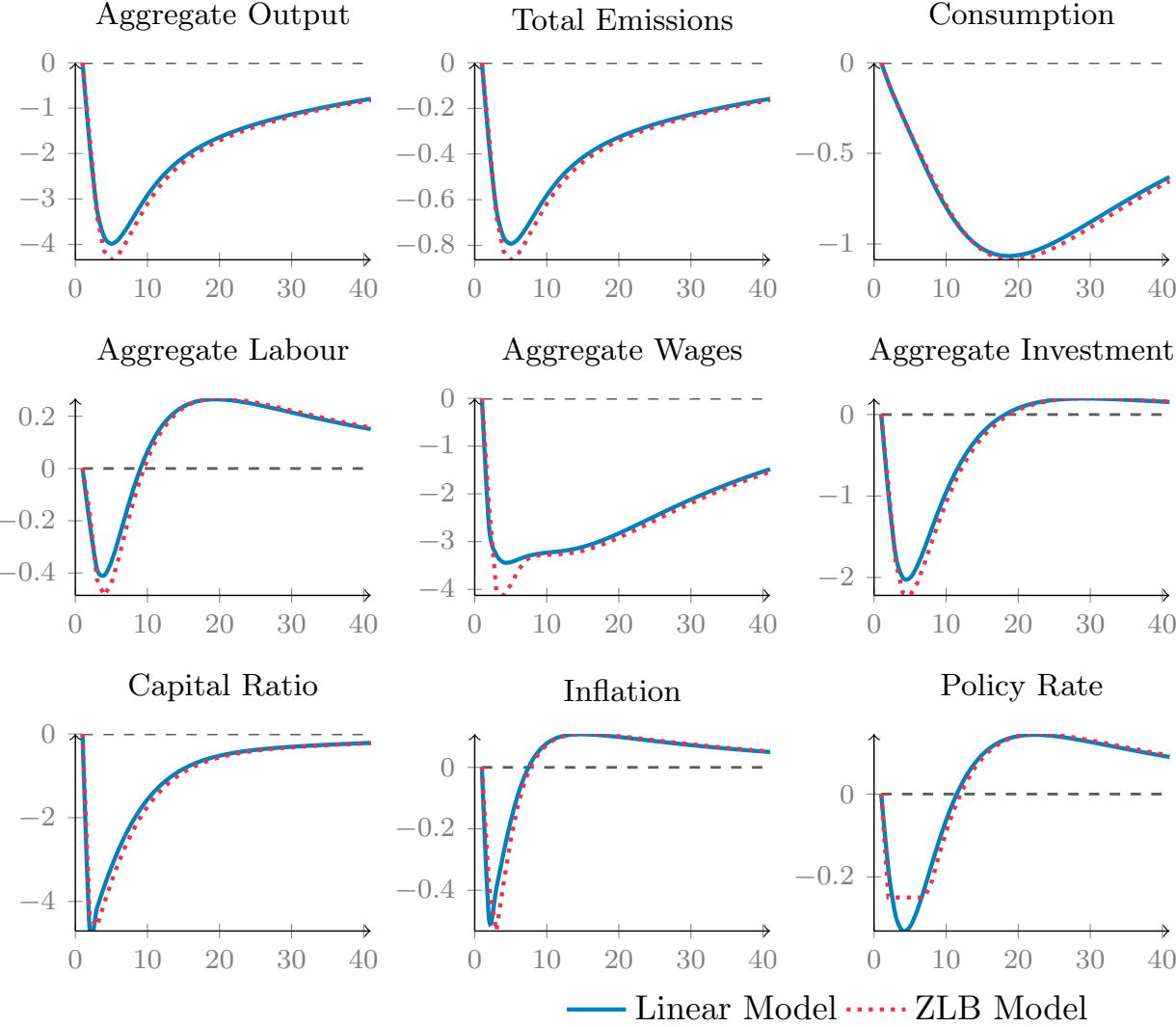
Figure 7: Effect of a positive green technology shock $(\varepsilon_t^{A_g})$ on selected variables between the baseline and tax policy scenarios - percentage deviations from steady state.
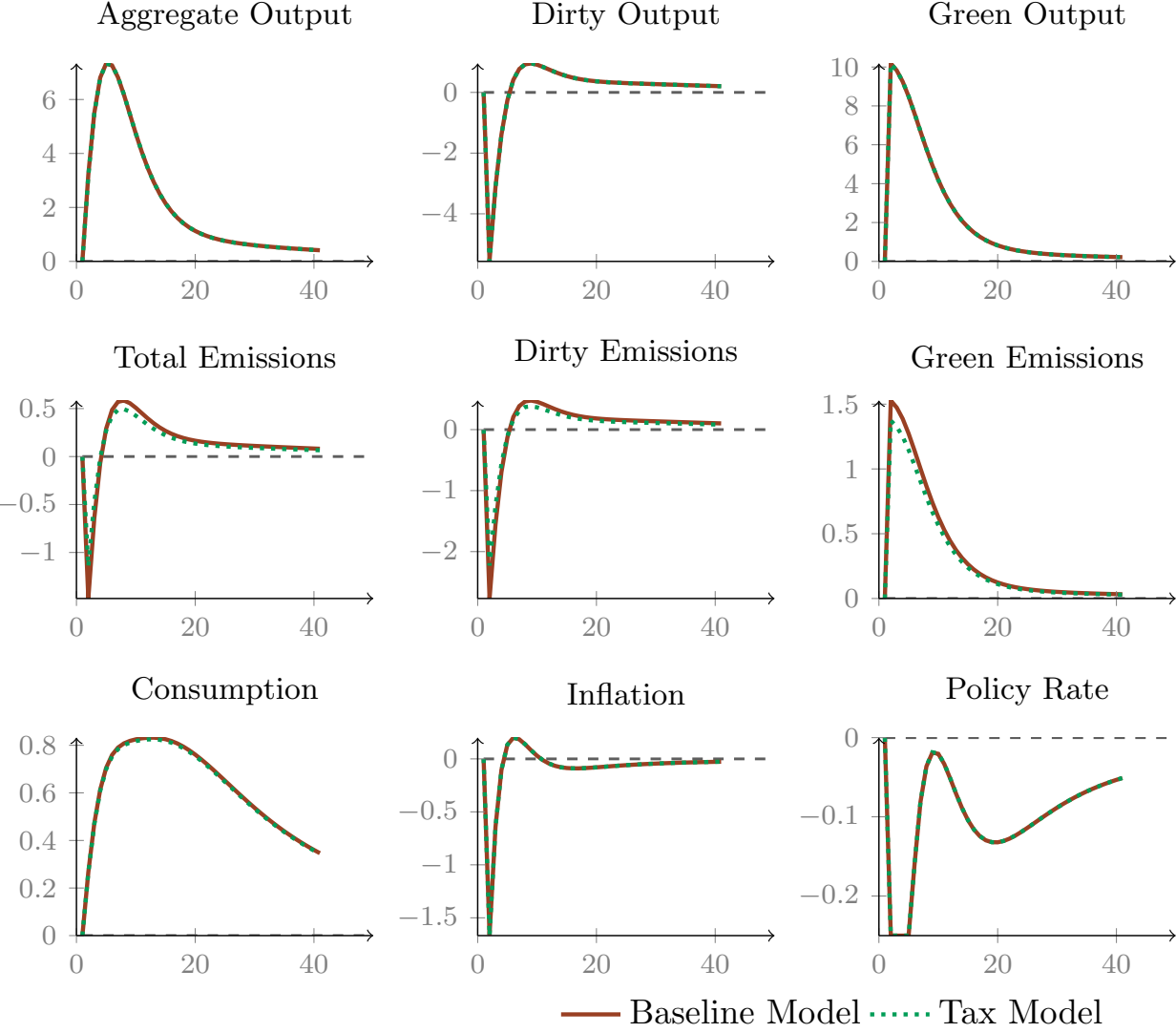
Figure 8: Effect of a positive dirty technology shock ($\varepsilon_t^{A_d}$) on selected variables between the baseline and tax policy scenarios - percentage deviations from steady state.
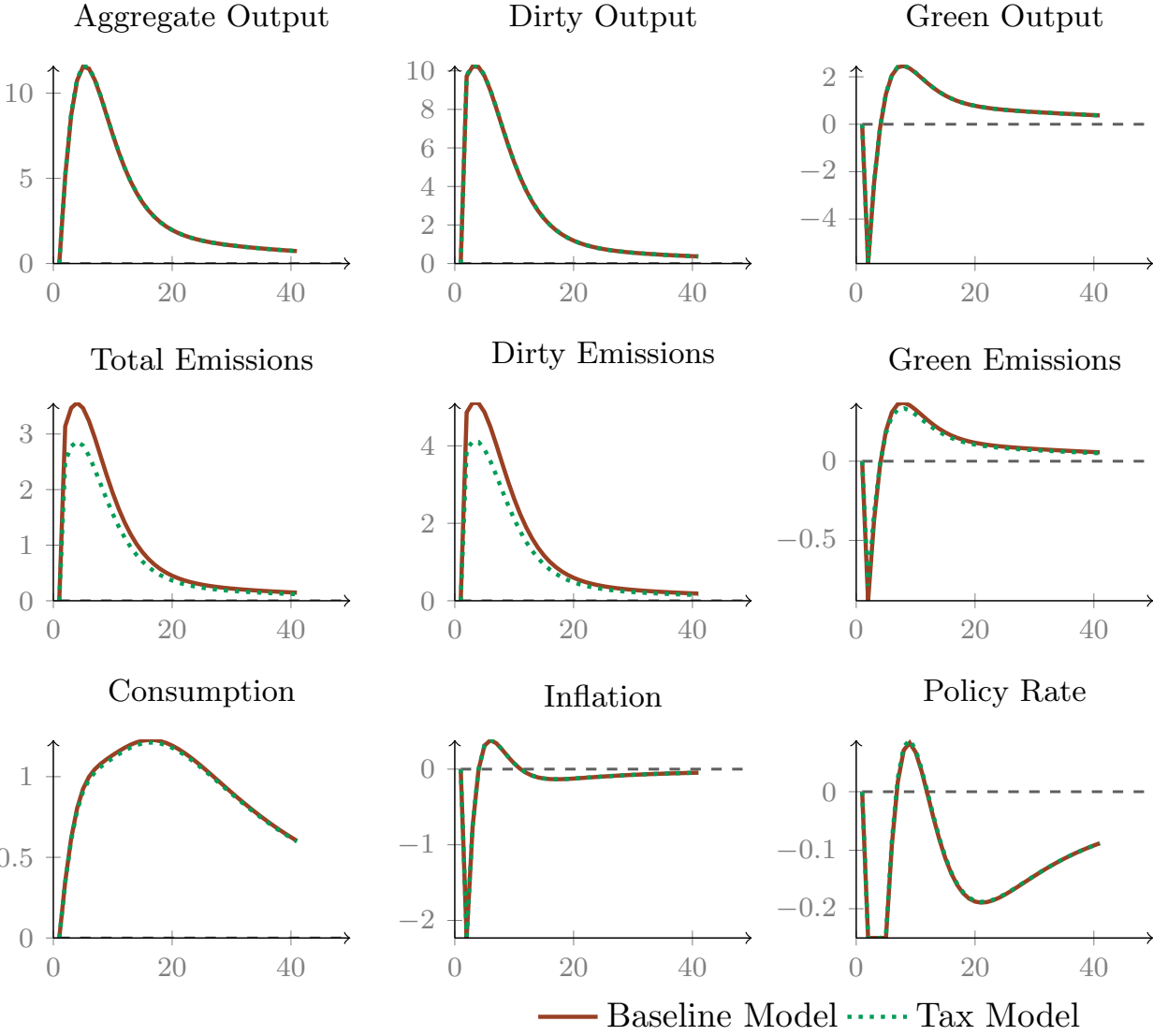

Aggregate Output

Dirty Output

Green Output

Total Emissions

Dirty Emissions

Green Emissions

Consumption

Inflation

Policy Rate

—— Baseline Model ····· Tax Model

Figure 9: Effect of a negative preference shock (ε_t^B) on selected variables between the baseline and tax policy scenarios - percentage deviations from steady state.
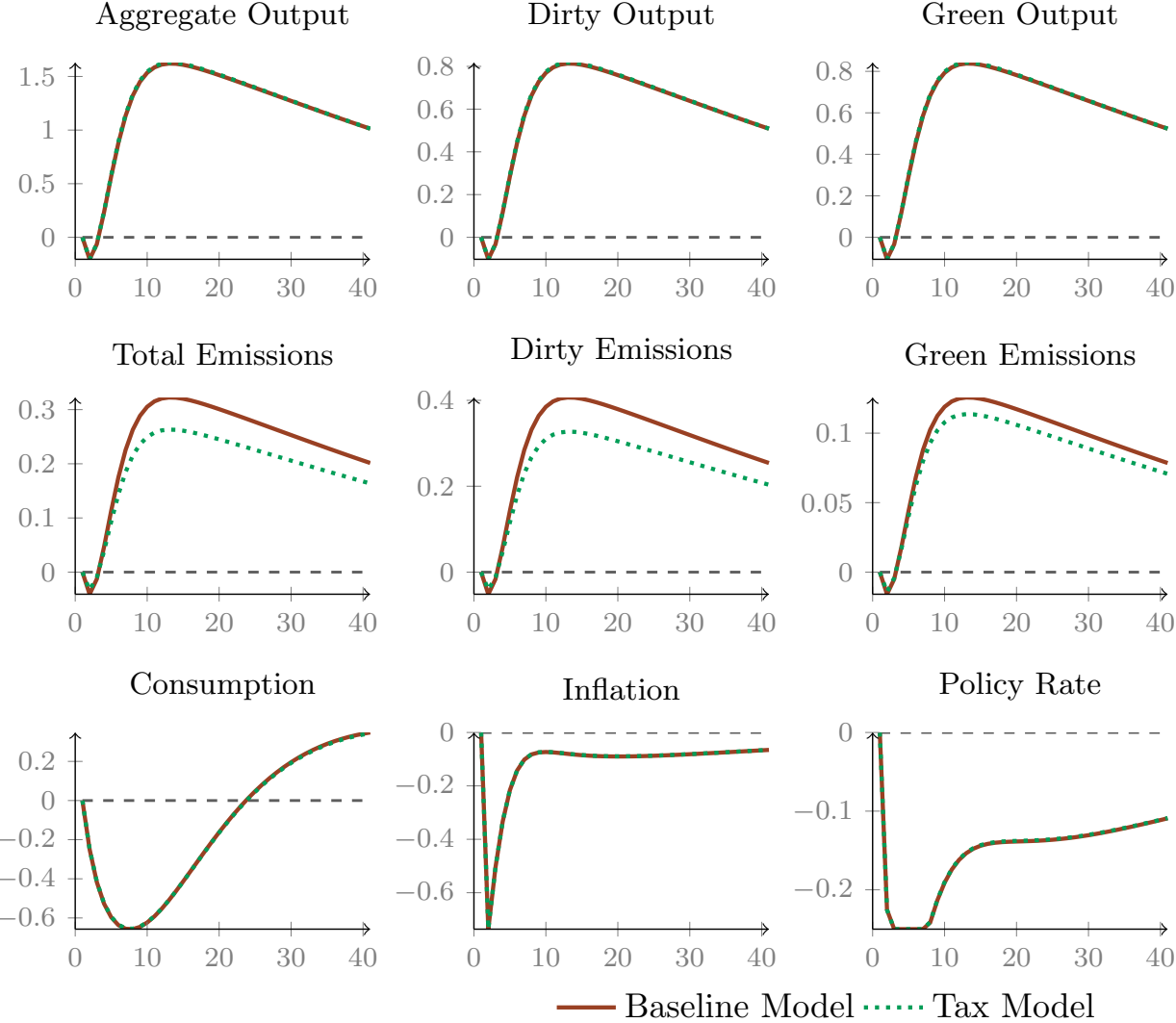

Aggregate Output   Dirty Output   Green Output

Total Emissions   Dirty Emissions   Green Emissions

Consumption   Inflation   Policy Rate

— Baseline Model ····· Tax Model

Figure 10: Effect of a negative preference preference shock (ε_t^B) on selected variables between the tax policy and macroprudential policy scenarios - percentage deviations from steady state.
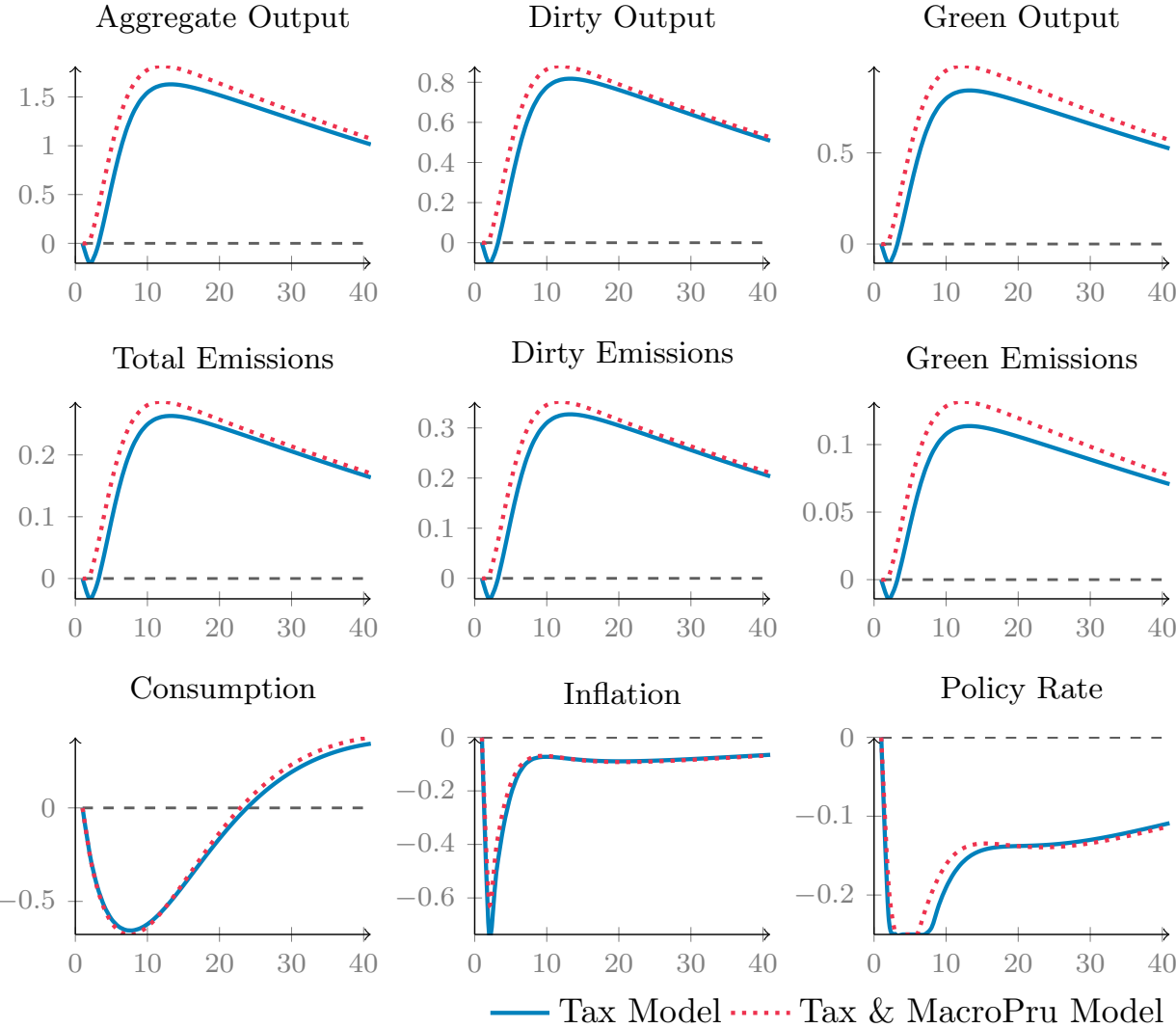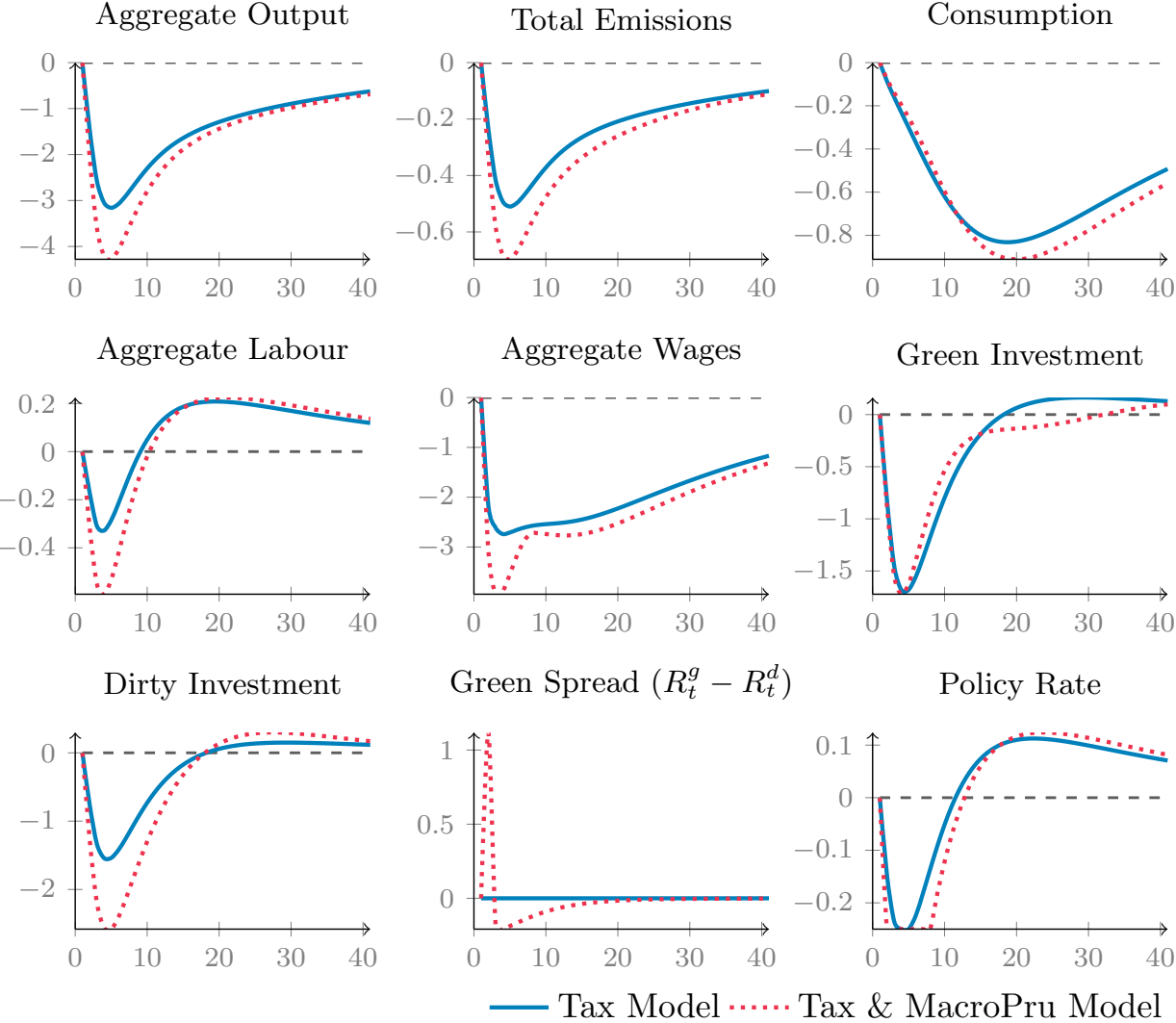

Aggregate Output · Dirty Output · Green Output · Total Emissions · Dirty Emissions · Green Emissions · Consumption · Inflation · Policy Rate

—— Tax Model ····· Tax & MacroPru Model

Figure 11: Effect of a negative capital quality shock (ε_t^K) on selected variables between the tax policy and macroprudential policy scenarios - percentage deviations from steady state.

Aggregate Output · Total Emissions · Consumption · Aggregate Labour · Aggregate Wages · Green Investment · Dirty Investment · Green Spread ($R_t^g - R_t^d$) · Policy Rate

—— Tax Model ····· Tax & MacroPru Model

Figure 12: Effect of a seris of positive dirty QE shock ($\varepsilon_t^{\psi_d}$) on selected variables - percentage deviations from steady state.
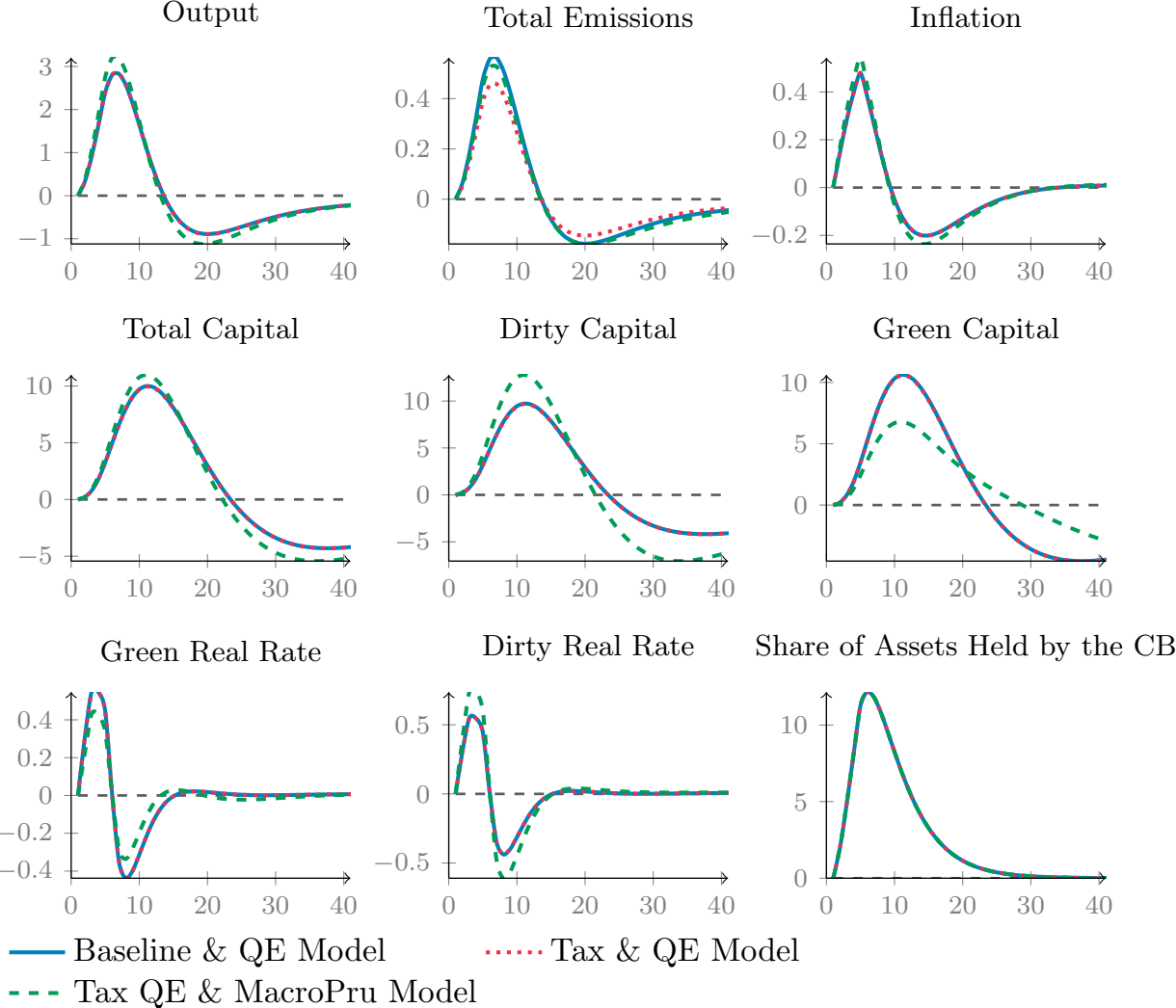

Output

Total Emissions

Inflation

Total Capital

Dirty Capital

Green Capital

Green Real Rate

Dirty Real Rate

Share of Assets Held by the CB

— Baseline & QE Model

· · · · Tax & QE Model

- - - Tax QE & MacroPru Model

Figure 13: Effect of a series of positive green QE shock ($\varepsilon_t^{\psi_g}$) on selected variables - percentage deviations from steady state.

— Baseline & QE Model   ····· Tax & QE Model

--- Tax QE & MacroPru Model
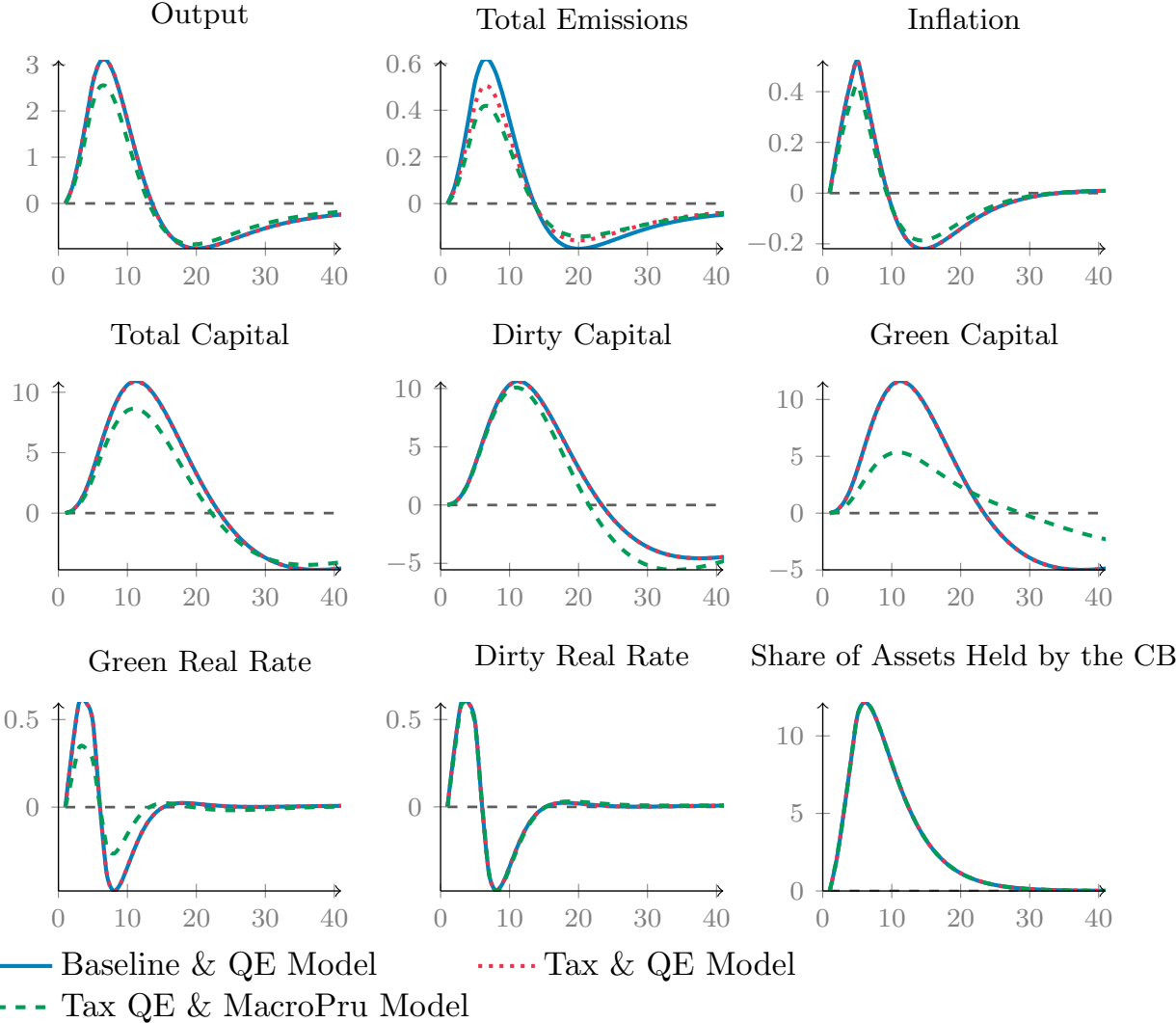
C   Appendix: Robustness Check

In this section we compare our banking modeling approach, which is built à la Pietrunti [2017], with the proposed model specification of Gertler et al. [2012]. The sections treating Firms (2.2) and Public Authorities (2.4) remain unchanged, while Household (2.1) and Financial intermediaries (2.3) are adjusted as shown in the following sections. We confirm that our findings regarding the broad dynamics of the model at the ZLB hold with this specification. For the additional parameters calibrations, we take the values of Gertler et al. [2012].

C.1   The Household

We modify our initial setup to allow for a supply of funds to banks through deposits and equity. Deposits, which are non-contingent, risk-less loans to banks, are remunerated at the same rate as government bonds. Given that they are both one-period riskless bonds, they are perfect substitutes. Following Gertler et al. [2012], equity funded by households are modeled as perfectly state-contingent debt and will be called *outside* equity hereafter.[21] We differ from their setup in that we allow households to provide two types of outside equity to banks that will be used to finance either green or dirty firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. The rest of the setup remains unchanged for the representative household.

Th new household maximization problem reads:

$$\max_{\{C_t, L_{t,k}, B_{t+1}, \bar{e}_{t+1,k}\}} E_t \sum_{i=0}^{\infty} \beta^i \frac{\varepsilon_{t+i}^B}{\varepsilon_t^B} \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \sum_k \frac{\chi_k}{1+\varphi} L_{k,t+i}^{1+\varphi} \right] \tag{57}$$

---

[21] As opposed to *inside* equity, which are banks' retained earnings.

s.t.

$$C_t + B_{t+1} + \sum_k Q_{et,k} \bar{e}_{t+1,k} =$$

$$\sum_k \left(\frac{W_{k,t}}{P_t} L_{k,t} + \Pi_{k,t} \right) + \frac{T_t}{P_t} + R_t B_t + \sum_k \left[\frac{R_{t,k}^K}{P_t} + (1-\delta) Q_{et,k} \right] \psi_{t,k} e_{t,k}, \tag{58}$$

The sole difference between the above budget constraint and the one used in the main model section is the introduction of equities. As in Gertler et al. [2012], we normalize the units of outside equity $e_{t,k}$ to allow for the equity in each sector to be a claim to the future returns of one unit of the asset that the bank holds. $R_{t,k}^K$ represents the nominal flow of returns generated by one unit of the specific sectoral bank's assets. $Q_{et,k}$ is the price of each type of outside equity. $\psi_{t,k}$ represents the shock on capital quality as in Gertler and Karadi [2011].

The new first order conditions read

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{59}$$

$$\varrho_t = \chi_k \frac{L_{t,k}^{\varphi}}{W_{t,k}/P_t}, \tag{60}$$

$$1 = \beta E_t \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \Lambda_{t,t+1} R_{t+1}, \tag{61}$$

$$1 = \beta E_t \frac{\varepsilon_{t+1}^B}{\varepsilon_t^B} \Lambda_{t,t+1} R_{et+1,k}, \tag{62}$$

where the stochastic discount factor (i.e. the expected variation in marginal utility of consumption) and the returns on sectoral equity $R_{et+1,k}$ read, respectively:

$$\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}, \tag{63}$$

$$R_{et+1,k} = \frac{[\frac{R_{t+1,k}^K}{P_{t+1}} + (1-\delta) Q_{et+1,k}] \psi_{t+1,k}}{Q_{et,k}}. \tag{64}$$

C.2 Financial Intermediaries

We modify the setup of Gertler et al. [2012] to allow financial intermediaries to invest in both green and carbon-intensive ('dirty') firms. They also issue two types of outside equity, depending on the type of firms they want to lend to. A representative bank's balance sheet can be depicted as[22]:

$$Q_{t,g}S_{t,g} + Q_{t,d}S_{t,d} = N_t + B_t + Q_{et,g}e_{t,g} + Q_{et,d}e_{t,d}, \tag{65}$$

where $S_{t,g}$ and $S_{t,d}$ are financial claims on green and dirty firms and $Q_{t,g}$ and $Q_{t,d}$ their respective relative price. On the liability side, N_t is the banks' net worth (also referred to as *inside* equity), B_t is debt to households, and $Q_{et,k}e_{t,k}$ is outside equity for each sector k[23]. Over time, the banks' equity capital evolves as follows :

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,d}Q_{t-1,d}S_{t-1,d} - R_{et,g}Q_{et-1,g}e_{t-1,g} - R_{et,d}Q_{et-1,d}e_{t-1,d} - R_tB_{t-1}. \tag{66}$$

Using equations (65) and (66) we can rewrite the banks' equity capital law of motion as follows:

$$
\begin{aligned}
N_t =& (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d} - R_t)Q_{t-1,d}S_{t-1,d} \\
& - (R_{et,g} - R_t)Q_{et-1,g}e_{t-1,g} - (R_{et,d} - R_t)Q_{et-1,d}e_{t-1,d} + R_tN_{t-1},
\end{aligned}
\tag{67}
$$

where $R_{t,k} = \frac{R_{k,t}^K/P_t - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denote the gross rate of return on a unit of the bank's assets from $t-1$ to t for sector k.[24]

The goal of a financial intermediary is to maximize the expected present value of the

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[22]As shown in Gertler et al. [2012] the results still hold for all the aggregate banking sector as well.

[23]The outside equity $e_{t,k}$ is the same as the equity held by household as we are interested in equilibrium demand by the household matches the supply from banks.

[24]Note that the depreciate capital has a value of one as adjustment costs only apply to net investment.

future terminal dividend. Thus, we can write the following objective function:

$$V_t = E_t \left\{ \sum_{i=1}^{\infty} \beta^i \Lambda_{t,t+i} (1 - \theta_B) \theta_B^{i-1} N_{t+1} \right\}. \tag{68}$$

Following Gertler et al. [2012], we assume that managers may divert a fraction of assets to their household once a bank obtains funds. As such possibility arise, households set limitations to the funds they lend to banks. Furthermore, the fraction of funds that could be diverted depends on the composition of the banks' balance sheets. In particular, as highlighted by Gertler et al. [2012], it is assumed "that at the margin it is more difficult to divert assets funded by short term deposits than by outside equity". While, short term deposits constrain the bank to meet a non-contingent payment, dividend payments on the other hand, are tied to the performance of the bank's assets, which is difficult for outsiders to monitor.

Let $x_{t,k}$ denote the fraction of bank assets funded by outside equity for each sector:

$$x_{t,k} = \frac{Q_{et,k} e_{t,k}}{Q_{t,k} S_{t,k}}. \tag{69}$$

Then we assume that after the bank has obtained funds it may divert the fraction $\lambda(x_t)$ of assets:

$$\lambda(x_{t,k}) = \lambda \left(1 + \lambda_1 x_{t,k} + \frac{\lambda_2}{2} x_{t,k}^2 \right). \tag{70}$$

We assume that households require that the discounted value of the bankers' net worth should be greater than or equal to the value they would be able to divert:

$$V_{t,k} \geq \lambda(x_{t,k}) Q_{t,k} S_{t,k}. \tag{71}$$

Using equation (67) and (69) we rewrite again the evolution of the net worth:

$$\begin{aligned} N_t =& (R_{t,g} - x_{t-1,g} R_{et,g} - (1 - x_{t-1,g}) R_t)) Q_{t-1,g} S_{t-1,g} \\ &+ (R_{t,d} - x_{t-1,d} R_{et,d} - (1 - x_{t-1,d}) R_t)) Q_{t-1,d} S_{t-1,d} + R_t N_{t-1}. \end{aligned} \tag{72}$$

Thus for easing the resolution of the maximization problem, we introduce $N_{t,k}$ the net worth for each sector k such that:

$$N_t = \sum_k N_{t,k},$$

(73)

and

$$N_{t,k} = (R_{t,k} - x_{t-1,k}R_{et,k} - (1 - x_{t-1,k})R_t))Q_{t-1,k}S_{t-1,k} + R_t N_{t-1,k}.$$

(74)

Thus, the franchise value of the bank at the end of period $t - 1$ should satisfy the following Bellman equation for each sector k:

$$V_{t-1,k}(S_{t-1,k}, x_{t-1,k}, N_{t-1,k}) = E_{t-1}\beta\Lambda_{t-1,t}\left\{(1 - \theta_B)N_{t,k} + \theta_B \max_{S_{t,k}, x_{t,k}} \left[V_{t,k}(S_{t,k}, x_{t,k}, N_{t,k})\right]\right\},$$

(75)

where θ_B is the banks probability to keep existing. We guess as in Gertler et al. [2012] that the value function is linear of the form:

$$V_{t,k}(S_{t,k}, x_{t,k}, N_{t,k}) = (\mu_{st,k} + x_{t,k}\mu_{et,k})Q_{t,k}S_{t,k} + \nu_{t,k}N_{t,k}.$$

(76)

In order to solve the above maximization problem with the conjectured value function linear form, we set the leverage ratio for each sector $\Phi_{t,k}$ (i.e. the maximum ratio of bank assets to net worth) such as:

$$\Phi_{t,k} = \frac{Q_{t,k}S_{t,k}}{N_{t,k}},$$

(77)

which indicates that when the borrowing constraint binds, the total quantity of private assets that a bank can intermediate is limited by its net worth $N_{t,k}$.

Maximization of the Bellman function subject to constraint (71) yields the following first order and slackness conditions:

$$\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1,k}(R_{t+1})\right\} = \nu_{t,k},$$

(78)

$$\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1,k}(R_{t+1,k} - R_{t+1})\right\} = \mu_{st,k},$$

(79)

$$\beta E_t \left\{ \Lambda_{t,t+1} \Omega_{t+1,k} (R_{t+1,k} - R_{et+1,k}) \right\} = \mu_{et,k}, \tag{80}$$

where ν_t is the multiplier for constraint (71), $\beta \Lambda_{t,t+1}$ is the banks' stochastic discount factor and $\Omega_{t+1,k} = 1 - \theta_B + \theta_B [\nu_{t+1,k} + \Phi_{t+1,k}(\mu_{et+1,k} + x_{t+1,k}\mu_{et+1,k})]$ the shadow value of a unit of net worth to the bank at $t+1$. Furthermore, we can rewrite the leverage ratio following the above first order conditions:

$$\Phi_{t,k} = \frac{\nu_{t,k}}{\lambda(x_{t,k}) - (\mu_{st,k} + x_{t,k}\mu_{et,k})}. \tag{81}$$

Solving the first order conditions on $x_{t,k}$ and $S_{t,k}$ we rewrite the fraction of assets financed by outside equity in each sector as the ratio of the excess value from substituting outside equity for deposit finance μ_{et} to the excess value on assets over the deposit μ_{st} as follows:

$$x_{t,k} = -\frac{\mu_{st,k}}{\mu_{et,k}} + \left[\frac{\mu_{st,k}}{\mu_{et,k}}^2 + \frac{2}{\lambda_2} \left(1 - \lambda_1 \frac{\mu_{st,k}}{\mu_{et,k}} \right) \right]^{\frac{1}{2}}. \tag{82}$$

Since the leverage ratio does not depend on bank-specific factors, we can aggregate equation (77) to obtain a relation between the aggregate demand for securities by banks $S_{pt,k}$ in each sector and aggregate net worth in the banking sector for each firms sector $N_{t,k}$.

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t = \sum_k \left\{ (\theta_B + \omega')[R_{t,k}^K + (1-\delta)Q_{t,k}]\psi_{t,k}S_{pt-1,k} - \theta_B[R_{t,k}^K + (1-\delta)Q_{et,k}]\psi_{t,k}e_{t-1,k} \right\} - \theta_B R_t B_{t-1}, \tag{83}$$

C.3 Robustness Check: Main Model Dynamic Results

Figure 14: Effect of a positive green technology shock ($\varepsilon_t^{A_g}$) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.
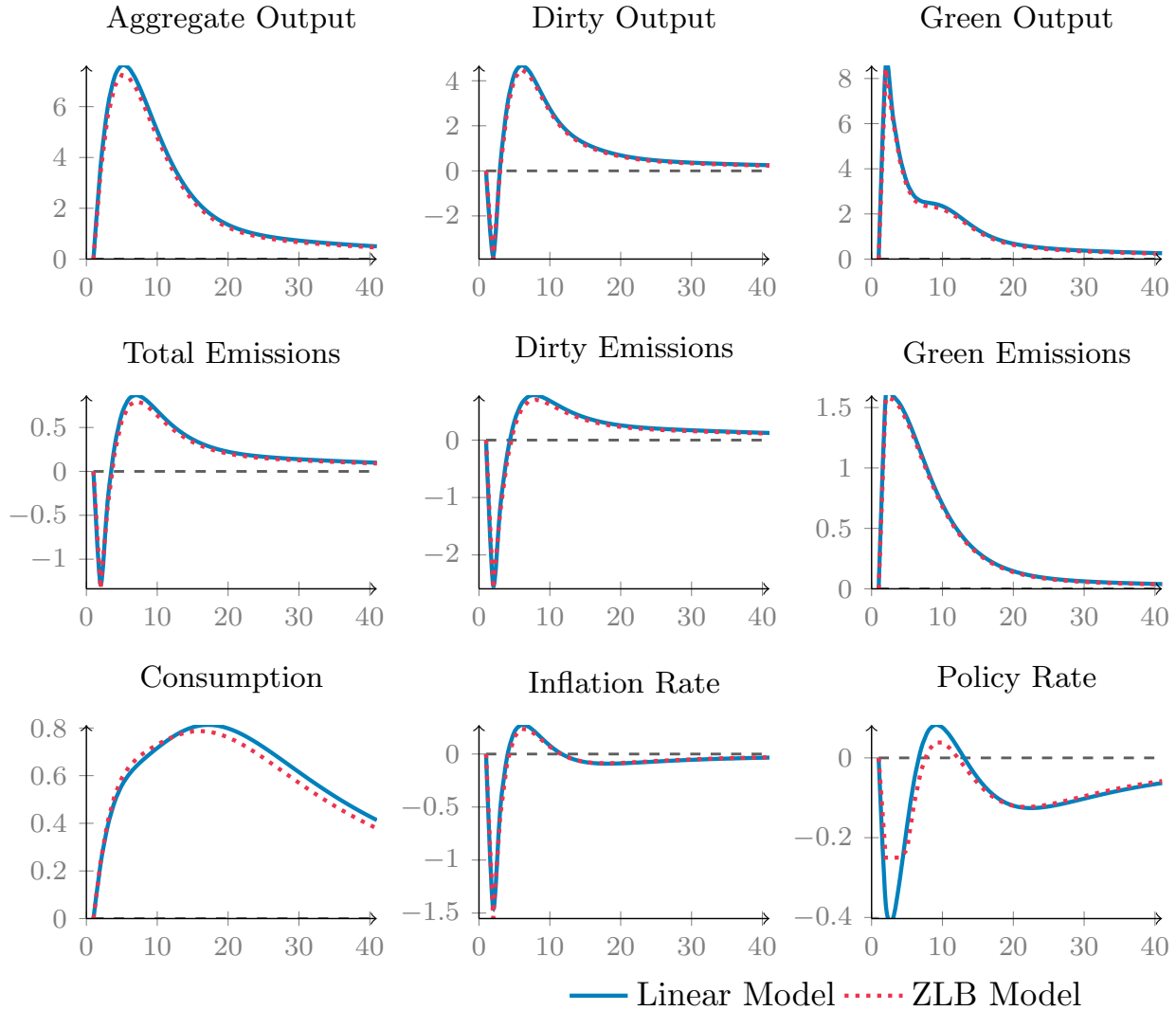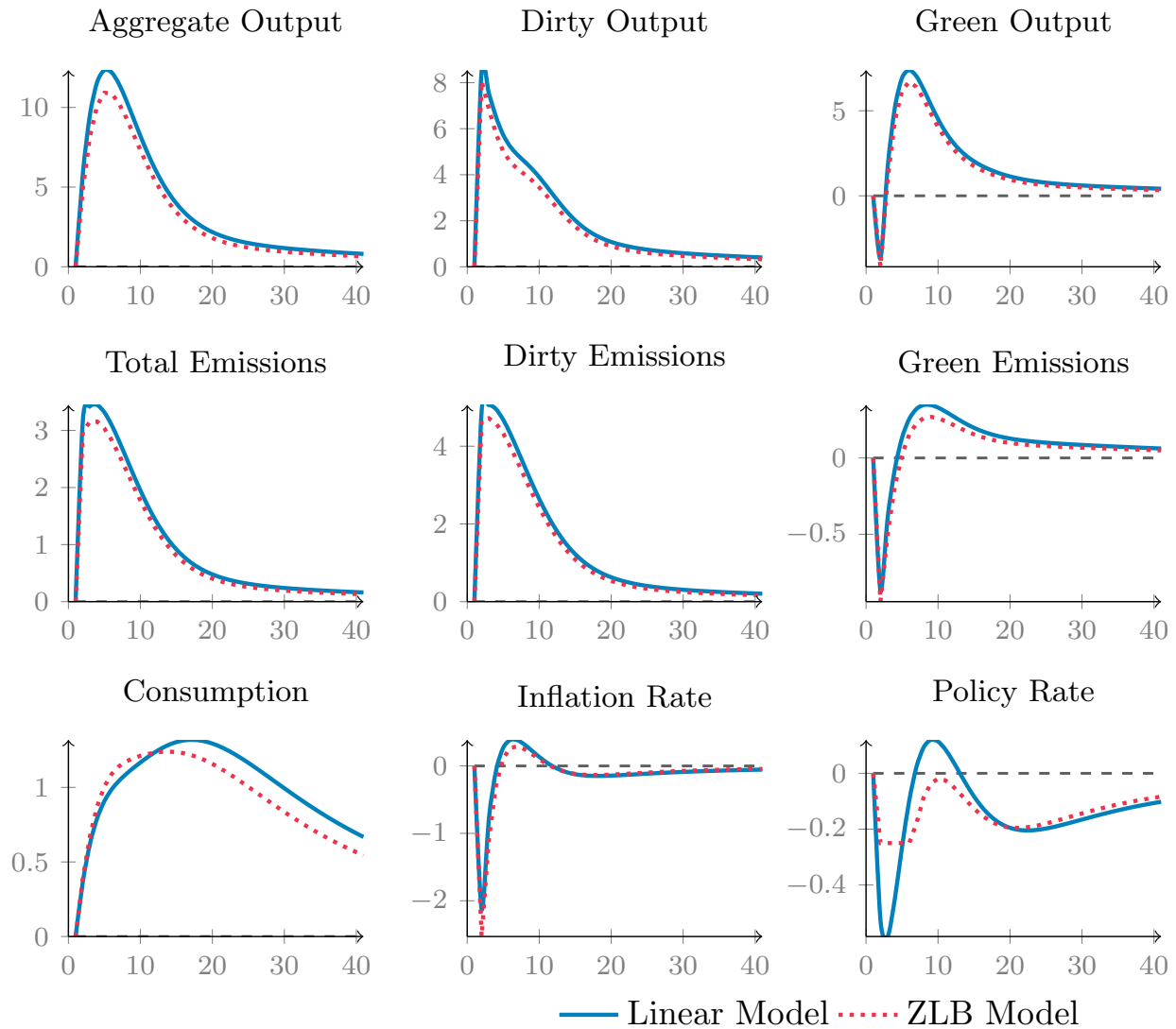
Figure 15: Effect of a positive dirty technology shock ($\varepsilon_t^{A_d}$) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.

Figure 16: Effect of a negative preference shock (ε_t^B) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.
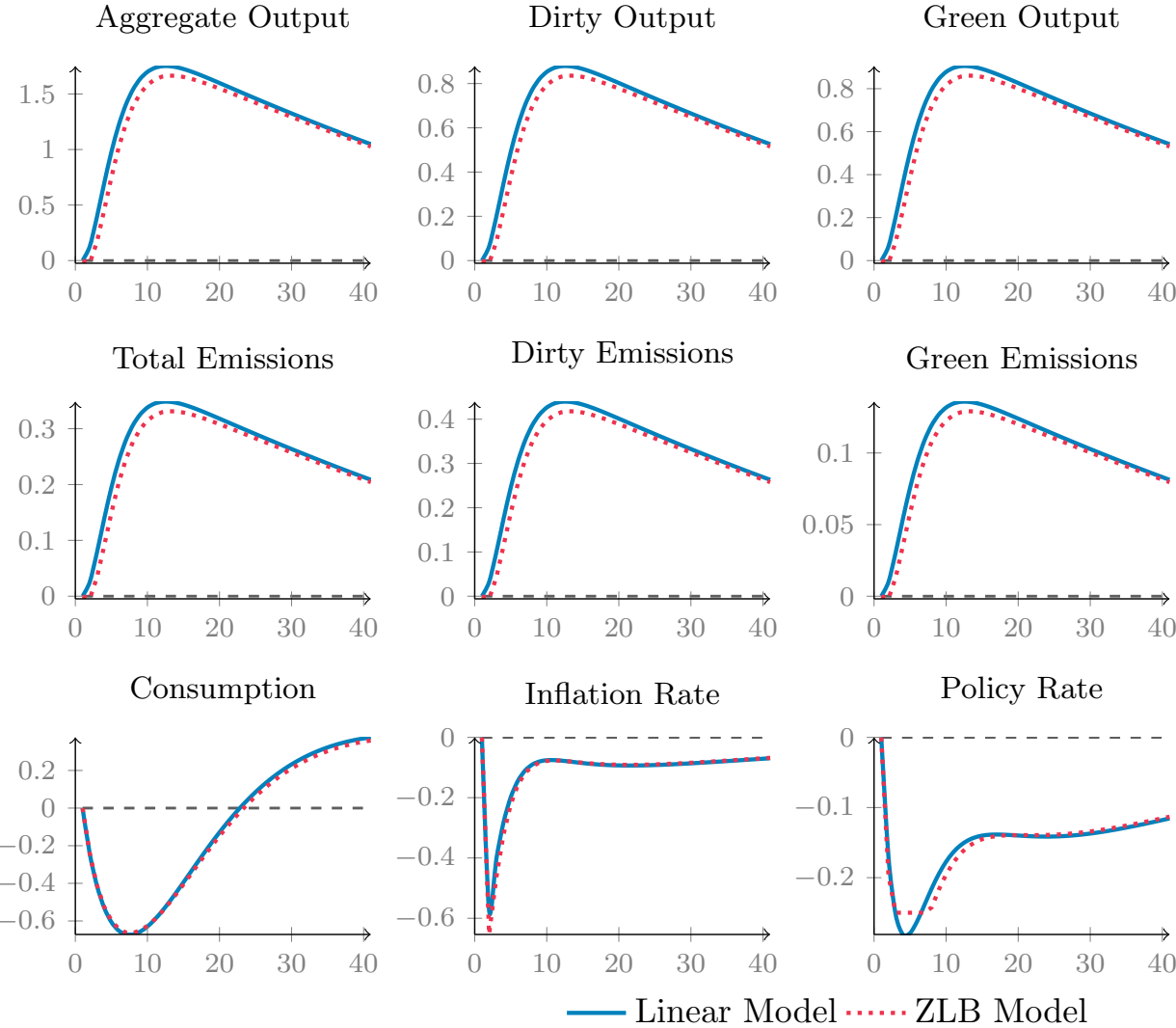
Figure 17: Effect of a negative capital shock (ε_t^K) on selected variables between the linear and non-linear models (following Gertler et al. [2012] banking specification) - percentage deviations from steady state.
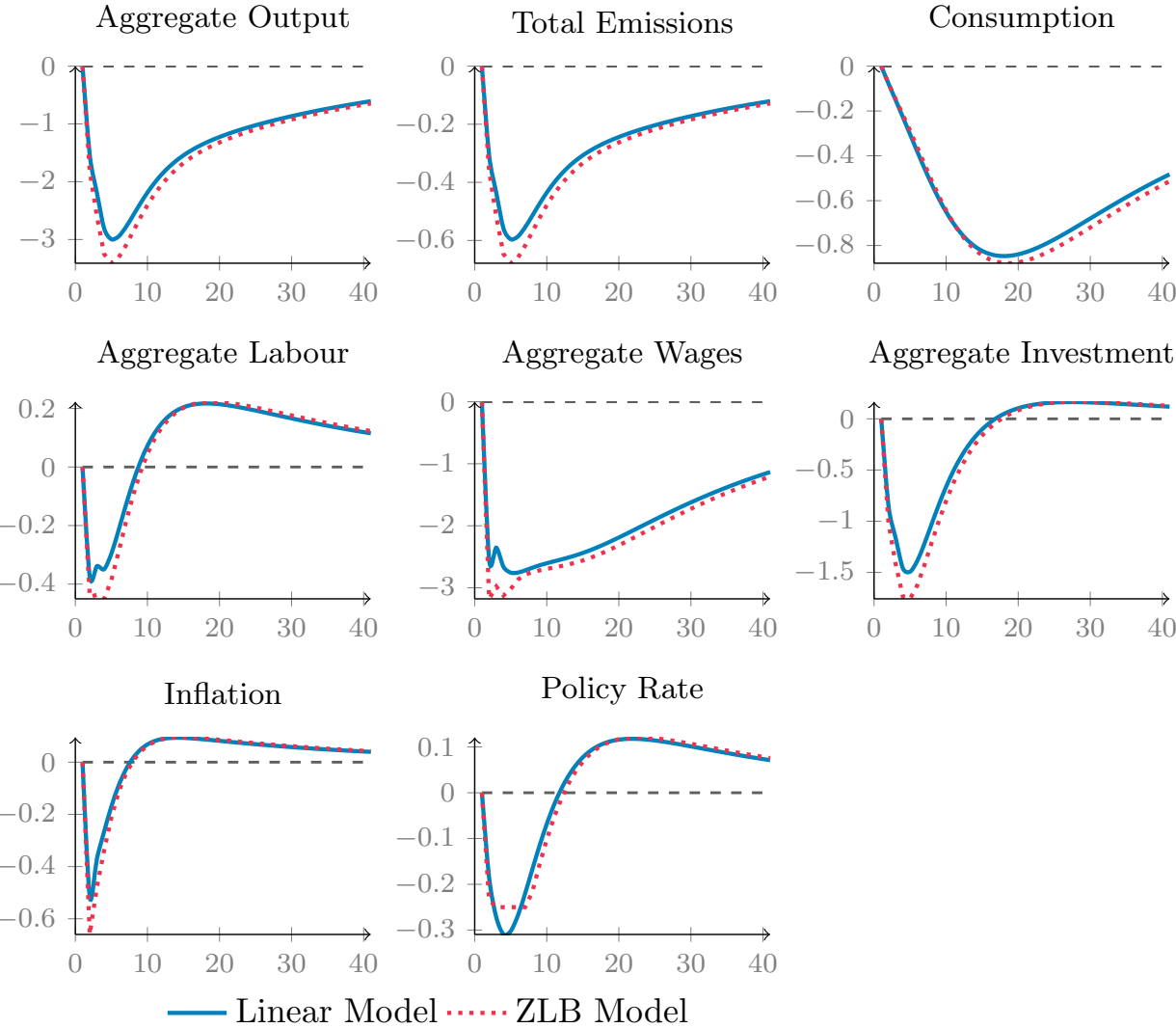

Aggregate Output

Total Emissions

Consumption

Aggregate Labour

Aggregate Wages

Aggregate Investment

Inflation

Policy Rate

—— Linear Model ····· ZLB Model

Notes: The impulse response for capital ratio is not presented as we do not have weights in the Gertler et al. [2012] banking specification.