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Elisa Cavatorta and Ben Groom

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Hyperbolic Discounting in the Absence of Credibility

Elisa Cavatorta* and Ben Groom†

March 25, 2019

Abstract

Hyperbolic discounting *behavior* can arise in experiments when expected utility maximizing subjects who discount exponentially doubt the credibility of future payoffs. We show theoretically that lack of credibility introduces a present bias, as subjects internalize the uncertainty. Hence, experiments that do not ensure credibility may erroneously conclude that observed behavior is driven by hyperbolic pure time preferences, rather than the rational response to non-credible payoffs.

1 Introduction

Hyperbolic discounting is seen as one of the chief departures from the standard rational agent model, and provides a theoretical explanation for all sorts of irrational inter-temporal behaviour (e.g. Loewenstein and Prelec, 1992; O'Donoghue and Rabin, 1999; Frederick et al., 2002) and self-control problems (Camerer et al., 2003). The associated present-bias explains drug-addiction and procrastination, e.g. in saving (Laibson, 1997); while time-inconsistent hyperbolic discounters who recognise their affliction demand commitment mechanisms in the form of financial instruments or collective saving funds (Ashraf et al., 2006; Ambec and Treich, 2007; Basu, 2011). On the supply side, hyperbolic discounters are subjected to price discrimination that exploits their characteristic present-bias, gym membership being a much-cited example (e.g., Akerlof, 1991; Della Vigna and Malmandier, 2006). Unsurprisingly, the theory of hyperbolic discounting has been accompanied by a glut of experimental evidence, from lab and field, revealing its prevalence among individuals.

In this paper we show that experimental evidence could be flawed if subjects perceive future payoffs in the decision task as lacking credibility, and unlikely to be paid in full. We show that an experimental subject that doubts the credibility of future payoffs will behave like a hyperbolic discounter even if they are an exponential discounter with a horizon-invariant pure time preference rate. Observing this behaviour, the experimenter, who does not doubt the credibility of their experiment, could explain this behaviour as arising from horizon-dependent pure time preferences, rather than exponential discounting coupled with the rational internalisation of the credibility risk. Having different beliefs to the subject,

*Elisa Cavatorta: Department of Political Economy, King's College London, Strand, London WC2R 2LS and Institute for Fiscal Studies, 7 Ridgmount Street London WC1E 7AE; e-mail: elisa.cavatorta@kcl.ac.uk.

†Ben Groom (corresponding author): Department of Geography and Environment, London School of Economics, Houghton Street, London WC2A 2AE, e-mail: b.groom@lse.ac.uk.
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the experimenter comes to the wrong conclusions about the structure of preferences and the determinants of observed inter-temporal behaviour.

We provide a general characterisation of non-credibility, and model future payoffs as being subject to uncertain decay rates. An expected utility maximizer will then factor-in the lack of credibility in her expected utility calculations, and internalise the possibility that the promise of future payoffs could be broken. We show that this internalisation leads to an *effective* discount rate that declines with the waiting-time for payoff.

Analogous examples of "payoff decay" come from the animal world. For instance, decay could represent the hazard associated with food sources being eaten by a competitor, or the risk of an animal dying before reproducing (Dasgupta and Maskin, 2005; Sozou, 1998). The characterization of lack of credibility as a decay rate function is distinct from the idea that pure time preference rates vary with the time horizon, which would lead to hyperbolic behaviour even when all parties agree that future payoffs are guaranteed. The "non-credibility effect" is an additional discounting effect, and additional to pure time preference.

The theory developed here adapts and extends the explanation for hyperbolic discounting that relies on uncertain hazard rates in a risk neutral world, advanced by Sozou (1998). There are two key results. First, we show that in presence of non-credible payoffs, subjects with exponential time preferences could be mistaken for hyperbolic discounters, with their departure from exponential discounting depending upon their risk preferences. Typically greater risk tolerance leads to more hyperbolic and present-biased behaviour. Second, if observed inter-temporal behavior follows the Matching Rule of Herrnstein (1961) this could be the outcome of exponential discounter applying a Bayesian prior over the uncertain payoff decay rate that depends on the subject's pure time and risk preferences. The analysis provides a precise behavioural meaning to the single parameter of the Matching Rule and shows that the departure from exponential discounting depends on the ratio of the pure time and risk preferences.

The conclusion is clear. Exponential discounters who doubt the credibility of future payoffs could be observationally described as hyperbolic discounters. Experimenters may then conflate hyperbolic behaviour with hyperbolic pure time preference, and conclude a departure from rationality where there is none. This is a potentially important distinction in the understanding of inter-temporal decision making and its behavioural anomalies. It highlights the need to disentangle risk and time preferences in experiments, and speaks directly to concerns about misclassification of discounting 'types' raised by Harrison and Lau (2005), Dohmen et al. (2012), Andreoni and Sprenger (2010) and others, when credibility in behavioural experiments is doubtful.

2 A model of non-credible payoffs

The extent to which experimental subjects perceive the experiment in which they participate to reflect credible 'real-life' decisions is often called into question. In turn a lack of credibility or 'reliability' undermines the applicability of observed experimental behavior to contexts external to the experiment. The perception of non-credibility could come from uncertainty about whether the promised payoffs will arrive in their entirety, if at all. Both lab and field experiments are potentially susceptible to perceptions of non-credibility if the credibility of the payment delivery mechanism is unclear and uncertain, perhaps subject to transactions costs (e.g. graft) in the future, or if the experimenters themselves are perceived to be untrustworthy. The issue of credibility is likely to be particularly important when subjects are asked to evaluate payoffs that are proposed to occur in the future. Of course, experimenters are not blind to this possibility, and the use of Front-End Delay (FED) in the measurement of time preferences, or commitment to well-established, credible payoff delivery methods are but some of the remedial approaches that have been taken.[1] Yet

[1] Andreoni and Sprenger (2010) use the trusted internal mail system of the university in which the experimental subjects were students to enhance the credibility of their experiment.

despite best efforts, the *perception* or belief of non-credibility may remain at the point in time at which the choice task is undertaken. Furthermore, FED, which is almost universally deployed in experiments on time preference, only solves issues of credibility that are not dependent on the time horizon. Intuitively, issues of credibility are likely to amplify with the time horizon of the payoff, as the delay increases and subjects are asked to consider payoffs that will be delivered further and further into the future (Harrison and Lau 2005). The following model captures the potential effects of the perception of non-credibility or unreliability on inter-temporal decisions in a parsimonious way.

Suppose an experimental subject faces a choice between a cash flow today and what they are told by the experimenter is a *sure* cash flow at time $\tau : \bar{c}_\tau$. However, the subject is uncertain about the credibility or reliability of this payoff, despite the assurances of the experimenter. One dimension of non-credibility is that the subject may not believe that the payoff will arrive in its entirety, and this effect increases with futurity of the payoff. A simple decay function captures these time related dimensions of non-credibility where the subject perceives the payoff at time τ as:

$$c_\tau = \bar{c}_\tau \exp(-\lambda\tau)$$

where $\lambda \geq 0$ is a time-invariant decay parameter which captures the subject's beliefs about the credible payoff at time τ. If $\lambda = 0$ then the subject believes that the payoff is fully credible.

Another element underpinning the perception of non-credibility is that subjects are typically uncertain about how credible the payoff is. To capture this uncertainty we suppose that the respondent is uncertain about the decay parameter, λ, believing that it is drawn from some subjective prior probability distribution, $f(\lambda)$, with support $\lambda \in [\lambda_{\min}, \lambda_{\max}]$. For the time being we consider a single respondent, and return to the issue of heterogeneous agents at a later stage. In this framework, the current value of the expected payoff at time τ is given by $E[c_\tau] = \int_{\lambda_{\min}}^{\lambda_{\max}} \bar{c}_\tau \exp(-\lambda\tau) f(\lambda) \, d\lambda$.

This simple model of uncertain decay is an appealing way to model credibility due to its simplicity. It is also somewhat general, having a number intuitive interpretations. For instance, the decay rate can be interpreted as reflecting the cumulation of the impact of discrete events which prevent payments from arriving in full: thefts, future transactions costs including graft, weather events or accidents, all of which could raise the costs of delivery and reduce the eventual future payoff. Such an interpretation may be particularly relevant in field experiments in developing countries, for instance.[2] Similarly, following Sozou (1998), in a risk neutral context the decay factor $\exp(-\lambda\tau)$ could reflect unreliable payoffs that are subject to an uncertain hazard rate. In this case $\exp(-\lambda\tau)$ reflects the conditional probability that the payoff arrives at the promised time τ if the hazard rate is λ.[3] Alternatively, the uncertain decay framework can be thought of as a general model of prior subjective beliefs about the future payoff, which embody those effects of non-credibility that vary with the time horizon, for whatever reason. The following section shows how, modelled this way and understood very generally, the absence of credibility can affect the nature and interpretation of experimental results on individual discount rates.

[2] In the Supplementary Material we explain how a special case of our decay-rate framework can be found in work by Martin and Pindyck (2013) on the onset of multiple catastrophic events which each remove a proportion of GDP. Our decay function could be interpreted in the same way as their expected cumulant function, reflecting the summation over time of multiple catastrophes. Something similar could be one way of thinking about the mechanism that drives the perception of the absence of credibility, albeit at the micro level.

[3] Sozou (1998) interprets the discount factor as the survivor function associated with a hazard rate λ. This probabilistic interpretation of the discount factor could also reflect a lack of credibility in payoffs, but is not the interpretation we take in this paper.

3 Hyperbolic discounting with non-credible payoffs and risk neutrality

Consider a risk neutral experimental subject who discounts future utility exponentially at a pure rate of time preference δ. If the subject doubts the credibility of the future payoff $\bar{c}_\tau$ then according to the model above their discounted expected utility at time τ is given by:

$$\begin{aligned} E[u(c_\tau)\exp(-\delta\tau)] &= E\left[u(\bar{c}_\tau \exp(-\lambda\tau))\exp(-\delta\tau)\right] \\ &= \bar{c}_\tau \int_{\lambda_{\min}}^{\lambda_{\max}} \exp(-(\delta+\lambda)\tau) f(\lambda) d\lambda \\ &= \bar{c}_\tau \beta(\tau) \end{aligned} \tag{1a}$$

where the last line defines $\beta(\tau) = \int_{\lambda_{\min}}^{\lambda_{\max}} \exp(-(\delta+\lambda)\tau) f(\lambda) d\lambda$.

The parameter $\beta(\tau)$ can be thought of as the effective discount factor applied by the subject to the payoff $\bar{c}_\tau$. This discount factor embodies both the pure rate of time preferences, δ, and the lack of credibility reflected by the uncertain λ. However, if the experimenter is ignorant of the subject's perception that the future payoff is non-credible or unreliable, they will attribute any observed discounting behavior to the subject's pure time preference alone. Defining the experimenter's erroneous perception of pure time preference for any particular time horizon τ, δ^*_τ, as follows:

$$\bar{c}\exp\left(-\delta^*_\tau \tau\right) = \bar{c}\beta(\tau) \tag{2}$$

means that $\exp\left(-\delta^* \tau\right) = \beta(\tau)$ and hence by rearrangement:

$$\delta^*_\tau = \delta - \frac{1}{\tau}\ln\left[\beta(\tau)\right] \tag{3}$$

This leads to Proposition 1.

Proposition 1 *With no loss of generality, if λ is distributed with a probability density function $f(\lambda)$ defined on support $\lambda \in [\lambda_{\min}, \lambda_{\max}]$, with mean $\bar{\lambda}$, δ^*_τ, as defined in equation (3), is declining hyperbolically with the time horizon, τ, of the payoff. Specifically: a) $\delta^*_0 = \delta + \bar{\lambda}$; b) $\lim_{\tau\to\infty} \delta^*_\tau = \delta + \lambda_{\min}$.*

Proof. See Appendix A.1. for the risk-neutral case where $\eta = 0$. ■

Proposition 1 means that the experimenter, assuming that δ^* represents pure time preference alone, will erroneously conclude that the subject is a hyperbolic discounter in an experiment performed repeatedly for different values of τ. Whereas, the estimate δ^*_τ is a biased estimate of δ, biased by the subject's perception of non-credibility.

Noticing that subjects behave in a way that suggests non-exponential discounting, the experimenter may turn to other commonly used parametric forms for the discount function to characterize behavior, such as the Matching Rule of Hernnstein (1961), the generalised hyperbolic model of Loewenstein and Prelec (1992), or the quasi-hyperbolic discounting model. Proposition 1 shows that this would be a mistake where payoffs are perceived as being non-credible, since exponential discounters behave as if they were non-exponential when they internalize the non-credibility into their response to the decision task. The following sections extend this result to the case where respondents are not risk-neutral and shows how the internalization of non-credibility depends on risk-preferences.

4 Hyperbolic discounting with non-credible payoffs and risk aversion

In this section we extend the analysis beyond risk-neutrality. By modelling the credibility of an experimental payoff we formalise concerns made in the experimental literature that the credibility or reliability of future payoffs will conflate risk and time preferences in the estimation of these parameters (Harrison et al., 2005; Andreoni and Sprenger 2011; Weber and Chapman 2005). The theory developed here provides a neat explanation of experimental results on the relationship between time preference, present-bias and risk aversion.

For simplicity assume the subject has iso-elastic utility, a parametric form commonly used in the analysis of decision tasks (e.g. Holt and Laury 2002). In this case the subject's discounted expected utility at time τ is:

$$\begin{aligned} E\left[u\left(c_{\tau}\right)\exp\left(-\delta\tau\right)\right] &= E\left[u\left(\bar{c}_{\tau}\exp\left(-\lambda_{i}\tau\right)\right)\exp\left(-\delta\tau\right)\right] \\ &= \left(1-\eta\right)^{-1}\bar{c}_{\tau}^{1-\eta}\int_{\lambda_{\min}}^{\lambda_{\max}}\exp\left(-\left(\lambda\left(1-\eta\right)+\delta\right)\tau\right)f\left(\lambda\right)d\lambda \\ &= \left(1-\eta\right)^{-1}\bar{c}_{\tau}^{1-\eta}\beta\left(\tau\right) \end{aligned} \tag{4}$$

where: $\beta(\tau) = \exp(-\delta\tau)\int_0^{\infty}\exp(-(\lambda(1-\eta))\tau)f(\lambda)d\lambda$. In this formulation $\beta(\tau)$ can again be thought of as the effective discount factor applied by the subject to the proposed future payoff $\bar{c}_{\tau}$. In this case, in addition to the pure rate of time preference, δ, and the uncertain decay rate, λ, risk aversion, η, also plays a part in the subject's evaluation of the future payoff. Specifically, the subject views non-credibility (uncertain decay rate) through the lens of their risk aversion, reflected by the scaling of the uncertain decay rate by exponent of the utility function: $\lambda(1-\eta)$.

As before, suppose that the experimenter assumes that the subject shares their convictions about the credibility of the future payoff and that any impatience exhibited by the subject is due to pure time preference alone. Defining the experimenter's erroneous perception of pure time preference for any particular time horizon τ, δ_{τ}^{*}, as follows:

$$u\left(\bar{c}_{\tau}\right)\exp\left(-\delta_{\tau}^{*}\tau\right) = \left(1-\eta\right)^{-1}\bar{c}_{\tau}^{1-\eta}\beta\left(\tau\right) \tag{5a}$$

which means $\exp(-\delta^{*}\tau) = \beta(\tau)$, and hence by rearrangement:

$$\delta_{\tau}^{*} = \delta - \frac{1}{\tau}\ln\left[\beta\left(\tau\right)\right] \tag{6}$$

This leads to Proposition 2.

Proposition 2 *With no loss of generality, if λ is distributed with a probability density function $f(\lambda)$ defined on support $\lambda \in [\lambda_{\min}, \lambda_{\max}]$, with mean $\bar{\lambda}$, δ_{τ}^{*}, as defined in equation (6), is declining with the time horizon, τ, of the payoff, unless $\eta = 1$. Specifically:*

a) $\eta < 1$: δ_{τ}^{*}, is decreasing hyperbolically in τ with $\delta_0^{*} = \delta + (1-\eta)E[\lambda]$ and $\lim_{\tau\to\infty}\delta_{\tau}^{*} = \delta + \lambda_{\min}(1-\eta)$;

b) $\eta > 1$: δ_{τ}^{*}, is decreasing in τ with $\delta_0^{*} = \delta + (1-\eta)E[\lambda]$ and $\lim_{\tau\to\infty}\delta_{\tau}^{*} = \delta + \lambda_{\max}(1-\eta)$. Finally, it is easy to see that:

c) $\eta = 1$: $\delta_{\tau}^{*} = \delta$, and discounting is exponential at the rate of pure time preference.

Proof. See Appendix A.1. ■

Proposition 2 means that where non-credibility in future payoffs is characterized as uncertain decay, exponential discounters who are not risk-neutral will behave *as if* their pure time preference for the sure payoff is declining with the time horizon τ. The extent of the present bias/hyperbolic behaviour is dependent on the extent of the perception of

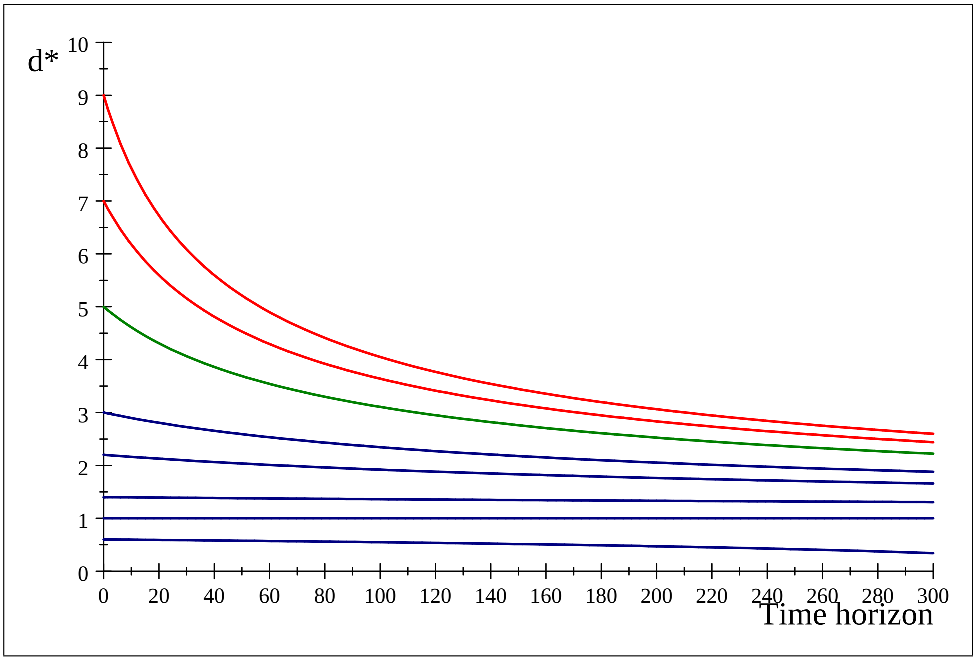

Figure 1: δ^*_τ with perception of non-credibility/non-reliability. The $\tau-$horizon discount rate with gamma distributed decay rate λ with a mean of 4% and a standard deviation of 0.3% Pure time preference is $\delta = 1\%$. Red schedules are risk loving respondents for whom from the top: $\eta = (-1.0, -0.5)$. The green schedule is the risk neutral case: $\eta = 0$, and the blue schedules are the risk averse cases: $\eta = (0.5, 0.7, 0.9, 1.0, 1.1)$.

non-credibility (the beliefs about λ), and their aversion to risk, η. The experimenters, on the other hand, believing that the payoff $\bar{c}_\tau$ is fully credible, will attribute such behaviour to pure time preference alone and conclude that the subjects are non-exponential, sometimes hyperbolic, discounters. This would only be the correct interpretation when $\eta = 1$.[4]

The model sheds light on how the estimation of risk and time preferences can be conflated in non-credible experiments. For ease of illustration Figures 1 and 2 provide an example in which non-credibility is reflected by a Gamma distributed λ.[5] In both cases, for $\eta < 1$ agents will appear more impatient and less exponential than they really are. For $\eta > 1$ subjects appear more patient and less exponential than they really are. There is a negative relationship between the discount rate and risk aversion. Contrasting Figures 1 and 2 reveals that as future payoffs become increasingly non-credible, as measured by an increasing variance of λ, the $\tau-$ horizon discount rates become more sharply hyperbolic when $\eta < 1$.[6]

5 Prior distributions of λ for hyperbolic discounters

Several parametric forms of hyperbolic discounting functions have been used in the literature to describe inter-temporal behaviour which differ from $\beta(\tau)$ and δ^*_τ defined in the previous section (e.g. Loewenstein and Prelec, 1992). In this section we as a different question: in the presence of non-credibility, which prior-beliefs about credibility, i.e. λ, must the exponential utility maximiser have used to give rise to the typical parametric form of hyperbolic discounting? To illustrate the point we use a special case of the generalized hyperbolic discount

[4]The risk neutral case is isomorphic to the analysis of Section 2.

[5]Derivations of the closed form solutions for the discount factor and discount rate when λ is Gamma distributed are shown in Appendix A.2.

[6]Figure 2 introduces a mean preserving spread to λ compared to 1.

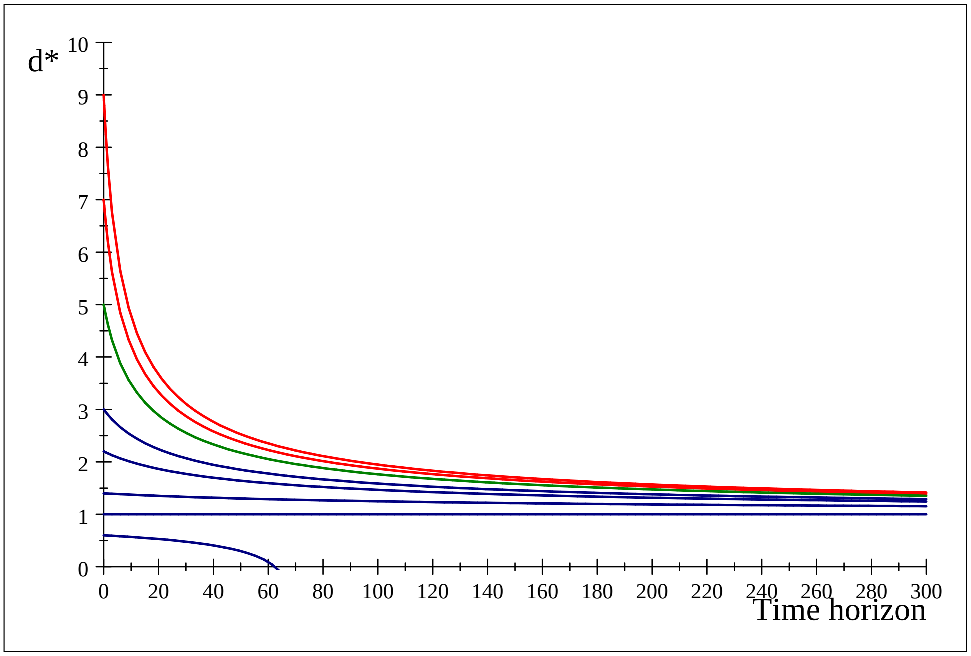

Figure 2: δ_τ^* with stronger perception of non-credibility/non-reliability: The $\tau-$horizon discount rate term structure with gamma distributed decay rate λ with a mean of 4% and a variance of 0.75% Pure time preference is $\delta = 1\%$. Red schedules are risk loving respondents for whom from the top: $\eta = (-1.0, -0.5)$. The green schedule is the risk neutral case: $\eta = 0$, and the blue schedules are the risk averse cases: $\eta = (0.5, 0.7, 0.9, 1.0, 1.1)$.

function: Hernnstein's Matching Rule (Hernnstein, 1961), which is particularly convenient.

The generalised hyperbolic discount function for time horizon τ is given by $(1+\alpha\tau)^{\frac{-\delta}{\alpha}}$, where parameters δ and α measure the extent to which individuals view units of time as being more *compressed,* meaning shorter, in the future. The notation for δ is deliberate since:

$$\lim_{\alpha\to 0}(1+\alpha\tau)^{\frac{-\delta}{\alpha}} = \exp(-\delta\tau)$$

so that the parameter δ can still be interpreted as the pure rate of time preference. α then measures the departure from the exponential discount function. In principle, each of these parameters represents a separate aspect of individual time preferences.

Hernnstein's Matching Rule is a special case of this function, which has been shown to have good explanatory power in human, as well as non-human, subjects. The Rule leads to a discount function (factor) of the following form:

$$\beta(\tau) = \frac{1}{1+\alpha\tau}$$

The parameter α again measures the extent of the deviation from exponential discounting. If α is large there will be a large disparity between short and long-horizon discount rates, a strong present-bias and a high likelihood of time-inconsistent decisions. The question in our context is: what prior belief about non-credibility, reflected in the prior distribution for the decay rate λ, leads to this kind of well-documented intertemporal behaviour? The answer is found by solving for $f(\lambda)$ using:

$$\beta(\tau) = \int_0^\infty \exp(-(\lambda(1-\eta)+\delta)\tau) f(\lambda)\, d\lambda = \frac{1}{1+\alpha\tau}$$

This leads to Proposition 3.

Proposition 3 *For an agent facing a non-credible future payoff $\bar{c}_\tau$ with iso-elastic utility with parameter η, pure time preference δ, the prior distribution of λ that leads to the posterior discount function reflected by the Matching Rule with $\alpha \neq 0$, is given by the preference and horizon-dependent exponential probability density function:*

$$f(\lambda) = \frac{1}{\alpha} \exp\left(-\frac{\lambda}{\alpha} + \tau(\delta - \lambda\eta) \right)$$

Proof. See Appendix A.4. ∎

Proposition 3 indicates that the prior on the uncertain decay rate is governed by the three preference parameters (δ, η, α) and the time horizon, τ. However, for this prior probability density function to be proper we need to check that $\int f(\lambda)\, d\lambda = 1$. This leads to Proposition 4:

Proposition 4 *For an agent with iso-elastic utility with risk aversion parameter η and pure time preference δ facing an uncertain decay parameter λ, the α-parameter of Hernnstein's Matching Rule is approximated by:*

$$\alpha \approx \frac{\delta}{\eta}$$

Proof. The proof comes from exploiting the fact that:

$$\int_0^\infty f(\lambda)\, d\lambda = \int_0^\infty \frac{1}{\alpha} \exp\left(-\frac{\lambda}{\alpha} + \tau(\delta - \lambda\eta) \right) d\lambda = 1$$

and solving out. See Appendix A.3. ∎

In the presence of uncertain decay rates, agents with iso-elastic utility that are hyperbolic discounters have a discount function that is determined by the ratio of their underlying preferences for risk and time. In this context, risk and time preferences work in opposite directions in determining departure from exponential discounting and hence the shape of the hyperbolic discount function. For a given level of risk aversion, more impatient individuals will be more 'hyperbolic', while for a given level of impatience, more risk averse agents will be less hyperbolic. To the extent that inter-temporal conflict is driven by the size of the α-parameter in the Matching Rule, this result states that when agents are uncertain about the decay rate of future payoffs, we are less likely to see time-inconsistent behaviour among patient and risk averse Matching Rule discounters, than among more impatient, risk-tolerant ones. This is a testable hypothesis.

The results are another illustration of how observed hyperbolic discounting behaviour can be a rational response to non-credible payoffs. More importantly, the results show how well-established, empirically motivated hyperbolic discount functions can be explained by traditional preference parameters.

6 Conclusion

Hyperbolic discounting has been widely used to provide an explanation of dynamic inconsistencies and self-control problems in economic behaviour. Choice experiments providing real payoffs have been an important tool to measure individual inter-temporal discounting and classify subjects in discounting types (e.g. exponential, hyperbolic, etc.). This paper has shown that hyperbolic discounting can arise when exponential discounters are faced with payoffs that lack credibility. Therefore, empirical findings of hyperbolic discounting may well be an experimental artefact reflecting a lack of credibility of future payoffs. Certainly, the *extent* of hyperbolic discounting could be exaggerated in such cases. The departure from exponential discounting is more pronounced the higher is subject's risk aversion. This would lead to 'biased' classification of subject discounting 'types'.

These findings are important as they indicate that some caution is warranted in the interpretation of previous empirical results. The theory developed here uncovers a potential source of the extent to which hyperbolic discounting might have been exaggerated in experimental work in which payoffs lacked credibility, due to the delivery mechanism, lab reputation or their hypothetical nature. The theory speaks directly to the quest for inter-temporal choice models which take into account subjects' time horizon-sensitivity in discounting observed in experiments (Dohmen et al., 2012). Future experimental work could quantify the extent to which credibility can affect discounting, and isolate how individuals construct their priors when credibility is lacking. Until then, this paper simply highlights the importance of gauging and ensuring reliability and credibility in the lab and field.

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A Appendix

A.1 Proof of Proposition 1:

Define the $\tau-$horizon discount rate, $\delta^* (\tau)$ as:

$$\beta(\tau) = \exp(-\delta^*(\tau)\tau) = \exp(-\delta\tau) E[\exp(-x\tau)]$$

and rearrange:

$$\delta^*(\tau) = \delta - \frac{1}{\tau}\ln[E[\exp(-x\tau)]]$$

First, when $x, \tau \geq 0$ it is well known that:

$$\lim_{\tau\to 0} \delta^*(\tau) = \delta + E[x]$$
$$\lim_{\tau\to\infty} \delta^*(\tau) = \delta + x_{\min}$$

Likewise, when $x < 0, \tau \geq 0$:

$$\lim_{\tau\to 0} \delta^*(\tau) = \delta - E[x]$$
$$\lim_{\tau\to\infty} \delta^*(\tau) = \delta - x_{\max}$$

This comes from the fact that for $\exp(-x(t)t) = E[\exp(-xt)]$, $x(0) = E[x]$, and $\lim_{t\to\infty} x(t) = x_{\min}$ while for $\exp(-x(t)t) = E[\exp(xt)]$, $x(0) = -E[x]$, and $\lim_{t\to\infty} x(t) = -x_{\max}$. The definition of $x = \lambda(1-\eta)$ means that $(1-\eta)$ determines the sign of x which then proves Proposition 1a) and b). These are well known results from the social discounting literature. See e.g. Hepburn and Groom (2007, Appendix). For Proposition 1c), when $\eta = 1$, then the discount rate is purely δ.

A.2 Proposition A.1: 'Gamma discounting'

Proposition 5 (A.1.) *For an agent with iso-elastic utility with parameter η and pure time preference δ, facing an uncertain payoff decay rate, λ,if the prior distribution of decay rates is a gamma distribution $\Gamma(\alpha, \beta)$, then the $\tau-$horizon discount factor, $\beta^G(\tau)$ and discount rate, $\delta^G(\tau)$, are given respectively by:*

$$\beta^G(\tau) = \exp(-\delta\tau)\left(\frac{\beta}{(1-\eta)\tau+\beta}\right)^{\alpha}$$

$$\delta^G(\tau) = \delta + \frac{\alpha}{\tau}\ln\left(\frac{(1-\eta)\tau+\beta}{\beta}\right)$$

$\delta^G(\tau)$ is a present biased, declining term structure of discount rates for all η except $\eta = 1$, as shown in Figures 1 and 2 and predicted by Proposition 1.

Proof. The $\tau-$period certainty equivalent discount factor is defined by:

$$\begin{aligned}\beta(\tau) = \exp(-\delta^*(\tau)\tau) &= \int_0^\infty \exp(-(\lambda(1-\eta)+\delta)\tau) f(\lambda) d\lambda \\ &= E[\exp(-(\lambda(1-\eta)+\delta)\tau)]\end{aligned}$$

the associated certainty equivalent discount rate is then given by:

$$\delta^*(\tau) = -\frac{1}{\tau}\ln[E[\exp(-(\lambda(1-\eta)+\delta)\tau)]]$$

Assuming a gamma distribution for $\lambda : f(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\lambda\beta}$, gives:

$$\begin{aligned}\beta^G(\tau) &= \frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty \exp(-(\lambda(1-\eta)+\delta)\tau)\lambda^{\alpha-1}e^{-\lambda\beta}d\lambda \\ &= \exp(-\delta\tau)\frac{\beta^\alpha}{\Gamma(\alpha)}\int_0^\infty \exp(-(\lambda((1-\eta)\tau+\beta)))\lambda^{1-\alpha}d\lambda\end{aligned}$$

It is a fact of calculus that:

$$\int \lambda^{\alpha-1}e^{-b\lambda}d\lambda = \frac{\Gamma(\alpha)}{b^\alpha}$$

so applying this, noting that with $b = (1-\eta)\tau+\beta > 0$ we get:

$$\beta^G(\tau) = \exp(-\delta\tau)\left(\frac{\beta}{(1-\eta)\tau+\beta}\right)^{\alpha}$$

The certainty equivalent discount rate is then:

$$\begin{aligned}\delta^G(\tau) &= -\frac{1}{\tau}\ln\left[\exp(-\delta\tau)\left(\frac{\beta}{(1-\eta)\tau+\beta}\right)^{\alpha}\right] \\ &= \delta + \frac{\alpha}{\tau}\ln\left(\frac{(1-\eta)\tau+\beta}{\beta}\right)\end{aligned}$$

■

A.3 Proof of Proposition 3:

The proof that the prior distribution of Hazard rates that leads to the hyperbolic discount factor is given by:

$$f(\lambda) = \frac{1}{\alpha} \exp\left(-\frac{\lambda}{\alpha} + \tau(\delta - \lambda\eta)\right)$$

goes as follows: Suppose the prior distribution is given by $\frac{1}{\alpha}\exp\left(\frac{X}{\alpha}\right)$. It must be the case that when this is multiplied by the payoff function $\exp(-(\lambda(1-\eta)+\delta)\tau)$ we end up with a function that looks like $\frac{1}{\alpha}\exp\left(-\lambda\left(\tau+\frac{1}{\alpha}\right)\right)$ since the definite integral of this between zero and infinity is equal to the hyperbolic discount function. This means that the exponents must add up as follows:

$$\begin{aligned}
-(\lambda(1-\eta)+\delta)\tau + \frac{X}{\alpha} &= -\lambda\tau - \frac{\lambda}{\alpha} \\
&\rightarrow \\
X &= \alpha((\lambda(1-\eta)+\delta)\tau) - \lambda\tau\alpha - \lambda \\
&= (\delta - \lambda\eta)\alpha\tau - \lambda
\end{aligned}$$

So the exponent of the prior distribution is $X/\alpha : f(\lambda) = \frac{1}{\alpha}\exp\left(-\frac{\lambda}{\alpha} + (\delta - \lambda\eta)\tau\right)$.

A.4 Proof of Proposition 4:

A proper p.d.f. must satisfy $\int_0^\infty \frac{1}{\alpha}\exp\left(-\frac{\lambda}{\alpha} + \tau(\delta - \lambda\eta)\right) d\lambda = 1$. Solving this gives:

$$\begin{aligned}
\left[-\frac{\exp\left(-\frac{1}{\alpha}(\lambda - \alpha\tau\delta + \alpha\lambda\tau\eta)\right)}{\alpha\tau\eta + 1}\right]_0^\infty &= 1 \\
\frac{\exp(\tau\delta)}{\alpha\tau\eta + 1} &= 1 \\
&\rightarrow \\
\alpha &= \frac{\exp(\tau\delta) - 1}{\tau\eta}
\end{aligned}$$

which can be approximated by $\alpha \approx \frac{\delta}{\eta}$.

B Supplementary Material

B.1 Pindyck and Martin (2013)

In this section we model the lack of credibility as an uncertain hazard rate. This hazard reflects an event that could take place that prevents all or some of the payoff being delivered to the subject: e.g. a theft or the disappearance of the experimenter. Following Martin and Pindyck (2015, p2954), suppose again that the experimental subject is offered a payoff $\bar{c}_\tau$ at time τ, and denote $\bar{C}_\tau = \ln(\bar{c}_\tau)$. Suppose now that in the period $[0, \tau]$ the subject thinks that some event will take place that removes an amount ϕ from their log payoff so that if the event takes place their log payoff would be $C_\tau = \bar{C} - \phi$. The parameter ϕ can be thought of as the fraction of the payoff that is lost. If the event does not take place then $c_\tau = \bar{c}_\tau$, if it does $c_\tau = \bar{c}_\tau \exp(-\phi)$. It is then natural to think of the probability of the event in terms of a Poisson distribution, with mean arrival rate λ. In this framework, the expected payoff can be seen as the result of a Poisson counting process which counts the number of such hazard events that takes place in the horizon τ, $Q(\tau)$:

$$C_\tau = \bar{C}_\tau - \sum_{n=1}^{Q(\tau)} \phi_n$$

where C_τ is the expected payoff in the event that $Q(\tau)$ such events occur. The effect of this lack of credibility of future payoffs on consumption can be summarised by the *cumulant-generating function* (CGF) (Martin and Pindyck, 2015; Martin, 2013). The CGF represents the expected utility that the subject of the experiment will anticipate when the future payoff is seen as lacking credibility due to some intervening event causing a loss ϕ. Denoting the CGF for horizon τ by $\kappa_\tau(\theta)$:

$$\kappa_\tau(\theta) = \log E \exp(C_\tau \theta) = \log E c_\tau^\theta$$

In the simple case of iso-elastic utility $\theta = (1-\eta)$. In this case Martin and Pindyck (2015) show that the per-period CGF simplifies to:[7]

$$\kappa(1-\eta) = \lambda\left(E\exp(-(1-\eta)\phi) - 1\right)$$

So that the subjects expected utility of the payoff at time τ is given by:

$$\begin{aligned} E\left[u(\bar{c}_\tau)\exp(-\delta t)\right] &= E\left[\frac{1}{1-\eta}c_\tau^{1-\eta}\exp(-\delta\tau)\right] \\ &= \frac{1}{1-\eta}\bar{c}_\tau^{1-\eta}\exp\left(-\left(\delta - \kappa(1-\eta)\right)\tau\right) \\ &= (1-\eta)^{-1}\,\bar{c}_\tau^{1-\eta}\beta(\tau) \end{aligned}$$

where $\beta(\tau)$ is the effective discount factor that the subject uses to discount future payoffs. Again, it reflects two reasons for discounting future payoffs: pure time preference, δ, and the expected loss in utility stemming from the absence of credibility, $\kappa(1-\eta)$. In this framework, subjects reveal an effective discount rate that is higher (lower) than their pure rate of time preference if $\kappa(1-\eta)$ is less than (greater than) zero. Proposition 5 in Appendix A.3. provides an example when the loss parameter is i.i.d. normal with mean μ_ϕ and variance σ_ϕ^2. In this case the subject will,appear more patient than their pure time preference if $\eta > 1 - 2\mu_\phi/\sigma_\phi^2$ which is more likely when the subject is highly risk averse (η is high) and average loss, ϕ, is high.

Again, an experimenter that overlooks the uncertainty arising from the lack of credibility will assume that the discount factor used by the subject reflects only pure time preference δ^*:

$$\beta(\tau) = \exp(-\delta^*\tau) \tag{7}$$

when in fact it reflects both pure time preference and internalisation of the cumulative hazard associated with the absence of credibility.[8]

So far, this framework does not explain the hyperbolic type preferences that are typically observed in experimental work. Following on from the previous section in which the lack of credibility was reflected by uncertain decay rates, suppose now that the arrival rate, λ, of these uncertain events is itself uncertain. This seems like a reasonable assumption given that the subject will have only a vague suspicion of the credibility of the experimenter, and will not know the arrival rate with certainty. Provided that the loss parameter ϕ and the arrival rate λ are independent, the conditional CGF becomes:

$$\kappa_i(1-\eta) = \lambda_i\left(E\exp(-(1-\eta)\phi) - 1\right)$$

and expected utility becomes:

$$\begin{aligned} E\left[\frac{1}{1-\eta}c_\tau^{1-\eta}\exp(-\delta\tau)\right] &= \frac{1}{1-\eta}\bar{c}_\tau^{1-\eta}\int_0^\infty \exp\left(-\left(\delta - \kappa_i(1-\eta)\right)\tau\right) f(\lambda)\,d\lambda \\ &= \frac{1}{1-\eta}\bar{c}_\tau^{1-\eta}\beta(\tau) \end{aligned}$$

[7] This is the one period CGF which scales linearly in time under these independence assumptions. Note that Martin and Pindyck (2015) assume that $\kappa(\theta)$ contains a growth parameter θg, which we have abstracted away from in our analysis.

[8] Note also that, just as in the gamma discounting case discussed above, for $\eta = 1$, $\kappa(0) = 0$, and the discount factor collapses to $\exp(-\delta t)$.

and the effective discount function on the utility from the payoff $\bar{c}_\tau$ becomes:

$$\beta(\tau) = \int_0^\infty \exp\left(-\left(\delta - \kappa_i\left(1-\eta\right)\right)\tau\right) f\left(\lambda\right) d\lambda$$

Again, the experimenter is unaware of the lack of credibility and assumes that the observed behaviour is attributed solely to pure time preference. Technically they assume that:

$$\beta(\tau) = \exp\left(-\delta^*\left(\tau\right)\tau\right)$$

As above, as shown in the Appendix, $\delta^*(\tau)$ declines with the time horizon leading the experimenter to conclude that the subjects are hyperbolic discounters.