Determinacy without the Taylor Principle*

George-Mario Angeletos[†] **Chen Lian**[‡]

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Abstract

Our understanding of how monetary policy works is complicated by an equilibrium-selection conundrum: because the same path for the nominal interest rate can be associated with multiple equilibrium paths for inflation and output, there is a long-lasting debate about what the right equilibrium selection is. We offer a potential resolution by showing that a small friction in memory and intertemporal coordination can remove the indeterminacy. The unique surviving equilibrium is the same as that conventionally selected by the Taylor principle, but it no more relies on it. By the same token, no space is left for the Fiscal Theory of the Price Level, as currently formulated.

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[†]MIT and NBER; angelet@mit.edu.

[‡]UC Berkeley and NBER; chen_lian@berkeley.edu.

1 Introduction

Can monetary policy regulate inflation and aggregate demand? Does the ZLB trigger a deflationary spiral? Does Ricardian equivalence hold when taxation is non-distortionary, markets are complete, and consumers have rational expectations and long horizons? One may be inclined to answer "yes" to all these questions. But the right answer, at least within the New Keynesian model, is that it depends on how equilibrium is selected.

The basic problem goes back to Sargent and Wallace (1975): the same path for the nominal interest rate can be consistent with multiple equilibrium paths for inflation and, in the presence of nominal rigidity, for output too. In the face of this problem, the standard practice is to select a specific equilibrium by assuming that monetary policy satisfies the Taylor principle (Taylor, 1993), or that it is "active" (Leeper, 1991). This amounts to a commitment by the monetary authority to "punish" the private sector with an explosion in inflation and the output gap, unless a particular equilibrium is selected.[1] The model's three "famous" equations then admit a unique solution, which is the one customarily used to interpret the data and guide policy.

But an alternative approach to equilibrium selection, known as the Fiscal Theory of the Price Level (FTPL), leads to a very different perspective on how the economy works. According to it, equilibrium is pinned down by a "non-Ricardian" fiscal policy. This amounts to a commitment to violate the government's solvency constraint unless a particular equilibrium is selected.[2] It causes Ricardian Equivalence to fail, not because of finite horizons, incomplete markets, etc., but rather by force of equilibrium selection. And it lets debt and deficits drive inflation and output gaps even when these objects do not enter the model's three famous equations.

There has been a long debate about which approach and corresponding conclusions are more sensible. This is hard to settle because, within the standard paradigm, both approaches reduce to assumptions about *off-equilibrium* strategies of the monetary and fiscal authorities, which are basically untestable. This explains why, as Kocherlakota and Phelan (1999, p.22) put it more provocatively, the debate has been "fundamentally a religious, not scientific, issue."

We offer a way out of this conundrum. We highlight how the relevant indeterminacy hinges on strong assumptions about memory and intertemporal coordination. Once we perturb these assumptions appropriately, the model's conventional solution, known as the fundamental or minimum state variable (MSV) solution (McCallum, 1983, 2009), emerges as the unique rational ex-

[1] To be precise, the Taylor principle is used to guarantee determinacy of *bounded* equilibria, which is our focus here. Unbounded equilibria, such as self-fulfilling hyper-inflations (Obstfeld and Rogoff, 1983, 2021; Cochrane, 2011) and self-fulfilling liquidity traps (Benhabib et al., 2002), are implicitly or explicitly ruled out by appropriate "exit clauses." See Atkeson et al. (2010), and especially their section on "hybrid" rules, for a careful treatment of this issue.

[2] See Kocherlakota and Phelan (1999). As discussed later, Bassetto (2002) and Cochrane (2005) object to this interpretation; but our own results are valid regardless of one's preferred interpretation of the non-Ricardian assumption.

pectations equilibrium regardless of monetary policy. This leaves little space for the FTPL as currently formulated. And it reinforces the logical foundations upon which one can answer "yes" to the questions raised in the beginning or, more broadly, interpret the data and guide policy.

Preview of results. A New Keynesian economy can be understood as a dynamic game among the consumers. One's optimal spending depends on others' spending via three GE channels: the feedback for aggregate spending to income (the Keynesian cross); the feedback for aggregate spending to inflation (the Phillips curve); and the response of monetary policy (the Taylor rule). The first two channels contribute to strategic complementarity, and in particular to a dynamic feedback strong enough to support multiple equilibria; the third pulls in the opposite direction.[3]

In Sections 2 and 3, we formalize this prism as simply and transparently as possible, and use it to translate the Taylor principle to the following requirement: let the third channel be strong enough so as to guarantee a unique equilibrium when consumers can perfectly coordinate their behavior over time.[4] In the rest of the paper, we instead accommodate a friction in such coordination and to show how it can guarantee a unique equilibrium regardless of monetary policy.

For our main result, developed in Section 4, we model the friction as follows. In each period, a consumer learns perfectly the concurrent fundamentals and sunspots; with probability $\lambda \in [0,1)$, she knows nothing else; and with the remaining probability, she inherits the information of another, randomly selected, player from the previous period. This lets λ parameterize the speed at which social memory "fades" with time: for any t, the fraction of the population who "remembers" and can condition their actions on the shocks realized at any $\tau \leq t$ is $(1-\lambda)^{t-\tau}$.

The standard, representative-agent, case is nested with $\lambda = 0$; it translates to common knowledge of the economy's history (which defines what "perfect" coordination means for us); and it admits a continuum of sunspot and backward-looking equilibria whenever the Taylor principle is violated. Proposition 2 shows that, as soon as $\lambda > 0$, all these equilibria unravel. Only the fundamental/MSV solution survives, regardless of whether the monetary policy is active or passive.

Strictly speaking, this result precludes direct observation of the actions of others, or of endogenous outcomes such as inflation and output. But because such outcomes are functions of the underlying shocks, in the limit as $\lambda \to 0$ nearly all consumers are nearly perfectly informed about nearly infinite histories of *both* shocks and outcomes. From this perspective, our result

[3]The second channel is shut off with rigid prices and the third one is shut off with interest rate pegs. But the first channel is always there—whether hidden behind the Euler condition of the representative consumer in the textbook New Keynesian model, or salient in the "intertemporal Keynesian cross" of HANK models (Auclert et al., 2018).

[4]The Taylor principle is sometimes described as follows: in response to self-fulfilling inflationary pressures, raise the interest rate enough to reduce aggregate demand and bring the economy back on track. But this confounds the stabilization and equilibrium selection functions of feedback policies. Once these functions are separated (King, 2000; Atkeson et al., 2010), it becomes clear that the Taylor principle regards *exclusively* the former. This is fully consistent with our game-theoretic translation.

illustrates the fragility of the model's non-fundamental solutions to the introduction of small, idiosyncratic noise in the knowledge of the economy's history.

We corroborate this message in Section 7 with two additional results, both of which are motivated by recursive representations of full-information equilibria. In such representations, finite memory of appropriately chosen endogenous variables helps replicate infinite memory of past shocks. For instance, pure sunspot equilibria are replicated by having agents keep track merely of today's sunspot and yesterday's inflation or output. Proposition 5 shows that this replication is itself fragile: it breaks as soon as we let the agents' observation of past inflation or output be contaminated with arbitrarily small idiosyncratic noise. Proposition 6 adds that a similar fragility can be present even when past outcomes are *perfectly* observed, provided that, for any period t, there is a small shock to fundamentals that is known at t but is "forgotten" at $t+1$.

The common thread between our results can be summarized as follows. All equilibria other than the MSV solution are sustained by the following infinite chain: in any period, agents are responding to a payoff-irrelevant variable (e.g., the current sunspot or the past rate of inflation) because they expect to be "rewarded" appropriately by future agents, who in turn are expected to act on the basis of a similar expectation about behavior further into the future, and so on. Because such purely self-fulfilling dynamic chains have no anchor on fundamentals, they can be exceedingly sensitive to perturbations of "social memory," or of common knowledge across time.

Interpreting our contribution. The logic behind our results echoes the literature on global games (Morris and Shin, 2002, 2003) and is subject to a related qualification: indeterminacy may strike back if markets generate enough common knowledge (Angeletos and Werning, 2006; Atkeson, 2000). But there is an important twist: in our context, the most relevant coordination is that of behavior over long periods of time. This explains both why the relevant perturbations relate, one way or another, to social memory and why the requisite type of common knowledge may be harder to reach in practice than in the case of, say, a self-fulfilling bank run.

All in all, we view our results as (i) a lens for understanding better the New Keynesian model's indeterminacy problem and (ii) as a reinforcement of the logical foundations of its conventional solution. To paraphrase Kocherlakota and Phelan (1999), there is no more space for a "religious" debate on what is the most sensible equilibrium selection. Under our perturbations, equilibrium is pinned down by the model's famous three equations regardless of whether monetary policy is active or passive; and fiscal policy *has* to be Ricardian, or else an equilibrium just fails to exist.

This of course does not mean that Ricardian equivalence has to hold in practice, nor does it contradict Sargent and Wallace (1981)'s "unpleasant arithmetic." But it refines the channels via which fiscal policy can drive inflation and output; it helps liberate the study of the fiscal-monetary interaction from the thorny equilibrium selection issue; and it invites new ways of thinking about the question of which authority is “dominant” and what exactly this means.

Finally, note that equilibrium uniqueness allows room for sunspot-like volatility due to noisy public news (Morris and Shin, 2002) or shocks to higher-order beliefs (Angeletos and La’O, 2013). From this perspective, our contribution is not to rule out “animal spirits” but rather to help recast policies that lean against them as a form of stabilization instead of equilibrium selection.

Related literature. Although Cochrane (2011, 2017, 2018) has been the most vocal advocate of the FTPL recently, this theory and the question of whether equilibrium is determined by an “active” monetary policy or a “non-Ricardian” fiscal policy go back to Leeper (1991), Sims (1994) and Woodford (1995). Canzoneri, Cumby, and Diba (2010) review the debate and discuss how it fits in the broader context of the study of the fiscal-monetary interaction.

Early criticisms of the FTPL by Kocherlakota and Phelan (1999), Buiter (2002), and Niepelt (2004) boil down to this idea: the non-Ricardian assumption is an off-equilibrium threat to “blow up” the economy, in the sense of violating the government’s solvency constraint. Bassetto (2002) and Cochrane (2005) object to this interpretation and articulate ways around it. Going on the offense, Cochrane (2011) argues that the Taylor principle itself is a blow-up threat, now in the sense of a threat to trigger an explosion in inflation and non-existence of continuation equilibrium. But Atkeson et al. (2010) show how to avoid this criticism with more “sophisticated” monetary policies than a plain-vanilla Taylor rule.

The bottom line is that, although the debate has morphed in different forms over time, it has never ended. This is because the underlying core issue has always been the same one: the indeterminacy implied by interest-rate pegs (Sargent and Wallace, 1975). What distinguishes our contribution is the attempt to resolve this indeterminacy—and to escape the endless “agree to disagree”—by introducing a friction in intertemporal coordination.

Our main approach (Propositions 2 and 4) brings to mind Morris and Shin (1998, 2003) and Abreu and Brunnermeier (2003). Although the application and the formal argument are different, there is a close resemblance in terms of the discontinuity of equilibria to perturbations of common knowledge and the role of higher-order beliefs. Our second approach (Proposition 6), on the other hand, is more closely connected to Bhaskar (1998) and Bhaskar, Mailath, and Morris (2012). These papers show that Markov Perfect Equilibria—the analogue of the MSV concept in our context—are the only equilibria that survive in a class of dynamic games when a purification in payoffs is combined with certain restrictions in social memory. The deep connections between such seemingly disparate approaches deserve further exploration.

A large literature has already incorporated information/coordination frictions in the New

Keynesian model (Mankiw and Reis, 2002; Woodford, 2003; Maćkowiak and Wiederholt, 2009; Angeletos and Lian, 2018). But it has *not* addressed the determinacy issue. Instead, it has focused on how information shapes the model's MSV solution and has assumed away all other solutions by invoking, implicitly or explicitly, the Taylor principle. We do the exact opposite: our perturbations remove all other solutions without necessarily affecting the MSV solution itself.

A different literature has attempted to refine the model's solutions by requiring that they are E-stable (McCallum, 2007; Christiano et al., 2018). This approach relies on specific assumptions about what it means for an equilibrium to be "learnable" and has had mixed success.[5] Still, we view this approach and ours as complements in that they both contribute towards reinforcing the logical foundations of the conventional, or "monetarist," approach.

Although we commit to Rational Expectations Equilibrium (REE) throughout, both the indeterminacy problem and our resolution of it extend to a larger class of solution concepts, including cognitive discounting (Gabaix, 2020), diagnostic expectations (Bordalo et al., 2018), and Bayesian equilibrium with mis-specified priors about one another's knowledge or rationality (Angeletos and Sastry, 2021). Relative to REE, these concepts relax the perfect coincidence of subjective beliefs and objective distributions and, in so doing, can modify the details of the Taylor principle (i.e., the critical threshold for the slope of the Taylor rule). But they do not change the essence of the problem, because they preserve a fixed-point relation between beliefs and behavior. By contrast, Level-K Thinking (García-Schmidt and Woodford, 2019; Farhi and Werning, 2019) produces a unique solution precisely because it shuts down the feedback from objective reality to subjective beliefs, which seems a strong assumption for stationary environments.[6]

Finally, let us emphasize that work with the *linearized* New Keynesian model and focus on *bounded* equilibria around a given steady state. The essence here is that we take for granted that expectations are "anchored" around the given steady state. In the literature, this has been justified by appropriate exit clauses, namely an off-equilibrium commitment to abandon the Taylor rule and start fixing the supply of money, or follow some other appropriate course of action, should inflation were to go out of bounds.[7] But whereas such exit strategies are insufficient for pinning down a unique equilibrium in the standard paradigm (there they have to be combined with the Taylor principle), they become sufficient under our approach.

[5]For example, sunspot equilibria can be E-stable if the interest rate rule is written as a function of expected inflation (Honkapohja and Mitra, 2004). And there is a debate on how the E-stability of backward-looking solutions depends on the observability of shocks (Cochrane, 2011; Evans and McGough, 2018).

[6]Furthermore, whenever the environment admits multiple REE, the Level-K solution becomes infinitely sensitive to the assumed Level-0 behavior as the depth of reasoning gets larger. In this sense, this concept translates one free variable (the sunspot or the equilibrium selection) to another free variable (the analyst's choice of Level-0 behavior). See Appendix B for a detailed explanation.

[7]See Atkeson et al. (2010)'s section on "hybrid" rules for a careful treatment of this issue.

2 A Simplified New Keynesian Model

In this section, we introduce our version of the New Keynesian model. This contains two unusual assumptions: a specific OLG structure for the consumers and an ad hoc Phillips curve. These assumptions ease the exposition, especially once we perturb common knowledge of the economy's history; but as discussed in Section 5, they do not drive the results.

An intertemporal Keynesian cross (aka a Dynamic IS equation)

Time is discrete and is indexed by t. There are overlapping generations of consumers, each living two periods. A consumer born at t has preferences given by

$$u(C^1_{i,t}) + \beta u(C^2_{i,t+1})e^{-\varrho_t},$$

where $C^1_{i,t}$ and $C^2_{i,t+1}$ are consumption when young and old, respectively, $u(C) \equiv \frac{1}{1-1/\sigma}C^{1-1/\sigma}$, $\beta \in (0,1)$ is a fixed scalar, ϱ_t is an intertemporal preference shock (the usual proxy for aggregate demand shocks), and $E_{i,t}$ is the consumer's expectation. Young and old consumers earn the same income. Young consumers can borrow or save using the single asset traded in the economy, a one-period nominal bond; old consumers pay out any outstanding debt, or eat their savings, before they die. The budget constraint of a consumer born at t are therefore given by $C^1_{i,t} + B_{i,t} = Y_t$ and $C^2_{i,t+1} = Y_{t+1} + \frac{I_t}{\Pi_{t+1}}B_{i,t}$, where $B_{i,t}$ is her saving/borrowing in the first period, I_t is the (gross) nominal interest rate between t and $t+1$, and Π_{t+1} is the corresponding inflation rate.

Old consumers are "robots:" they face no optimizing margin, their consumption mechanically adjusts to meet their end-of-life budget. Young consumers, instead, are "strategic:" they optimally choose consumption and saving/borrowing, given their available information. After the usual log-linearization,[8] this translates to the following consumption function:

$$c^1_{i,t} = E_{i,t}\left[\tfrac{1}{1+\beta}y_t + \tfrac{\beta}{1+\beta}y_{t+1} - \tfrac{\beta}{1+\beta}\sigma(i_t - \pi_{t+1} - \varrho_t)\right]. \tag{1}$$

This is basically the Permanent Income Hypothesis. The only subtlety is that we have allowed young consumers to be imperfectly informed about, or inattentive to, current income and current interest rates—which explains why y_t and i_t appear inside the expectation operator.

Pick any t. Because the average saving/borrowing of the young has to be zero, $\int c^1_{i,t}di = y_t$; and because the average net wealth of old has to be zero as well, $\int c^2_{i,t}di = y_t$. Combining, we infer that the two groups consume the same—or equivalently that aggregate consumption, c_t, coincides with the average consumption of the young. Computing the latter from (1), and imposing $y_t = c_t$, we infer that, for any process of interest rate and inflation, the process for aggregate

[8] Throughout, we log-linearize around the steady state in which $\varrho_t = 0$, $\Pi_t = 1$, and $I_t = \beta^{-1}$; and we use lower-case variables to denote log-deviations from steady state.

spending must satisfy the following equation:

$$c_t = \bar{E}_t \left[\tfrac{1}{1+\beta} c_t + \tfrac{\beta}{1+\beta} c_{t+1} - \tfrac{\beta}{1+\beta} \sigma (i_t - \pi_{t+1} - \varrho_t) \right], \tag{2}$$

where $\bar{E}_t[\cdot] = \int E_{i,t}[\cdot]\, di$ is the average expectations of the young.

As evident from its derivation, this equation combines consumer optimality with market clearing; and it encapsulates the positive feedback between aggregate spending and income, holding constant the real interest rate. This equation can thus be read as a special case of the "intertemporal Keynesian cross" (Auclert et al., 2018), or as a Dynamic IS equation.

Connection to standard New Keynesian model

Although equation (2), our version of the Dynamic IS equation, looks different from the familiar textbook counterpart, it actually nests it when there is full information. Indeed, in this benchmark $\bar{E}_t$ can be replaced by $\mathbb{E}_t$, which henceforth denotes the full-information rational expectation; and because full information implies knowledge of concurrent outcomes in any rational expectations equilibrium, equation (2) reduces in this case to

$$c_t = \tfrac{1}{1+\beta} c_t + \tfrac{\beta}{1+\beta} \mathbb{E}_t[c_{t+1}] - \tfrac{\beta}{1+\beta} \sigma (i_t - \mathbb{E}_t[\pi_{t+1}] - \varrho_t),$$

or equivalently

$$c_t = \mathbb{E}_t[c_{t+1}] - \sigma (i_t - \mathbb{E}_t[\pi_{t+1}] - \varrho_t).$$

Clearly, this is the same as the Euler condition of a representative, infinitely-lived consumer.

This clarifies the dual role of the adopted micro-foundations. With full information, they let our model translate to the standard, representative-agent, New Keynesian model. And away from this benchmark, they ease the exposition by letting the intertemporal Keynesian cross take a particularly simple form and by equating the players in our upcoming game representation to the young consumers. These simplifications are relaxed in Section 5, without changing the essence.

A Phillips curve and a Taylor rule

For the main analysis, we abstract from optimal price-setting behavior (firms are "robots") and impose the following, ad hoc Phillips curve:

$$\pi_t = \kappa (y_t + \xi_t), \tag{3}$$

where $\kappa \geq 0$ is a fixed scalar and ξ_t is a "supply" or "cost-push" shock. As discussed in Section 5, our results are robust to replacing (3) with the fully micro-founded, forward-looking, New Keynesian Phillips curve; these same is true if we employ a Neoclassical Phillips curve a la Lucas (1972).

In all cases, the essence (for our purposes) is that there is a positive GE feedback from aggregate output to inflation. Equation (3) merely stylizes this feedback in a convenient form.

We finally assume that monetary policy follows a Taylor rule:

$$i_t = z_t + \phi \pi_t, \tag{4}$$

where z_t is a random variable, possibly correlated with ϱ_t and ξ_t, and $\phi \geq 0$ is a fixed scalar that parameterizes how aggressively the monetary authority raises the interest rate in response to inflationary pressures. As is well known and will be reviewed shortly, $\phi > 1$ is necessary and sufficient for the uniqueness of bounded equilibrium in the standard paradigm—but *not* under our perturbations. Our results will indeed apply even if $\phi = 0$, which nests interest rate pegs.[9]

The model in one equation—and the economy as a game

From (3) and (4), we can readily solve for π_t and i_t as simple functions of y_t, which itself equals c_t. Replacing into (2), we conclude that the model reduces to the following equation:

$$c_t = \bar{E}_t\left[(1-\delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1}\right] \tag{5}$$

where δ_0, δ_1 are fixed scalars and θ_t is a random variable, defined by

$$\delta_0 \equiv \frac{1-\beta\sigma\phi\kappa}{1+\beta} < 1, \quad \delta_1 \equiv \frac{\beta+\beta\sigma\kappa}{1+\beta} > 0, \quad \theta_t \equiv -\frac{1}{1+\phi\kappa\sigma}\left(\sigma z_t - \sigma\varrho_t + \sigma\phi\kappa\xi_t - \sigma\kappa\mathbb{E}_t[\xi_{t+1}]\right).$$

By construction, equation (5) summarizes private sector behavior and market clearing, for any information structure and any monetary policy. Different information structures change the properties of $\bar{E}_t$ but do not change the equation itself. Similarly, different monetary policies map to different values for δ_0 or different stochastic processes for θ_t, via the choice of, respectively, a value for ϕ or a stochastic process for z_t. But for any given monetary policy, we can understand equilibrium in the private sector by studying equation (5) alone.

Equation (5) and the micro-foundations behind it also facilitate the interpretation of the economy as a certain infinite-horizon game. In this game, the only players acting at t are the young consumers of that period (old consumers, firms, and the monetary authority are "robots," in the sense already explained) and their best responses are obtained by combining their optimal consumption functions with first-order knowledge of market clearing, the Phillips curve, and the Taylor rule. This gives the *individual* best response at t as

$$c_{i,t} = E_{i,t}\left[(1-\delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1}\right], \tag{6}$$

[9] Note that an interest rate peg can be state-contingent, via z_t; and that the latter can be correlated, possibly in an optimal way, with ϱ_t and ξ_t. Similarly to King (2000) and Atkeson et al. (2010), this allows disentangling the stabilization and equilibrium selection functions of Taylor rules: the former is served by the design of z_t, the latter by the restriction $\phi > 1$.

and recasts (5) as the period-t average best response function. Under this prism, δ_0 and δ_1 parameterize, respectively, the intra-temporal and the inter-temporal degree of strategic complementarity, while θ_t identifies the game's fundamental (i.e., the only payoff-relevant exogenous random variable). Finally, by regulating the strength of the underlying GE feedbacks, different values for β, κ, and ϕ map to different degrees of strategic complementarity.

This game-theoretic prism is not strictly needed for proving our results, which work directly with (5). But it allows a one-to-one mapping between the Rational Expectations Equilibria of our economy and the Perfect Bayesian Equilibria of the game described above; it helps build insightful connections to the literatures on global games and beauty contests; and, once we add fiscal policy to the model (Section 6), it helps clear some of the confusion that the existing literature on the FTPL has created about how the non-Ricardian assumption works and what constitutes a "fundamental" in the New Keynesian model.

Fundamentals, sunspots, and the equilibrium concept

Aggregate uncertainty is of two sources: fundamentals and sunspots. As already mentioned, the former are herein conveniently summarized in θ_t.[10] We assume that this variable is a stationary, zero-mean, Gaussian process, admitting a finite-state representation.

Assumption 1 (***Fundamentals***)**.** *The fundamental θ_t admits the following representation:*

$$\theta_t = q' x_t \qquad \textit{with} \qquad x_t = R x_{t-1} + \varepsilon_t^x, \tag{7}$$

where $q \in \mathbb{R}^n$ is a vector, R is an $n \times n$ matrix of which all the eigenvalues are within the unit circle (to guarantee stationarity), $\varepsilon_t^x \sim \mathcal{N}(\mathbf{0}, \Sigma_\varepsilon)$, and Σ_ε is a positive definite matrix.

This directly nests the case in which (ϱ_t, ξ_t, z_t) follows a VARMA of any finite length. It also allows x_t to contain "news shocks," or forward guidance about future monetary policy. We henceforth refer to x_t as the *fundamental state.*

We next introduce a sunspot variable:

Assumption 2 (***Sunspots***)**.** *The only source of aggregate uncertainty other than that behind x_t is a sunspot. This is given by a random variable $\eta_t \sim \mathcal{N}(0,1)$, which is independent of past, current and future fundamentals and is distributed independently and identically over time.*[11]

[10] The fact that θ_t contains an expectation term does interfere with our results. For instance, it suffices to assume that the fundamental state x_t, introduced below, is a sufficient statistic $(z_t, \varrho_t, \xi_t, \bar{E}_t[\xi_{t+1}])$ and therefore also for θ_t.

[11] Although we are restricting η_t to be uncorrelated over time, we are not ruling out persistent sunspot fluctuations: such fluctuations are still possible insofar as agents condition their behavior on past sunspots. Furthermore, as discussed at the end of Section 4, our results are robust to letting η_t itself be persistent, except for a degenerate case.

Let h^t capture the history of all shocks, fundamental or not, up to and including period t. To simplify the exposition, we assume that histories are infinite and, accordingly, focus on stationary equilibria. More precisely, we let $h^t \equiv \{x_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$ and we define an equilibrium as follows:

Definition 1 (**Equilibrium**)**.** *An equilibrium is any solution to equation (5) along which: expectations are rational, although potentially based on imperfect and heterogeneous information about h^t; the outcome is a stationary, linear function of the underlying shocks, or*

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma'_k x_{t-k} \tag{8}$$

where $a_k \in \mathbb{R}$ and $\gamma_k \in \mathbb{R}^n$ are known coefficients for all k; and the outcome is bounded in the sense that $Var(c_t)$ is finite.[12]

Recall that consumer optimality, firm behavior, and market clearing have already been embedded in equation (5). It follows that the above definition is the standard definition of a Rational Expectations Equilibrium (REE), except for the addition of three "auxiliary" restrictions: stationarity, linearity, and boundedness. The stationarity restriction, which comes hand-in-hand with the assumption of infinite history, can readily be relaxed. The linearity restriction is strictly needed for tractability, but we do not have any reason to believe that it drives our results, plus it is commonplace in the literature.

The last requirement is our version of "local determinacy" or "bounded equilibria." This is herein treated as a primitive; but as usual, it can be justified by an "exit" strategy along the lines of Taylor (1993), Christiano and Rostagno (2001) and Atkeson et al. (2010), namely a commitment to switch from the Taylor rule to a money-growth-targeting regime, or to whatever it takes for keeping inflation (and the output gap) within some bounds.[13]

Finally, and circling back to our game-theoretic prism, note that the following is true: because every agent is infinitesimal, one's deviations are of no consequence for others, and there is hence no need to specify off-equilibrium beliefs. It follows that the economy's Rational Expectations Equilibria (REE) basically coincide with the corresponding game's Perfect Bayesian Equilibria (PBE).[14]

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[12] Note that $Var(c_t)$ can be finite only if there exists a scalar $M > 0$ such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k, where $\|\cdot\|_1$ is the L^1-norm. Our upcoming result actually uses only this weaker form of boundedness.

[13] The credibility of such exit strategies, their precise formulation, and the subtlety of whether they amount to a threat of "equilibrium non-existence" (Cochrane, 2007) or a more "sophisticated" implementation (Atkeson et al., 2010), are important topics beyond the scope of our paper. The relevant observation for our purposes is, instead, the following: whereas the boundedness requirement must be combined with the Taylor principle in order to deliver global determinacy in the standard paradigm, it will alone do the job under our perturbations.

[14] Of course, the stationarity, linearity, and bounded restrictions embedded in our REE definition must be extended to its PBE counterpart for this equivalence to be exact.

3 The Standard Paradigm and the Taylor Principle

In this section, we consider the full-information version of our model, which is, in essence, the standard New Keynesian model. We first identify the model's fundamental/MSV solution; we next show how its determinacy hinges, under full information, on the Taylor principle; and we finally contextualize our departures from this benchmark.

Full information

Suppose that all consumers know the entire h^t, at all t. As shown earlier, it is then *as if* there is a representative, fully informed and infinitely lived, consumer—just as in the textbook case. Accordingly, equation (5), which summarizes equilibrium, reduces to the following:

$$c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}], \tag{9}$$

where $\mathbb{E}_t[\cdot] \equiv \mathbb{E}[\cdot|h_t]$ is the rational expectation conditional on full information and

$$\delta \equiv \frac{\delta_1}{1-\delta_0} = \frac{1+\kappa\sigma}{1+\phi\kappa\sigma} > 0.$$

Although δ is necessarily positive, it can be on either side of 1, depending on ϕ. We will see momentarily how this relates to equilibrium determinacy. Also note that the above is a single, first-order, difference equation in c_t alone. By contrast, the textbook New Keynesian model maps to a system of *two* such equations in the vector (c_t, π_t). What affords the present reduction in dimensionality is the omission of a forward-looking term in the Phillips curve. But as it will become clear in Section 5, this simplification is inessential. All we have done thus far is to reduce the standard model's determinacy question from a two-dimensional eigenvalue problem to the simpler question of whether δ, or equivalently the sum $\delta_0 + \delta_1$, is higher or lower than 1.

The fundamental/MSV Solution

Because equation (9) is purely forward looking and x_t is a sufficient statistic for both the concurrent θ_t and its expected future values, it is natural to look for a solution in which c_t is a function of x_t alone. Thus guess $c_t = \gamma' x_t$ for some $\gamma \in \mathbb{R}^n$; use this to compute $\mathbb{E}_t[c_{t+1}] = \gamma' R x_t$; and substitute into (9) to get $c_t = \theta_t + \delta\gamma' R x_t = [q' + \delta\gamma' R] x_t$. Clearly, the guess is verified if and only if γ' solves $\gamma' = q' + \delta\gamma' R$, which in turn is possible if and only if $I - \delta R$ is invertible (where I is the $n \times n$ identity matrix) and $\gamma' = q'(I - \delta R)^{-1}$.

This is known as the model's "fundamental" or "minimum state variable (MSV)" solution (McCallum, 1983). To guarantee its existence, we henceforth impose the following assumption:

Assumption 3. *The matrix $I - \delta R$ is invertible.*

And we write this solution as $c_t = c_t^F$, where

$$c_t^F \equiv q' \left(I - \delta R\right)^{-1} x_t. \tag{10}$$

As shown momentarily, other solutions are possible when, and only when, $\delta \geq 1$. But let us first note the following property of the MSV solution. Provided that the infinite sum $\sum_{k=0}^{\infty} \delta^k R^k$ exists, we can rewrite this solution as

$$c_t^F = \sum_{k=0}^{\infty} \delta^k \mathbb{E}_t[\theta_{t+k}].$$

This illustrates that c_t^F can depend on the economy's history only insofar as this pins down the current θ_t or helps forecast its future values. And it verifies that c_t^F maps to what Blanchard (1979) calls the "forward-looking solution," namely the solution of iterating (9) forward.[15]

Determinacy under full information and the Taylor Principle

We now turn attention to the question of whether there exist equilibria other than the MSV one. Let us first fix the language:

Definition 2 (**Taylor principle**). *The Taylor principle is defined by the restriction $\phi > 1$.*[16]

Note that $\phi > 1$ translates to $\delta_0 + \delta_1 < 1$ and, equivalently, $\delta < 1$. The former can be read as "the overall degree of strategic complementarity is small to guarantee a unique equilibrium," the latter as "the dynamics are forward-stable." And conversely, $\phi < 1$ translates to "the complementarity is large enough to support multiple equilibria" ($\delta_0 + \delta_1 > 1$) and the "dynamics are backward-stable" ($\delta > 1$). This underscores the tight connection between our way of thinking about determinacy (the size of the strategic complementarity) and the standard way (the size of the eigenvalue). The next proposition verifies this point and also characterizes the type of equilibria that emerge in addition to the MSV solution once the Taylor principle is violated.

Proposition 1 (**Full-information benchmark**). *Suppose that h^t is known to every i for all t, which means in effect that there is a representative, fully informed, agent. Then:*

(i) There always exist an equilibrium, given by the fundamental/MSV solution c_t^F, as in (10).

[15] What if $\sum_{k=0}^{\infty} \delta^k R^k$ does not exist (i.e., the sum fails to converge)? In this case, c_t^F remains an REE but is no more solvable by forward induction; and its correlation with θ_t can switch sign. This relates to whether the MSV solution can feature "neo-Fisherian" effects (Cochrane, 2017; García-Schmidt and Woodford, 2019), a question that is interesting but separate from that considered here. For our purposes, the relevant quality of the MSV solution is this: in this solution, history matters only insofar as it is part of x_t, the fundamental state variable. This contrasts with the model's other solutions, along which payoff-irrelevant histories serve as correlation devices.

[16] Recall that we have restricted $\phi \geq 0$ and therefore also $\delta > 0$. Had we allowed $\delta < 0$, which is possible if ϕ itself is *sufficiently* negative, the proposition and the discussion after it continue to hold, provided that we recast the Taylor principle as $\delta \in (-1, 1)$.

(ii) When the Taylor principle is satisfied ($\phi > 1$), the above equilibrium is the unique one.

(iii) When this principle is violated ($\phi < 1$), there exist a continuum of equilibria, given by

$$c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta, \tag{11}$$

where $a, b \in \mathbb{R}$ are arbitrary scalars and c_t^B, c_t^η are given by

$$c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k} \qquad \textit{and} \qquad c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k}. \tag{12}$$

To understand the type of non-fundamental equilibria documented in part (iii) above, take equation (9), backshift it by one period, and rewrite it as follows:

$$\mathbb{E}_{t-1}[c_t] = \delta^{-1}(c_{t-1} - \theta_{t-1}). \tag{13}$$

Since η_t is unpredictable at $t-1$, the above is clearly satisfied with

$$c_t = \delta^{-1}(c_{t-1} - \theta_{t-1}) + a\eta_t, \tag{14}$$

for any $a \in \mathbb{R}$. As long as $\delta > 1$, we can iterate backwards to obtain

$$c_t = -\sum_{k=1}^{\infty} \delta^{-k}\theta_{t-k} + a\sum_{k=0}^{\infty} \delta^{-k}\eta_{t-k} = c_t^B + ac_t^\eta. \tag{15}$$

This is both bounded, thanks to $\delta > 1$, and a rational-expectations solution to (13), by construction. This verifies that $c_t^B + ac_t^\eta$ constitutes an equilibrium, for any $a \in \mathbb{R}$. Part (iii) of the Proposition adds that the same is true if we replace c_t^B with any mixture of it and the MSV solution.

When there are no fundamental shocks, $c_t^F = c_t^B = 0$. The solution obtained above reduces to a pure sunspot equilibrium, of arbitrary aptitude a. In this equilibrium, agents respond to the sunspot because and only because they expect future agents to keep reacting to it, in perpetuity.[17]

In the presence of fundamental shocks, the indeterminacy takes an additional, perhaps more disturbing, form: the same path for interest rates and other fundamentals can result to different paths for aggregate spending and inflation, even if we switch off the sunspots. Consider, for example, the solution given by $c_t = c_t^B$. Along it, the outcome is pinned down by past fundamentals and is invariant to both the current value of θ_t and any news about its future path—which is the exact opposite of what happens along c_t^F, the MSV solution.

The logic behind c_t^B is basically the same as that behind sunspot equilibria: agents respond to past shocks that are payoff-irrelevant looking forward, because and only because they expect future agents to keep doing the same, in perpetuity. This statement extends to any equilibrium of the form (11) for $b \neq 0$, and explains why all such equilibria can be thought of as both non-fundamental and backward-looking.[18]

[17] This is the same as a traditional, rational-expectations bubble, except that it is not explosive, thanks to $\delta > 1$.

[18] Blanchard (1979) refers to the analogue of c_t^B in his analysis as a "backward-looking fundamental equilibrium;"

Beyond the full-information benchmark: a challenge and the way forward

Consider conditions (14) and (15). Clearly, these are equivalent representations of the same equilibrium: the first is recursive, the second is sequential. This equivalence means that all the equilibria that can be supported by perfect knowledge of $h_t = \{x_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$ coincide with those that can be supported by perfect knowledge of $(x_t, \eta_t; x_{t-1}, c_{t-1})$, or $(x_t, \eta_t; \theta_{t-1}, c_{t-1})$.[19] But what if agents lack such perfect knowledge, as it is bound to the case in reality?

Regardless of what agents know or don't, one can *always* represent any equilibrium in a sequential form, or as in equation (8). This is simply because c_t *has* to be measurable in the history of exogenous aggregate shocks, fundamental or otherwise. But it is far from clear if and when there is an equivalent recursive representation. In fact, a finite-state recursive representation is generally impossible when agents observe noisy signals of endogenous outcomes, due to the infinite-regress problem first highlighted by Townsend (1983).[20]

This poses a challenge for what we want to do in this paper. On the one hand, we seek to highlight how fragile all non-fundamental solutions can be to perturbations of the aforementioned kinds of common knowledge, or to small frictions in coordination. On the other hand, we need to make sure that these perturbations do not render the analysis intractable.

To accomplish this dual goal, in the rest of the paper we follow two strategies. Our main one, in Sections 4 and 5, takes off from (15), or the sequential representation. Our second strategy, in Section 7, circles back to (14), the recursive representation. Both strategies illustrate the fragility of non-fundamental equilibria, each one from a different angle.

4 Uniqueness with Fading Social Memory

This section contains our main result. We introduce a friction in social memory and show how it yields a unique equilibrium no matter ϕ, or the size of the strategic complementarity.

Main assumption

For the purposes of this and the next section, we replace the assumption of a representative, fully-informed agent with the following, incomplete-information variant:

but this is not *really* fundamental, in the sense we just explained.

[19] Note that θ_{t-1} is measurable in x_{t-1} and that, from (15), only θ_{t-1} and not the whole x_{t-1} is needed for recursive replication.

[20] See Huo and Takayama (2021) for a detailed study of the issue.

Assumption 4 (**Social memory**). *In every period t, a consumer's information set is given by*

$$I_{i,t} = \{(x_t, \eta_t), \cdots, (x_{t-s}, \eta_{t-s})\},$$

where $s \in \{0, 1, \cdots\}$ is drawn from a geometric distribution with parameter λ, for some $\lambda \in (0, 1]$.

To understand this assumption, note that herein s indexes the length of the history of shocks that the consumer knows. Next, recall that the geometric distribution means that $s = 0$ with probability λ, $s = 1$ with probability $(1-\lambda)\lambda$, and more generally $s = k$ with probability $(1-\lambda)^k\lambda$, for any $k \geq 0$. By the same token, the fraction of agents who know *at least* the past k realizations of shocks is given by $\mu_k \equiv (1-\lambda)^k$.

One can visualize this as follows under the preceding micro-foundations. At every t, the typical player (young consumer) learns the concurrent shocks; with probability λ, she learns nothing more; and with the remaining probability, she inherits the information of another, randomly selected player from the previous period (a currently old consumer). In this sense, λ parameterizes the speed at which social memory (or common-p belief of past shocks) fades over time.

Main result

The full-information benchmark can be nested with $\lambda = 0$, which translates to $I_{i,t} = h_t$ (perfect knowledge of the infinite history) for all i and t. But the question of interest is what happens for $\lambda > 0$, and in particular as $\lambda \to 0^+$. In this limit, the friction becomes vanishingly small, in the sense that almost every agent knows the history of shocks up to an arbitrarily distant point in the past. But the following is also true: no matter how small λ is, as long as it is not exactly zero, we have that $\lim_{k\to\infty} \mu_k = 0$, which means that shocks are expected to be "forgotten" in the very distant future. As shown next, this causes all non-fundamental equilibria to unravel.

Proposition 2 (**Determinacy without the Taylor principle**). *Suppose that social memory is imperfect in the sense of Assumption 4, for any $\lambda > 0$. Regardless of ϕ, or of δ_0 and δ_1, the equilibrium is unique and is given by the fundamental/MSV solution.*[21]

A detailed proof is provided in Appendix A. Here, we illustrate the main idea for the special case in which there are no fundamental disturbances, so the task reduces to checking for the

[21] Note that the fundamental/MSV solution remains the same as we move away from $\lambda = 0$ thanks to the assumption that $I_{i,t}$ contains x_t always. As mentioned in the Introduction, this helps isolate our contribution from the existing literature on informational frictions, which focuses on how the MSV solution is influenced by imperfect information about x_t but does not address the determinacy issue. Here, we do the exact opposite, but one could have it both ways: modify Assumption 4 so as to remove perfect information about x_t and reshape the MSV solution, while also preserving our argument for uniqueness.

existence of pure sunspot equilibria. That is, we specialize our equilibrium condition to

$$c_t = \delta_0 \bar{E}_t[c_t] + \delta_1 \bar{E}_t[c_{t+1}]; \tag{16}$$

we search for solutions of the form $c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k}$; and we verify that $a_k = 0$ for all k.

By Assumption 4, we have that, for all $k \geq 0$,

$$\bar{E}_t[\eta_{t-k}] = \mu_k \eta_{t-k}$$

where $\mu_k \equiv (1-\lambda)^k$ measures the fraction of the population at any given date that know, or remember, a sunspot realized k periods earlier. Future sunspots, on the other hand, are known to nobody. It follows that, along any candidate solution, average expectations satisfy

$$\bar{E}_t[c_t] = \bar{E}_t\left[\sum_{k=0}^{\infty} a_k \eta_{t-k}\right] = \sum_{k=0}^{+\infty} a_k \mu_k \eta_{t-k}$$

and similarly

$$\bar{E}_t[c_{t+1}] = \bar{E}_t\left[a_0 \eta_{t+1} + \sum_{k=1}^{\infty} a_k \eta_{t+1-k}\right] = 0 + \sum_{k=0}^{+\infty} a_{k+1} \mu_k \eta_{t-k}.$$

By the same token, condition (16) rewrites as

$$\underbrace{\sum_{k=0}^{+\infty} a_k \eta_{t-k}}_{c_t} = \delta_0 \underbrace{\sum_{k=0}^{+\infty} a_k \mu_k \eta_{t-k}}_{\bar{E}_t[c_t]} + \delta_1 \underbrace{\sum_{k=0}^{+\infty} a_{k+1} \mu_k \eta_{t-k}}_{\bar{E}_t[c_{t+1}]}.$$

For this to be true for all sunspot realizations, it is necessary and sufficient that, for all $k \geq 0$,

$$a_k = \mu_k(\delta_0 a_k + \delta_1 a_{k+1}). \tag{17}$$

Since $\delta_0 < 1$, $\delta_1 > 0$, and $\mu_k \in (0,1)$, the above is equivalent to

$$a_{k+1} = \frac{1 - \delta_0 \mu_k}{\delta_1 \mu_k} a_k; \tag{18}$$

and because $\mu_k \to 0$ and hence $\frac{1-\delta_0\mu_k}{\delta_1\mu_k} \to \infty$ as $k \to \infty$, we have that $|a_k|$ explodes to infinity as $k \to \infty$ (and hence so does the variance of c_t), unless $a_0 = 0$. But $a_0 = 0$ implies $a_k = 0$ for all k. We conclude that the unique bounded equilibrium is $a_k = 0$ for all k, which herein corresponds to the MSV solution, since we have switched off the fundamental shocks. The proof in Appendix A extends the argument to the presence of such shocks.

Comparison to full information and the boundedness restriction

We expand on the intuition behind the above argument momentarily. But first, it is useful to repeat it for the knife-edge case with $\lambda = 0$. In this case, $\mu_k = 1$ for all k and condition (18) becomes

$$a_{k+1} = \delta^{-1} a_k,$$

where, recall, $\delta \equiv \frac{\delta_1}{1-\delta_0} = \frac{1+\kappa\sigma}{1+\phi\kappa\sigma}$. When $\delta < 1$ (equivalently $\phi > 1$), this still explodes as $k \to \infty$ unless $a_0 = 0$, which means that the unique bounded solution is once again $a_k = 0$ for all k. But when $\delta > 1$, the above remains bounded, and indeed converges to zero as $k \to \infty$, for arbitrary $a_0 = a \in \mathbb{R}$. This recovers the standard model's sunspot equilibria.

Note how *both* the standard argument with $\lambda = 0$ and our variant with $\lambda > 0$ use the boundedness assumption, namely that a_k does not explode. But whereas this assumption must be complemented with the Taylor principle in order to rule out sunspot equilibria in the standard case, it *alone* does the job under our perturbation. The Taylor principle has become redundant.[22]

Intuition and additional remarks

Although the adopted micro-foundations pin down the values for δ_0 and δ_1 as specific functions of the underlying preference, technology, and policy parameters, our argument did not rely at all on these restrictions. With this in mind, let us momentarily ignore these restrictions, set $\delta_0 = 0$ and $\delta_1 = \delta$ for arbitrary δ (possibly even negative), and simplify condition (17) to

$$a_k = \delta \mu_k a_{k+1}. \tag{19}$$

Focus now on the effects of the first-period sunspot and let $\{\frac{\partial c_t}{\partial \eta_0}\}_{t=0}^{\infty}$ stand for the corresponding impulse response function (IRF). We can then rewrite condition (19) as

$$\frac{\partial c_t}{\partial \eta_0} = \delta \mu_t \frac{\partial c_{t+1}}{\partial \eta_0}.$$

This is the same condition as that characterizing the IRF of c_t to η_0 in a "twin" representative-agent economy, in which condition (5) is modified as follows:

$$c_t = \tilde{\delta}_t \mathbb{E}_t[c_{t+1}], \quad \text{with} \quad \tilde{\delta}_t \equiv \delta \mu_t.$$

Under this prism, it is *as if* we are back to the standard New Keynesian model but the relevant eigenvalue, or the overall strategic complementarity, has become time-varying and has been reduced from δ to $\tilde{\delta}_t$. Furthermore, because $\mu_t \to 0$ as $t \to \infty$, we have that there is T large enough but finite so that $0 < \tilde{\delta}_t < 1$ for all $t \geq T$, regardless of δ. In other words, the twin economy's dynamic feedback becomes weak enough that c_t cannot depend on η_0 after T. By induction then,

[22] Recall the standard justification of the boundedness assumption: the monetary authority commits to follow a Taylor rule as long as π_t, or c_t, stays within some bounds, and to switch from such an interest-rate-setting regime to a money-supply-setting regime, or to some other appropriately specified exit strategy, if inflation goes outside these bounds. As shown most clearly in Atkeson et al. (2010), in the standard paradigm such "hybrid" rules avoid the ad hoc boundedness assumption but continue to require the Taylor principle to guarantee determinacy. Our result, instead, suggests that such hybrid rules can do the job *without* the Taylor principle: it suffices to have a consensus that the monetary authority will "do whatever it takes" to keep inflation, or the output gap, within some bounds.

c_t cannot depend on η_0 before T either.[23]

This interpretation of our result must be clarified as follows. In the above argument, we studied the response of c_t to η_0. This means that our "twin" economy is defined from the perspective of period 0, and that $\tilde{\delta}_t = \mu_t \delta$ measures the feedback from $t+1$ to t in a very specific sense: as perceived from agents in period 0, when they contemplate whether to react to η_0. To put it differently, in this argument t indexes not the calendar time but rather the belief order, or how far into the future agents reason about the effects of an innovation today.

Let us explain. Because η_0 is payoff irrelevant in every single period, period-0 agents have an incentive to respond to it *if and only if* they are confident that period-1 agents will also respond to it, which can be true only if they are also confident that period-1 will themselves be confident that period-2 agents will do the same, and so on, ad infinitum. It is this kind of "infinite chain" that supports sunspot equilibria when $\lambda = 0$. And conversely, the friction we have introduced here amounts to the typical period-0 agent reasoning as follows:

> "I can see η_0. And I understand that it would make sense to react to it if I were confident that all future agents will keep conditioning their behavior on it *in perpetuity*. But I worry that future agents will fail to do so, either because they will be unaware of it, or because they may themselves worry that agents further into the future will not react to it. By induction, I am convinced that it makes sense not to react to η_0 myself."

Three remarks complete the picture. First, the reasoning articulated above, and the proof given earlier, can be understood as a chain of contagion effects from "remote types" (uninformed agents in the far future) to "nearby types" (informed agents in the near future) and thereby to present behavior. This underscores the high-level connection between our approach and the global games literature (Morris and Shin, 1998, 2003). Second, the aforementioned worries don't have to be "real" (objectively true). That is, we can reinterpret Assumption 4 as follows: agents don't necessarily forget themselves but believe that others will forget.[24] Finally, consider how such worries influence the response to a persistent innovation in the fundamental. Even if all future agents fail to react to it, current agents have an incentive to react to it, because it has a direct effect on their own payoffs. This highlights the following point: although all full-information equilibria, including the MSV solution, embed perfect coordination, the MSV solution is not as fragile as all other equilibria to the friction under consideration.

[23] Although this argument assumed $\delta_0 = 0$, it readily extends to $\delta_0 \neq 0$. In this case, the twin economy has both δ_0 and δ_1 replaced by, respectively, $\mu_t \delta_0$ and $\mu_t \delta_1$. That is, both types of strategic complementarity are attenuated.

[24] Strictly speaking, this requires a modification of the solution concept: from REE to PBE with misspecified priors about one another's knowledge, along the lines of Angeletos and Sastry (2021). But the essence is the same.

Alternative monetary policies and observation of current interest rates

Proposition 4 directly extends if we add a forward-looking term to the Taylor rule, for instance if we let $i_t = z_t + \phi\mathbb{E}_t[\pi_{t+1}]$. This merely changes the exact values of δ_0 and δ_1, which were never used in our argument. What we cannot readily nest is a policy of the form $i_t = z_t + \phi_\pi \pi_{t-1}$, because (5) does not allow for backward-looking terms. See, however, Appendix B for an illustration of why this does not upset our result, insofar as, of course, Assumption 4 is maintained.

In principle, this assumption requires that the consumer be uncertain about, or inattentive to, the current interest rate i_t. An important exception to this statement is when $\phi = 0$: in this case, knowledge of z_t translates to knowledge of i_t. That is, for the special case of interest rate pegs, our uniqueness result is consistent with *perfect* knowledge of the policy instrument. As for the more general case in which $\phi \neq 0$, any uncertainty about the current interest rate becomes vanishingly small in the limit as $\lambda \to 0$. This is a direct implication of Proposition 3 below.

As similar point applies to the uncertainty consumers face about current income, or current prices: this uncertainty, too, vanishes as $\lambda \to 0$. What is more, we can reconcile our uniqueness result with *perfect* knowledge of current outcomes if we do one of the following: (i) we abstract from the possibility that consumers may try to extract information from concurrent outcomes about the past sunspots, as explained in Appendix B; or (ii) we allow for such signal extraction but invoke a different perturbation argument, that developed in Section 4.

Persistent sunspots and endogenous state variables

Let us now revisit the assumption that the sunspot η_t is uncorrelated over time. Proposition 2 readily extends to an arbitrary ARMA process for the sunspot, except for one knife-edge case: when η_t follows an AR(1) process with autocorrelation *exactly* equal to δ^{-1}. In this case, $c_t = c_t^F + a\eta_t$ is an equilibrium for any a and is supported by knowledge of (x_t, η_t) alone.

Of course, such a situation seems exceedingly unlikely if the sunspot is an exogenous random variable. But could it be that an *endogenous* variable, such as c_{t-1} or π_{t-1}, can serve the same function? We will return to this question in Section 7, but we offer a preliminary answer here:

Proposition 3 (Nearly perfect information about endogenous outcomes). *Under Assumption 4, almost all agents become arbitrarily well informed about arbitrarily long histories of c_t as $\lambda \to 0$: for any mapping from h^t to c_t as in Definition 1, any $K < \infty$ arbitrarily large but finite, and any $\epsilon, \epsilon' > 0$ arbitrarily small but positive, there exists $\hat{\lambda} > 0$ such that, whenever $\lambda \in (0, \hat{\lambda})$, we have $Var\left(E_t^i[c_{t-k}] - c_{t-k}\right) \leq \epsilon$ for all $k \in \{0, 1, \cdots, K\}$, for at least a mass $1 - \epsilon'$ of agents and for every t.*

In this sense, Proposition 2 is compatible with nearly perfect knowledge of *both* current and past outcomes: it is *as if* agents have received arbitrarily precise signals about $\{c_t, c_{t-1}, ..., c_{t-K}\}$, and by extension about $\{\pi_t, \pi_{t-1}, ..., \pi_{t-K}\}$ and $\{i_t, i_{t-1}, ..., i_{t-K}\}$, too, for arbitrarily large K.

5 A Generalization

In this section we extend Proposition 2 to a more flexible class of games, featuring rich forward-looking behavior; we next explain how this helps nest more standard New Keynesian economies, where consumers are long-lived and firms set prices optimally; and we finally expand on the interpretation of Assumption 4 in a market context.

An abstract generalization

At this point, it should be clear that the micro-foundations of equation (5) played no essential role in our argument. This suggests the following generalization. Maintain our assumptions about stochasticity and information but replace equation (5) with the following:

$$c_t = \bar{E}_t \left[\theta_t + \sum_{k=0}^{+\infty} \delta_k c_{t+k} \right] \tag{20}$$

for some scalars $\{\delta_k\}_{k=0}^{\infty}$, with $\delta_0 < 1$ and $\Delta \equiv \delta_0 + \sum_{k=1}^{\infty} |\delta_k| < \infty$. And interpret this equation as the average best response of a dynamic game in which: (i) a continuum of players acts in each period; (ii) a player's optimal strategy is given by $c_{i,t} = E_{i,t}\left[\theta_t + \sum_{k=0}^{+\infty} \delta_k c_{t+k}\right]$ for any t, any realization of her information set $I_{i,t}$, and any strategy played by other players; and (iii) the coefficient δ_k identifies the slope of an agent's best response with respect to the average action k periods later.

This generalization allows outcomes to depend on expectations of outcomes in the entire infinite future, not just the next period; and this dependence could be of arbitrary size and sign.[25] The analogue of the sum $\delta_0 + \delta_1$ from our main analysis, a measure of the overall strength of strategic interdependence, is now given by Δ. With $\Delta > 1$, multiple self-fulfilling equilibria can be supported under full information, in a similar fashion as in Section 3. But they unravel once $\lambda > 0$, because this again breaks the "infinite chain" behind them.

We verify this claim below. The proof is more convoluted than that of Proposition 4, and is delegated to Appendix A. But the basic logic is the same.

Proposition 4 (**Generalized result**). *Consider the generalization above, impose Assumption 4, and let $\lambda > 0$. Whenever an equilibrium exists, it is unique and is given by the MSV solution.*[26]

[25] We are only restricting $\delta_0 < 1$. This is necessarily true in the (extended) New Keynesian model we describe next as long as $\phi \geq 0$ and it means that multiplicity can originate only from the dynamic feedback between c_t and $\{\bar{E}_t[c_{t+1}]\}_{k=1}^{\infty}$, as opposed to the static feedback between c_t and $\bar{E}_t[c_t]$.

[26] The MSV solution is now given by $c_t^F = \gamma' x_t$, where γ solves $\gamma' = q' + \delta_0 \gamma' + \sum_{k=1}^{\infty} \delta_k R^k \gamma'$. Clearly, this solution

Nesting a larger class of New Keynesian economies

We now sketch how the above generalization helps accommodate a larger class of New Keynesian economies than the specific one employed in our main analysis. In particular, let us make the following "minimal" assumption about aggregate consumption: that it can be expressed as a linear function of the average expectations of income and interest rates, namely

$$c_t = \mathscr{C}\left(\{\bar{E}_t[y_{t+k}]\}_{k=0}^{\infty}, \{\bar{E}_t[r_{t+k}]\}_{k=0}^{\infty}\right), \tag{21}$$

where $r_t \equiv i_t - \pi_{t+1}$ and $\mathscr{C}$ is a linear function. This generalizes equation (1) from our baseline model, allowing aggregate consumption to depend on expectations about interest rates and income at all future periods, not just the next period. Below we will show how to obtain (21) from a fully micro-founded New Keynesian economy, in which consumers have infinite horizons. For now, take equation (21) as given and think of it as a linear but otherwise flexible specification of the intertemporal Keynesian cross (Auclert et al., 2018).

Consider next the supply side. We now replace our baseline model's ad hoc, static Phillips with the standard, micro-founded, and forward-looking New Keynesian Phillips curve:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \xi_t, \tag{22}$$

where $\kappa \geq 0$ and $\beta \in (0,1)$ are fixed scalars and ξ_t is, again, a cost-push shock. The micro-foundations of (21) are omitted because they are entirely standard: whenever given the opportunity by the "Calvo fairy," firms optimally reset their prices under rational expectations and with full information.[27] Finally, we let the Taylor rule be

$$i_t = z_t + \phi_y y_t + \phi_\pi \pi_t, \tag{23}$$

for some fixed scalars $\phi_c, \phi_\pi \geq 0$ and some random variable z_t.

The "famous" three equations are now given by (21), (22) and (23), along with $y_t = c_t$ (by market clearing). Solving (22) and (23) for inflation and the interest rate, and replacing these solutions into (21), we obtain c_t as a linear function of $\left\{\bar{E}_t[y_{t+k}]\right\}_{k=0}^{\infty}$, or equivalently of $\left\{\bar{E}_t[c_{t+k}]\right\}_{k=0}^{\infty}$. That is, the economy is reduced to a special case of equation (20). Similar to equation (5) in our baseline model, this equation conveniently summarizes all the underlying GE feedbacks and helps translate the economy to a game among the consumers.[28] And via Proposition 4, our uniqueness

exists if and only if $(1-\delta_0)I - \sum_{k=1}^{\infty} \delta_k R^k$ exists and is invertible, which is the present analogue of Assumption 3.

[27] The assumption that firms, unlike consumers, have full information simplifies the exposition and maximizes proximity to the standard New Keynesian model, without affecting the essence. For, as long as the informational friction is present in the consumer side, it is not necessary to "double" it in the production side.

[28] Accordingly, the coefficients $\{\delta_k\}_{k=0}^{\infty}$ can be expressed as functions of the following "deeper" parameters, which regulate these feedbacks: the MPCs out of current and future income, $\{\frac{\partial C}{\partial y_k}\}_{k=0}^{\infty}$; the sensitivities of consumption to current and future real interest rates, $\{\frac{\partial C}{\partial y_k}\}_{k=0}^{\infty}$; the slope, κ, and the forward-lookingness, β, of the NKPC; and the

result directly extends to the larger class of New Keynesian economies considered here, provided of course that Assumption 4 or an appropriate variant thereof holds.

Micro-foundations, markets, and information

We now expand on the micro-foundations behind (21), our intertemporal Keynesian cross assumed above. We only sketch the main ideas here; the proof of Corollary 1 contains all details.

Consider a "perpetual youth," overlapping generations economy along the lines of Blanchard (1985). Preferences are standard, given by expected lifetime utility, and the survival rate is invariant to age, given by $\omega \in (0,1]$. When a consumer dies, she gets replaced by a newborn consumer, who has zero wealth. As in Blanchard (1985), consumers can trade actuarily fair annuities, whose return conditional on survival is given in equilibrium by the risk-free rate plus ω. Furthermore, consumers have perfect recall over their lifetime. But unlike Blanchard (1985), information is not necessarily transferred from dying consumers to newborn consumers. This allows us to think of the decay in social memory, namely Assumption 4, as a byproduct of natural death. But this interpretation is, of course, not strictly needed.[29]

On top of the aggregate shocks, there are various kinds of idiosyncratic shocks. This allows not only for more realism but also for a natural reason for why individual outcomes, even if observed perfectly, may not reveal the underlying aggregate shock (which for purposes translates to guaranteeing lack of common knowledge of the payoff irrelevant histories). At the same time, this opens the door to the possibility that consumers confuse aggregate shocks for idiosyncratic ones (Lucas, 1972), and more specifically that they confuse a sunspot for an idiosyncratic income or rate-of-return shock (Benhabib et al., 2015). This possibility is not only costly to deal with but also orthogonal to the issues of concern here.[30] We thus assume it away.

policy coefficients, ϕ_π and ϕ_c. For instance, in the micro-founded OLG economy that we describe momentarily, equation (22) specializes to equation (24), implying the following formula for δ_k :

$$\delta_k \equiv \left(1-\beta\omega-\beta\omega\sigma\phi_y\right)\left(\beta\omega\right)^k + \omega\sigma\kappa\left(-\beta\phi_\pi + \left(1-\beta\omega\phi_\pi\right)\frac{1-\omega^k}{1-\omega}\right)\beta^k;$$

Note then that $\delta_0 < 1$ and $\Delta < \infty$, which means that the only restrictions imposed on (20) are readily satisfied.

[29]Strictly speaking, the above interpretation restricts $\lambda = 1-\omega$, where $1-\omega$ is the probability of death. But we could have $\lambda < 1-\omega$ if newborn consumers inherit some of the information of the dying consumers. And conversely, we could justify $\lambda > 1-\omega$ by letting consumers be altruistic towards future generations. For instance, if consumers are "dynasties" as in Barro (1974), they choose consumption *as if* they are infinitely lived ($\omega = 0$), but we can still justify $\lambda > 0$ as the product of physical death. Last but not least, we can think of $\lambda > 0$ as the by-product of bounded recall *within* the lifecycle of an individual. This would add a behavioral flavor to our approach, which we welcome but do not strictly require.

[30]For instance, when agents are rationally confused between aggregate and idiosyncratic shocks, multiple equilibria may emerge in the feedback between others' behavior and one's optimal signal extraction problem; see Benhabib et al. (2015) and Gaballo (2017) for examples. But this kind of multiplicity is clearly orthogonal to the one we are concerned with in this paper.

By aggregating the individual consumption functions, and using the above assumption, we can arrive at the following aggregate consumption function:

$$c_t = (1-\beta\omega)\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k \bar{E}_t\left[y_{t+k}\right]\right\} - \beta\omega\sigma\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k \bar{E}_t\left[i_{t+k}-\pi_{t+k+1}\right]\right\}, \tag{24}$$

where $\sigma > 0$ is the elasticity of intertemporal substitution and $\beta \in (0,1)$ is the subjective discount factor. This equation has a simple interpretation: the first term captures permanent income; the second term captures intertemporal substitution.[31] Clearly, this equation is directly nested in (21). The following is then an immediate implication of the earlier results in this section.

Corollary 1. *Consider the micro-founded OLG economy described above. The equilibrium is unique and is given by the MSV solution regardless of monetary policy, provided (i) that consumers do not misperceive aggregate for idiosyncratic shocks and (ii) that we impose Assumption 4 with regard to what agents know about the aggregate shocks.*

We think that these assumptions are the right ones for our purposes: the one rules out misperceptions of sunspots for idiosyncratic fundamentals, the other formalizes the friction of interest. But they also create a tension: in a realistic market context, the available information may naturally confound different shocks (Lucas, 1972; Benhabib et al., 2015), plus one's information is endogenous to others' choices, and our approach cannot accommodate these possibilities.

In our view, this tension is moderated by the fact that we have concentrated on the limit as $\lambda \to 0^+$, which, as explained at the end of the previous section, translates to nearly perfect, private knowledge of arbitrarily long histories of aggregate outcomes, or the average actions of others. But let us be clear: so far we have established equilibrium uniqueness for a sequence of *exogenous*-information economies that converges to the standard, full-information benchmark as $\lambda \to 0$, and it is not clear how exactly this maps to *endogenous*-information economies.

This consideration, along with our earlier discussion about recursive equilibria, motivates the analysis of Section 7. There, we show how our message goes through if replace Assumption 4 with two other assumptions, which allow for direct and even perfect signals of the aggregate outcomes (and hence also for endogenous coordination devices). But before exhausting the readers' patience with these robustness exercises, we discuss what our results mean for applied purposes.

[31] The only subtlety behind this equation is that we have used the aforementioned assumption—that consumers do not confuse aggregate shocks for idiosyncratic shocks—to replace the average expectations of *individual* conditions, which is what matters at the micro level, with the average expectations of the corresponding *aggregate* variables. See the proof of Corollary 1 for the formal statement of this assumption and for a detailed derivation of equation (24).

6 Discussion: FTPL, stabilization, and flexible prices

In this section, we extend the analysis to allow for fiscal policy and explain in detail what our result means vis-a-vis the FTPL. We also offer a refined take on the equilibrium selection and stabilization functions of monetary policy,

On the Fiscal Theory of the Price Level

Let us momentarily go back to the basics: the textbook, three-equation, New Keynesian model. Add now a fourth equation, the government's intertemporal budget constraint, written compactly (and in levels) as follows:

$$\frac{B_{t-1}}{P_t} = PVS_t, \tag{25}$$

where B_{t-1} denotes the outstanding nominal debt, P_t denotes the nominal price level, and PVS_t denotes the present discounted value of primary surpluses. Does the incorporation of this equation make a difference for the model's predictions about inflation and output?

The conventional approach says no by assuming that fiscal policy is Ricardian, in the sense that it adjusts to make sure that (25) holds along the MSV solution of the model's other three equations. The FTPL argues the opposite by letting (25) be satisfied for a *different* solution, and by letting that solution identify the model's overall equilibrium.

To illustrate, consider a negative shock to tax revenue. Because such a shock does not enter the New Keynesian model's three famous equations, it does not change its MSV solution. But it of course reduces PVS_t, other things equal. Thus suppose that the following is true after the shock:

$$\frac{B_{t-1}}{P_t^{MSV}} > PVS_t,$$

where P_t^{MSV} denotes the price level predicted by the MSV solution. How is equilibrium restored? One possibility (the Ricardian regime) is that government raises taxes and/or cuts spending so as to make sure that (25) is satisfied in the MSV solution. But another possibility (the non-Ricardian regime) is that the government fails to do so, and yet the government remains solvent, because a different solution obtains and along it the price level adjusts to

$$P_t = P_t^{FTPL} \equiv \frac{B_{t-1}}{PVS_t} > P_t^{MSV}.$$

In a nutshell, the FTPL selects a particular sunspot equilibrium and uses it to clear the government's intertemporal budget.

As mentioned in the Introduction, Kocherlakota and Phelan (1999), Buiter (2002), and others have argued that a non-Ricardian policy amounts to an off-equilibrium threat by the government to "blow up" its budget. But Cochrane (2005) firmly objects to this interpretation. He proposes that (25) must be read as a market valuation equation, instead of an actual constraint on fiscal policy. This recasts (25) as an equilibrium condition, that is, as an equation that *by definition* must only hold in equilibrium and not off equilibrium. Under this prism, the blow-up criticism of the FTPL becomes inapplicable; and one is pressed to think that, perhaps, the FTPL solution is the *only* full equilibrium.

Our analysis helps resolve this confusion. From the preceding analysis, we know that, as long as our informational assumptions are satisfied, the MSV solution of the model's famous three equations is the unique REE of the economy when there is no government. What may not be immediately obvious is how this result and the micro-foundations behind it extend once we add fiscal policy and incorporate the model's fourth equation. The details are worked out in Appendix A , but the basic ideas are summarized here.

We make the following modifications to the micro-foundations spelled out in the previous section. Finally, we let consumers have infinite horizons ($\omega = 1$), or be "dynasties" as in Barro (1974). Second, we add government spending as an exogenous random variable, include it in the economy's fundamentals, and specify the fiscal authority's policy rule for taxes and new debt issuances as follows:

$$(\tau_t, b_t) = F\left(\{c_{t-k}, g_{t-k}, \tau_{t-k}, p_{t-k}, b_{t-k-1}\}_{k=0}^{\infty}; h^t\right),$$

where τ_t denotes taxes, g_t denotes government spending, b_t denotes the quantity of *nominal* bonds issued at t, and F is arbitrary.

Finally, we impose the government budget (25). Following the previous discussion, we remain agnostic on whether (25) is a "true" constraint on fiscal policy or a valuation equation. Instead, we make the following key observation. As long as consumers understand that (25) must hold and do not misperceive an aggregate change in fiscal policy as an idiosyncratic shock, aggregation of their optimal consumption functions yields the following log-linearized condition for c_t:

$$c_t = (1-\beta)\left\{\sum_{k=0}^{+\infty}\beta^k \bar{E}_t\left[y_{t+k} - g_{t+k}\right]\right\} - \beta\sigma\left\{\sum_{k=0}^{+\infty}\beta^k \bar{E}_t\left[i_{t+k} - \pi_{t+k+1}\right]\right\}. \tag{26}$$

This is the same as equation (24) in the previous section, except for two small twists: $\omega = 1$, and y_{t+k} is replaced with $y_{t+k} - g_{t+k}$, because consumers understand that the government absorbs part of the aggregate output.[32] Crucially, neither the level of government debt nor the expected path of taxes shows up in this condition; and this is true despite the fact that no assumption has been made thus far about how consumers form expectations regarding one another's behavior or any aggregate variable. In other words, to reach condition (26) we have *not* used the full bite of REE; we have only assumed that consumers have first-order knowledge of condition (25) and

[32]See the proof of Corollary 2 in Appendix A for a detailed derivation.

do not confuse aggregate changes in the timing of taxes for idiosyncratic shocks.

In any REE, the following two properties hold in addition to (26): first, consumers understand that the goods markets must clear; and second, consumers understand that inflation obeys the NKPC (22) and that monetary policy follows the Taylor rule (23). The first property allows us to replace the expectations of $\{y_{t+k} - g_{t+k}\}$ in condition (26) with those of $\{c_{t+k}\}$; the second allows us to do the same for expectations of $\{\pi_{t+k}\}$ and $\{i_{t+k}\}$.[33]

Putting everything together, we arrive at the *same* fixed-point relation between c_t and the average expectations of $\{c_{t+k}\}$, or the same "game" among the consumers, as when fiscal policy is absent. That is, the equilibrium process for c_t must still solve equation (20);[34] under our informational assumptions, the MSV solution of this equation continues to identify the unique possible equilibrium process for c_t; conditional on the latter, the processes for π_t and i_t are uniquely pinned down by the NKPC curve and the Taylor rule; and the fiscal authority's policy rule, F, does not enter the determination of any of these objects.

Corollary 2. *Whenever an equilibrium exists, it corresponds to the MSV solution of equation (20) and has the following property: for a given Taylor rule, inflation and output are invariant to both the outstanding level of debt and to the fiscal rule F, which describes how deficits adjust to economic conditions. Finally, this is true regardless of whether monetary policy is active* $(\phi > 1)$ *or passive* $(\phi < 1)$.

The corollary starts with the qualification "whenever an equilibrium exists" to account for the following: even though equation (20) is well defined for *every* fiscal rule F and is invariant to it, the government may of course be insolvent for *some* fiscal rules. This translates as follows:

Corollary 3. *Fiscal policy has to be "Ricardian," or else it leads to equilibrium non-existence.*

Table 1 helps position this lesson in the literature. The left panel, which is basically reproduced from Leeper (1991), summarizes the state of the art. According to it, the non-Ricardian assumption is consistent with equilibrium existence, and uniquely pins down inflation and output when monetary policy is passive. The right panel summarizes our own take on the issue: the non-Ricardian assumption is equated to equilibrium non-existence regardless of whether monetary policy is active or passive. This explains the sense in which our approach transforms the rejection of the FTPL from a "religious choice" to a logical necessity—provided, of course, that one accommodates the type of informational/coordination friction we have formalized here.

[33] To be precise, although the expectations of $\{g_{t+k}\}$ drop out in the first step, they reemerge in the second step as long as $\kappa > 0$, because government spending enters the NKPC as a cost push shock. But this amounts to a redefinition of ξ_t, or θ_t, and is of no consequence for our purposes.

[34] Minor qualification: g_t must now be included in the definition of θ_t, but this makes not difference for the argument made here.

Standard Result		
	Fiscal Policy is	
	Ricardian	Non-Ricardian
Taylor Principle holds	Determinacy	No equilibrium
does not hold	Multiplicity	Determinacy

Our Result		
	Fiscal Policy is	
	Ricardian	Non-Ricardian
Taylor Principle holds	Determinacy	No equilibrium
does not hold	Determinacy	No equilibrium

Table 1: Standard Paradigm vs Our Approach

In closing, let us iterate that the entire argument presented here is valid whether condition (25) represents a "true" constraint on fiscal policy, which is the conventional take, or a valuation equation, which is Cochrane (2005)'s preferred interpretation. Furthermore, because we work directly with the optimal consumption functions (which embed not only the consumers' Euler conditions but also their transversality conditions and their budget constraints), we avoid the criticism that the New Keynesian model's DIS equation is an incomplete description of consumer behavior. We hope that these points help resolve some of the confusion surrounding the FTPL.[35]

Having said that, let us also emphasize that our results leave ample room for fiscal considerations, such as seigniorage or the real debt burden, to enter the monetary authority's choice of $\{z_t\}$ and thereby the unique equilibrium. For instance, even if markets are complete and Ricardian equivalence holds, higher deficits could trigger an inflationary boom today insofar as they are (rationally) interpreted as news about lax monetary policy in the future. In other words, the model's conventional/fundamental equilibrium *itself* is logically consistent with the "unpleasant arithmetic" of Sargent and Wallace (1981), the Ramsey literature on how monetary policy can substitute for fiscal policy and/or ease tax distortions (e.g., Chari et al., 1994; Benigno and Woodford, 2003), and some topical discussions. Perhaps this is what the FTPL is *meant* to be about, once freed up from the equilibrium selection conundrum.

Feedback rules, and equilibrium selection vs stabilization

We now shift attention to another issue: the conventional separation of the equilibrium selection and stabilization functions of monetary policy, and our fresh take on it.

Go back to the textbook, full-information, case and let $\{i_t^o, \pi_t^o, c_t^o\}$ denote the optimal path for interest rates, inflation, and output, as a function of the underlying demand and supply shocks.[36]

[35] In the same vein, consider the following narrative from Cochrane (2005) about how one can think about the off-equilibrium adjustment from $P_t \neq P_t^{FTPL}$, say $P_t < P_t^{FTPL}$, to $P_t = P_t^{FTPL}$. Cochrane (2005) proposes that, when $P_t < P_t^{FTPL}$, or equivalently $B_{t-1}/P_t > PVS_t$, the consumers perceive an increase in their wealth and demand more goods, which in turns causes "inflationary pressures" and pushes P_t towards P_t^{FTPL}. Under the prism of our analysis, this narrative requires either a departure from rationality, in the sense that consumers must not understand the validity of equation (25), or a confusion in the tradition of Lucas (1972).

[36] We do not need to specify the objective under which this path is optimal because the arguments made here hold no matter this objective.

If the monetary authority observes these shocks, the optimal path can be implemented as the economy's unique equilibrium by having the monetary authority obey the following feedback rule, for any $\phi > 1$:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o),$$

which in turn is nested in (4) with $z_t = i_t^o - \phi\pi_t^o$. Note then that the precise value of ϕ is indeterminate, subject only to the restriction that $\phi > 1$, and it does not affect the properties of the optimum. That is, the feedback from π_t to i_t serves only the role of equilibrium selection; macroeconomic stabilization is instead achieved via the optimal design of z_t, and in particular via its correlation with the underlying demand and supply shocks.[37]

What if the monetary authority does not observe these shocks? Feedback rules may then be useful for the purpose of replicating the optimal contingency of interest rates on shocks, or for optimal stabilization. And, in general, this function could be at odds with that of equilibrium selection. See Galí (2008, p.101) for an illustration with cost-push shocks, and Loisel (2021) for a general formulation. Seen from this perspective, our results help ease the potential conflict between equilibrium selection and stabilization: because feedback rules are no more needed for equilibrium selection, they are "free" to be used for stabilization.

At the same time, our results pave the way for recasting the *spirit* of the Taylor principle as a form of stabilization instead of a form of equilibrium selection. By this, we mean the following. When the equilibrium is unique (whether thanks to our perturbations or otherwise), sunspot-like volatility may still obtain from overreaction to noisy public news (Morris and Shin, 2002) or shocks to higher-order beliefs (Angeletos and La'O, 2013). In particular, suppose that we relax Assumption 4 in our main analysis so as to remove common knowledge of the fundamental state, x_t, and accommodate independent shocks to higher-order beliefs of future monetary policy or other fundamentals. Then, we can maintain the MSV solution as the economy's unique equilibrium but also let this solution fluctuate in response to these shocks. In the eyes of an outside observer, the economy may appear to be ridden with "animal spirits." And a policy that "leans against the wind" may well help contain the effects of such animal spirits basically in the same as it does with other, less exotic, demand and supply shocks.

Sticky vs flexible prices

Our analysis has allowed the Phillips curve to have an arbitrary slope $\kappa \in [0, \infty)$. In this sense, our results do not depend on the degree of nominal rigidity, and they allow in particular the limit with

[37]While some textbook treatments stop here, the most careful ones combine the Taylor principle with escape clauses that take care of unbounded equilibria. See, e.g., Atkeson, Chari, and Kehoe (2010).

nearly flexible prices ($\kappa \to \infty$). But what about the knife-edge case in which prices are *perfectly* flexible ("$\kappa = \infty$")?

To ease the exposition, let us address this question in our baseline model. Maintain our assumptions about consumers and monetary policy, but modify the production side so that prices are truly flexible and output is given by a fixed endowment (so that $c_t = y_t = 0$ in log-deviations). Clearly, our characterization of the individual optimal consumption function in (1) is still valid, and so does the intertemporal Keynesian cross obtained in condition (2), which we repeat below:

$$c_t = \bar{E}_t \left[\tfrac{1}{1+\beta} c_t + \tfrac{\beta}{1+\beta} c_{t+1} - \tfrac{\beta}{1+\beta} \sigma (i_t - \pi_{t+1} - \varrho_t) \right]$$

But now the assumption of a fixed endowment together with market clearing implies that $c_t = 0$, which in turns means that the above condition reduces to

$$\bar{E}_t \left[i_t - \pi_{t+1} - \varrho_t \right] = 0.$$

This is no other than the Fisher equation, only adapted to heterogeneous information.

For simplicity, switch off the discount rate shock, so that $\varrho_t = 0$, and let monetary policy peg the nominal interest rate at its steady-state value, so that $\phi = 0$ and $i_t = z_t = 0$. Recall that these restrictions are consistent with our main result, which guaranteed uniqueness for an arbitrary degree of nominal rigidity. But now that prices are perfectly flexible, these restrictions imply that the Fisher equation reduces to

$$\bar{E}_t \left[\pi_{t+1} \right] = 0.$$

Two properties are then evident. First, there is no feedback from expectations of future outcomes to current outcomes, or no intertemporal coordination of the type that has been at the core of our analysis thus far. And second, equilibrium pins down only the average expectation of inflation and not its precise realizations. In particular, $\pi_t = a\eta_t$, where η_t is the sunspot and $a \in \mathbb{R}$ is an arbitrary scalar, is an REE under our main assumption for every $\lambda > 0$ and, more generally, for every information structure such that $I_{i,t}$ merely contains η_t. In a nutshell, our uniqueness result does not apply and we are basically back to Sargent and Wallace (1981).

Although this clarifies the applicability of our result, we suspect that it ultimately speaks to an inherent "bug" of the baseline RBC model, or equivalently of the flexible-price core of the New Keynesian model. By design, this otherwise important conceptual benchmark is not well suited for understanding how nominal prices are determined: the nominal price level is both payoff-irrelevant and set by an "invisible hand." By contrast, the New Keynesian model ties the adjustment in nominal prices to the optimizing behavior of specific players, the firms, and allows one to recast the whole economy as a game between the firms and the consumers.[38] The pres-

[38] This point might have been blurred by our choice to solve out firm behavior and reduce the economy to a game

ence of *some* nominal rigidity was essential for obtaining such a game in the first place. But once we got to this point, that is, once we properly accounted for both real output and nominal prices as the average actions of specific players, our analysis could proceed without any restriction on how large or small the nominal rigidity might be.

What about models that assume away nominal rigidities but let nominal prices be otherwise relevant, such as models with money in the utility function or Samuelson's classic about money as a bubble? Proposition 5 has already hinted at what it takes for our uniqueness result to apply: the multiplicity has to be sustained by an infinite chain of intertemporal coordination. But the translation of this insight to the aforementioned or other applications is an open question.

7 Robustness: Observing Past Outcomes

In the end of Section 4, we highlighted that, although our key assumption excluded direct observation of the past endogenous outcomes, such as output and inflation, it allowed agents to face arbitrarily little uncertainty about them, in the sense of Proposition 3. We now push this argument further, by showing how uniqueness can obtain if we let agents have direct, and possibly even perfect, knowledge of the past outcomes. This also circles back to our discussion of sequential and recursive representations of full-information equilibria.

Recursive sunspot equilibria: an example of fragility

Consider our baseline model, where consumption choices must solve

$$c_{i,t} = E_{i,t}[(1-\delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1}]. \tag{27}$$

In the full-information case, this boils down to

$$c_t = \theta_t + \delta \mathbb{E}_t[c_{t+1}],$$

where $\delta = \frac{\delta_1}{1-\delta_0}$. Let us momentarily shut down the fundamentals, assume $\delta > 1$, and focus on the following, pure sunspot equilibrium:

$$c_t = c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^k \eta_{t-k}. \tag{28}$$

As noted earlier, this can be represented in recursive form as

$$c_t = \eta_t + \delta^{-1} c_{t-1}. \tag{29}$$

among the consumers alone. But recall that this game embeded the best-responses of the firms, via the NKPC, which translates as follows: what we *really* did in this paper was to study the game played by both consumers and firms, for any given monetary policy rule.

It follows that perfect knowledge of yesterday's outcome can readily substitute for perfect knowledge of the infinite history of past sunspots. Intuitively, c_{t-1} serves as a sufficient statistic of the infinite history of sunspots.

Taken at face value, this challenges our message that multiplicity hinges on "infinite" memory: it suffices that agents have very short memories, provided that they keep track of the past average action, here c_{t-1} (or π_{t-1}). But as shown next, this logic itself is fragile.

Abstract from shocks to the fundamentals, let sunspots be Gaussian, and let information sets be given by

$$I_{i,t} = \{\eta_t, s_{i,t}\}, \qquad \text{with} \qquad s_{i,t} = c_{t-1} + \varepsilon_{i,t}.$$

Here, $s_{i,t}$ is a private signal of the past aggregate outcome, $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2)$ is idiosyncratic noise, and $\sigma \geq 0$ is a fixed parameter. When $\sigma = 0$, we nest the case studied above. When instead $\sigma > 0$ but arbitrarily small, agents' knowledge of the past outcome is only slightly blurred by idiosyncratic noise. As shown next, this causes sunspot equilibria to unravel.[39]

Proposition 5. *Consider the economy described above. For any $\sigma > 0$, not matter how small, and regardless of δ_0 and δ_1, there is a unique equilibrium and it corresponds to the MSV solution.*

The proof is actually quite simple. Since information sets are given by $I_{i,t} = \{\eta_t, s_{i,t}\}$, any (stationary) strategy can be expressed as

$$c_{i,t} = a\eta_t + bs_{i,t},$$

for some coefficients a and b. Then, $c_{t+1} = a\eta_{t+1} + bc_t$; and since agents have no information about the *future* sunspot, $E_{i,t}[c_{t+1}] = bE_{i,t}[c_t]$. Next, note that $E_{i,t}[c_t] = a\eta_t + b\chi s_{it}$, where

$$\chi \equiv \frac{Var(c_{t-1})}{Var(c_{t-1}) + \sigma^2} \in (0, 1].$$

Combining these facts, we infer that condition (27), the individual best response, reduces to

$$c_{i,t} = E_{i,t}[\delta_0 c_t + \delta_1 c_{t+1}] = (\delta_0 + \delta_1 b)E_{i,t}[c_t] = (\delta_0 + \delta_1 b)\left\{a\eta_t + b\chi s_{i,t}\right\}.$$

It follows that a strategy is a best response to itself if and only if

$$a = (\delta_0 + \delta_1 b)a \qquad \text{and} \qquad b = (\delta_0 + \delta_1 b)b\chi. \tag{30}$$

Clearly, $a = b = 0$ is always an equilibrium, and it corresponds to the MSV solution. To have a sunspot equilibrium, on the other had, it must be that $a \neq 0$ (and also that $|b| < 1$, for it to be bounded). From the first part of condition (30), we see that this $a \neq 0$ if and only if $\delta_0 + \delta_1 b = 1$,

[39] The noise is herein assumed to be entirely idiosyncratic, but Proposition 5 continues to hold if the noise has both aggregate and idiosyncratic components. It is only in the knife-edge case where the noise is perfectly correlated across agents that multiplicity comes back.

which is equivalent to $b = \delta^{-1}$. But then the second part of this condition reduces to $1 = \chi$, which in turn is possible if and only if $\sigma = 0$ (since $Var(c_{t-1}) > 0$ whenever $a \neq 0$).

Let us connect this result to our earlier discussion of sequential and recursive equilibria. In the main analysis, we took off from the sequential form (28), represented information in terms of direct signals of the infinite history of shocks, and showed how a small perturbation in this domain results in a unique equilibrium. Here, we took off from the recursive form (29), represented information in terms of the relevant endogenous state variable, and showed how a small perturbation in this domain results, once again, in a unique equilibrium.

At this point one may raise the following question: could it be that multiple equilibria are supported by noisy idiosyncratic observations of *longer* histories of aggregate consumption or inflation? Or perhaps of some other, more "intelligently" chosen, endogenous state variable? We cannot address this question in full generality, because of the complexities we alluded to earlier (signal-extraction and infinite regress). But we offer a complementary approach next, which lets past outcomes be observed *perfectly* and nevertheless obtains uniqueness.

Breaking the infinite chain even when past outcomes are perfectly observed

In the above exercise we focused on pure sunspot equilibria. Let us now bring back the fundamental shocks and consider any of the equilibria of the form $c_t^B + ac_t^\eta$, which, recall, were obtained by "solving the model backwards." These can replicated by letting $I_{i,t} \supseteq \{\eta_t, c_{t-1}, \theta_{t-1}\}$ and by having each consumer play the following recursive strategy:

$$c_{i,t} = \delta^{-1}(c_{t-1} - \theta_{t-1}) + a\eta_t. \tag{31}$$

Contrary to the strategy that supported the pure sunspot equilibrium, the above strategy requires that the agents at t know not only c_{t-1} but also θ_{t-1}. Why is knowledge of θ_{t-1} necessary? Because this is what it takes for agents at t to know how to undo the direct, intrinsic effect of θ_{t-1} on the incentives of the agents at $t-1$, or to "reward" them for not responding to their intrinsic impulses.

This suggests that the "infinite chain" that supports all backward-looking equilibria—and all sunspot equilibria, as well— breaks if the agents at t do not know what exactly it takes to reward the agents at $t-1$. To make this point crisply, we proceed as follows.

First, we introduce a new fundamental disturbance, denoted by ζ_t; we modify equation (5) to

$$c_{i,t} = E_{i,t}[(1-\delta_0)(\theta_t + \zeta_t) + \delta_0 c_t + \delta_1 c_{t+1}]; \tag{32}$$

and we let ζ_t be drawn independently over time, as well as independently of any other shock in the economy, from a uniform distribution with support $[-\varepsilon, +\varepsilon]$, where ε is positive but arbitrarily small. This let us parameterize the payoff perturbation by ε, or the size of the support of ζ_t.

Second, we abstract from informational heterogeneity *within* periods, that is, we let $I_{i,t} = I_t$ for all i and all t. This guarantees that $c_{it} = c_t$ for all i and t, and therefore that we can think of the economy as a sequence of representative agents, or a sequence of players, one for each period. Under the additional, simplifying assumption that I_t contains both θ_t and ζ_t, we can then write the best response of the period-t representative agent as

$$c_t = \theta_t + \zeta_t + \delta E[c_{t+1}|I_t]. \tag{33}$$

where $\delta \equiv \frac{\delta_1}{1-\delta_0}$, as always, and $E[\cdot|I_t]$ is the rational expectation conditional on I_t. This is similar to the standard, full-information benchmark, except that we have allowed for the possibility that today's representative agent does not inherit all the information of yesterday's representative agent: I_t does not necessarily nest I_{t-1}.

Finally, we let I_t contain perfect knowledge of arbitrary long histories of the endogenous outcome, the sunspots, and the "main" fundamental; but we preclude knowledge of the past values of the payoff perturbation introduced above. Formally:

Assumption 5. *For each t, there is a representative agent whose information is given by*

$$I_t = \{\zeta_t\} \cup \{x_t, \cdots, x_{t-K_\theta}\} \cup \{\eta_t, \cdots, \eta_{t-K_\eta}\} \cup \{c_{t-1}, \cdots, c_{t-K_c}\}$$

for finite but possibly arbitrarily large K_η, K_c, and K_θ.

When $\varepsilon = 0$ (the ζ_t shock is absent), Assumption 5 allows replication of all sunspot and backward-looking equilibria with extremely short memory, namely with $K_\eta = 0$ and $K_\theta = K_c = 1$. This is precisely the recursive representation of these equilibria in the standard paradigm. But there is again a discontinuity: once $\varepsilon > 0$, all the non-fundamental equilibria unravel, no matter how long the memory may be.

Proposition 6. *Suppose that Assumption 5 holds and $\varepsilon > 0$. Regardless of δ, there is unique equilibrium and is given by $c_t = c_t^F + \zeta_t$, where c_t^F is the same MSV solution as before.*

To further illustrate the logic behind this result, abstract from the θ_t shock (but of course keep the ζ_t shock) and let $I_t = \{\zeta_t, \eta_t, c_{t-1}\}$. In this case, "solving the model backwards," which literally means having the agents at $t+1$ create indifference for the agents t, requires that

$$\mathbb{E}[c_{t+1}|I_t] = \delta^{-1}(-\zeta_t + c_t).$$

Since the only "news" contained in I_{t+1} relative to I_t are η_{t+1} and ζ_{t+1}, the above is true if and only if c_{t+1} satisfies

$$c_{t+1} = a\eta_{t+1} + d\zeta_{t+1} + \delta^{-1}(-\zeta_t + c_t)$$

for some $a, d \in \mathbb{R}$. As noted before, the agents at $t+1$ may extract information about ζ_t from their knowledge of c_t. But since ζ_t is not *directly* known and c_{t+1} has to be measurable in $I_{t+1} = \{\zeta_{t+1}, \eta_{t+1}, c_t\}$, the above condition can hold only if c_t itself is measurable in ζ_t and not in any other shock, such as the sunspots realized at t or earlier. In short, because the agents at $t+1$ does not know a (small) component of the "preferences" of the agents at t, it is impossible to support the aforementioned chain of indifference. The proof in Appendix A shows that an extension of this logic rules out all equilibria but the MSV solution.[40]

High-level connections between our results, and between them and the literature

Our last result, Proposition 5, is closely connected to Bhaskar (1998) and Bhaskar, Mailath, and Morris (2012). These works have shown that only Markov Perfect Equilibria (which in our context translate to the MSV solution) survive in a certain class of games when a purification in payoffs is combined with "finite" social memory (the latter being defined in a manner analogous to Assumption 5 here). Even though our environment is different, Proposition 5 is a close cousin of these earlier results. But this is not the case for our first result, Proposition 2 or 4. There, the key assumption was of different kind (contrast Assumption 4 to Assumption 5), and so was the formal argument: there was a contagion from "remote" type (agents in the far future) to "nearby" types (agent in the near future), akin to those found in the global games literature.

A broad lesson of this literature is that determinacy ultimately hinges on whether information is private versus public (where "public" means not merely publicly available but common knowledge, or at least high common-p belief). The results of Mailath and Morris (2002) and Pęski (2012), which like the aforementioned works by Bhaskar (1998) and Bhaskar, Mailath, and Morris (2012) shift the focus to Markov Perfect equilibria in dynamic games, seem consistent with this logic: Mailath and Morris (2002) relies on "almost public monitoring" to support multiple, non-Markovian equilibria, and Pęski (2012) goes in the opposite direction to rule them out. But the precise relation between these literatures remains unclear, at least to us.

One particular difference is that we work with a continuum of players per period, which seems to shut down some complications with individual-level mixed strategies that instead take a central place in the aforementioned works. Furthermore, our Proposition 5 offered an example that looked like almost public monitoring (everybody observed the past aggregate action with only

[40]Note that $c_t = c_t^F + \zeta_t$ is MSV solution of the perturbed model. This differs from c_t^F, the original MSV solution, because the relevant fundamentals now include ζ_t. But as $\varepsilon \to 0$, the new solution converges to the old one. Also note that the argument given above goes through even if the ζ_t shock occurs only every, say, 10 periods rather than every single period, because once there is a chance that the chain will break at some future date the whole thing unravels. Finally, the argument goes through even that the agents at $t+1$ know ζ_t perfectly, provided that agents at t are (incorrectly) worried that this may not be the case.

tiny idiosyncratic noise) and nevertheless obtained uniqueness. This is all to say that there are not only deep connections but also subtle differences, all of which deserve further study.

Going to an even higher level of abstraction, the results of Weinstein and Yildiz (2007) suggest that, although multiple equilibria may be "degenerate" in an appropriate topology, this statement by itself can be vacuous: with enough freedom in choosing priors and information structures, one can recast equilibrium indeterminacy as strategic uncertainty in a unique equilibrium. Under this prism, a key task for theory is to understand how a model's determinacy and its predictions more generally depend on strong common knowledge assumptions, and what are plausible relaxations thereof. We hope that our paper has made some progress in this direction.

8 Conclusion

In this paper, we revisited the indeterminacy issue of the New Keynesian model. We highlighted how all sunspot and backward-looking equilibria hinge on an infinite coordination chain between current and future behavior. And we showed how "easy" it can be to break this chain by relaxing the model's assumptions about memory and intertemporal coordination.

We thus provided a rationale for why equilibrium can be determinate in the New Keynesian model even with interest rate pegs—or why monetary policy may be able to regulate aggregate demand without a strict reliance on the kind of off-equilibrium threats embedded in the Taylor principle and related approaches. What was needed was only a consensus that inflation and output gaps are "bounded," which itself can be justified by the kind of exit strategies articulated in, inter alia, Christiano and Rostagno (2001) and Atkeson et al. (2010).

By the same token, our results left no space for the FTPL within the New Keynesian model. They equated the "non-Ricardian" assumption to equilibrium non-existence regardless of whether monetary policy is "active" or "passive. And they lend support to the conventional practice of using the model's MSV solution as the "right" lens for interpreting the data and guiding policy.

Throughout, we concentrated on the New Keynesian framework. The degree of nominal rigidity was allowed to be very small, but not exactly zero: some nominal rigidity was needed for understanding nominal prices as the choices of agents inside the economy, as opposed to the choice of an invisible hand outside the economy. Whether some version of our results can be obtained for models where nominal prices are perfectly flexible but otherwise payoff relevant, such as in Samuelson's classic or models where money offers liquidity services, is an open question.

At the same time, it should be clear that our *formal* arguments did not rely on the specific application. In particular, the analysis of Section 5 can be readily extended to settings in which c_t is a vector and, accordingly, the coefficients $\{\delta_k\}$ are matrices. This suggests that an extension of

Proposition 4 may offer a rationale for selecting the MSV solution in a general class of linear REE models (as in the classics by Blanchard, 1979, and Blanchard and Kahn, 1980), or equivalently for selecting the Markov Perfect Equilibrium in a general class of dynamic, continuum-player games with linear best responses ("dynamic beauty contests"). We leave this idea, and any further exploration of the possible links between our contribution and other attempts in game theory to justify Markov Perfect Equilibria, for future work.

Let us close with another possible return from our contribution. Consider the question of whether the Fed should internalize the fiscal ramifications of its actions, or which authority is "dominant." Such questions seem to call for modeling interaction between the monetary and the fiscal authority as a game, for example as a game of attrition. But this requires in the first place the existence of a unique mapping from those player's actions—interest rates and government deficits, respectively—to their payoffs. Such a unique mapping is missing in the standard paradigm, because the same paths for interest rates and government deficits can be associated with multiple equilibria within the private sector. By providing a possible fix to this "bug," our paper paves the way for a fresh approach to the aforementioned kind of questions.

Appendix A: Proofs

As discussed after Definition 1, our proofs use a weaker boundedness criterion than the requirement of a finite $Var(c_t)$. The next lemma verifies that that the latter implies the former. The rest of the Appendix provides the proofs for all the results.

Lemma 1. *Consider any candidate equilibrium, defined as in Definition 1. There exist a finite scalar $M > 0$ such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k, where $\|\cdot\|_1$ is the L^1-norm.*

Substituting (7) into (8), we have that any candidate equilibrium can be rewritten as

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \Gamma'_k \varepsilon^x_{t-k}, \tag{34}$$

where, for all $k \geq 0$,

$$\Gamma'_k \equiv \sum_{l=0}^{k} \gamma'_{k-l} R^l. \tag{35}$$

Since η_t and ε^x_t are independent of each other as well as independent over time, we have

$$Var(c_t) = \sum_{k=0}^{\infty} \left(a_k^2 + \Gamma'_k \Sigma_\varepsilon \Gamma_k \right).$$

This can be finite only if $\lim_{k\to+\infty} |a_k| = 0$ and $\lim_{k\to+\infty} \|\Gamma_k\|_1 = 0$.[41] From (35), $\gamma'_k = \Gamma'_k - \Gamma'_{k-1} R$ for all $k \geq 1$. It follows that $\lim_{k\to+\infty} \|\gamma_k\|_1 = 0$ as well. We conclude that there exist a scalar $M > 0$, large enough but finite, such that $|a_k| \leq M$ and $\|\gamma_k\|_1 \leq M$ for all k.

Proof of Proposition 1

Part (i) follows directly from the fact that $c^F_t \equiv q' (I - \delta R)^{-1} x_t$ satisfies (9).

Consider part (ii). Let $\{c_t\}$ be any equilibrium and define $\hat{c}_t = c_t - c^F_t$. From (9),

$$\hat{c}_t = \delta \mathbb{E}_t[\hat{c}_{t+1}]. \tag{36}$$

From Definition 1,

$$\hat{c}_t = \sum_{k=0}^{\infty} \hat{a}_k \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}'_k x_{t-k},$$

with $|\hat{a}_k| \leq \hat{M}$ and $\|\hat{\gamma}'_k\|_1 \leq \hat{M}$ for all k, for some finite $\hat{M} > 0$. From Assumptions 1–2, we have

$$\mathbb{E}_t[\hat{c}_{t+1}] = \sum_{k=0}^{+\infty} \hat{a}_{k+1} \eta_{t-k} + \sum_{k=0}^{\infty} \hat{\gamma}'_{k+1} x_{t-k} + \hat{\gamma}'_0 R x_t.$$

[41] To prove the latter statement, note that, because Σ_ε is positive definite, there exists an invertible L such that $\Sigma_\varepsilon = L'L$ by Cholesky decomposition. The finiteness of $Var(c_t)$ then implies $\lim_{k\to+\infty} \|L\Gamma_k\|_1 = 0$, which implies $\lim_{k\to+\infty} \|\Gamma_k\|_1 = 0$.

The equilibrium condition (36) can thus be rewritten as

$$\sum_{k=0}^{\infty}\hat{a}_k\eta_{t-k}+\sum_{k=0}^{\infty}\hat{\gamma}'_k x_{t-k}=\delta\left(\sum_{k=0}^{+\infty}\hat{a}_{k+1}\eta_{t-k}+\sum_{k=0}^{\infty}\hat{\gamma}'_{k+1}x_{t-k}+\hat{\gamma}'_0 R x_t\right).$$

For this to be true for all t and all states of nature, the following restrictions on coefficients are necessary and sufficient:

$$\hat{a}_k=\delta\hat{a}_{k+1}\ \forall k\geq 0,\qquad \hat{\gamma}'_0=\delta\hat{\gamma}'_1+\delta\hat{\gamma}'_0 R\qquad\text{and}\qquad \hat{\gamma}'_k=\delta\hat{\gamma}'_{k+1}\ \forall k\geq 1.$$

When the Taylor principle is satisfied ($|\delta|<1$), $\hat{a}_k$ and $\hat{\gamma}_k$ explodes unless $\hat{a}_0=0$ and $\hat{\gamma}'_1=0$. Since $I-\delta R$ is invertible from Assumption 3, $\hat{\gamma}'_0=0$ too. We know that the only bounded solution of (36) is $\hat{c}_t=0$. As a result, c_t^F is the unique equilibrium.

Finally, consider part (iii). $c_t^B\equiv-\sum_{k=1}^{\infty}\delta^{-k}\theta_{t-k}$ and $c_t^{\eta}\equiv\sum_{k=0}^{\infty}\delta^{-k}\eta_{t-k}$ are bounded (the infinite sums converge) when the Taylor principle is violated ($|\delta|>1$). c_t^B satisfies (9). So does $c_t=(1-b)c_t^F+bc_t^B+ac_t^{\eta}$ for arbitrary $b,a\in\mathbb{R}$.

Proof of Proposition 2

Since the sunspots $\{\eta_{t-k}\}_{k=0}^{\infty}$ are orthogonal to the fundamental states $\{x_{t-k}\}_{k=0}^{\infty}$, the argument in the main text proves that $a_k=0$ for all k. We can thus focus on solutions of the following form:

$$c_t=\sum_{k=0}^{\infty}\gamma'_k x_{t-k}. \tag{37}$$

And the remaining task is to show that $\gamma'_0=q'(I-\delta R)^{-1}$ and $\gamma'_k=0$ for all $k\geq 1$, which is to say that only the MSV solution survives.

To start with, note that, since x_t is a stationary Gaussian vector given by (7), the following projections apply for all $k\geq s\geq 0$:

$$\mathbb{E}\left[x_{t-k}|I_t^s\right]=W_{k,s}x_{t-s},$$

where $I_t^s\equiv\{x_t,\ldots,x_{t-s}\}$ is the period-t information set of an agent with memory length s and

$$W_{k,s}=\mathbb{E}\left[x_{t-k}x'_{t-s}\right]\mathbb{E}\left[x_t x'_t\right]^{-1}=\mathbb{E}\left[x_t x'_t\right]\left(R'\right)^{k-s}\mathbb{E}\left[x_t x'_t\right]^{-1}$$

is an $n\times n$ matrix capturing the relevant projection coefficients.

Next, note that

$$\|W_{k,s}\|_1\leq\|\mathbb{E}\left[x_t x'_t\right]\|_1\|\left(R'\right)^{k-s}\|_1\|\mathbb{E}\left[x_t x'_t\right]^{-1}\|_1, \tag{38}$$

where $\|\cdot\|_1$ is the 1-norm. Since all the eigenvalues of R are within the unit circle, we know the spectral radius $\rho(R)=\rho\left(R'\right)<1$. From Gelfand's formula, we know that there exists $\bar{\Lambda}\in(0,1)$ and $M_1>0$ such that

$$\|\left(R'\right)^{k-s}\|_1\leq M_1\bar{\Lambda}^{k-s},$$

for all $k \geq s \geq 0$. Together with the fact that $E\left[x_t x_t'\right]$ is invertible (because Σ_ε is positive definite and $\rho(R) < 1$), we know that there exists $M_2 > 0$ such that

$$\|W_{k,s}\|_1 \leq M_2 \bar{\Lambda}^{k-s}. \tag{39}$$

Now, from Assumption 4, we know

$$\bar{E}_t\left[x_{t-k}\right] = (1-\lambda)^k x_{t-k} + \sum_{s=0}^{k-1} \lambda (1-\lambda)^s \mathbb{E}\left[x_{t-k} | I_t^s\right] \equiv \sum_{s=0}^{k} V_{k,s} x_{t-s}, \tag{40}$$

where, for all $k \geq s \geq 0$,

$$V_{k,k} = (1-\lambda)^k I_{n\times n} \quad \text{and} \quad V_{k,s} = \lambda (1-\lambda)^s W_{k,s}.$$

Together with (39), we know that there exits $M_3 > 0$ and $\Lambda = \max\left\{1-\lambda, \bar{\Lambda}\right\} \in (0,1)$ such that for all $k \geq s \geq 0$,

$$\|V_{k,s}\|_1 \leq M_3 \Lambda^k. \tag{41}$$

Now consider an equilibrium in the form of (37). From equilibrium condition (5), we know

$$\begin{aligned}
\sum_{k=0}^{+\infty} \gamma_k' x_{t-k} &= (1-\delta_0)\theta_t + \delta_0 \bar{E}_t\left[\sum_{k=0}^{+\infty} \gamma_k' x_{t-k}\right] + \delta_1 \bar{E}_t\left[\sum_{k=0}^{+\infty} \gamma_k' x_{t+1-k}\right] \\
&= \left((1-\delta_0) q' + \delta_0 + \delta_1 \gamma_0' R + \delta_1 \gamma_1'\right) x_t + \bar{E}_t\left[\sum_{k=1}^{+\infty} \left(\delta_0 \gamma_k' + \delta_1 \gamma_{k+1}'\right) x_{t-k}\right] \\
&= \left((1-\delta_0) q' + \delta_0 + \delta_1 \gamma_0' R + \delta_1 \gamma_1'\right) x_t + \sum_{k=1}^{+\infty} \left(\delta_0 \gamma_k' + \delta_1 \gamma_{k+1}'\right) \left(\sum_{s=0}^{k} V_{k,s} x_{t-s}\right),
\end{aligned}$$

where we use the fact that all agents at t know the values of the fundamental state x_t.

For this to be true for all states of nature, we can compare coefficients on each x_{t-k}, we have

$$\begin{aligned}
\gamma_0' &= (1-\delta_0) q' + \delta_0 \gamma_0' + \delta_1 \gamma_0' R + \delta_1 \gamma_1' \\
\gamma_k' &= \sum_{l=k}^{+\infty} \left(\delta_0 \gamma_l' + \delta_1 \gamma_{l+1}'\right) V_{l,k} \quad \forall k \geq 1.
\end{aligned} \tag{42}$$

From Definition 1, we know that there is a scalar $M > 0$ such that $\|\gamma_k'\|_1 \leq M$ for all $k \geq 0$, where $\|\cdot\|_1$ is the 1-norm. From (41) and (42), we know that, for all $k \geq 1$,

$$\|\gamma_k'\|_1 \leq (|\delta_0| + |\delta_1|) \sum_{l=k}^{+\infty} \|V_{l,k}\|_1 M \leq (|\delta_0| + |\delta_1|) M_3 \frac{\Lambda^k}{1-\Lambda} M. \tag{43}$$

Because $\lim_{k\to\infty} \Lambda^k = 0$, there necessarily exists an $\hat{k}$ finite but large enough $(|\delta_0| + |\delta_1|) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} < 1$. So we know that, for all $k \geq \hat{k}$,

$$\|\gamma_k'\|_1 \leq (|\delta_0| + |\delta_1|) M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} M.$$

Now, we can use the above formula and (42) to provide a tighter bound of $\|\gamma'_k\|_1$: for all $k \geq \hat{k}$,

$$\|\gamma'_k\|_1 \leq \left((|\delta_0| + |\delta_1|)\, M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} \right)^2 M.$$

We can keep iterating. For for all $k \geq \hat{k}$ and $l \geq 0$,

$$\|\gamma'_k\|_1 \leq \left((|\delta_0| + |\delta_1|)\, M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} \right)^l M.$$

Since $(|\delta_0| + |\delta_1|)\, M_3 \frac{\Lambda^{\hat{k}}}{1-\Lambda} < 1$, we then have $\gamma'_k = 0$ for all $k \geq \hat{k}$. Using (42) and doing backward induction, we then know $\gamma'_k = 0$ for all $k \geq 1$ and

$$\gamma'_0 = (1-\delta_0)\, q' + \delta_0 \gamma'_0 + \delta_1 \gamma'_0 R,$$

which means $\gamma'_0 = q' \left(I - \frac{\delta_1}{1-\delta_0} R \right)^{-1} = q' \,(I - \delta R)^{-1}$, where I use $\delta_0 < 1$. Together, this means that the equilibrium is unique and is given by $c_t = c_t^F$, where c_t^F is defined in (10).

Proof of Proposition 3

Consider a candidate equilibrium c_t in Definition 1. We first notice that, for the period-t agent with memory length s, her information set I_t^s in Assumption 4 can be written equivalently as

$$I_t^s = \{\eta_{t-s}, ..., \eta_t, x_{t-s}, \varepsilon_{t-s+1}^x, \cdots, \varepsilon_t^x\},$$

where ε_t^x is the innovation in x_t in Assumption 1. From (34) in the proof of Lemma 1, we know that c_t can be written as

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \Gamma'_k \varepsilon_{t-k}^x,$$

where Γ'_k is given by (35). From the law of total variances, we have

$$Var\left(E_t\left[c_t | I_t^s\right] - c_t\right) \leq Var\left(\sum_{k=s+1}^{\infty} a_k \eta_{t-k} + \sum_{k=s}^{\infty} \Gamma'_k \varepsilon_{t-k}^x \right).$$

Since η_t and ε_t^x are independent of each other as well as independent over time, the finiteness of $Var\,(c_t)$ implies that

$$\lim_{s \to +\infty} Var\left(\sum_{k=s+1}^{\infty} a_k \eta_{t-k} + \sum_{k=s}^{\infty} \Gamma'_k \varepsilon_{t-k}^x \right) = 0.$$

As a result, for any $\epsilon > 0$ arbitrarily small but positive, there exists $\hat{s}_0$, such that

$$Var\left(E_t\left[c_t | I_t^s\right] - c_t\right) \leq \epsilon$$

for all $s \geq \hat{s}_0$ and every t. Similarly, for each $k \leq K$, there exists $\hat{s}_k$, such that

$$Var\left(E_t\left[c_{t-k} | I_t^s\right] - c_{t-k}\right) \leq \epsilon$$

for all $s \geq \hat{s}_k$ and every t. Now, for any $\epsilon' > 0$ arbitrarily small but positive, we can find $\hat{\lambda} > 0$ such that $\left(1-\hat{\lambda}\right)^{\hat{s}_k} \geq 1-\epsilon'$ for all $k \in \{0, \cdots, K\}$. Together, this means that whenever $\lambda \in (0, \hat{\lambda})$, $Var\left(E_t^i[c_{t-k}] - c_{t-k}\right) \leq \epsilon$ for all $k \leq K$, for at least a fraction $1-\epsilon'$ of agents, and for every period t.

Proof of Proposition 4

We first find the MSV solution $c_t^F = \gamma' x_t$ for some $\gamma \in \mathbb{R}^n$. From (5), we have

$$\gamma' = q' + \gamma' \left(\sum_{k=0}^{+\infty} \delta_k R^k \right).$$

Since $I - \sum_{k=0}^{+\infty} \delta_k R^k$ is invertible, the unique solution is $\gamma' = q'(I - \sum_{k=0}^{+\infty} \delta_k R^k)^{-1}$. We henceforth denote this solution as

$$c_t^F \equiv q'(I - \delta \sum_{k=0}^{+\infty} R^k)^{-1} x_t. \tag{44}$$

Consider an equilibrium taking the form of (8). We use (5):

$$\sum_{l=0}^{+\infty} a_l \eta_{t-l} + \sum_{l=0}^{\infty} \gamma_l' x_{t-l} = q' x_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k \left(\sum_{l=0}^{+\infty} a_l \eta_{t+k-l} + \sum_{l=0}^{\infty} \gamma_l' x_{t+k-l} \right) \right]. \tag{45}$$

We know

$$\bar{E}_t[\eta_{t-l}] = \begin{cases} \mu_l \eta_{t-l} & \text{if } l \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\mu_l = (1-\lambda)^l$ is the measure of agents who remember a sunspot realized l periods earlier as in the proof of Proposition 2. Comparing coefficient in front of η_{t-l} and using the facts that each sunspot is orthogonal to all fundamentals:

$$a_l = \mu_l \sum_{k=0}^{+\infty} \delta_k a_{k+l} \quad \forall l \geq 0. \tag{46}$$

Because $\lim_{l \to \infty} \mu_l = 0$, there necessarily exists an $\hat{l}$ finite but large enough $\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| < 1$.[42]

Since we are focusing bounded equilibria as in Definition 1, there exists a scalar $M > 0$, arbitrarily large but finite, such that $|a_l| \leq M$ for all l. From (46), we then know that, for all $l \geq \hat{l}$,

$$|a_l| \leq \mu_{\hat{l}} M \sum_{k=0}^{+\infty} |\delta_k|, \tag{47}$$

where we also use the fact that the sequence $\{\mu_l\}_{l=0}^{\infty}$ is decreasing. Now, we can use (46) and (47) to provide a tigehter bound of $|a_l|$. That is, for all $l \geq \hat{l}$,

$$|a_l| \leq \left(\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| \right)^2 M.$$

[42] $\sum_{k=0}^{\infty} |\delta_k| < \infty$ because $\Delta < \infty$.

We can keep iterating. Since $\mu_{\hat{l}} \sum_{k=0}^{\infty} |\delta_k| < 1$, we then have $a_l = 0$ for all $l \geq \hat{l}$. Using (46) and doing backward induction, we then know $a_l = 0$ for all l.

Now, (45) can be simplified as

$$\sum_{l=0}^{\infty} \gamma_l' x_{t-l} = q' x_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{\infty} \gamma_l' x_{t+k-l} \right]. \tag{48}$$

$$= q' x_t + \sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{k} \gamma_l' R^{k-l} x_t + \bar{E}_t \left[\sum_{l=1}^{+\infty} \left(\sum_{k=0}^{+\infty} \delta_k \gamma_{k+l}' \right) x_{t-l} \right].$$

For this to be true for all states of nature, we can compare coefficients on each x_{t-l}:

$$\gamma_0' = q' + \sum_{k=0}^{+\infty} \delta_k \sum_{l=0}^{k} \gamma_l' R^{k-l} \tag{49}$$

$$\gamma_l' = \sum_{s=l}^{+\infty} \left(\sum_{k=0}^{+\infty} \delta_k \gamma_{k+s}' \right) V_{s,l} \quad \forall l \geq 1, \tag{50}$$

where $V_{s,l}$ is defined in (40). The above two equations can be re-written as:

$$\gamma_0' = \left(q' + \sum_{k=1}^{+\infty} \delta_k \sum_{l=1}^{k} \gamma_l' R^{k-l} \right) \left(I - \sum_{k=0}^{+\infty} \delta_k R^k \right)^{-1} \tag{51}$$

$$\gamma_l' = \left(\sum_{k=l+1}^{+\infty} \sum_{s=l}^{k} \delta_{k-s} \gamma_k' V_{s,l} \right) \left(I - \delta_0 V_{l,l} \right)^{-1} \quad \forall l \geq 1, \tag{52}$$

where, from (40), we know that $I - \delta_0 V_{l,l} = \left[1 - \delta_0 (1-\lambda)^l \right] I$ is invertible.

From Definition 1, we know that there is a scalar $M > 0$ such that $\|\gamma_l'\|_1 \leq M$ for all $l \geq 0$, where $\|\cdot\|_1$ is the 1-norm. From (50), we know, for all $l \geq 1$

$$\|\gamma_l'\|_1 \leq \left(\sum_{k=0}^{+\infty} |\delta_k| \right) \left(\sum_{s=l}^{+\infty} \|V_{s,l}\|_1 \right) M \leq \left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^l}{1-\Lambda} M, \tag{53}$$

where M_3 and Λ are defined in (40) Because $\lim_{l \to \infty} \Lambda^l = 0$, there necessarily exists an $\hat{l}$ finite but large enough such that $\left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} < 1$. So we know that, for all $l \geq \hat{l}$,

$$\|\gamma_l'\|_1 \leq \left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} M.$$

Now, we can use the above formula and (50) to provide a tighter bound of $\|\gamma_l'\|_1$: for all $l \geq \hat{l}$,

$$\|\gamma_l'\|_1 \leq \left(\left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} \right)^2 M.$$

We can keep iterating. Since $\left(\sum_{k=0}^{+\infty} |\delta_k| \right) M_3 \frac{\Lambda^{\hat{l}}}{1-\Lambda} < 1$, we then have $\gamma_l' = 0$ for all $l \geq \hat{l}$. Using (52) and doing backward induction, we then know $\gamma_l' = 0$ for all $l \geq 1$ and, from (49),

$$\gamma_0' = q' + \gamma_0' \left(\sum_{k=0}^{+\infty} \delta_k R^k \right),$$

which means $\gamma_0' = q'\left(I - \delta\sum_{k=0}^{+\infty}\delta_k R^k\right)^{-1}$. Together, this means that the equilibrium is unique and is given by $c_t = c_t^F$, where c_t^F is defined in (44). This proves the Proposition.

Proof of Corollary 1 (and some additional discussion)

As mentioned in the main text, we let consumers be subject to idiosyncratic shocks in their income, wealth, nominal interest rate, and nominal prices. And we let them choose consumption on the basis of imperfect information about, or with inattention to, these variables and the underlying shocks. Regardless of what their available information might be, we can express an individual's optimal consumption function, in log-linearized form, as follows:

$$c_{i,t} = E_{i,t}\left[(1-\beta\omega)\,w_{i,t} - \beta\omega\sigma\sum_{k=0}^{+\infty}(\beta\omega)^k\left(i_{i,t+k} - \pi_{i,t+k+1}\right) + (1-\beta\omega)\sum_{k=0}^{+\infty}(\beta\omega)^k\, y_{i,t+k}\right], \tag{54}$$

where $w_{i,t}$, $y_{i,t}$, $i_{i,t}$, $\pi_{i,t}$ are the individual's wealth, income, interest rate, and inflation rate. The above generalizes (1), the consumption function in our baseline model. The first term inside the expectation operator captures wealth, the second term captures intertemporal substitution, and the last term captures permanent income. $\beta\omega$ is the effective discount factor (inclusive of mortality risk) and $1-\beta\omega$ is the MPC (out of financial wealth and permanent income alike).

We now wish to aggregate (54) and, from there, to arrive us to (24), the aggregate consumption function stated in the main text. This task is complicated by the fact that aggregation of (54) yields the average expectations of *individual* income and other variables, while (21) contains the average expectations of the corresponding *aggregate* variables. To merge this gap, we impose the following assumption:

Assumption 6 (**No average misperception**)**.** *$\int E_{i,t}[x_{i,t} - x_t]\,di$ is invariant to h_t, where $x_{i.t}$ stands for an individual's income, wealth, interest rate, or inflation, and x_t for the corresponding aggregate.*

This is the formal statement of the no-misperceptions assumption mentioned in the main text. To interpret it, note that $E_{it}[x_{it} - x_t]$ captures *$i's$ belief* about her relative position in the population. A consumer's *actual* position is only a function of idiosyncratic shocks, and every consumer knows this fact. But to the extent a consumer confuses an aggregate shock for an idiosyncratic shock, she may rationally mis-perceive a change in her relative position. Assumption 6 rules out precisely this possibility, at least for the average consumer. And it leaves the consumers' information otherwise unrestricted.

We are now in business. By the above assumption, we have that $\int E_{i,t}[x_{i,t}]\,di = \bar{E}_t[x_t]$ for every $x \in \{y, i, \pi, w\}$.[43] Aggregating (54) across i, using the property $\int E_{i,t}[x_{i,t}]\,di = \bar{E}_t[x_t]$ from above,

[43] To be precise, the assumption directly implies that $\int E_{i,t}[x_{i,t}]\,di = \bar{E}_t[x_t]$ up to a constant. But this constant has

and noting that aggregate wealth is zero (because there is no capital), we arrive at equation (24), which we repeat here for convenience:

$$c_t = (1-\beta\omega)\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k \bar{E}_t\left[y_{t+k}\right]\right\} - \beta\omega\sigma\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k \bar{E}_t\left[i_{t+k}-\pi_{t+k+1}\right]\right\}.$$

Using market clearing to replace $y_{t+k} = c_{t+k}$, and using also (22) and (23) to substitute the average expectations of $\{i_{t+k}\}$ and $\{\pi_{t+k}\}$ as functions of the average expectations of $\{c_{t+k}\}$, we conclude that equilibrium is summarized by the following equation:

$$c_t = (1-\delta_0)\theta_t + \sum_{k=0}^{\infty}\delta_k \bar{E}_t[c_{t+k}], \tag{55}$$

where, for all $k \geq 0$,

$$\delta_k \equiv \left(1-\beta\omega-\beta\omega\sigma\phi_y\right)(\beta\omega)^k + \omega\sigma\kappa\left(-\beta\phi_\pi + \left(1-\beta\omega\phi_\pi\right)\frac{1-\omega^k}{1-\omega}\right)\beta^k.$$

Note then that $\delta_0 < 1$ and $\Delta < \infty$, which means that the only restrictions imposed on (20) are readily satisfied. The proof of the corollary is completed by invoking Assumption 4 and applying Proposition 4.

For this last step, we interpret Assumption 4 as a restriction solely on what consumers know about the aggregate shocks (i.e., leaving unrestricted what they know about idiosyncratic shocks). More formally, we do not require that the entire $I_{i,t}$ satisfies Assumption 4, which would literally mean that agents know nothing about idiosyncratic shocks. We only require that the average expectations of the past sunspots satisfy the property $\bar{E}_t[\eta_{t-k}] = (1-\lambda)^k\eta_{t-k}$, and similarly for the past fundamentals.

We conclude with the following remark, which echoes a similar point made in the end of Section 4. So far, we have allowed consumers to be inattentive to, or face uncertainty about, their concurrent wealth, income, and interest rates—this why these variables appear inside the expectation operator in (54), along with future income and future returns. This assumption plays no role along the MSV solution: because we preserve perfect knowledge of x_t, the aggregate outcomes along this solution are the same as with full information. This assumption is used only away from the MSV solution and only for one and only one purpose: to bypass the complication that consumers may be able extract information about the economy's payoff-irrelevant history from their observations of current outcomes.

This in turn suggests the following path for deriving a close cousin to condition (55) and for arriving at our uniqueness result. Suppose, contrary to what we have assumed so far, that each consumer observes perfectly $w_{i,t}$, $i_{i,t}$ and $y_{i,t}$ at t. Suppose further that idiosyncratic shocks are

to be zero, or otherwise the unconditional expectation of $x_{i,t}$ would not coincide with the unconditional expectation of x_t, violating rationality.

purely transitory, so that $E_{i,t}[x_{i,t+k}] = E_{i,t}[x_{t+k}]$ for all $k \geq 1$ and any variable $x \in \{c, y, \pi, i\}$. Then, the individual consumption function (54) rewrites as follows:

$$c_{i,t} = (1-\beta\omega)\, w_{i,t} + \beta\omega\, y_{i,t} - \beta\omega\sigma(i_{i,t} - E_{i,t}[\pi_{t+1}]) + \\ + E_{i,t}\left[-\beta\omega\sigma \sum_{k=1}^{+\infty} (\beta\omega)^k (i_{t+k} - \pi_{t+k+1}) + (1-\beta\omega) \sum_{k=1}^{+\infty} (\beta\omega)^k y_{t+k}\right].$$

If we aggregate this condition, impose market clearing in all periods, use the NKPC and the Taylor rule to substitute $\{\pi_{t+k}\}_{k=0}^{\infty}$ and $\{i_{t+k}\}_{k=0}^{\infty}$ as functions of $\{c_{t+k}\}_{k=0}^{\infty}$, we arrive at the following equation:

$$c_t = (1-\delta_0)\theta_t + \delta_0 c_t + \sum_{k=1}^{\infty} \delta_k \bar{E}_t[c_{t+k}], \tag{56}$$

where the coefficients $\{\delta_k\}$ are defined as before. That is, we have arrived at *exactly* the same equation as equation (55) above, except that now $\bar{E}_t[c_t]$ is replaced by c_t itself.

In effect, what we have done so far is is to shut down the coordination friction *within* time but preserve it *across* time: the consumers act, on average, as if they knows what other consumers are doing today, but they still need to form expectations about how future consumption will be determined and, more specifically, how it may depend on payoff-irrelevant histories. By the same token, if we solve (56) for c_t, we can recast the equilibrium in the following game form:

$$c_t = \theta_t + \sum_{k=1}^{\infty} \frac{\delta_k}{1-\delta_0} \bar{E}_t[c_{t+k}],$$

which is again nested in (20). It follows that Proposition 4 continues to apply, provided of course that the average expectations of past sunspots and past fundamentals satisfied our usual "fading" restrictions.

Last but not least, note that in the above derivations we have no more used Assumption **??**. But we have effectively abstracted from the possibility that consumers extract information about payoff-irrelevant histories from their own wealth, income and interest rates, in order to make sure that we can invoke Assumption 4 vis-a-vis such histories. This further clarifies that the key to our result is the "fading" in the average expectation of past sunspots and past fundamentals. As discussed in the main text (see the concluding points of Sections 4 and 5), any remaining tension between our assumptions and realism seems to be minimized by taking the limit as $\lambda \to 0$, or by considering the variant perturbation of Section 7.

Proof of Corollary 2

Let us revisit our characterization of optimal consumption. Relative to what we did in the previous section, there are exactly four changes: first, we let $\omega = 0$ so that consumers are infinitely lived and fiscal policy does not redistribute wealth across generations (a possibility that is empirically plausible but orthogonal to the FTPL); second, aggregate disposable income is $Y_{i,t} - T_t$ instead of Y_t, where Y_t are the taxes; third, the consumers' aggregate financial wealth is $W_t \equiv \int W_{i,t} di = B_{t-1}/P_t$ instead of 0, where B_{t-1}/P_t is the real value of the outstanding nominal debt; and finally, we shut down idiosyncratic shocks in interest rates and inflation for simplicity. Accordingly, the consumer's budget constraint is given by

$$\sum_{k=0}^{+\infty} \left\{ \left[\prod_{j=1}^{k} \left(\frac{I_{t+j-1}}{\Pi_{t+j}} \right)^{-1} \right] C_{i,t+k} \right\} = W_{i,t} + \sum_{k=0}^{+\infty} \left\{ \left[\prod_{j=1}^{k} \left(\frac{I_{t+j-1}}{\Pi_{t+j}} \right)^{-1} \right] \left(Y_{i,t+k} - T_{t+k} \right) \right\} .$$

The government's budget in (25) can be written as

$$B_{t-1}/P_t = \sum_{k=0}^{+\infty} \left\{ \left[\prod_{j=1}^{k} \left(\frac{I_{t+j-1}}{\Pi_{t+j}} \right)^{-1} \right] (T_{t+k} - G_{t+k}) \right\} . \tag{57}$$

In an REE, since a consumer understand (57) holds, she understands that her budget can be written as

$$\sum_{k=0}^{+\infty} \left\{ \left[\prod_{j=1}^{k} \left(\frac{I_{t+j-1}}{\Pi_{t+j}} \right)^{-1} \right] C_{i,t+k} \right\} = W_{i,t} - W_t + \sum_{k=0}^{+\infty} \left\{ \left[\prod_{j=1}^{k} \left(\frac{I_{t+j-1}}{\Pi_{t+j}} \right)^{-1} \right] \left(Y_{i,t+k} - G_{t+k} \right) \right\} .$$

The consumer's optimal consumption function, in log-linearized form, as follows:

$$c_{i,t} = E_{i,t} \left[(1-\beta) \mu_b (w_{i,t} - w_t) - \sigma\beta \sum_{k=0}^{+\infty} \beta^k E_{i,t} [i_{t+k} - \pi_{t+k+1}] + (1-\beta) \sum_{k=0}^{+\infty} \beta^k E_{i,t} [y_{i,t+k} - g_{t+k}] \right], \tag{58}$$

where lowercase variables represent log-variables from the steady state ($G_t = 0$, $Y_t = C_t = Y^*$, and $T^* = (1-\beta) B^* > 0$) and $\mu_b = \frac{(1-\beta) b^*}{c^*}$.[44]

This is basically the same equation as (54) before, now adjusted for taxes and government debt. Aggregating and using Assumption 6, we arrive at (26). The rest of the proof follows from the argument in the main text.

Proof of Proposition 6

Given Assumption 5, an possible equilibrium takes the form of

$$c_t = \sum_{k=0}^{K_\eta} a_k \eta_{t-k} + \sum_{k=1}^{K_\beta} \beta_k c_{t-k} + \sum_{k=0}^{K_\theta} \gamma_k' x_{t-k} + \chi \zeta_t .$$

[44] The following exception applies: g_t represents G_t/Y^*. This is a standard trick in the literature on fiscal multipliers (e.g., Woodford, 2011) and it simply takes care of the issue that the log-deviation of the government spending is not well defined when its steady-state value is 0.

From (33), we have that

$$\sum_{k=0}^{K_\eta} a_k\eta_{t-k}+\sum_{k=1}^{K_\beta}\beta_k c_{t-k}+\sum_{k=0}^{K_\theta}\gamma'_k x_{t-k}+\chi\zeta_t=\theta_t+\zeta_t+\delta\mathbb{E}[\sum_{k=0}^{K_\eta-1}a_{k+1}\eta_{t-k}+\sum_{k=0}^{K_{\beta-1}}\beta_{k+1}c_{t-k}+\sum_{k=0}^{K_\theta-1}\gamma'_{k+1}x_{t-k}|I_t]$$
$$=q'x_t+\zeta_t+\delta\left[\sum_{k=0}^{K_\eta-1}a_{k+1}\eta_{t-k}+\sum_{k=1}^{K_{\beta-1}}\beta_{k+1}c_{t-k}+\sum_{k=0}^{K_\theta-1}\gamma'_{k+1}x_{t-k}+\gamma'_0Rx_t\right]$$
$$+\delta\beta_1\left[\sum_{k=0}^{K_\eta}a_k\eta_{t-k}+\sum_{k=1}^{K_\beta}\beta_k c_{t-k}+\sum_{k=0}^{K_\theta}\gamma'_k x_{t-k}+\chi\zeta_t\right]$$

where we use Assumptions 1–2 and the fact that ζ_t is drawn independently over time. For this to be true for all states of nature, we can compare coefficients:

$$a_k=\delta a_{k+1}+\delta\beta_1 a_k \quad \forall k\in\{0,\cdots,K_\eta-1\} \quad\text{and}\quad a_{K_\eta}=\delta\beta_1 a_{K_\eta} \tag{59}$$
$$\beta_k=\delta\beta_{k+1}+\delta\beta_1\beta_k \quad \forall k\in\{1,\cdots,K_\beta-1\} \quad\text{and}\quad \beta_{K_\beta}=\delta\beta_1\beta_{K_\beta} \tag{60}$$
$$\gamma'_k=\delta\gamma'_{k+1}+\delta\beta_1\gamma'_k \quad \forall k\in\{1,\cdots,K_\theta-1\} \quad\text{and}\quad \gamma'_{K_\theta}=\delta\beta_1\gamma'_{K_\theta} \tag{61}$$
$$\gamma'_0=q'+\delta\gamma'_1+\delta\beta_1\gamma'_0+\gamma'_0R \quad\text{and}\quad \chi=1+\delta\beta_1\chi. \tag{62}$$

First, from the second equation in (62), we know $\delta\beta_1\neq 1$. Then, from the second parts of (59)–(61), we know $a_{K_\eta}=0$, $\beta_{K_\beta}=0$, and $\gamma'_{K_\theta}=0$. From backward induction on (59)–(62), we know that all a,b,γ are zero except for the following:

$$\gamma'_0=q'+\gamma'_0R,$$

which means $\gamma'_0=q'(I-R)^{-1}$. We also know that $\chi=1$. We conclude that the unique solution is

$$c_t=c_t^F+\zeta_t,$$

where c_t^F is given by (10).

Appendix B: Additional Discussion

In this section, we illustrate the robustness of our main result to different policy rules for the monetary authority and observability of current outcomes. We also expand on the connection of our paper to two literatures: the one on discounted Euler conditions; and the one on Level-k Thinking.

Alternative monetary policies

In the main analysis, we specify the monetary policy (4) where the nominal interest rate responds to *current* inflation. In the literature (e.g. Bullard and Mitra, 2002), variants of such rules have been proposed. One may wonder whether the alternative specifications change our lessons on determinacy. The answer is no.

For example, one specification is that the the nominal interest rate responds to forecasts of future inflation:

$$i_t = z_t + \phi \bar{E}_t [\pi_{t+1}], \tag{63}$$

where $\phi \geq 0$. A system consisting of (2), (3), and (63) can be nested by the general environment (20), and the determinacy result in Proposition 4 directly applies.

Another specification is that the nominal interest rate responds to lagged values of inflation:

$$i_t = z_t + \phi \pi_{t-1}, \tag{64}$$

where $\phi \geq 0$. Even though this case is not directly nested in Proposition 4, the result about how frictions in intertemporal coordination results in determinacy remains to hold. Specifically, consider the systems consisting of (2), (3), and (64). Finally, shut down fundamentals shocks $\varrho_t = \xi_t = z_t = 0$, so the MSV solution is $c_t = 0$. Proposition 2 can be recast as the follows:

Proposition 7 (**Alternative monetary policies**)**.** *Suppose Assumption 4 holds, there are no shocks to fundamentals, and monetary policy takes the form of (64). The equilibrium is unique and is given by $c_t = 0$.*

Proof: From (2), (3), and (64), we have that any equilibrium must satisfy

$$c_t = \bar{E}_t \left[\tfrac{1}{1+\beta} c_t - \tfrac{\beta}{1+\beta} \sigma \phi \kappa c_{t-1} + \tfrac{\beta}{1+\beta} (1 + \sigma\kappa) c_{t+1} \right]; \tag{65}$$

and since there are no shocks to fundamentals, we search for solutions of the form $c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k}$. The goal is to verify that $a_k = 0$ for all k.

By Assumption 4, we have that, for all $k \geq 0$,

$$\bar{E}_t[\eta_{t-k}] = \mu_k \eta_{t-k}$$

where $\mu_k \equiv (1-\lambda)^k$ measures the fraction of the population at any given date that know, or remember, a sunspot realized k periods earlier. Future sunspots, on the other hand, are known to nobody. It follows that, along any candidate solution, average expectations satisfy

$$\bar{E}_t[c_t] = \sum_{k=0}^{+\infty} a_k \mu_k \eta_{t-k}$$

and similarly

$$\bar{E}_t[c_{t-1}] = \sum_{k=1}^{+\infty} a_{k-1}\mu_k \eta_{t-k}.$$

$$\bar{E}_t[c_{t+1}] = \sum_{k=0}^{+\infty} a_{k+1}\mu_k \eta_{t-k}$$

For condition (31) to be true for all sunspot realizations, it is necessary and sufficient that,

$$a_0 = (1+\sigma\kappa)\, a_1,$$

and, for $k \geq 1$,

$$a_k = \mu_k \left(\frac{1}{1+\beta} a_k - \tfrac{\beta}{1+\beta}\sigma\phi\kappa\, a_{k-1} + \tfrac{\beta}{1+\beta}(1+\sigma\kappa)\, a_{k+1} \right).$$

We hence have, for $k \geq 1$,

$$a_{k+1} = \frac{\frac{1}{\mu_k} - \frac{1}{1+\beta}}{\frac{\beta}{1+\beta}(1+\sigma\kappa)} a_k + \frac{\sigma\phi\kappa}{1+\sigma\kappa} a_{k-1}. \tag{66}$$

Since $\frac{1}{\mu_k} - \frac{1}{1+\beta} > 0$, we know that, all $\{a_k\}_{k=0}^{+\infty}$ have the same sign if $a_0 \neq 0$. But because $\mu_k \to 0$, we have that $|a_k|$ explodes to infinity as $k \to \infty$ from 66 unless $a_0 = 0$. But $a_0 = 0$ implies $a_k = 0$ for all k. We conclude that the unique bounded equilibrium is $a_k = 0$ for all k, which herein corresponds to the MSV solution.

Variant specification with perfect knowledge of current outcomes but no signal extraction

We now spell out a variant of our baseline model that allows perfect observation of the concurrent interest rate, income and prices, but abstracts from any signal-extraction thereof.

Revisit the consumption function (1) and let consumers know their income and interest rates. Then, aggregate consumption can be expressed as

$$c_t = \tfrac{1}{1+\beta} y_t - \tfrac{\beta}{1+\beta}\sigma(i_t - \varrho_t) + \bar{E}_t \left[\tfrac{\beta}{1+\beta} y_{t+1} + \tfrac{\beta}{1+\beta}\sigma\pi_{t+1} \right].$$

Combining this with market clearing ($y_t = c_t$ and $y_{t+1} = c_{t+1}$), and solving out for c_t we get

$$c_t = -\sigma\left(i_t - \bar{E}_t[\pi_{t+1}] - \varrho_t\right) + \bar{E}_t\left[c_{t+1}\right].$$

That is, the DIS curve is now the same as in the representative-agent benchmark, modulo the replacement of that agents' full-information expectation with the average, incomplete-information expectation in the population. By the same token, once we substitute out the interest rate and inflation, our game representation becomes

$$c_t = \theta_t + \delta \bar{E}_t[c_{t+1}].$$

The argument presented in the main text then goes through, and Proposition 4 continues to hold, provided that the consumers' average expectations of the past sunspots satisfy the restriction $\bar{E}_t[\eta_{t-k}] = \mu_k \eta_{t-k}$ with $\lim_{k\to\infty} \mu_k = 0$ (and similarly for the past fundamentals).

Strictly speaking, this variant specification—which is actually the one adopted in an earlier draft—combines our assumption of "fading social memory" with a mild form of bounded rationality: agents are not allowed to extract information about the economy's payoff-irrelevant history from their observation of current outcomes. Such signal extraction *is* allowed in our second perturbation, that developed in Section 7.

Discounted Euler equations

Suppose we replace our IS equation (2) with the following variant:

$$c_t = -m_i i_t + m_\pi \bar{E}_t[\pi_{t+1}] + m_c \bar{E}_t[c_{t+1}] + \varrho_t, \tag{67}$$

for some positive scalars m_i, m_π, m_c. When $m_c < 1$, this nests the "discounted" Euler equations generated by liquidity constraints in McKay et al. (2017) and by cognitive discounting in Gabaix (2020). The opposite case, $m_c > 1$, is consistent with the broader HANK literature (Werning, 2015; Bilbiie, 2020), as well as with over-extrapolation or "cognitive hyperopia". Finally, $m_i \neq m_\pi$ could capture differential attention to (or salience of) nominal interest rates and inflation.

With these modifications, the entire analysis goes through modulo the following adjustment in the definition of δ :

$$\delta = \frac{m_\pi \sigma\kappa + m_c}{1 + m_i \sigma\phi\kappa}$$

The Taylor principle is still the same in the δ space, but of course changes in the ϕ space: we now have that $|\delta| < 1$ if and only if $\phi \in (-\infty, \underline{\phi}) \cup (\overline{\phi}, +\infty)$, where

$$\underline{\phi} \equiv -\frac{m_\pi}{m_i} - \frac{1+m_c}{\sigma\kappa m_i} \qquad \text{and} \qquad \overline{\phi} \equiv \frac{m_\pi}{m_i} + \frac{m_c - 1}{\sigma\kappa m_i}$$

Depending on the m's, these thresholds can be either smaller or larger than the ones in the main analysis. In this sense, the model's region of indeterminacy may either shrink or expand by the above modifications. For instance, Gabaix (2020) assumes $m_i = m_\pi$ and $m_c < 1$, obtains $\overline{\phi} < 1$, and uses this to argue that cognitive discounting relaxes the Taylor principle and, thereby, eases the potential conflict between the stabilization and equilibrium selection functions of monetary policy. From this perspective, that paper and ours are complements. But none of these enrichments changes the fact that indeterminacy remains for sufficiently "passive" monetary policy, and this is where our approach offers a potential way out.

Alternative Solution Concepts

Throughout, we have preserved Rational Expectations Equilibrium (REE), relaxing only the assumption of perfect information about the past. REE is defined by the requirement that the agents' subjective model of the economy *exactly* coincides with the true model generated by their behavior. One can capture bounded rationality by allowing a discrepancy between the former and the latter. But as long as one allows for a two-way feedback between them, the kind of indeterminacy we have studied here remains possible, and so does our resolution to it.

This circles back our earlier discussion of Gabaix (2020): the solution concept in that paper allows the objective model to feed into the subjective model, albeit with a distortion relative to REE. The same is true for Diagnostic Expectations (Bordalo et al., 2018); for Perfect Bayesian Equilibrium with mis-specified priors (Angeletos and Sastry, 2021); and for Woodford (2019)'s model of "finite planning horizons," at least once learning is allowed (Xie, 2019). All these concepts are close cousins of REE in the sense that they preserve the two-way feedback between beliefs and outcomes, thus also preserving the indeterminacy problem we have addressed in this paper.

Contrast this class of concepts with Level-K Thinking (García-Schmidt and Woodford, 2019; Farhi and Werning, 2019). The latter pins down a unique solution by shutting down the feedback from objective truth to subjective beliefs. But this begs the question of how agents adjust their behavior over time, in the light of repeated, systematic discrepancies between what they expect to happen and what actually happens. Accordingly, we believe that Level-K Thinking is more appropriate for unprecedented experiences (e.g., the recent ZLB experience) than for the kind of stationary environments we are concerned with in this paper.

Furthermore, one may argue that Level-K Thinking does not "really" resolve the indeterminacy problem and, instead, only translates it to a different dimension: whenever $|\delta| > 1$, the level-k outcome becomes *infinitely* sensitive to the arbitrary level-0 outcome as $k \to \infty$. To see this, consider what Level-K Thinking means in our setting. First, level-0 behavior is exogenously specified, by a random process $\{c_t^0\}$. Level-1 behavior is then defined as the best response to the belief that others play according to level-0 behavior, that is, $c_t^1 \equiv \theta_t + \delta \mathbb{E}_t[c_{t+1}^0]$,where $\mathbb{E}_t$ is the full-information expectation operator. This amounts to using the "wrong" beliefs about what other players do but the "correct" beliefs about the random variables θ_t and c_{t+1}^0. Iterating K times, for any finite K, gives the level-K outcome as $c_t^K \equiv \sum_{k=0}^K \delta^k \mathbb{E}_t[\theta_{t+k}] + \delta^K \mathbb{E}_t[c_{t+K}^0]$. The solution concept says that actual behavior is given by $c_t = c_t^K$ for all periods and states of nature, where both K and $\{c_t^0\}$ are free variables for the modeler to choose. Clearly, $\{c_t^K\}$ is uniquely determined for any given K and any given $\{c_t^0\}$. But because $\{c_t^0\}$ is a free variable, the original indeterminacy issue is effectively transformed to the modeler's (or the reader's) uncertainty about $\{c_t^0\}$. Furthermore, the bite of this uncertainty is most severe precisely when the indeterminacy issue is present: whenever $|\delta| > 1$, the sensitivity of $\{c_t^K\}$ to $\{c_t^0\}$ explodes to infinity as $K \to \infty$.

This explains the sense in which Level-K Thinking replaces one free variable in beliefs (the sunspot) with another free variable (the analyst's specification of the level-0 behavior). By contrast, our approach leaves neither kind of freedom in specifying beliefs.

This is not to say that our approach is "better." One may question the realism of both our main informational assumption and our approach's heavy reliance on REE. Furthermore, the two approaches are ultimately complementary in two regards: highlighting the role of higher-order beliefs; and solidifying the logical foundations of the MSV solution. Thus, while the above discussion clarifies the differences in the two approaches, perhaps their common ground is what matters the most for applied purposes.

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