Controlling Inflation with Central Bank News[*]

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Abstract

What determines the stochastic path of inflation? We study this question in a monetary economy featuring imperfect information and rational expectations. The central bank targets inflation and releases noisy information about future non-monetary fundamentals through its non-systematic component. We show that real interest rate increases following such information releases lead the private sector to revise upward its expectations of future output – an outcome arising from "Fed information effects". Through this channel, central bank communication influences market expectations about the economic outlook, adding further constraints to the equilibrium and ultimately determining the stochastic path of inflation. We propose a novel role for Fed information effects: their ability to serve as a mechanism for equilibrium selection.

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1 Introduction

The ability of monetary policy to control the path of inflation is both theoretically and practically important. Failure to control inflation can cause suboptimal fluctuations. Sargent and Wallace (1975) discussed the indeterminacy of inflation and output under interest rate policy. The New Keynesian framework achieves local determinacy of equilibrium by assuming that monetary policy satisfies the Taylor principle (Taylor, 1993). Alternative selection arguments based on the fiscal theory of the price level emphasise the role of debt and deficits as key drivers of inflation and output (Cochrane, 2018).

This paper studies the role of central bank announcements about non-monetary fundamentals in controlling the stochastic path of inflation. Monetary policy targets inflation and also conveys information about future non-monetary fundamentals through deviations from its systematic component. This builds on Nakamura and Steinsson (2018), who show that monetary policy shocks convey information about underlying aggregate fundamentals, thereby shaping market expectations of economic conditions (termed "Fed information effects" by Romer and Romer, 2000). We highlight a new role for Fed information effects: their capacity to control inflation and act as a mechanism for equilibrium selection.

We develop a model with informational frictions and rational expectations, featuring a representative consumer/worker and producer. Productivity consists of a permanent and temporary component. Consumers learn current productivity but take time to recognise permanent changes in aggregate fundamentals. The central bank targets inflation and communicates noisy signals about the next period's temporary component of productivity through policy surprises. Producers are separated from consumers and do not learn productivity during production decisions, leading to sluggish adjustment in aggregate variables. Inspired by Phelps (1970) and Lucas (1972), our model yields a Lucas-type Philips curve where fluctuations are generated from the producer's expectational errors in prices.[1]

The central bank transmits information about next period's temporary productivity through current policy shocks. To respect producers' informational constraints, policy is set only after production, once current productivity is commonly observed. Agents then infer the central bank's private signal from deviations relative to the inflation targeting rule.

Unlike Lorenzoni (2009) and Melosi (2016), who introduce such deviations to prevent agents (including the central bank) from inferring current fundamentals from prices, we repurpose them to

[1] See also Mankiw and Reis (2002), Reis (2006), Lorenzoni (2010), among others, for alternative models where lags in informational adjustment give rise to nominal rigidities.

convey information about future non-monetary fundamentals, in line with the findings of Nakamura and Steinsson (2018).

More broadly, the paper shows that when *unexpected* policy changes convey information about future fundamentals, they act as an equilibrium selection device. To isolate this effect, we assume the central bank observes prices when setting policy, so deviations from inflation targeting transmit the central bank's private forecasts rather than being used to support informational asymmetries. Imperfect information equilibria are supported by conditioning production on past policy actions, thereby respecting agents' informational constraints.

The baseline scenario of inflation targeting assumes that no information transmission occurs in equilibrium. The resulting multiplicity reflects the classic result that inflation (and the initial price level) are indeterminate under an interest rate rule (Sargent and Wallace, 1975). Economic dynamics are naturally shaped by agents' forecast errors regarding productivity given gradual learning of permanent activity. In the minimum state variable (MSV) solution, monetary policy anchors the sensitivity of equilibrium dynamics to these forecast errors. In contrast, non-MSV solutions permit inflation dynamics to depend also on past inflation. Monetary policy controls the response to past inflation, leaving the sensitivity to forecast errors unanchored. Sunspot shocks have purely nominal effects in such solutions.

The existence of an MSV-type equilibrium is not robust to the introduction of policy shocks. Such an equilibrium requires that both prices and producer expectations of prices respond to the central bank's current signals, implying simultaneity between policy actions and production decisions. This simultaneity violates the key consistency condition: the central bank must not possess an informational advantage over producers regarding prices. The information effects mechanism—consistent with the findings and modelling approach in Nakamura and Steinsson (2018)—relies on the central bank's private forecasts being transmitted through *unexpected* policy actions. But in our setting, prices cannot be indexed to current unexpected policy shocks without violating informational constraints. As a result, the interaction between information frictions and information effects unravels the existence of the MSV-type equilibrium.

Equilibria where prices are measurable to past shocks (non-MSV-type) avoid the earlier problem. Policy announcements influence the real interest rate, shaping output and inflation dynamics by imposing the restrictions necessary for uniqueness. In particular, policy surprises that raise the real rate lead consumers to revise expectations of future output upward. This introduces *additional constraints* in equilibrium and anchors the sensitivity of dynamics to consumer forecast errors and determines the stochastic paths of inflation and output.[2] Unlike the standard New Keynesian model—where monetary tightening reduces expected output—this mechanism aligns with findings by Nakamura and Steinsson (2018), who show that unexpected real rate hikes increase survey-based expectations of output growth. A natural interpretation is that central bank announcements affect beliefs about both future policy and fundamentals (the "Fed information effects," per Romer and Romer, 2000).[3]

Our results are robust across a range of counterfactual scenarios. The central mechanism is that information effects operate through unanticipated policy actions. When we abstract for unanticipated actions—either by reverting to pure inflation targeting, as in the baseline, or by allowing the central bank to issue binding commitments about *future* policy deviations that carry no informational content about non-monetary fundamentals—the MSV-type equilibrium exists and multiplicity is restored. Alternatively, we can mute information effects by allowing for policy surprises that convey no information about fundamentals—the standard assumption in the literature—in which case multiplicity also re-emerges.

Additionally, we leverage the model's timing structure and the information effects mechanism to distinguish between *Central Bank* signals (private information held by the central bank) and *non-Central Bank* signals (information released by other institutions). Abstracting from Central Bank news, the MSV-type equilibrium exists under non-Central Bank signals and multiplicity re-emerges.

1.1 Related Literature

Our paper contributes to the vast literature on indeterminacy of monetary equilibria, including works by Sargent and Wallace (1975), Leeper (1991), Sims (1994), Woodford (1994), Clarida et al. (2000), Nakajima and Polemarchakis (2005), Cochrane (2011), Magill and Quinzii (2014), Adao et al. (2014), Castillo-Martinez and Reis (2019), among others. We identify a novel channel for equilibrium determinacy, highlighting a new role for "Fed information effects" in controlling inflation and output. Indeterminacy does not derive from stability of a deterministic steady state, rendering the Taylor principle irrelevant.[4] Moreover, fiscal policy is Ricardian and does not impose non-trivial

[2]Recent empirical literature also demonstrates the power of central bank communication to shape private-sector inflation expectations (Medina et al., 2024).

[3]Hansen et al. (2019) show that news about economic uncertainty significantly affects the yield curve—a channel we abstract from in this paper. Instead, we focus on news that shifts market expectations of levels, and show that this mechanism can support a unique equilibrium.

[4]Eusepi and Preston (2010) characterise communication strategies that restore the Taylor principle when agents have an incomplete model of the macroeconomy.

equilibrium restrictions.[5]

A strand of literature links determinacy to incomplete information and strategic interactions. Angeletos and Lian (2023), building on the results of Morris and Shin (1998), show that frictions in social memory restore uniqueness in a New Keynesian setting framed as a coordination game. While our analysis abstracts from coordination issues central to the global games literature,[6] it underscores the relevance of studying determinacy in settings with information frictions, showing how their interaction with policy announcements unravels certain equilibria.

The central bank's possession of private information is central to the cheap talk framework in Bassetto (2019), which identifies conditions under which such information can be credibly communicated to improve outcomes. In contrast, we abstract from cheap talk conflict and take as given that central bank announcements do transmit information about fundamentals. We then show that, within a fully specified general equilibrium environment, the presence of information effects leads to a unique equilibrium outcome.

Our analysis is connected to the news-driven business cycle literature (see Beaudry and Portier, 2014, for a review), as it examines how news about future productivity shape economic decisions. We offer a framework that distinguishes between Central Bank and non-Central Bank news and show that the presence of Central Bank news—unlike non-Central Bank news—is crucial for controlling the paths of inflation and output.

Related literature, see Lorenzoni (2009, 2010), Forni et al. (2017), explores the role of noise shocks in fluctuations. Our focus, however, is on how central bank information about future fundamentals can determine the stochastic path of inflation and output. Policy announcements serve as forecasts of the future economic outlook, conveyed through noisy signals about future temporary productivity, allowing agents to learn permanent aggregate activity gradually.

Campbell et al. (2019) examine the limits of forward guidance under imperfect central bank communication, modelling it as noisy signals about future deviations from systematic policy. In contrast, following Nakamura and Steinsson (2018), we interpret deviations from current systematic policy as conveying private forecasts about future temporary productivity. The precision of the central bank's forecast (controlled by the variance of noise in signals) does not influence our key result about equilibrium selection.[7]

[5]See Woodford (1994, 1996) and Benhabib et al. (2001, 2002) for examples of non-Ricardian fiscal policies.

[6]Notable contributions include Morris and Shin (2002) and Angeletos and Pavan (2007).

[7]Vereda et al. (2024) develop a theory-based approach to assess how central banks' inflation forecasts shape private inflation expectations. In their framework, the central bank receives noisy signals about the rational expectation of inflation and forms its own forecast. By contrast, we assume the central bank releases noisy information about

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 characterises the baseline case of pure inflation targeting. Section 4 provides intuition behind multiplicity. Section 5 shows how central bank information effects ensure uniqueness. Finally, Section 6 concludes.

2 Model

2.1 Set-up

Time is discrete and infinite with each period denoted by $t = 0, 1, ...$ The economy is populated by a representative consumer/worker, a representative firm and the central bank. The household supplies labor to the firm, trades one-period nominal bonds in zero net supply and consumes a single non-storable good. Consumer preferences are represented by the utility function

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{N_t^{1+\zeta}}{1+\zeta}\right)\right], \tag{1}$$

where C_t denotes consumption and N_t denotes labor supply at period t. The parameter $\gamma > 0$ is the coefficient of relative risk aversion, $\zeta > 0$ is the inverse of Frisch elasticity of labor supply and $\beta \in (0, 1)$ is the discount factor. The consumer faces a sequence of budget constraints:

$$P_tC_t + Q_tB_{t+1} = B_t + W_tN_t + \Psi_t,\ t = 0, 1, ... \tag{2}$$

where P_t denotes commodity prices, B_{t+1} denotes holdings of nominal bonds purchased at period t and maturing at $t + 1$, Q_t denotes the nominal bond price, W_t denotes the nominal wage and Ψ_t denotes the firm's nominal profits. The firm's technology and profits are given respectively by:

$$Y_t = A_tN_t \tag{3}$$

$$\Psi_t = P_tY_t - W_tN_t, \tag{4}$$

where Y_t is aggregate output and A_t denotes productivity. The Central bank targets current inflation and sets the nominal bond price according to the following rule:

$$Q_t = \ \Pi_t^{-\chi} \cdot e^{-\eta_t}, \tag{5}$$

primitives via monetary policy shocks, however, we show that the precision of information conveyed from these signals is not central to our main finding.

where $\chi > 0$ and $\Pi_t \equiv P_t/P_{t-1}$ is the rate of inflation between $t-1$ and t. The feedback rule (5) is a commonly used Taylor rule, augmented with monetary policy shocks η_t, where the nominal interest rate, which is the inverse of the bond price Q, increases with the inflation rate. Incorporating this rule allows us to compare our results with the broader monetary economics literature. However, our results are robust to alternative interest rate rules, including those targeting output and inflation, both expected and current inflation, as well as interest rate pegs.

2.2 Shocks and Signals

Let $a_t = \log(A_t)$ and define similarly any lowercase variable henceforth. Aggregate productivity consists of a permanent component, x_t, and a temporary component, ϵ_t:

$$a_t = \log(A_t) = x_t + \epsilon_t \quad \text{with} \quad \epsilon_t \sim N(0, \sigma_\epsilon^2).$$

The household observes a_t but not its decomposition. The permanent component of aggregate productivity follows an AR(1) process:

$$x_t = \rho x_{t-1} + e_t \quad \text{with} \quad e_t \sim N(0, \sigma_e^2), \quad \rho \in (0, 1].$$

At each date t, the central bank receives noisy signals about the temporary component of productivity next period:

$$s_t = \epsilon_{t+1} + u_t \quad \text{with} \quad u_t \sim N(0, \sigma_u^2).$$

Consistent with Nakamura and Steinsson (2018), we assume that monetary policy shocks convey information about non-monetary fundamentals. In their framework, current policy shocks influence consumers' beliefs about the future path of the natural rate of interest. A key parameter governs the sensitivity of these beliefs to changes in the policy shock, determining the extent to which monetary announcements have information effects rather than traditional effects. In our setting, the simplest way to model this is by defining policy shocks as

$$\eta_t = s_t,$$

where policy surprises provide noisy signals about the temporary component of productivity in the next period. The sensitivity of beliefs to these shocks is governed by the precision of the signal s_t, as discussed below. The results section examines scenarios in which policy shocks are disentangled from signals.

All shocks are mutually independent and serially uncorrelated. Agents know the distribution of shocks.

2.3 Timing and information

Each period is divided in two stages: labor decisions are made in stage 1, while consumption and savings decisions are made in stage 2. Payments materialise in stage 2 and are perfectly enforceable. All actions that are taken in any given stage are simultaneous.

In stage 1, the consumer/worker learns a_t, decides on her labour supply and production takes place. We assume that the producer and the consumer are physically separated at the time production takes place, and the representative producer, taking prices as given, maximise expected profits (as we discuss in detail below). Production decisions are made with incomplete information about current productivity and stage 2 outcomes (this is a common assumption in the literature, see, for example, Angeletos and La'O, 2013). We demonstrate that this behaviour leads to aggregate fluctuations driven by producer's expectational error in prices, as described by the Lucas-type Phillips curve discussed in Section 3.[8]

In stage 2, the Central Bank sets the nominal interest rate (which is the inverse of the bond price Q) according to the targeting rule (5) and effectively transmits noisy signal s_t (about ϵ_{t+1}) through policy surprises, and commodity and bonds markets open. The consumer decides on her consumption/saving decisions at the given prices. With production pre-determined from stage 1, the good's price adjust to clear the good's market and the bonds market clears residually. All agents learn a_t, but the permanent component, x_t, remains unobserved.

Let the information of agent i, with $i \in \{p, c\}$ denoting respectively the producer and consumer, be denoted by $I^i_{t,s}$ with $s \in \{1, 2\}$ denoting the stage within a given period t. Our baseline case examines equilibria under pure inflation targeting, with policy shocks set to zero. In this case, agents' information sets are:

$$I^c_t = \{a_0, a_1, ..., a_{t-1}, a_t\}\,, \; I^p_{t,1} = \{a_0, a_1, ..., a_{t-1}\}\,, \; I^c_t = I^p_{t,2}. \tag{6}$$

[8] The producer can not infer a_t upon observing the nominal wage due to linearity in production that requires the equilibrium wage to be independent of the amount of labor. Alternatively, we could assume decreasing returns in production and introduce idiosyncratic shocks to labour supply to get a similar result.

When the central bank conveys information through policy surprises, the information sets modify:

$$\begin{gathered} I^c_{t,1} = \{a_0, a_1, .., a_{t-1}, a_t, s_0, .., s_{t-1}\}\,,\ I^p_{t,1} = \{a_0, a_1, .., a_{t-1}, s_0, .., s_{t-1}\}\,, \\ I^c_{t,2} = I^p_{t,2} = \{a_0, a_1, .., a_{t-1}, a_t, s_0, .., s_{t-1}, s_t\}\,. \end{gathered} \tag{7}$$

The timing of policy actions is important. Specifically, when the Central Bank sets the policy rate in period t, it must not possess any informational advantage over producers regarding prices and underlying fundamentals. To that end, policy actions occur at $(t, 2)$, when productivity is commonly observed. Since producers base decisions on past policy actions, and the public signal s_{t-1} does not reveal a_t, public information ensures equilibrium uniqueness without removing the informational frictions producers face.

This timing structure and the information effects hypothesis will allow us also to identify the difference between Central Bank signals—information privately held by the central bank—and non-Central Bank signals—public information released by other institutions—as we discuss in the results section.

With slight abuse of notation we denote the expectation of agent $i \in \{p, c\}$ at date t and stage s, conditional on their information set $I^i_{t,s}$, with $E^i_{t,s}[.] \equiv E^i_{t,s}[.|I^i_{t,s}]$. We focus on linear equilibria where the stochastic paths of inflation and output depend on agents' expectation about productivity, a, and its permanent component, x. In turn, agents use past and current realisations of observables to update their information about x recursively through the use of the Kalman filter. Specifically, the consumer's estimate about x_t is given by

$$E^c_{t,1}[x_t] = E^c_{t,2}[x_t] = E^c_{t-1,2}[x_t] + \mu \left(a_t - a_{t|t-1}\right),$$

where μ (Kalman gain) is a constant that depends on the variance parameters and $a - a_{t|t-1}$ is the gain in information upon observing a_t, with prior mean given by $a_{t|t-1} \equiv E^c_{t-1,2}[a_t] = E^c_{t-1,2}[x_t] + \psi s_{t-1}$, where ψ depends on variance parameters as well.[9] This parameter captures the precision of public signals and governs the extent to which monetary announcements generate information effects as opposed to traditional policy effects. Notice that there is no informational gain between stages in period t relevant to estimating x_t, so $E^c_{t,1}[x_t] = E^c_{t,2}[x_t]$. The producer does not observe the current realisation of a at $(t, 1)$, and as a consequence, their estimate about x_t is given by

$$E^p_{t,1}[x_t] = E^c_{t-1,2}[x_t],$$

[9] See Appendix A for detailed derivations.

which coincides with the consumer's estimate of x_t at the end of $t-1$. Furthermore, upon observing public signals at each date, the consumer and the producer update beliefs about aggregate productivity, a, as follows:

$$
\begin{aligned}
E^c_{t,2}[a_{t+1}] &= \rho E^c_{t,2}[x_t] + \psi s_t, && (8)\\
E^p_{t,1}[a_t] &= E^p_{t,1}[x_t] + \psi s_{t-1}. && (9)
\end{aligned}
$$

2.4 Equilibrium and Optimality Conditions

A rational expectations equilibrium under the interest rate rule (5) is given by a collection of prices, $\{P_t, W_t, Q_t\}_{t=0}^{\infty}$, and allocations, $\{C_t, B_t, N_t, Y_t\}_{t=0}^{\infty}$, such that agents' decisions are optimal, at the stated prices, and markets clear, $Y_t = C_t$, $B_t = 0$, $\forall t$, with initial condition $B_0 = 0$.

In the rest of this section, we describe in detail each agents' optimal decision problem. The consumer maximises expected, discounted utility (1), subject to the sequence of flow budget constraints (2), and the usual natural debt limit on borrowing. Consider first the consumption decisions of the household during stage 2 of period t. Let $\beta^t \lambda_t$ denote the Lagrange multiplier on its flow budget. Optimal consumption choices satisfy

$$
\begin{aligned}
\frac{C_t^{-\gamma}}{P_t} &= \lambda_t && (10)\\
Q_t &= \beta E^c_{t,2}\left[\frac{1}{\Pi_{t+1}}\left(\frac{C_t}{C_{t+1}}\right)^{\gamma}\right]. && (11)
\end{aligned}
$$

Equation (10) equates the marginal utility of consumption per dollar to the Lagrange multiplier. Equation (11) is the standard Euler equation, equating the marginal cost of one additional unit of (real) savings, $Q_t(C_t^{-\gamma}/P_t)$, to its discounted expected benefit, $\beta E^c_{t,2}\left[C_{t+1}^{-\gamma}/P_{t+1}\right]$. The expectation is based on the information set $I^c_{t,2}$, which may include signals s_t about the temporary component of a_{t+1}.

The consumer's labor-supply decision in stage 1 of period t (anticipating correctly the equilibrium in stage 2) is given by the following first-order condition:

$$
N_t^{\zeta} = \frac{W_t}{P_t} C_t^{-\gamma}, \tag{12}
$$

which equates the real wage in terms of consumption units to the marginal disutility of labor

In stage 1 of period t, the producer observes the nominal wage rate W_t and makes production decisions with incomplete information about stage 2 outcomes. A possible justification for the information asymmetry is that the consumer owns the technology but hires a manager (the producer) to oversee production decisions. The manager is uncertain about certain aspects of the technology, specifically the owner's productivity, and must operate the firm as the consumer/owner would want. The producer maximises expected profits, $E^p_{t,1}[\lambda_t \Psi_t]$, evaluated using the consumer's/owner's marginal utility of wealth, $\lambda_t = C_t^{-\gamma}/P_t$, as the appropriate discount rate.[10] The optimality condition reduces to

$$W_t = E^p_{t,1}\left[\tilde{\lambda}_t P_t A_t\right], \tag{13}$$

with $\tilde{\lambda}_t \equiv \lambda_t / E^p_{t,1}[\lambda_t]$. Due to linearity in technology, the firm accommodates any labour supplied at the given wage as long as expected profits are zero (realised profits are typically not zero since the real wage is not equal to productivity).

Our results can also be explained by nominal wage stickiness due to information frictions. Nominal wages are set based on the producer's expectations in stage $(t, 1)$, meaning wages are predetermined at the beginning of each period using past information. Real indeterminacy is attributed to nominal rigidities caused by information frictions, thus placing our findings within the broader context of discussions on nominal rigidities and monetary policy.

3 Inflation targeting

Our baseline scenario characterises equilibria under inflation targeting, with policy shocks set to zero ($\eta_t = 0\ \forall t$). The assumption of separable, isoelastic preferences and Gaussian shocks allows us to derive linear rational expectations equilibria in closed form. We conjecture linear rules for the paths of inflation and output as function of shocks, and then verify that they are indeed rational expectations equilibria. Importantly, we show that in a linear equilibrium the agents optimality conditions can be written in a linear form (see Lorenzoni, 2010, for details on the computation of linear rational expectations equilibria).

Equilibrium is characterised by two equilibrium blocks, the Fisher equation (FE), which represents the Euler equation in linear form, and a Lucas-type Philips curve relationship (PC), arising due to informational frictions. We fix $E^c_{t,2}[\cdot] \equiv E^c_t[\cdot]$ and $E^p_{t,1}[\cdot] \equiv E^p_t[\cdot]$ for the remainder of the analysis.

[10]Notice that the consumer/owner Lagrange multiplier can reveal perfectly a_t. However, as we discussed above, we assume that the consumer and the producer are separated at the time production decisions take place, which allow us to abstract from the "Lucas-Phelps" islands framework and consider only one "island" in its place. Furthermore, by maximising the firm's profits evaluated with the consumer's Lagrange multiplier, the producer operates the firm in the way the consumer/owner would want her to (Magill and Quinzii, 1996).

There exist two distinct classes of stationary linear equilibria: the minimum state variable (MSV) solution, where inflation is determined solely by the current realisation of shocks, and non-MSV solutions of the stochastic Fisher equation, where current inflation depends on past shock realisations and arbitrary mean-zero shocks.

The class of non-MSV linear rational expectations equilibria is as follows:

$$\pi_t \;=\; \xi_0 \,+\, \xi_1\pi_{t-1} \,+\, \xi_2 a_{t-1} \,+\, \xi_3 E^p_{t-1}[a_{t-1}] \,+\, \xi_4 a_t \,+\, \xi_5 E^c_t[x_t],\;\; t = 0, 1, ... \tag{14}$$

$$y_t \;=\; \theta_0 \,+\, \theta_1 a_t \,+\, \theta_2 E^p_t[a_t],\;\; t = 0, 1, ..., \tag{15}$$

where $\xi \equiv \{\xi_0, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5\}$ and $\theta \equiv \{\theta_0, \theta_1, \theta_2\}$ are vectors of coefficients to be determined. We verify that (14) and (15) satisfy the Fisher equation and the Phillips curve relationship.

The path of inflation depends on past inflation (π_{t-1}), past and current aggregate activity (a_{t-1}, a_t), producers' expectations about past productivity ($E^p_{t-1}[a_{t-1}]$), and consumer's expectations about the permanent component of productivity ($E^c_t[x_t]$), which provides the best estimate about future aggregate activity. We revisit (14) and (15) after presenting the model's equilibrium blocks in linear form.

In a linear equilibrium, we show that combining the interest rate rule (5) and agents' Euler equation (11), together with market clearing, yields

$$E^c_t[\pi_{t+1}] \;=\; \chi\pi_t \;-\; \gamma\,(E^c_t[y_{t+1}] \;-\; y_t) \;+\kappa_c, \tag{FE}$$

where κ_c is a known quadratic function of θ and ξ (see Appendix B). Equilibrium condition (FE) represents the stochastic Fisher equation.

Next, combining intratemporal optimality (12), market clearing, technology, and the producer's optimality condition (13), yields

$$(\gamma + \zeta)y_t \;=\; E^p_t[a_t] + E^p_t[\pi_t] - \pi_t + \zeta a_t \;+\kappa_y, \tag{PC}$$

which represents the Lucas-type Philips curve relationship in the economy, and shows that aggregate fluctuations are driven by the producer's expectational error in prices, $E^p_t[\pi_t] - \pi_t$. The term κ_y is a known quadratic functions of coefficients θ and ξ.

Output dynamics depend on current productivity, a_t and the producer's estimate of it, $E^p_t[a_t]$, as both influence the (PC) block. As we show below, the producer's price expectation error, $E^p_t[\pi_t] - \pi_t$, is independent of the consumer's estimate of the current long-run component, $E^c_t[x_t]$, and since the producer observes past productivity and prices, it is also independent of π_{t-1}. The (FE) block links inflation and output, requiring the producer's productivity estimates in inflation dynamics for equilibrium.

The following proposition formally characterises non-MSV equilibria without communication.

Proposition 1 *(Non-MSV solutions) A continuum of non-MSV equilibria exists, indexed by* ξ_5*:*

$$\begin{aligned}\pi_t = \kappa_c + \chi\pi_{t-1} + \tfrac{\gamma}{\gamma+\zeta}\left(\xi_5 + \tfrac{\gamma(1+\zeta)+\zeta\left[\frac{\gamma-\mu}{1-\mu}+\zeta\right]}{\gamma+\zeta}\right)a_{t-1} + \tfrac{\gamma\left(\frac{\zeta(1-\gamma)}{(\gamma+\zeta)(1-\mu)}-\xi_5-\rho(1+\zeta)\right)}{\rho(\gamma+\zeta)}E^c_{t-1}[x_t] \\ + \underbrace{\left(\frac{\gamma(1+\zeta)}{\gamma+\zeta}+\xi_5(1-\mu)\right)\left(E^c_{t-1}[x_t]-a_t\right)}_{\textit{unanchored forecast error}},\end{aligned} \tag{16}$$

$$(\gamma+\zeta)y_t = \kappa_y + \zeta a_t + E^c_{t-1}[x_t] - \underbrace{\left(\frac{\gamma(1+\zeta)}{\gamma+\zeta}+\xi_5(1-\mu)\right)\left(E^c_{t-1}[x_t]-a_t\right)}_{\textit{unanchored forecast error}}. \tag{17}$$

Proof. See Appendix B.1 for detailed computations of non-MSV linear rational expectations equilibria. ■

It is essential to outline the proof of Proposition 1 (and Proposition 2 below), as it plays a key role in the argument presented in Section 5, where we examine the impact of public information. The equilibrium system provides one fewer independent restriction than the number of coefficients. Considering the (FE) equilibrium block and the conjectured dynamics (14) and (15), the one-step-ahead forecasts of inflation and output are:

$$E^c_t[\pi_{t+1}] = \xi_0 + \xi_1\pi_t + \xi_2 a_t + \xi_3 E^p_t[a_t] + (\xi_4+\xi_5)\rho E^c_t[x_t] \tag{18}$$

$$E^c_t[y_{t+1}] = \theta_0 + (\theta_1+\theta_2)\,\rho E^c_t[x_t]. \tag{19}$$

The key property for deriving these forecasts is $E^c_t[a_{t+1}] = E^c_t[x_{t+1}] = \rho E^c_t[x_t]$.

Substituting these forecasts into (FE) block, setting $\xi_1 = \chi$ for stationarity (so that π_t cancels out), and matching coefficients term by term, we obtain:

$$\text{constants}: \xi_0 = \kappa_c, \tag{20}$$

$$a_t : \xi_2 = \gamma\theta_1, \tag{21}$$

$$E^p_t[a_t] : \xi_3 = \gamma\theta_2, \tag{22}$$

$$E_t^c[x_t] : \xi_4 + \xi_5 = -\gamma\left(\theta_1 + \theta_2\right). \tag{23}$$

We obtain four equations in five unknowns $\{\xi_0, \xi_2, \xi_3, \xi_4, \xi_5\}$, implying that the Fisher equation block determines only $\xi_4 + \xi_5$, not each coefficient separately.

Next, by substituting the conjectured inflation dynamics (14) into the producer's expectational price error $E_t^p[\pi_t] - \pi_t$, and using the Kalman filter to express $E_t^p\left[E_t^c[x_t]\right] = E_t^c[x_t] + \mu\left[E_t^p[a_t] - a_t\right]$, we obtain

$$E_t^p[\pi_t] - \pi_t \;=\; \left(\xi_4 + \mu\xi_5\right)\left(E_t^p[a_t] - a_t\right). \tag{24}$$

Substituting the conjectured output dynamics (equation 15) and the expectational error (equation 24) into the (PC) block, and matching coefficients term by term, we obtain

$$a_t : (\gamma + \zeta)\theta_1 \;=\; \zeta \;-\; \left(\xi_4 + \mu\xi_5\right), \tag{25}$$

$$E_t^p[a_t] : (\gamma + \zeta)\theta_2 \;=\; 1 \;+\; \xi_4 \;+\; \mu\xi_5, \tag{26}$$

$$\text{constants} : (\gamma + \zeta)\theta_0 \;=\; \kappa_y. \tag{27}$$

We obtain three equations in three unknowns, $\{\theta_0, \theta_1, \theta_2\}$.

Using the equilibrium restrictions (20)-(23) and (25)-(27), we express coefficients in terms of primitives and ξ_5. Using the Kalman filter, we express inflation and output in terms of the consumer's forecast error of productivity. Additionally, we show in the appendix how to express $E_{t-1}^p[a_{t-1}]$ in the inflation dynamics (14) as a function of $E_{t-1}^c[x_t]$ and a_{t-1}, facilitating a direct comparison with the MSV solution.[11]

The MSV linear rational expectations equilibrium takes the form:

$$\pi_t \;=\; \hat{\xi}_0 \;+ \hat{\xi}_1 E_t^p[a_t] \;+\; \hat{\xi}_2 a_t \;+\; \hat{\xi}_3 E_t^c[x_t],\;\; t = 0, 1, ... \tag{28}$$

$$y_t \;=\; \hat{\theta}_0 \;+\; \hat{\theta}_1 a_t \;+\; \hat{\theta}_2 E_t^p[a_t],\;\; t = 0, 1, ... \tag{29}$$

where $\hat{\xi} \equiv \{\hat{\xi}_0, \hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3\}$ and $\hat{\theta} \equiv \{\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2\}$ are vectors of coefficients to be determined.

The following proposition characterises the MSV equilibrium.

[11] In the appendix, we show that indexing inflation conjecture (14) with $E_t^p[a_t]$ instead of $E_{t-1}^p[a_{t-1}]$ implies non-existence of non-MSV equilibria with information frictions.

Proposition 2 *(MSV solution) As long as $\chi \neq 1$, a unique MSV equilibrium exists, given by:*

$$\pi_t = \frac{\kappa_c}{1-\chi} + \left(-\frac{1-\rho}{\chi-\rho}\frac{\gamma(1+\zeta)}{\gamma+\zeta}\right) E^p_{t-1}[x_t] + \frac{\frac{(\chi-1)(1+\zeta)(\chi(\gamma+\zeta)-\gamma(1-\mu))}{(\chi/\gamma)(\chi/\rho-1)(\gamma+\zeta)} - \gamma\zeta}{\chi(\gamma+\zeta)-\gamma} a_t$$
$$- \underbrace{\left(\frac{\frac{\mu(\chi-1)(1+\zeta)(\chi(\gamma+\zeta)+\gamma-1)}{(\chi/\gamma)(\chi/\rho-1)(\gamma+\zeta)} - \gamma\zeta}{\chi(\gamma+\zeta)-\gamma}\right)\left(E^c_{t-1}[x_t] - a_t\right)}_{\textit{anchored forecast error}}, \tag{30}$$

$$(\gamma+\zeta)y_t = \kappa_y + \zeta a_t + E^p_{t-1}[x_t] + \underbrace{\left(\frac{\frac{\mu(\chi-1)(1+\zeta)(\chi(\gamma+\zeta)+\gamma-1)}{(\chi/\gamma)(\chi/\rho-1)(\gamma+\zeta)} - \gamma\zeta}{\chi(\gamma+\zeta)-\gamma}\right)\left(E^c_{t-1}[x_t] - a_t\right)}_{\textit{anchored forecast error}}. \tag{31}$$

Proof. See Appendix B.2 for detailed computations of the MSV linear rational expectations equilibrium. ■

The equilibrium system under the MSV solution imposes the same number of restrictions as the number of unknown coefficients. Matching coefficients term by term in the (FE) block gives:

$$\text{constants}: \hat{\xi}_0(1-\chi) = \kappa_c, \tag{32}$$
$$a_t: \chi\hat{\xi}_2 + \gamma\hat{\theta}_1 = 0, \tag{33}$$
$$E^p_t[a_t]: \chi\hat{\xi}_1 + \gamma\hat{\theta}_2 = 0, \tag{34}$$
$$E^c_t[x_t]: \rho(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3) = \chi\hat{\xi}_3 - \gamma\rho\left(\hat{\theta}_1 + \hat{\theta}_2\right). \tag{35}$$

Two key observations follow: equations (32)-(35) form a system of four equations in four unknowns, $\{\hat{\xi}_0, \hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3\}$. Additionally, for existence, it is necessary that $\chi \neq 1$, as otherwise, equation (32) would be violated.

Similarly, matching coefficients term by term in the (PC) block yields:

$$a_t: (\gamma+\zeta)\hat{\theta}_1 = \zeta - \left(\hat{\xi}_2 + \mu\hat{\xi}_3\right), \tag{36}$$
$$E^p_t[a_t]: (\gamma+\zeta)\hat{\theta}_2 = 1 + \hat{\xi}_2 + \mu\hat{\xi}_3, \tag{37}$$
$$\text{constants}: (\gamma+\zeta)\hat{\theta}_0 = \kappa_y. \tag{38}$$

This results in a system of three equations in three unknowns, $\{\hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2\}$, ensuring that the MSV equilibrium is unique. Furthermore, using the Kalman filter and appropriately rearranging terms, we demonstrate in Appendix B.2 how to derive expressions (30) and (31).

Without public communication about future fundamentals, equilibrium multiplicity is pervasive. A continuum of non-MSV equilibria emerges, where the consumer's forecast error remains unanchored and arbitrarily affects allocations, while a unique MSV equilibrium anchors expectations. Comparing inflation paths, non-MSV dynamics depend on past inflation and shocks, whereas MSV dynamics are independent of past realisations (compare terms highlighted in red across the two propositions).

In the next section, we examine the mechanism underlying multiplicity and connect it to established results in the literature. We then demonstrate how the communication of the central bank's information about the economy's future fundamentals through its policy surprises ensures the uniqueness of equilibrium.

4 Mechanism behind multiplicity

The intuition behind multiplicity is standard and is best understood by examining the solutions to the stochastic Fisher equation and linking them to the equilibria characterised earlier.

To do so, let us revisit the equilibrium block given by (FE):

$$E_t^c[\pi_{t+1}] \,=\, \chi\pi_t \,-\, \gamma\left(E_t^c[y_{t+1}] \,-\, y_t\right) + \kappa_c.$$

The solutions to this equation can be expressed as follows (see Cochrane, 2011, for details):

$$\pi_t \,=\, \chi\pi_{t-1} \,-\, \gamma\left(E_{t-1}^c[y_t] \,-\, y_{t-1}\right) \,+\, \kappa_c \,+\, \delta_t, \quad E_{t-1}^c[\delta_t] = 0. \tag{39}$$

Equation (39) admits multiple solutions, where the stochastic path of inflation is indexed by arbitrary mean-zero shocks δ_t. This observation underpins the classical result that inflation (and the initial price level) is indeterminate under an interest rate target (Sargent and Wallace, 1975).[12]

We align the solutions in (39) with those in Propositions 1 and 2 by linking δ_t to fundamental and policy parameters, and free coefficients. The multiplicity of inflation paths directly translates into multiplicity of output and consumption paths via the Lucas-type Phillips curve (PC). We allow for nominal explosions as equilibrium outcomes, while output remains bounded, as established

[12]Focusing on stationary solutions, we discipline the initial price level according to (14) under non-MSV solutions or (28) under the MSV solution. However, we formally demonstrate in this section that even if the initial price level π_0 is indexed to arbitrary shocks unrelated to economic fundamentals under non-MSV solutions, this results solely in nominal indeterminacy and leaves our results about uniqueness unchanged.

in Propositions 1 and 2. The intuition behind multiple equilibria stems from the multiplicity of solutions in (39) under an interest rate target (the Sargent and Wallace, 1975, classic doctrine) and the role of the Phillips curve in translating nominal to real indeterminacy.

Matching solutions in (39) with those in Propositions 1 under the non-MSV solutions requires the shock δ_t to satisfy:

$$\delta_t \equiv \underbrace{\left(\frac{\gamma(1+\zeta)}{\gamma+\zeta} + \xi_5(1-\mu)\right)\left(E^c_{t-1}[x_t] - a_t\right)}_{\text{unanchored forecast error}}, \tag{40}$$

which coincides with the unanchored forecast error in Proposition 1. Since δ_t depends on the free coefficient ξ_5, it remains arbitrary, leading to multiple inflation paths indexed by δ_t. Moreover, output is determined by the producer's price error, $E^p_t[\pi_t] - \pi_t$, which itself depends on $E^p_t[\delta_t] - \delta_t$. Given that the latter expectational error satisfies $E^p_t[\delta_t] - \delta_t = -\delta_t$, due to $E^p_t[E^c_{t-1}[x_t] - a_t] = E^p_t[E^c_{t-1}[x_t]] - E^p_t[a_t] = E^p_t[a_t] - E^p_t[a_t] = 0$, it follows that the output path is also indexed by the arbitrary shock δ_t.

Our results remain robust even if conjecture (14) is indexed by mean-zero sunspot shocks. Let $h_t \sim N(0, \sigma^2_h)$ denote a *publicly observed* sunspot shock (unrelated to productivity) that realises in stage 1 of period t (equilibrium existence would fail if public sunspots were introduced at stage 2). Equation (40) then modifies to:

$$\delta_t \equiv \underbrace{\left(\frac{\gamma(1+\zeta)}{\gamma+\zeta} + \xi_5(1-\mu)\right)\left(E^c_{t-1}[x_t] - a_t\right)}_{\text{unanchored forecast error}} + \underbrace{h_t}_{\text{sunspot}}. \tag{41}$$

However, output dynamics remain *unaffected* by sunspots. The producer's price error, $E^p_t[\pi_t] - \pi_t$, depends on $E^p_t[h_t] - h_t$, which equals zero ($E^p_t[h_t] = h_t$). Sunspots only scale nominal prices and wages proportionally in period t leaving consumer budgets and producer decisions unchanged. Indeterminacy is *purely nominal* and does not translate into multiplicity of real allocations. Thus, in this context, sunspot shocks serve purely as normalisations.[13]

[13]In economies with incomplete markets and nominal assets, both nominal and real indeterminacy typically coexist (Geanakoplos and Mas-Colell, 1989). There are changes in commodity and asset prices and asset demands that leave agents' budgets unaffected, but there are also dimensions of price variations that affect real decisions. This calls for government intervention to control prices and determine the real allocation. We show that information frictions call for government intervention to control the inflation component due to the "unanchored forecast error", while the "sunspot" component is irrelevant. The key point is that nominal wage stickiness stems from information frictions about productivity, while nominal prices adjust instantly to public sunspots, leaving real allocations unchanged.

Finally, aligning solutions in (39) with the MSV solution requires δ_t to satisfy:

$$\delta_t = -\underbrace{\left(\frac{\frac{\mu(\chi-1)(1+\zeta)(\chi(\gamma+\zeta)+\gamma-1)}{(\chi/\gamma)(\chi/\rho-1)(\gamma+\zeta)} - \gamma\zeta}{\chi(\gamma+\zeta)-\gamma}\right)\left(E^c_{t-1}[x_t] \, - \, a_t\right)}_{\text{anchored forecast error}}, \tag{42}$$

which coincides with the anchored forecast error in Proposition 2. Here, δ_t is uniquely determined by fundamental parameters and the Taylor rule coefficient χ. Furthermore, sunspot shocks do not affect the MSV solution.

5 Information effects

This section presents the main result, outlining the mechanism behind information effects that ensures uniqueness of equilibrium. We conduct robustness checks and demonstrate that multiplicity re-emerges under various counterfactual scenarios.

5.1 Mechanism

Suppose the central bank releases information about non-monetary fundamentals through its policy surprises. Specifically, monetary policy shocks are given by $\eta_t = s_t$, with $s_t = \epsilon_{t+1} + u_t$. In this formulation, policy surprises are constructed to provide noisy signals about the temporary component of productivity in the following period. These shocks influence the equilibrium through two distinct channels.

The first is the traditional channel, whereby policy shocks affect the real interest rate via their direct impact on the nominal interest rate. The second is the information channel, through which policy announcements lead agents to revise their expectations about future productivity. These updated beliefs, in turn, shape expectations regarding future output and prices.

Specifically, through the information channel, central bank announcements that tighten monetary policy by raising the real interest rate lead consumers to revise their expectations of future output upward. This revision, in turn, introduces equilibrium restrictions that shape the trajectory of inflation and output. As we argue below, such effects cannot be induced by the traditional transmission channel.

The next proposition is the main result of this section.

Proposition 3 *Let us normalise sunspot shocks to zero ($h_t = 0, \forall t$). The unique equilibrium is* *given by:*

$$\pi_t = \kappa_c + \chi\pi_{t-1} + \tfrac{\gamma}{\gamma+\zeta}\frac{\gamma(1+\zeta)+\zeta\left[\frac{\gamma-\mu}{1-\mu}+\zeta\right]}{\gamma+\zeta}a_{t-1} + + \frac{\gamma\left[\frac{\zeta(1-\gamma)}{(\gamma+\zeta)(1-\mu)}-\rho(1+\zeta)\right]}{\rho(\gamma+\zeta)}E^c_{t-1}[x_t]$$

$$+ \tfrac{\gamma\psi}{\gamma+\zeta}\left(\tfrac{\zeta(1-\gamma)}{(\gamma+\zeta)(1-\mu)}s_{t-2} - (1+\zeta)s_{t-1}\right) + \underbrace{\frac{\gamma(1+\zeta)}{\gamma+\zeta}\left(E^c_{t-1}[x_t] + \psi s_{t-1} - a_t\right)}_{\textit{anchored forecast error}} + \underbrace{s_{t-1}}_{\textit{policy shock}} \tag{43}$$

$$(\gamma+\zeta)y_t \;=\; \kappa_y \;+\; \zeta a_t \;+ E^c_{t-1}[x_t] + \psi s_{t-1} - \underbrace{\frac{\gamma(1+\zeta)}{\gamma+\zeta}\left(E^c_{t-1}[x_t] + \psi s_{t-1} - a_t\right)}_{\textit{anchored forecast error}}. \tag{44}$$

Proof. See Appendix C for detailed computations. ■

We outline the proof of Proposition 3, which proceeds in two steps. First, we demonstrate that central bank announcements "destroy" existence of the MSV-type equilibrium. Subsequently, we show that communication uniquely selects an equilibrium from the continuum of non-MSV-type equilibria identified in Proposition 1.

Before proceeding, we note that the Fisher equation (FE) block is modified to incorporate policy shocks, and is now given by:

$$E^c_t[\pi_{t+1}] \;=\; \underbrace{\chi\pi_t}_{\text{systematic}} \;+\; \underbrace{\eta_t}_{\text{non-systematic}} \;-\gamma\left(E^c_t[y_{t+1}] - y_t\right) \;+\; \kappa_c, \tag{45}$$

whereas the Philips curve (PC) block is unchanged.

In an MSV-type equilibrium, satisfying the (FE) and (PC) blocks requires that prices respond to current announcements. However, this creates an informational advantage for the policymaker over the uninformed producer, which ultimately undermines the mechanism supporting rational expectations equilibria under imperfect information.

Let the conjectured dynamics in (28) and (29) under the MSV-type solution be modified as follows:

$$\pi_t \;=\; \hat{\xi}_0 \;+ \hat{\xi}_1 E^p_t[a_t] \;+\; \hat{\xi}_2 a_t \;+\; \hat{\xi}_3 E^c_t[x_t] \;+\; \hat{\xi}_4 s_t,\;\; t = 0, 1, ... \tag{46}$$

$$y_t \;=\; \hat{\theta}_0 \;+\; \hat{\theta}_1 a_t \;+\; \hat{\theta}_2 E^p_t[a_t],\;\; t = 0, 1, ... \tag{47}$$

The producers' expectational errors in prices are independent of the term $\hat{\xi}_4 s_t$, so that output dynamics are not influenced by s_t. This follows because both conjectured prices and producers' expectations of these prices are measurable with respect to public central bank announcements. As a result, these announcements do not influence expectational errors.

The central bank's announcements lead consumers to update their beliefs about productivity, prompting a revision of their expectations regarding future inflation and output. These updated expectations are given by:

$$E_t^c[\pi_{t+1}] = \hat{\xi}_0 + \rho(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3)E_t^c[x_t] + \underbrace{\psi s_t\left(\hat{\xi}_1 + \hat{\xi}_2\right)}_{\text{information effects}}, \tag{48}$$

$$E_t^c[y_{t+1}] = \hat{\theta}_0 + \rho\left(\hat{\theta}_1 + \hat{\theta}_2\right)E_t^c[x_t] + \underbrace{\psi s_t\left(\hat{\theta}_1 + \hat{\theta}_2\right)}_{\text{information effects}}, \tag{49}$$

where the parameter ψ captures the precision of the signal and governs the strength of the central bank's informational influence.

Matching coefficients in the Fisher equation imposes the same restrictions as before, with an additional condition on the signal term s_t:

$$s_t: \ \chi\hat{\xi}_4 = \psi\left[\hat{\xi}_1 + \hat{\xi}_2 + \gamma\left(\hat{\theta}_1 + \hat{\theta}_2\right)\right] - 1. \tag{50}$$

Since the restrictions implied by the Phillips curve block (PC) remain unchanged, this condition simplifies to:

$$s_t: \ \hat{\xi}_4 = \psi\gamma\frac{1+\zeta}{\gamma+\zeta}\frac{\chi-1}{\chi^2} - \frac{1}{\chi}, \tag{51}$$

which shows that $\hat{\xi}_4$ is fully determined by the model's primitive parameters.[14] Since announcements influence consumer expectations—see equations (48) and (49)—the Fisher equation can only hold if current prices, via the inflation targeting component, are measurable with respect to current announcements. The key insight is that satisfying equilibrium conditions necessitates prices being measurable with respect to current signals. In what follows, we argue that this requirement unravels the mechanism underpinning the existence of the MSV-type solution.

For current prices to be measurable with respect to s_t, given that consumers determine their optimal labor supply in stage 1 while *correctly anticipating* the market-clearing price in stage 2, the central bank must communicate its forecasts in stage 1. However, this introduces simultaneity

[14] We treat the systematic policy parameter χ as given throughout the analysis. Since determinacy in our framework does not rely on the stability properties of a deterministic steady state, the value of χ is irrelevant for equilibrium selection.

between policy actions and production decisions, contradicting our assumption that the central bank intervenes in stage 2 to avoid gaining an informational advantage over producers. Consequently, prices cannot be conditioned on current signals, thus violating the equilibrium conditions necessary for the existence of an MSV-type solution.

The information effects mechanism relies on agents observing the policy rate and inferring the central bank's private information through *unanticipated* policy actions. This aligns with the findings of Nakamura and Steinsson (2018), who show that current policy surprises reveal information about non-monetary fundamentals. In our setting, prices cannot be indexed to current *unexpected* policy shocks without violating informational constraints, and the MSV-type equilibrium fails to exist.

Abstracting from unanticipated policy actions allows us to disentangle information effects from policy shocks. In the purely "Odyssean" case—where the central bank has no superior information relative to the public—the existence of the MSV-type equilibrium is restored, and real allocations match the baseline. This can be captured by splitting stage 2 into two sub-stages: in the first, all actions described in the main text occur; in the second, after asset markets close, the central bank issues binding commitments to next period's policy deviations. Producers may adjust expectations immediately, while consumers incorporate the news at the start of the next period.

Our framework accommodates interest rate rules that feature both information effects (through current policy surprises) and past guidance that communicate the central bank's intended actions regarding nominal variables. For simplicity, we abstract from the latter policy and focus on information effects by tying policy shocks to signals, and adopting the pure inflation targeting case as the baseline.[15]

We now show how central bank announcements uniquely select an equilibrium from the continuum of non-MSV-type solutions described in Proposition 1. To do so, we modify the conjectured dynamics in equations (14) and (15) as follows:

$$\pi_t = \xi_0 + \xi_1 \pi_{t-1} + \xi_2 a_{t-1} + \xi_3 E^p_{t-1}[a_{t-1}] + \xi_4 a_t + \xi_5 E^c_t[x_t] + \eta_{t-1}, \quad t = 0, 1, ... \tag{52}$$

$$y_t = \theta_0 + \theta_1 a_t + \theta_2 E^p_t[a_t], \quad t = 0, 1, ... \tag{53}$$

[15]Our distinction between information effects and forward guidance echoes the argument in Bassetto (2019), who, while sidestepping the issue of indeterminacy, distinguish between forward guidance and broader notions of transparency. If the central bank holds superior information about the state of the economy, the natural course is to communicate directly about those economic conditions—rather than indirectly through intended policy actions, which may serve as a poor proxy for the informational needs of the private sector. Bassetto illustrates this with an example where policy announcements are uninformative, while direct statements about economic conditions enhance coordination among private agents.

Here, current prices are indexed to lagged policy shocks ($\eta_{t-1} = s_{t-1}$).

The one-step-ahead forecasts of inflation and output in (18) and (19) are modified to:

$$E_t^c\left[\pi_{t+1}\right] = \xi_0 + \xi_1\pi_t + \xi_2 a_t + \xi_3 E_t^p[a_t] + \rho(\xi_4 + \xi_5)E_t^c[x_t] + \underbrace{\xi_4\psi s_t}_{\text{information effects}} + \underbrace{s_t}_{\text{policy shock}}, \tag{54}$$

$$E_t^c\left[y_{t+1}\right] = \theta_0 + \rho(\theta_1 + \theta_2)E_t^c[x_t] + \underbrace{(\theta_1 + \theta_2)\psi s_t}_{\text{information effects}}. \tag{55}$$

Matching coefficients in the Fisher equation imposes the same set of restrictions as before, along with an *additional condition* on the signal term s_t:

$$s_t: \quad \xi_4 = -\gamma(\theta_1 + \theta_2). \tag{56}$$

Combining this with equation (23) yields $\xi_5 = 0$. Since the Phillips curve block (PC) imposes the same restrictions as previously derived (equations (25)-(27)), we identify a unique equilibrium within the continuum described in Proposition 1. Using the Kalman filter, the equilibrium dynamics can be appropriately re-expressed as in Proposition 3.

The intuition can be understood through the stochastic Fisher equation, which we modify as follows:

$$\underbrace{\gamma\left(E_t^c[y_{t+1}] - y_t\right)}_{\substack{\text{expected output/consumption growth} \\ \equiv \Delta y_t}} = \underbrace{\chi\pi_t + \eta_t - E_t^c[\pi_{t+1}]}_{\substack{\text{real interest rate} \\ \equiv R_t}} + \kappa_c.$$

Fix a non-MSV equilibrium from the continuum. Let us study how the introduction of policy announcements affect expected output and the real rate. We obtain

$$\frac{\partial \Delta y_t}{\partial s_t} = \frac{\partial\left(\gamma E_t^c[y_{t+1}]\right)}{\partial s_t} = \gamma\left(\theta_1 + \theta_2\right)\cdot\psi \quad \text{and} \quad \frac{\partial R_t}{\partial s_t} = -\xi_4 \cdot \psi, \tag{57}$$

where we differentiate (54) and (55) with respect to s_t. Satisfaction of equilibrium requires

$$\frac{\partial\left(\gamma E_t^c[y_{t+1}]\right)}{\partial s_t} = \frac{\partial R_t}{\partial s_t} \iff -\xi_4 = \gamma\left(\theta_1 + \theta_2\right) \Rightarrow \xi_5 = 0, \tag{58}$$

which corresponds to restriction (56). Thus, equilibrium selection is pinned down uniquely by this condition: only the equilibrium satisfying $\xi_5 = 0$ survives, while the rest unravel.

The announced signal s_t affects decisions through two channels: (i) by influencing consumers' expectations about the future economic outlook at stage $(t, 2)$, and (ii) by shaping producers' expectations about productivity and prices at stage $(t+1,1)$. However, only the first channel is relevant for equilibrium selection.

The central bank influences the real interest rate by releasing information about future fundamentals, guiding the real rate in the central bank's desired direction. This, in turn, shapes output and inflation dynamics by imposing equilibrium restrictions – as shown in (58) – to ensure uniqueness. Public information that raises the real rate ($\partial R_t / \partial s_t > 0$), since the equilibrium requires $\xi_4 < 0$, can subsequently boosts output growth ($\partial \Delta y_t / \partial s_t > 0$) by elevating the agents' forecasts about future output ($\partial \left(\gamma E_t^c[y_{t+1}] \right) / \partial s_t > 0$).

This mechanism diverges from standard New-Keynesian logic, which predicts the opposite outcome after monetary tightening. However, Nakamura and Steinsson (2018) show that monetary policy shocks not only reflect policy tightening but also convey information about economic fundamentals. Specifically, they find that unexpected increases in the real interest rate lead to higher survey-based estimates of expected output growth. A natural interpretation is that central bank announcements prompt the private sector to revise its beliefs about both the future path of monetary policy and non-monetary economic fundamentals (referred to as "Fed information effects" by Romer and Romer, 2000).[16]

In our framework, monetary policy includes both a systematic inflation-targeting component and a non-systematic component, which – consistent with empirical findings – provides information about future non-monetary fundamentals, modelled here as noisy signals s_t about the future temporary component of productivity.

Our findings reveal that central bank announcements raise estimates of expected output by increasing the real interest rate, thereby introducing the necessary restrictions to achieve uniqueness. More fundamentally, we uncover a new role for "Fed information effects": their capacity to act as an equilibrium selection device.

The following remarks are in order.

Remark 1 *The concept of Fed information effects rests on the idea that the central bank possesses superior information about the economy or provides forecasts that the private sector finds valuable. Is this a reasonable assumption? Recent evidence, for example Ricco and Savini (2025), shows that the Fed holds an information advantage regarding the future economic outlook—its forecasts outperform those of all professional forecasters—and that private forecasters update their own projections following policy announcements. They argue that the "information channel" hypothesis*

[16]Campbell et al. (2012) also find evidence that an unexpected tightening of future interest rates lowers expectations of future unemployment.

aligns more closely with the empirical evidence than the "Fed response to news" hypothesis as proposed by Bauer and Swanson (2023).

Remark 2 *Public communication attains the unique equilibrium for any public information precision level $\psi > 0$. When $\psi = 0$, information effects are muted, and the continuum of non-MSV equilibria re-emerges.*

Remark 3 *Under the selected equilibrium in Proposition 3, output dynamics are independent of the monetary policy parameter χ, rendering conventional monetary policy ineffective as a stabilisation tool. This contrasts with the MSV-type equilibrium under the baseline in Proposition 2, where real allocations respond to the systematic policy parameter.*

Remark 4 *There are also ineffective communication policies that fail to select a unique non-MSV equilibrium from the continuum of solutions under the baseline of pure inflation targeting. For example, consider a case where policy surprises η_t convey only noisy signals about the innovations e_{t+1} that influence the dynamics of the permanent component x_{t+1}. In this case, expectation updating yields $E_t^c\left[E_{t+1}^p[a_{t+1}]\right] = E_t^c[x_{t+1}] = E_t^c[a_{t+1}]$, and it can be shown that the additional restriction associated with the signal is identical to the restriction involving $E_t^c[x_t]$, as in equation (23) above.*

5.2 Central Bank vs non-Central Bank signals, and uninformative shocks

The model's timing structure and the information effects mechanism allows us to distinguish between *Central Bank* signals or news—private information held by the central bank—and *non-Central Bank* signals or news—public information released by other institutions. Information effects arise when credible signals about future fundamentals are conveyed at stage 2 through deviations from the monetary policy rule, reflecting the central bank's private forecasts. By contrast, any credible signal revealed at stage 1, before the policy rate is set, must originate outside the central bank and is therefore classified as "non-Central Bank" news.

The transmission of Central Bank news through policy surprises—Fed information effects—must occur in stage 2, after production is completed, thereby conditioning production on past policy actions and respecting producers' informational constraints. In essence, Fed information effects *microfound* the news transmission mechanism in stage 2, enabling us to distinguish the impact of different information sources across stages on real allocations.

In the absence of Central Bank signals, the MSV-type equilibrium exists under non-Central Bank signals—a variant of condition (51) applies—but information effects have no bite: announcements no longer steer the real rate to shape expectations, as clarified by restriction (58). Instead, causality reverses—non-Central Bank signals affect expected output growth, and the real rate adjusts accordingly—allowing the MSV equilibrium to coexist with a unique non-MSV equilibrium that resembles the equilibrium previously selected through information effects.

Our framework could include non-Central Bank signals in stage 2, but adding them alongside Central Bank news does not change the main results in Section 5.1. Expectations would reflect a convex combination of Central Bank and non-Central Bank signals, with weights given by their relative precisions, yet the logic behind MSV nonexistence remains the same, and the argument for selecting a unique non-MSV equilibrium from many possible solutions still holds. Because adding non-Central Bank news at stage 2 is redundant, we omit it for simplicity.

What matters for uniqueness is not the presence of policy shocks per se, but the presence of information effects—specifically, the transmission of the central bank's private information about fundamentals. When we strip policy surprises of informational content, multiplicity re-emerges (see Remark 2). Similarly, if the central bank issues only binding commitments to future deviations from the policy rule—without revealing anything about fundamentals, as in the purely "Odyssean" case—the equilibrium set and real allocations match the baseline. This confirms that deviations from the policy rule alone are not critical—uninformative policy surprises or purely "Odyssean" guidance allow multiplicity.[17]

6 Conclusion

This paper studies a monetary economy with imperfect information and rational expectations, framing the classic determinacy problem as one of signal extraction. The potential for multiple equilibria arises from the classic doctrine that inflation and the initial price level are indeterminate under an interest rate peg (Sargent and Wallace, 1975). Our framework accommodates both nominally explosive equilibria and those influenced by "sunspots"—though, as argued, such shocks affect prices but not real allocations.

Our first result shows that the interaction between information frictions and announcements unravels the existence of equilibria that require prices to be measurable with respect to current policy announcements. In such cases, producer expectations about prices are influenced by the same announcements that guide monetary policy, implying that policy actions occur simultaneously with producers' optimal production decisions. This simultaneity violates a key consistency condition: the

[17]Even without information effects, the presence of (uninformative) policy surprises still precludes the existence of the MSV equilibrium, since its existence requires prices to be measurable with respect to information or shocks known in stage 1. However, an equilibrium-selection mechanism based on this reasoning is not robust, as we have already argued that the existence of the MSV equilibrium is restored under purely Odyssean guidance.

central bank must not hold an informational advantage over producers regarding current prices. The information effects mechanism—consistent with the framework and findings of Nakamura and Steinsson (2018)—depends on agents inferring the central bank's private information from *unanticipated* policy actions. However, in our framework, indexing prices to current unanticipated shocks is incompatible with informational constraints. Consequently, the interaction between information frictions and information effects precludes the existence of an MSV-type equilibrium.

Equilibria in which prices depend on past shocks and announcements are immune to the previous problem. In this setting, we show that monetary policy announcements about non-monetary fundamentals—specifically, noisy forecasts of future temporary productivity—enables the central bank to raise the real interest rate and thereby increase expectations of future output. This mechanism aligns with what Nakamura and Steinsson (2018) term "Fed information effects." Our results demonstrate that central bank announcements can raise expected output by tightening policy, thus imposing the restrictions needed for uniqueness. More broadly, we identify a novel role for Fed information effects: their ability to serve as an equilibrium selection device.

Appendix

A Kalman filter

Consider first the case without public announcements. Let the consumer's prior at date t about x and a be given by:

$$x_t | I^c_{t-1} \sim \mathcal{N}\left(x_{t|t-1}, \sigma^2_{x|t-1}\right) \tag{A.1}$$

$$a_t | I^c_{t-1} \sim \mathcal{N}\left(x_{t|t-1}, \sigma^2_{x|t-1} + \sigma^2_{\epsilon}\right), \tag{A.2}$$

where $x_{t|t-1} := E[x_t|I^c_{t-1}]$ and $\sigma^2_{x|t-1} := Var[x_t|I^c_{t-1}]$. Upon observing a_t, the consumer's updated beliefs about x_t are given by:

$$x_t | I^c_t \sim \mathcal{N}\left(x_{t|t-1} + \mu_t\left(a_t - a_{t|t-1}\right), \quad \left(\frac{1}{\sigma_{x,t-1}} + \frac{1}{\sigma_{\epsilon}}\right)^{-1}\right) \quad \text{where} \quad \mu_t = \frac{\frac{1}{\sigma^2_{\epsilon}}}{\frac{1}{\sigma^2_{x|t-1}} + \frac{1}{\sigma^2_{\epsilon}}}. \tag{A.3}$$

Their expectation about x_{t+1} are given by:

$$x_{t+1} | I^c_t \sim \mathcal{N}\left(\rho x_{t|t}, \sigma^2_{x|t}\right), \tag{A.4}$$

where

$$\sigma^2_{x|t} = \left(\frac{1}{\sigma^2_{x|t-1}} + \frac{1}{\sigma^2_{\epsilon}}\right)^{-1} + \sigma^2_e. \tag{A.5}$$

Let σ^2_x denote the solution (fixed point) of the Riccati equation $(A.5)$. We assume that at period 0, the agents' learning problem is at their steady state $x_{-1} \sim \mathcal{N}\left(\hat{x}_{-1}, \sigma^2_x\right)$. The Kalman gain will be also time invariant:

$$\mu = \frac{\frac{1}{\sigma^2_{\epsilon}}}{\frac{1}{\sigma^2_x} + \frac{1}{\sigma^2_{\epsilon}}}. \tag{A.6}$$

Moreover, the posterior distribution of $a_{t+\tau}$, $\tau \geq 1$, is given by

$$a_{t+\tau} | I^c_t \sim \mathcal{N}\left(\rho^{\tau} x_{t|t}, \sigma^2_x + \sigma^2_{\epsilon}\right), \tag{A.7}$$

with $x_{t|t} = (1-\mu)x_{t|t-1} + \mu a_t$.

The producer does not observe a at $(t,1)$, and their expectation about the permanent component coincide with the expectation of the consumer about x at the end of $t-1$, $E_t^p[x_t] = E_{t-1}^c[x_t] = x_{t|t-1}$. However, they learn realised productivity at $(t,2)$, and their beliefs are then given by the posterior distribution in $(A.3)$.

Next, we proceed to characterise the agents' learning problem when the central bank releases noisy signals about the temporary component next period at every date. Let the consumer's prior at date t about x and a be given by:

$$x_t|I_{t-1}^c \sim \mathcal{N}\left(x_{t|t-1}, \sigma_{x|t-1}^2\right), \tag{A.8}$$

$$a_t|I_{t-1}^c \sim \mathcal{N}\left(x_{t|t-1} + \psi s_{t-1}, \sigma_{x|t-1}^2 + \left(\frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon^2}\right)^{-1}\right) \tag{A.9}$$

where

$$\psi = \frac{\frac{1}{\sigma_u^2}}{\frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon^2}} \tag{A.10}$$

denotes the precision of the signal s_{t-1} that agents receive at $t-1$ about the temporary component of productivity at t.

Upon observing a_t, the consumer's updated beliefs about x_t are given by:

$$x_t|I_t^c \sim \mathcal{N}\left(x_{t|t-1} + \mu_t\left(a_t - a_{t|t-1}\right), \quad \left(\frac{1}{\sigma_{x,t-1}} + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon}\right)^{-1}\right) \quad \text{where} \quad \mu_t = \frac{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_u^2}}{\frac{1}{\sigma_{x|t-1}^2} + \frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_u^2}}. \tag{A.11}$$

Following similar steps as shown above, the posterior distribution of x_{t+1} is given by

$$x_{t+1}|I_t^c \sim \mathcal{N}\left(\rho x_{t|t}, \sigma_{x|t}^2\right), \tag{A.12}$$

with

$$\sigma_{x|t}^2 = \left(\frac{1}{\sigma_{x,t-1}} + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon}\right)^{-1} + \sigma_e^2 \tag{A.13}$$

and, as before, σ_x^2 denote the solution (fixed point) of the Riccati equation $(A.13)$.

The posterior distributions for the consumer are given by

$$a_{t+1}|I_t^c \sim \mathcal{N}\left(\rho x_{t|t} + \psi s_t, \sigma_x^2 + \left(\frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon^2}\right)^{-1}\right), \tag{A.14}$$

and the relevant posteriors for the producer are given by

$$a_t|I_{t,1}^p \sim \mathcal{N}\left(x_{t|t-1} + \psi s_{t-1}, \sigma_x^2 + \left(\frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon^2}\right)^{-1}\right). \tag{A.15}$$

B Inflation targeting

B.1 Non-MSV solution

Define $x_t \equiv \log(X_t)$. Consider linear equilibria of the form:

$$\pi_t \;=\; \xi_0 \;+ \xi_1 \pi_{t-1} \;+\; \xi_2 a_{t-1} \;+\; \xi_3 E_{t-1}^p[a_{t-1}] \;+\; \xi_4 a_t \;+\; \xi_5 E_t^c[x_t],\; t = 0, 1, ... \tag{B.1}$$

$$y_t \;=\; \theta_0 \;+\; \theta_1 a_t \;+\; \theta_2 E_t^p[a_t],\; t = 0, 1, ... \tag{B.2}$$

Let us start with the labor optimality condition. Substituting (13) into (12), taking into account that $\lambda_t = P_t^{-1} C_t^{-\gamma}$, and multiplying and dividing the right-hand side of (12) by P_{t-1}, yields

$$N_t^\zeta \;=\; \frac{1}{\Pi_t C_t^\gamma} \frac{E_t^p\left[\frac{A_t}{C_t^\gamma}\right]}{E_t^p\left[\frac{1}{\Pi_t C_t^\gamma}\right]}. \tag{B.3}$$

In turn, taking into account that, in turn, technology and market clearing in logs can written as $y_t = n_t \,+\, a_t$ and $c_t = y_t$, (B.3) can be written as follows:

$$e^{(\gamma+\zeta)y_t - \zeta a_t + \pi_t} \;=\; \frac{E_t^p\left[e^{a_t - \gamma y_t}\right]}{E_t^p\left[e^{-\pi_t - \gamma y_t}\right]}. \tag{B.4}$$

The following result will be useful throughout the analysis that follows:

$$E_t^p\left[E_t^c[x_t]\right] \;=\; x_{t|t-1} \;=\; E_t^c[x_t] + \mu\left[E_t^p[a_t] - a_t\right]. \tag{B.5}$$

Conditional on the producer's information set at $(t, 1)$, the exponent of the numerator in the right-hand side of (B.4) is normally distributed with mean given by

$$E_t^p[a_t - \gamma y_t] = -\gamma\theta_0 \; + \; (1 - \gamma[\theta_1 + \theta_2])E_t^p[a_t] \tag{B.6}$$

and variance given by

$$Var_t^p[a_t - \gamma y_t] = (1 - \gamma\theta_1)^2 \left(\sigma_x^2 + \sigma_\epsilon^2\right), \tag{B.7}$$

where σ_x^2 solves the Riccati equation (A.5). Similarly, the exponent of the denominator is normally distributed with mean given by

$$E_t^p\left[-(\pi_t + \gamma y_t)\right] \; = \; \Delta \; - \; (\gamma\theta_1 + \gamma\theta_2 + \xi_4)\, E_t^p[a_t] \; - \; \xi_5 E_t^p\left[E_t^c[x_t]\right], \tag{B.8}$$

where

$$\Delta = -\left\{\gamma\theta_0 + \xi_0 + \xi_1\pi_{t-1} + \xi_2 a_{t-1} + \xi_3 E_{t-1}^p(a_{t-1})\right\}$$

and variance given by

$$Var_t^p[\pi_t + \gamma y_t] = (\xi_4 + \xi_5\mu + \gamma\theta_1)^2 \left(\sigma_x^2 + \sigma_\epsilon^2\right). \tag{B.9}$$

The random variables inside the expectation operators in the numerator and denominator of (B.4) are log-normally distributed, so we obtain

$$e^{(\gamma+\zeta)y_t - \zeta a_t + \pi_t} \; = \; \frac{E_t^p\left[e^{a_t - \gamma y_t}\right]}{E_t^p\left[e^{-\pi_t - \gamma y_t}\right]} \; = \; e^{E_t^p[a_t - \gamma y_t] + E_t^p[\pi_t + \gamma y_t] + \frac{1}{2}Var_t^p[a_t - \gamma y_t] - \frac{1}{2}Var_t^p[\pi_t + \gamma y_t]}, \tag{B.10}$$

which, in turn, can be equivalently written as follows:

$$(\gamma + \zeta)y_t \; = \; E_t^p[a_t] + \zeta a_t + E_t^p[\pi_t] - \pi_t + \kappa_y, \tag{B.11}$$

with

$$\kappa_y = \frac{1}{2}\left(\sigma_x^2 + \sigma_\epsilon^2\right)\left[(1 - \gamma\theta_1)^2 - (\xi_4 + \xi_5\mu + \gamma\theta_1)^2\right]. \tag{B.12}$$

Taking into account (B.5) and substituting conjectures (B.1)-(B.2) into (B.11), yields

$$a_t : (\gamma + \zeta)\theta_1 \; = \; \zeta \; - \; (\xi_4 + \mu\xi_5), \tag{B.13}$$

$$E_t^p[a_t] : (\gamma + \zeta)\theta_2 \; = \; 1 \; + \; \xi_4 \; + \; \mu\xi_5, \tag{B.14}$$

$$\text{constants} : (\gamma + \zeta)\theta_0 \; = \; \kappa_y. \tag{B.15}$$

Note that substituting (B.1) and (B.5) into $E_t^p[\pi_t] - \pi_t$ implies that the expectational error is independent of $E_t^c[x_t]$, and thus, we only need to match coefficients between a_t and $E_t^p[a_t]$.

Next, we derive the Fisher equation in the main text. The interest rate rule (5) can be written as

$$e^{q_t} \;=\; e^{-\chi\pi_t}. \tag{B.16}$$

The non-linear Euler equation (11) can be written as follows:

$$e^{q_t} = e^{\log\beta} E_t^c\left[e^{-\,\pi_{t+1} + \gamma(y_t\,-\,y_{t+1})}\right] = e^{\log\beta + \gamma y_t + E_t^c[-(\pi_{t+1}+\gamma y_{t+1})] + \frac{1}{2}Var_t^c[-(\pi_{t+1}+\gamma y_{t+1})]}, \tag{B.17}$$

where we have used the equilibrium condition $y_t = c_t$, and the fact that $\pi_{t+1} + \gamma y_{t+1}$ is normally distributed (hence $e^{\pi_{t+1}+\gamma y_{t+1}}$ is log-normally distributed).

Combining (B.16),(B.17) to eliminate q_t, yields

$$E_t^c[\pi_{t+1}] \;=\; \chi\pi_t \;-\; \gamma\left(E_t^c[y_{t+1}] \;-\; y_t\right) + \kappa_c, \tag{B.18}$$

with

$$\kappa_c = \log\beta \;+\; \frac{1}{2}\left(\gamma\theta_1 + \xi_4 + \xi_5\mu\right)^2\left(\sigma_x^2 + \sigma_\epsilon^2\right). \tag{B.19}$$

The one-step-ahead forecasts of inflation and output are given by

$$E_t^c[\pi_{t+1}] \;=\; \xi_0 \;+\; \xi_1\pi_t \;+\; \xi_2 a_t \;+\xi_3 E_t^p[a_t] \;+\; \rho(\xi_4+\xi_5)E_t^c[x_t], \tag{B.20}$$

$$E_t^c[y_{t+1}] \;=\; \theta_0 \;+\; \rho\left(\theta_1 \;+\; \theta_2\right)E_t^c[x_t]. \tag{B.21}$$

We used that $E_t^c[a_{t+1}] = E_t^c[x_{t+1}] = \rho E_t^c[x_t] = E_t^c\left[E_{t+1}^c[x_{t+1}]\right]$ and $E_t^c\left[E_{t+1}^p[a_{t+1}]\right] = E_t^c[x_{t+1}] = \rho E_t^c[x_t]$ in the previous computations.

Substituting (B.1),(B.2),(B.20),(B.21) into (B.18), yields

$$\begin{aligned}\underbrace{\xi_0 - \kappa_c - (\chi-\xi_1)\xi_0}_{\text{constant terms}} &+ a_t\left(\xi_2 - \gamma\theta_1 - (\chi-\xi_1)\xi_4\right) + E_t^p[a_t]\left(\xi_3 - \gamma\theta_2\right)\\ &+ E_t^c[x_t]\left(\rho(\xi_4+\xi_5) + \gamma\rho(\theta_1+\theta_2) - (\chi-\xi_1)\xi_5\right)\\ &- (\chi-\xi_1)\underbrace{\left(\xi_1\pi_{t-1} \;+\; \xi_2 a_{t-1} + \xi_3 E_{t-1}^p[a_{t-1}]\right)}_{\text{predetermined terms}} \;=\; 0.\end{aligned} \tag{B.22}$$

Stationarity requires $\xi_1 = \chi$, so that predetermined terms vanish in (B.22). Matching coefficients yields

$$
\begin{aligned}
\text{constants}: \xi_0 &= \kappa_c, && \text{(B.23)}\\
a_t: \xi_2 &= \gamma\theta_1, && \text{(B.24)}\\
E_t^p[a_t]: \xi_3 &= \gamma\theta_2, && \text{(B.25)}\\
E_t^c[x_t]: \; -\rho(\xi_4+\xi_5) &= \gamma\rho\left(\theta_1+\theta_2\right). && \text{(B.26)}
\end{aligned}
$$

Combining (B.13)-(B.15) and (B.23)-(B.26), we obtain

$$
\begin{aligned}
\theta_0 &= \frac{\kappa_y}{\gamma+\zeta} && \text{(B.27)}\\
\theta_1 &= \frac{\zeta}{\gamma+\zeta}+\frac{\gamma(1+\zeta)}{(\gamma+\zeta)^2}+\frac{\xi_5(1-\mu)}{\gamma+\zeta} && \text{(B.28)}\\
\theta_2 &= \frac{\zeta(1-\gamma)}{(\gamma+\zeta)^2}-\frac{\xi_5(1-\mu)}{\gamma+\zeta}, && \text{(B.29)}\\
\xi_0 &= \kappa_c, && \text{(B.30)}\\
\xi_1 &= \chi, && \text{(B.31)}\\
\xi_2 &= \gamma\left(\frac{\zeta}{\gamma+\zeta}+\frac{\gamma(1+\zeta)}{(\gamma+\zeta)^2}+\frac{\xi_5(1-\mu)}{\gamma+\zeta}\right), && \text{(B.32)}\\
\xi_3 &= \gamma\left(\frac{\zeta(1-\gamma)}{(\gamma+\zeta)^2}-\frac{\xi_5(1-\mu)}{\gamma+\zeta}\right) && \text{(B.33)}\\
\xi_4 &= -\xi_5-\frac{\gamma(1+\zeta)}{\gamma+\zeta}. && \text{(B.34)}
\end{aligned}
$$

and the coefficient ξ_5 cannot be determined by the equilibrium system.

We derive the characterisation in Proposition 1. Beginning with the output dynamics, we substitute (B.13)-(B.15) and (B.34) into the linear conjecture (B.2), while considering that $E_t^p[a_t] = E_{t-1}^c[x_t]$, which yields

$$
(\gamma+\zeta)y_t \;=\; \kappa_y \;+\; \zeta a_t \;+\; E_{t-1}^c[x_t] \;-\; \left(\frac{\gamma(1+\zeta)}{\gamma+\zeta}+\xi_5(1-\mu)\right)\left(E_{t-1}^c[x_t] \;-\; a_t\right), \qquad \text{(B.35)}
$$

which coincides with the expression in Proposition 1.

Next, substituting (B.30), (B.31) and (B.34) into conjecture (B.1), and taking into account that $E_t^c[x_t] = (1-\mu)E_{t-1}^c[x_t] + \mu a_t$, yields

$$
\pi_t = \kappa_c + \chi\pi_{t-1} + \xi_2 a_{t-1} + \xi_3 E_{t-1}^p[a_{t-1}] + \left(-\tfrac{\gamma(1+\zeta)}{\gamma+\zeta}\right)a_t + \xi_5(1-\mu)\left(E_{t-1}^c[x_t] - a_t\right). \qquad \text{(B.36)}
$$

Next, we use the Kalman filter to express $E^p_{t-1}[a_{t-1}]$ in terms of $E^p_t[a_t]$ and a_{t-1}:

$$E^p_{t-1}[a_{t-1}] = \frac{1}{\rho(1-\mu)} E^p_t[a_t] - \frac{\mu}{1-\mu} a_{t-1}. \tag{B.37}$$

This expression is derived by combining the relationships $E^p_{t-1}[a_{t-1}] = E^c_{t-2}[x_{t-1}]$ and $E^p_t[a_t] = E^c_{t-1}[x_t] = \rho E^c_{t-1}[x_{t-1}] = \rho\left((1-\mu)E^c_{t-2}[x_{t-1}] + \mu a_{t-1}\right)$.

The inflation dynamics in (B.36) reduce to

$$\pi_t \;=\; \kappa_c + \chi\pi_{t-1} \;+\; \Xi_2 a_{t-1} \;+\; \Xi_3 E^p_t[a_t] \;+\; \left(-\frac{\gamma(1+\zeta)}{\gamma+\zeta}\right) a_t + \xi_5(1-\mu)\left(E^c_{t-1}[x_t] - a_t\right), \tag{B.38}$$

where $\Xi_3 \equiv \xi_3/\rho(1-\mu)$ and $\Xi_2 \equiv \xi_2 - \xi_3\mu/(1-\mu)$ and ξ_2, ξ_3 are given by (B.32) and (B.33). Next, adding and subtracting $[\gamma(1+\zeta)/(\gamma+\zeta)]E^c_{t-1}[x_t]$ while considering that $E^p_t[a_t] = E^c_{t-1}[x_t]$, we obtain

$$\pi_t \;=\; \kappa_c + \chi\pi_{t-1} \;+\; \Xi_2 a_{t-1} \;+\; \left(\Xi_3 - \frac{\gamma(1+\zeta)}{\gamma+\zeta}\right) E^c_{t-1}[x_t] \;+\; \left(\frac{\gamma(1+\zeta)}{\gamma+\zeta} + \xi_5(1-\mu)\right)\left(E^c_{t-1}[x_t] - a_t\right), \tag{B.39}$$

and using (B.32) and (B.33) to express Ξ_2 and Ξ_3 in terms of fundamentals and the free coefficient ξ_5, we obtain the expression in Proposition 1.

Remark 5 *Combining (B.1) and (B.37) we obtain*

$$\pi_t \;=\; \xi_0 \;+\; \xi_1\pi_{t-1} \;+\; \tilde{\xi}_2 a_{t-1} \;+\; \tilde{\xi}_3 E^p_t[a_t] \;+\; \xi_4 a_t \;+\; \xi_5 E^c_t[x_t]. \tag{B.40}$$

Using (B.40) and (B.2) as our initial conjectures to compute non-MSV equilibria establishes that the equilibrium output satisfies

$$y_t = a_t\left(\frac{1+\zeta}{\gamma+\zeta}\right), \tag{B.41}$$

which coincides with the equilibrium output under the symmetric benchmark case. Consequently, in this setting, information asymmetries do not affect the equilibrium real allocation. To see this, consider conjectures (B.40) and (B.2) and compute the one-step-ahead forecasts of inflation and output:

$$E^c_t[\pi_{t+1}] \;=\; \xi_0 \;+\; \xi_1\pi_t \;+\; \tilde{\xi}_2 a_t \;+\; \rho(\tilde{\xi}_3 + \xi_4 + \xi_5)E^c_t[x_t], \tag{B.42}$$

$$E^c_t[y_{t+1}] \;=\; \theta_0 \;+\; \rho\,(\theta_1 \;+\; \theta_2)\,E^c_t[x_t]. \tag{B.43}$$

Matching coefficients in (B.18) with $\xi_1 = \chi$ gives $\theta_2 = 0$, as there is no corresponding term to match. From (B.14), $\xi_4 + \mu\xi_5 = -1$, leading to $\kappa_y = 0$ and $\theta_0 = 0$. Finally, (B.13) gives $\theta_1 = (1+\zeta)/(\gamma+\zeta)$. Substituting into (B.2) yields (B.41).

B.2 MSV solution

Consider linear equilibria of the form:

$$\pi_t \;=\; \hat{\xi}_0 \;+ \hat{\xi}_1 E_t^p[a_t] \;+\; \hat{\xi}_2 a_t \;+\; \hat{\xi}_3 E_t^c[x_t],\;\; t = 0, 1, ... \tag{B.44}$$

$$y_t \;=\; \hat{\theta}_0 \;+\; \hat{\theta}_1 a_t \;+\; \hat{\theta}_2 E_t^p[a_t],\;\; t = 0, 1, ... \tag{B.45}$$

The derivation of the Phillips curve remains unchanged, and the equilibrium restrictions (B.13)-(B.15) hold as well. For clarity, we restate these equilibrium restrictions below:

$$a_t : (\gamma + \zeta)\hat{\theta}_1 \;=\; \zeta \;-\; \left(\hat{\xi}_2 + \mu\hat{\xi}_3\right), \tag{B.46}$$

$$E_t^p[a_t] : (\gamma + \zeta)\hat{\theta}_2 \;=\; 1 \;+\; \hat{\xi}_2 \;+\; \mu\hat{\xi}_3, \tag{B.47}$$

$$\text{constants} : (\gamma + \zeta)\hat{\theta}_0 \;=\; \kappa_y. \tag{B.48}$$

Following similar steps as before, the Fisher equation is given by

$$E_t^c[\pi_{t+1}] \;=\; \chi\pi_t \;-\; \gamma\left(E_t^c[y_{t+1}] \;-\; y_t\right) \;+ \kappa_c, \tag{B.49}$$

with

$$\kappa_c = \log\beta \;+\; \frac{1}{2}\left(\gamma\hat{\theta}_1 + \hat{\xi}_2 + \hat{\xi}_3\mu\right)^2\left(\sigma_x^2 + \sigma_\epsilon^2\right). \tag{B.50}$$

The one-step-ahead forecasts of inflation and output are given by

$$E_t^c[\pi_{t+1}] \;=\; \hat{\xi}_0 \;+ \rho(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3)E_t^c[x_t], \tag{B.51}$$

$$E_t^c[y_{t+1}] \;=\; \hat{\theta}_0 \;+\; \rho\left(\hat{\theta}_1 \;+\; \hat{\theta}_2\right)E_t^c[x_t]. \tag{B.52}$$

Substituting (B.44), (B.45), (B.51), (B.52) into (B.49) and matching coefficients, yields

$$\text{constants} : \hat{\xi}_0(1 - \chi) = \kappa_c, \tag{B.53}$$

$$a_t : \chi\hat{\xi}_2 + \gamma\hat{\theta}_1 = 0, \tag{B.54}$$

$$E_t^p[a_t] : \chi\hat{\xi}_1 + \gamma\hat{\theta}_2 = 0, \tag{B.55}$$

$$E_t^c[x_t] : \rho(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3) = \chi\hat{\xi}_3 - \gamma\rho\left(\hat{\theta}_1 + \hat{\theta}_2\right). \tag{B.56}$$

Combining (B.46)-(B.48) and (B.53)-(B.56), we obtain

$$\hat{\theta}_0 = \frac{\kappa_y}{\gamma + \zeta}, \tag{B.57}$$

$$\hat{\theta}_1 = \frac{\zeta - \mu\frac{(\chi-1)\gamma\rho}{\chi(\chi-\rho)} \cdot \frac{1+\zeta}{\gamma+\zeta}}{\zeta + \frac{\gamma(\chi-1)}{\chi}}, \tag{B.58}$$

$$\hat{\theta}_2 = \frac{1 - \frac{\gamma}{\chi} \cdot \frac{\zeta - \mu\frac{(\chi-1)\gamma\rho}{\chi(\chi-\rho)} \cdot \frac{1+\zeta}{\gamma+\zeta}}{\zeta + \frac{\gamma(\chi-1)}{\chi}} + \mu\frac{(\chi-1)\gamma\rho}{\chi(\chi-\rho)} \cdot \frac{1+\zeta}{\gamma+\zeta}}{\gamma + \zeta}, \tag{B.59}$$

$$\hat{\xi}_0 = \frac{\kappa_c}{1 - \chi}, \tag{B.60}$$

$$\hat{\xi}_1 = -\frac{\gamma}{\chi} \cdot \frac{1 - \frac{\gamma}{\chi} \cdot \frac{\zeta - \mu\frac{(\chi-1)\gamma\rho}{\chi(\chi-\rho)} \cdot \frac{1+\zeta}{\gamma+\zeta}}{\zeta + \frac{\gamma(\chi-1)}{\chi}} + \mu\frac{(\chi-1)\gamma\rho}{\chi(\chi-\rho)} \cdot \frac{1+\zeta}{\gamma+\zeta}}{\gamma + \zeta}, \tag{B.61}$$

$$\hat{\xi}_2 = -\frac{\gamma}{\chi} \cdot \frac{\zeta - \mu\frac{(\chi-1)\gamma\rho}{\chi(\chi-\rho)} \cdot \frac{1+\zeta}{\gamma+\zeta}}{\zeta + \frac{\gamma(\chi-1)}{\chi}}, \tag{B.62}$$

$$\hat{\xi}_3 = \frac{(\chi - 1)(1 + \zeta)}{(\chi/\gamma)(\chi/\rho - 1)(\gamma + \zeta)}. \tag{B.63}$$

To conclude, we derive the characterisation of Proposition 2. Starting with (B.44), we substitute $E_t^c[x_t] = (1 - \mu)E_{t-1}^c[x_t] + \mu a_t$ and add and subtract $\hat{\xi}_3 a_t$, yielding:

$$\pi_t = \hat{\xi}_0 + \hat{\xi}_1 E_t^p[a_t] + (\hat{\xi}_2 + \hat{\xi}_3)a_t + \hat{\xi}_3(1 - \mu)\left(E_{t-1}^c[x_t] - a_t\right). \tag{B.64}$$

Next, add and subtract $(\hat{\xi}_2 + \hat{\xi}_3)E_{t-1}^c[x_t]$ and substitute $E_t^p[a_t] = E_{t-1}^c[x_t]$, to get

$$\pi_t = \hat{\xi}_0 + \left(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3\right) E_{t-1}^c[x_t] + (\hat{\xi}_2 + \hat{\xi}_3)a_t - \left(\mu\hat{\xi}_3 + \hat{\xi}_2\right)\left(E_{t-1}^c[x_t] - a_t\right). \tag{B.65}$$

By substituting (B.60)-(B.63) into (B.65), we obtain expression (30) in Proposition 2.

Next, we analyze (B.45). Using $E_t^p[a_t] = E_{t-1}^c[x_t]$ and (B.46), (B.47), we obtain

$$(\gamma + \zeta)y_t = \hat{\theta}_0(\gamma + \zeta) + \zeta a_t + E_t^p[a_t] + \left(\mu\hat{\xi}_3 + \hat{\xi}_2\right)\left(E_{t-1}^c[x_t] - a_t\right). \tag{B.66}$$

Substituting (B.57), (B.62), and (B.63) into (B.66), we obtain expression (31) in Proposition 2.

C Information effects

First, we show that existence of MSV-type equilibrium requires prices and producer expectations of prices to be measurable to current announcements. Subsequently, we show that communication uniquely selects an equilibrium from the continuum of non-MSV-type equilibria.

Let the conjectured dynamics in (B.44) and (B.45) under the MSV-type solution be modified as follows:

$$\pi_t = \hat{\xi}_0 + \hat{\xi}_1 E_t^p[a_t] + \hat{\xi}_2 a_t + \hat{\xi}_3 E_t^c[x_t] + \hat{\xi}_4 s_t, \; t = 0, 1, ... \tag{C.1}$$

$$y_t = \hat{\theta}_0 + \hat{\theta}_1 a_t + \hat{\theta}_2 E_t^p[a_t], \; t = 0, 1, ... \tag{C.2}$$

The one-step-ahead forecasts in (B.51) and (B.52) modify as follows:

$$E_t^c[\pi_{t+1}] = \hat{\xi}_0 + \rho(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3) E_t^c[x_t] + \psi s_t \left(\hat{\xi}_1 + \hat{\xi}_2\right), \tag{C.3}$$

$$E_t^c[y_{t+1}] = \hat{\theta}_0 + \rho \left(\hat{\theta}_1 + \hat{\theta}_2\right) E_t^c[x_t] + \psi s_t \left(\hat{\theta}_1 + \hat{\theta}_2\right), \tag{C.4}$$

with $\psi > 0$. Matching coefficients in the Fisher equation yields

$$\text{constants} : \hat{\xi}_0(1 - \chi) = \kappa_c, \tag{C.5}$$

$$a_t : \chi\hat{\xi}_2 + \gamma\hat{\theta}_1 = 0, \tag{C.6}$$

$$E_t^p[a_t] : \chi\hat{\xi}_1 + \gamma\hat{\theta}_2 = 0, \tag{C.7}$$

$$E_t^c[x_t] : \rho(\hat{\xi}_1 + \hat{\xi}_2 + \hat{\xi}_3) = \chi\hat{\xi}_3 - \gamma\rho\left(\hat{\theta}_1 + \hat{\theta}_2\right), \tag{C.8}$$

$$s_t : \chi\hat{\xi}_4 = \psi\left[\hat{\xi}_1 + \hat{\xi}_2 + \gamma\left(\hat{\theta}_1 + \hat{\theta}_2\right)\right] - 1. \tag{C.9}$$

Combining (C.6), (C.7) and (C.9) with (B.46) and (B.47), we obtain

$$\hat{\xi}_4 = \gamma\frac{1 + \zeta}{\gamma + \zeta}\frac{\chi - 1}{\chi^2} - \frac{1}{\chi}, \tag{C.10}$$

which shows that $\hat{\xi}_4$ is fully determined by the model's primitive parameters.

Next, the conjectured dynamics in equations (B.1) and (B.2) are modified as follows:

$$\pi_t = \xi_0 + \xi_1\pi_{t-1} + \xi_2 a_{t-1} + \xi_3 E_{t-1}^p[a_{t-1}] + \xi_4 a_t + \xi_5 E_t^c[x_t] + \eta_{t-1}, \quad t = 0, 1, ... \tag{C.11}$$

$$y_t = \theta_0 + \theta_1 a_t + \theta_2 E_t^p[a_t], \quad t = 0, 1, ... \tag{C.12}$$

The constant term κ_y in (B.11) reduces to

$$\kappa_y = \frac{1}{2}\left(\sigma_x^2 + \left(\frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon^2}\right)^{-1}\right)\left[(1-\gamma\theta_1)^2 - (\gamma\theta_1 + \xi_4 + \mu\xi_5)^2\right]. \tag{C.13}$$

As before, substituting (C.11)-(C.12) into (B.11) and matching coefficients, yields

$$a_t : (\gamma+\zeta)\theta_1 \;=\; \zeta \;-\; (\xi_4 + \mu\xi_5)\,, \tag{C.14}$$

$$E_t^p[a_t] : (\gamma+\zeta)\theta_2 \;=\; 1 \;+\; \xi_4 \;+\; \mu\xi_5, \tag{C.15}$$

$$\text{constants} : (\gamma+\zeta)\theta_0 \;=\; \kappa_y. \tag{C.16}$$

Next, we turn to the Fisher equation. The constant term κ_c reduces to

$$\kappa_c = \log\beta + \frac{1}{2}(\xi_4 + \mu\xi_5 + \gamma\theta_1)^2\left(\sigma_x^2 + \left(\frac{1}{\sigma_u^2} + \frac{1}{\sigma_\epsilon^2}\right)^{-1}\right). \tag{C.17}$$

The one-step-ahead forecasts are given by

$$E_t^c\left[\pi_{t+1}\right] \;=\; \xi_0 \;+\; \xi_1\pi_t \;+\; \xi_2 a_t \;+\; \xi_3 E_t^p[a_t] \;+\; \rho(\xi_4+\xi_5)E_t^c[x_t] \;+\; \underbrace{\xi_4\psi s_t}_{\text{information effects}} \;+\; \underbrace{s_t}_{\text{policy shock}}, \tag{C.18}$$

$$E_t^c\left[y_{t+1}\right] \;=\; \theta_0 \;+\; \rho(\theta_1+\theta_2)E_t^c[x_t] \;+\; \underbrace{(\theta_1+\theta_2)\psi s_t}_{\text{information effects}}, \tag{C.19}$$

where we have used the following results

$$E_t^c[a_{t+1}] \;=\; E_t^c[x_{t+1}] \;+\; \psi s_t \;=\; \rho E_t^c[x_t] \;+\; \psi s_t, \tag{C.20}$$

$$E_t^c\left[E_{t+1}^c[x_{t+1}]\right] \;=\; x_{t+1|t} \;+\; E_t^c\left[\mu(a_{t+1} - a_{t+1|t})\right] \;=\; E_t^c[x_{t+1}] = \rho E_t^c[x_t], \tag{C.21}$$

$$E_t^c\left[E_{t+1}^p[a_{t+1}]\right] \;=\; x_{t+1|t} \;+\; \psi s_t \;=\; E_t^c[a_{t+1}] \;=\; \rho E_t^c[x_t] \;+\; \psi s_t. \tag{C.22}$$

Imposing $\xi_1 = \chi$ and matching coefficients in the Fisher equation, yields

$$\text{constants} : \xi_0 \;=\; \kappa_c, \tag{C.23}$$

$$a_t : \xi_2 \;=\; \gamma\theta_1, \tag{C.24}$$

$$E_t^p[a_t] : \xi_3 \;=\; \gamma\theta_2, \tag{C.25}$$

$$E_t^c[x_t]: \; -(\xi_4+\xi_5) \;=\; \gamma\left(\theta_1+\theta_2\right), \tag{C.26}$$

$$s_t: \; -\xi_4 \;=\; \gamma\left(\theta_1+\theta_2\right). \tag{C.27}$$

Combining (C.14)-(C.16) and (C.23)-(C.27), we obtain

$$\theta_0 \;=\; \frac{\kappa_y}{\gamma+\zeta} \tag{C.28}$$

$$\theta_1 \;=\; \frac{\zeta}{\gamma+\zeta}+\frac{\gamma(1+\zeta)}{(\gamma+\zeta)^2} \tag{C.29}$$

$$\theta_2 \;=\; \frac{\zeta(1-\gamma)}{(\gamma+\zeta)^2}, \tag{C.30}$$

$$\xi_0 \;=\; \kappa_c, \tag{C.31}$$

$$\xi_1 \;=\; \chi, \tag{C.32}$$

$$\xi_2 \;=\; \gamma\left(\frac{\zeta}{\gamma+\zeta}+\frac{\gamma(1+\zeta)}{(\gamma+\zeta)^2}\right), \tag{C.33}$$

$$\xi_3 \;=\; \gamma\frac{\zeta(1-\gamma)}{(\gamma+\zeta)^2} \tag{C.34}$$

$$\xi_4 \;=\; -\frac{\gamma(1+\zeta)}{\gamma+\zeta}, \tag{C.35}$$

$$\xi_5 \;=\; 0. \tag{C.36}$$

We derive the characterisation in Proposition 3, assuming that sunspot shocks are normalised to zero. Beginning with the output dynamics, using Kalman updating, we obtain $E_t^p[a_t]=E_t^p[x_t]+\psi s_{t-1}$ and taking into account also (C.14), (C.15) and rearranging, we obtain

$$(\gamma+\zeta)y_t \;=\; \kappa_y \;+\; \zeta a_t \;+ E_{t-1}^c[x_t]+\psi s_{t-1} \;-\; \frac{\gamma(1+\zeta)}{\gamma+\zeta}\left(E_{t-1}^c[x_t]+\psi s_{t-1}-a_t\right), \tag{C.37}$$

which coincides with the characterisation in Proposition 3.

Next, substituting (C.31), (C.32), (C.35) and (C.36) into (C.11) we obtain

$$\pi_t=\kappa_c+\chi\pi_{t-1}+\xi_2 a_{t-1}+\xi_3 E_{t-1}^p[a_{t-1}]+\left(-\tfrac{\gamma(1+\zeta)}{\gamma+\zeta}\right)a_t+s_{t-1}. \tag{C.38}$$

We use the Kalman filter to express $E_{t-1}^p[a_{t-1}]$ in terms of $E_t^p[a_t]$ and a_{t-1}:

$$E_{t-1}^p[a_{t-1}]=\frac{1}{\rho(1-\mu)}E_t^p[a_t]-\frac{\mu}{1-\mu}a_{t-1}+\frac{\psi\left(\rho s_{t-2}-s_{t-1}\right)}{\rho(1-\mu)}. \tag{C.39}$$

This expression is derived by combining the relationships $E^p_{t-1}[a_{t-1}] = E^c_{t-2}[x_{t-1}] + \psi s_{t-2}$ and $E^p_t[a_t] = E^c_{t-1}[x_t] + \psi s_{t-1} = \rho E^c_{t-1}[x_{t-1}] + \psi s_{t-1} = \rho\left((1-\mu)E^c_{t-2}[x_{t-1}] + \mu\left[a_{t-1} - \psi s_{t-2}\right]\right) + \psi s_{t-1}$.

Substituting (C.39) into (C.38) and rearranging, we obtain

$$\pi_t \;=\; \kappa_c + \chi\pi_{t-1} \;+\; \Xi_3\psi\left(\rho s_{t-2} - s_{t-1}\right) \;+\; \Xi_2 a_{t-1} \;+\; \Xi_3 E^p_t[a_t] \;+\; \left(-\frac{\gamma(1+\zeta)}{\gamma+\zeta}\right) a_t + s_{t-1}, \quad \text{(C.40)}$$

where $\Xi_3 \equiv \xi_3/\rho(1-\mu)$ and $\Xi_2 \equiv \xi_2 - \xi_3\mu/(1-\mu)$ and ξ_2, ξ_3 are given by (C.33) and (C.34). Using the Kalman filter once more to express $E^p_t[a_t] = E^p_t[x_t] + \psi s_{t-1}$, adding and subtracting $[\gamma(1+\zeta)/(\gamma+\zeta)]\left(E^c_{t-1}[x_t] + s_{t-1}\right)$, and rearranging, we obtain the characterisation in Proposition 3.

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